Inclusive processes from lattice QCD

Shoji Hashimoto (KEK)

April 13, 2018

@ MITP workshop "Challenges in Semileptonic B Decays," Mainz Institute for Theoretical Physics, Johannes Gutenberg University

計算基礎科学連携拠点
Joint Institute for Computational Fundamental Science
JICFuS

国立大学法人
総合研究大学院大学
The Graduate University for Advanced Studies [SOKENDAI]

Inclusive b→c

- Theory involves some assumptions/approximations.
  - Perturbation theory: an expansion in α_s
    - Is α_s small enough?
  - Heavy quark expansion: in $1/m_b$
    - Is m_b large enough?
  - Quark-hadron duality (local or global)
    - Is the process sufficiently inclusive? Quantitative measure?
- Can we test the method using LQCD?
  - Consistency check within theory.
  - Freedom to vary parameters: m_b, m_c, etc.

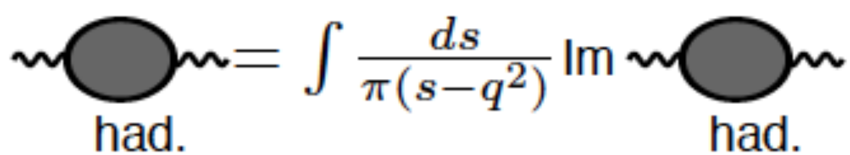
Another idea: talk by H. Meyer.

"inclusive" process

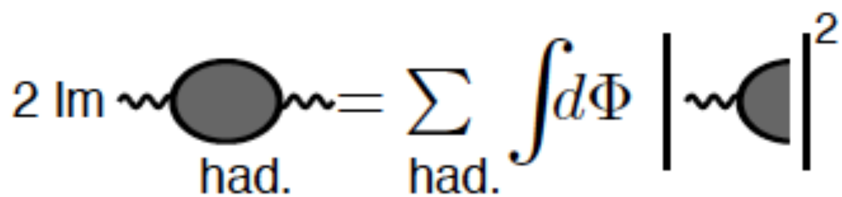
= sum over final states

- Best-known example: R ratio $\dfrac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

Dispersion relation:

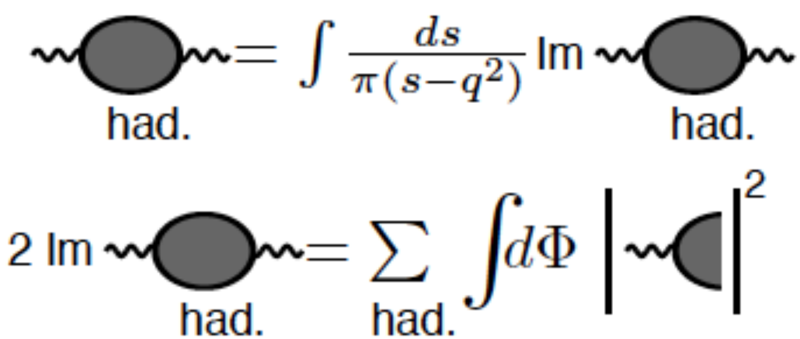


Optical theorem:

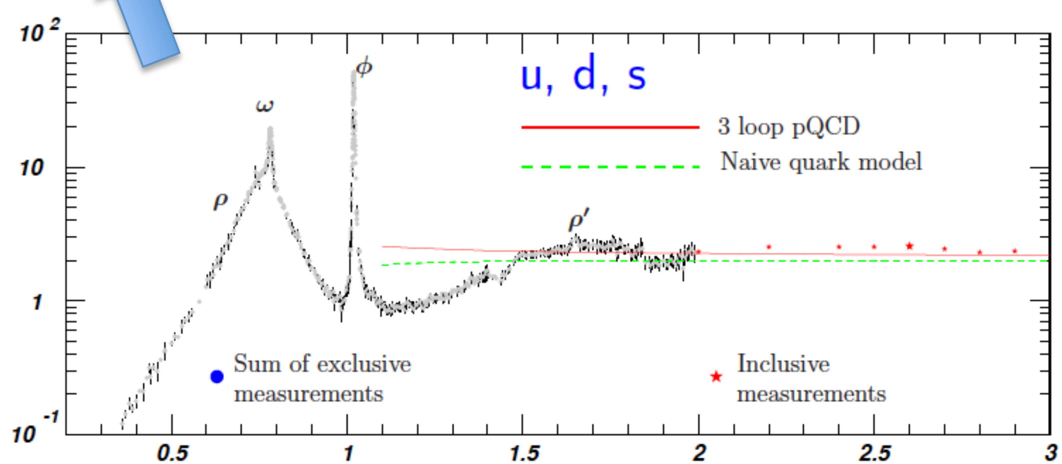


“inclusive” process

and experiment



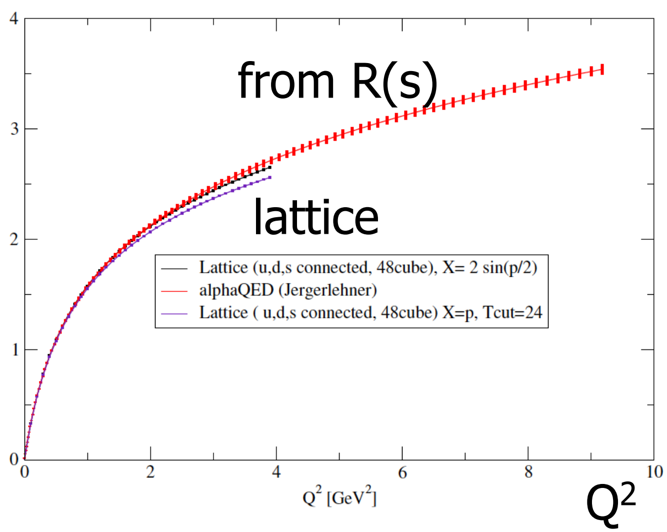
time-like region: $s=q^2 > 4m_\pi^2$



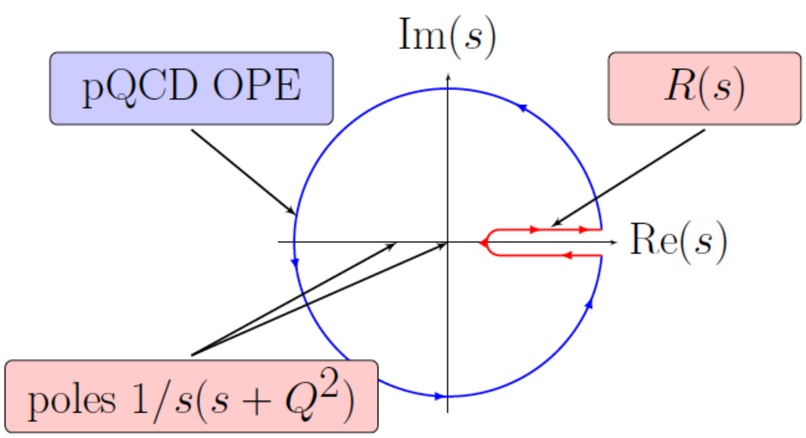
Analytic continuation

- Basis of the SVZ sum rule.
- Also calculable on the lattice.

RBC/UKQCD:
Izubuchi @ g-2 WS (2017)

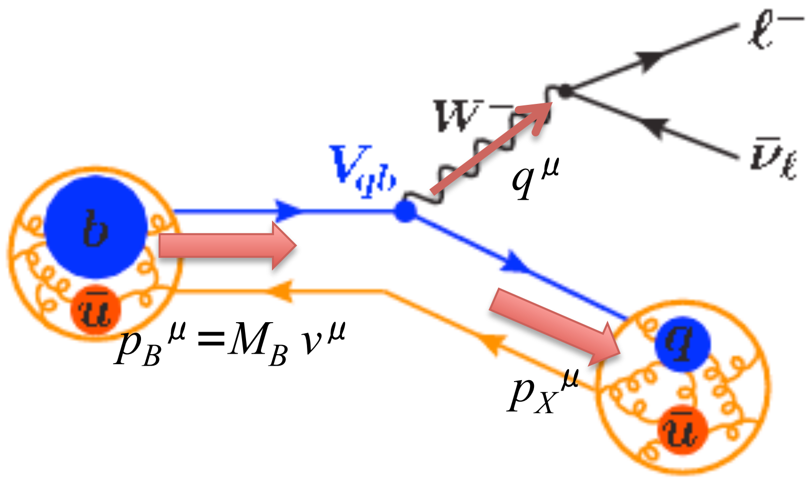


$$\hat{\Pi}(Q^2) \;=\; Q^2 \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}$$



Inclusive semi-leptonic B decays

– Additional complication due to …



2 Kinematical variables:

- q^2 : lepton pair inv mass
- v.q : energy taken by leptons

Inclusive decays =

- $p_X{}^2 = m_X{}^2$ arbitrary

Need to specify the four-momentum:
$p_X{}^\mu = (\omega, \mathbf{p}_X)$.
Use the analytic continuation for
$\omega^2 < m_D{}^2 + \mathbf{p}_X{}^2$

Inclusive semi-leptonic B decays

- Standard analysis (formulation borrowed from DIS)
  - Decay rate

$$|\mathcal{M}|^2 = |V_{qQ}|^2 G_F^2 M_B l^{\mu\nu} W_{\mu\nu}$$

function of q^2 and v.q

  - Structure functions

$$W_{\mu\nu} = \sum_X (2\pi)^3 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

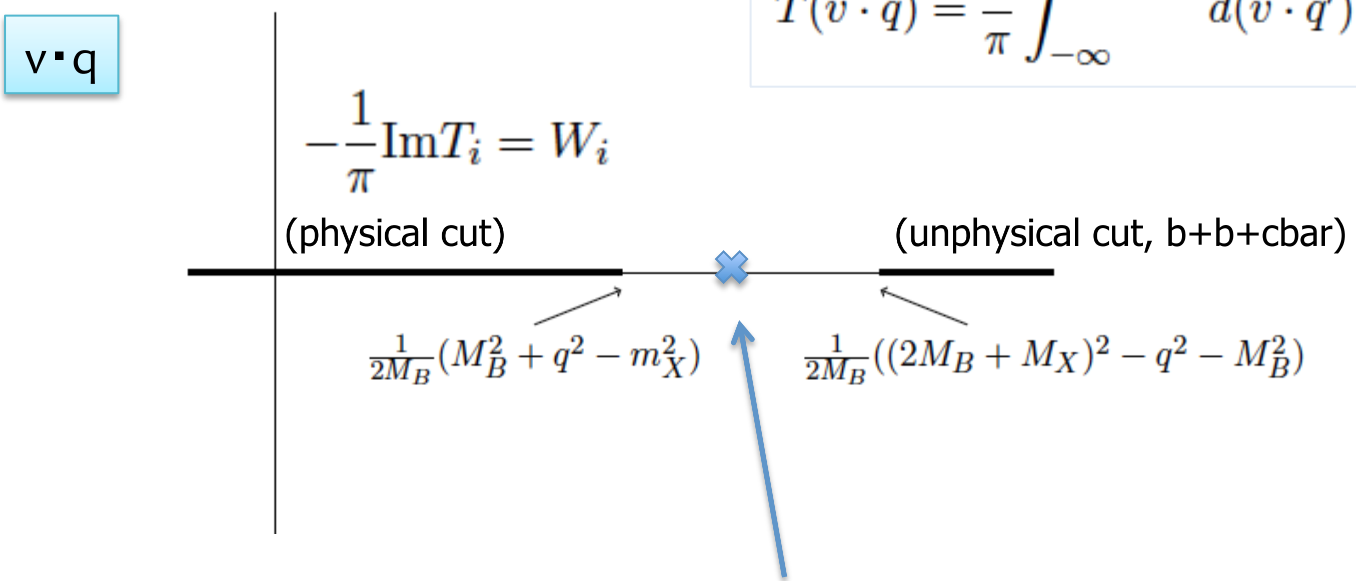
$$W_{\mu\nu} = -W_1 g_{\mu\nu} + W_2 v_\mu v_\nu + iW_3 \epsilon_{\mu\nu\alpha\beta} v^\alpha q^\beta + W_4 q_\mu q_\nu + W_5(q_\nu v_\mu + q_\mu v_\nu)$$

