Towards NNLO Monte Carlo

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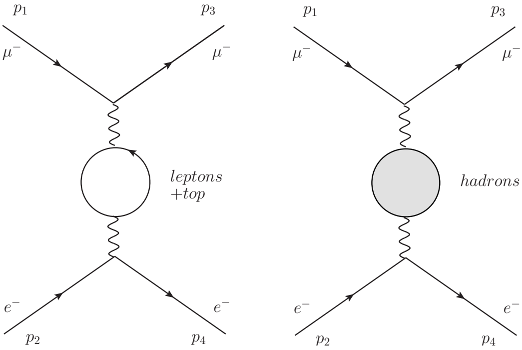
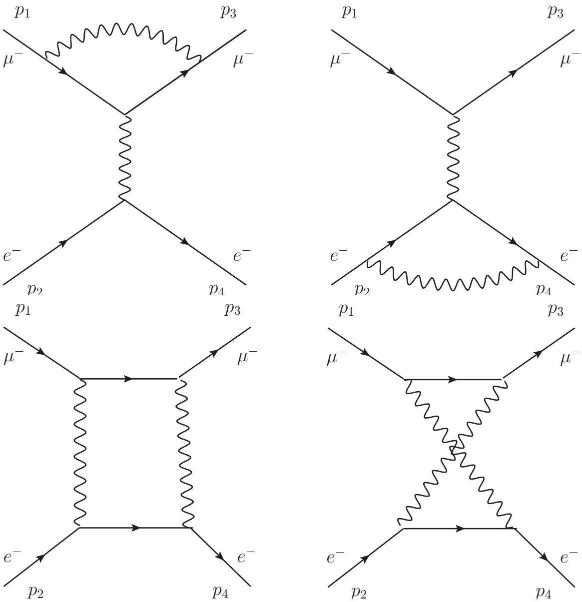
Already with NLO some useful results

- three gauge invariant classes of photonic corrections
  - radiation (virtual and real) along the μ line
  - radiation (virtual and real) along the e line
  - interference between radiation (v. and r.) from μ and e lines
- useful exercise to quantify the three classes of effects for two different (extreme) event selections
  - only detector acceptance cuts
    - $0 \leq \vartheta_{e/\mu} \leq 100$ mrad
    - $E_e \geq 200$ MeV
  - imposing additionally “elasticity” cuts
    - maximum acoplanarity cut (0.35 mrad)
    - NA7-like distance in $\vartheta_\mu - \vartheta_e$ plane from the Born-lile correlation curve (implemented for simplicity in the c.m. frame) $d < 0.5$ mrad

