Muon electron scattering: a NLO QED MC event generator

Massimo Alacevich

University of Pavia & INFN Pavia, Italy

with C.M. Carloni Calame, M. Chiesa,
G. Montagna O. Nicrosini, F. Piccinini

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Hadronic Contribution to the Muon Anomalous Magnetic Moment

MC event generator features

- the complete NLO QED μ-e scattering amplitudes are built in
- both μ^- and μ^+ cases can be chosen
- fully-exclusive events are generated in the CM frame and then they are boosted to the Target Rest frame
- Literature:

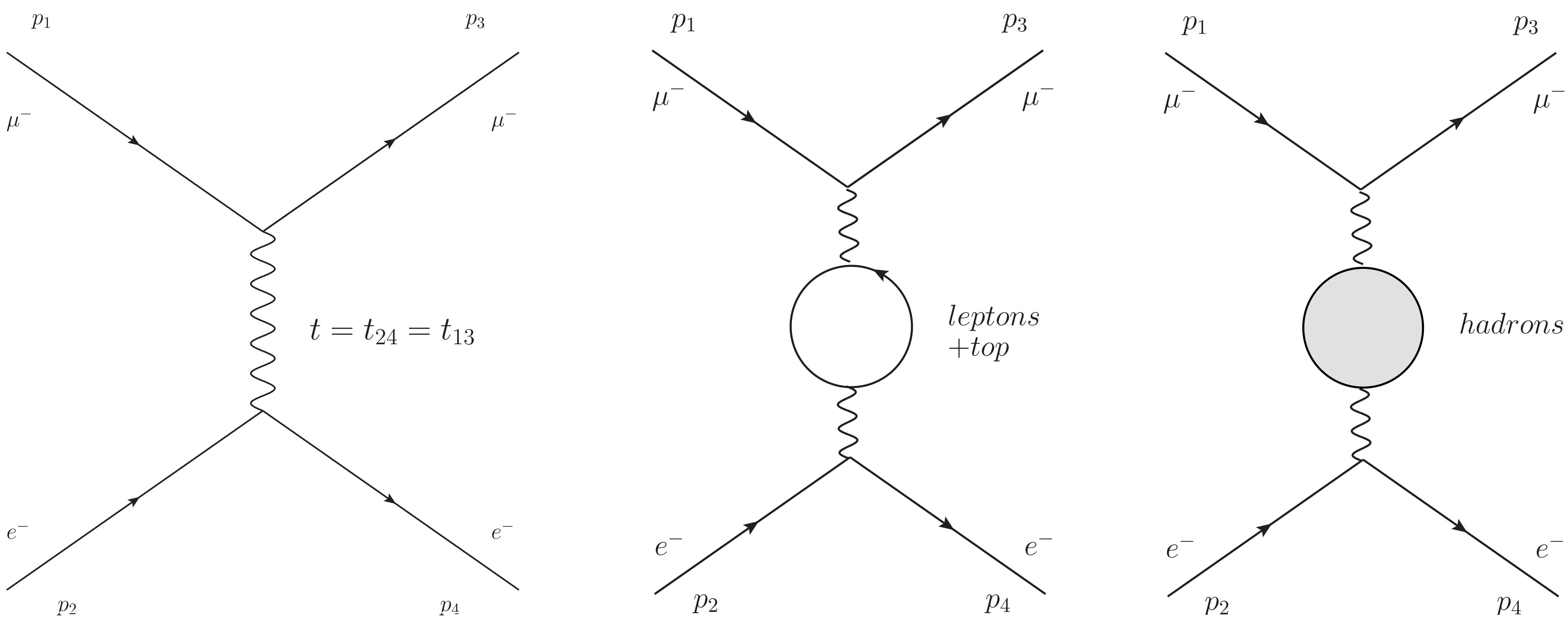
G. Balossini, C.M. Carloni Calame, G. Montagna, O. Nicrosini and F. Piccinini
Matching perturbative and parton shower corrections to Bhabha process at flavor factories Nucl. Phys. B **758** (2006) 227 - hep-ph/0607181

M. Cacciari, G. Montagna, O. Nicrosini and F. Piccinini
SABSPV: A Monte Carlo integrator for small angle Bhabha scattering
Comput. Phys. Commun. **90** (1995) 301 - hep-ph/9507245