  - Forward-scattering matrix element

$$T_{\mu\nu} = i\int d^4x e^{-iqx} \frac{1}{2M_B} \langle B|T\{J_\mu^\dagger(x) J_\nu(0)\}|B\rangle$$

$$T_{\mu\nu} = i\int d^4x e^{-iqx} \frac{1}{2M_B} \langle B | T\{J_\mu^\dagger(x) J_\nu(0)\} | B \rangle$$

$$T(v\cdot q) = \frac{1}{\pi}\int_{-\infty}^{(v\cdot q)_{\max}} d(v\cdot q') \frac{\mathrm{Im}T(v\cdot q')}{v\cdot q' - v\cdot q}$$

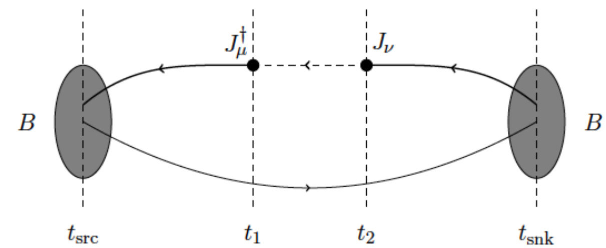


calculable on the lattice
in the unphysical kinematical region
(like the space-like HVP)

SH, arXiv:1703.01881

Lattice calculation: recipe

1. Four-point function:
   – calculate on the lattice



2. after taking appropriate ratios to cancel the external B meson source, we construct

$$C_{\mu\nu}^{JJ}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0})|J_\mu^\dagger(\mathbf{x},t) J_\nu(0)|B(\mathbf{0})\rangle$$

3. do the "Fourier transform" in the time direction

$$T_{\mu\nu}^{JJ}(\omega,\mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C_{\mu\nu}^{JJ}(t;\mathbf{q})$$

– Corresponds to $T_{\mu\nu}(v\cdot q, q^2)$ at $p_X=(\omega,-\boldsymbol{q})$, $q=(m_B-\omega,\boldsymbol{q})$

Ensembles from JLQCD

- With Mobius domain-wall fermion (2012~)
  - 2+1 flavor (uds)
  - Mobius domain-wall fermion [with stout link]
    - residual mass < O(1 MeV)
  - lattice spacing : $1/a$ = 2.4, 3.6, 4.5 GeV
  - volume : L = 2.7 fm（32^3, 48^3, 64^3 lattices）
  - light quark mass : m_π = 230, 300, 400, 500 MeV
  - statistics : 50-200 measurements
- Valence sector
  - heavy (MDW) + strange (MDW)
  - tuned charm + (unphysical) bottom $m_b = (1.25)^2\, m_c, (1.25)^4\, m_c$

    (up to 3.4 GeV B_s meson)
  - on Oakforest-PACS with IROIRO++

β = 4.17, 1/a ~ 2.4 GeV, 32^3x64 (x12)

m_{ud}	m_π [MeV]	MD time
m_s = 0.030		
0.007	310	10,000
0.012	410	10,000
0.019	510	10,000
m_s = 0.040		
0.0035	230	10,000
0.0035 (48^3x96)	230	10,000
0.007	320	10,000
0.012	410	10,000
0.019	510	10,000

β = 4.35, 1/a ~ 3.6 GeV, 48^3x96 (x8)

m_{ud}	m_π [MeV]	MD time
m_s = 0.018		
0.0042	300	10,000
0.0080	410	10,000
0.0120	500	10,000
m_s = 0.025		
0.0042	300	10,000
0.080	410	10,000
0.0120	510	10,000

