Decay Constants from Lattice QCD

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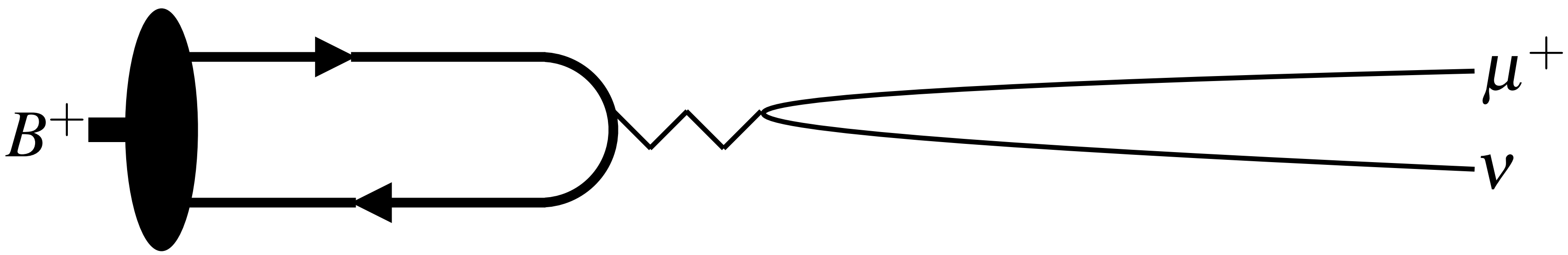
Challenges in Semileptonic *B* Decays v2.0
MITP, Mainz | April 12, 2018

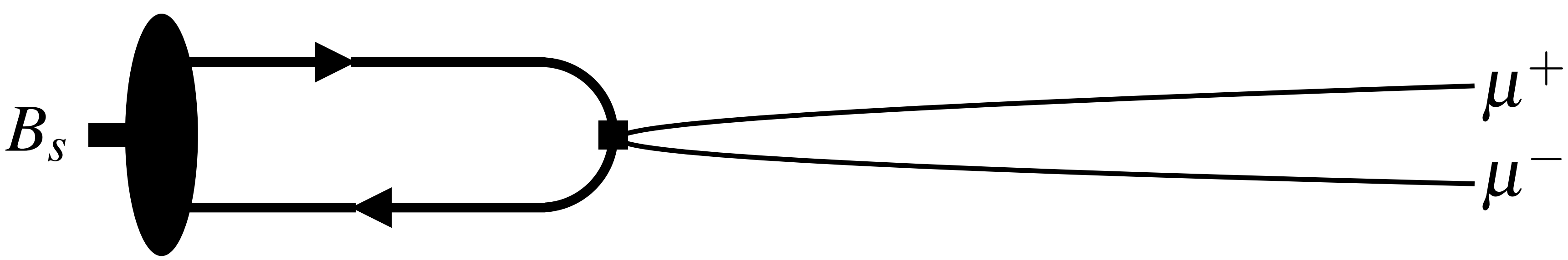
Leptonic Decays

$$if_B p^\mu = \langle 0|A^\mu_{qb}|B\rangle + \text{PCAC} \Rightarrow$$

$$f_B M_\pi^2 = \langle 0|(m_b + m_q)P_{qb}|B\rangle$$



$$\Gamma = \frac{M_{B^+}}{8\pi} f_{B^+}^2 |G_F V_{ub}|^2 m_\mu^2 \left(1 - \frac{m_\mu^2}{M_{B^+}}\right)^2$$



$$\Gamma(t=0) = \frac{M_{B_s}}{8\pi} f_{B_s}^2 |V_{tb}V_{ts}|^2 |C_{10}|^2 m_\mu^2 \left(1 - \frac{4m_\mu^2}{M_{B_s}^2}\right)^{1/2}$$

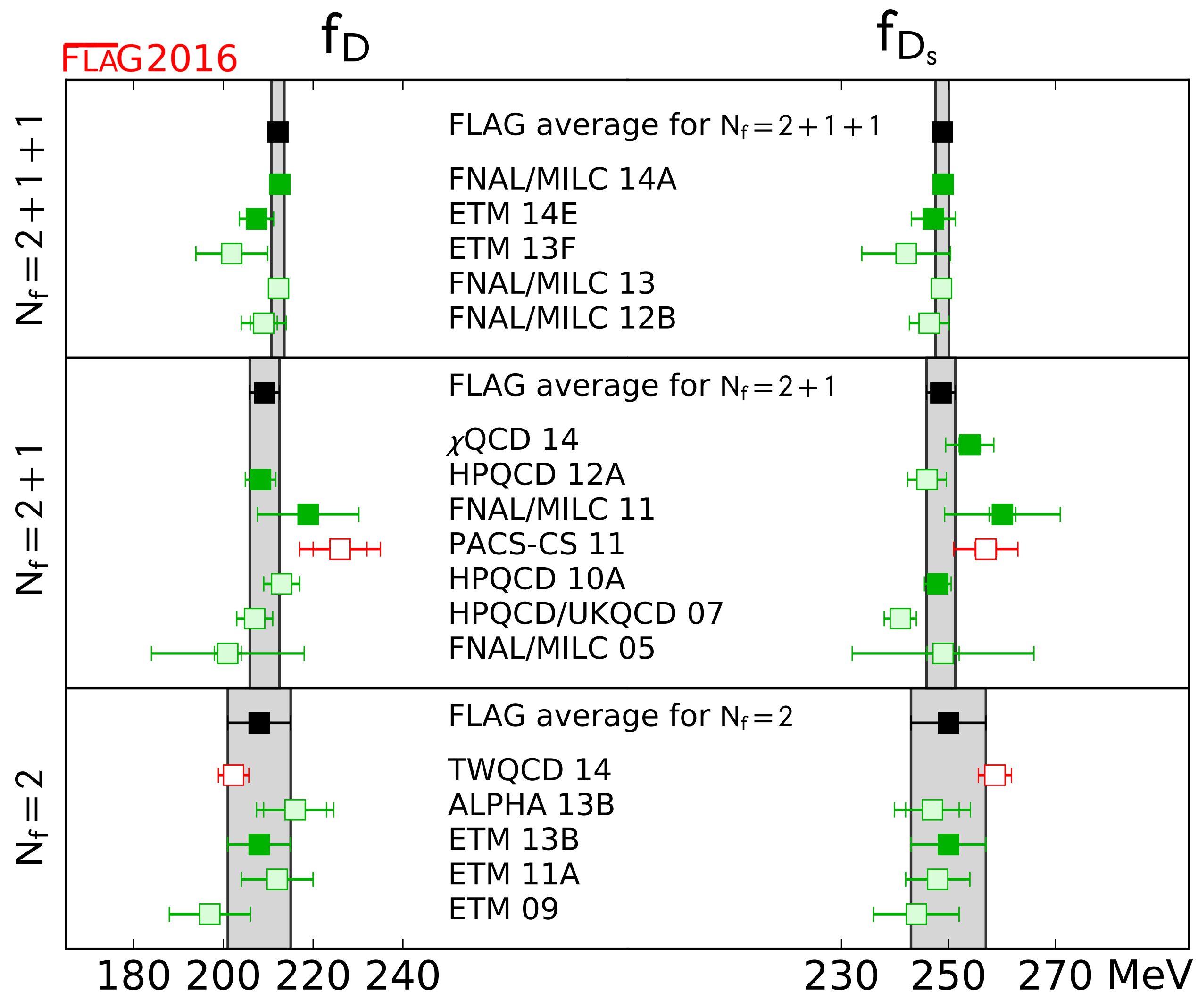
Motivations

- Another way to obtain $|V_{xq}|$ (charged) or constrain new physics (neutral).
- For lattice QCD, the simplest electroweak matrix element (harbinger of precision to come).
- Outline
  - Compendia and comments
  - New results from HPQCD and Fermilab/MILC
  - Electromagnetism

