Monte Carlo generators for flavour factories:

Updates on the Phokhara and Ekhara event generators

H. CZYŻ, IF, UŚ, Chorzów, and Helmholtz Institute, Mainz

The Evaluation of the Leading Hadronic Contribution to the Muon Anomalous Magnetic Moment ,
MITP, Mainz, February 2018

Outline

⇒ PHOKHARA and EKHARA in brief

⇒ Recent developments in PHOKHARA and EKHARA

- ⇒ χ_{c_i} production: PHOKHARA and EKHARA
- ⇒ Models: $\mathcal{L}_{\gamma\gamma P}$, $\mathcal{L}_{\gamma V}$,$\mathcal{L}_{V\gamma P}$, $\mathcal{L}_{VVP}$
- ⇒ PHOKHARA: $e^+e^- \to P\gamma(\gamma)$
- ⇒ EKHARA: $e^+e^- \to e^+e^-P$
- ⇒ $a_\mu(P)$

⇒ Radiative corrections in EKHARA

⇒ Radiative corrections in PHOKHARA

MC for $e^+e^- \to e^+e^-$ -luminosity

BHWIDE

- **YFS exponentiation**

[S. Jadach, W. Placzek and B. Ward]

Phys.Lett. B390 (1997) 298-308

MCGPJ

- **ISR, structure functions, e^+e^-, $\mu^+\mu^-$, $\pi^+\pi^-$, K^+K^-, $\bar{K}^0K^0$**

[A. Arbuzov, G. Fedotovich, F. Ignatov, E. Kuraev and A. Sibidanov]

Eur.Phys.J. C46 (2006) 689-703

BABAYAGA@NLO

- **parton showers, NLO,NNLO**

[G. Balossini, C. M. Carloni Calame, G. Montagna, O. Nicrosini and F. Piccinini]

Nucl. Phys. B758 (2006)

MC for $e^+e^- \to hadrons(muons)$

KKMC

- ISR,YFS exponentiation, muons, 'hadrons'

[S. Jadach, B. Ward and Z. Was]

Comp.Phys. Comm. 130 (2000) 260;Phys. Rev. D 63(2001)113009

MCGPJ

- ISR, structure functions, e^+e^-, $\mu^+\mu^-$, $\pi^+\pi^-$, K^+K^-, $\bar{K}^0K^0$

[A. Arbuzov, G. Fedotovich, F. Ignatov, E. Kuraev and A. Sibidanov]

Eur.Phys.J. C46 (2006) 689-703

PHOKHARA 8.0

- ISR, fixed order (NNLO), muons, hadrons

[H.Czyz, M. Gunia, J.H. Kuhn]

JHEP08(2013)110.

MC for $e^+e^- \to hadrons(muons)$

LUARLW

- adopted LUND model

[H. Hu, B. Anderson]

hep-ph/9910285

ZRC

- structure functions, narrow resonances

[D. Zhang, G. Rong, J.C. Chen]

Phys.Rev. D74 (2006) 054012

carlomat_3.1

- LO, CHPT, multi-hadronic states, ISR,FSR,automatic code generation

[F. Jegerlehner, K.Kolodziej]

CPC(2015)563.Eur.Phys.J. C77 (2017) no.4, 254

MC for $\gamma^{(*)} - \gamma^{(*)}$

no name

- $e^+e^- \rightarrow e^+e^-\pi^0\pi^0$, DAPHNE energy

[F. Nguyen,F.Piccinini,A.D.Polosa]

Eur.Phys.J. C47 (2006) 65

TwoGam

- ??

[D.Coffman, V.Savinov]

CLEO

TREPS

- equivalent photon approximation

[S. Uehara]

KEK-REPORT-96-11, arXiv:1310.0157

MC for $\gamma^{(*)} - \gamma^{(*)}$

GALUGA

- $e^+e^- \to e^+e^-X$, LEP energy

[G.A.Schuler]

Comput.Phys.Commun. 108 (1998) 279

EKHARA

- exact matrix element at LO

[H.Czyz,S.Ivashyn,E.Nowak]

Comput.Phys.Commun. 182 (2011) 1338

GGRESRC

- approximated radiative correction

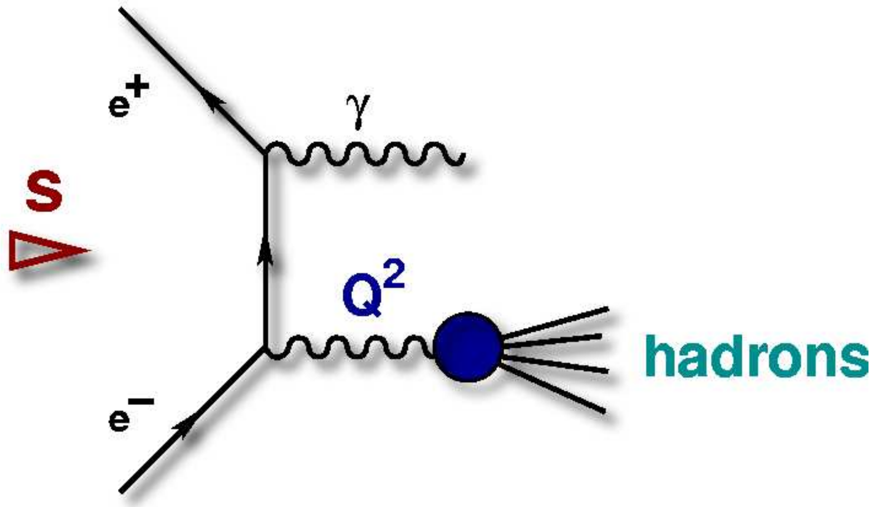
[Druzhinin, V. P. and Kardapoltsev, L. V. and Tayursky, V.A.]

Comput.Phys.Commun. 185 (2014) 236

THE RADIATIVE RETURN METHOD

$$d\sigma(e^+e^- \to \text{hadrons} + \gamma(ISR)) =$$

$$H(Q^2, \theta_\gamma)\; d\sigma(e^+e^- \to \text{hadrons})(s = Q^2)$$



- measurement of $R(s)$ over the full range of energies, from threshold up to $\sqrt{s}$
- large luminosities of factories compensate α/π from photon radiation
- radiative corrections essential (NLO,...)

High precision measurement of the hadronic cross-section at meson-factories

NLO-ISR vs. SF

Structure functions to be used carefully

	$\sqrt{s}$ =1.02 GeV	4 GeV	10.6 GeV
Born	2.1361 (4)	0.12979 (3)	0.011350 (3)
SF	2.0192 (4)	0.12439 (5)	0.010526 (3)
NLO (1)	2.0332 (5)	0.12526 (5)	0.010565 (4)
NLO (2)	2.4126 (7)	0.14891 (9)	0.012158 (9)

	$s^{1/2} = 1.02\,\mathrm{GeV}$	$4\,\mathrm{GeV}$	$10.6\,\mathrm{GeV}$
E_γ^{min} (GeV)	0.01	0.1	1
θ_γ (degrees)	$[5, 21]$	$[10, 170]$	$[25, 155]$
θ_π (degrees)	$[55, 125]$	$[20, 160]$	$[30, 150]$
$M^2_{\pi^+\pi^-\gamma}$ (GeV^2)	0.9	12	90

PHOKHARA MC generator

EVA: $e^+e^- \to \pi^+\pi^-\gamma$

- tagged photon ($\theta_\gamma > \theta_{cut}$)
- ISR at LO + Structure Function
- FSR: point-like pions

[Binner et al.]

$e^+e^- \to 4\pi + \gamma$

- ISR at LO + Structure Function

[Czyż, Kühn,2000]

F. Campanario, H.C., J. Gluza,

A. Grzelińska, M. Gunia, P. Kisza,

J. H. Kühn, E. Nowak-Kubat, T. Riemann,

G. Rodrigo, Sz. Tracz, A. Wapienik,

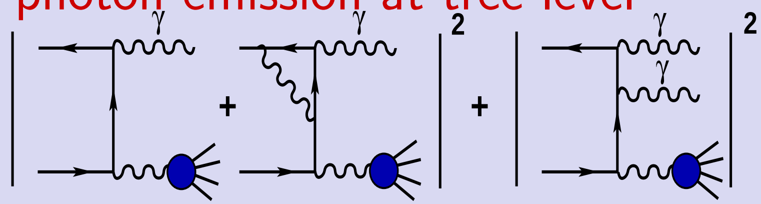
V. Yundin, D. Zhuridov

PHOKHARA 9.3: $\pi^+\pi^-$,$\mu^+\mu^-$,

4π, $\bar{N}N$, 3π, KK,$\Lambda\bar{\Lambda}$, $P\gamma$

$J/\psi, \psi(2S)$, χ_{c1}, χ_{c2}

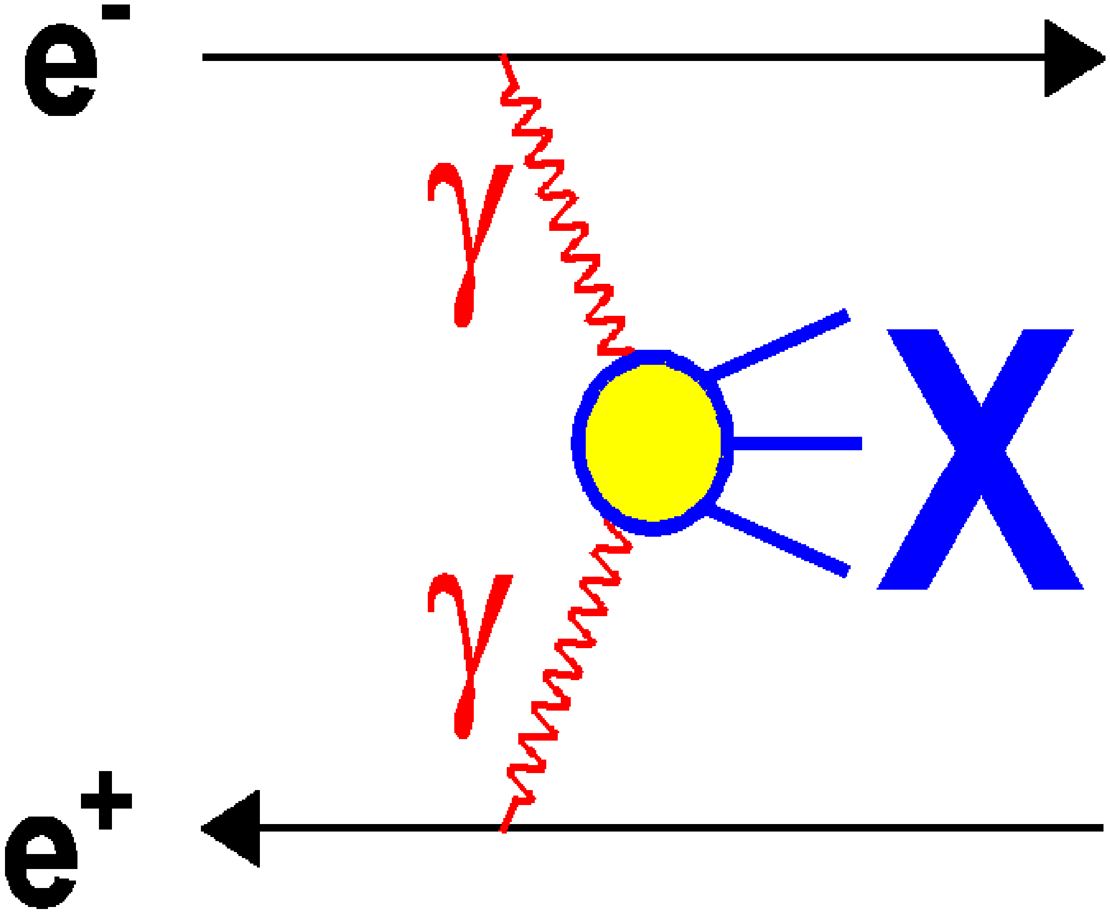
- **ISR at NLO:** virtual corrections to one photon events and two photon emission at tree level