NLO QED μ–e scattering: calculation features

- the exact and complete NLO QED amplitudes calculation is performed in the on-shell renormalization scheme
- Feynman's diagrams have been manipulated with the help of FORM, rewriting 2-3-4 points function of 1-loop integrals as Lorentz-covariant tensor coefficients (J.Vermaseren, https://nikhef.nl/~form)
- tensors coefficients are decomposed into a combination of scalar coefficient functions, which are evaluated numerically with LOOPTOOLS libraries (T.Hahn, https://www.feynarts.de/looptools)
- full μ and e mass dependency is kept
- fermions' helicity is kept explicit
- IR singularities are regularized with the introduction of a vanishingly small photon mass λ

NLO QED Amplitudes



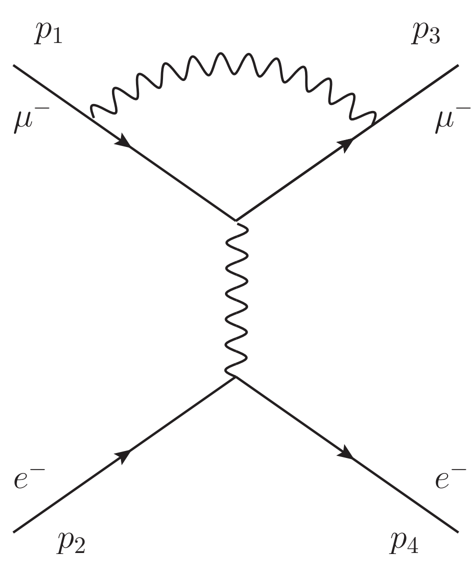
Born — **NLO Vacuum Polarization**

$\mathcal{M}_0$ $\qquad$ $\mathcal{M}_{VP}^{lep+top}$ $\qquad$ $\mathcal{M}_{VP}^{had}$