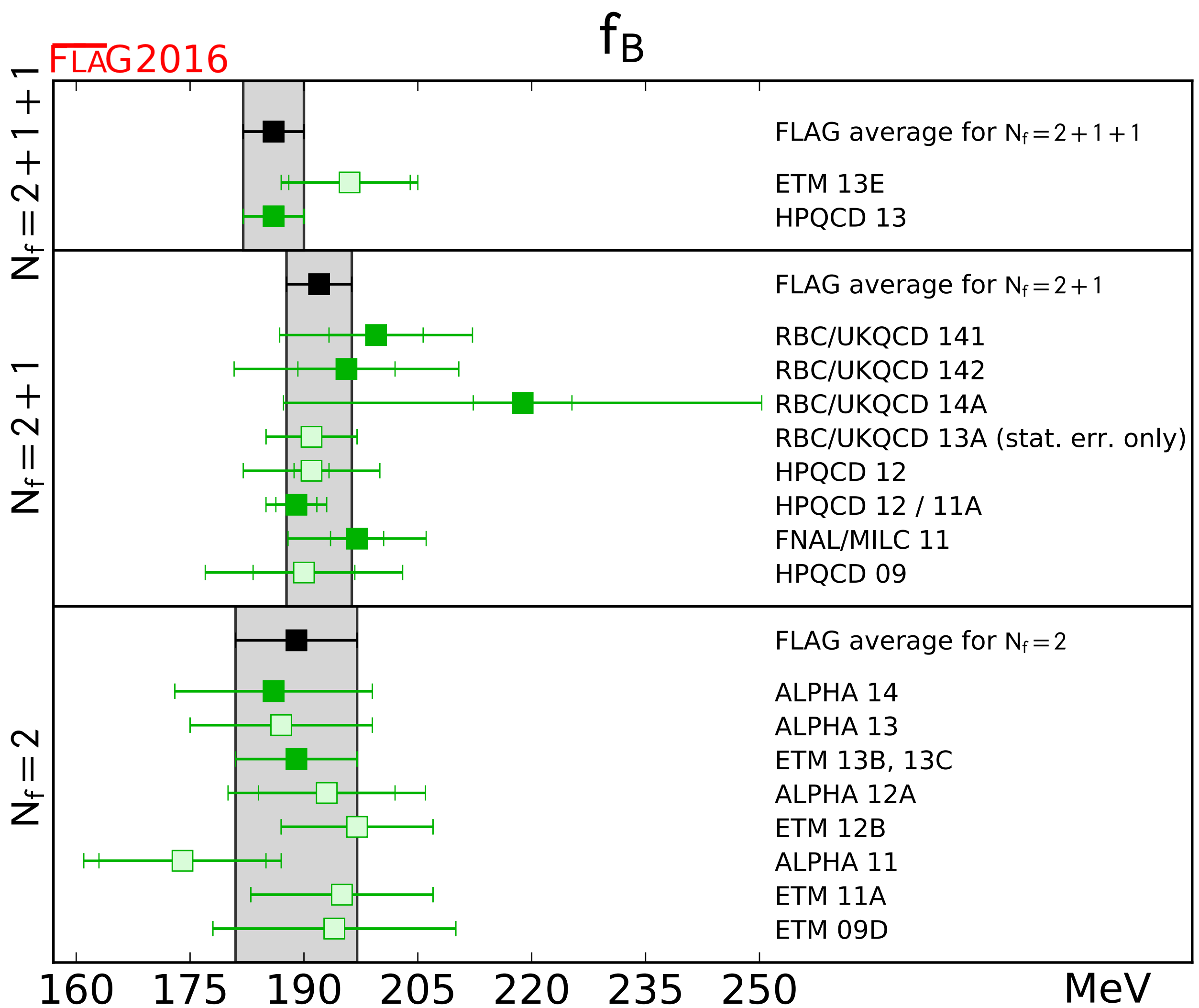
Compendia

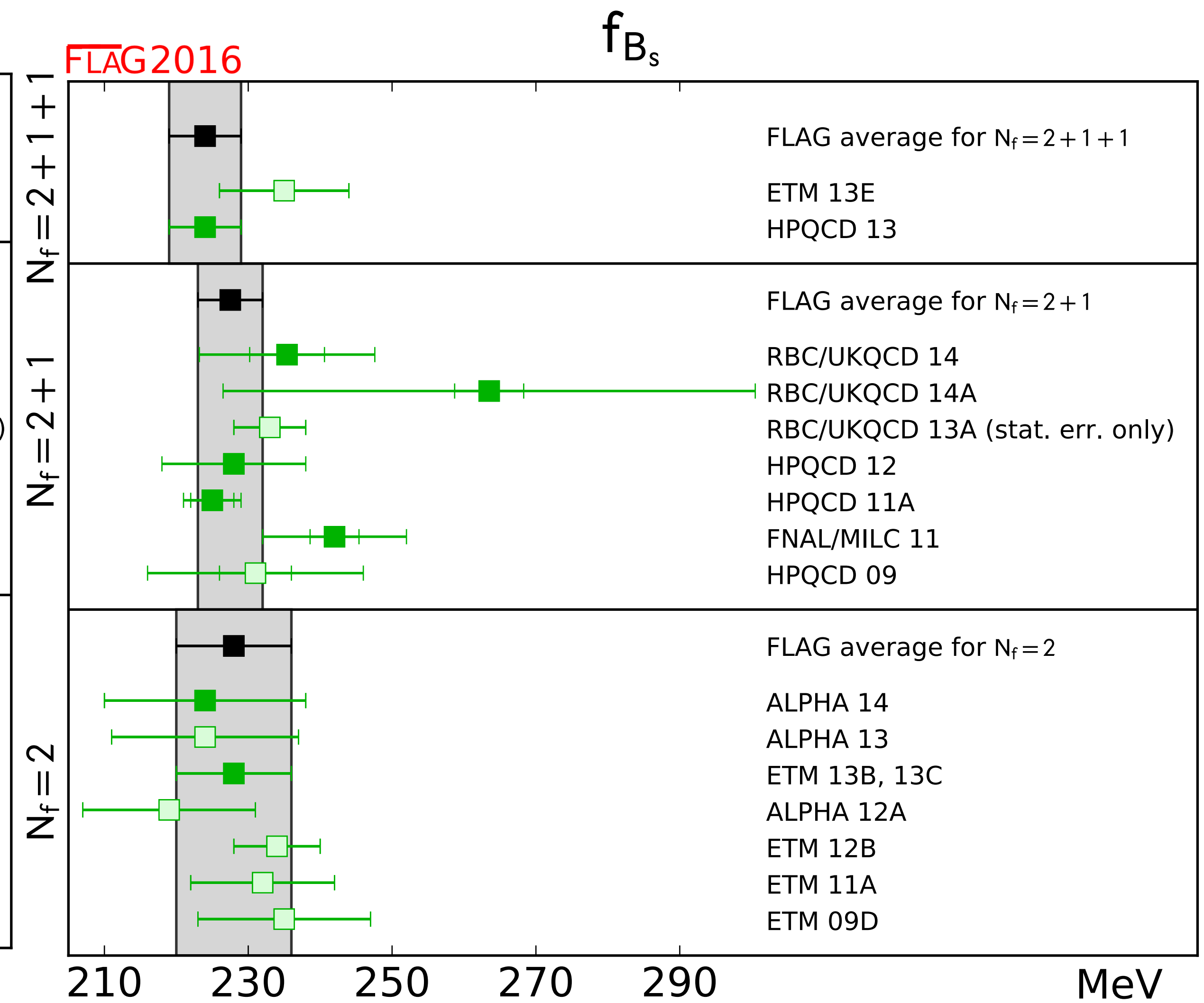
and updates

- Members drawn from various lattice-QCD collaborations, worldwide.
- Separate averages for different sea-quark content.
- Attempt to "grade" quality of calculations.
- Consider correlations between calculations (*e.g.*, HPQCD and Fermilab/MILC both use MILC ensembles).



and updates





- FLAG policy: don't just cite the report! Cite the underlying literature!
  - Links to BibTeX

PDG

- Similar procedures, with some differences:
  - no $n_f = 2$;
  - no grading: include those calculations with full error budgets;
  - after obtaining 2+1+1 and 2+1 averages separately, combine them;
  - different treatments of correlations of systematic uncertainties (if you want to get this deep into the weeds).
- Compatible averages (e.g., 2+1):

$$f_{B,\mathrm{FLAG}} = 192.0(4.3)\ \mathrm{MeV} \qquad f_{B,\mathrm{PDG}} = 189.9(4.2) + 1.9\ \mathrm{MeV}$$

remove isospin correction to B^+

Since the Compendia

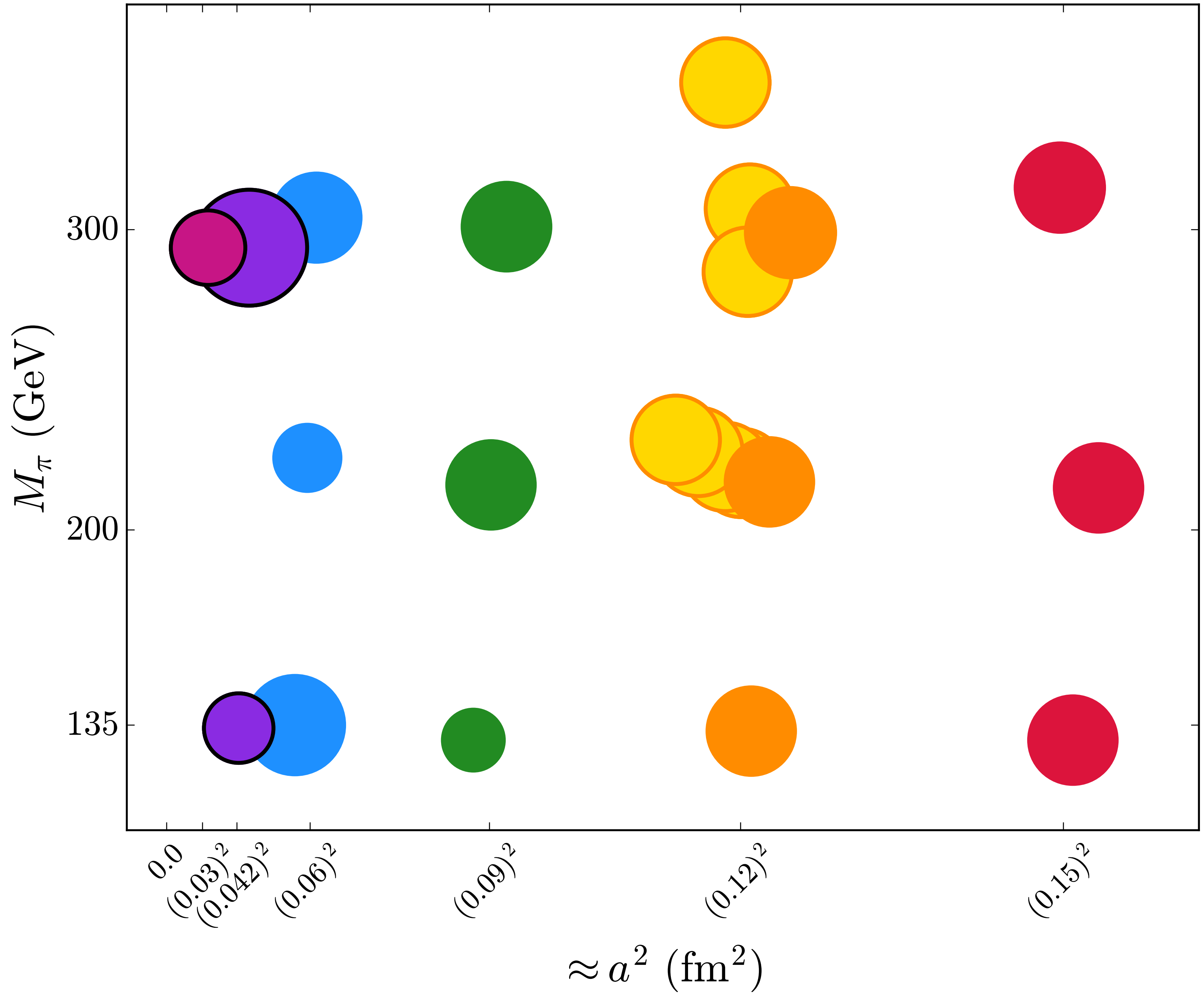
2017 Calculations

- Hughes, Davies, Monahan (HPQCD), arXiv:1711.09981:
  - lattice NRQCD *b* quark ⊕ HISQ light quark;
  - 9 of MILC's HISQ 2+1+1-flavor ensembles @ 3 lattice spacings;
  - pseudoscalar density with different matching wrt. (earlier) axial current.
- Fermilab Lattice & MILC Collaborations, arXiv:1712.09262:
  - HISQ for both valence quarks w/ physical *b* at smaller lattice spacings;
  - all 24 of MILC's HISQ 2+1+1-flavor ensembles @ 6 lattice spacings;
  - absolutely normalized pseudo scalar density—no matching!

