On the decomposition of 2-loop μe scattering via Adaptive Integrand Decomposition

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The Evaluation of the Leading Hadronic Contribution to a_μ
20.02.2018, MITP, Mainz, Germany.


Disclaimer

Motivation

- QED provides more than 99.99% of the standard-model value of a_μ
- Simplify calculations in high perturbative orders
- Cross checks from μe scattering
  — One of the cleanest processes
  — Pure t-channel
- Theoretical work needed —> NNLO QED correction unknown

>> KNECHT'S talk

>> VENANZONI'S talk

>> PASSERA'S talk

Motivation

- Status after the fist calculation of a_μ by Schwinger in 1948

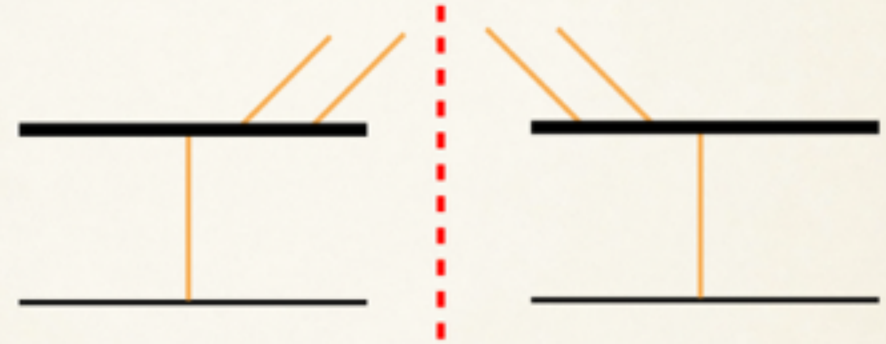
{Abdalgabar, Aboubrahim, Accioly, Adachi, Addazi, Adkins, Agashe, Aghababaei, Ahmadi, Ahmadzadegan, Akhiezer, Albino, Albrecht, Aldaya Martin, Alkofer, Alvegard, Alves, Ambrosino, Anchordoqui, Anger, Anulli, Aoki, Aoyama, ARBOR, Arbuzov, Artru, Artuso, Atag, Athron, Attaourti, Babu, Babukhadia, Babusci, Baez, Bagrov, Bahar, Baier, Baikov, Bailey, Baker, Bakhoum, Balachandran, Banerjee, Barbieri, Barish, Barker, Barnes, Bashir, Bauke, Beier, Belokogne, Belov, Bélusca–Maïto, Bennett, Berestetsky, Berger, Berghaus, Bergman, Bermudez, Bern, Bernabeu, Bernard, Bernstein, Berry, Bertsche, Besson, Bethe, Bettini, Bhatti, Bianco, Biebel, Biggio, Billur, Bini, Binosi, Biraben, Biswas, Blaum, Blokland, Blum, Böhler, Bohm, Bohr, Bordovitsyn, Borer, Borisov, Bouchendira, Bousquet, Bowcock, Boyle, Braghin, Brambilla, Broadhurst, Brodsky, Broglia, Bronner, Brown, Buchmann, Bufalo, Bunce, Bussone, Butler, Cacciapaglia, Caffo, Cakir, Calvo–Mozo, Canuto, Cao, Capra, Carey, Carroll, Carruth, Casey, Cassidy, Castro Nunes Fiolhais, Cavalcanti, Cesar, Chabysheva, Chang, Charlton, Charmchi, Charpak, Chauhan, Chazelle, Cheng, Cheoun, Chishtie, Chiu, Chizhov, Chongqing, Choudhury, Chraplyvy, Christ, Chu, Cirelli, Cirigliano, Cladé, Cloet, Cohen, Collister, Combley, Commins, Concha, Consonni, Conti, Couchot, Crane, Cristadoro, Cudell, Culatti, Cushman, Cvitanovic, Czarnecki, Danby, Darewych, Das, Da Silva, Das Sarma, Davier, Debevec, Debierre, de Carvalho, Dehmelt, Deile, del Aguila, de la Incera, Del Debbio, Delemontex, Della Morte, Della Valle, Deng, Denig, Deninger, Deo, de Queiroz, Deraad, de Rafael, Derwent, De Sangro, Deshpande, Devi, de Vries, Dhawan, Diaz, Dickinson, Dixon, Djouadi, Dobrigkeit Chinellato, Dogangun, Dorokhov, Dorr, Dowling, Drell, Drumm, Druzhinin, Ducu, Dukes, Duong, Durand, Dutta, Dyson, Eads, Eberly, Echenard, Eck, Efstathiadis, Eichmann, Eidelman, Eides, Ejili, El–Bennich, Elhandi, Elias, Elmetenawee, El–Mezeini, Elmfors, Emelyanov, Englert, Epifanov, Eriksson, Erler, Ermolaev, Escalier, Esposito, Evans, Evetts, Fabbri, Fael, Fajans, Farley, Farzinnia, Fassio–Canuto, Fayazbakhsh, Fedosov, Fedotovich, Ferrer, Field, Fienga, Fischer, Fitch, Flay, Flegel, Fleming, Flowers, Fornal, Forshaw, Fortson, Fraga, Frampton, Frank, Franke, Frederiksen, Freitas, French, Friesen, Frolov, Fujiwara, Gabrielse, Gaete, Gamboa, Gaponenko, Garwin, Garziano, Gastaldi, Gauzzi, Gensini, Gevorkyan, Ghose, Ghosh, Giacomini, Gibbs, Gill, Giovannella, Giron, Gisin, Giudice, Glazov, Glenzinski, Göcke, Goertz, Goncalves, Gonzalez, Gonzalez–Alonso, Gonzalez–Martin, Gonzalez–Sprinberg, González–Sprinberg, Gousheh, Govaerts, Gover, Govindarajan, Graesser, Grancagnolo, Grange, Gray, Green, Greiner, Gribov, Grigoriev, Grinstein, Grojean, Groote, Grosse–Perdekamp, Grossman, Grossmann, Grotch, Group, Guellati–Khélifa, Guevara, Gutierrez, Gutiérrez–Guerrero, Haffner, Hagar, Haghighat, Hangst, Hanif, Hanneke, Hansen, Hardy, Hare, Harman, Harnik, Harris, Hartin, Hattersley, Hayakawa, Hayden, Helayel–Neto, Hermanspahn, Hertzog, Hessler, Heupel, Hiller, Hisakado, Hitlin, Hocker, Hofmann, Hollik, Holman, Home, Honkanen, Hoogerheide, Horbatsch, Hounkonnou, Hsieh, Hu, Huang, Hughes, Hwang, Ibrahim, Ignatov, Ilderton, Illana, Indelicato, Isaac, Ishida, Islam, Itami, Iurato, Iwamoto, Iwasaki, Izubuchi, Jager, Jäger, Jansen, Japaridze, Jarlskog, Jauch, Jegerlehner, Jentschura, Jin, Johnert, Johnson, Jones, Jonsell, Jung, Jungmann, Juttner, Kagan, Kajino, Kamal, Karplus, Karshenboim, Kataev, Katkov, Kawall, Kawamura, Kazama, Kazes, Keitel, Kennedy, Kermiche, Kerrane, Khazin, Khomovsky, Kibble, Kiburg, Kindem, Kinoshita, Kirby, Kiril'tseva, Kluge, Knecht, Knight, Knoepfel, Kochelev, Kockum, Köhler–Langes, Koksal, Köksal, Kondratyev, Kopp, Kopylova, Korner, Krebs, Krein, Krienen, Kroll, Kronkvist, Kubota, Kulikova, Kumar, Kumar Rai, Kuno, Kuo, Kurchaninov, Kurilin, Kurkcuoglu, Kürkçüoğlu, Kurz, Kwang, Kwon, Lai, Laidet, Lam, Lamb, Lancaster, Lange, LANGENBERG, Laporta, Larsen, Lautrup, Lebedev, Lebee, Lee, Lee Roberts, Lehner, Lemke, Lemmon, Leutwyler, Levine, Li, Lim, Lin, Lindgren, Liu, Logashenko, López Castro, Lu, Luo, Lusanna, MacDowell, Mac Gregor, Madsen, Maier, Malcles, Manaut, Mane, Maniatis, Manreza Paret, Mao, Marciano, Marino, Maris, Marquard, Martin, Martinez, Martínez, Maru, Maruyama, Marvik, Mathers, Mathews, Maurer, Maxwell, Mbelek, McCartor, McCoy, McKeen, McKenna, McKeon, McMillan, McNabb, Mehra, Melnikov, Menary, Méndez, Menezes, Meng, Mercolli, Meric, Merola, Messineo, Meuren, Mi, Michan, Michel, Migdal, Mikhailov, Miller, Mills, Milotti, Milton, Mishima, Mitra, Mittag, Mizumachi, Mohr, Moll, Momose, Mooney, Moore, Morais, Morais Smith, Morse, Moshinsky, Motie, Muccifora, Mulders, Muller, Munich, Nakamura, Nambu, İnan, Nascimento, Nath, N'Dolo, Negrini, Neuberger, Neuman, Neves, Newman, Newton, Nez, Nguyen, Nikas, Nio, Nolan, Nori, Novaes, Novitski, Nyffeler, Nyiri, O'Brien, O'Connell, Ohnuki, Oishi, Okada, Olchanski, Olin, Onderwater, Ono, Oreshkina, Orlov, Osland, Öttinger, Oufni, Oury, Ousmane Samary, Overview, Ozben, Özgüven, Pacetti, Pachucki, Padmanabhan, Pagels, Paley, Palle, Palumbo, Pancheri, Papavassiliou, Paradisi, Paret, Park, Parke, Parker, PARKER, Parsons, Passera, Passeri, Paston, Pecina–Cruz, Peierls, Peng, Pengo, Peressutti, Perez–Gonzalez, Pérez Martínez, Perez–Victoria, Persson, Pestana Morais, Petermann, Petley, Petrucci, Pettersen, Piana, Picasso, Pich, Piclum, Pidd, Pietschmann, Pike, Pilaftsis, Pimentel, Pivovarov, Plunien, Pollock, Polly, Polosa, Portillo, Pospelov, Prades, Pretz, Prigl, Prokhvatilov, Puig Navarro, Pusa, Qian, Quint, Quiroz, Radici, Radzhabov, Raha, Rahmat, Raklev, Ramsey–Musolf, Rashid, Rasmussen, Raval, Ray, Raychaudhuri, Redin, Remiddi, Riad, Rich, Riemann, Rigolin, Rind, Roberts, Robicheaux, Rodionov, Röhrig, Rohrlich, Roig, Rojas, Rominsky, Roodman, Roscoe, Rosencwaig, Roskies, Rosner, Rossi, Ruffini, Runolfsson, Ruoso, Ryskulov, Ryu, Sabatier, Sacramento, Sadooghi, Saimpert, Saito, Salomonson, Sameed, Sampayo, Sanchez, Sanchez–Puertas, Sanchis–Alepuz, Santamaria, Sarazin, Sarid, Sarma, Savasta, Savitskaya, Scatena, Schappacher, Schmauch, Schneider, Schnetz, Schupp, Schwartz, Schweber, Schwinger, Sedykh, Selyugin, Semertzidis, Sens, Serino, Serna, Settles, Shabaev, Shagin, Shatunov, Shelyuto, Shtabovenko, Sichtermann, Sikora, Silveira, Singh, Sinha Roy, Six, Skagerstam, Skiba, Skrzypek, Smirnov, Smith, So, Soff, Solodov, Sossong, Spannowsky, Spiesberger, Spira, Sprague, Srivastava, Stahl, Stanev, Stassi, Stefano, Steinhauser, Steinmetz, Stockinger, Stöckinger, Stoeckinger, Stoffer, Stracka, Stroscio, Stroynowski, Studenikin, Sturm, Stutter, Sulak, Sunnergren, Suura, Suzuki, Sveshnikov, Sweetman, Szafron, Taboada, Taj, Talby, Talman, Tandy, Tanedo, Tapia, Taylor, TAYLOR, Teese, Tejeda–Yeomans, Telegdi, Terazawa, Terekidi, Ternick, Ternov, Teryaev, Tesanovic, Teubner, Tharp, Thomas, Thompson, 't Hooft, Throckmorton, Timmermans, Todtli, Tom, Tommasini, Torres, Trofimov, Troian, Troyan, Tsai, Tucker–Smith, Turcati, Turrini, Ugai, Uhlenbrock, Umezawa, Unruh, Urban, Urner, Vaidya, Vairo, van der Werf, Van De Water, Van Hooydonk, Vary, Vaurynovich, Venanzoni, Veneziano, Verdier, Verdu, Vereshchagin, Vidal, Violini, Vladlen, Vogel, Volkov, Volotka, von Ruden, von Walter, Wang, Warburton, Watanabe, Weber, Weiland, Weinberg, Weinstein, Weiss, Weisskopf, Wellmann, Welton, Wen, Wensley, Wentzel, Werth, Wesley, West, Wichmann, Widom, Wilkinson, Williams, Williamson, Wimett, Winn, Winter, Wise, Wittig, Wojcicki, Wong, Woodard, Wouters, Wright, Wu, Wurtele, Xue, Yablon, Yamamoto, Yang, Yao, Yavin, Yelkhovsky, Yennie, Yerokhin, Yndurain, Yokoya, Zambrano, Zanotti, Zatorski, Zavada, Zavattini, Zeman, Zerwas, Zhang, Zhao, Zhevlakov, Zhu, Zhukovsky, Zichichi, Zimmerman, Zuevsky, zu Putlitz, Zvolský, Zwanziger}

Mathematica routine by J.J. Carrasco
http://prettyquestions.com/notes/2017_Mathematica_Prettifier/

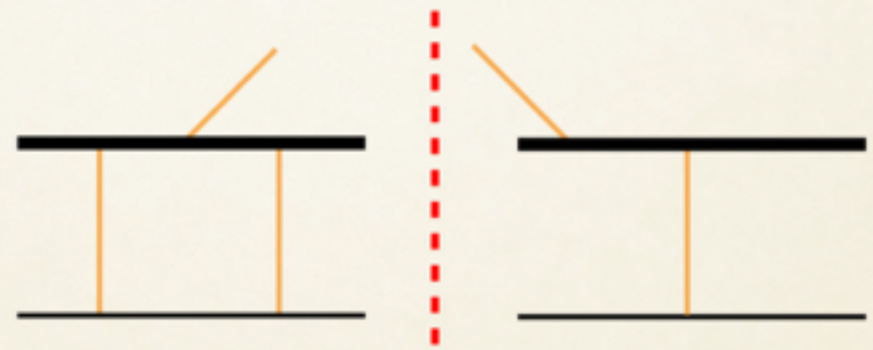
Introduction

What do we need for NNLO?