NLO amplitudes: vacuum polarization

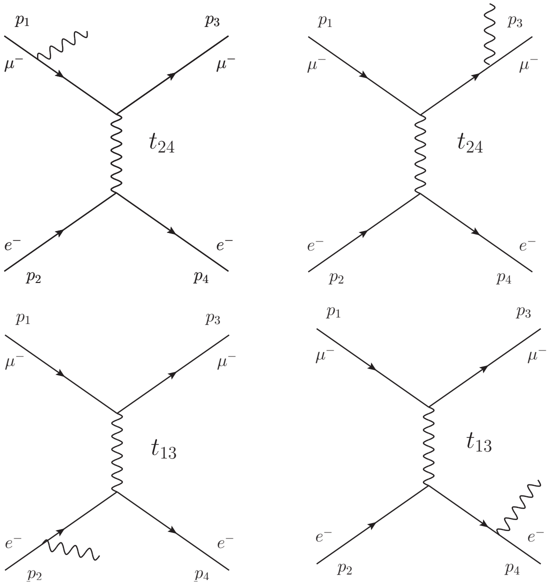


NLO QED virtual diagrams $\mathcal{A}_{NLO}^{virtual}$

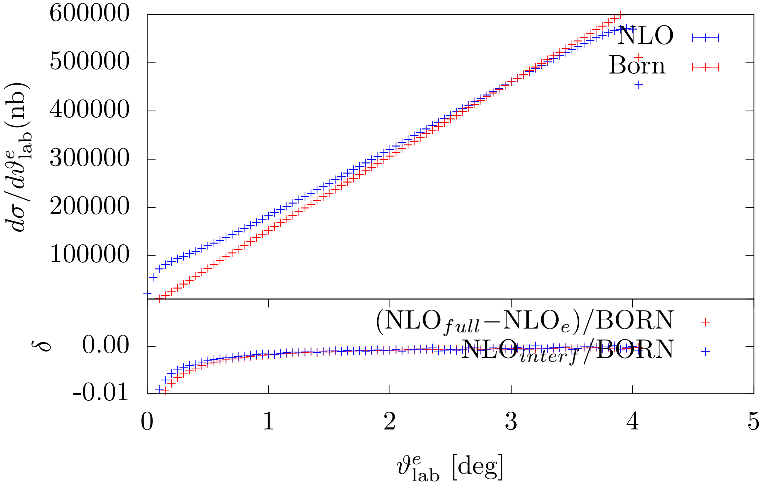


+ counterterms

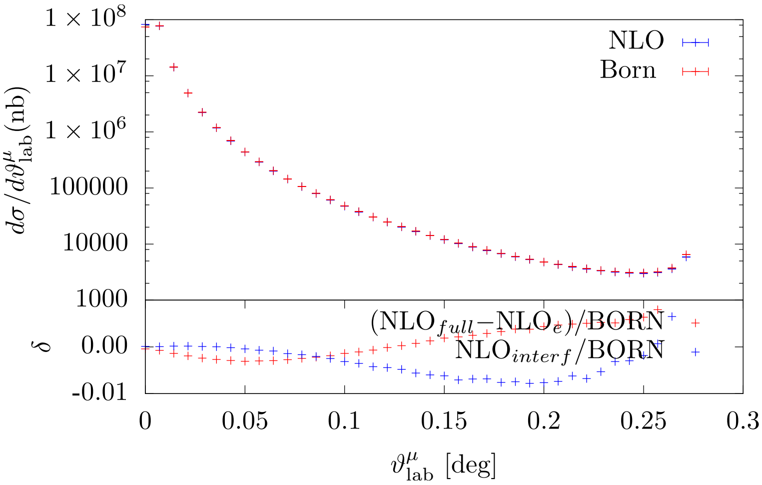
NLO real diagrams $\mathcal{A}^{1\gamma}_{NLO}$



