$\bar{B} \to X_u \, l \, \bar{\nu}$ theory

Gil Paz

Department of Physics and Astronomy,

Wayne State University,

Detroit, Michigan, USA

Charge

"We would like to invite you to give a review talk on the B$-$ $>$Xu l v theory, discussing (very quickly) the available methods, [and] the (very little) recent progress and the open problems."

Outline

- Reminders
- Available methods
- Recent progress
- Open problems and future progress

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- **If** we could measure total $\Gamma(\bar{B} \to X_u\, l\, \bar{\nu})$ we could use a **local** OPE

$$d\Gamma \sim \sum_n c_n \frac{\langle O_n \rangle}{m_b^n}$$

c_n perturbative, $\langle O_n \rangle$ non-perturbative **numbers**

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$M_X^2 \sim m_b^2$	local OPE	("OPE region")
$M_X^2 \sim m_b \Lambda_{\rm QCD}$	Non local OPE	("end point region")
$M_X^2 \sim \Lambda_{\rm QCD}^2$	No inclusive description	("resonance region")

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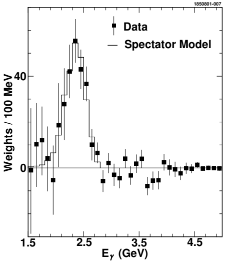
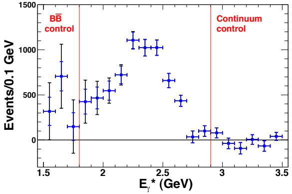
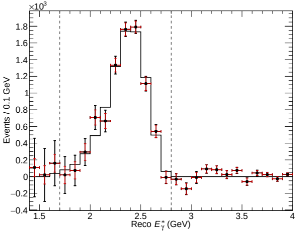


BaBar (2012)



Belle (2016)



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- At subleading power in $\Lambda_{\rm QCD}/m_b$:

- Several subleading shape functions (SSF) appear
- Different linear combinations for $\bar{B} \to X_u \, l \, \bar{\nu}$ and $\bar{B} \to X_s \, \gamma$
- $\bar{B} \to X_s \, \gamma$ has unique SSF ("resolved photon contributions")

- Shape functions moments are related to HQET parameters:
  E.g. leading shape function: 1^{st} moment $\leftrightarrow m_b$, 2^{nd} moment $\leftrightarrow \mu_\pi^2$

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- Methods with flexible cuts must include shape functions effects:

- BLNP
  [Lange, Neubert, GP, PRD **72**, 073006, (2005)]
- GGOU
  [Gambino, Giordano, Ossola, Uraltsev, JHEP **10**, 058, (2007)]
- DGE
  [Andersen, Gardi, JHEP **01**, 097, (2006)]
- ADFR
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- In the following only discuss BLNP, GGOU, DGE

BLNP

- Based on

$$d\Gamma \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \ldots$$

- Factorize perturbative coefficient into hard H and jet J function
- Leading power $H \cdot J \otimes S$ at $\mathcal{O}(\alpha_s)$
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- Many BLNP NNLO calculations are known:
  H, J at $\mathcal{O}(\alpha_s^2)$, j_i/m_b at $\mathcal{O}(\alpha_s)$, resolved photon contributions
  Not fully combined yet

GGOU

- Based on

$$W_i \sim F_i \otimes W_i^{pert}$$

- W_i structure functions that give $d\Gamma$
- W_i^{pert} known perturbative quantities
- $F_i(k_+, q^2, \mu)$ OPE-constrained non-perturbative distribution functions
- uses kinetic scheme, Wilsonian cutoff $\mu \sim 1$ GeV

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- Improvements to GGOU discussed later

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- $m_b(\overline{MS})$ used as an input
- See also Gardi's talk at MITP 2015 workshop

Available methods: $|V_{ub}|$

- Latest HFLAV summary [arXiv:1612.07233]

- Table 91: Summary of inclusive determinations of $|V_{ub}|$
  The errors quoted on $|V_{ub}|$ correspond to experimental and theoretical uncertainties