- **Double-real Radiation**
  tree-level matrix elements
  $d\hat{\sigma}^{RR}_{NNLO}$

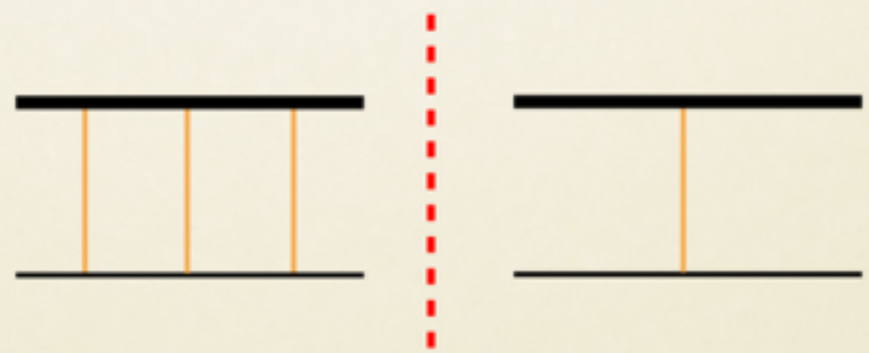


- **Single-real Radiation**
  1-loop matrix elements
  $d\hat{\sigma}^{RV}_{NNLO}$



>> GREINER'S talk

- **Virtual 2-loop matrix element**
  $d\hat{\sigma}^{VV}_{NNLO}$



$$d\hat{\sigma}_{NNLO} \sim \int_{d\Phi_{m+2}} d\hat{\sigma}^{RR}_{NNLO} + \int_{d\Phi_{m+1}} d\hat{\sigma}^{RV}_{NNLO} + \int_{d\Phi_{m}} d\hat{\sigma}^{VV}_{NNLO}$$

+ Subtractions and MC integration

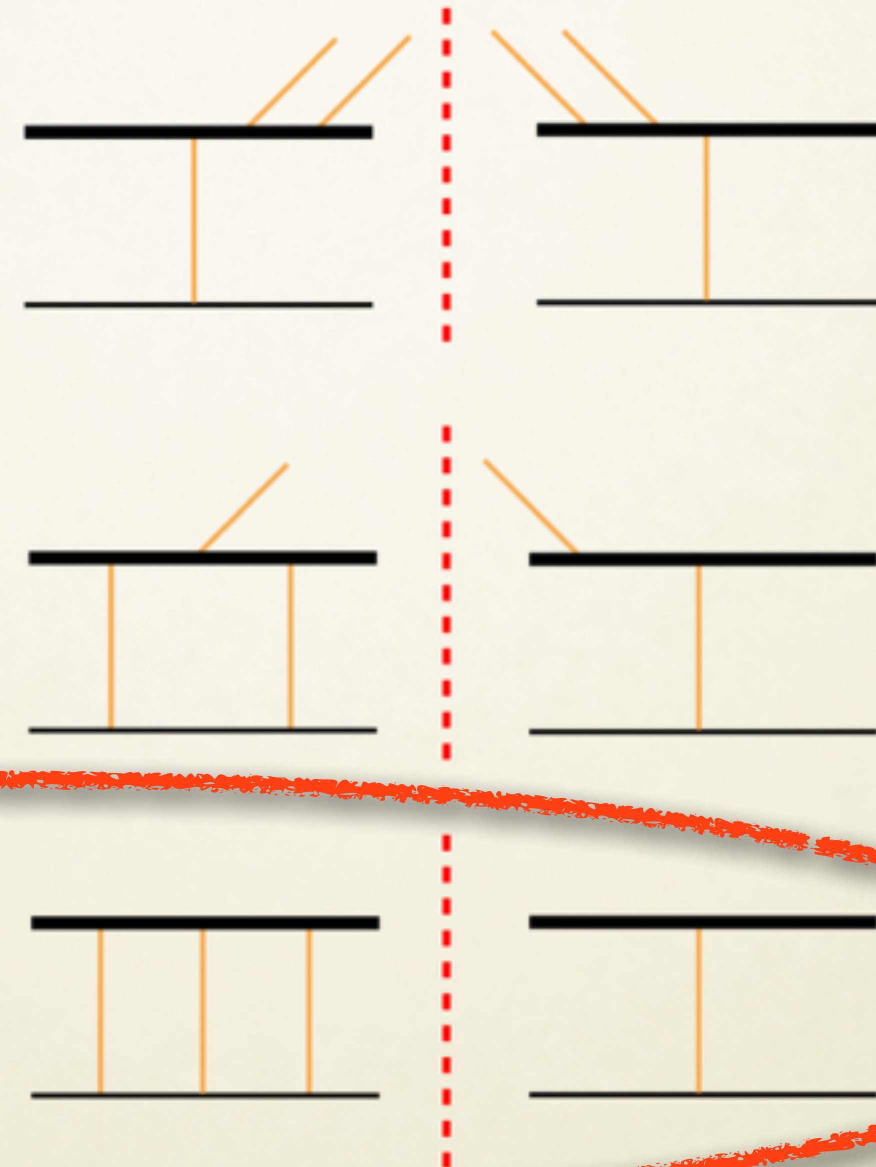
Introduction

What do we need for NNLO?

- **Double-real Radiation tree-level matrix elements**
  $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$

- **Single-real Radiation 1-loop matrix elements**
  $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$

- **Virtual 2-loop matrix element**
  $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$



>> GREINER'S talk

$$\mathrm{d}\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

+ Subtractions and MC integration

Outline

- Calculation of multi-loop scattering amplitudes
  — d dimensional generalised unitarity
  — Integrand reduction methods
  — Automated 1- and 2-loop reduction for any generic processes
- Results
- Conclusions & Outlook

Multi-loop Scattering amplitudes

Dimensional regularisation schemes

For all dimensional schemes $$\int \frac{\mathrm{d}^4 l}{(2\pi)^4} \to \mu_{\mathrm{DS}}^{4-d} \int \frac{\mathrm{d}^d \bar{l}}{(2\pi)^d}$$

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[Gnendiger (RADCOR 2017)]

To d, or not to d?

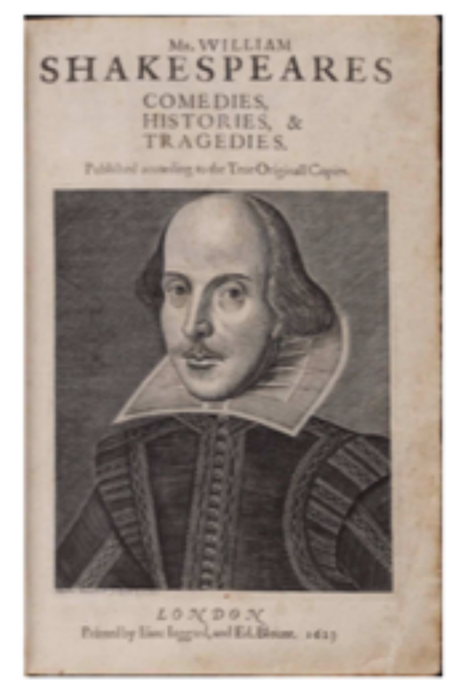
traditional dimensional schemes
- 't Hooft / Veltman (HV) '72
- conventional dim. reg. (CDR) '73
- dim. reduction (DRED) '79
- four-dim. helicity (FDH) '92

reformulations of dimensional schemes
- six-dim. formalism (SDF) '09
- four-dim. formalism (FDF) '14

non-dimensional schemes
- implicit reg. (IREG) '98
- loop regularization (LORE) '03
- four-dim. reg. / ren. (FDR) '12
- four-dim. unsubtraction (FDU) '16

- mathematical consistency
- unitarity, causality (equivalence to $\overline{\mathrm{MS}}$ / BPHZ)
- symmetries (gauge invariance, SUSY, ...)

- computational efficiency (analytical/numerical automation)



Dimensional regularisation schemes

For all dimensional schemes $\int \frac{d^4 l}{(2\pi)^4} \to \mu_{DS}^{4-d} \int \frac{d^d \bar{l}}{(2\pi)^d}$

[Gnendiger (RADCOR 2017)]

To d, **or not to d?**

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[Fazio, Mastrolia, Mirabella, W.J.T. (2014)]

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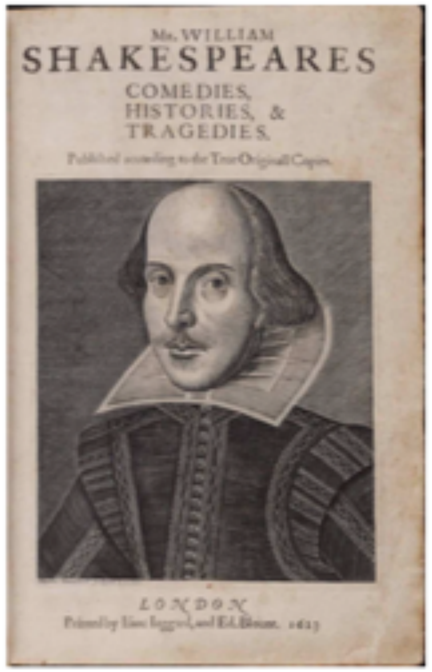


Dimensional regularisation schemes

For all dimensional schemes $$\int \frac{\mathrm{d}^4 l}{(2\pi)^4} \to \mu_{\mathrm{DS}}^{4-d} \int \frac{\mathrm{d}^d \bar{l}}{(2\pi)^d}$$

with the unified framework

$$S_{[4]} \subset QS_{[d]} \subset QS_{[d_s]} \equiv QS_{[d]} \oplus QS_{[n_\epsilon]}$$

$S_{[4]}$	$QS_{[d]}$	$QS_{[d_s]}$	$QS_{[n_\epsilon]}$
strictly four-dim. (unregularized)	quasi d-dim. (PS integration)	quasi d_s-dim. (usually $d_s = 4$)	'evanescent' space

[Gnendiger, *et al* (W.J.T.) (2017)]

The most used DS can be understood as

	CDR	HV	FDH	DRED
singular vector fields (1PI; soft / collinear in initial / final state)	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[d_s]}$	$g^{\mu\nu}_{[d_s]}$
regular vector fields (all other VFs)	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[4]}$	$g^{\mu\nu}_{[4]}$	$g^{\mu\nu}_{[d_s]}$

$$n_\epsilon = d_s - d$$

$$\mathrm{CDR/HV}: n_\epsilon = 0$$

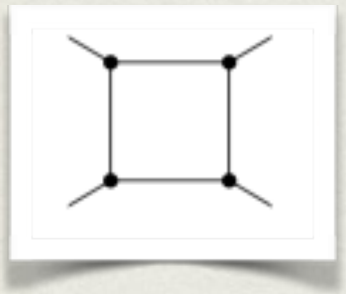
$$\mathrm{FDH/DRED}: n_\epsilon = 2\epsilon$$

[Signer, Stöckinger (2008)]

One-loop scattering amplitudes

Deal with with integrals of the form

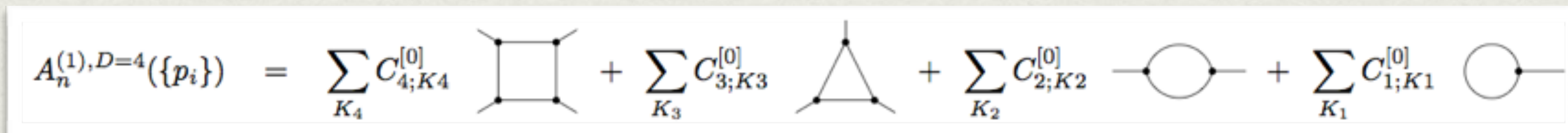
$\bar{l}^2,\ \bar{l}\cdot p_i,\ \bar{l}\cdot \varepsilon_i$



$$I_{i_1\cdots i_k}\left[\mathcal{N}(\bar{l},p_i)\right] = \int d^d\bar{l}\, \frac{\mathcal{N}_{i_1\cdots i_k}(\bar{l},p_i)}{D_{i_1}\cdots D_{i_k}}$$

Numerator and denominators are polynomials in the integration variable

Tensor reduction

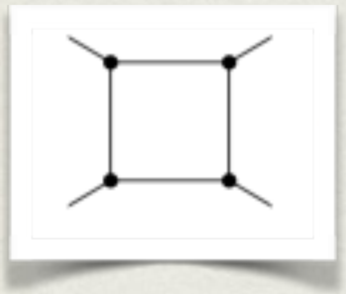


[Passarino - Veltman (1979)]

- ☑ Cut-constructible amplitude -> determined by its branch cuts
- ☑ All one-loop amplitudes are cut-constructible in dimensional regularisation.
- ☑ Master integrals are known

One-loop scattering amplitudes

Deal with with integrals of the form



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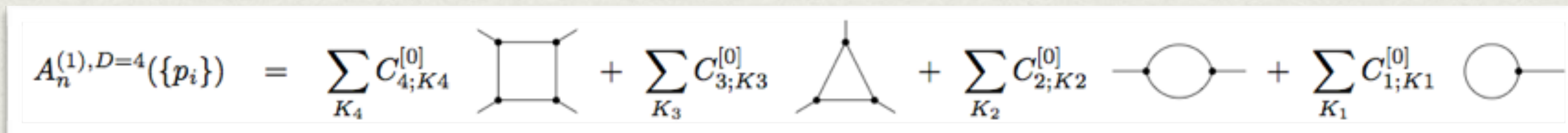
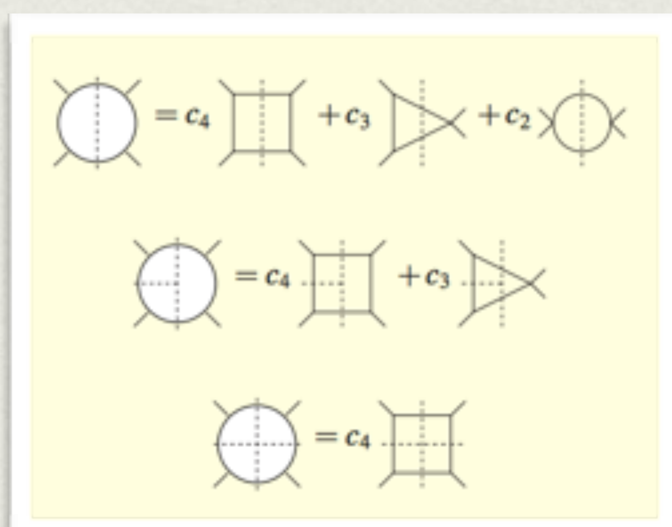


$$A_n^{(1),D=4}(\{p_i\}) = \sum_{K_4} C^{[0]}_{4;K4} + \sum_{K_3} C^{[0]}_{3;K3} + \sum_{K_2} C^{[0]}_{2;K2} + \sum_{K_1} C^{[0]}_{1;K1}$$

[Passarino - Veltman (1979)]

Unitarity based methods $\quad 2\pi\delta^{(+)}\left(p^2 - m^2\right) \rightarrow \frac{i}{p^2 - m^2 - i\epsilon}$



cut-4 :: Britto Cachazo Feng

cut-3 :: Forde
Bjerrum-Bohr, Dunbar, Ita, Perkins
Mastrolia

cut-2 :: Bern, Dixon, Dunbar, Kosower.
Britto, Buchbinder, Cachazo, Feng.
Britto, Feng, Mastrolia.

Isolate the leading discontinuity!