- FSR at NLO:$\pi^+\pi^-$, $\mu^+\mu^-$,K^+K^-, $\bar{p}p$
- tagged or untagged photons
- $e^+e^- \to hadrons$ ($muons$) ISR at NNLO
- Modular structure

http://ific.uv.es/~rodrigo/phokhara/

Photon-photon interactions



EKHARA MC generator

1.0:
$e^+e^- \to \pi^+\pi^-e^+e^-$

- background to $e^+e^- \to \pi^+\pi^-\gamma$
- Henryk Czyż, Elżbieta Nowak-Kubat, Phys. Lett. B 634, 493 (2006),

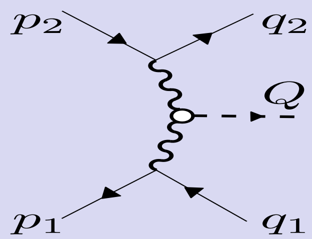
2.1: $e^+e^- \to \pi^0e^+e^-$

- Henryk Czyż, Sergiy Ivashyn, Com.Phys.Commun. 182 (2011) 1338

+ A. Korchin, O. Shekhovtsova, P. Kisza

↗

EKHARA 2.2: $\pi^+\pi^-$, π^0, η, η', χ_{c_i}, $\chi_{c_i} \to J/\psi(\to \mu^+\mu^-)\gamma$



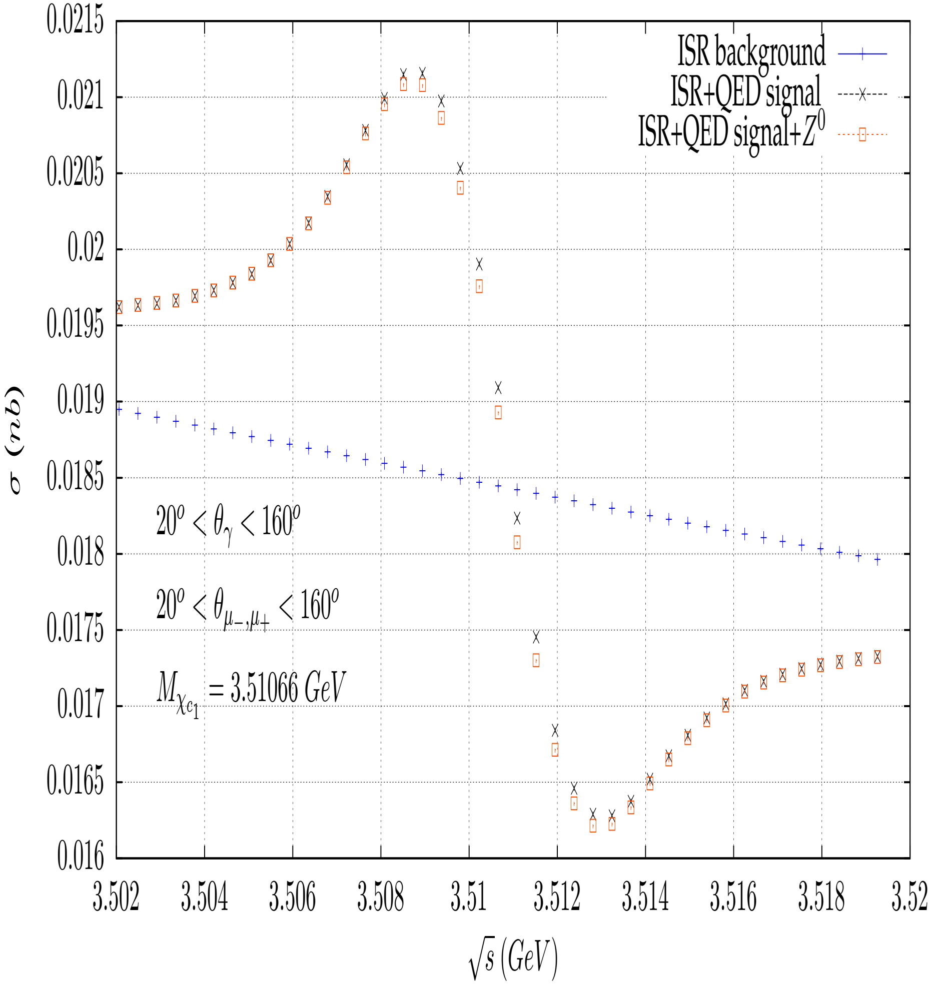
- Modular structure
- radiative correction to be included soon

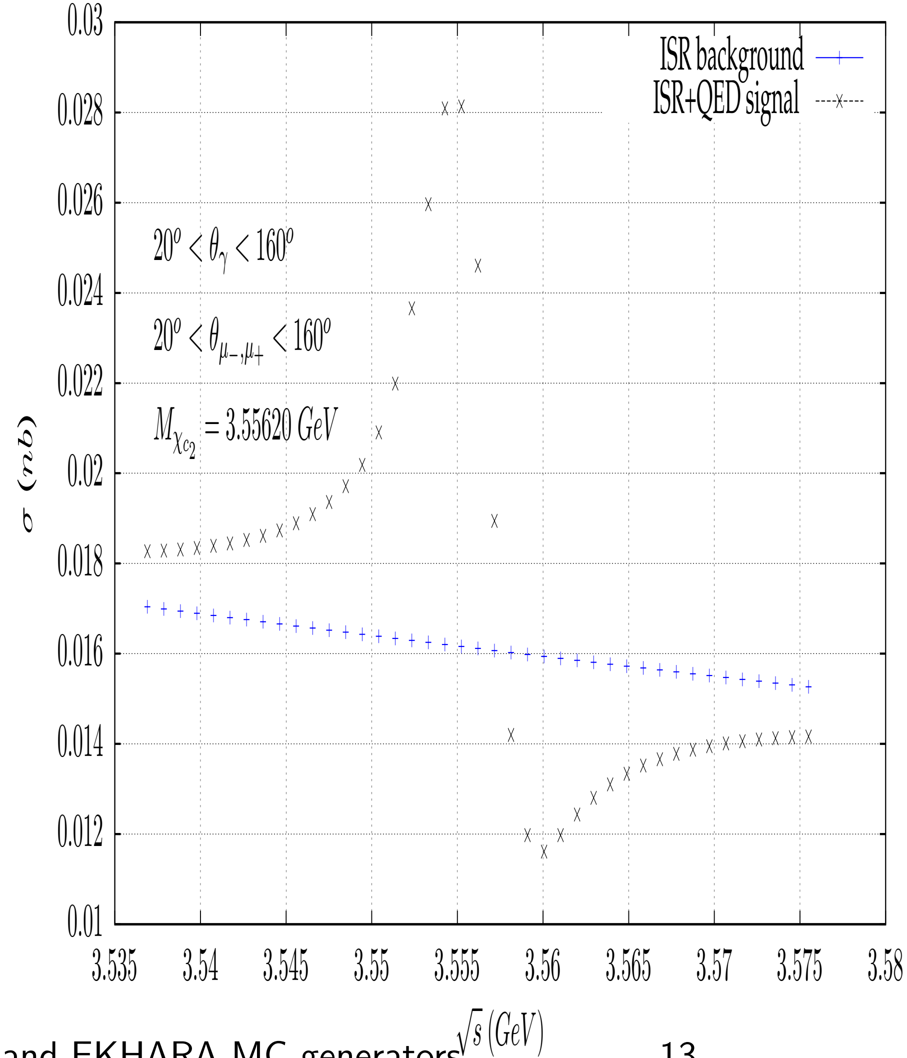
http://prac.us.edu.pl/˜ ekhara/

χ_{c1} and χ_{c2} production at e^+e^- colliders.

H. Czyż, J. H. Kühn, Sz. Tracz, Phys. Rev. D94 (2016), 034033

$$e^+e^- \to \mu^+\mu^-\gamma,\ e^+e^- \to \chi_{c_i}(\to J/\psi(\to \mu^+\mu^-)\gamma)$$