Framework	$\lvert V_{ub}\rvert[10^{-3}]$
BLNP	$4.44 \pm 0.15^{+0.21}_{-0.22}$
DGE	$4.52 \pm 0.16^{+0.15}_{-0.16}$
GGOU	$4.52 \pm 0.15^{+0.11}_{-0.14}$
ADFR	$4.08 \pm 0.13^{+0.18}_{-0.12}$
BLL (m_X/q^2 only)	$4.62 \pm 0.20 \pm 0.29$

- This doesn't include latest BaBar analysis
  [BaBar PRD **95** 072001 (2017)]

Recent progress

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I apologize if I missed other progress

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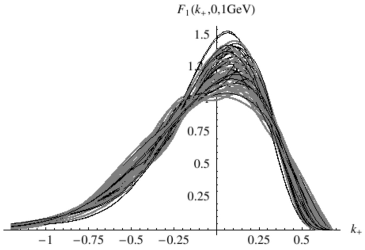
- Motivation: About 100 forms considered in GGOU
  but each parameterized by simple 2-parameter functional forms
  is that good enough?



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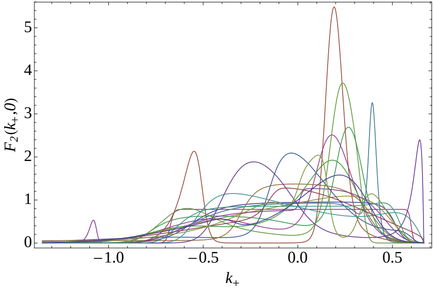
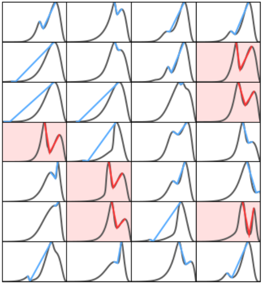
- Proof of principle
  [Gambino, Healey, Mondino PRD **94** 014031 (2016)]
- Selection of NN replicas of $F_2(k_+, 0)$ trained on first 3 moments only

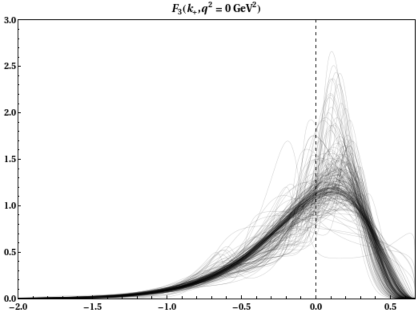


Demonstrates NN capability to properly sample the functional space

The NNVub Project

- After further pruning, e.g. keep only one dominant peak





- The results are used to extract $|V_{ub}|$ in the GGOU framework

- Good agreement is found with original GGOU and 2014 HFLAV

Recent progress:

Power corrections

[Gambino, Healey, Turczyk PLB **763** 60 (2016)]

Power corrections

- Inclusive $\bar{B} \to X_c \ell \bar{\nu}_\ell$ decays allow to extract HQET parameters
- Moments of shape function(s) are related to these parameters
- Dimension 7 and 8 HQET operators contribution to $\bar{B} \to X_c \ell \bar{\nu}_\ell$
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]
- $|V_{cb}|$ extraction from inclusive B decays
  uses dimension 7 and 8 HQET operators
  [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

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Table 2
Default fit results: the second and third columns give the central values and standard deviations.

m_b^{kin}	4.546	0.021	r_1	0.032	0.024
$\overline{m}_c(3\text{ GeV})$	0.987	0.013	r_2	−0.063	0.037
μ_π^2	0.432	0.068	r_3	−0.017	0.025
μ_G^2	0.355	0.060	r_4	−0.002	0.025
ρ_D^3	0.145	0.061	r_5	0.001	0.025
ρ_{LS}^3	−0.169	0.097	r_6	0.016	0.025
$\overline{m}_1$	0.084	0.059	r_7	0.002	0.025
$\overline{m}_2$	−0.019	0.036	r_8	−0.026	0.025
$\overline{m}_3$	−0.011	0.045	r_9	0.072	0.044
$\overline{m}_4$	0.048	0.043	r_{10}	0.043	0.030
$\overline{m}_5$	0.072	0.045	r_{11}	0.003	0.025
$\overline{m}_6$	0.015	0.041	r_{12}	0.018	0.025
$\overline{m}_7$	−0.059	0.043	r_{13}	−0.052	0.031
$\overline{m}_8$	−0.178	0.073	r_{14}	0.003	0.025
$\overline{m}_9$	−0.035	0.044	r_{15}	0.001	0.025
χ^2/dof	0.46		r_{16}	0.001	0.025
$BR(\%)$	10.652	0.156	r_{17}	−0.028	0.025
$\mathbf{10^3\lvert V_{cb}\rvert}$	**42.11**	**0.74**	r_{18}	−0.001	0.025