One-loop scattering amplitudes

In D=4-2ε we can do the decomposition

$$\bar{\ell}^\nu = \ell^\nu + \tilde{\ell}^\nu$$

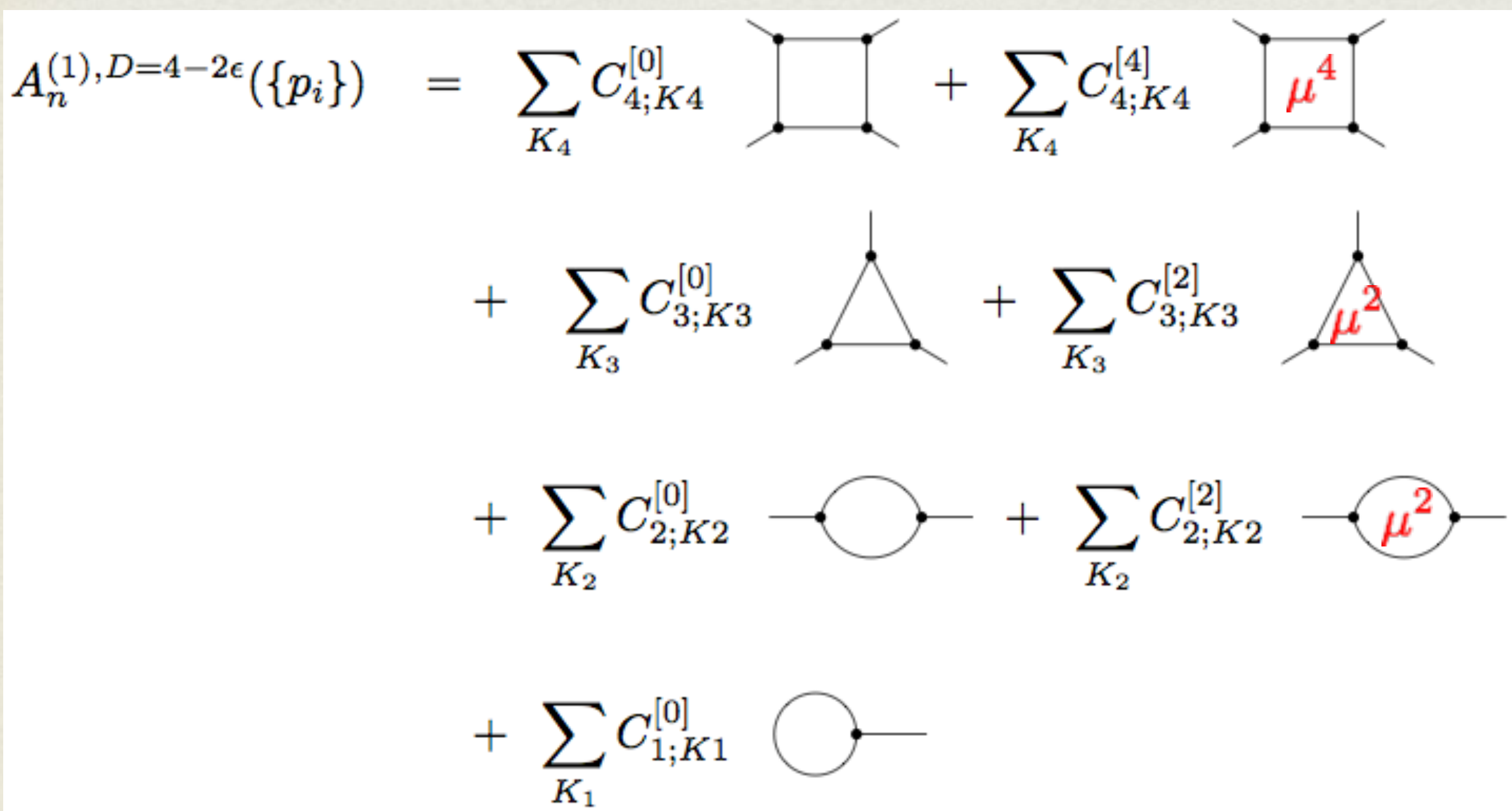
D=4 D=-2ε

At integral level $$\int \frac{d^d\bar{l}}{(2\pi)^d} \equiv \int \frac{d^4 l}{(2\pi)^4}\frac{d^{-2\epsilon}\mu}{(2\pi)^{-2\epsilon}}$$

The on-shell condition $$\bar{\ell}^2 = \ell^2 - \mu^2 = 0 \rightarrow \ell^2 = \mu^2$$

Mass term

Any one-loop amplitude becomes



[Ossola,Papadopoulos,Pittau (2006)]
[Giele,Kunszt,Melnikov (2008)]
[Badger (2008)]
[Mastrolia, Mirabella, Peraro (2012)]

One-loop scattering amplitudes

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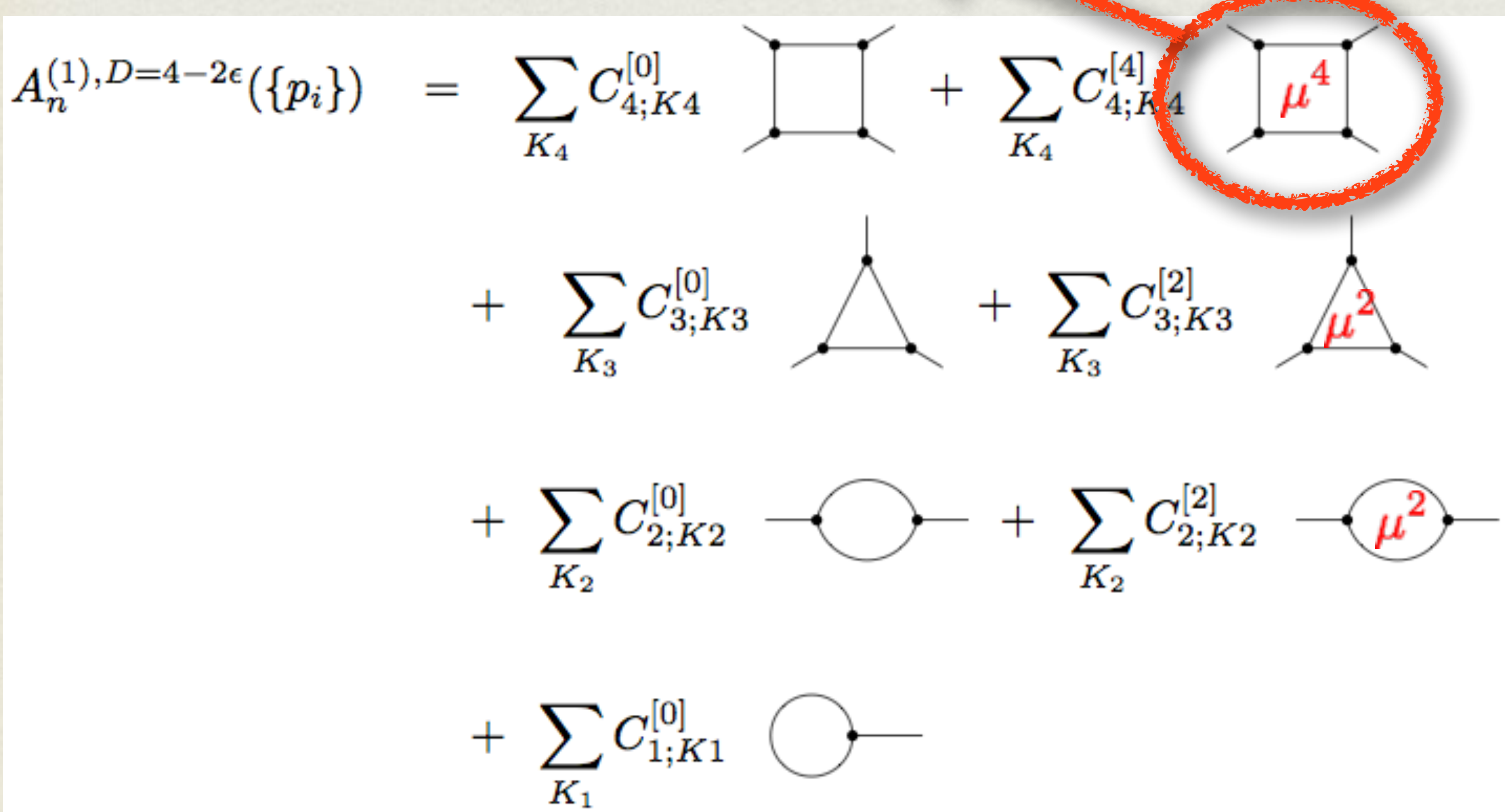
Th

$$\int d^4 l_1 d^{-2\epsilon}\mu \frac{\mu^4}{l^2 (l-K_1)^2 (l-K_1-K_2)^2 (l+K_4)^2} = -\frac{1}{6} = \mu^2$$

Mass term

Any one-loop amplitude becomes



$$A_n^{(1),D=4-2\epsilon}(\{p_i\}) = \sum_{K_4} C^{[0]}_{4;K4} [\text{box}] + \sum_{K_4} C^{[4]}_{4;K4} [\text{box } \mu^4] + \sum_{K_3} C^{[0]}_{3;K3} [\text{triangle}] + \sum_{K_3} C^{[2]}_{3;K3} [\text{triangle } \mu^2] + \sum_{K_2} C^{[0]}_{2;K2} [\text{bubble}] + \sum_{K_2} C^{[2]}_{2;K2} [\text{bubble } \mu^2] + \sum_{K_1} C^{[0]}_{1;K1} [\text{tadpole}]$$

$$I_n^{(1)\,d}[\mu^2] = -\epsilon I_n^{(1)\,d+2}[1]$$
$$I_n^{(1)\,d}[\mu^4] = -\epsilon(1-\epsilon) I_n^{(1)\,d+4}[1]$$

[Bern, Morgan (1995)]

[Ossola,Papadopoulos,Pittau (2006)]
[Giele,Kunszt,Melnikov (2008)]
[Badger (2008)]
[Mastrolia, Mirabella, Peraro (2012)]

One-loop integrand decomposition

[Ossola, Papadopoulos, Pittau (2006)]
[Ellis, Giele, Kunszt, Melnikov (2007)]
[Mastrolia, Ossola, Papadopoulos, Pittau (2008)]

Recall

$$\int d^4\bar{l}\,\frac{\mathcal{N}(l)}{D_1\cdots D_n} = \sum_{i\ll m} c_{ijkm}\int d^4\bar{l}\,\frac{1}{D_iD_jD_kD_m} + \sum_{i\ll k} c_{ijk}\int d^4\bar{l}\,\frac{1}{D_iD_jD_k} + \sum_{i<j} c_{ij}\int d^4\bar{l}\,\frac{1}{D_iD_j} + \sum_{i} c_{i}\int d^4\bar{l}\,\frac{1}{D_i}$$

Find an identity between integrands. Moreover,

$$\frac{\mathcal{N}(l)}{D_1\cdots D_n} \neq \sum_{i\ll m} c_{ijkm}\frac{1}{D_iD_jD_kD_m} + \sum_{i\ll k} c_{ijk}\frac{1}{D_iD_jD_k} + \sum_{i<j} c_{ij}\frac{1}{D_iD_j} + \sum_{i} c_{i}\frac{1}{D_i}$$

Suppose a multipole decomposition

$$\frac{\mathcal{N}(l)}{D_1\cdots D_n} = \sum_{i\ll m} \tilde{c}_{ijkm}\frac{\Delta_{ijkm}(l)}{D_iD_jD_kD_m} + \sum_{i\ll k} \tilde{c}_{ijk}\frac{\Delta_{ijk}(l)}{D_iD_jD_k} + \sum_{i<j} \tilde{c}_{ij}\frac{\Delta_{ij}(l)}{D_iD_j} + \sum_{i} \tilde{c}_{i}\frac{\Delta_i(l)}{D_i}$$

- ☑ Residues Δ are made of **I**rreducible **S**calar **P**roducts
- ☑ Can we find parametric expressions for Δ's in 4- or d-dimensions?
- ☑ Parametric expressions

One-loop integrand decomposition

[Ossola, Papadopoulos, Pittau (2006)]
[Ellis, Giele, Kunszt, Melnikov (2007)]
[Mastrolia, Ossola, Papadopoulos, Pittau (2008)]

Recall

$$\int d^4\bar{l}\,\frac{\mathcal{N}(l)}{D_1\cdots D_n} = \sum_{i\ll m} c_{ijkm}\int d^4\bar{l}\,\frac{1}{D_iD_jD_kD_m} + \sum_{i\ll k} c_{ijk}\int d^4\bar{l}\,\frac{1}{D_iD_jD_k} + \sum_{i<j} c_{ij}\int d^4\bar{l}\,\frac{1}{D_iD_j} + \sum_{i} c_{i}\int d^4\bar{l}\,\frac{1}{D_i}$$

Find an identity between integrands. Moreover,

$$\frac{\mathcal{N}(l)}{D_1\cdots D_n} \neq \sum_{i\ll m} c_{ijkm}\frac{1}{D_iD_jD_kD_m} + \sum_{i\ll k} c_{ijk}\frac{1}{D_iD_jD_k} + \sum_{i<j} c_{ij}\frac{1}{D_iD_j} + \sum_{i} c_{i}\frac{1}{D_i}$$

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- ☑ Residues Δ are made of **I**rreducible **S**calar **P**roducts
- ☑ Can we find parametric expressions for Δ's in 4- or d-dimensions?
- ☑ Parametric expressions coefficients are fixed by sampling the numerators on the cut

Yes. General way —> Use multivariate polynomial division

One-loop integrand decomposition

Loop parametrisation

$$l_i^\alpha = p_i^\alpha + x_1 e_1^\alpha + x_2 e_2^\alpha + x_3 e_3^\alpha + x_4 e_4^\alpha$$
$$\mathcal{N}\left(\bar{l}\right) = \mathcal{N}\left(l, \mu^2\right) = \mathcal{N}\left(\boldsymbol{z}\right) \qquad \boldsymbol{z} = \{x_1, x_2, x_3, x_4, \mu^2\}$$

Multivariate polynomial division

[Mastrolia, Ossola (2011)]
[Badger, Frellesvig, Zhang (2012)]
[Zhang (2012)]
[Mastrolia, Mirabella, Ossola, Peraro (2012)]

Consider the **ideal** generated by the set of denominators

$$\mathcal{J}_{1\cdots n} \equiv \langle D_1, \cdots, D_n \rangle = \left\{ \sum_{k=1}^{n} h_k(\mathbf{z}) D_k(\mathbf{z}) \;:\; h_k(\mathbf{z}) \in P[\mathbf{z}] \right\}$$

Choose a monomial order and build a Groebner basis $\mathcal{G}\left(\mathbf{z}\right) = \{g_1\left(\mathbf{z}\right), \ldots, g_m\left(\mathbf{z}\right)\}$

$$D_1(\mathbf{z}) = \cdots = D_n(\mathbf{z}) = 0 \Longleftrightarrow g_1(\mathbf{z}) = \cdots = g_m(\mathbf{z}) = 0$$

Perform the multivariate polynomial division of $\mathcal{N}\left(\boldsymbol{z}\right)$ module $\mathcal{G}\left(\mathbf{z}\right)$

$$\mathcal{N}_{1\cdots n}(\mathbf{z}) = \sum_{k=1}^{m} \underbrace{\Gamma_{1\cdots k-1\, k+1\cdots m}(\mathbf{z}) g_k(\mathbf{z})}_{\text{Quotient}} + \underbrace{\Delta_{1\cdots n}(\mathbf{z})}_{\text{Remainder}}$$

Express back the elements of $\mathcal{G}\left(\mathbf{z}\right)$ in terms of denominators

$$\mathcal{N}_{1\cdots n}(\mathbf{z}) = \sum_{k=1}^{n} \underbrace{\mathcal{N}_{i\cdots k-1\, k+1\cdots n}(\mathbf{z})}_{\text{Subtopology}} D_k(\mathbf{z}) + \underbrace{\Delta_{1\cdots n}(\mathbf{z})}_{\text{Residue}}$$

One-loop integrand decomposition

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[Mastrolia, Ossola (2011)]
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Multivariate polynomial division

