Recent News in $B \to X_c \ell \bar{\nu}$

(Including $B \to X_c \tau \bar{\nu}$)

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Contents

Introduction: Status of V_{cb} and the HQE Fits

- Standard tool: Heavy Quark Expansion
- Structure of the expansion (@ tree):

$$d\Gamma = d\Gamma_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 d\Gamma_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 d\Gamma_3 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4 d\Gamma_4$$
$$+ d\Gamma_5 \left(a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 \right)$$
$$+ \ldots + d\Gamma_7 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4$$

- Power counting $m_c^2 \sim \Lambda_{\rm QCD} m_b$

Status of the calculation:

- Tree level terms up to and including $1/m_b^5$ known
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the leading term known
- $\mathcal{O}(\alpha_s)$ for the μ_π^2/m_b^2 and μ_G^2/m_b^2 known
- On the way: α_s/m_b^3 corrections
- Modelling for the HQE matrix elements beyond $1/m^3$

Basis of Dimension Seven Matrix Elements at $1/m^4$

Dim-7 Matrix elements = four derivatives
Spin-independent basic parameters of dimension 7

$$2M_B\, m_1^4 = \langle B|\bar b_v\, iD_\rho iD_\sigma iD_\lambda iD_\delta\, b_v|B\rangle$$

$$\frac{1}{3}\left(\Pi^{\rho\sigma}\Pi^{\lambda\delta} + \Pi^{\rho\lambda}\Pi^{\sigma\delta} + \Pi^{\rho\delta}\Pi^{\sigma\lambda}\right) = \langle \left((\vec p)^2\right)^2\rangle$$

$$2M_B\, m_2^4 = \langle B|\bar b_v \left[iD_\rho, iD_\sigma\right]\left[iD_\lambda, iD_\delta\right] b_v|B\rangle\; \Pi^{\rho\delta} v^\sigma v^\lambda = g^2\langle \vec E^2\rangle$$

$$2M_B\, m_3^4 = \langle B|\bar b_v \left[iD_\rho, iD_\sigma\right]\left[iD_\lambda, iD_\delta\right] b_v|B\rangle\; \Pi^{\rho\lambda}\Pi^{\sigma\delta} = g^2\langle \vec B^2\rangle$$

$$2M_B\, m_4^4 = \langle B|\bar b_v \left\{iD_\rho, \left[iD_\sigma, \left[iD_\lambda, iD_\delta\right]\right]\right\} b_v|B\rangle\; \Pi^{\sigma\lambda}\Pi^{\rho\delta}$$

$$= g\langle \vec p \cdot \mathrm{rot}\, \vec B\rangle$$

$$\Pi_{\mu\nu} = g_{\mu\nu} - v_\mu v_\nu$$

Spin-dependent basic parameters of dimension 7

$$2M_B\, m_5^4 = \langle B|\bar{b}_v \left[iD_\rho, iD_\sigma\right]\left[iD_\lambda, iD_\delta\right]\left(-i\sigma_{\alpha\beta}\right) b_v|B\rangle\ \Pi^{\alpha\rho}\Pi^{\beta\delta} v^\sigma v^\lambda$$

$$2M_B\, m_6^4 = \langle B|\bar{b}_v \left[iD_\rho, iD_\sigma\right]\left[iD_\lambda, iD_\delta\right]\left(-i\sigma_{\alpha\beta}\right) b_v|B\rangle\ \Pi^{\alpha\sigma}\Pi^{\beta\lambda}\Pi^{\rho\delta}$$

$$2M_B\, m_7^4 = \langle B|\bar{b}_v \left\{\left\{iD_\rho, iD_\sigma\right\}, \left[iD_\lambda, iD_\delta\right]\right\}\left(-i\sigma_{\alpha\beta}\right) b_v|B\rangle \\ \Pi^{\sigma\lambda}\Pi^{\alpha\rho}\Pi^{\beta\delta}$$