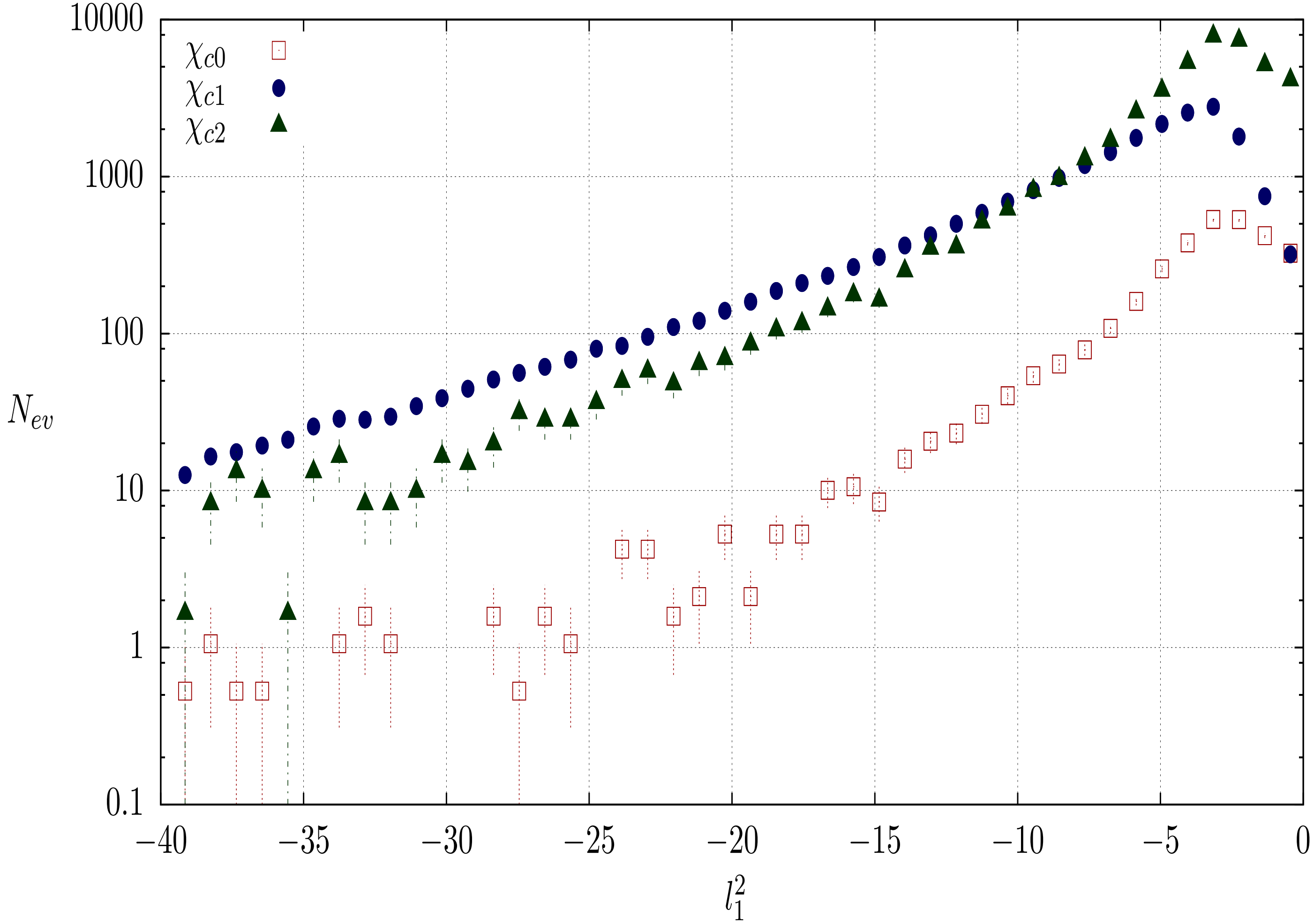




BELLE II event rates

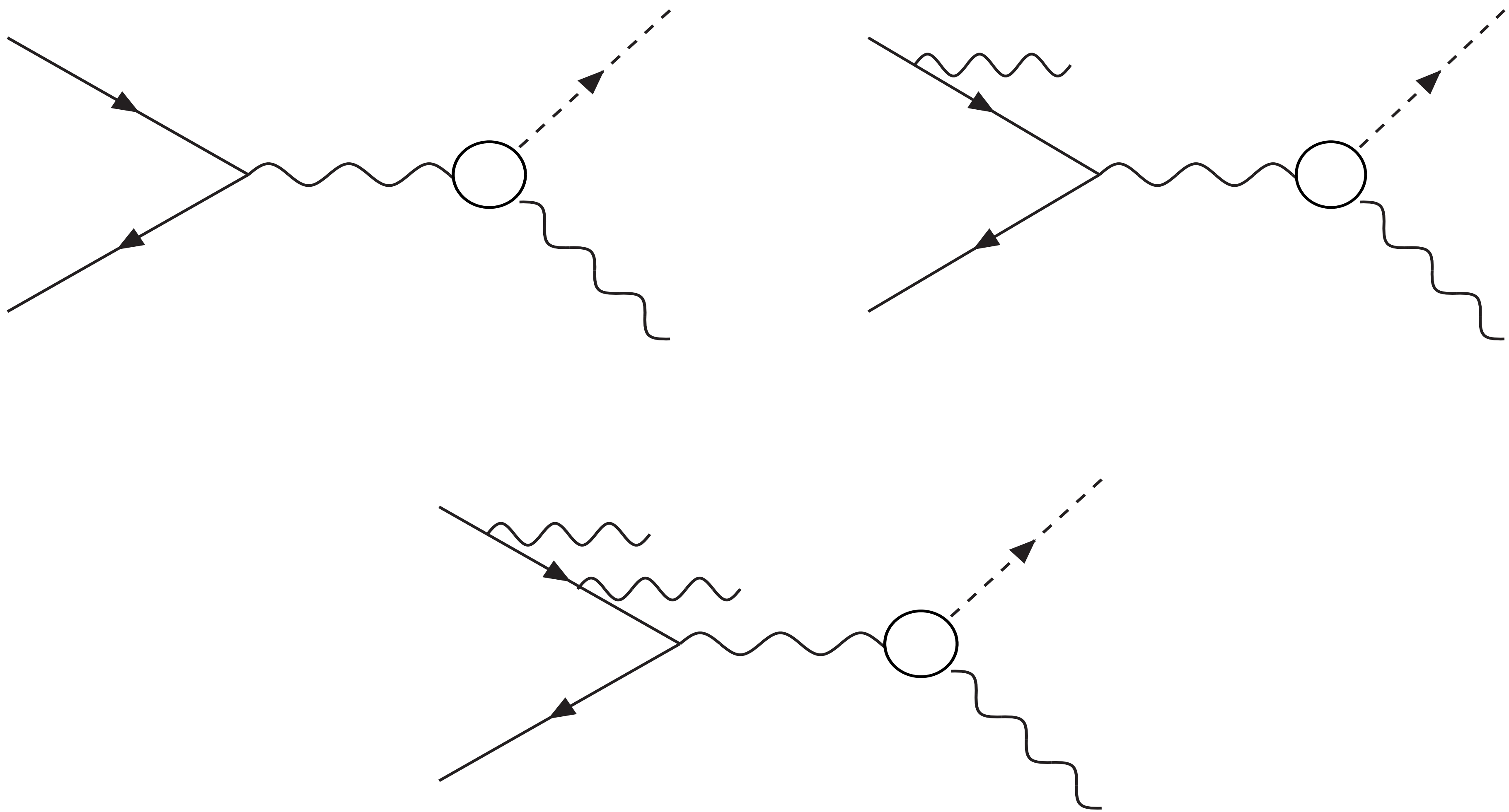
H. Czyż, P. Kisza, Phys.Lett. B771 (2017) 487

$$e^+e^- \to e^+e^-\chi_{c_i}(\to J/\psi(\to \mu^+\mu^-)\gamma)$$



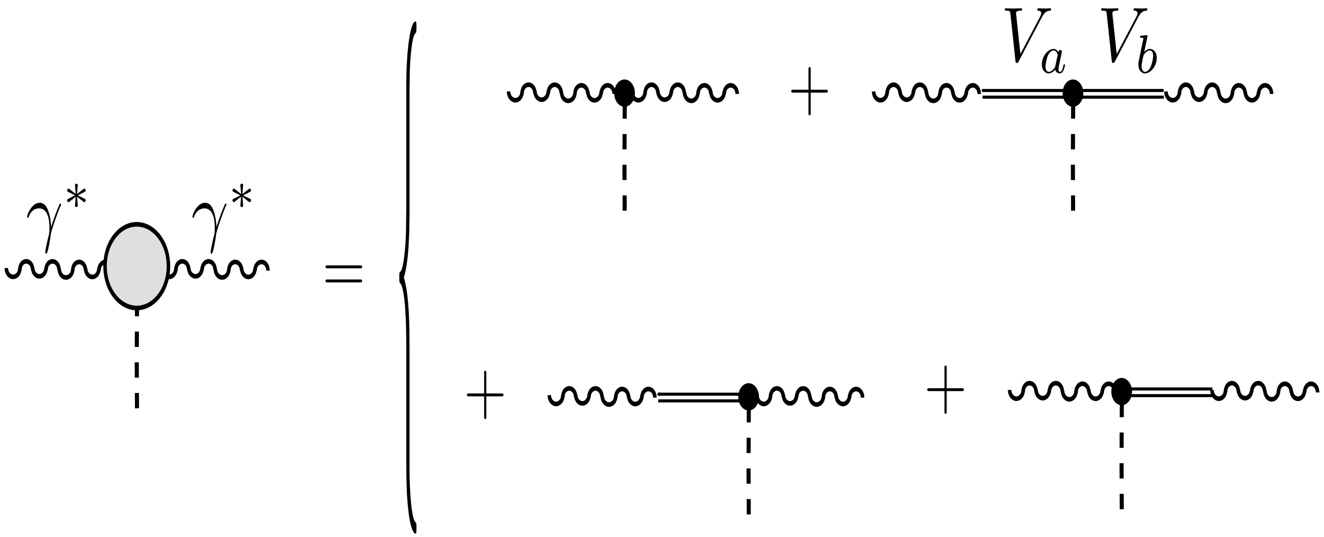
PHOKHARA: $e^+e^- \to P\gamma(\gamma)$

H. Czyż, P.Kisza, Sz. Tracz, Phys.Rev. D97 (2018), 016006



The $P - \gamma^* - \gamma^*$ form factors

H.C., S. Ivashyn, A. Korchin, O. Shekhovtsova, Phys.Rev. D85 (2012) 094010



We need

$$\mathcal{L}_{\gamma\gamma P},\ \mathcal{L}_{\gamma V},\mathcal{L}_{V\gamma P},\ \mathcal{L}_{VVP}$$

The lagrangians

$$\mathcal{L}_{\gamma\gamma P} = \frac{-e^2 N_c}{24\pi^2 f_\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu B_\nu \partial_\alpha B_\beta \Big[\pi^0 + \eta(\frac{5}{3}C_q - \frac{\sqrt{2}}{3}C_s) + \eta'(\frac{5}{3}C'_q + \frac{\sqrt{2}}{3}C'_s)\Big] .$$

$$\mathcal{L}_{\gamma V} = -e\sum_{i=1}^{3} f_{V_i}\partial_\mu B_\nu \Big(\tilde{\rho}_i^{\mu\nu} + \frac{1}{3}F_{\omega_i}\tilde{\omega}_i^{\mu\nu} - \frac{\sqrt{2}}{3}F_{\phi_i}\tilde{\phi}_i^{\mu\nu}\Big),$$

$$\mathcal{L}_{V\gamma\pi^0} = -\sum_{i=1}^{n}\frac{4\sqrt{2}eh_{V_i}}{3f_\pi}\epsilon_{\mu\nu\alpha\beta}\partial^\alpha B^\beta \Big(\rho_i^\mu + 3H_{\omega_i}\omega_i^\mu - \frac{3}{\sqrt{2}}A_i^{\pi^0}\phi_i^\mu\Big)\partial^\nu\pi^0 ,$$

$$\mathcal{L}_{V\gamma\eta} = -\sum_{i=1}^{n}\frac{4\sqrt{2}eh_{V_i}}{3f_\pi}\epsilon_{\mu\nu\alpha\beta}\partial^\alpha B^\beta \Big[(3\rho_i^\mu + \omega_i^\mu)C_q + 2\phi_i^\mu C_s - (\frac{5}{\sqrt{2}}C_q - C_s)A_i^\eta\phi_i^\mu\Big]\partial^\nu\eta ,$$

$$\mathcal{L}_{V\gamma\eta'} = -\sum_{i=1}^{n}\frac{4\sqrt{2}eh_{V_i}}{3f_\pi}\epsilon_{\mu\nu\alpha\beta}\partial^\alpha B^\beta \Big[(3\rho_i^\mu + \omega_i^\mu)C'_q - 2\phi_i^\mu C'_s - (\frac{5}{\sqrt{2}}C'_q + C'_s)A_i^{\eta'}\phi_i^\mu\Big]\partial^\nu\eta' ,$$