Write the numerator in terms of Irreducible polynomials

$$\mathcal{I} \equiv \frac{\mathcal{N}}{D_0 \cdots D_{n-1}} = \sum_{k=1}^{5} \sum_{\{i_1, \cdots, i_k\}} \frac{\Delta_{i_1 \cdots i_k}}{D_{i_1} \cdots D_{i_k}}$$

sum of integrands with five or less denominators

$\Delta_{i_1 \cdots i_k}$ Made of **I**rreducible **S**calar **P**roducts
Cannot be expressed in terms of denominators

Generic structure of the residue

$$\begin{aligned}
\Delta_{i_1 i_2 i_3 i_4 i_5} &= c_0\, \mu^2, \\
\Delta_{i_1 i_2 i_3 i_4} &= c_0 + c_1\, x_{4,v} + \mu^2 \left(c_2 + c_3\, x_{4,v} + \mu^2\, c_4\right), \\
\Delta_{i_1 i_2 i_3} &= c_0 + c_1\, x_4 + c_2\, x_4^2 + c_3\, x_4^3 + c_4\, x_3 + c_5\, x_3^2 + c_6\, x_3^3 + +\mu^2 \left(c_7 + c_8\, x_4 + c_9\, x_3\right), \\
\Delta_{i_1 i_2} &= c_0 + c_1\, x_1 + c_2\, x_1^2 + c_3\, x_4 + c_4\, x_4^2 + c_5\, x_3 + c_6\, x_3^2 + c_7\, x_1 x_4 + c_8\, x_1 x_3 + c_9\, \mu^2, \\
\Delta_{i_1} &= c_0 + c_1\, x_1 + c_2\, x_2 + c_3\, x_3 + c_4\, x_4,
\end{aligned}$$

Adaptive integrand decomposition (AID)

$$d = d_{\parallel} + d_{\perp}$$

$$d_{\parallel} = n - 1 \qquad d_{\perp} = (5-n) - 2\epsilon$$

- Splits d=4-2ε into parallel and orthogonal directions
- Nice properties for less than 5 external legs

[Collins (1984)]
[van Neerven and Vermaseren (1984)]
[Kreimer (1992)]

Loop momenta

$$\bar{l}_i^{\alpha} = \bar{l}_{\parallel i}^{\alpha} + \lambda_i^{\alpha} \longrightarrow \bar{l}_i^{\alpha} = \sum_{j=1}^{d_{\parallel}} x_{ji}\, e_j^{\alpha}\,, \qquad \lambda_i^{\alpha} = \sum_{j=d_{\parallel}+1}^{4} x_{ji}\, e_j^{\alpha} + \mu_i^{\alpha}\,, \qquad \lambda_{ij} = \sum_{l=d_{\parallel}+1}^{4} x_{li} x_{lj} + \mu_{ij}$$

- Numerator and denominators depend on different variables

[Mastrolia, Peraro, Primo (2016)]

$$\int \prod_i d^{d_{\parallel}} \bar{l}_{\parallel i} \int \prod_{1 \le i \le j \le \ell} d\lambda_{ij}\, G(\lambda_{ij})^{\frac{d_{\perp}-1-\ell}{2}} \int d\Theta_{\perp} \frac{\mathcal{N}(\bar{l}_{\parallel i}, \lambda_{ij} \Theta_{\perp})}{D_1(\bar{l}_{\parallel i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

Straightforward integration of transverse components

Expand in Gegenbauer polynomials

$$\int d\,\Theta_{\perp} = \int_{-1}^{1} \prod_{i=1}^{4-d_{\parallel}} \prod_{j=1}^{\ell} d\cos\theta_{i+j-1\,j} (\sin\theta_{i+j-1\,j})^{d_{\perp}-i-j-1}$$

$$\int_{-1}^{1} d\cos\theta (\sin\theta)^{2\alpha-1} C_n^{(\alpha)}(\cos\theta) C_m^{(\alpha)}(\cos\theta) = \delta_{mn} \frac{2^{1-2\alpha}\pi\Gamma(n+2\alpha)}{n!(n+\alpha)\Gamma^2(\alpha)}$$

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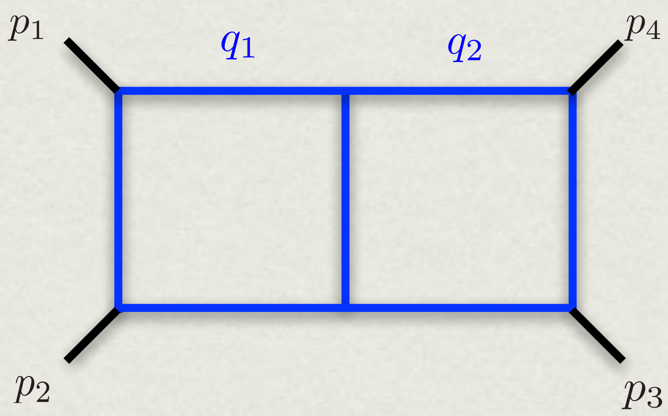
and identification of spurious terms

Adaptive integrand decomposition (AID)

Example 1



- Decompose q_i^α in long/transv components:

$$q_{i\,\parallel}^\alpha = x_{i1}e_1^\alpha + x_{i2}e_2^\alpha + x_{i3}e_3^\alpha$$

$$\lambda_i^\alpha = x_{4i}e_4^\alpha + \mu_i^\alpha$$

$$D_i = l_{\parallel\, i}^2 + \sum_{j,k} \alpha_{ij}\alpha_{ik}\,\lambda_{jk} + m_i^2$$

$$d_\parallel = 3 \to e_4 \cdot p_i = 0$$

- Parametrise the integral as

$$I_4^{d\,(2)}[\mathcal{N}] = \frac{2^{d-6}}{\pi^5\Gamma(d-5)} \int d^3q_{1\,\parallel} \int d^3q_{2\,\parallel} \int d\lambda_{11}d\lambda_{22}d\lambda_{12}[G(\lambda_{ij})]^{\frac{d-6}{2}} \times \int_{-1}^{1} d\cos\theta_{11}d\cos\theta_{22}(\sin\theta_{11})^{d-6}(\sin\theta_{11})^{d-7}\frac{\mathcal{N}}{D_1\cdots D_7}$$

with

$$G(\lambda_{ij}) = \lambda_{11}\lambda_{22} - \lambda_{12}^2$$

$$\begin{cases} x_{41} = \sqrt{\lambda_{11}}\cos\theta_{11} \\ x_{42} = \sqrt{\lambda_{22}}(\cos\theta_{11}\cos\theta_{12} + \sin\theta_{12}\cos\theta_{22}) \end{cases}$$

- Integrate away transverse directions

$$I_4^{d\,(2)}[\,x_{41}^{\alpha_4}x_{42}^{\beta_4}\,] = 0 \qquad \alpha_4 + \beta_4 = 2n+1$$

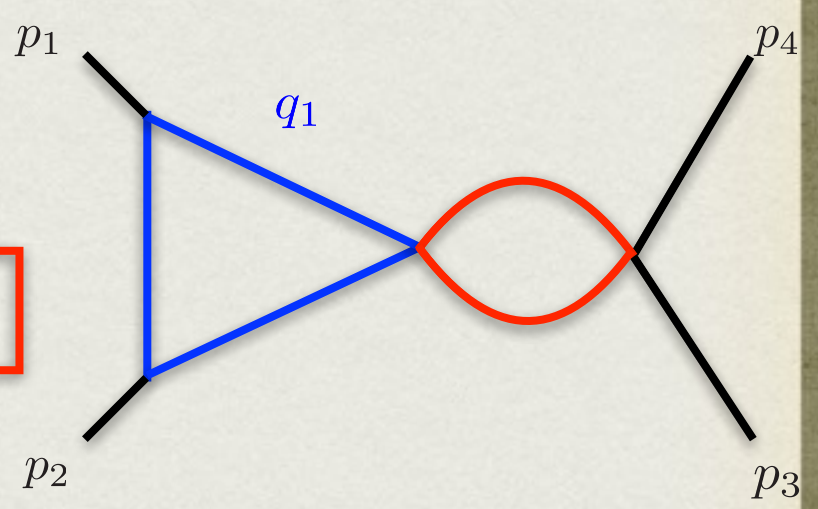
$$I_4^{d\,(2)}[\,x_{42}^3x_{41}^3\,] = \frac{3}{(d-3)(d-1)(d+1)}I_4^{d\,(2)}[\lambda_{12}(2\lambda_{12}^2 + 3\lambda_{11}\lambda_{22})]$$

$$I_4^{d\,(2)}[\,x_{41}^2x_{42}^2\,] = \frac{3}{(d-3)(d-1)}I_4^{d\,(2)}[\,2\lambda_{12}^2 + \lambda_{11}\lambda_{22}]$$

. . .

Adaptive integrand decomposition (AID)

Example 2



- Decompose q_i^α in long/transv components:

$d_{\|} = 2 \to e_{3,4} \cdot p_{1,2} = 0$

$$q_{1\,\|}^\alpha = x_{11} e_1^\alpha + x_{12} e_2^\alpha$$
$$\lambda_1^\alpha = x_{13} e_3^\alpha + x_{14} e_4^\alpha + \mu_1^\alpha$$

$d_{\|} = 1 \to e_{2,3,4} \cdot (p_3 + p_4) = 0$

$$q_{2\,\|}^\alpha = x_{21} \hat{e}_1^\alpha$$
$$\lambda_2^\alpha = x_{22} \hat{e}_2^\alpha + x_{23} \hat{e}_3^\alpha + x_{24} \hat{e}_4^\alpha + \mu_2^\alpha$$

- Parametrise the integral as

$$\mathcal{N}(q_1, q_2) = (\mu_{12})^\alpha \mathcal{N}(q_{1\,[4]}, \mu_{11}) \mathcal{N}(q_{2\,[4]}, \mu_{22})$$

$$I_4^{d\,(2)}[\mathcal{N}] = \Omega_d \int d^2 q_{1\,\|} \int d\lambda_{11} [\lambda_{11}]^{\frac{d-4}{2}} \int dc_{\theta_{11}} dc_{\theta_{12}} (s_{\theta_{11}})^{d-5} (s_{\theta_{12}})^{d-6} \frac{\mathcal{N}_1}{D_1 D_2 D_3}$$
$$\times \int dq_{2\,\|} \int d\lambda_{22} [\lambda_{22}]^{\frac{d-3}{2}} \int dc_{\theta_{21}} dc_{\theta_{22}} dc_{\theta_{23}} (s_{\theta_{21}})^{d-4} (s_{\theta_{22}})^{d-5} (s_{\theta_{23}})^{d-6} \frac{\mathcal{N}_2}{D_4 D_5}$$

with

$$\begin{cases} x_{13} = & \sqrt{\lambda_{11}} c_{\theta_{11}} \\ x_{14} = & \sqrt{\lambda_{11}} s_{\theta_{11}} c_{\theta_{12}} \end{cases} \qquad \begin{cases} x_{22} = & \sqrt{\lambda_{22}} c_{\theta_{21}} \\ x_{23} = & \sqrt{\lambda_{22}} s_{\theta_{21}} c_{\theta_{22}} \\ x_{24} = & \sqrt{\lambda_{22}} s_{\theta_{21}} s_{\theta_{22}} c_{\theta_{23}} \end{cases}$$

Adaptive integrand decomposition (AID)

[Mastrolia, Peraro, Primo (2016)]

[Mastrolia, Peraro, Primo, W.J.T. (2016)]

Algorithm

- For each integrand, adapt longitudinal and parallel components
- Denominators depend on the minimal set of variables
- Loop components expressed as linear combination of denominators
- Poly division and integration reduced to substitution rules
- Extra dimension variables are always reducible

Recipe in **3** steps

1) Divide and get $\Delta(\bar{l}_{\parallel i}, \lambda_{ij}, \Theta_\perp)$
2) Integrate out transverse variables $\Theta_\perp$
3) Divide again to get rid of λ_{ij}

Features

- Final decomposition in terms of ISPs
- No need for TID
- Output ready to apply IBPs
- @1L no need of any integral identity

$$\frac{\mathcal{N}(\bar{l}_{\parallel i}, \lambda_{ij}, \Theta_\perp)}{D_1(\bar{l}_{\parallel i}, \lambda_{ij})\cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

1)

$$\frac{\Delta(\bar{l}_{\parallel i}, \lambda_{ij}, \Theta_\perp)}{D_1(\bar{l}_{\parallel i}, \lambda_{ij})\cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

2)

$$\frac{\Delta^{\text{int}}(\bar{l}_{\parallel i}, \lambda_{ij})}{D_1(\bar{l}_{\parallel i}, \lambda_{ij})\cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

3)

$$\frac{\Delta'(\bar{l}_{\parallel i})}{D_1(\bar{l}_{\parallel i}, \lambda_{ij})\cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

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Algorithm requires **automation**

AIDA: a Mathematica implementation

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]

[W.J.T. (2018)]

AMPLITUDE GENERATOR
(FeynArts+FeynCalc, QGRAF+FORM…)

AIDA
(Adaptive Integrand Decomposition Algorithm)

IBPs REDUCTION CODE
(Reduze, FIRE, Kira, Azurite…)

COMPUTE MIs

Analytically
(Loopedia)

Numerically
(SecDec, FIESTA…)

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Cut solution parametrisation

- Identify parent topology
- Analyse kinematics for all cuts
- Store adaptive parametrisation
- Organise cuts in Jobs

Read numerators

Divide
(apply substitution rules)

Numerators = $\Delta^{\text{int}}(\bar{l}_{\parallel i}, \lambda_{ij})$

Residues depend on $\Theta_\perp$?