$$2M_B\, m_8^4 = \langle B|\bar{b}_v \left\{\left\{iD_\rho, iD_\sigma\right\}, \left[iD_\lambda, iD_\delta\right]\right\}\left(-i\sigma_{\alpha\beta}\right) b_v|B\rangle \\ \Pi^{\rho\sigma}\Pi^{\alpha\lambda}\Pi^{\beta\delta}$$

$$2M_B\, m_9^4 = \langle B|\bar{b}_v \left[iD_\rho, \left[iD_\sigma, \left[iD_\lambda, iD_\delta\right]\right]\right]\left(-i\sigma_{\alpha\beta}\right) b_v|B\rangle \\ \Pi^{\rho\beta}\Pi^{\lambda\alpha}\Pi^{\sigma\delta}\,.$$

Physical interpretation

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$

$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$

$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$

$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$

$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

Model: Lowest State Saturation Assumption (LSSA)

Express all matrix elements by μ_π, μ_G and the excitation energies $\epsilon_{1/2}$, $\epsilon_{3/2}$.

$$\rho_D^3 = \frac{1}{3}\epsilon_{1/2}(\mu_\pi^2 - \mu_G^2) + \frac{1}{3}\epsilon_{3/2}(2\mu_\pi^2 + \mu_G^2),$$
$$\rho_{LS}^3 = \frac{2}{3}\epsilon_{1/2}(\mu_\pi^2 - \mu_G^2) - \frac{1}{3}\epsilon_{3/2}(2\mu_\pi^2 + \mu_G^2).$$

	Expression			Expression	
m_1	$\frac{5}{9}\mu_\pi^4$	9.5	m_2	$-\frac{\epsilon_{1/2}^2}{3}(\mu_\pi^2 - \mu_G^2) - \frac{\epsilon_{3/2}^2}{3}(2\mu_\pi^2 + \mu_G^2)$	-8.2
m_3	$-\frac{2}{3}\mu_G^4$	-7.7	m_4	$\mu_G^4 + \frac{4}{3}\mu_\pi^4$	34.4
m_5	$-\frac{2\epsilon_{1/2}^2}{3}(\mu_\pi^2 - \mu_G^2) + \frac{\epsilon_{3/2}^2}{3}(2\mu_\pi^2 + \mu_G^2)$	7.0	m_6	$\frac{2}{3}\mu_G^4$	7.7
m_7	$-\frac{8}{3}\mu_\pi^2\mu_G^2$	-37.5	m_8	$-8\mu_\pi^2\mu_G^2$	-112.6
m_9	$\mu_G^4 - \frac{10}{3}\mu_\pi^2\mu_G^2$	-35.4			