$$\mathcal{L}_{VV\pi^0} = -\sum_{i=1}^{n}\frac{4\sigma_{V_i}}{f_\pi}\epsilon_{\mu\nu\alpha\beta}\Big[\frac{1}{F_{\omega_i}}\pi^0\partial^\mu\omega_i^\nu\partial^\alpha\rho_i^\beta + \frac{3(F_{\omega_i}H_{\omega_i} - 1 - A^{\pi^0}_{\phi\omega,i})}{2F^2_{\omega_i}}\pi^0\partial^\mu\omega_i^\nu\partial^\alpha\omega_i^\beta$$
$$+\frac{3(A_i^{\pi^0} - A^{\pi^0}_{\phi\omega,i}/F_{\phi_i})}{4F_{\phi_i}}\pi^0\partial^\mu\phi_i^\nu\partial^\alpha\phi_i^\beta - \frac{3A^{\pi^0}_{\phi\omega,i}}{\sqrt{2}F_{\omega_i}F_{\phi_i}}\pi^0\partial^\mu\phi_i^\nu\partial^\alpha\omega_i^\beta\Big],$$

$$\mathcal{L}_{VV\eta} = -\sum_{i=1}^{n}\frac{4\sigma_{V_i}}{f_\pi}\epsilon_{\mu\nu\alpha\beta}\eta\Big[(\partial^\mu\rho_i^\nu\partial^\alpha\rho_i^\beta + \frac{1}{F_{\omega_i}}\partial^\mu\omega_i^\nu\partial^\alpha\omega_i^\beta)\frac{1}{2}C_q - \frac{9A^\eta_{\phi\omega_i}}{F^2_{\omega_i}}\partial^\mu\omega_i^\nu\partial^\alpha\omega_i^\beta - \frac{1}{F_{\phi_i}}\partial^\mu\phi_i^\nu\partial^\alpha\phi_i^\beta\frac{1}{\sqrt{2}}C_s$$
$$-\frac{9A^\eta_{\phi\omega,i}}{2F^2_{\phi_i}}\partial^\mu\phi_i^\nu\partial^\alpha\phi_i^\beta + \frac{A_i^\eta}{6F_{\phi_i}}(\frac{15}{2}C_q - \frac{3}{\sqrt{2}}C_s)\partial^\mu\phi_i^\nu\partial^\alpha\phi_i^\beta - \frac{9\sqrt{2}A^\eta_{\phi\omega,i}}{F_{\omega_i}F_{\phi_i}}\partial^\mu\phi_i^\nu\partial^\alpha\omega_i^\beta\Big],$$

$$\mathcal{L}_{VV\eta'} = -\sum_{i=1}^{n}\frac{4\sigma_{V_i}}{f_\pi}\epsilon_{\mu\nu\alpha\beta}\eta'\Big[(\partial^\mu\rho_i^\nu\partial^\alpha\rho_i^\beta + \frac{1}{F_{\omega_i}}\partial^\mu\omega_i^\nu\partial^\alpha\omega_i^\beta)\frac{1}{2}C'_q + \frac{1}{F_{\phi_i}}\partial^\mu\phi_i^\nu\partial^\alpha\phi_i^\beta\frac{1}{\sqrt{2}}C'_s$$
$$+\frac{A_i^{\eta'}}{6F_{\phi_i}}(\frac{15}{2}C'_q + \frac{3}{\sqrt{2}}C'_s)\partial^\mu\phi_i^\nu\partial^\alpha\phi_i^\beta\Big],$$

The data

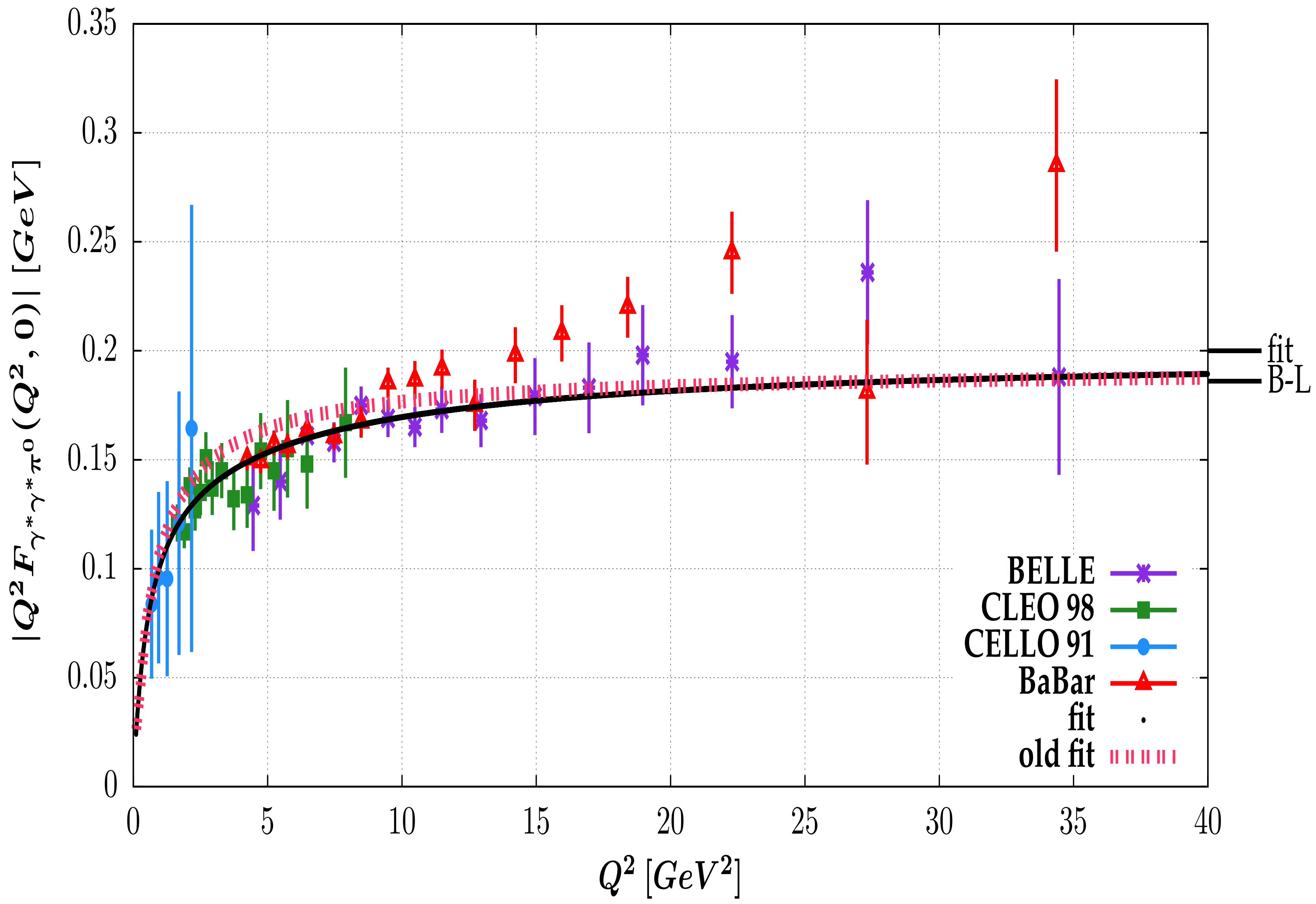
Experiment	nep	χ^2,fit 1	χ^2,fit 2	Experiment	nep	χ^2,fit 1	χ^2,fit 2
space-like form-factors							
BELLE (π^0)[42]	15	9.96	6.72	CLEO98(η) [44]	19	15.8	15.5
CELLO91(π^0) [43]	5	0.34	0.24	BaBar(η') [57]	11	5.4	3.70
CLEO98(π^0) [44]	15	10.6	6.82	CELLO91(η') [43]	5	0.73	0.56
BaBar(η) [57]	11	7.34	7.5	CLEO98(η') [44]	29	25.1	24.4
CELLO91(η) [43]	4	0.16	0.16				
e^+e^- cross sections							
CMD2$(\pi^0\gamma)$ [47]	46	54.1	54.1	SND$(\eta\gamma)$ [45]	78	68.7	59.8
SND$(\pi^0\gamma)$ [46]	62	65.5	54.2	BaBar$(\eta\gamma,\eta'\gamma)$ [58]	2	0.18	1.57
CMD2 $(\eta\gamma)$ [47]	42	25.4	25.6				