MILC HISQ Ensembles

arXiv:1212.4768 + update in arXiv:1712.09262

- 2+1+1 sea quarks;
- 24 ensembles
- 5 w/ $M_\pi = 135$ MeV;
- down to $a = 0.03$ fm;
- typically 1000×4 samples;
- $M_\pi L > 3.2$, often > 5;
- up to $144^3 \times 288$.



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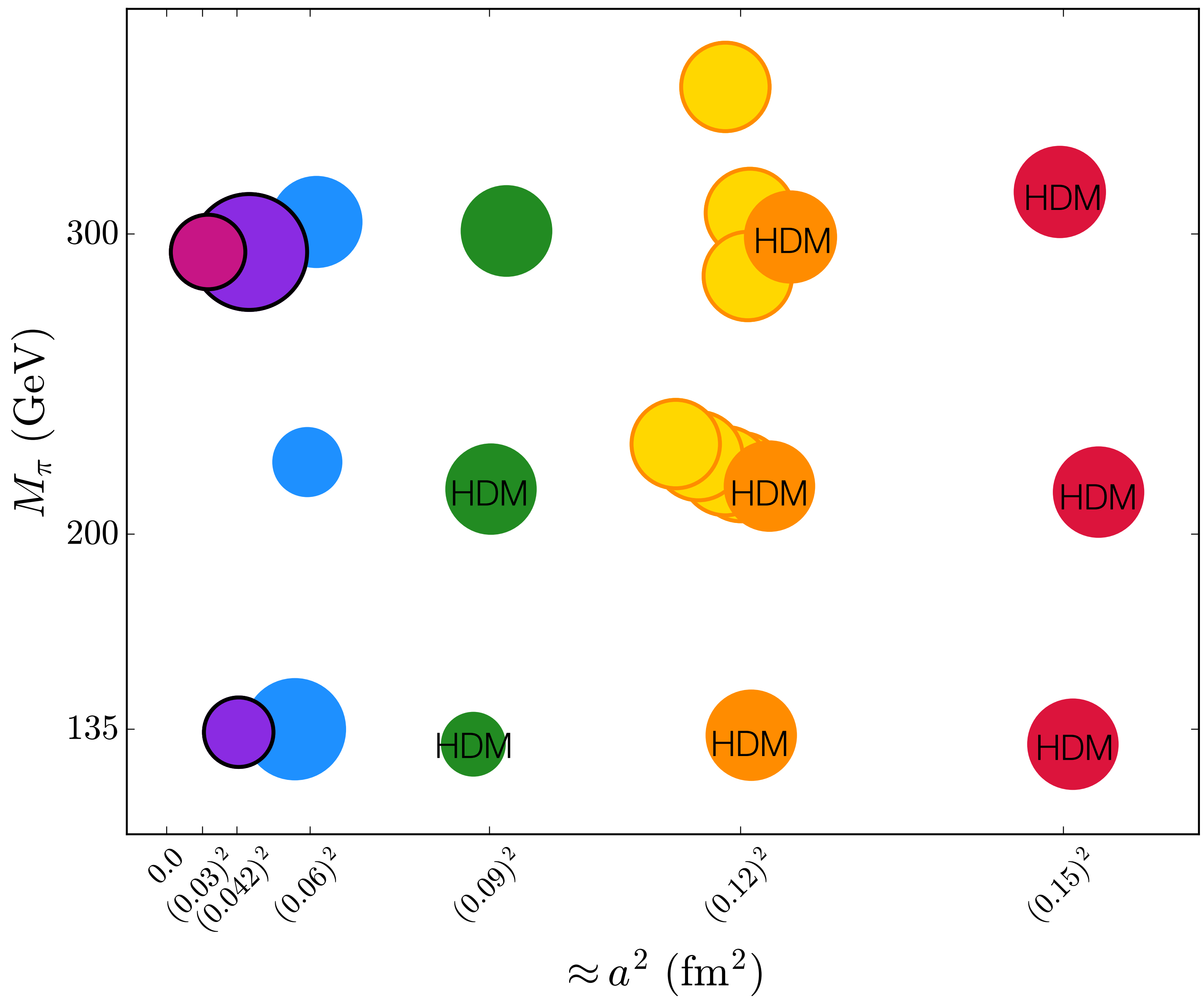


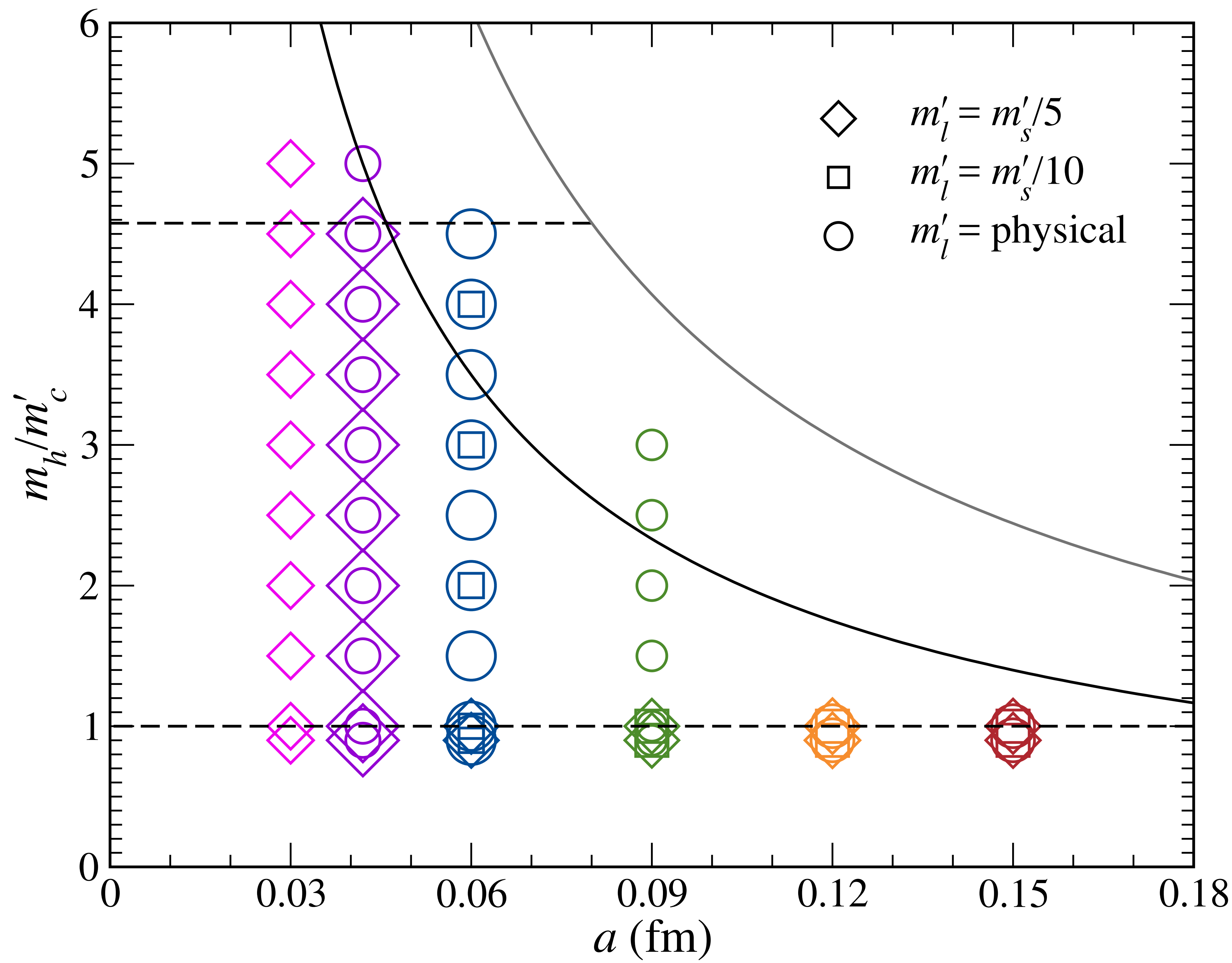
HISQ Ensembles: 2+1+1

MILC, arXiv:1212.4768 + further runs

a (fm)	size	$am'_l/am'_s/am'_c$	# confs	# sources	notes
≈ 0.15	$16^3\times 48$	0.0130/0.065/0.838	1020	4	
≈ 0.15	$24^3\times 48$	0.0064/0.064/0.828	1000	4	
≈ 0.15	$32^3\times 48$	0.00235/0.0647/0.831	1000	4	physical
≈ 0.12	$24^3\times 64$	0.0102/0.0509/0.635	1040	4	
≈ 0.12	$32^3\times 64$	0.00507/0.0507/0.628	1020	4	also 24^3, 40^3
≈ 0.12	$48^3\times 64$	0.00184/0.0507/0.628	999	4	physical
≈ 0.12	$24^3\times 64$	0.0102/0.03054/0.635	1020	4	$m'_s < m_s$
≈ 0.12	$24^3\times 64$	0.01275/0.01275/0.640	1020	4	$m'_s = m'_l$
≈ 0.12	$32^3\times 64$	0.00507/0.0304/0.628	1020	4	$m'_s < m_s$
≈ 0.12	$32^3\times 64$	0.00507/0.022815/0.628	1020	4	$m'_s < m_s$
≈ 0.12	$32^3\times 64$	0.00507/0.012675/0.628	1020	4	$m'_s \ll m_s$
≈ 0.12	$32^3\times 64$	0.00507/0.00507/0.628	1020	4	$m'_s = m'_l$
≈ 0.12	$32^3\times 64$	0.0088725/0.022815/0.628	1020	4	$m'_s < m_s$
≈ 0.09	$32^3\times 96$	0.0074/0.037/0.440	1005	4	
≈ 0.09	$48^3\times 96$	0.00363/0.0363/0.430	999	4	
≈ 0.09	$64^3\times 96$	0.0012/0.0363/0.432	484	4	physical
≈ 0.06	$48^3\times 144$	0.0048/0.024/0.286	1016	4	
≈ 0.06	$64^3\times 144$	0.0024/0.024/0.286	572	4	
≈ 0.06	$96^3\times 192$	0.0008/0.022/0.260	842	6	physical
≈ 0.042	$64^3\times 192$	0.00316/0.0158/0.188	1167	6	
≈ 0.042	$144^3\times 288$	0.000569/0.01555/0.1827	429	6	physical
≈ 0.03	$96^3\times 288$	0.00223/0.01115/0.1316	724	4	