from Heinonen, M., 1407.4384

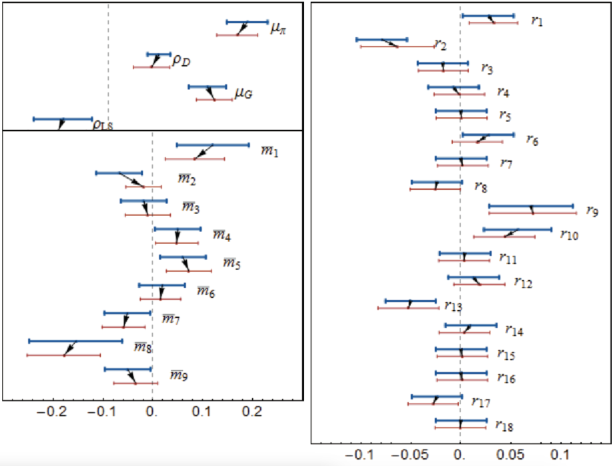
Current fit

Gambino, Healy Turzcyk, 1606.06174

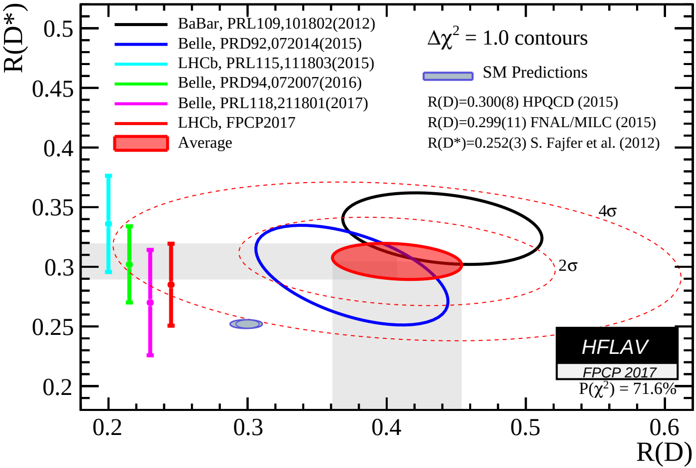
m_b^{kin}	4.546	0.021	r_1	0.032	0.024
$\overline{m}_c(3\text{GeV})$	0.987	0.013	r_2	−0.063	0.037
μ_π^2	0.432	0.068	r_3	−0.017	0.025
μ_G^2	0.355	0.060	r_4	−0.002	0.025
ρ_D^3	0.145	0.061	r_5	0.001	0.025
ρ_{LS}^3	−0.169	0.097	r_6	0.016	0.025
$\overline{m}_1$	0.084	0.059	r_7	0.002	0.025
$\overline{m}_2$	−0.019	0.036	r_8	−0.026	0.025
$\overline{m}_3$	−0.011	0.045	r_9	0.072	0.044

$\overline{m}_4$	0.048	0.043	r_{10}	0.043	0.030
$\overline{m}_5$	0.072	0.045	r_{11}	0.003	0.025
$\overline{m}_6$	0.015	0.041	r_{12}	0.018	0.025
$\overline{m}_7$	−0.059	0.043	r_{13}	−0.052	0.031
$\overline{m}_8$	−0.178	0.073	r_{14}	0.003	0.025
$\overline{m}_9$	−0.035	0.044	r_{15}	0.001	0.025
χ^2/dof	0.46		r_{16}	0.001	0.025
$BR(\%)$	10.652	0.156	r_{17}	−0.028	0.025
$\mathbf{10^3\|V_{cb}\|}$	**42.11**	**0.74**	r_{18}	−0.001	0.025

Shift in the parameters from 2014 → 2016 fits



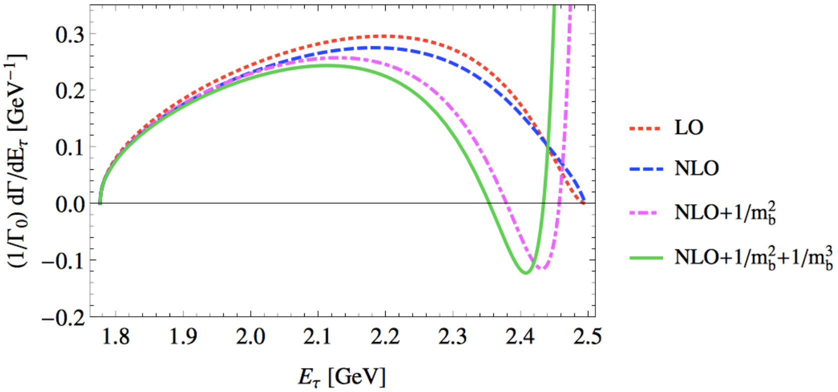
Inclusive $B \to X_c \tau \bar{\nu}$ Decays



What does this mean for the inclusive decays?