NLO QED Amplitudes

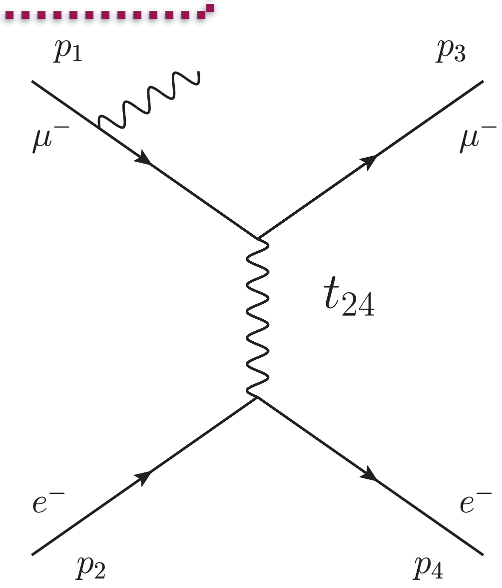
▶ muon line

virtual correction

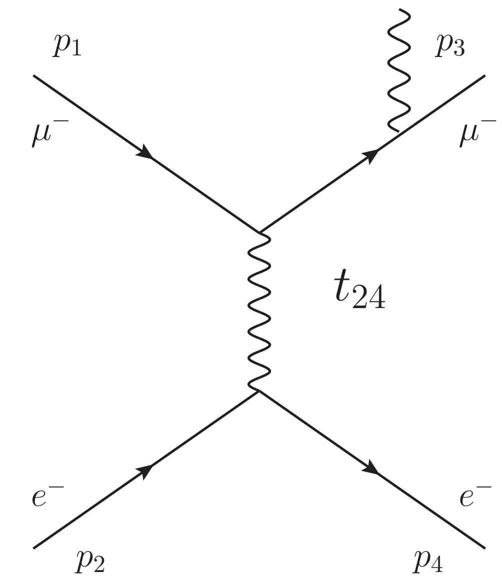


$\mathcal{M}_{V_\mu}$

real radiation



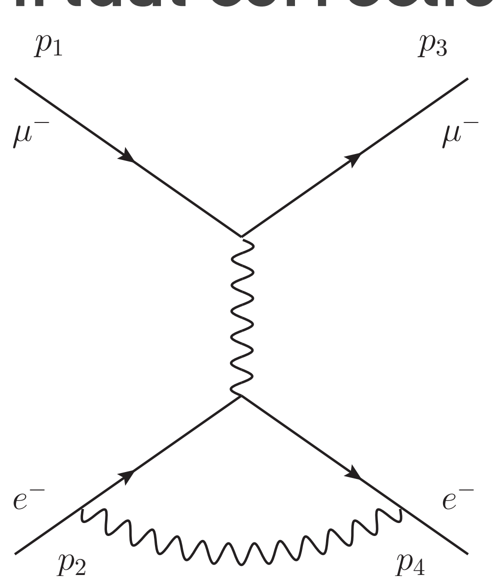
$\mathcal{M}_{R_1}$



$\mathcal{M}_{R_3}$

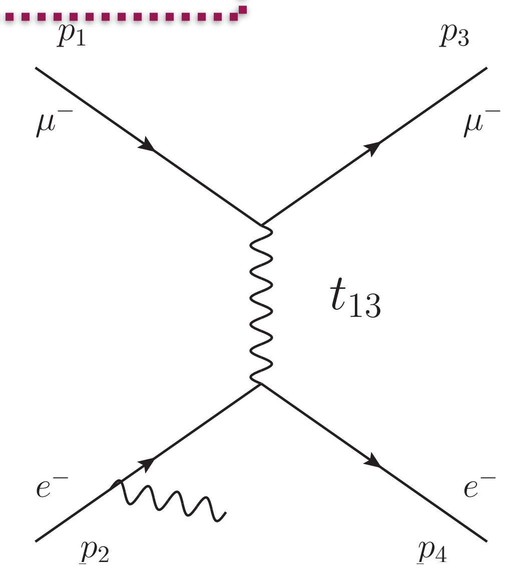
▶ electron line

virtual correction

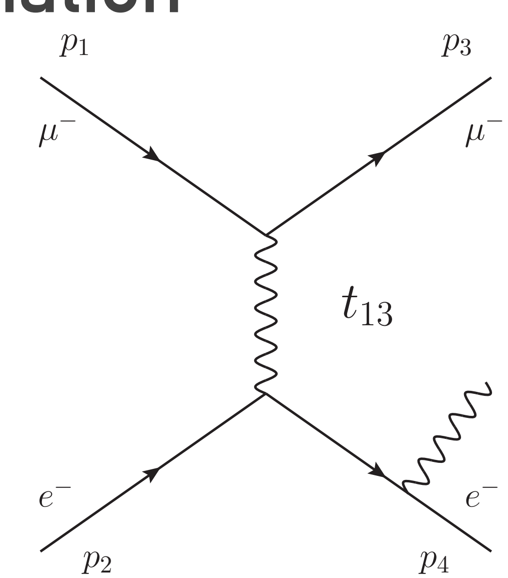


$\mathcal{M}_{V_e}$

real radiation



$\mathcal{M}_{R_2}$

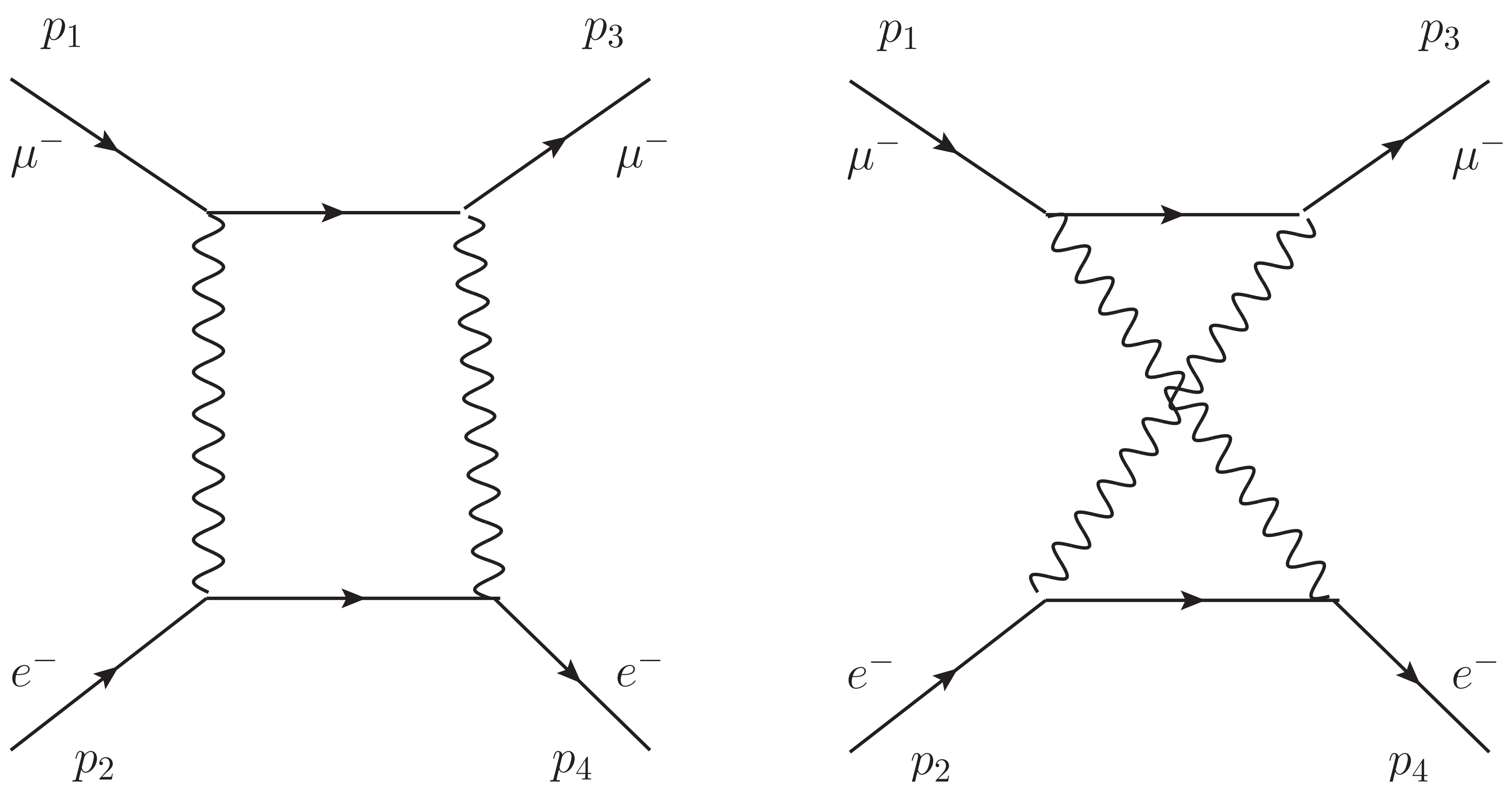


$\mathcal{M}_{R_4}$

NLO QED Amplitudes

▶ box diagrams

virtual correction



$\mathcal{M}_X$ $\mathcal{M}_{XX}$

NLO calculation: details

the total cross section can be divided into two contribution by the different number of final state particles

$$\sigma^{NLO} = \sigma_{\mu e} + \sigma_{\mu e\gamma} = \sigma_{2\to 2} + \sigma_{2\to 3}$$

$$\sigma_{2\to 2} = \sigma^{LO} + \sigma^{NLO}_{virtual} = \frac{1}{F}\int d\Phi_2 \left\{ \left|\mathcal{A}_{LO}\right|^2 + 2\,Re\left[\mathcal{A}^*_{LO} \times \mathcal{A}^{virtual}_{NLO}(\lambda)\right] \right\}$$

$$\sigma_{2\to 3} = \frac{1}{F}\int_{E_\gamma > \lambda} d\Phi_3 \left|\mathcal{A}^{real}_{NLO}\right|^2$$

the integration over the real contribution can be done using slicing techniques with soft-limit approximation by the introduction of an arbitrary small photon energy cutoff ω_s

$$\sigma_{2\to 3} = \frac{1}{F}\int_{\lambda < E_\gamma < \omega_s} d\Phi_3 \left|\mathcal{A}^{real}_{NLO}\right|^2 + \frac{1}{F}\int_{E_\gamma > \omega_s} d\Phi_3 \left|\mathcal{A}^{real}_{NLO}\right|^2$$

$$= \Delta_s(\lambda, \omega_s)\frac{1}{F}\int d\Phi_2 \left|\mathcal{A}_{LO}\right|^2 + \frac{1}{F}\int_{E_\gamma > \omega_s} d\Phi_3 \left|\mathcal{A}^{real}_{NLO}\right|^2$$

soft factorization

NLO calculation: radiation setup

NLO muon-line correction and NLO electron-line correction are 2 different gauge-invariant subset of the whole total cross-section