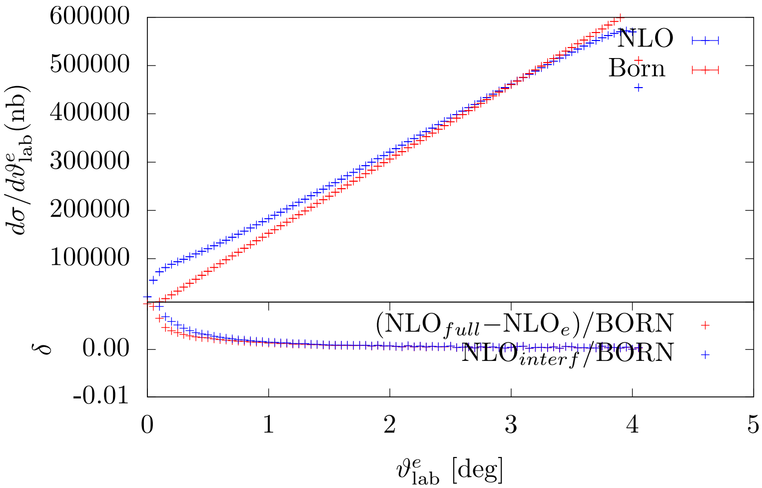
Acceptance cuts, μ^- beam, ϑ_e



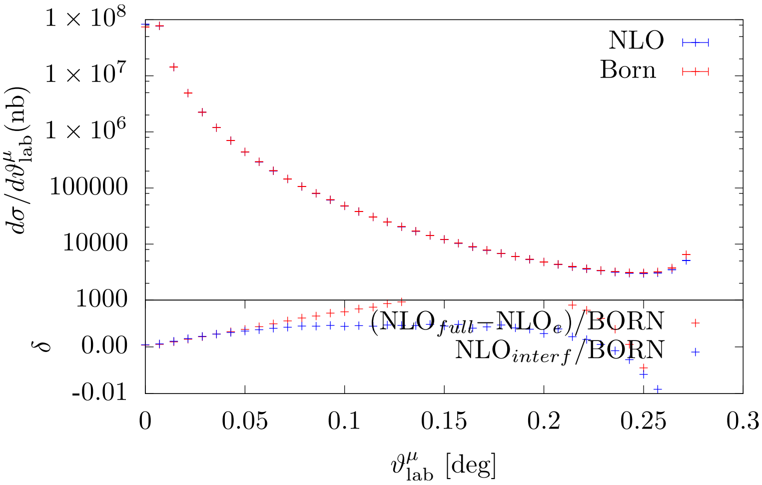
Acceptance cuts, μ^- beam, ϑ_μ



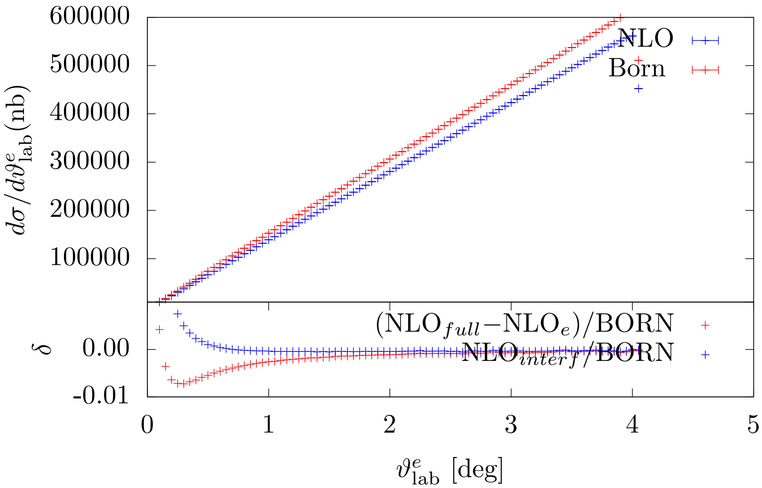
Acceptance cuts, μ^+ beam, ϑ_e



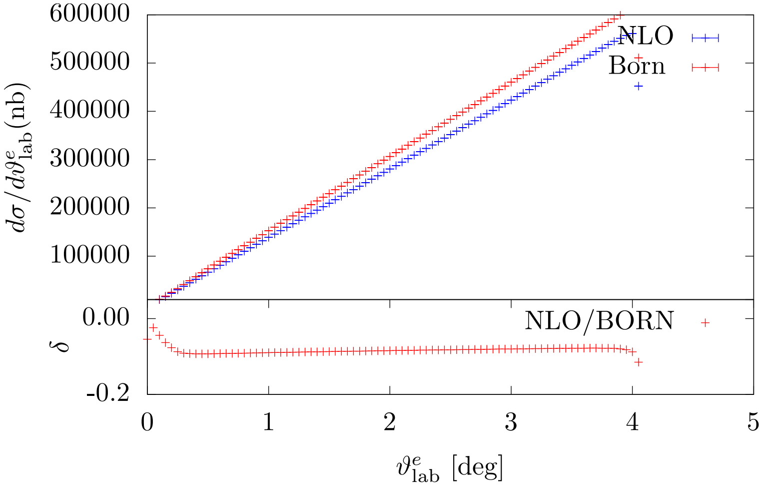
Acceptance cuts, μ^+ beam, ϑ_μ



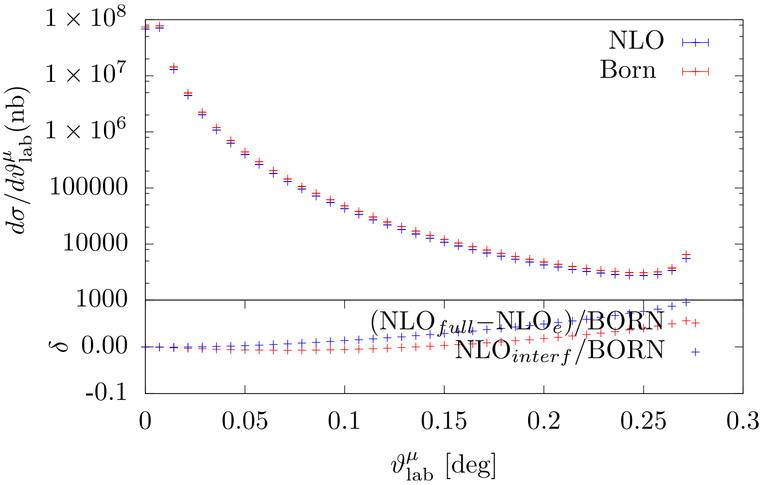
Acceptance+elasticity cuts, μ^- beam, ϑ_e