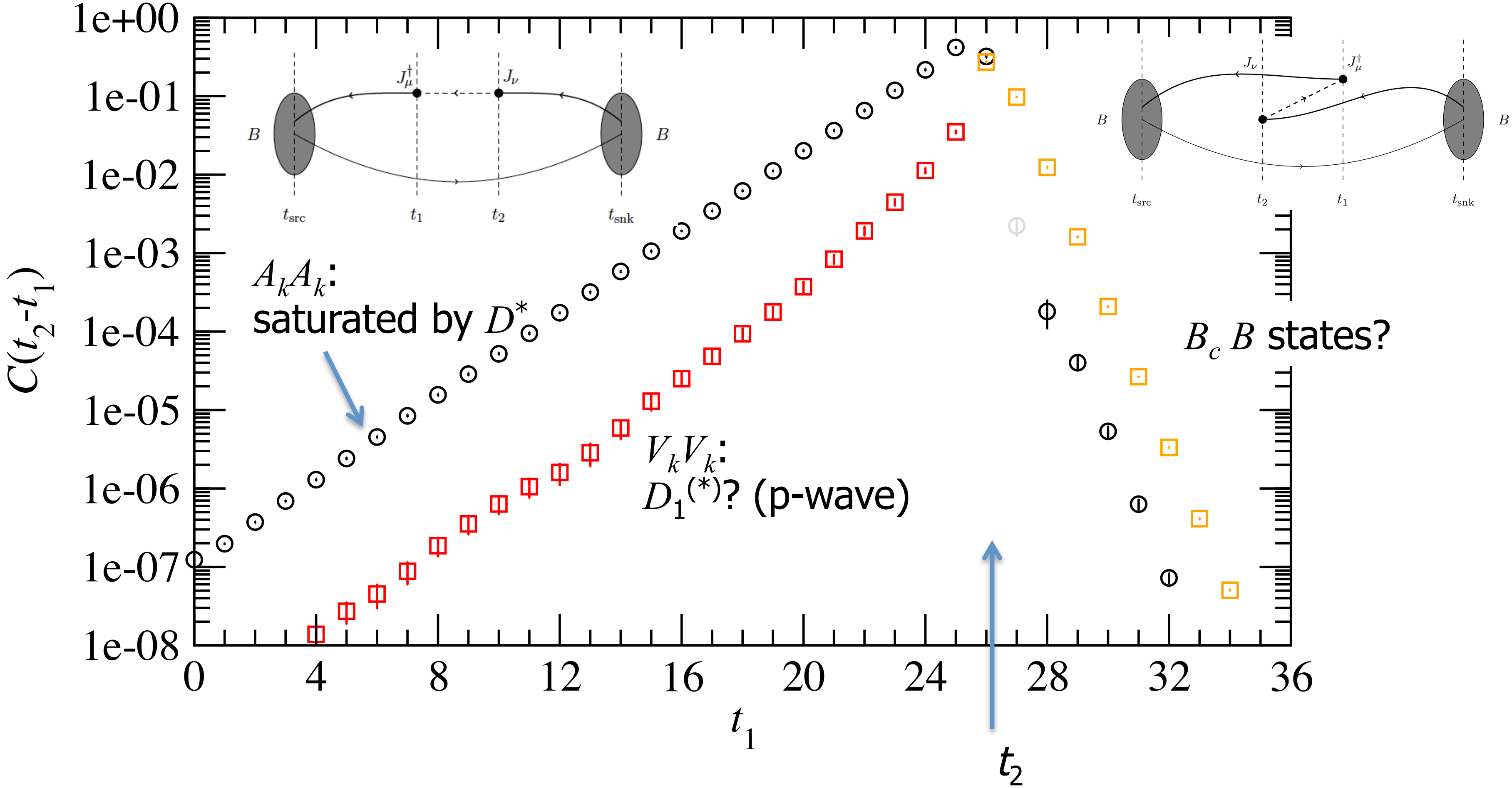
β = 4.47, 1/a ~ 4.6 GeV, 64^3x128 (x8)

0.0030	~ 300	10,000

a glance at the lattice data:

$$C^{JJ}_{\mu\nu}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0})|J^\dagger_\mu(\mathbf{x},t)J_\nu(0)|B(\mathbf{0})\rangle \qquad J_\mu = \bar{b}\gamma_\mu c \quad \text{or} \quad \bar{b}\gamma_\mu\gamma_5 c$$

m_b=$1.25^4 m_c$, zero recoil (**q**=**0**) $1/a$ = 3.6 GeV



"Fourier transform"

- Time direction should be analytically continued to go time-like, i.e. energy ω:

$$e^{ip_0 t} \longrightarrow e^{\omega t} \quad : \quad T_{\mu\nu}^{JJ}(\omega, \mathrm{q}) = \int_0^\infty dt\, e^{\omega t} C_{\mu\nu}^{JJ}(t; \mathrm{q})$$

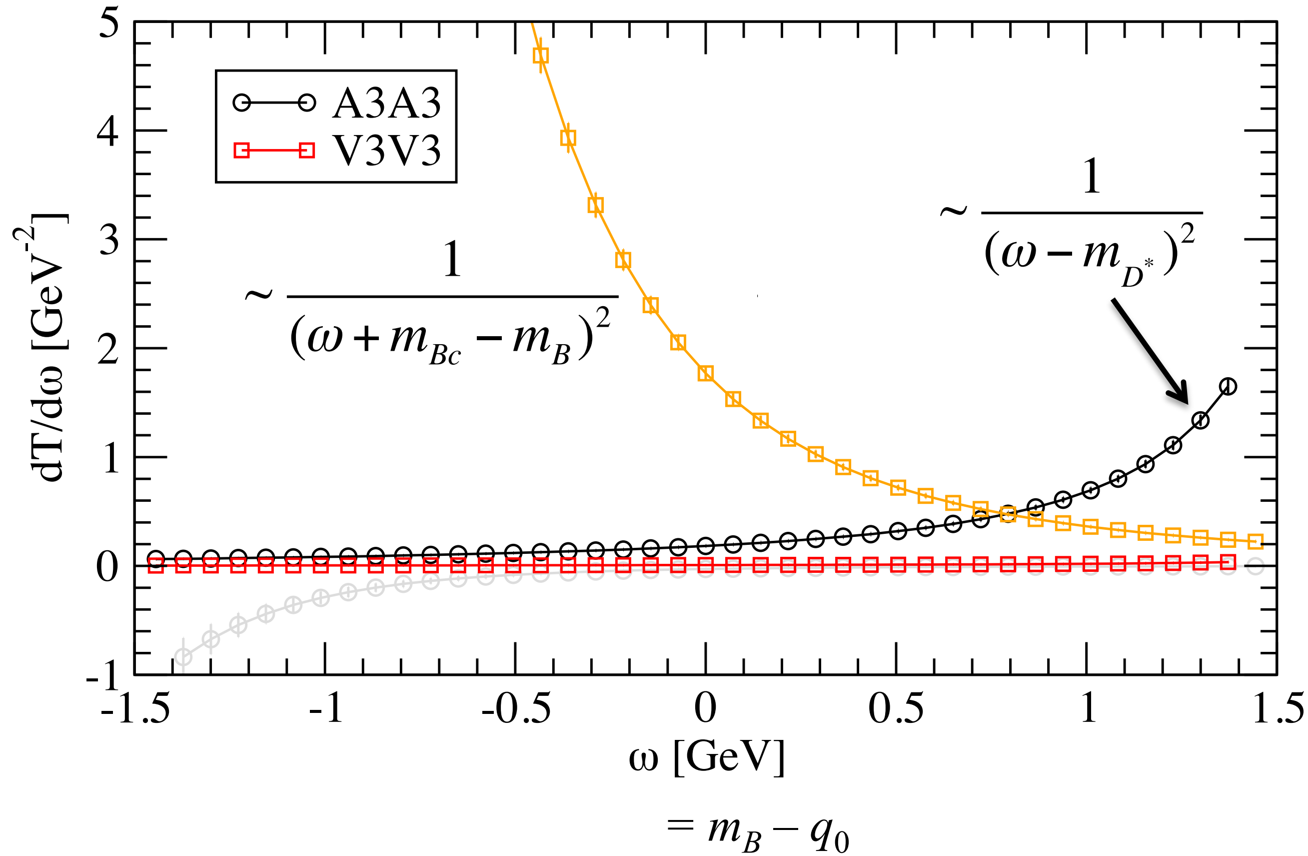
  - Can be understood by a Taylor expansion in p_0, and then reconstruct with $ip_0=\omega$.
  - Only below any singularity: pole, cut, …
  - Obviously,