- Measurement at LEP:
  $\mathrm{Br}(b\text{-admix} \to X \tau \bar{\nu}) = (2.41 \pm 0.23)\%$
- HQE Calculation to $O(1/m^2)$ (1S scheme) (LIgeti, Tackman, Ruderman)
  $\mathrm{Br}(B^- \to X_c \tau \bar{\nu}) = (2.42 \pm 0.05)\%$
- HQE Calculation to $O(1/m^3)$ (kinetic scheme) (M., Rusov, Shahriaran)
  $\mathrm{Br}(B^- \to X_c \tau \bar{\nu}) = (2.26 \pm 0.05)\%$
- Sum over exclusive states: (HFAG + PDG)
  $\mathrm{Br}(B \to [D + D^*] \tau \bar{\nu}) = (2.68 \pm 0.16)\%$

Differential Rates



Define inclusive observables including the τ decay.

New Physics in $R(D)$ and $R(D^*)$?

This will also have an impact on the inclusive rate!

Simple ansatz to test this:

$$\mathcal{H}_{\rm NP} = \frac{G_F V_{cb}}{\sqrt{2}} \left(\alpha \, O_{V+A} + \beta \, O_{S-P}\right)$$

$$O_{V+A} = (\bar{c}\gamma_\mu(1+\gamma_5)b)\,(\bar{\tau}\gamma^\mu(1-\gamma_5)\nu)\,,$$

$$O_{S-P} = (\bar{c}(1-\gamma_5)b)\,(\bar{\tau}(1-\gamma_5)\nu)$$

NB.: These correspond to SMEFT dim-8 operators

$$O'_{V+A} = (\bar{c}_R\gamma_\mu b_R)\left((\bar{L}\cdot\phi^\dagger)\gamma^\mu(\tilde{\phi}\cdot L)\right)$$

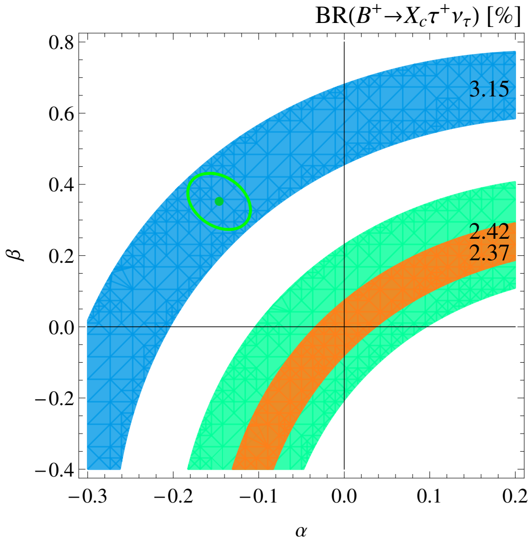
$$O'_{S-P} = \left(\bar{c}_R(\phi^\dagger\cdot Q)\right)\left(\bar{\tau}_R(\tilde{\phi}^\dagger\cdot L)\right)$$

Fit α and β to the values of $R(D)$ and $R(D^*)$

$$\alpha = -0.15 \pm 0.04, \quad \beta = 0.35 \pm 0.08$$

	SM	NP	Experiment
$\mathrm{Br}(B^+ \to D^0 \tau^+ \nu_\tau)$	(0.75 ± 0.13) %	0.93 %	(0.91 ± 0.11) %
$\mathrm{Br}(B^+ \to D^{*0} \tau^+ \nu_\tau)$	(1.25 ± 0.09) %	1.65 %	(1.77 ± 0.11) %
$\mathrm{Br}(B^+ \to X_c \tau^+ \nu_\tau)$	(2.37 ± 0.08) %	(3.15 ± 0.19) %	(2.41 ± 0.23) %

Significant enhancement of the total rate (with this ansatz)


BR(B⁺→X_cτ⁺ν_τ) [%]
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
β
3.15
2.42
2.37
−0.3
−0.2
−0.1
0.0
0.1
0.2
α

Reparametrization in HQE (T.M. and K. K. Vos: 1802.09409)

(pretty well known subject: Dugan, Golden, Grinstein, Chen, Luke, Manohar, ...