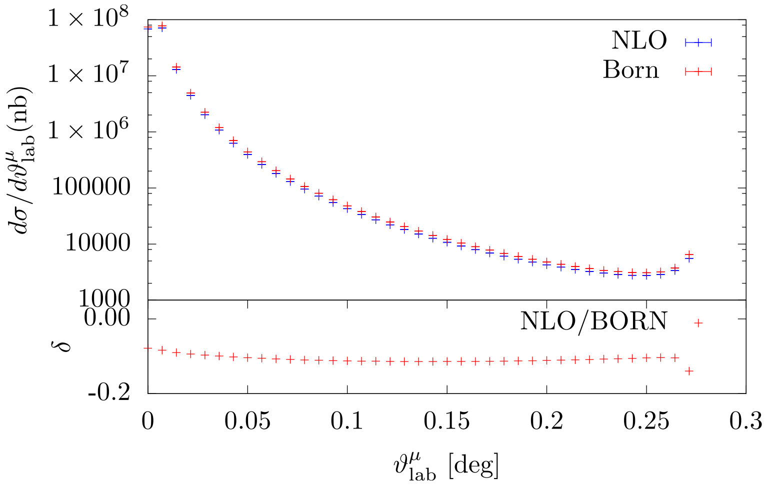
Acceptance+elasticity cuts, μ^- beam, ϑ_e



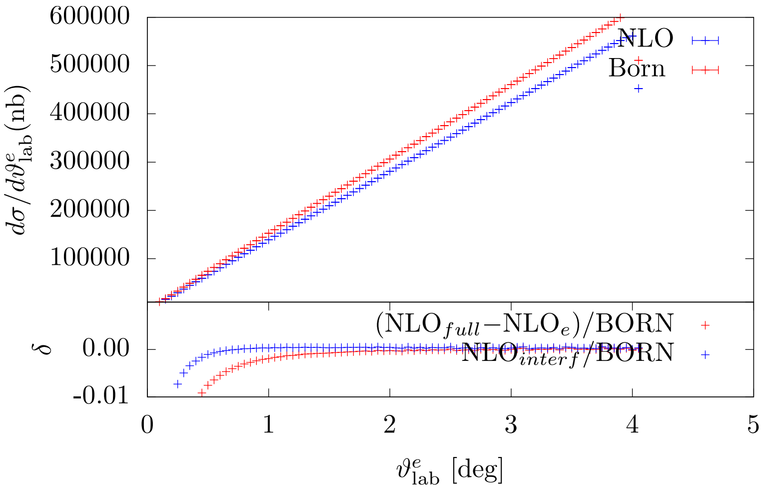
Acceptance+elasticity cuts, μ^- beam, ϑ_μ



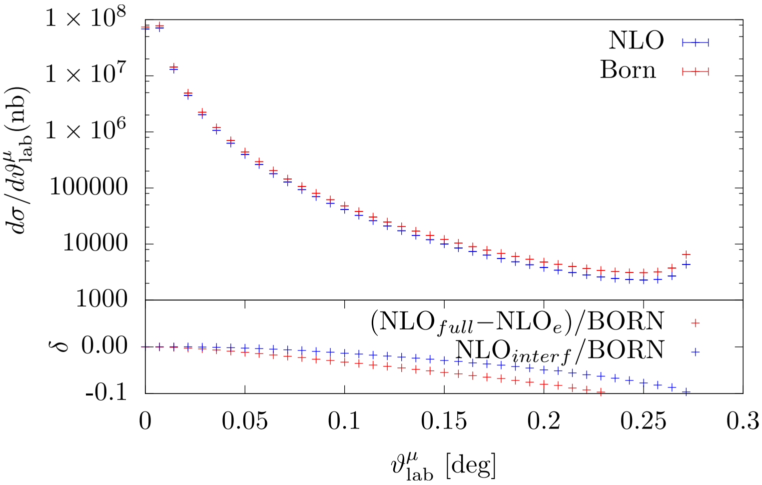
Acceptance+elasticity cuts, μ^- beam, ϑ_e