Heavy-Quark Masses

- always $0.9m_c$, m_c;
- up to $5m_c$;
- omit $am_c \geq 0.9$ from heavy-quark fits (need $< \pi/2$);
- omit 0.15 fm in base fit;
- 492 data points (498 w/ 0.15 fm).



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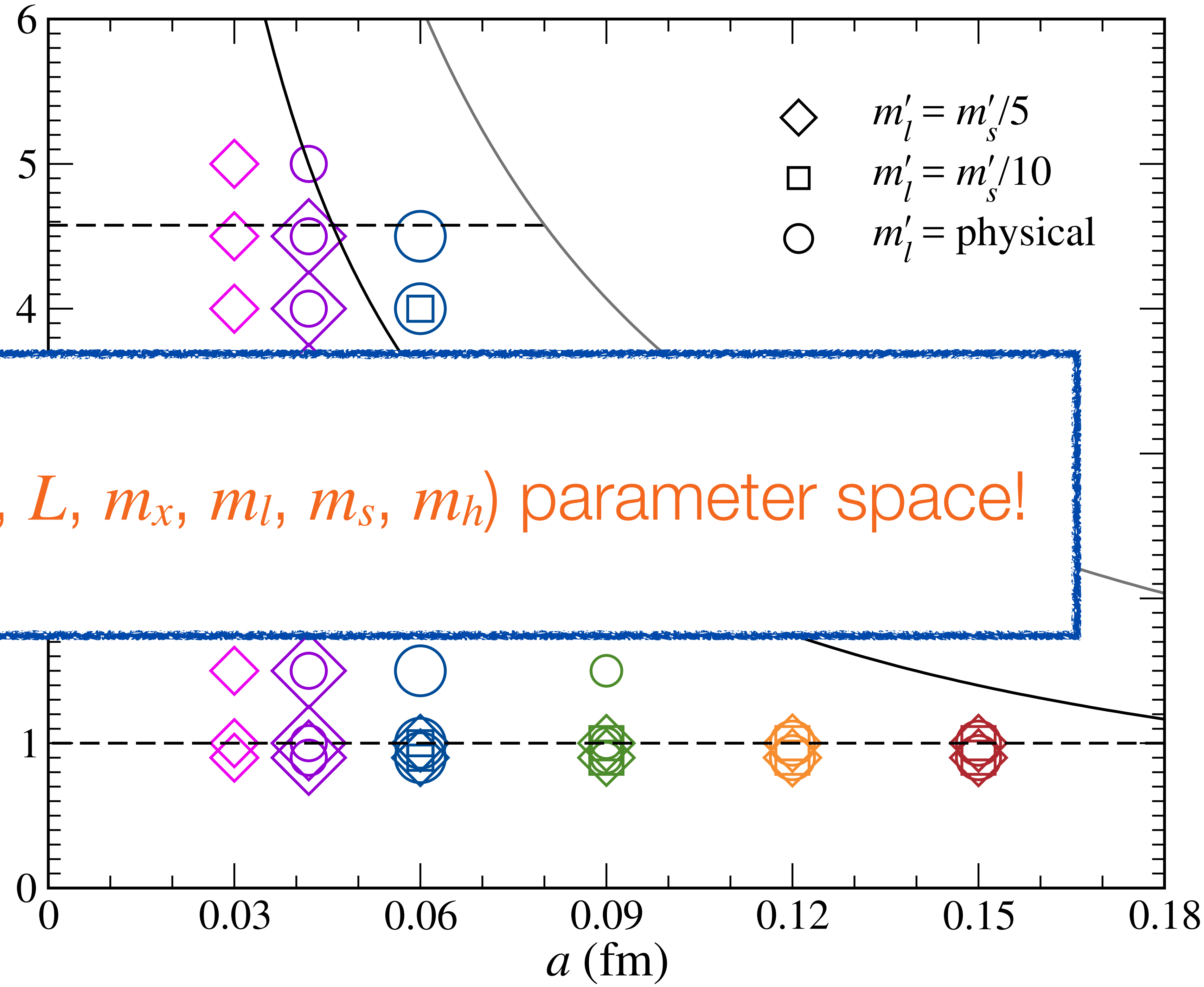


Huge slab in (a, L, m_x, m_l, m_s, m_h) parameter space!

Frozen Topology

- Continuum gauge fields: topological charge Q cannot change with an infinitesimal change in the gauge field.

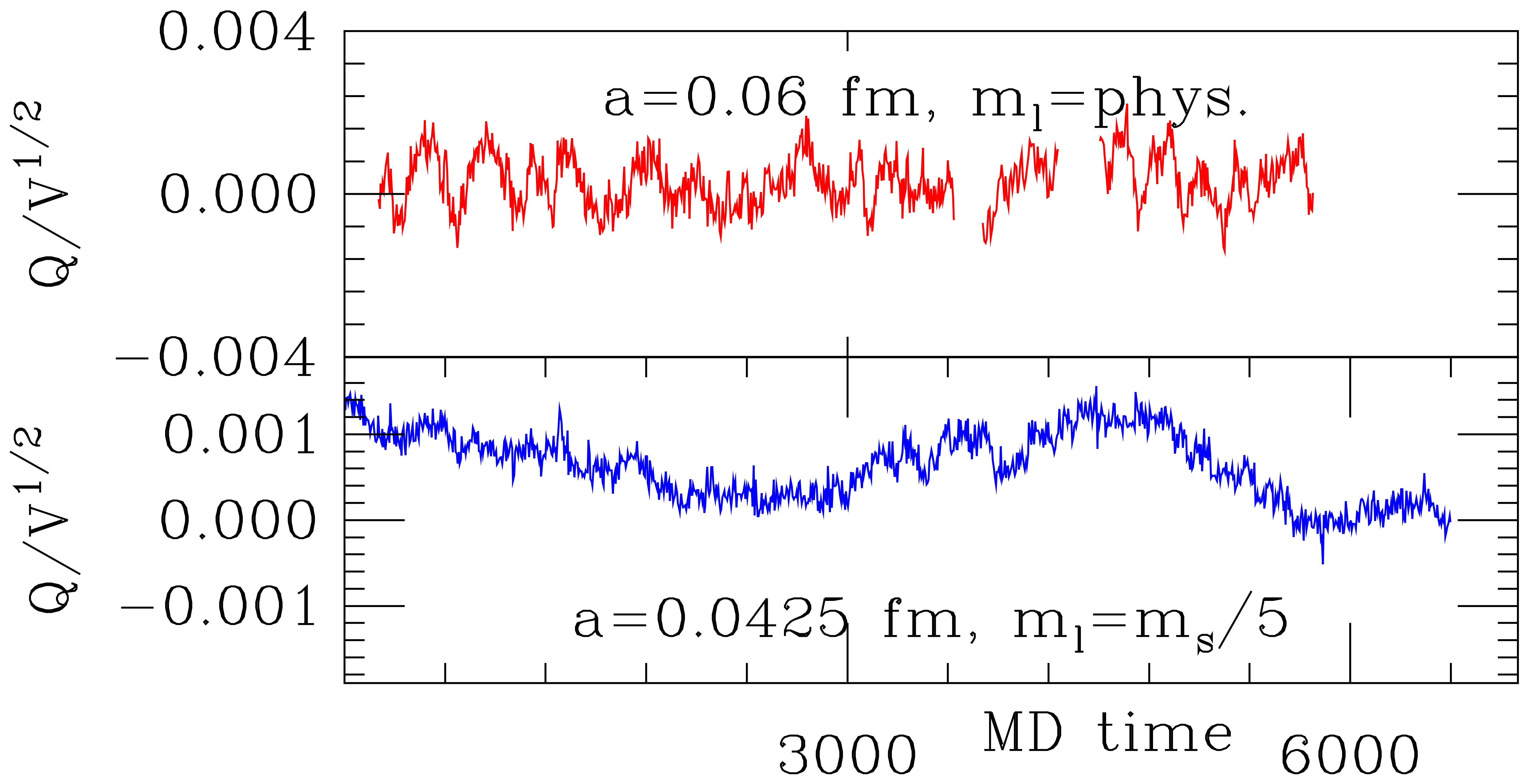
- Evolution of lattice gauge fields in CPU time consists of small steps that (in physical units) become smaller and smaller as lattice spacing $a \to 0$.