No

Yes

Integrate
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The code is designed for general **one-** and **two**-loop **numerical** and **analytical** calculations

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AIDA for muon-electron scattering

- Recent proposal for the determination of the hadronic contribution to the muon from the measurement of muon-electron scattering $g-2$

[Carloni Calame, Passera, Trentadue, Venanzoni (2015)]
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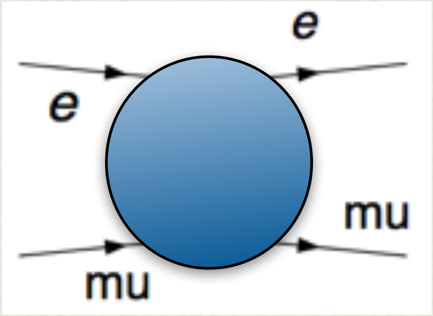
- In the massless electron limit, 4-point process depending on 3 scales



$$s = (p_1 + p_2)^2 \qquad t = (p_2 + p_3)^2$$
$$m_e^2 \simeq 0 \qquad u = -s - t + 2m^2$$

$e(p_1) + \mu(p_4) \to e(-p_2) + \mu(-p_3)$

- NNLO virtual contribution with adaptive integrand decomposition

[Ossola, Mastrolia, Peraro, Primo, Ronca, Schubert, W.J.T. (work in progress)]

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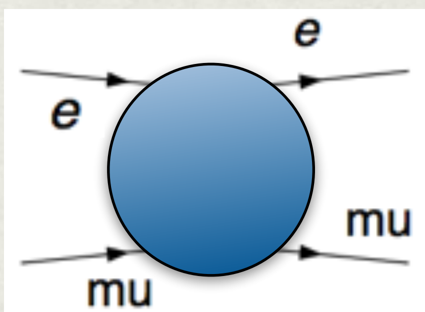

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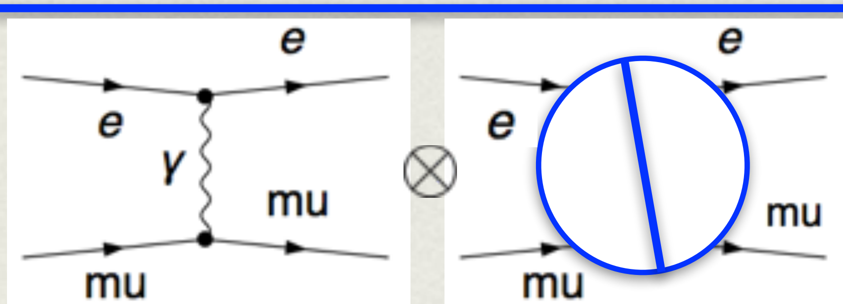


$$= \sum_{k=2}^{7} \sum_{i_1 \cdots i_k} \frac{\Delta_{i_1 \cdots i_k}(q_i)}{D_{i_1} \cdots D_{i_k}}$$

$$\Delta_{i_1 \cdots i_k}(q_i) \to c_{i_1 \cdots i_k}(s, t, m^2, d) \prod_{i,j} (q_i \cdot p_j)^{\alpha_{ij}}$$

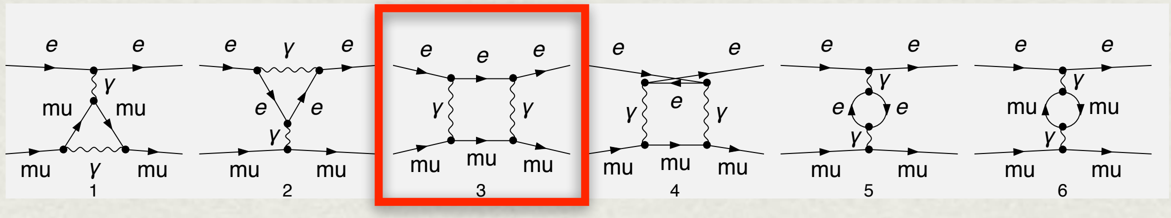
AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]

Initialisation

- Identify parent topologies from Feynman graphs

e.g. 1 Loop



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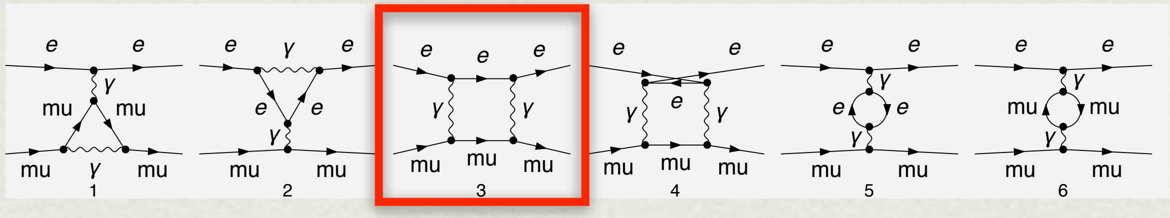
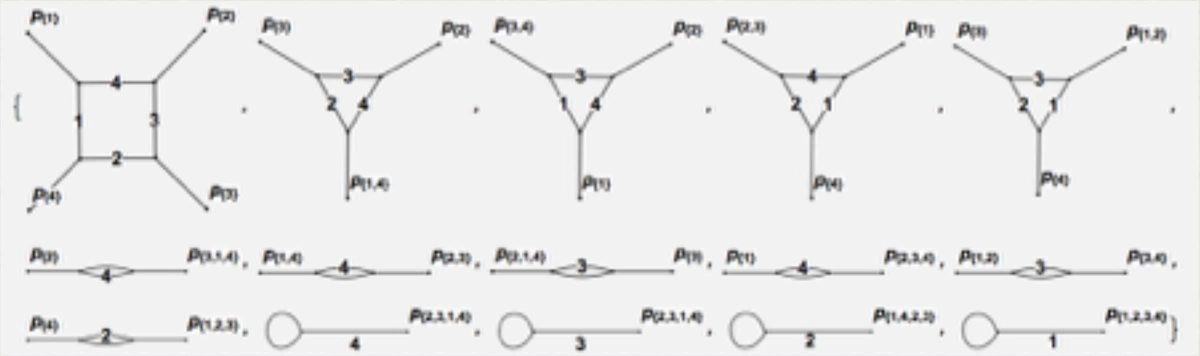


- Generate all cuts and analyse their kinematics



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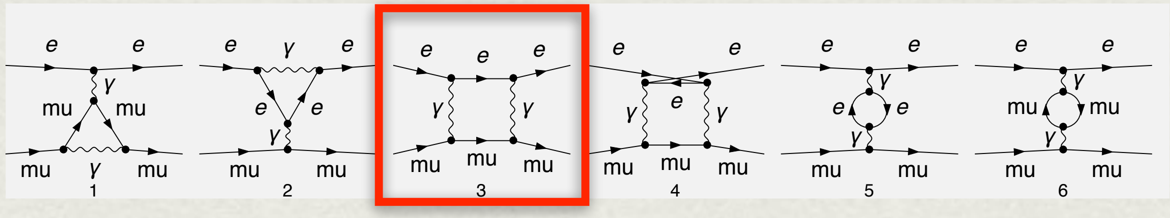
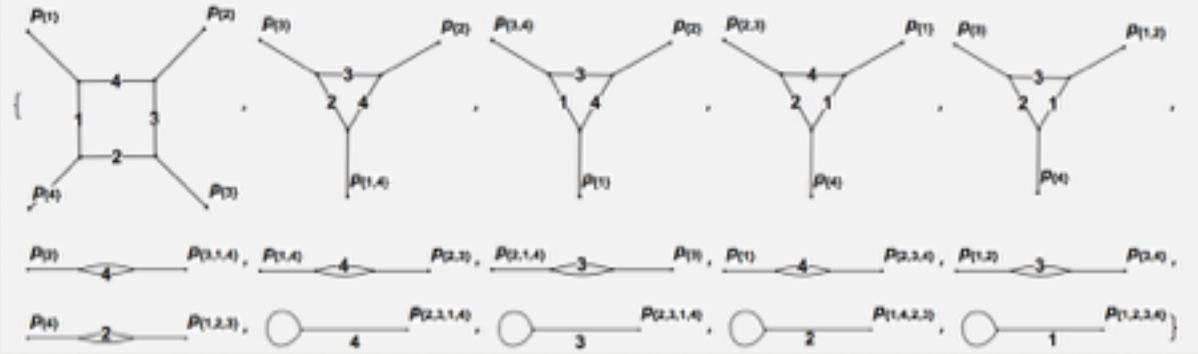


- Generate all cuts and analyse their kinematics



- Define adaptive variables and prepare substitution rules for all cuts

$$x_{1,\{1,2,3\}} \to \frac{-s+d[1]-2\,d[2]+d[3]}{-4\,m^2+s}$$

$$x_{2,\{1,2,3\}} \to \frac{2\,m^2\,s+2\,m^2\,d[1]-s\,d[1]+s\,d[2]-2\,m^2\,d[3]}{\left(4\,m^2-s\right)\,s}$$

$$\lambda^2_{\{1,2,3\}} \to \frac{m^2\,s^2-2\,m^2\,s\,d[1]+m^2\,d[1]^2+s^2\,d[2]-s\,d[1]\,d[2]+s\,d[2]^2-2\,m^2\,s\,d[3]-2\,m^2\,d[1]\,d[3]+s\,d[1]\,d[3]-s\,d[2]\,d[3]+m^2\,d[3]^2}{s\left(-4\,m^2+s\right)}$$

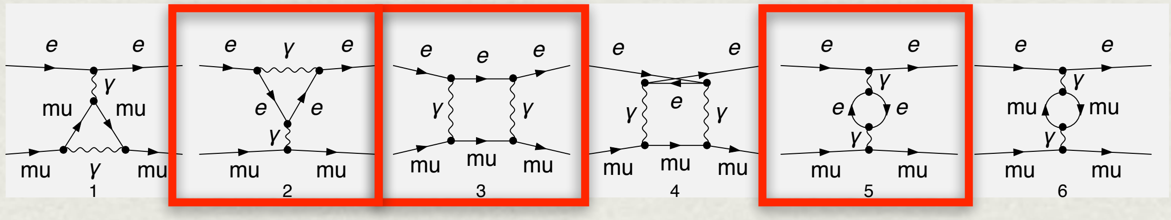
AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]

Job structure

- Group diagrams belonging to the same parent topology

e.g. 1 Loop



AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]

Job structure

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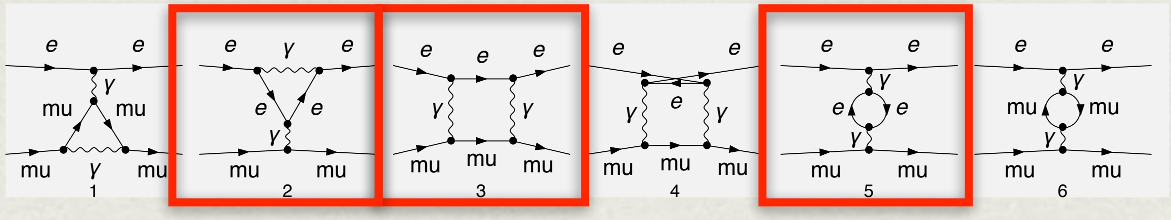
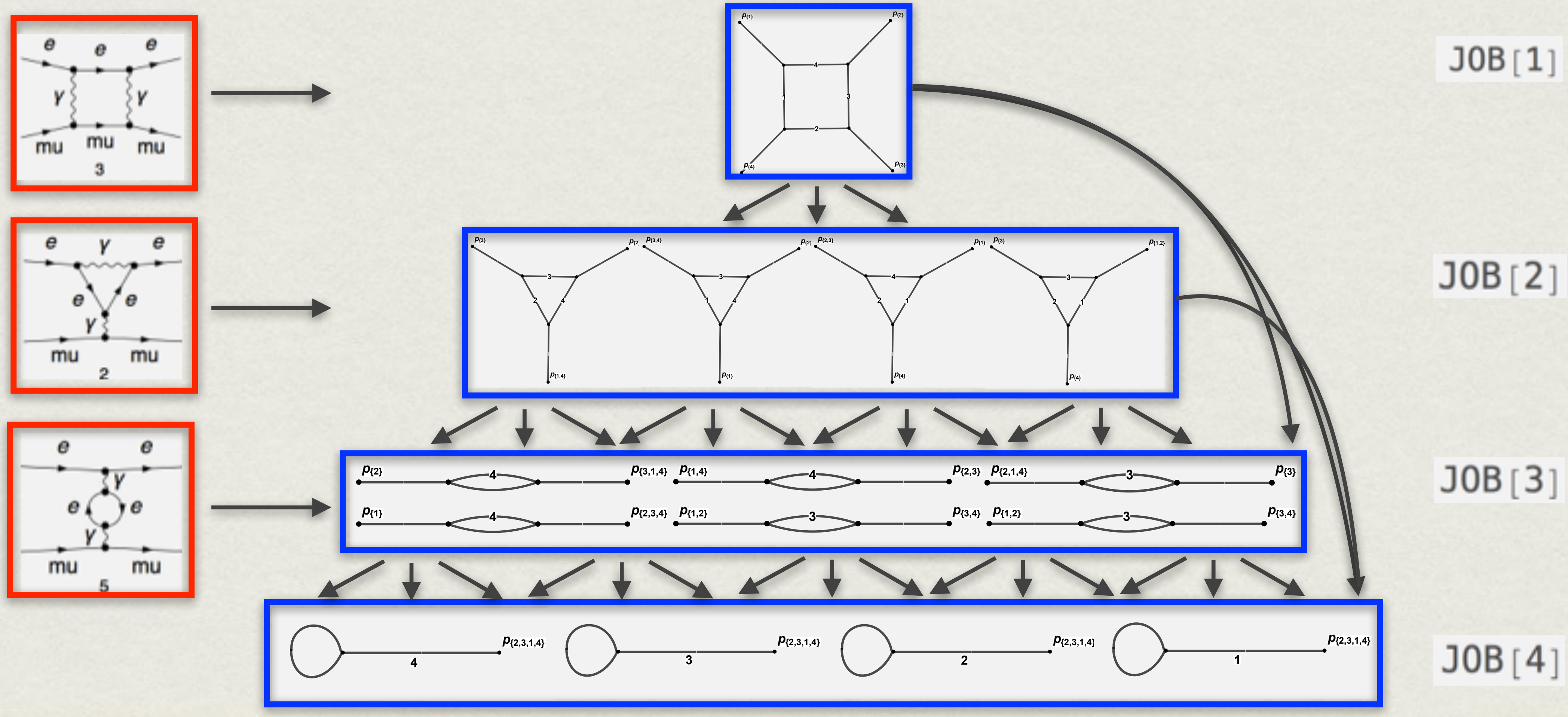


- Organise all cuts of the parent topology in **Jobs**

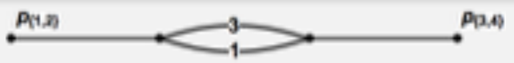


AIDA for muon-electron scattering

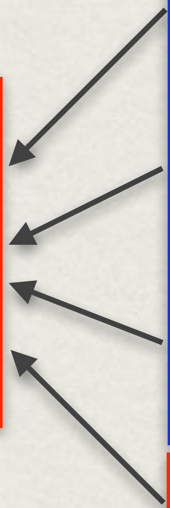
[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]

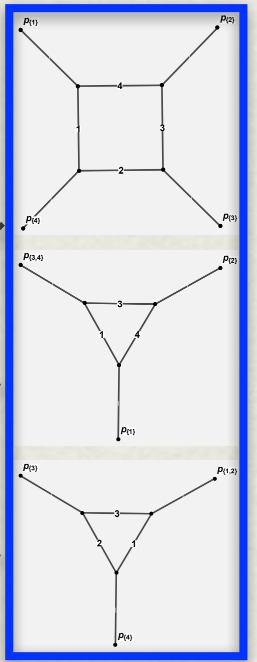
Divide - Integrate - Divide

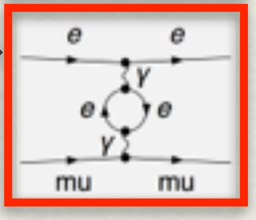
- For every Job, build the numerators of the corresponding cuts



$$\mathrm{Num}[\{\{1,3\}\},\{1,0,1,0\}] \to \begin{array}{l} \mathrm{Num}[\{\{1,3\}\},\{1,0,1,0\}]_{\{\{1,2,3,4\}\},\{1,1,1,1\}} \\ \mathrm{Num}[\{\{1,3\}\},\{1,0,1,0\}]_{\{\{1,3,4\}\},\{1,0,1,1\}} \\ \mathrm{Num}[\{\{1,3\}\},\{1,0,1,0\}]_{\{\{1,2,3\}\},\{1,1,1,0\}} \\ \mathrm{Num}[\{\{1,3\}\},\{1,0,1,0\}]_{\{\{1,3\}\},\{1,0,1,0\}} \end{array}$$







- Apply substitution rules to the numerator

$$x_{1,\{1,3\}} \to \frac{s+d[1]-d[3]}{2\,s}$$

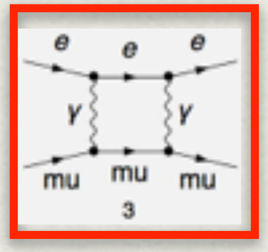
$$\lambda^2_{\{1,3\}} \to \frac{-s^2+2\,s\,d[1]-d[1]^2+2\,s\,d[3]+2\,d[1]\,d[3]-d[3]^2}{4\,s}$$

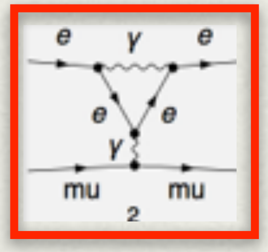
- Collect powers of denominators to read off residue and numerators of lower cuts
- Integrate (substitute) transverse vars appearing in the residues
- Division again, using as input numerators the residues!