Acceptance+elasticity cuts, μ^+ beam, ϑ_e



Acceptance+elasticity cuts, μ^+ beam, ϑ_μ



two strategies to be developed

- fixed order calculation at NNLO
- matching NLO calculation with Parton Shower $\Longrightarrow$ NLOPS accuracy
- eventually build a NNLOPS event generator

- something similar (first two items) happened for Bhabha scattering
  - small angle Bhabha at LEP
  - more recently, large angle Bhabha at flavour factories

Matching NLO and PS in **BabaYaga@NLO**

Exact $\mathcal{O}(\alpha)$ (NLO) soft+virtual (SV) corrections and hard-bremsstrahlung (H) matrix elements can be combined with QED PS *via* a matching procedure

- $d\sigma^{\infty}_{LL} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$
- $d\sigma^{\alpha}_{LL} = [1 + C_{\alpha,LL}]\,|\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \equiv d\sigma^{SV}_{LL}(\varepsilon) + d\sigma^{H}_{LL}(\varepsilon)$
- $d\sigma^{\alpha}_{\rm NLO} = [1 + C_{\alpha}]\,|\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1 \equiv d\sigma^{SV}_{\rm NLO}(\varepsilon) + d\sigma^{H}_{\rm NLO}(\varepsilon)$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,LL}) \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}$

$$d\sigma^{\infty}_{\rm matched} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,LL}|^2\ d\Phi_n$$

$d\Phi_n$ is the exact phase space for $n+2$ final-state particles

Matching NLO and PS in BabaYaga@NLO

- F_{SV} and $F_{H,i}$ are infrared/collinear safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of LL
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\rm NLO}$
- resummation of higher orders LL contributions is preserved
- the cross section is still fully differential in the momenta of the final state particles (e^+, e^- and $n\gamma$)
  (F's correction factors are applied on an event-by-event basis)
- as a by-product, part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV\ |\ H,i} \times LL$

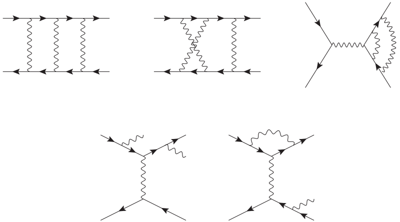
G. Montagna et al., **PLB** 385 (1996)

- the theoretical error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO, 2 loop) not infrared, singly collinear terms: very naively and roughly (for photonic corrections)

$$\frac{1}{2}\alpha^2 L \equiv \frac{1}{2}\alpha^2 \log\frac{s}{m_e^2} \sim 5\times 10^{-4}$$

NNLO Bhabha calculations

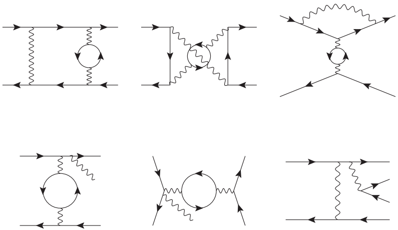
- Photonic corrections A. Penin, PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185



- Electron loop corrections

R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280

S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26

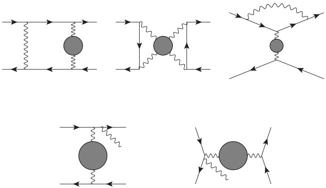


NNLO Bhabha calculations

- Heavy fermion and hadronic loops

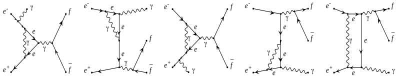
R. Bonciani, A. Ferroglia and A. Penin, PRL **100** (2008) 131601
S. Actis, M. Czakon, J. Gluza and T. Riemann, PRL **100** (2008) 131602