- Some reactions:

  - “Oh, my! Physics is now impossible!”—anonymous

  - “Physical quantities will suffer a systematic error, and we need to either correct for this error or account for it in our error budgets.”
    —Bernard & Toussaint [arXiv:1707.05430]

Good vs. Bad Sampling



- Instead of exponentially suppressed ($e^{-kM_\pi L}$) volume effects, poorly sampled topological charge leads to effects suppressed by

spacetime volume
$V = L^3T$

Q of fixed-Q sector, or
average of Q^2 in the simulation

pre-factor from
m_q dependence

$$\frac{1}{2\chi_T}\frac{1}{V}\left(1 - \frac{Q^2}{\langle Q^2 \rangle}\right)$$

topological susceptibility:
$\chi_T = \langle Q^2 \rangle / V$
in χPT, $\chi_T \propto f_\pi^2 M_\pi^2$
if $M_\pi L \sim$ const, $\chi_T V \propto f_\pi^2 LT$

vev of Q^2 in
$\theta = 0$ vacuum

References:
Leutwyler, Smilga [PRD46 (1992) 5607];
Brower *et alia* [hep-lat/0302005];
Aoki *et alia* [arXiv:0707.0396];
Aoki, Fukaya [arXiv:0906.4852].

Typical Corrections

Bernard & Toussaint, arXiv:1707.05430

$x(\sigma_x)$ $[\Delta_Q x]$	$m_l' = m_s'/5$	m_l' = physical
$\langle Q^2 \rangle_{\text{ens}} / \langle Q^2 \rangle_{\chi\text{PT}}$	1.30	0.65
f_K/f_π	1.20508(0.00250) [–0.01271]	1.19680(0.00114) [0.00015]
aM_π	0.031147(0.000172) [–0.000707]	0.028964(0.000020) [0.000008]
aM_D	0.048858(0.000261) [–0.000552]	0.045389(0.000245) [0.000006]
af_D	0.409786(0.000391) [–0.000044]	0.400678(0.000258) [0.000001]
aM_{Ds}	0.054828(0.000068) [–0.000001]	0.053582(0.000025) [0.000000]
af_{Ds}	0.430966(0.000116) [–0.000004]	0.422041(0.000037) [0.000000]

- Must be examined ensemble by ensemble.

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Tiny, and sometimes significant.

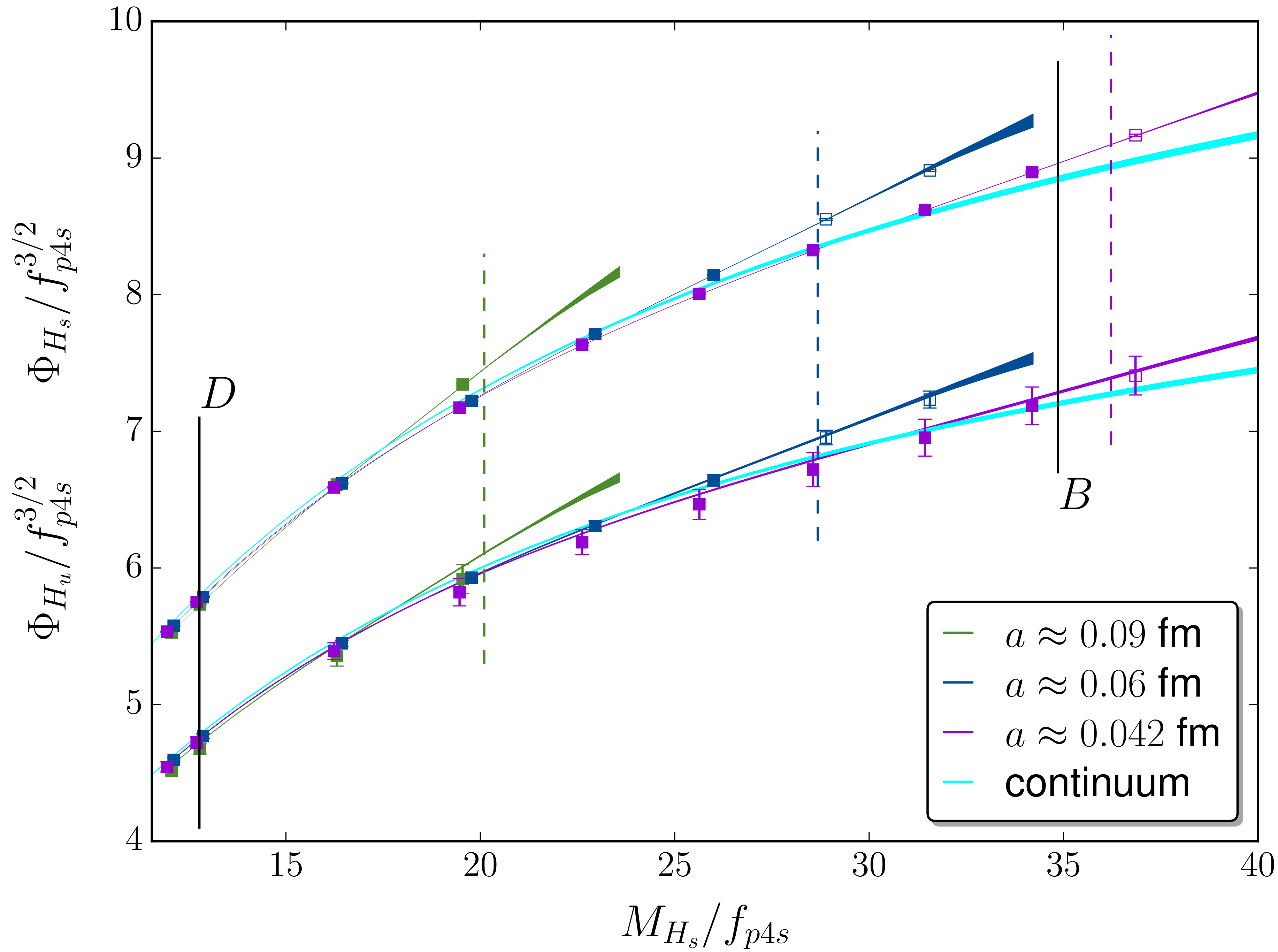
- Must be examined ensemble by ensemble.

HQET ⊕ Symanzik EFT ⊕ χPT Fits

- As noted, the slab of parameter space (5-dimensional) is huge.
- The raw statistical precision of the simulation data is
  - 0.04–1.4% for heavy-light meson decay constants;
  - 0.005–0.12% for heavy-light meson masses.
- It is insufficient to have a simple function to fit the dependence on (a, m_x, m_l, m_s, m_h).
- Functional form follows power-counting and builds in leading chiral logs and HQET anomalous dimension.