Start from the operator:

$$R(q) = \int d^4x\, e^{iqx}\, T[\bar{Q}(x)\Gamma q(x)\ \bar{q}(0)\Gamma^\dagger Q(0)]$$

and replace $Q(x) = \exp(-im(v \cdot x))Q_v(x)$

$$R(q) = \int d^4x\, e^{-iSx}\, T[\bar{Q}_v(x)\Gamma q(x)\ \bar{q}(0)\Gamma^\dagger Q_v(0)]$$

with $S = mv - q$.
These expressions are independent of v!

Perform the OPE $\longrightarrow$ HQE

$$R(q) = \sum_{n=0}^{\infty} \left[C^{(n)}_{\mu_1 \cdots \mu_n}(S) \right]_{\alpha\beta} \bar{Q}_{v,\alpha} (iD_{\mu_1} \cdots iD_{\mu_n}) Q_{v,\beta}$$
$$= \sum_{n=0}^{\infty} C^{(n)}_{\mu_1 \cdots \mu_n}(S) \otimes \bar{Q}_v (iD_{\mu_1} \cdots iD_{\mu_n}) Q_v$$

These expressions are still invariant under reparametrization of v:

$$\delta_{\mathrm{RP}}\, v_\mu = \delta v_\mu \quad \text{with} \quad v \cdot \delta v = 0$$
$$\delta_{\mathrm{RP}}\, iD_\mu = -m \delta v_\mu$$
$$\delta_{\mathrm{RP}}\, Q_v(x) = im(x \cdot \delta v) Q_v(x) \quad \text{in particular} \quad \delta_{\mathrm{RP}}\, Q_v(0) = 0\ .$$

The RP connects different orders in $1/m$, which yields relations between the coefficients $n = 0, 1, 2, \ldots$

$$\delta_{\rm RP} C^{(n)}_{\mu_1 \cdots \mu_n} = m\, \delta v^\alpha \left(C^{(n+1)}_{\alpha \mu_1 \cdots \mu_n} + C^{(n+1)}_{\mu_1 \alpha \mu_2 \cdots \mu_n} + \cdots + C^{(n+1)}_{\mu_1 \cdots \mu_n \alpha} \right)$$

This holds for differential rates
but it is very simple for total rates

$$R = \sum_{n=0}^{\infty} c^{(n)}_{\mu_1 \cdots \mu_n}(v) \otimes \bar{Q}_v (iD_{\mu_1} \cdots iD_{\mu_n}) Q_v$$

Tensor decomposition of the $c^{(n)}_{\mu_1 \cdots \mu_n}(v)$ in terms of v_α, $g_{\alpha\beta}$ and Dirac matrices

Making use of RPI ...

New strategy for the HQE:

- Add in higher order terms
  to make the result manifestly RPI
- RPI enforces relations between different orders
- Resummation of towers of terms from different orders
- Express everything in terms of operators
  (and states) of full QCD
- For selected observables:
  Reduction of HQE parameters

HQE parameters (for the total rate) to $O(1/m^4)$

$$2m_H\mu_3 = \langle H(p)|\bar{Q}_v Q_v|H(p)\rangle = \langle \bar{Q}_v Q_v\rangle$$
$$2m_H\mu_G = \langle \bar{Q}_v (iD^\mu)(iD^\nu)(-i\sigma_{\mu\nu})Q_v\rangle$$
$$2m_H\rho_D = \langle \bar{Q}_v \left[(iD^\mu), \left[\left((ivD) + \frac{(iD)^2}{2m}\right), (iD_\mu)\right]\right] Q_v\rangle$$
$$2m_H r_G^4 = \langle \bar{Q}_v \left[(iD_\mu), (iD_\nu)\right]\left[(iD^\mu), (iD^\nu)\right] Q_v\rangle$$
$$2m_H r_E^4 = \langle \bar{Q}_v \left[(ivD), (iD_\mu)\right]\left[(ivD), (iD^\mu)\right] Q_v\rangle$$
$$2m_H s_B^4 = \langle \bar{Q}_v \left[(iD_\mu), (iD_\alpha)\right]\left[(iD^\mu), (iD_\beta)\right](-i\sigma^{\alpha\beta}) Q_v\rangle$$
$$2m_H s_E^4 = \langle \bar{Q}_v \left[(ivD), (iD_\alpha)\right]\left[(ivD), (iD_\beta)\right](-i\sigma^{\alpha\beta}) Q_v\rangle$$
$$2m_H s_{qB}^4 = \langle \bar{Q}_v \left[iD_\mu, \left[iD^\mu, \left[iD_\alpha, iD_\beta\right]\right]\right](-i\sigma^{\alpha\beta}) Q_v\rangle$$