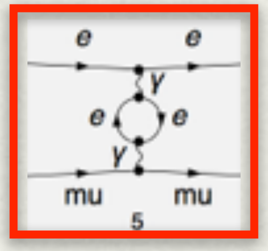
AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]

Input numerators



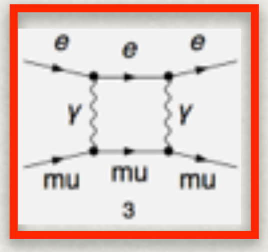


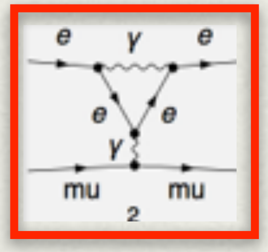


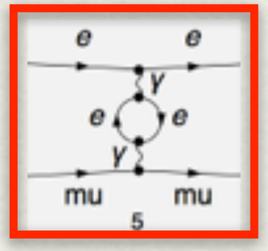
AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]

Input numerators







$$\Delta[\{\{1,2,3,4\}\},\{1,1,1,1\}] \to \frac{2\left(m^2-t\right)\left(16 m^4+(-8+3d)\, s^2-32 m^2 t+8 s t+16 t^2\right)}{s}$$

$$\Delta[\{\{2,3,4\}\},\{0,1,1,1\}] \to \frac{2\left(m^2-t\right)(d\, s+8 t)}{s}$$

$$\Delta[\{\{1,3,4\}\},\{1,0,1,1\}] \to \frac{1}{s^3}$$

$$2\left(s^2\left(32 m^4+(-16+7d)\, s^2-64 m^2 t+24 s t+32 t^2\right)+16(-2+d)\left(m^2-t\right)^2\left(m^4(1+8s)+m^2\left(5s^2-2t-16 s t\right)+t(s+t+8 s t)\right)^2 x^2_{3,\{\{1,3,4\}\}}+64(-2+d)\left(m^2-t\right)^2\left(m^4(1-8s)+t(s+t-8 s t)+m^2\left(-5s^2-2t+16 s t\right)\right)^2 x^2_{4,\{\{1,3,4\}\}}\right)$$

$$\Delta[\{\{1,2,4\}\},\{1,1,0,1\}] \to \frac{2\left(m^2-t\right)(d\, s+8 t)}{s}$$

$$\Delta[\{\{1,2,3\}\},\{1,1,1,0\}] \to \frac{2\left(8 m^4((-2+d)\, s-8 t)+8 m^2\left((-3+d)\, s^2-(-8+d)\, s t+8 t^2\right)-s\left((-8+3d)\, s^2+8 s t+16 t^2\right)\right)}{\left(4 m^2-s\right) s}$$

$$\Delta[\{\{3,4\}\},\{0,0,1,1\}] \to -\frac{8\left(4 m^4+(-2+d)\, s^2-8 m^2 t+4 s t+4 t^2\right)}{s^2}$$

$$\Delta[\{\{2,4\}\},\{0,1,0,1\}] \to \frac{4\left(2(-2+d)\, m^4-m^2((-8+3d)\, s-4(-2+d)\, t)+t((-8+3d)\, s+2(-2+d)\, t)\right)}{s\left(m^2-t\right)}$$

$$\Delta[\{\{2,3\}\},\{0,1,1,0\}] \to -\frac{4 m^2\left(4(-2+d)\, m^4+(8-3d)\, s^2+2(-2+d)\, s t+4(-2+d)\, t^2+2 m^2((-14+5d)\, s-4(-2+d)\, t)\right)}{\left(4 m^2-s\right) s\left(m^2-t\right)}$$

$$\Delta[\{\{1,4\}\},\{1,0,0,1\}] \to -\frac{8\left(4 m^4+(-2+d)\, s^2-8 m^2 t+4 s t+4 t^2\right)}{s^2}$$

$$\Delta[\{\{1,3\}\},\{1,0,1,0\}] \to$$

$$-\frac{4\left(16(-8+d)\, m^6-4(-8+d)\, m^4(s+8t)-s\left(\left(28-14d+d^2\right) s^2+2(-14+d)\, s t+4(-8+d)\, t^2\right)+2 m^2\left(\left(54-27d+2d^2\right) s^2+12(-8+d)\, s t+8(-8+d)\, t^2\right)\right)}{\left(4 m^2-s\right) s^2}+128\, x^2_{2,\{\{1,3\}\}}+$$

$$\frac{512\left(m^2-t\right)^2\left(m^4(1+8s)+m^2\left(5s^2-2t-16 s t\right)+t(s+t+8 s t)\right)^2 x^2_{3,\{\{1,3\}\}}}{s^4}+\frac{8192\left(m^2-t\right)^2\left(m^4(-1+8s)-m^2\left(5s^2+2t-16 s t\right)+t(-t+s(-1+8t))\right)^2 x^2_{4,\{\{1,3\}\}}}{s^4}$$

$$\Delta[\{\{1,2\}\},\{1,1,0,0\}] \to -\frac{4 m^2\left(4(-2+d)\, m^4+(8-3d)\, s^2+2(-2+d)\, s t+4(-2+d)\, t^2+2 m^2((-14+5d)\, s-4(-2+d)\, t)\right)}{\left(4 m^2-s\right) s\left(m^2-t\right)}$$

$$\Delta[\{\{4\}\},\{0,0,0,1\}] \to \frac{8(-2+d)}{s}$$

$$\Delta[\{\{3\}\},\{0,0,1,0\}] \to -\frac{4\left(8 m^4+(-8+3d)\, s^2-16 m^2 t+8 s t+8 t^2\right)}{s^3}$$

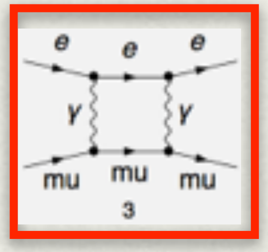
$$\Delta[\{\{2\}\},\{0,1,0,0\}] \to 0$$

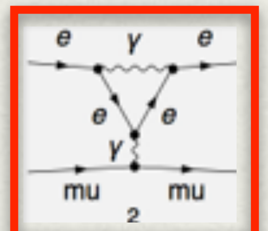
$$\Delta[\{\{1\}\},\{1,0,0,0\}] \to -\frac{4\left(8 m^4+(-8+3d)\, s^2-16 m^2 t+8 s t+8 t^2\right)}{s^3}$$

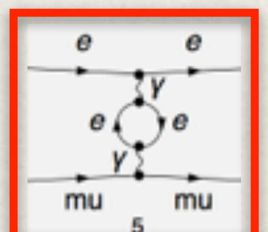
AIDA for muon-electron scattering

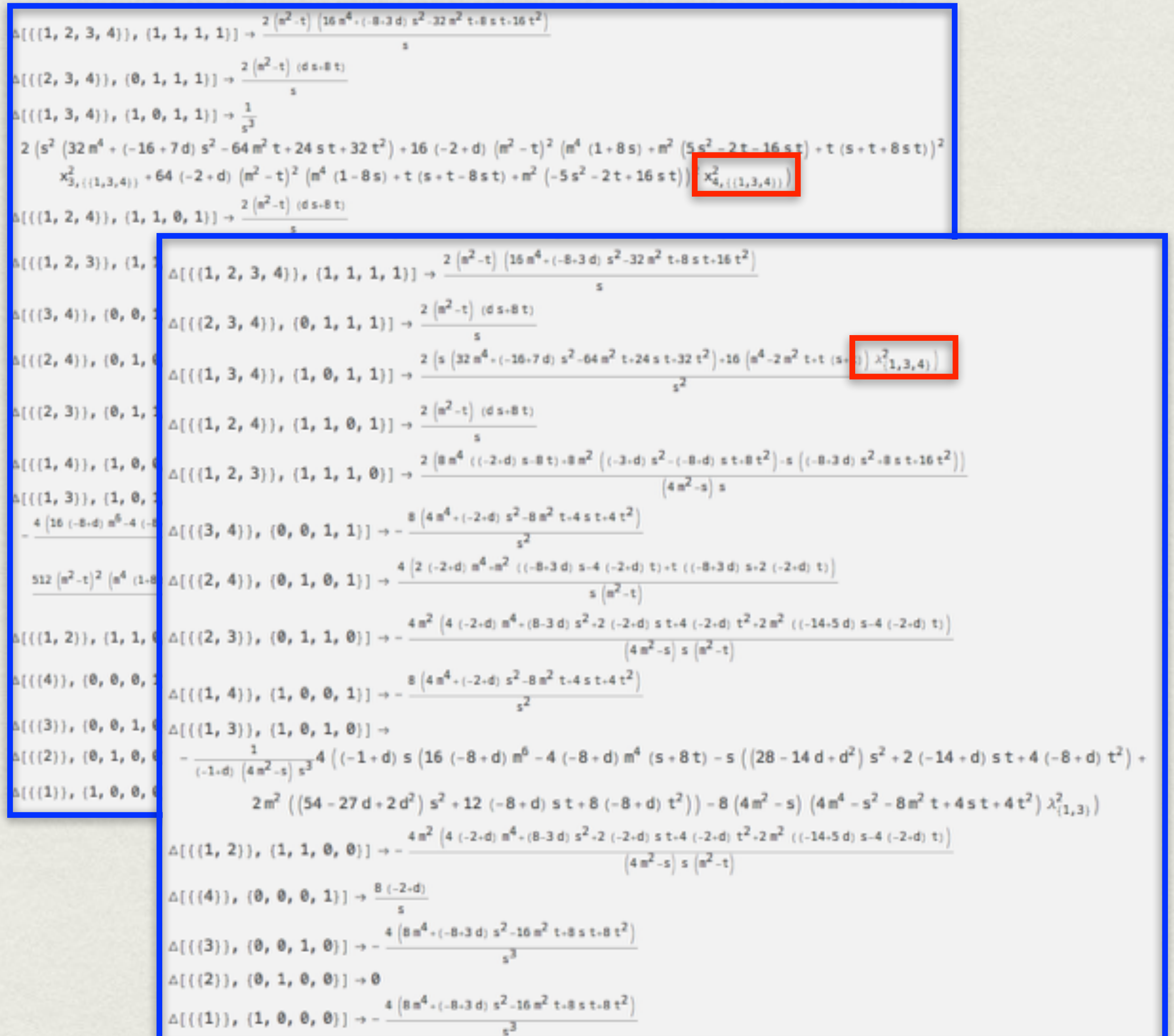
[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]

Input numerators









AIDA for muon-electron scattering

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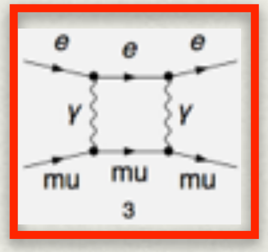
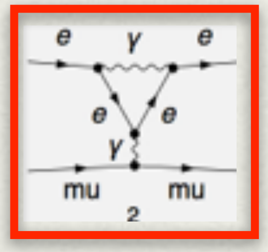
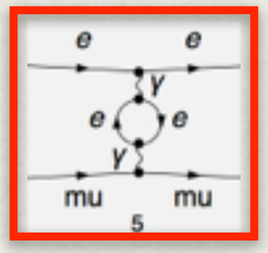
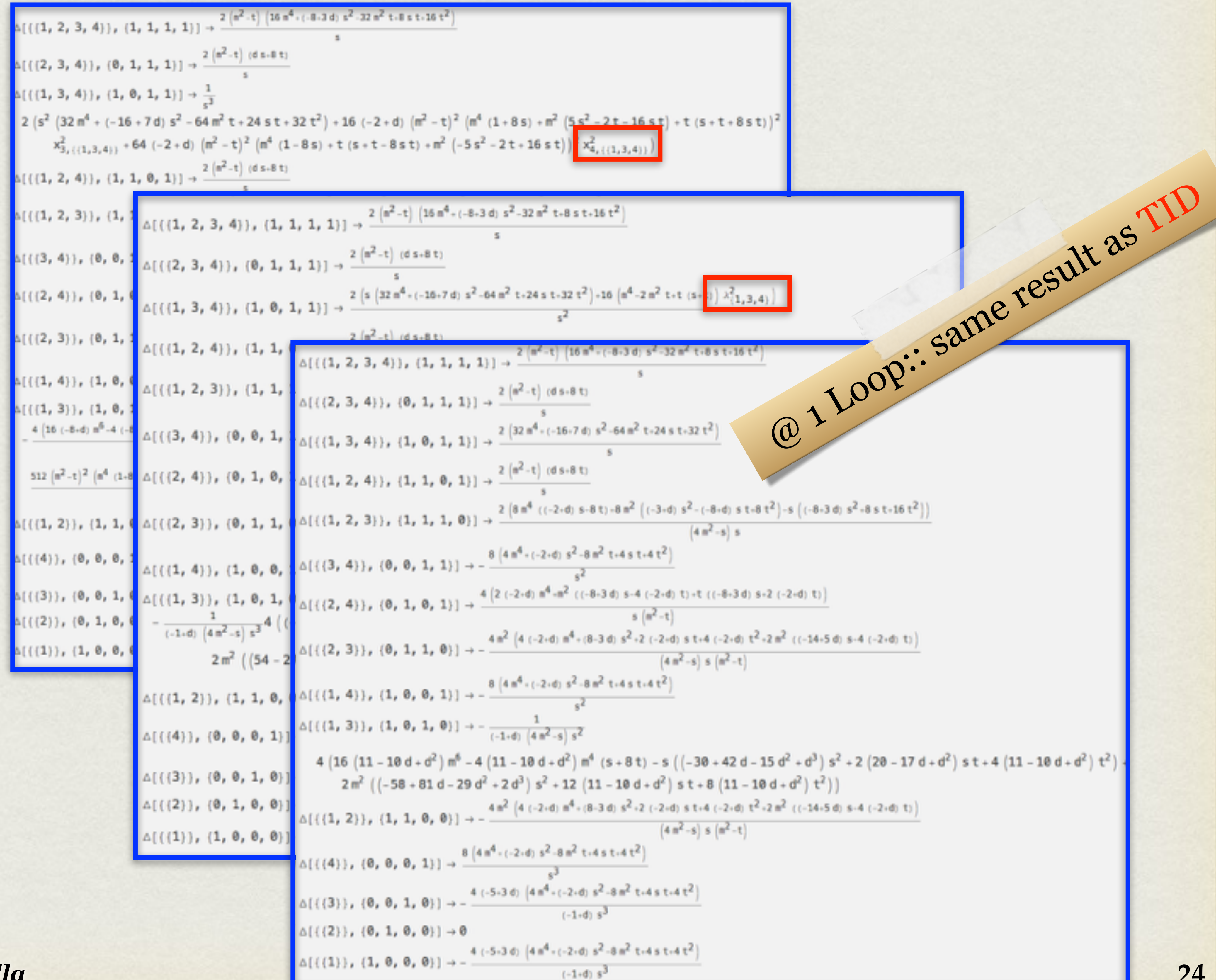