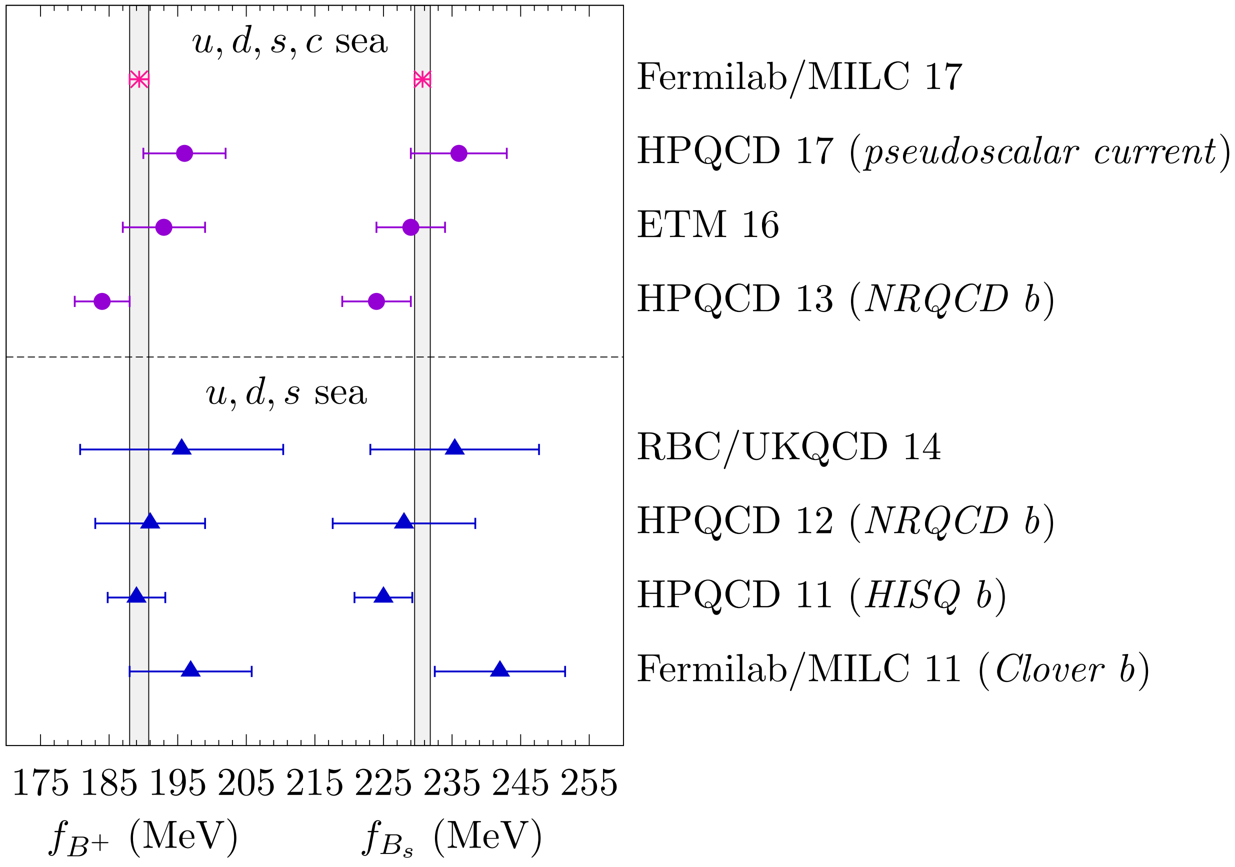
Snapshot of Decay Constants

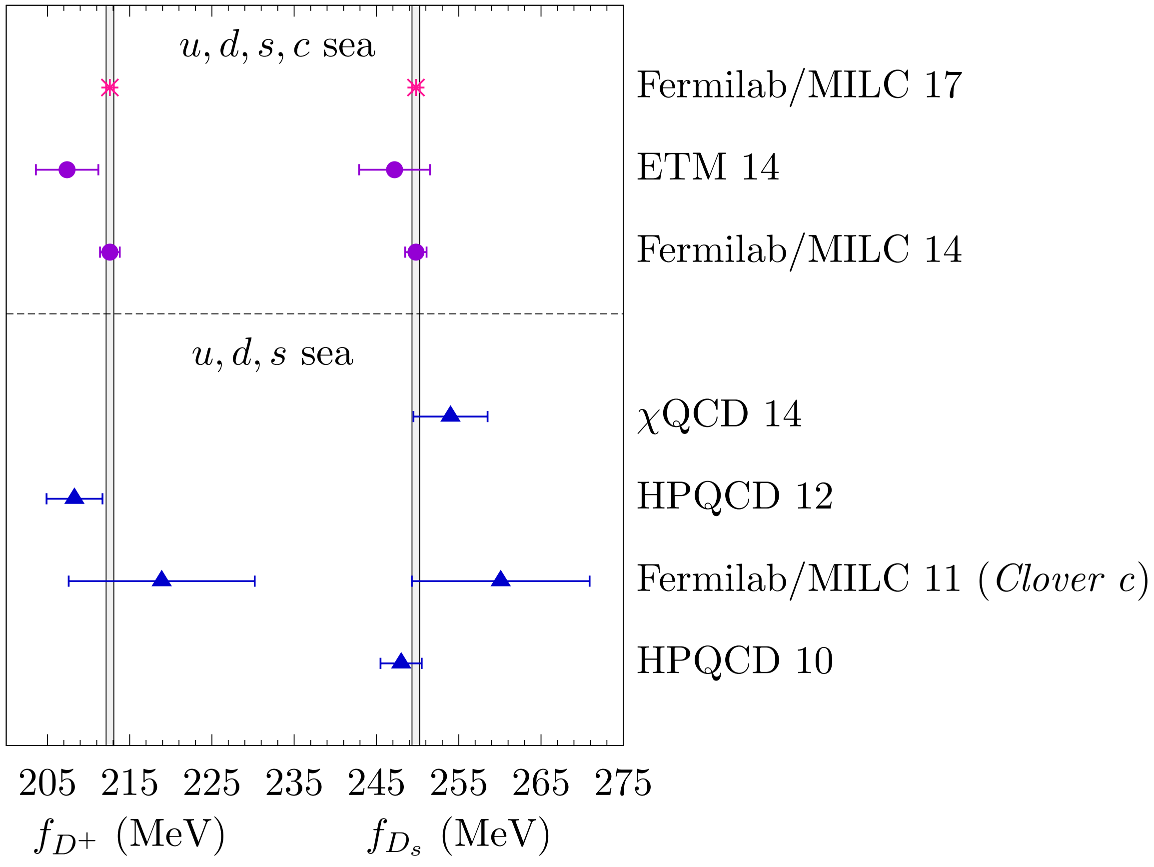
- 492 data pts;
- 60 parameters;
- $\chi^2/\text{dof} = 466/432$;
- $p = 0.12$;
- stable under fit variations;
- extra errors for FV, topology, EM.



Results for Decay Constants

- Fermilab Lattice & MILC [arXiv:1712.09262]:





Results for Decay Constants

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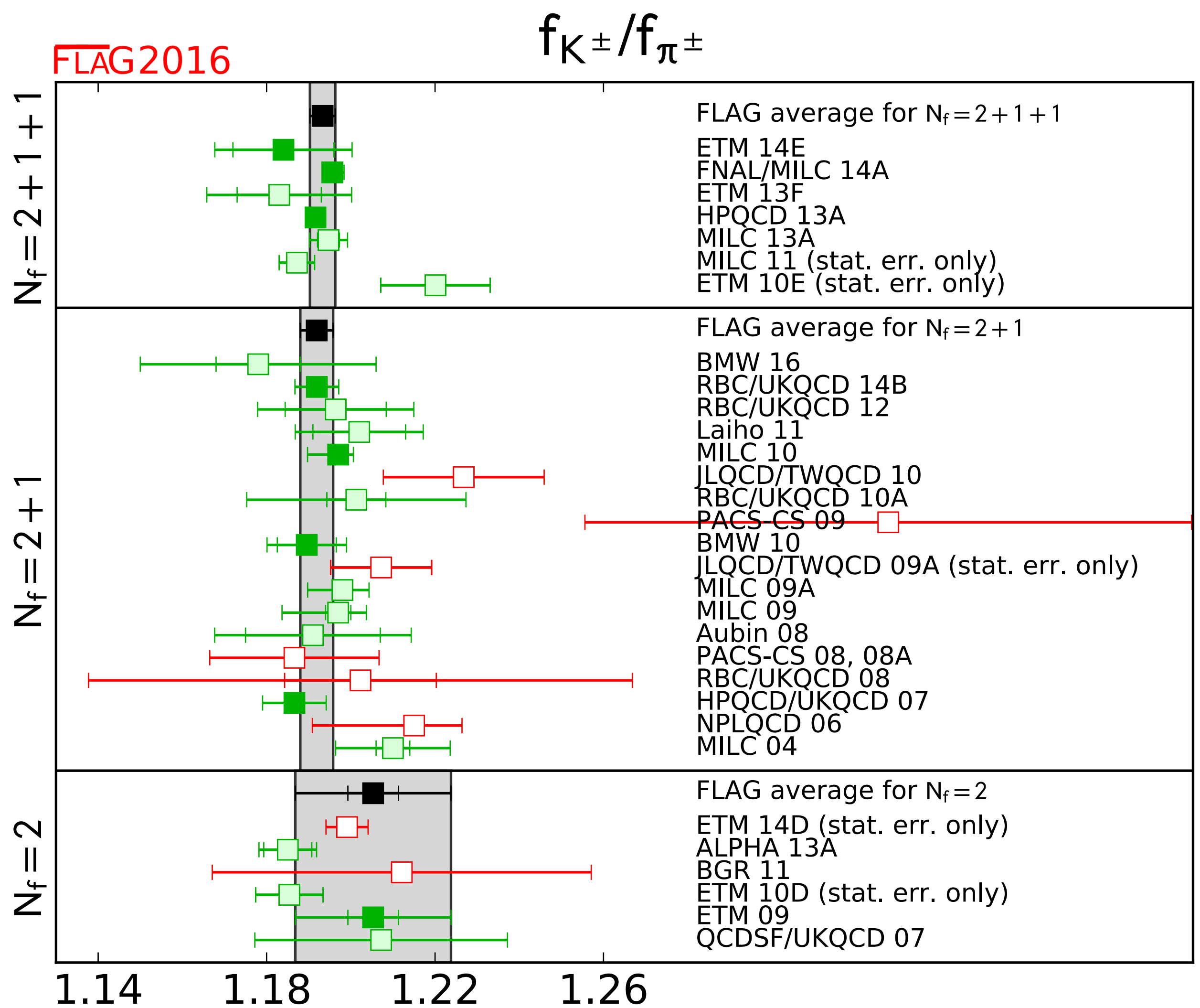
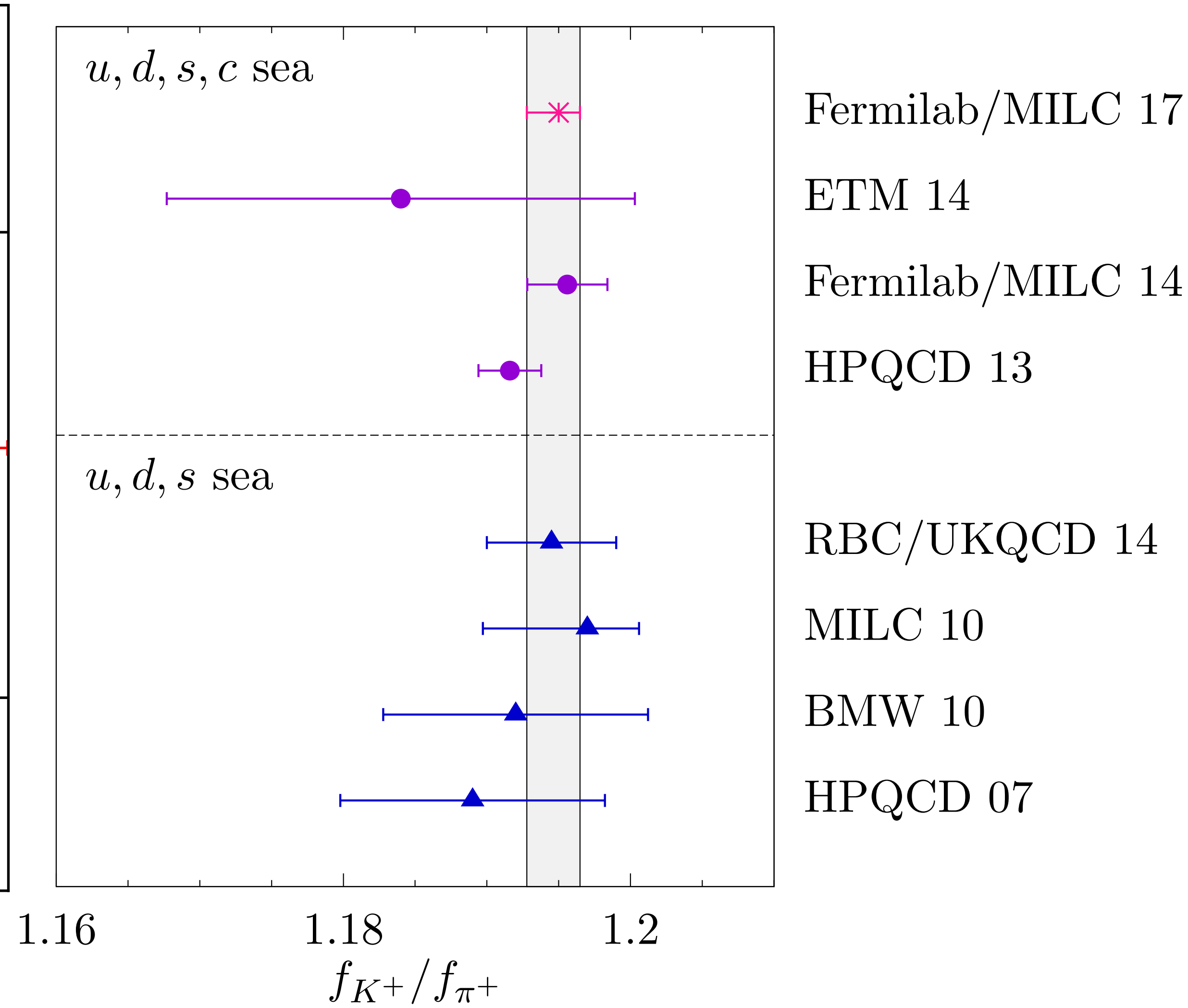
$$f_{D^0} = 211.5(0.3)_{\mathrm{stat}}(0.3)_{\mathrm{syst}}(0.2)_{f_{\pi,\mathrm{PDG}}}[0.1]_{\mathrm{EM\ scheme}}\ \mathrm{MeV}$$
$$f_{D^+} = 212.6(0.3)_{\mathrm{stat}}(0.3)_{\mathrm{syst}}(0.2)_{f_{\pi,\mathrm{PDG}}}[0.1]_{\mathrm{EM\ scheme}}\ \mathrm{MeV}$$
$$f_{D_s} = 249.8(0.3)_{\mathrm{stat}}(0.2)_{\mathrm{syst}}(0.2)_{f_{\pi,\mathrm{PDG}}}[0.1]_{\mathrm{EM\ scheme}}\ \mathrm{MeV}$$
$$f_{B^+} = 189.4(0.8)_{\mathrm{stat}}(1.1)_{\mathrm{syst}}(0.3)_{f_{\pi,\mathrm{PDG}}}[0.2]_{\mathrm{EM\ scheme}}\ \mathrm{MeV}$$
$$f_{B^0} = 190.5(0.8)_{\mathrm{stat}}(1.0)_{\mathrm{syst}}(0.3)_{f_{\pi,\mathrm{PDG}}}[0.2]_{\mathrm{EM\ scheme}}\ \mathrm{MeV}$$
$$f_{B_s} = 230.7(0.8)_{\mathrm{stat}}(0.7)_{\mathrm{syst}}(0.2)_{f_{\pi,\mathrm{PDG}}}[0.2]_{\mathrm{EM\ scheme}}\ \mathrm{MeV}$$