J.H. Kühn and S. Uccirati, Nucl. Phys. **B806** (2009) 300



- One-loop soft+virtual corrections to single hard bremsstrahlung

S. Actis, P. Mastrolia and G. Ossola, Phys. Lett. **B682** (2010) 419

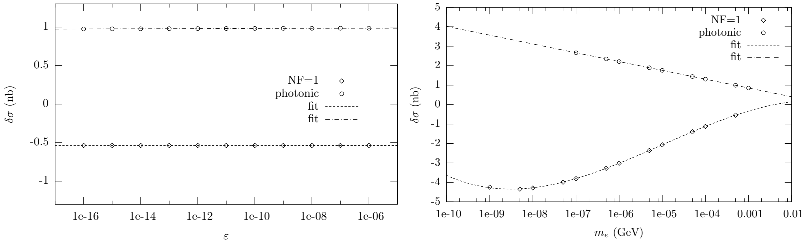


Comparison with NNLO calculation for $\sigma_{\rm SV}^{\alpha^2}$

Using realistic cuts for luminosity @ KLOE

Comparison of $\sigma_{\rm SV}^{\alpha^2}$ calculation of **BabaYaga@NLO** with

- Penin (photonic): function of the logarithm of the soft photon cut–off (left plot) and a fictitious electron mass (right plot)



★ differences are infrared safe, as expected

★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{\rm SV}^{\alpha^2}(\text{Penin}) - \sigma_{\rm SV}^{\alpha^2}(\textbf{BabaYaga@NLO}) < 0.02\% \times \sigma_0$$

Lepton and hadron loops & pairs at NNLO

- The exact NNLO soft+virtual corrections and $2 \to 4$ matrix elements $e^+e^- \to e^+e^-(l^+l^-)$ $[l = e, \mu, \tau]$, $e^+e^- \to e^+e^-(\pi^+\pi^-)$ are available
- Compared to the *approximation* in **BabaYaga@NLO**, using realistic luminosity cuts ($S_i \equiv \sigma_i^{\text{NNLO}}/\sigma_{BY}$)

	$\sqrt{s}$		σ_{BY}	$S_{e^+e^-}$ [‰]	S_{lep} [‰]	S_{had} [‰]	S_{tot} [‰]
KLOE	1.020	NNLO		-3.935(4)	-4.472(4)	1.02(2)	-3.45(2)
		BB@NLO	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.650	NNLO		-1.469(9)	-1.913(9)	-1.3(1)	-3.2(1)
		BB@NLO	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BB@NLO	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BB@NLO	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

⋆ The uncertainty due to lepton and hadron pair NNLO corrections is at the level of a few units in 10^{-4}

Carloni, Czyz, Gluza, Gunia, Montagna, Nicrosini, Piccinini, Riemann *et al.*, JHEP **1107** (2011) 126

Error budget for Bhabha luminometry

main conclusion of the Luminosity Section of the WG Report

Putting the sources of uncertainties (in large-angle Bhabha) all together:

Source of error (%)	$\Phi-$factories	$\sqrt{s}$ = 3.5 GeV	$B-$factories
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [Jegerlehner]	0.00	0.01	0.03
$\lvert\delta^{\rm err}_{\rm VP}\rvert$ [HMNT]	0.02	0.01	0.02
$\lvert\delta^{\rm err}_{{\rm SV},\alpha^2}\rvert$	0.02	0.02	0.02
$\lvert\delta^{\rm err}_{{\rm HH},\alpha^2}\rvert$	0.00	0.00	0.00
$\lvert\delta^{\rm err}_{{\rm SV,H},\alpha^2}\rvert$	0.05	0.05	0.05
$\lvert\delta^{\rm err}_{\rm pairs}\rvert$	0.03	0.016	0.03
$\lvert\delta^{\rm err}_{\rm total}\rvert$ linearly	0.12	0.1	0.13
$\lvert\delta^{\rm err}_{\rm total}\rvert$ in quadrature	0.07	0.06	0.06

★ The present error estimate appears to be rather robust and sufficient for high–precision luminosity measurements. It is comparable with that achieved for small–angle Bhabha luminosity monitoring at LEP/SLC

- For the experiments on top of and closely around the narrow resonances (J/ψ, Υ,...), the accuracy quickly deteriorates, because of the differences between the predictions of independent $\Delta\alpha^{(5)}_{\rm had}(q^2)$ parameterizations and/or their intrinsic error *[see extra slides]*

Last on $\mu \to \mathrm{e}$: weak contributions?

- Z-exchange at tree level
  - dimensionally $\sim \frac{m_\mu^2}{M_Z^2} \sim 10^{-6}$
  - checked with two independent implementations, to be well below 10^{-5} also for differential distributions

- complete NLO electroweak corrections
  - matrix elements calculated by means of RECOLA (Actis, Denner, Hofer, Scharf, Uccirati), with all finite masses
  - running code