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Motivation

- Original motivation: How to express moments of shape function(s) in terms of $m_1, ...m_9$ and $r_1, ...r_{18}$?
  (See also appendix A of [Heinonen, Mannel, arXiv:1609.01334])

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- Method of [Gunawardana, GP, JHEP **1707** 137 (2017)] allows to

1) Find such relations
2) List HQET parameters, in principle, to *arbitrary* dimension
3) Construct NRQED and NRQCD bilinear operators, in principle, to *arbitrary* dimension

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- Structure of effective field theories is simpler than we think:

- SM EFT
  [Henning, Lu, Melia, Murayama, JHEP **1708**, 016 (2017)]
- NRQED/NRQCD/HQET
  [Gunawardna, GP JHEP **1707** 137 (2017)]
  [Kobach, Pal PLB **772** 225 (2017)]

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\- Paz et al. arXiv:1702.0890 v1 $\rightarrow$ v2
  "*discussion of operators with multiple color structures was added*"

General method: Summary

- [Gunawardana, GP, JHEP **1707** 137 (2017)] method

  Consider matrix elements of the form

  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n}\, h|H\rangle$

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- Express them in terms of $v^{\mu_i}, \Pi^{\mu_i \mu_j} = g^{\mu\nu} - v^{\mu} v^{\nu}$, and $\epsilon^{\rho\sigma\alpha\beta} v_{\rho}$ using

- Orthogonality: $v_{\mu_1} = v_{\mu_n} = v_{\lambda} = 0$
- P, T, and Hermitian conjugation:

  SI (SD) matrix elements are sym. (anti-sym.) under inversion
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- [Kobach, Pal PLB **772** 225 (2017)] pointed out extra operators with multiple color structures

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  - Orthogonality: $v_{\mu_1} = v_{\mu_n} = v_{\lambda} = 0$
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    SI (SD) matrix elements are sym. (anti-sym.) under inversion
  - Four dimensions:
    not all tensors are linearly independent
- [Kobach, Pal PLB **772** 225 (2017)] pointed out extra operators with multiple color structures
- Published and $v \geq 2$ of [Gunawardana, GP, JHEP **1707** 137 (2017)]
  - Checking possible multiple color structures

New Result: Dimension 9 HQET operators

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$$\frac{1}{2M_H}\langle H|\bar h\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, iD^{\mu_6}\, h|H\rangle = a^{(9)}_{12,34}\,\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}\Pi^{\mu_5\mu_6} +$$
$$+a^{(9)}_{12,35}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}\Pi^{\mu_4\mu_6}+\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}\Pi^{\mu_5\mu_6}) + a^{(9)}_{12,36}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}\Pi^{\mu_4\mu_5}+\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}\Pi^{\mu_5\mu_6}) +$$
$$+a^{(9)}_{13,25}\,\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}\Pi^{\mu_4\mu_6} + a^{(9)}_{13,26}\,(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}\Pi^{\mu_4\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_6}) + a^{(9)}_{14,25}\,\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_6} +$$
$$+a^{(9)}_{14,26}\,(\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_6}) + a^{(9)}_{15,26}\,\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_4} + a^{(9)}_{16,23}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_5} +$$
$$+a^{(9)}_{16,24}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_5} + a^{(9)}_{16,25}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_4} + b^{(9)}_{12,36}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_4}\Pi^{\mu_5\mu_6}v^{\mu_2}v^{\mu_3}) +$$
$$+b^{(9)}_{12,46}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_6}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_3}\Pi^{\mu_5\mu_6}v^{\mu_2}v^{\mu_4}) + b^{(9)}_{12,56}\,\Pi^{\mu_1\mu_2}\Pi^{\mu_5\mu_6}v^{\mu_3}v^{\mu_4} +$$
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$$+b^{(9)}_{16,23}\,(\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_6}\Pi^{\mu_4\mu_5}v^{\mu_2}v^{\mu_3}) + b^{(9)}_{16,24}\,(\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_5}v^{\mu_2}v^{\mu_4}) +$$
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$$+b^{(9)}_{16,25}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}v^{\mu_3}v^{\mu_4} + b^{(9)}_{16,34}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_4}v^{\mu_2}v^{\mu_5} + c^{(9)}\,\Pi^{\mu_1\mu_6}v^{\mu_2}v^{\mu_3}v^{\mu_4}v^{\mu_5}$$