- Overall uncertainty: ~0.2% for D mesons, ~0.7% for B mesons.

- NB, use $f_{\pi,\mathrm{PDG}}$ to convert to MeV, so assumes $|V_{ud}|_{0^+\to 0^+}$.

f_K/f_π





$$f_{K^+}/f_{\pi^+} = 1.196(4)\text{MeV}$$

$$f_{K^+}/f_{\pi^+} = 1.1950(^{15}_{22})[3]_{\text{EM scheme}}\ \text{MeV}$$

What's Next?

What's Left Out?

- Uncertainties are now such that any future work will need the small/tiny effects of electromagnetism and sea isospin breaking (i.e., 1+1+1+1).

- Electromagnetic effects:

  - hard electroweak, $E_{\gamma,W,Z} \gg \Lambda_{\text{QCD}}$, known;

  - structure-dependent, $E_\gamma \sim \Lambda_{\text{QCD}}$, needs lattice QCD⊕QED;

  - universal soft radiation, $E_\gamma \ll \Lambda_{\text{QCD}}$, see below.

- (Of course, whether this is needed for B mesons….)

Lattice QCD⊕QED

- Charged states are not QED-gauge invariant, so gauge fixing needed.
- Photons are massless, so finite-volume effects are power-law, not exponentially, suppressed.
- Avid area of research in lattice gauge theory.
- Rome312-Sachrajda strategy [arXiv:1711.06537 and refs. therein]:
  - $$\begin{aligned}\Gamma(K^+ \to \ell^+ \nu(\gamma)) &= \Gamma_{0\gamma} + \Gamma_{1\gamma}^{\text{pt}}(E_\gamma) \\ &= \lim_{L\to\infty} \left[\Gamma_{0\gamma}(L) - \Gamma_{0\gamma}^{\text{pt}}(L)\right]_{\text{lattice QCD}\oplus\text{ QED}} \\ &+ \lim_{\mu_\gamma\to 0} \left[\Gamma_{0\gamma}^{\text{pt}}(\mu_\gamma) + \Gamma_{1\gamma}^{\text{pt}}(E_\gamma, \mu_\gamma)\right]_{\text{O}(\alpha)}\end{aligned}$$

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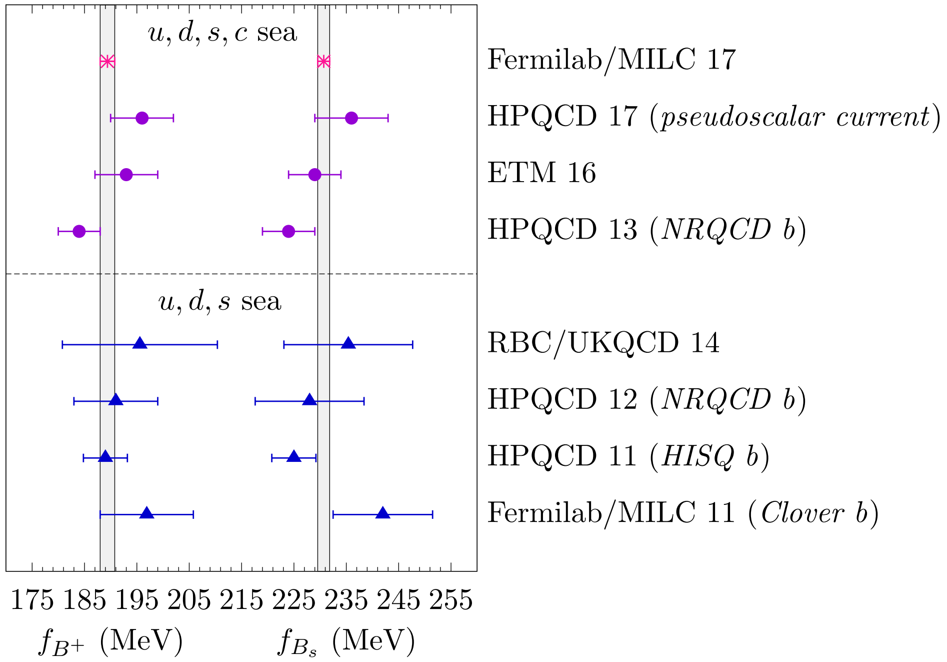
scalar QED

First Result for Leptonic Decays

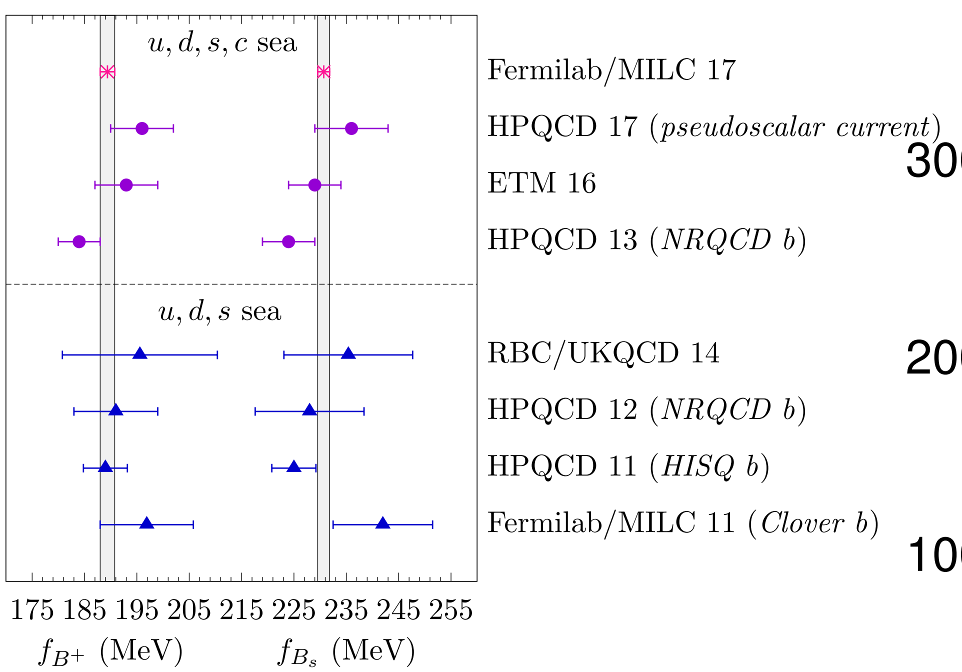
- Rome$_{312}$-Sachrajda strategy [arXiv:1711.06537 and refs. therein]:
  - $$\Gamma(K^+ \to \ell^+ \nu(\gamma)) = \Gamma_{0\gamma} + \Gamma^{\text{pt}}_{1\gamma}(E_\gamma)$$
    $$= \lim_{L\to\infty} \left[\Gamma_{0\gamma}(L) - \Gamma^{\text{pt}}_{0\gamma}(L)\right]_{\text{lattice QCD}\oplus\text{ QED}}$$
    $$+ \lim_{\mu_\gamma\to 0} \left[\Gamma^{\text{pt}}_{0\gamma}(\mu_\gamma) + \Gamma^{\text{pt}}_{1\gamma}(E_\gamma, \mu_\gamma)\right]_{\text{O}(\alpha)}$$
    $$\delta\frac{\Gamma_{K_{\ell 2}}}{\Gamma_{\pi_{\ell 2}}} = -1.22(16)\% \quad \text{vs} \quad -1.12(21)_{\chi\text{PT}}$$
  - In kaon (pion), maximum E_γ is 235 MeV (29 MeV).
- For heavy-light mesons, we'll have to worry about virtual photons with $E_\gamma \sim m_Q$.