$$e^{-mt} \xrightarrow{F.T.} \frac{1}{\omega - m}$$

$$T^{JJ}_{\mu\nu}(\omega,\mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t;\mathbf{q})$$

$m_b = 1.25^4 m_c$, zero recoil (**q**=**0**) $1/a$ = 3.6 GeV

derivative to avoid divergence (contact term)



Saturation by D$^{(*)}$?

Four-point function:

$$C_{\mu\nu}^{JJ}(t;0) = \frac{1}{2M_B}\sum_X \langle B(0)|J_\mu^\dagger|X(0)\rangle \frac{e^{-E_X t}}{2E_X}\langle X(0)|J_\nu|B(0)\rangle$$

Ground-state contribution:

$$\begin{aligned}\langle D(0)|V^0|B(0)\rangle &= 2\sqrt{M_B M_D}\, h_+(1),\\ \langle D^*(0)|A^k|B(0)\rangle &= 2\sqrt{M_B M_{D^*}}\, h_{A_1}(1)\,\varepsilon^{*k}.\end{aligned}$$

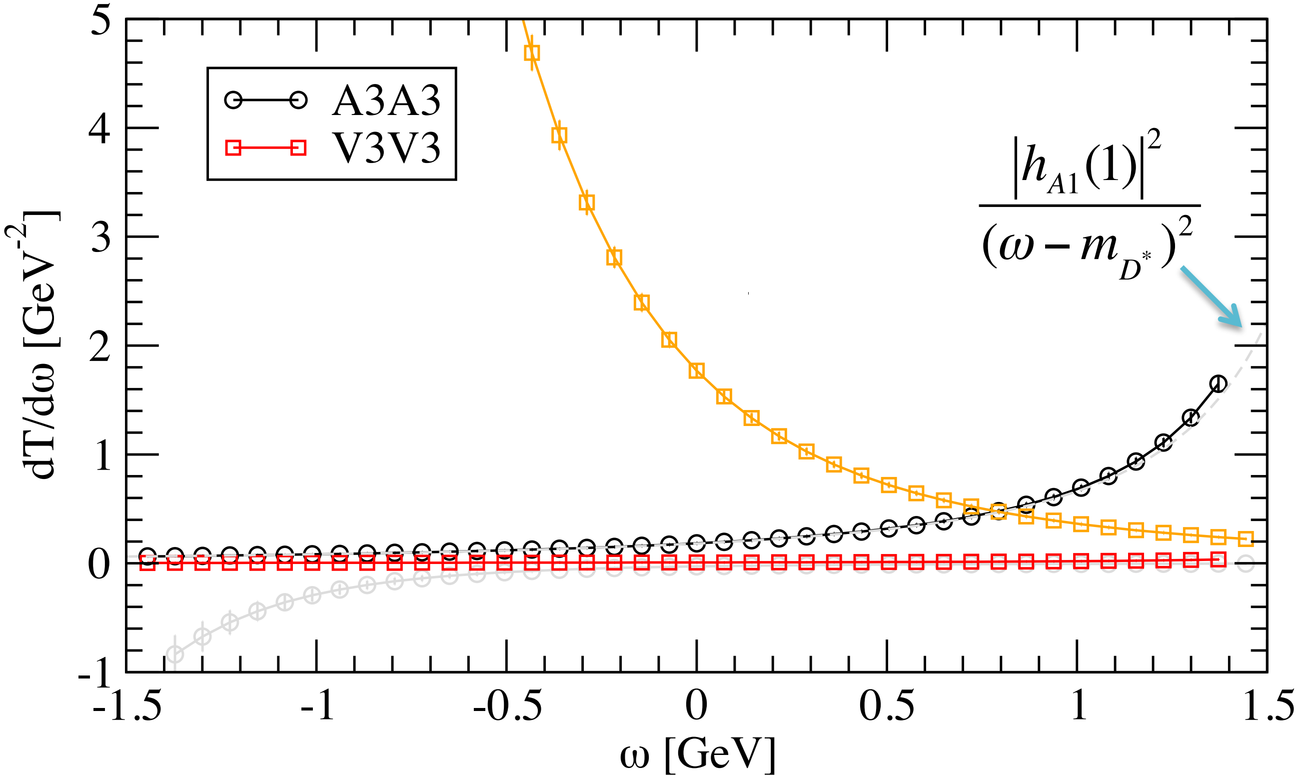
$$\begin{aligned}T_{00}^{VV}(\omega,0) &= \frac{|h_+(1)|^2}{M_D-\omega},\\ T_{kk}^{AA}(\omega,0) &= \frac{|h_{A_1}(1)|^2}{M_{D^*}-\omega}.\end{aligned}$$

zero-recoil form factors h(1)
~ Isgur-Wise function $\xi(1)=1$

$$T^{JJ}_{\mu\nu}(\omega,\mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t;\mathbf{q})$$

m_b=1.25^4m_c , zero recoil (**q**=**0**)   1/a = 3.6 GeV

derivative to avoid divergence (contact term)