The data

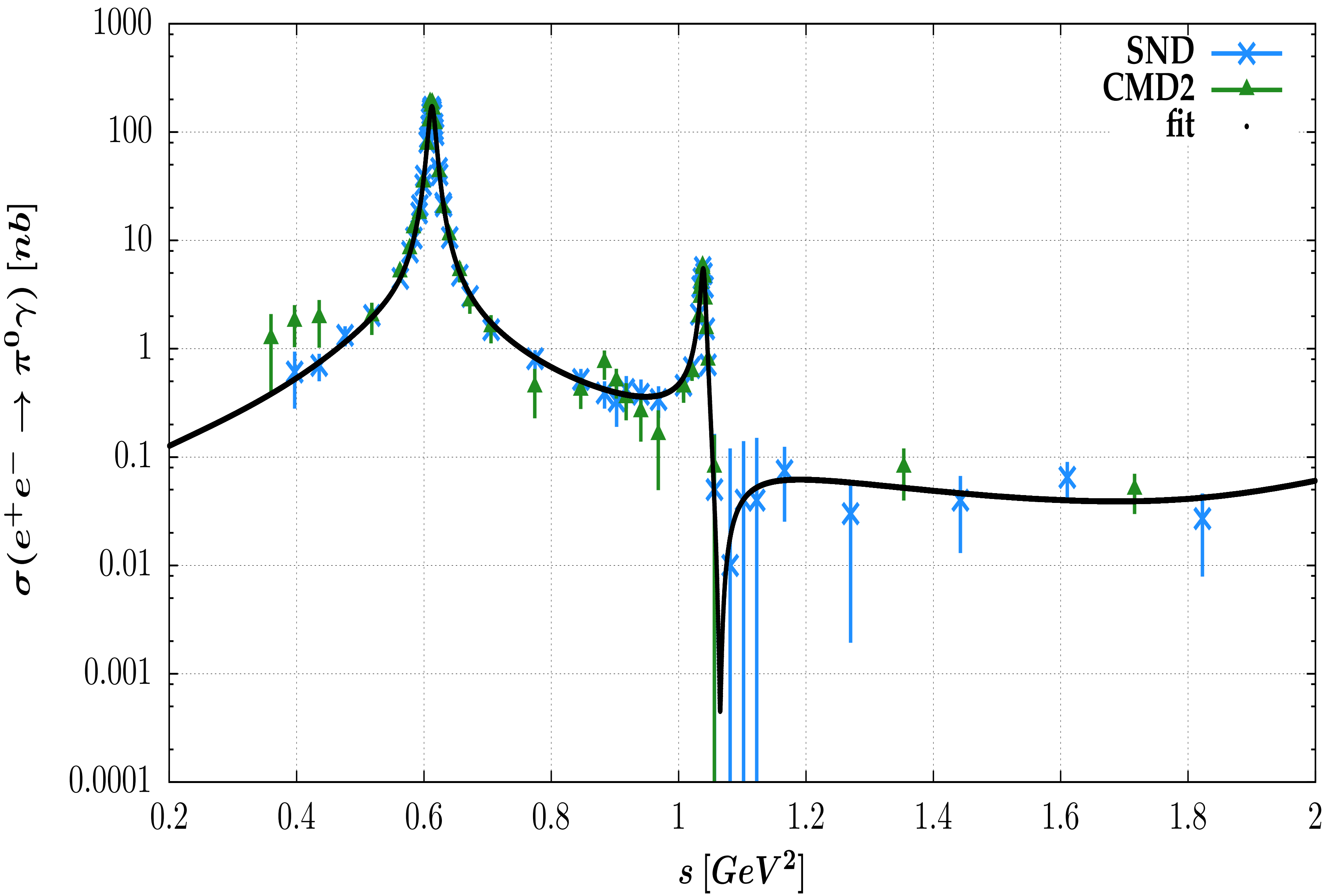
3-body decays							
A2($\pi^0 \to \gamma e^+ e^-$) [48]	18	0.32	0.34	A2($\omega \to \pi^0 e^+ e^-$) [49]	14	2.14	2.12
A2($\eta \to \gamma e^+ e^-$) [49]	34	10.2	11.1	KLOE-2($\phi \to \pi^0 e^+ e^-$) [51]	15	4.33	4.33
A2 ($\eta \to \pi^0 \gamma\gamma$) [53]	7	26.6	19.5	KLOE-2($\phi \to \eta e^+ e^-$) [52]	92	95.1	95.1
BESIII($\eta' \to \gamma e^+ e^-$)[50]	8	2.39	2.13				
2-body decays							
$\Gamma(\pi^0 \to \gamma\gamma)$ [54]	1	0.36	0.1	$\Gamma(\rho \to \pi^0 \gamma)$ [54]	1	1.17	0.42
$\Gamma(\eta \to \gamma\gamma)$ [54]	1	0.78	2.73	$\Gamma(\omega \to \pi^0 \gamma)$ [54]	1	4.08	1.56
$\Gamma(\eta' \to \gamma\gamma)$ [54]	1	1.05	0.44	$\Gamma(\phi \to \pi^0 \gamma)$ [54]	1	0.08	0.06
$\Gamma(\eta' \to \rho\gamma)$ [54]	1	3.0	0.77	$\Gamma(\rho \to \eta\gamma)$ [54]	1	3.32	6.8
$\Gamma(\eta' \to \omega\gamma)$ [54]	1	0.00	0.54	$\Gamma(\omega \to \eta\gamma)$ [54]	1	6.86	3.04
$\Gamma(\rho \to e^+ e^-)$ [54]	1	0.23	0.05	$\Gamma(\phi \to \eta\gamma)$ [54]	1	1.63	1.17
$\Gamma(\omega \to e^+ e^-)$ [54]	1	0.56	0.73	$\Gamma(\phi \to \eta'\gamma)$ [54]	1	0.01	0.00
$\Gamma(\phi \to e^+ e^-)$ [54]	1	0.69	0.46				
				Total	536	454	415

Number of free parameters 17(22).

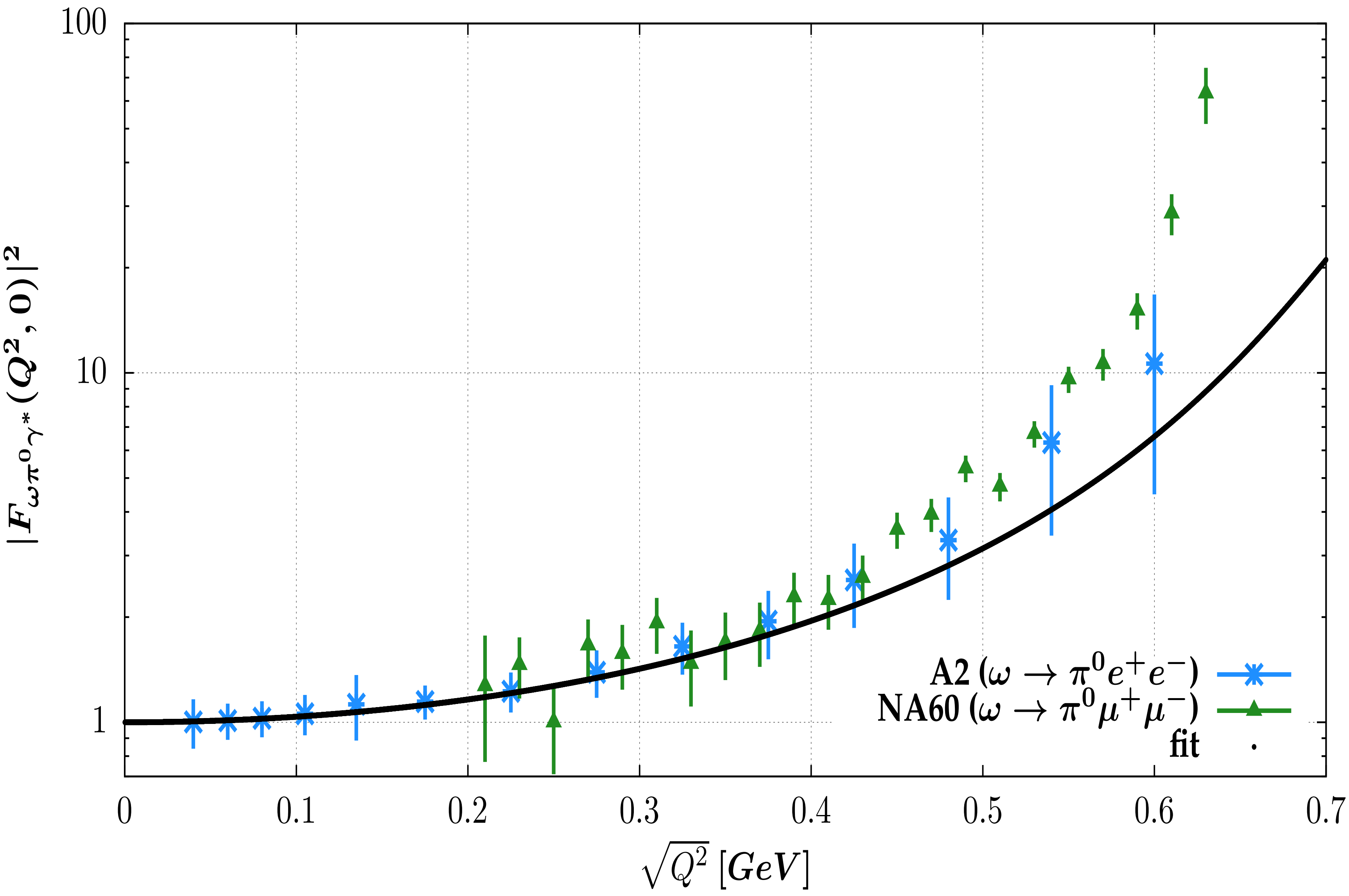
Transition form factors



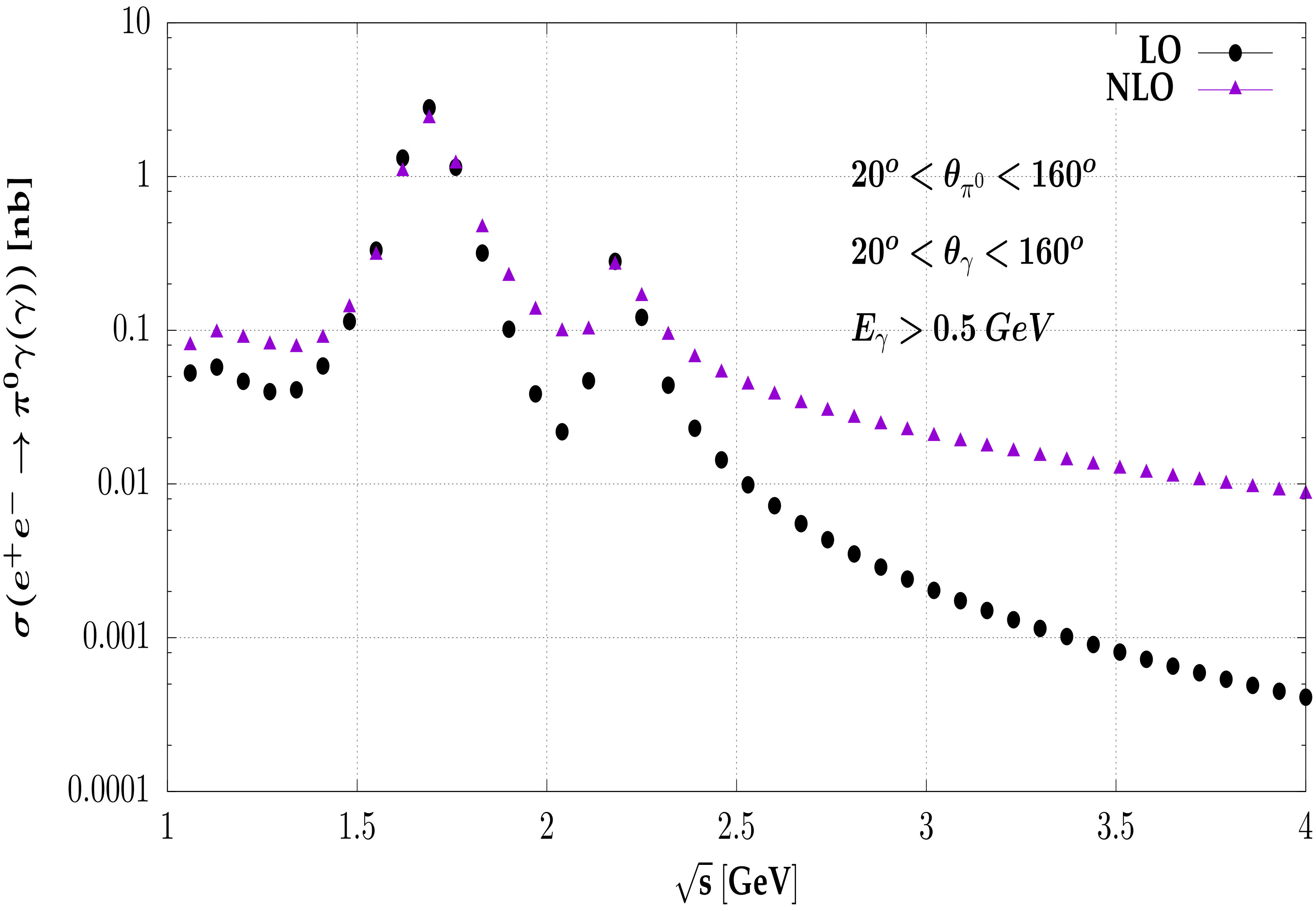
Cross sections



3-body decays



Radiative corrections in $e^+e^- \to P\gamma$



$a_\mu(P)$

Model	$a_\mu^{\pi^0}$	a_μ^{η}	$a_\mu^{\eta'}$	a_μ^{P}
fit 1	58.80 ± 0.27	13.56 ± 0.10	12.97 ± 0.09	85.32 ± 0.30
fit 2	56.96 ± 0.94	13.35 ± 0.45	12.55 ± 0.48	82.85 ± 1.15
fit 3	59.07 ± 0.17	13.52 ± 0.09	12.96 ± 0.09	85.55 ± 0.22
fit 4	57.79 ± 0.90	13.31 ± 0.19	12.31 ± 0.21	83.41 ± 0.94
[70]	57.4 ± 6.0	13.4 ± 1.6	11.9 ± 1.4	82.7 ± 6.4
[71]	58 ± 10	13 ± 1	12 ± 1	83 ± 12
[72]	-	-	-	85 ± 13
[73]	76.5 ± 6.5	18 ± 1.4	18 ± 1.5	114 ± 10
[74]	$62.7 - 66.8$	-	-	-
[10, 75]	72 ± 12	14.5 ± 4.8	12.5 ± 4.2	99 ± 16
[76]	68.8 ± 1.2	-	-	-
[77]	66.6 ± 2.1	20.4 ± 4.4	17.7 ± 2.3	104.7 ± 5.4
[78]	65.0 ± 8.3	-	-	-