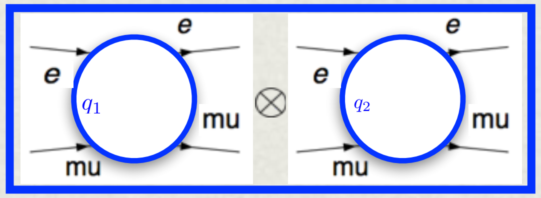


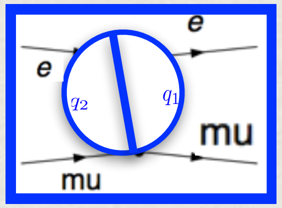
AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]

Two-loop features

- Different treatment for Factorised and non factorised topologies

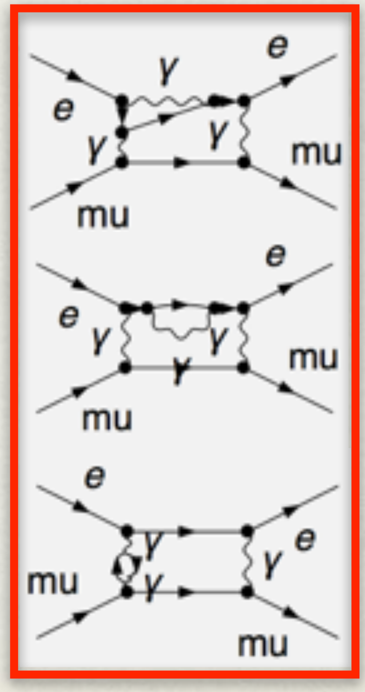




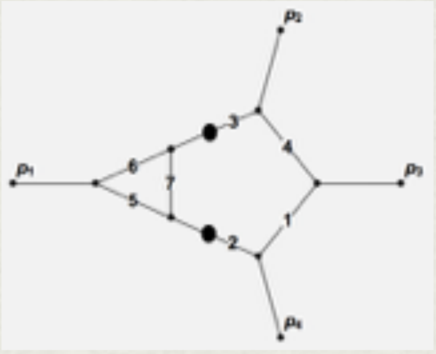
Independent　　　Same

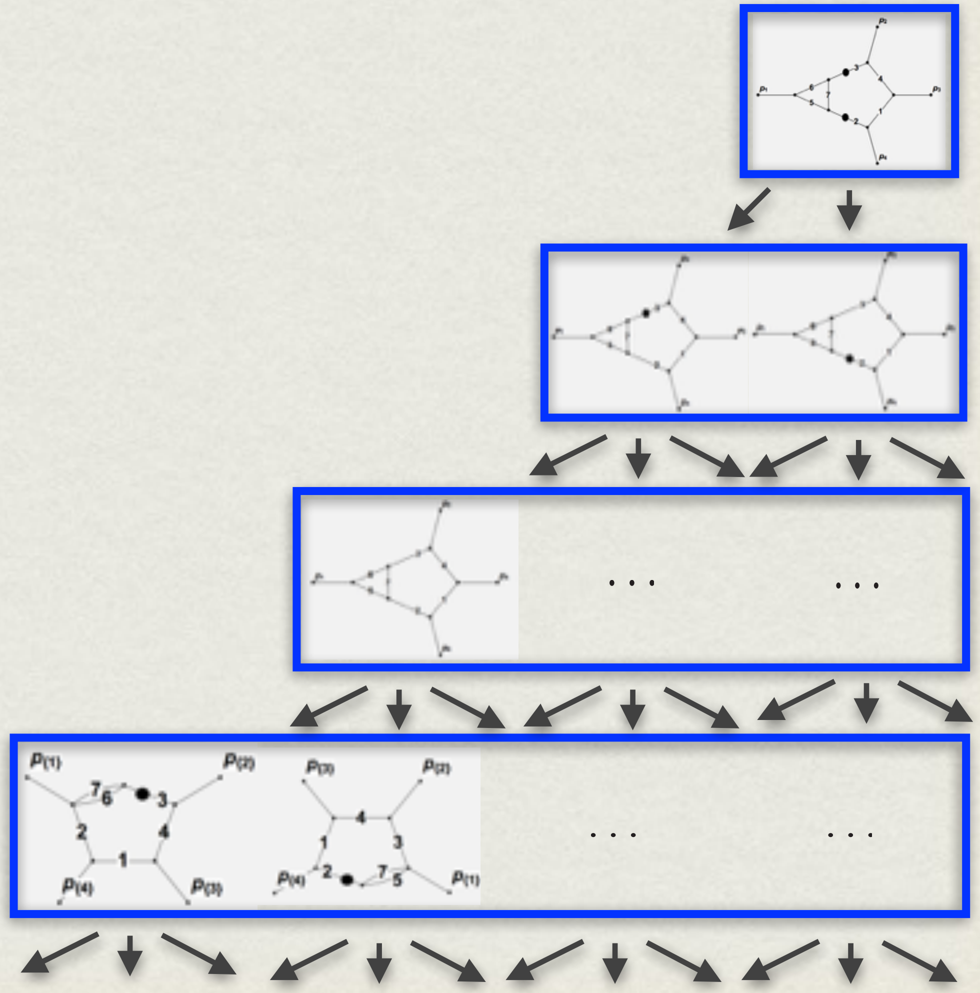
parametrisation for each loop

- Squared propagators affect Jobs organisation



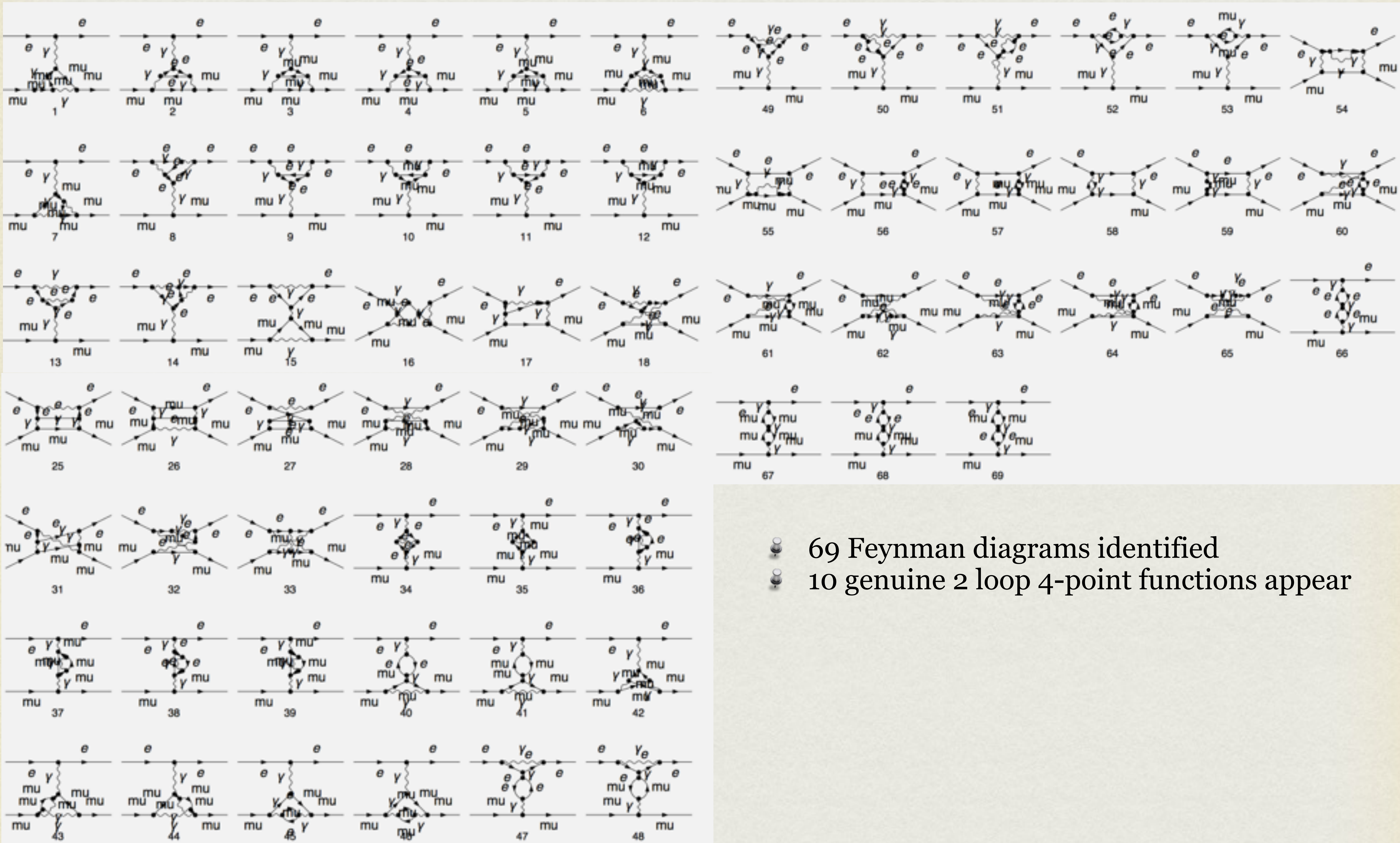
Parent Topology





AIDA for muon-electron scattering

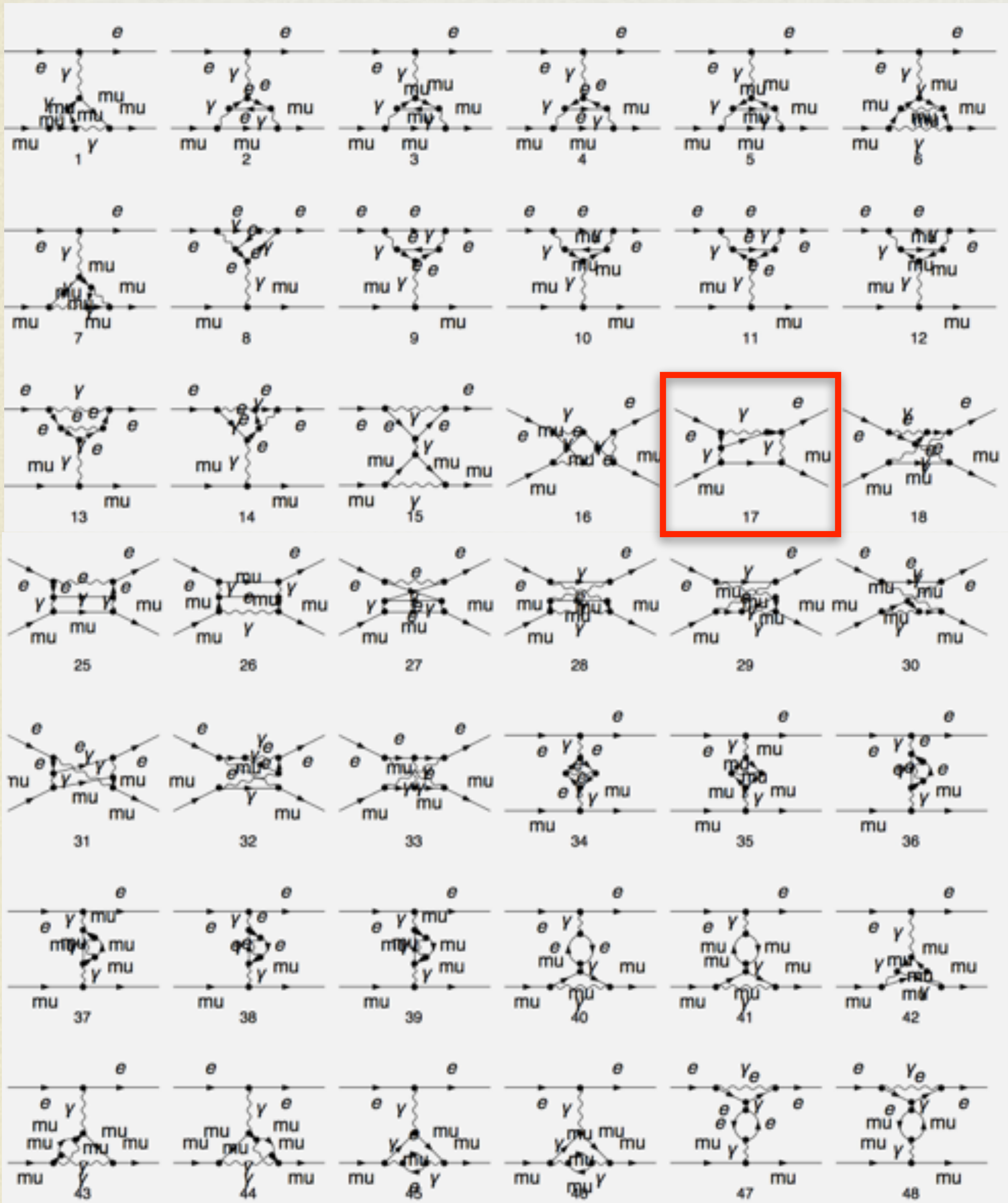
[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]

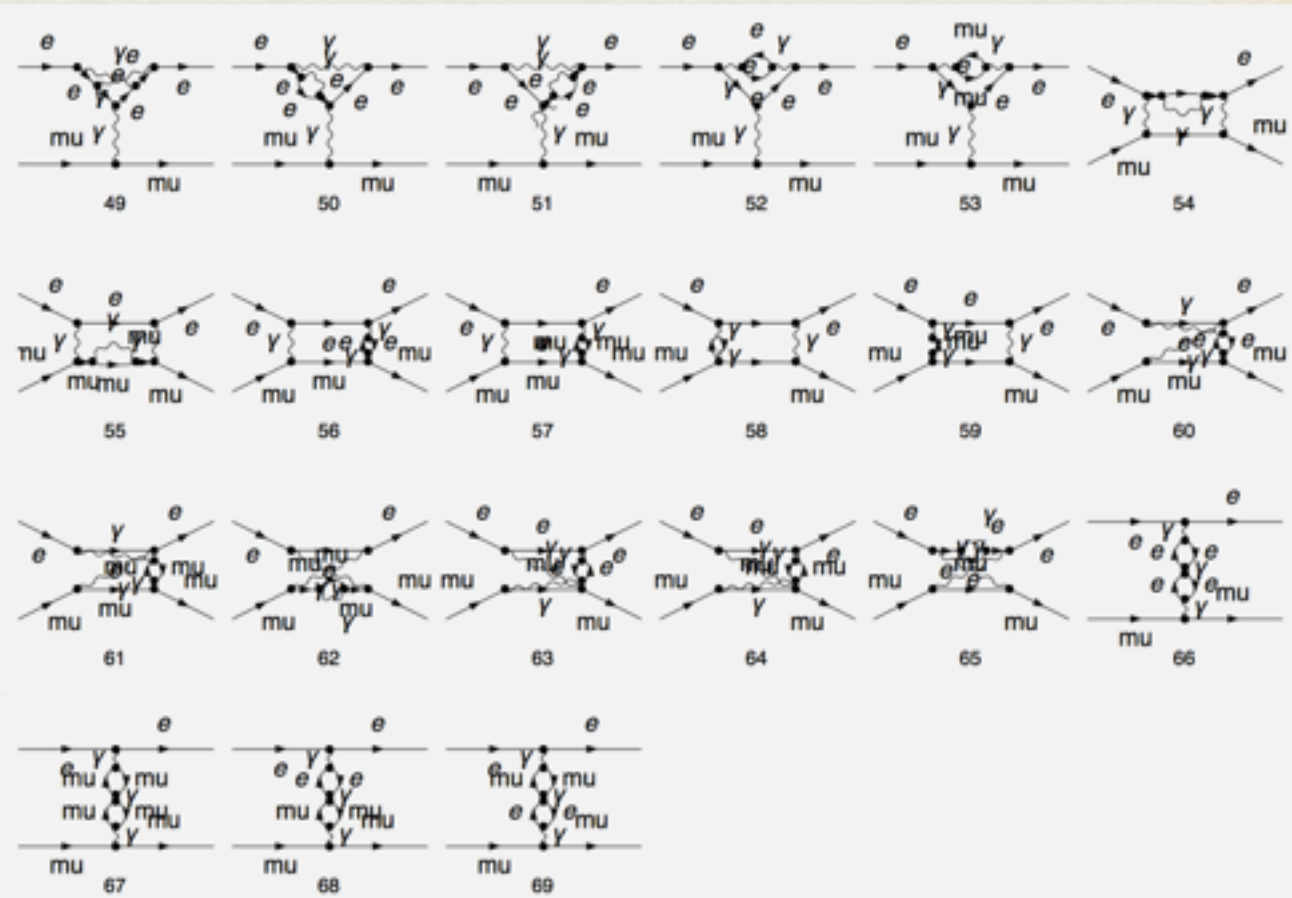


- 69 Feynman diagrams identified
- 10 genuine 2 loop 4-point functions appear

AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]