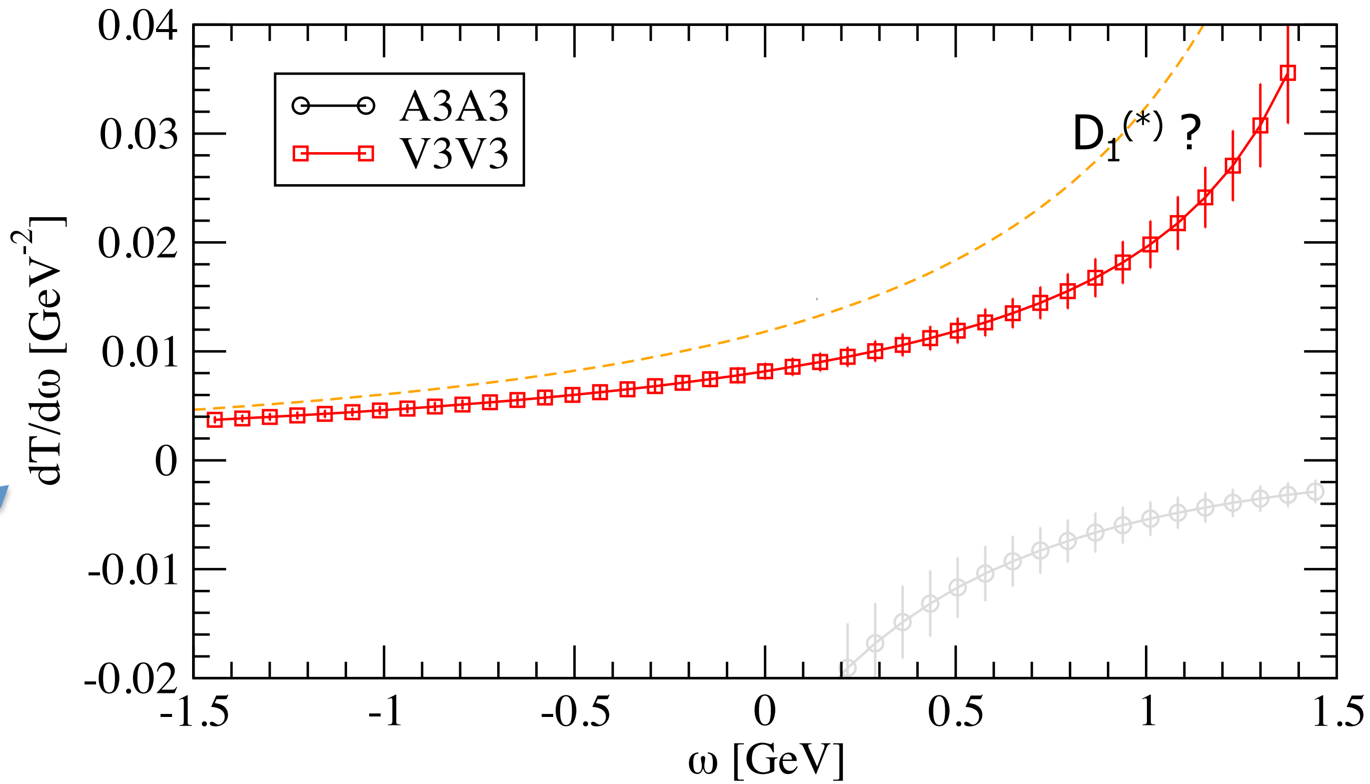


$= m_B - q_0$

$$T^{JJ}_{\mu\nu}(\omega, \mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t; \mathbf{q})$$

m_b=1.25$^4 m_c$, zero recoil (**q**=**0**) 1/a = 3.6 GeV

derivative to avoid divergence (contact term)



$= m_B - q_0$

Wrong parity channel?

$B \rightarrow D_1^{(*)}$ (at zero recoil): p-wave, 1^+ states
D_1^*: s_l=1/2, 2427 MeV (broad)
D_1 : s_l=3/2, 2421 MeV (narrow)

$$\langle D_1^* | \bar{c}\gamma_\mu b | B \rangle = \sqrt{m_{D_1^*} m_B}\, g_{V_1}(1)\, \varepsilon_\mu^*$$

$$\langle D_1 | \bar{c}\gamma_\mu b | B \rangle = \sqrt{m_{D_1} m_B}\, f_{V_1}(1)\, \varepsilon_\mu^*$$

$$\Rightarrow \quad \frac{dT_{kk}^{VV}}{d\omega} \sim \frac{\left| g_{V_1} \right|^2}{\left(m_{D_1^*} - \omega \right)^2} + \frac{\left| f_{V_1} \right|^2}{\left(m_{D_1} - \omega \right)^2}$$