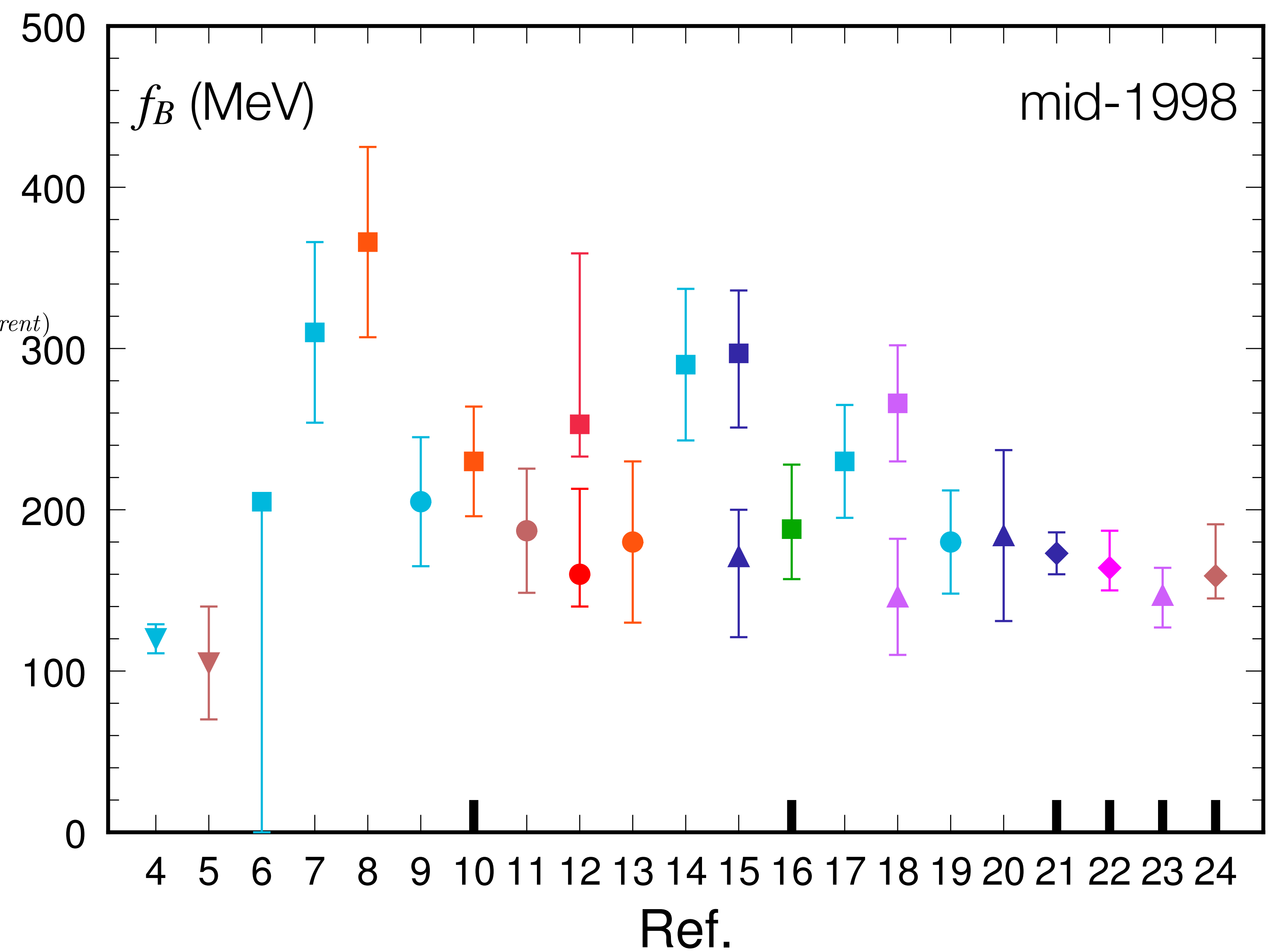
Rückblick and Outlook

Archaeology



Archaeology





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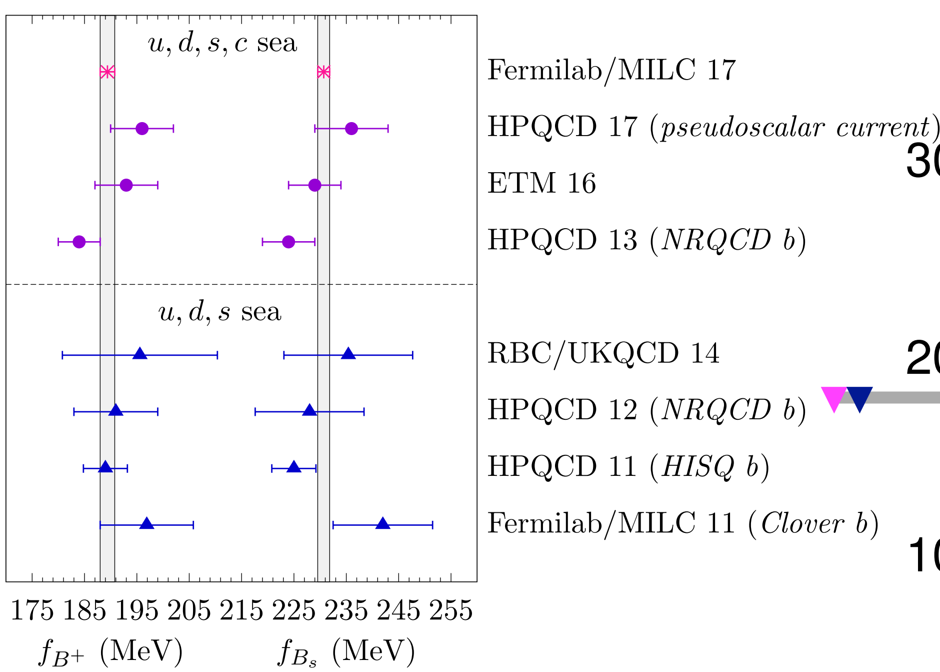

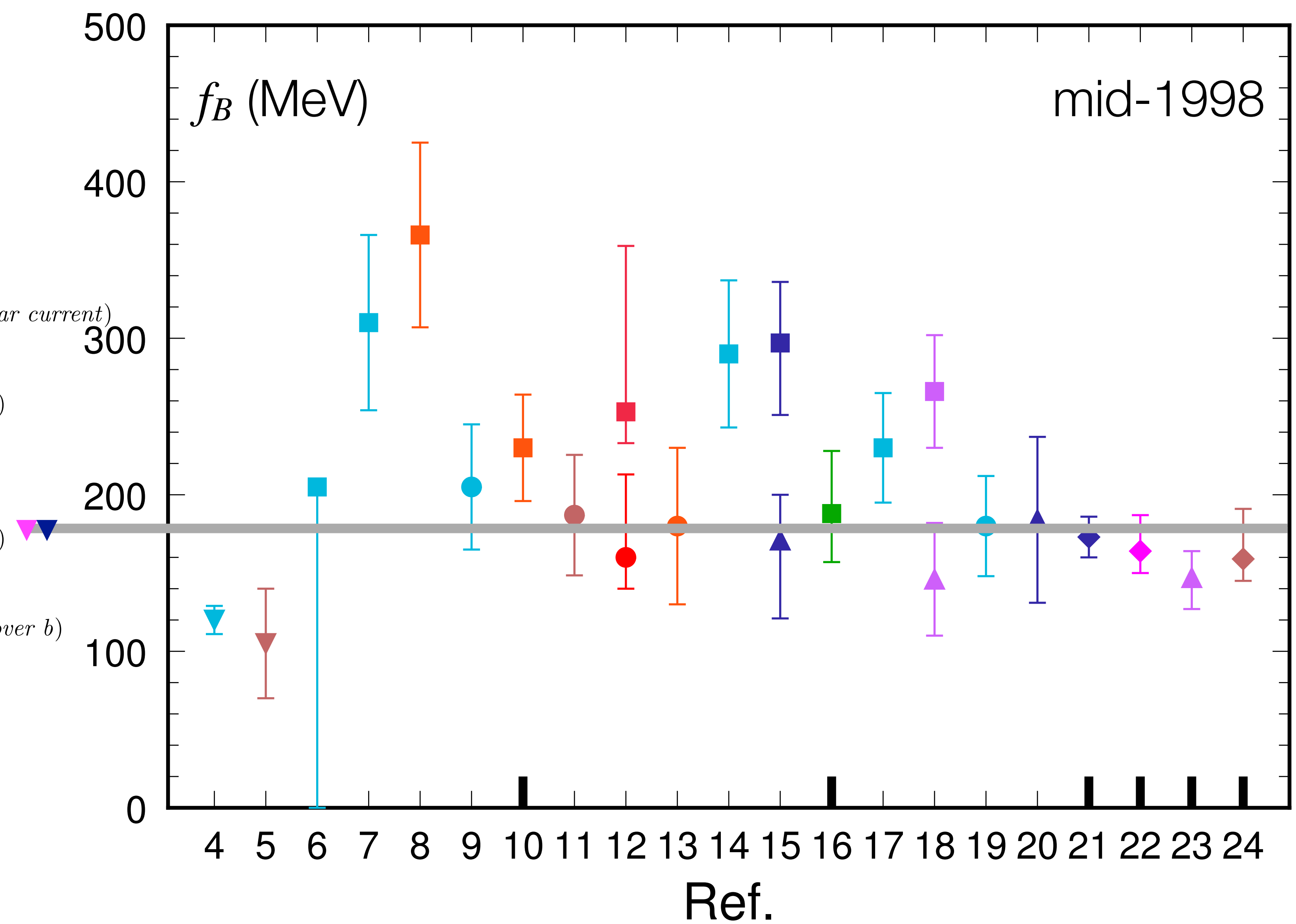

Outlook

- Decay constants are “done”?
    - QCD uncertainties (for *B* and *D*) well below experimental uncertainties;
    - independent calculations needed.
- As discussed in the experimenters’ talks, it is more interesting to focus on $B \to \ell\nu\gamma$.