Radiative corrections in EKHARA

H. Czyz, S. Ivashyn, P. Kisza, in preparation

$e^+e^- \to e^+e^- P(\gamma)$

$\Rightarrow$ The code is ready

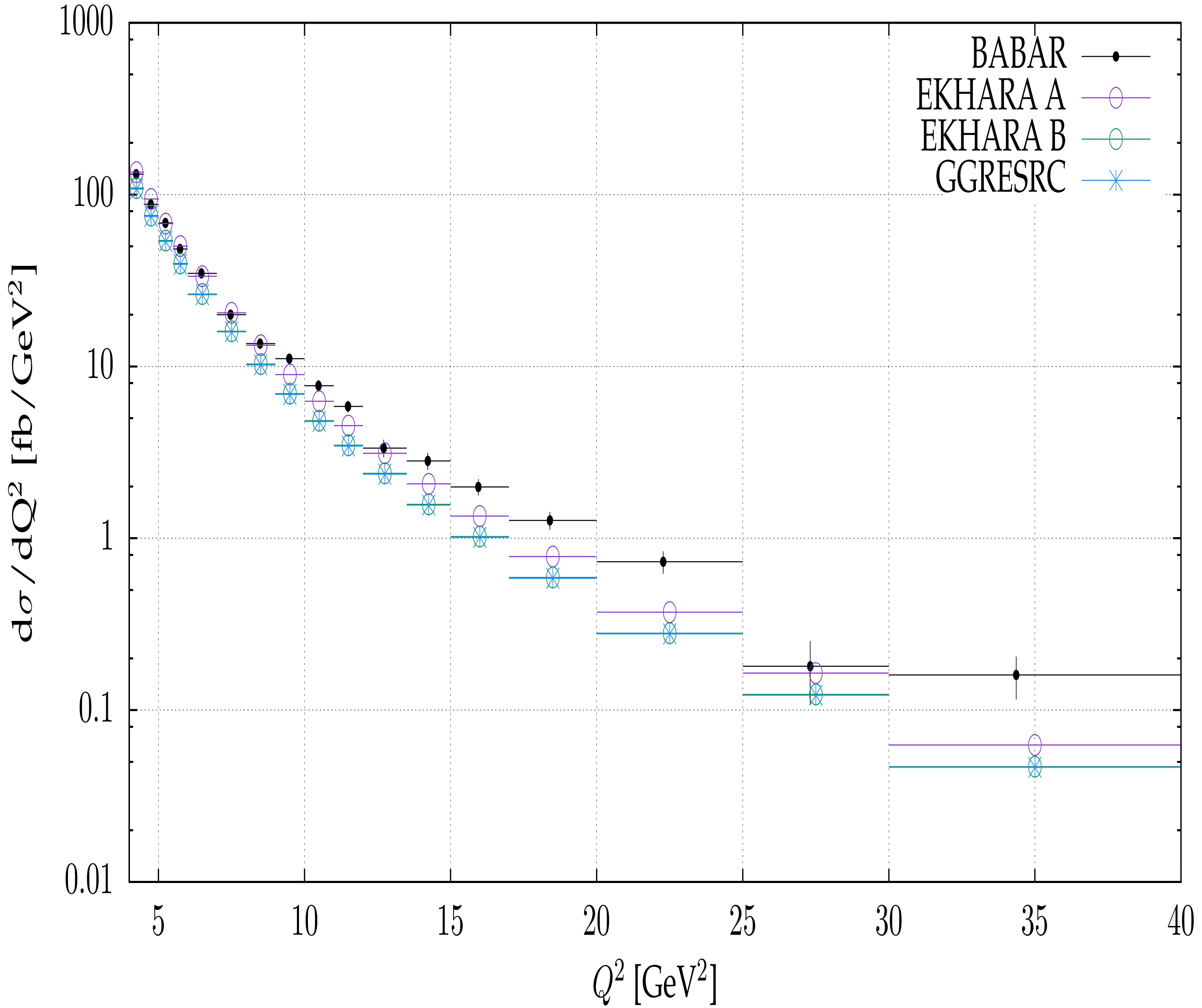
$\Rightarrow$ Last tests finished few days ago

$\Rightarrow$ To be done:

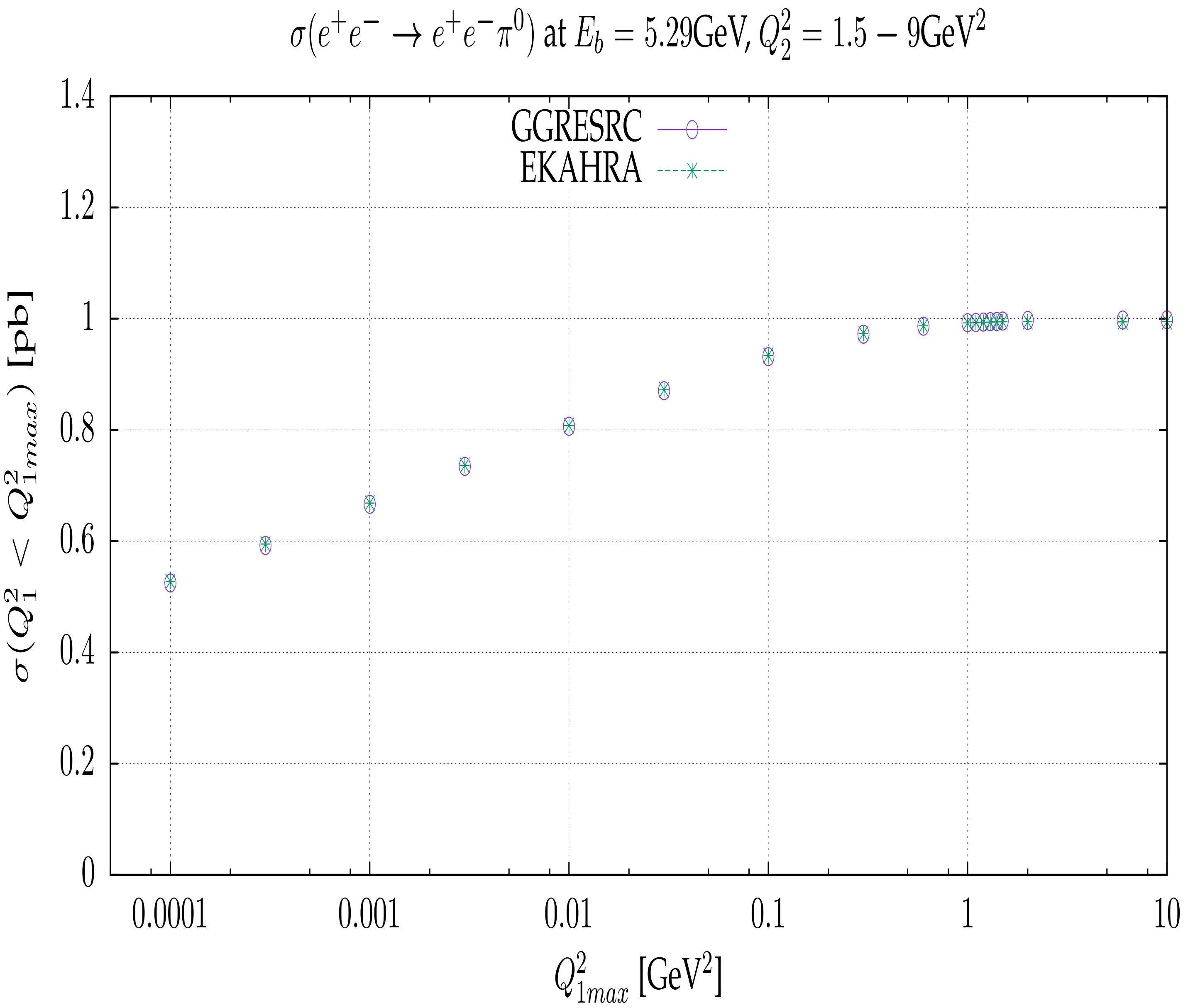
$\Rightarrow$ Comparisons with GGRESRC

V. P. Druzhinin, L. V. Kardapoltsev, V.A. Tayursky, Comput.Phys.Commun. 185 (2014) 236-243

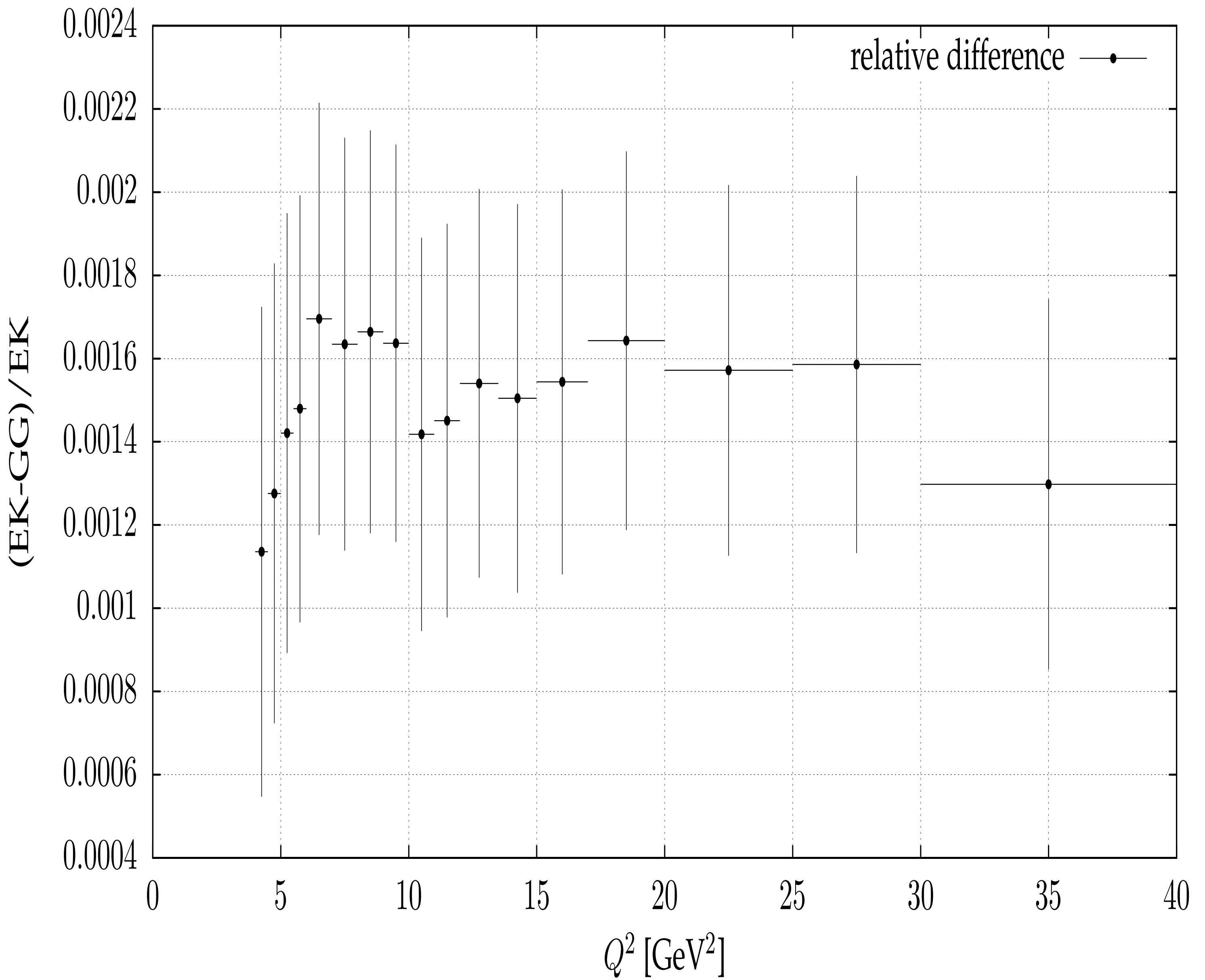
EKHARA vs. GGRESRC LO



EKHARA vs. GGRESRC LO



EKHARA vs. GGRESRC LO



PHOKHARA NLO: The team

F. Campanario, G. Rodrigo (Valencia)

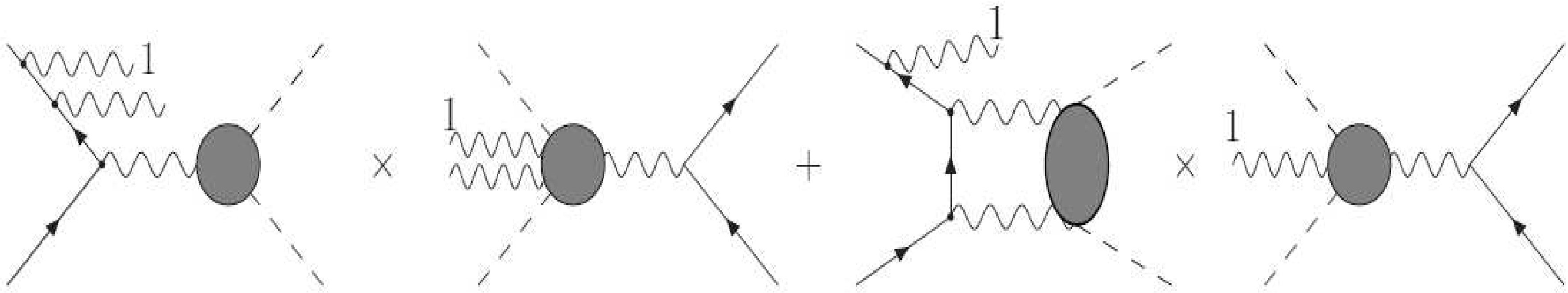
H.C., J. Gluza, T. Jeliński, Sz. Tracz, D. Zhuridov (Katowice)

Status

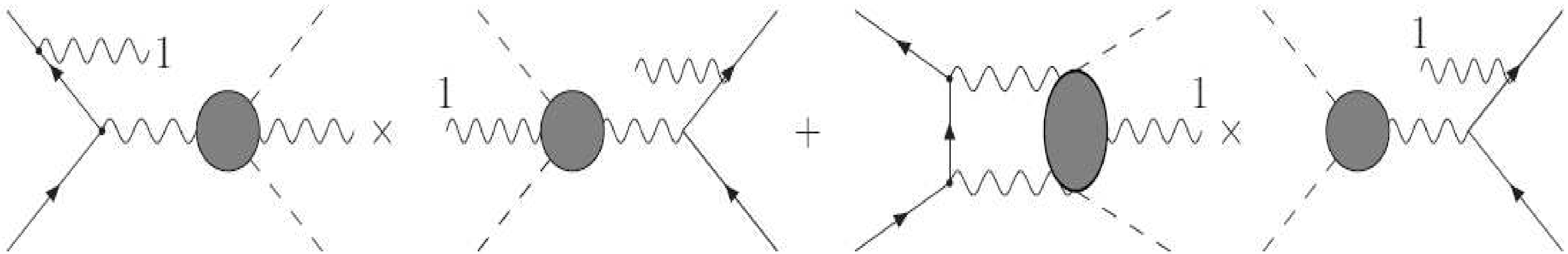
$\Rightarrow$ sQED + form factors:
pentaboxes ready and fully tested

$\Rightarrow$ pending: FSR at NLO, LL enough?

PENTABOXES for pions



(E)



PENTABOXES for pions - tests

⇒ two independent codes for the new hard part

⇒ the virtual corrections implementation:

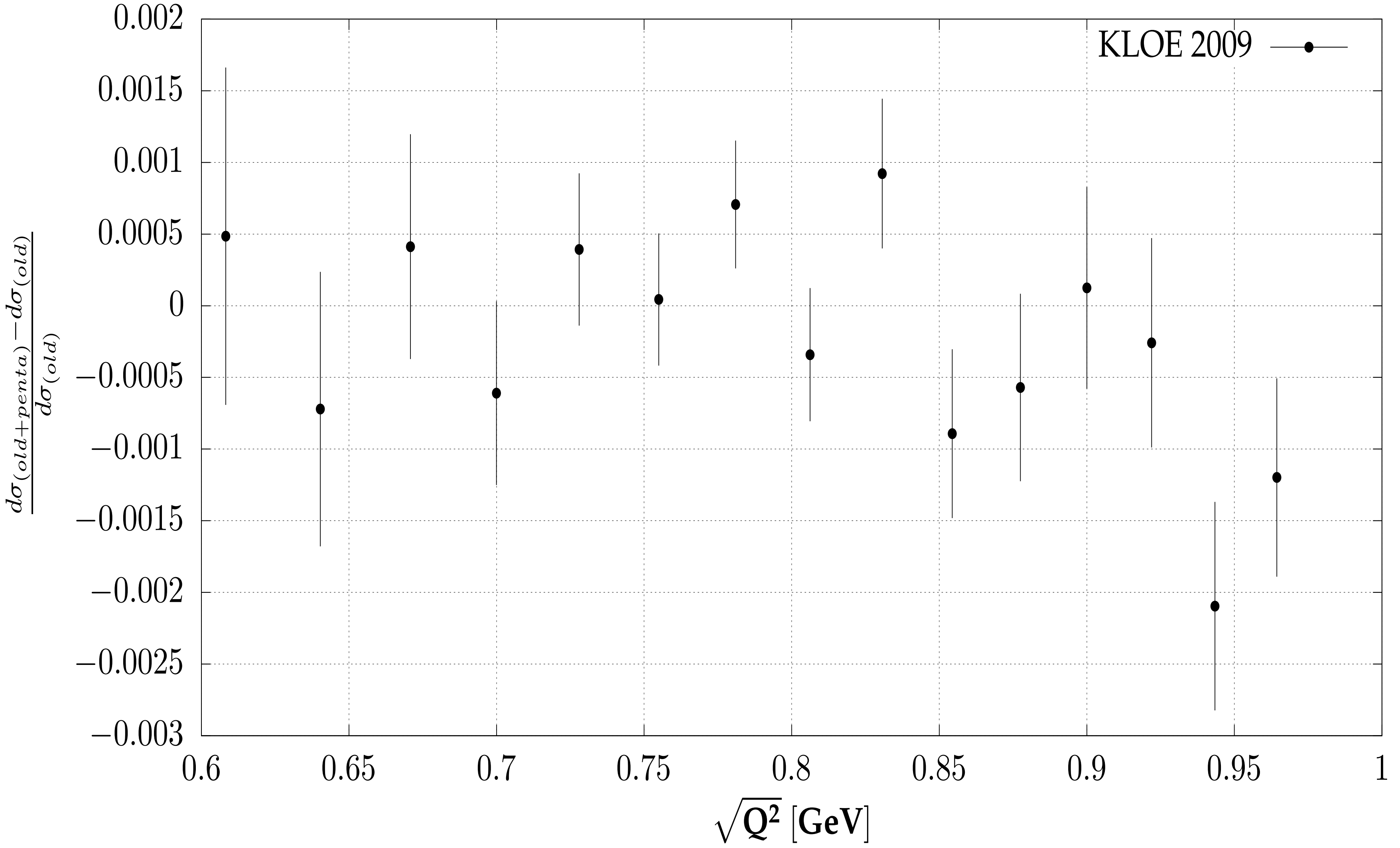
the tensor reduction and the amplitude (trace) in quadrupole precision
with scalar integrals in double precision (QCDLOOP)

⇒ Tests performed:

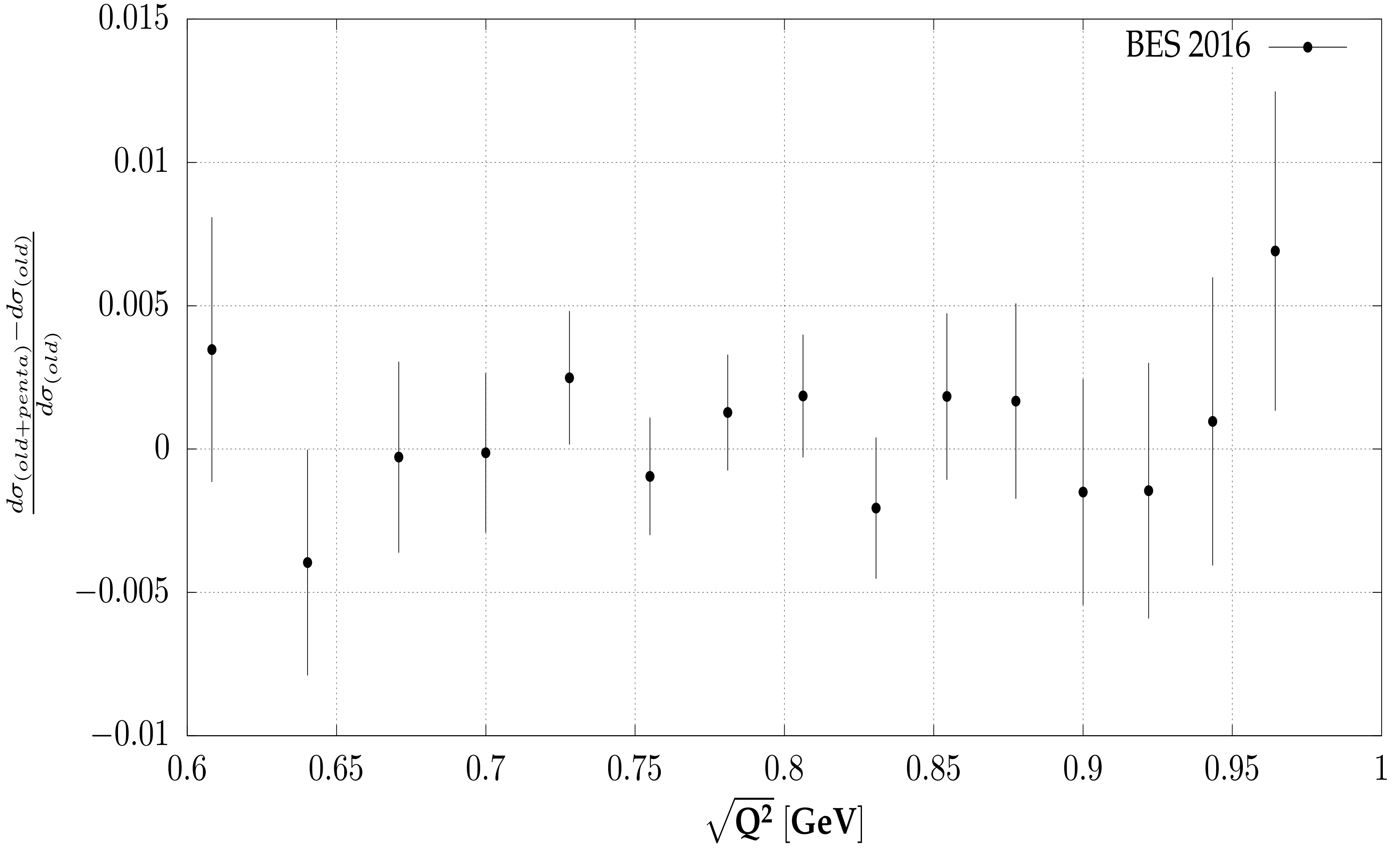
Comparison with LOOPTOOLS full quadrupole precision
within Mathematica; accuracy:$\mathbf{10^{-5}}$
Comparison with between the results
calculated with trace and helicity methods

⇒ Soft divergencies tests

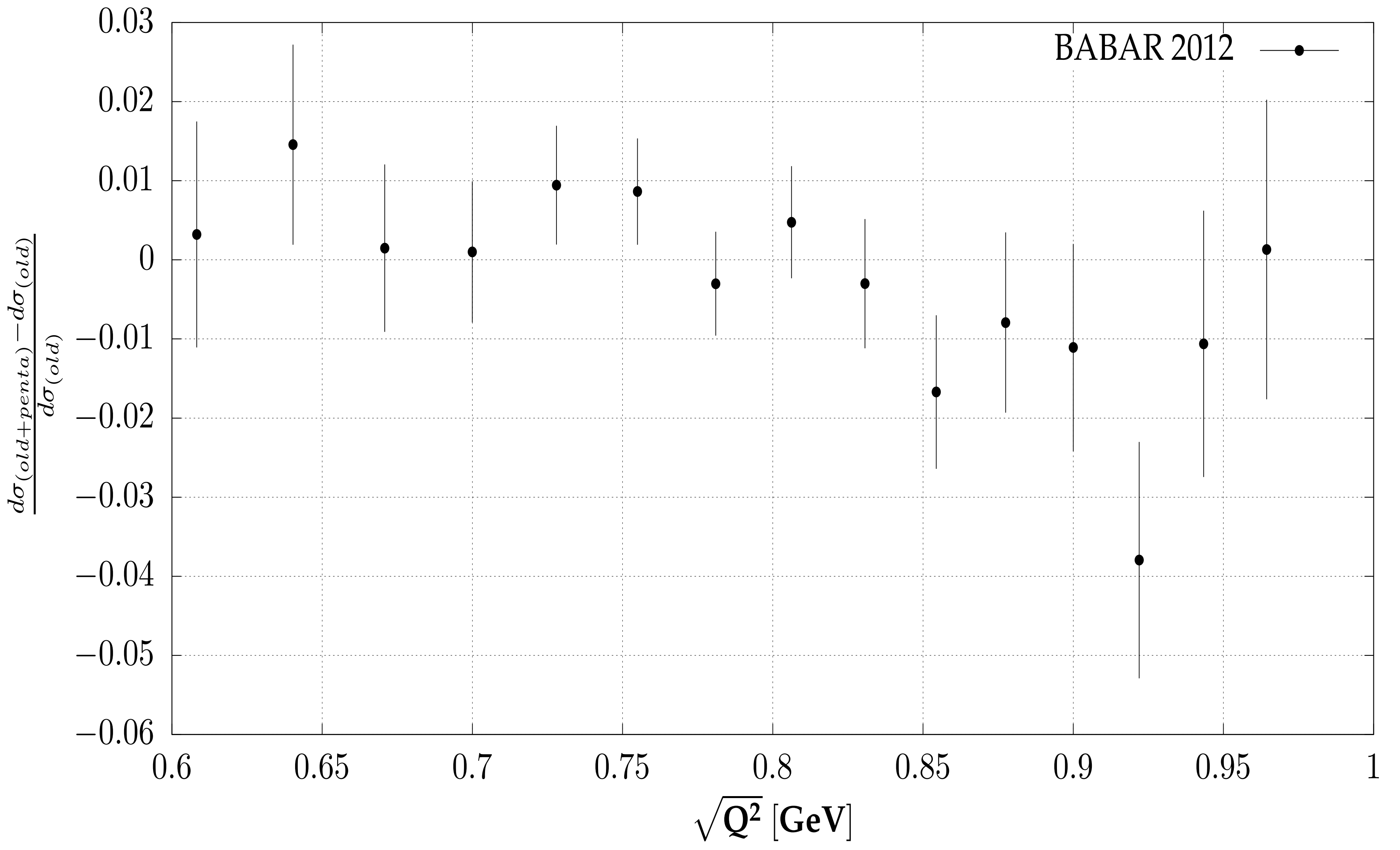
PENTABOXES for pions



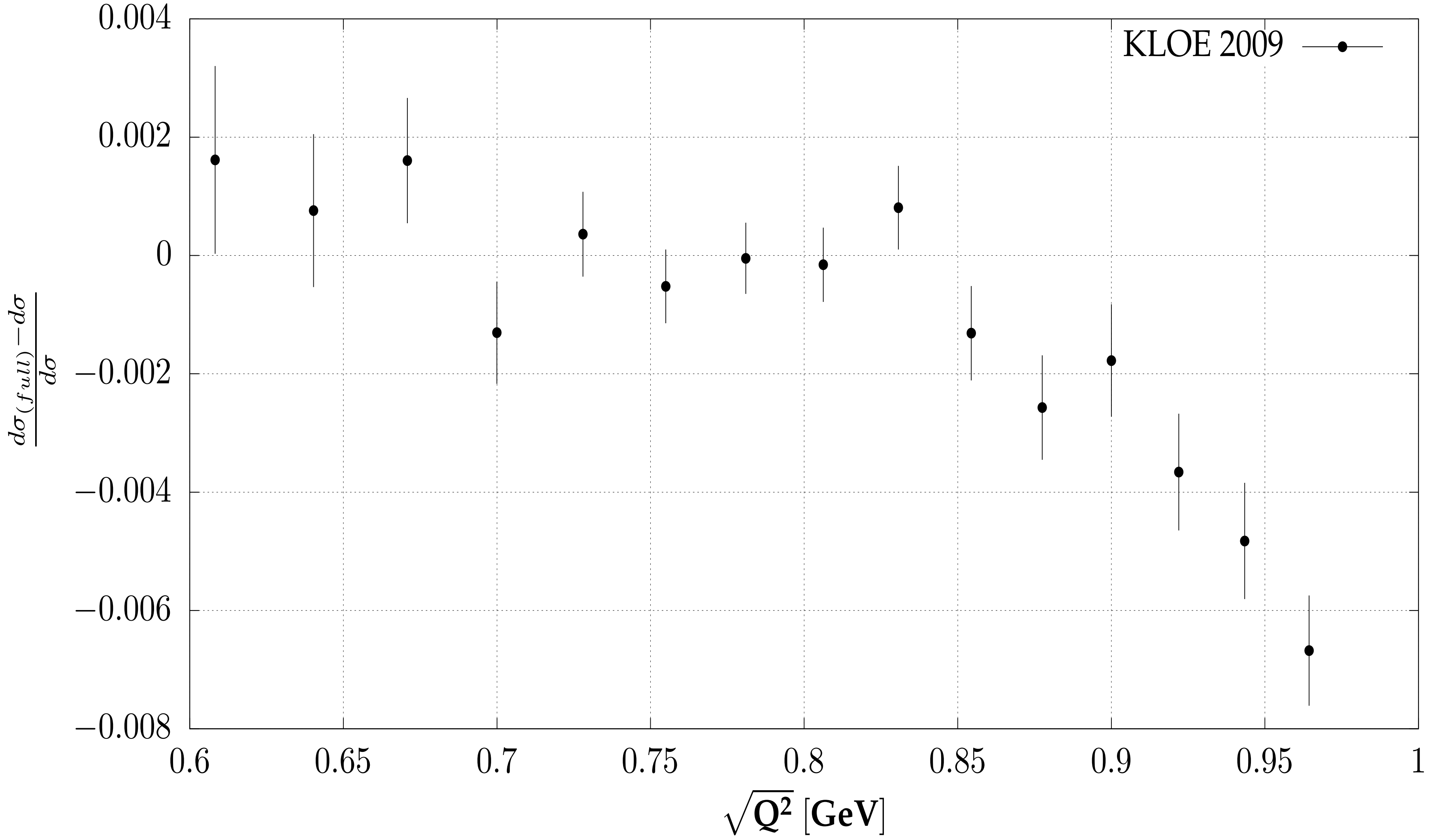
PENTABOXES for pions



PENTABOXES for pions



NLO FSR for pions



Concluding remarks

$\Rightarrow$ Slow progress,
but hoping to be of help

$\Rightarrow$ In about 1 year the accuracy of PHOKHARA should be at 0.1-0.2%

$\Rightarrow$ The release of new versions in about one month

$\Rightarrow$ Next in the waiting queue: $e^+e^- \to e^+e^-\pi\pi$