Bernlochner, Ligeti, Robinson, arXiv:1711.03110

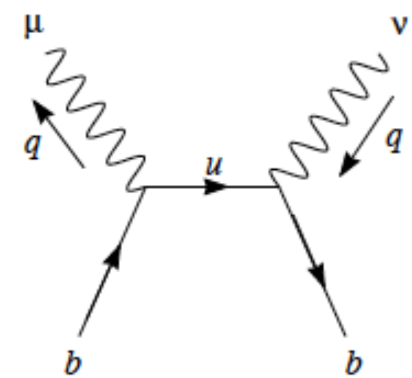
$$g_{V_1}(1) = \left(\varepsilon_c - 3\varepsilon_b \right)\left(\bar{\Lambda}^* - \bar{\Lambda} \right)\zeta(1)$$

$$f_{V_1}(1) = -\frac{8}{\sqrt{6}}\varepsilon_c \left(\bar{\Lambda}' - \bar{\Lambda} \right)\tau(1)$$

Comparison with Continuum

(Based on discussions with P. Gambino)

- Heavy Quark Expansion (tree-level formulae):
  Blok, Koyrakh, Shifman, Vainshtein, PRD49, 3356 (1994).
  Manohar, Wise, PRD49, 1310 (1993).
  Falk, Ligeti, Neubert, Nir, PLB326, 145 (1994)
  Balk, Korner, Pirjol, Schilcher, ZP C64, 37 (1994).

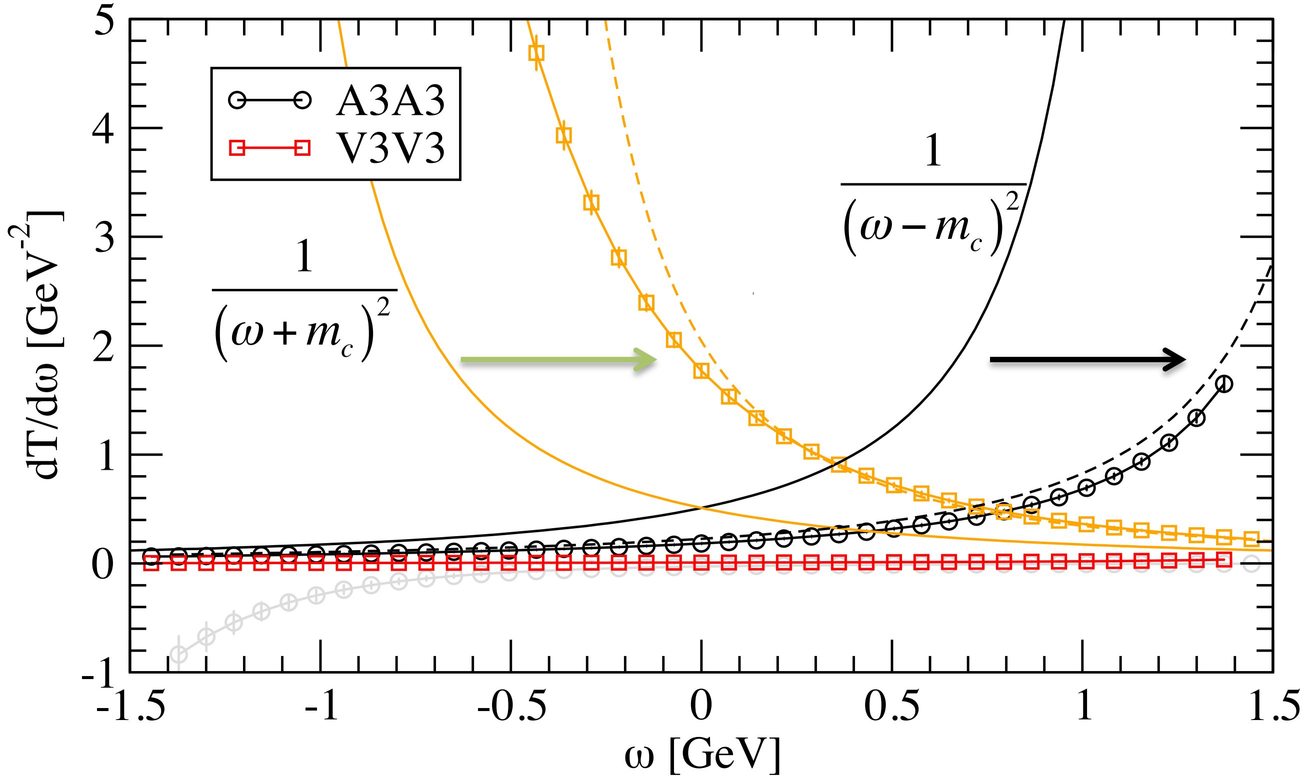


- Expand $\dfrac{1}{m_b \not{v} - \not{q} + \not{k} - m_c}$ in small *k*.

- Zero-recoil limit ($V_k V_K$ or $A_k A_k$ channel, leading order)

$$T_1^{VV} = -\frac{\omega - m_c}{\omega^2 - m_c^2}, \quad T_1^{AA} = -\frac{\omega + m_c}{\omega^2 - m_c^2} \qquad (\omega = m_B - q_0)$$

… pole at $\omega = -m_c$ ($V_k V_k$) or $\omega = m_c$ ($A_k A_k$)