- Multiple color structures arise from combining pure color octets: $[iD^{\mu_i}, iD^{\mu_j}]$, $\quad [iD^{\mu_i}, [iD^{\mu_j}, iD^{\mu_k}]]$, $\quad [iD^{\mu_i}, [iD^{\mu_j}, [iD^{\mu_k}, iD^{\mu_l}]]]$

- For $|V_{ub}|$ and $|V_{cb}|$ with tree level dimension ≥ 7 power corrections Only T^aT^b color structure is needed

New Result: Moments of the leading power shape function

- Moments of the shape function are related to HQET parameters
  The matrix elements decomposition makes their calculation easy

$$2M_B \int d\omega\, \omega^k\, S(\omega) = n_{\mu_1} ... n_{\mu_k} \langle \bar{B}(v)|\bar{h}\, iD^{\mu_1} ... iD^{\mu_k}\, h|\bar{B}(v)\rangle$$

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$$\int d\omega\, S(\omega) = 1, \qquad \int d\omega\, \omega\, S(\omega) = 0, \qquad \int d\omega\, \omega^2\, S(\omega) = -a^{(5)} = -\lambda_1/3,$$

$$\int d\omega\, \omega^3\, S(\omega) = -a^{(6)} = -\rho_1/3,$$

$$\int d\omega\, \omega^4\, S(\omega) = a^{(7)}_{12} + a^{(7)}_{13} + a^{(7)}_{14} - b^{(7)} = m_1/5 - m_2/3,$$

$$\int d\omega\, \omega^5\, S(\omega) = 2a^{(8)}_{12} + 2a^{(8)}_{13} + 2a^{(8)}_{15} + b^{(8)}_{12} + b^{(8)}_{14} + b^{(8)}_{15} - c^{(8)} = $$
$$= \left(-8r_1 + 2r_2 + 2r_3 + 2r_4 + r_5 + r_6 + r_7\right)/15,$$

$$\begin{aligned}\int d\omega\, \omega^6\, S(\omega) = & -a^{(9)}_{12,34} - 2a^{(9)}_{12,35} - 2a^{(9)}_{12,36} - a^{(9)}_{13,25} - 2a^{(9)}_{13,26} - a^{(9)}_{14,25} - 2a^{(9)}_{14,26} - a^{(9)}_{15,26} \\ & -a^{(9)}_{16,23} - a^{(9)}_{16,24} - a^{(9)}_{16,25} + 2b^{(9)}_{12,36} + 2b^{(9)}_{12,46} + b^{(9)}_{12,56} + 2b^{(9)}_{13,26} + b^{(9)}_{13,46} \\ & +2b^{(9)}_{14,26} + b^{(9)}_{14,36} + b^{(9)}_{15,26} + 2b^{(9)}_{16,23} + 2b^{(9)}_{16,24} + b^{(9)}_{16,25} + b^{(9)}_{16,34} - c^{(9)}\end{aligned}$$

- Future: moments of other SSF [Gunawardana, GP, *in progress*)]

Open problems and Future progress

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The future looks promising for $\bar{B} \to X_u \, l \, \bar{\nu}$ and inclusive $|V_{ub}|$!