- μ radiation setup

$$|\mathcal{A}_{LO}|^2 = |\mathcal{M}_0|^2$$

$$2\,Re\big[\mathcal{A}^*_{LO} \times \mathcal{A}^{virtual}_{NLO}\big] = 2\,Re\big[\mathcal{M}^*_0 \times (\mathcal{M}^{lep+top}_{VP} + \mathcal{M}^{had}_{VP} + \mathcal{M}_{V_\mu})\big]$$

$$\begin{aligned} |\mathcal{A}^{real}_{NLO}|^2 &= |\mathcal{M}_{R_1} + \mathcal{M}_{R_3}|^2 \\ &= |\mathcal{M}_{R_1}|^2 + |\mathcal{M}_{R_3}|^2 + 2\,Re\big[(\mathcal{M}^*_{R_1} \times \mathcal{M}_{R_3})\big] \end{aligned}$$

NLO calculation: radiation setup

NLO muon-line correction and NLO electron-line correction are 2 different gauge-invariant subset of the whole total cross-section

▸ e radiation setup

$$|\mathcal{A}_{LO}|^2 = |\mathcal{M}_0|^2$$

$$2\,Re\left[\mathcal{A}_{LO}^* \times \mathcal{A}_{NLO}^{virtual}\right] = 2\,Re\left[\mathcal{M}_0^* \times (\mathcal{M}_{VP}^{lep+top} + \mathcal{M}_{VP}^{had} + \mathcal{M}_{V_e})\right]$$

$$\begin{aligned} |\mathcal{A}_{NLO}^{real}|^2 &= |\mathcal{M}_{R_2} + \mathcal{M}_{R_4}|^2 \\ &= |\mathcal{M}_{R_2}|^2 + |\mathcal{M}_{R_4}|^2 + 2\,Re\left[(\mathcal{M}_{R_2}^* \times \mathcal{M}_{R_4})\right] \end{aligned}$$

NLO calculation: radiation setup

NLO muon-line correction and NLO electron-line correction are 2 different gauge-invariant subset of the whole total cross-section

▶ μ-e radiation setup

$$\left|\mathcal{A}_{LO}\right|^2 = \left|\mathcal{M}_0\right|^2$$

$$\begin{aligned} 2\,Re\left[\mathcal{A}^*_{LO} \times \mathcal{A}^{virtual}_{NLO}\right] &= 2\,Re\left[\mathcal{M}^*_0 \times (\mathcal{M}^{lep+top}_{VP} + \mathcal{M}^{had}_{VP})\right] + \\ &+ 2\,Re\left[\mathcal{M}^*_0 \times (\mathcal{M}_{V_\mu} + \mathcal{M}_{V_e} + \mathcal{M}_X + \mathcal{M}_{XX})\right] \end{aligned}$$

$$\begin{aligned} \left|\mathcal{A}^{real}_{NLO}\right|^2 &= \left|\mathcal{M}_{R_1} + \mathcal{M}_{R_2} + \mathcal{M}_{R_3} + \mathcal{M}_{R_4}\right|^2 = \\ &= \left|\mathcal{M}_{R_1} + \mathcal{M}_{R_3}\right|^2 + \left|\mathcal{M}_{R_2} + \mathcal{M}_{R_4}\right|^2 + \\ &+ 2\,Re\left[(\mathcal{M}^*_{R_1} + \mathcal{M}^*_{R_3}) \times (\mathcal{M}_{R_2} + \mathcal{M}_{R_4})\right] \end{aligned}$$

Numerical cross-checks

- Feynman's amplitudes have been calculated independently by at least two of us
- both LOOPTOOLS and COLLIER libraries have been used to evaluate 1-loop tensor coefficients and scalar 2-3-4 points function in the NLO calculations (A.Denner, S.Dittmaier, L. Hofer, https://collier.hepforge.org)
- UV finiteness, λ and ω_s independence are verified with high numerical accuracy
- 3 independent implementation of the 3-body phase-space have been done to cross-check the final results

▶ **perfect agreement !**

Conclusions

- the comparison with other independent results gives a high-accuracy perfect agreement (M. Fael, M. Passera et al.)
- 3 different setup for NLO event generation are available:
  1. only μ radiation,
  2. only e radiation,
  3. both μ-e radiation
- any type of experimental cut can be implemented and reproduced through the fully-exclusive event generation

Thank you !

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