Significant shift due to $m_c \rightarrow m_D$

or to $m_c \rightarrow m_{Bc}$-m_B

Non-perturbative effect

Significant shift due to $m_c \to m_D$
or to $m_c \to m_{Bc}\text{-}m_B$

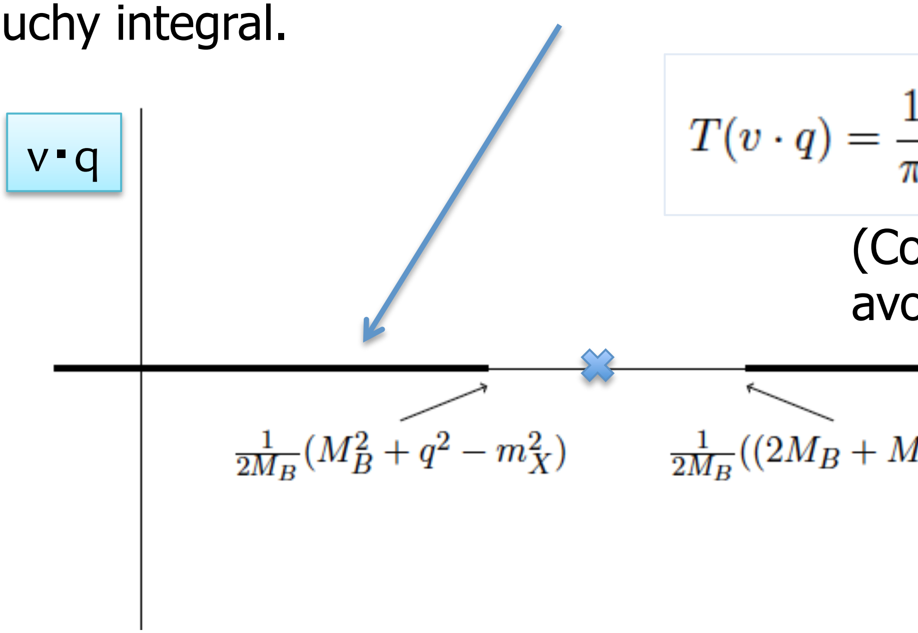
Understandable by the $1/m_b$ expansion?

- Probably not, because the effect should survive in the heavy quark limit.
- Then, what is missing in the heavy quark expansion?
- How much does perturbation theory show the sign of the non-perturbative effects?
- Or, m_b (in my calculation) is too close to m_c to induce really "inclusive" decays?

c.f. Boyd, Grinstein, Manohar (1996): consistency between exclusive and 1/m expansion was found in the SV limit m_b, $m_c >> m_b\text{-}m_c >> \Lambda_{QCD}$.

Going to one-loop

- One-loop corrections:
  Trott, PRD70, 073003 (2004).
  Aquila, Gambino, Ridolfi, Uraltsev, NPB719, 77 (2005).
  - Available only for the Imaginary part (cut); Needed to perform the Cauchy integral.



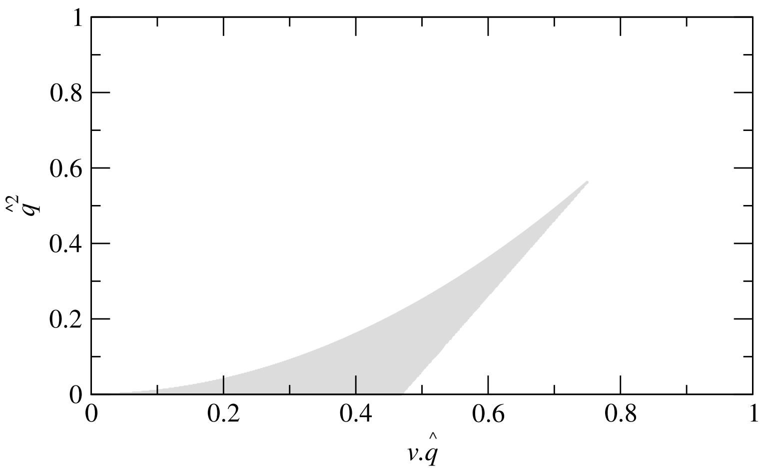
$$T(v\cdot q)=\frac{1}{\pi}\int_{-\infty}^{(v\cdot q)_{\max}} d(v\cdot q')\frac{\mathrm{Im}T(v\cdot q')}{v\cdot q'-v\cdot q}$$

(Consider its first derivative to avoid divergence.)

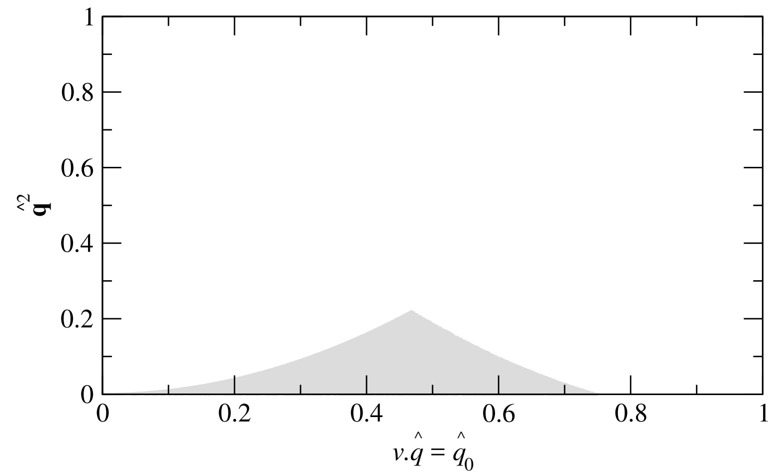
To be performed.

Then, to experiment

fixed q^2, or



fixed recoil mom



- Need 2D distribution of the experimental data.
- Unavailable region must be supplemented by perturbation theory.

Summary and Outlook (1)

- Inclusive structure functions calculable on the lattice, but at unphysical kinematics; Use Cauchy integral to match Exp
- Numerical test of the b→c transition on 2+1 flavor configs with DW heavy. The b quark mass is lighter than physical: m_b~2.5 m_c. Physical other than that.

- The region of ground state saturation (D or D*) is consistent with expectation (from form factors). The wrong parity channel too.
- May be compared to HQE. Large contribution from the unphysical cut. Non-perturbative effect needs to be understood.

Summary and Outlook (2)

- Range of applications:
  - B physics
    - b→c semileptonic and $|V_{cb}|$
    - b→c zero-recoil sum-rule
    - b→u semileptonic and $|V_{ub}|$
  - Nucleon structure

    see also, QCDSF, arXiv:1703.01153
    - Direct calculation of structure functions, but at unphysical kinematics: x>1
    - Not-so-deep inelastic scattering
  - Crucial to analyze experimental data accordingly!