- Less that had been identified before
- Depend on the quark mass
- Are expressed NOT in terms of $iD_\perp$, rather "full" derivatives
- Can be expressed in terms of full QCD operators via

$$ivD\, Q_v \to \left(ivD + \frac{(iD)^2}{m} \right) Q_v = \frac{1}{2m}((iD)^2 - m^2)Q$$

Thus

$$2m_H \mu_3 = \langle \bar{Q} Q \rangle$$
$$2m_H \mu_G = \langle \bar{Q} (iD^\mu)(iD^\nu)(-i\sigma_{\mu\nu}) Q \rangle$$
$$2m_H \rho_D = \frac{1}{2m} \langle \bar{Q} \left[(iD^\mu) \, , \, \left[(iD)^2 \, , \, (iD_\mu) \right] \right] Q \rangle$$
$$\cdots\cdot$$

How does this happen?

Look at the partonic result for the rate

$$R(p) = R(p^2) = R((mv+k)^2) = R(m^2 + 2m(vk) + k^2)$$
$$= R(m^2) + R'(m^2)(2m(vk) + k^2) + \frac{1}{2} R''(m^2)(2m(vk) + k^2)^2$$
$$= R(m^2)$$

if there are no gluons

$$(2m(vk) + k^2) \to 2m(iv\partial) + (i\partial)^2 \to 0$$

All the fully symmetrized (zero gluon) contributions are contained in the partonic result

Example: $b \to s\gamma$ (O_7 contribution only)

$$\Gamma_{b\to s\gamma} = \frac{\lambda^2 m^3}{4\pi}\left[\mu_3 - \frac{2}{m^2}\mu_G^2 - \frac{10\rho_D^3}{3m^3} - \frac{1}{3m^4}\left(4r_G^4 + 4r_E^4 + \frac{1}{4}s_{qB}^4 - 4s_E^4\right) + O(1/m^5)\right]$$

Why this might be useful ...

Alternative Normalization

Usual way: $\mu_3 = 1 + O(1/m^2)$

Alternative Normalization:

Use the trace of the energy momentum tensor

$$\Theta^{\mu}_{\ \mu} = m\,\bar{Q}Q + \frac{\beta(\alpha_s)}{4\pi} G^a_{\mu\nu} G^{\mu\nu,\,a}$$

and take the forward matrix element

$$\langle \Theta^{\mu}_{\ \mu} \rangle = 2m_H^2 = m\langle \bar{Q}Q \rangle + \frac{\beta(\alpha_s)}{4\pi} \langle G^a_{\mu\nu} G^{\mu\nu,\,a} \rangle \ ,$$

thus

$$m\,\mu_3 = m_H - \bar{\Lambda}$$

Replaces one power of m by the the hadron mass.

Conclusion

- The HQE in $b \to c$ transition is in a very good shape:
  - The determination of V_{cb} is “converging”
  - ... and approaches the 1% level
  - $B \to X_c \tau \bar{\nu}$ is computed to $1/m^3$
  - waiting for a more precise measurement
- The structure of the HQE at higher orders is (re)considered
  - Redefinition of HQE parameters
  - For some observables a reduction of their number