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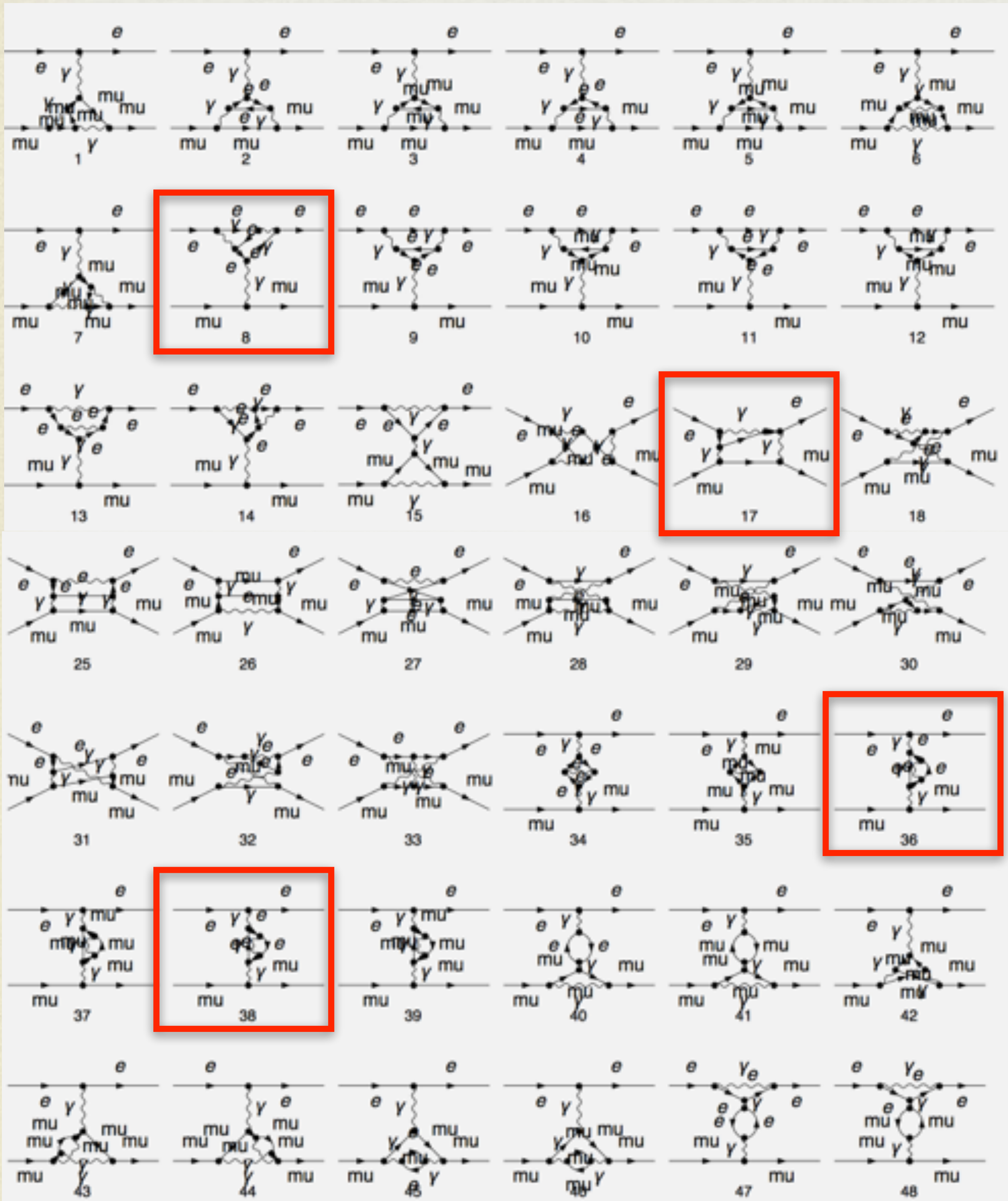
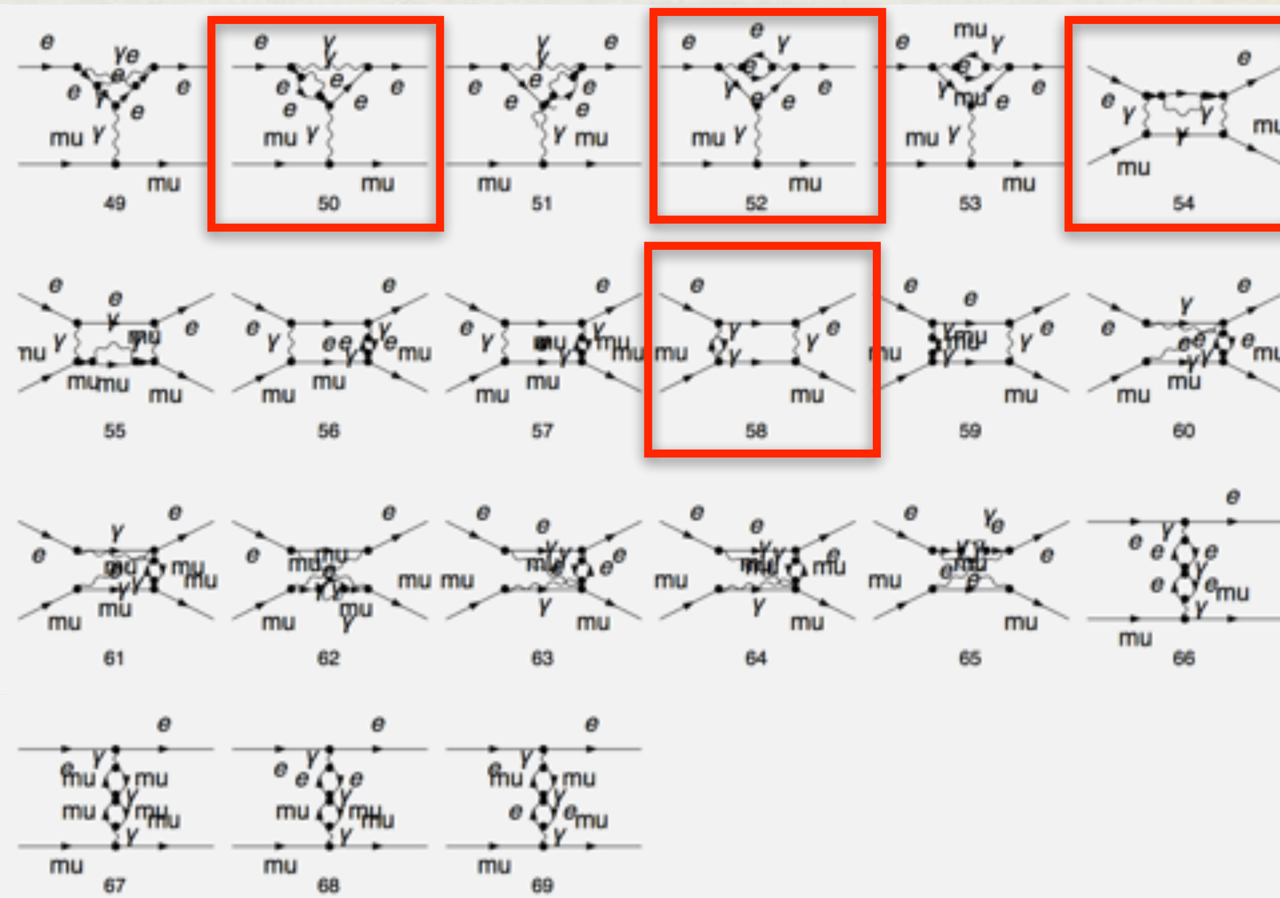
AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]





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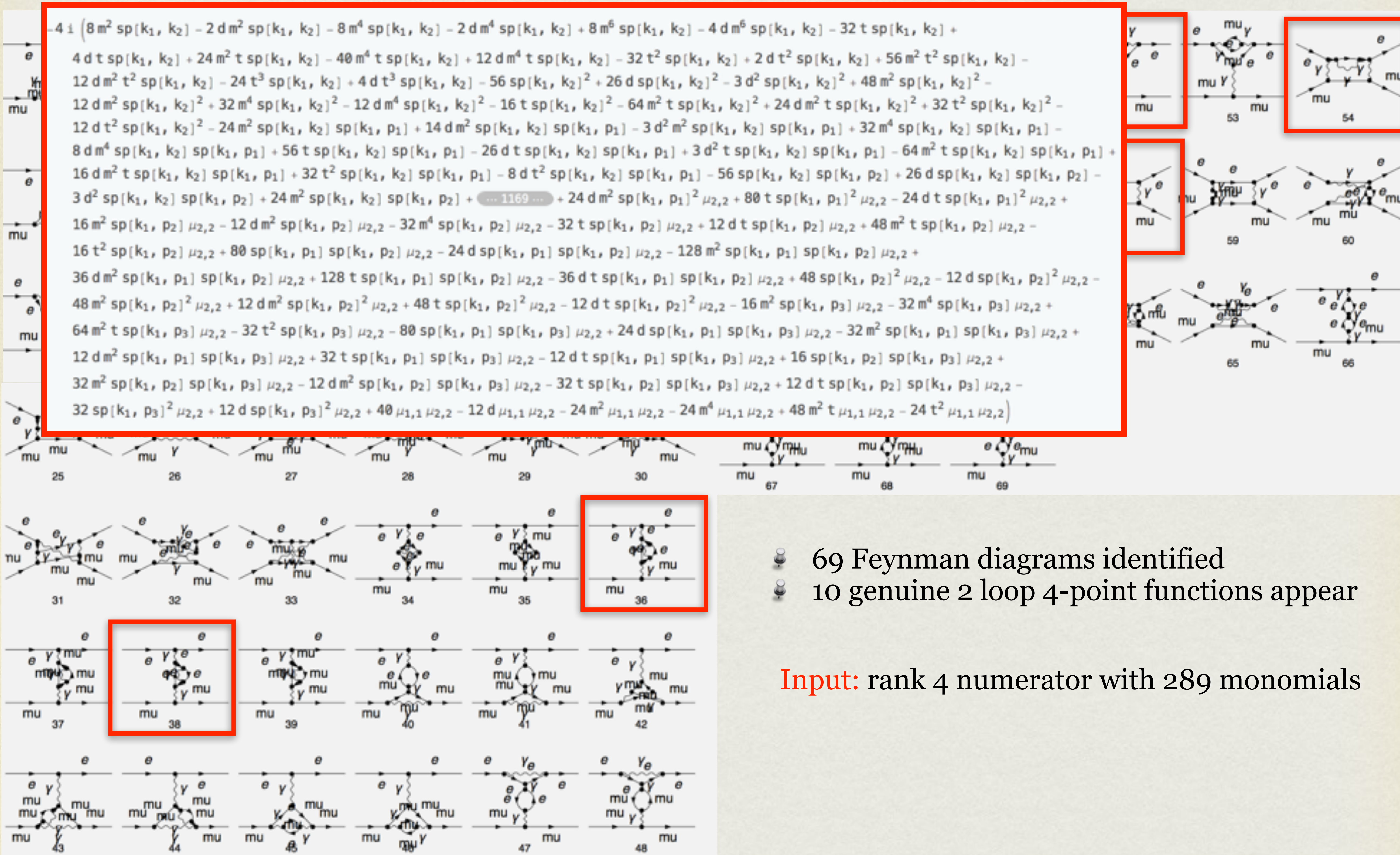
AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]



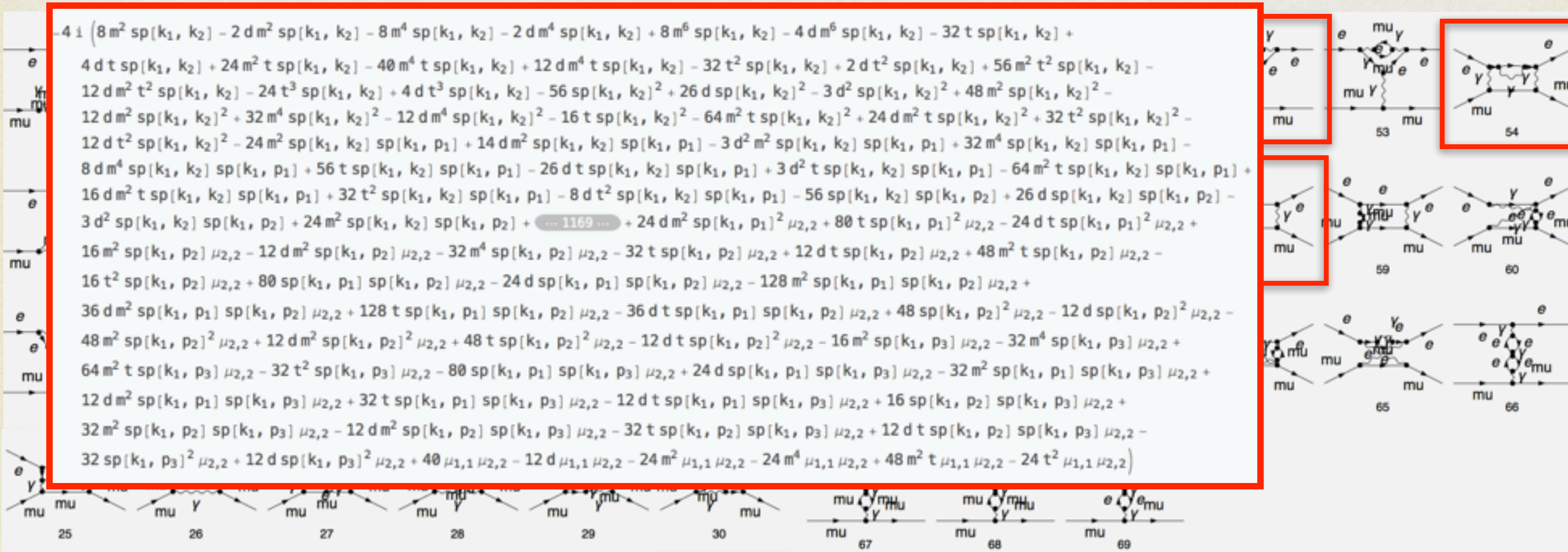
$$
\begin{aligned}
-4\,i\,\big(&8 m^2 sp[k_1,k_2] - 2 d m^2 sp[k_1,k_2] - 8 m^4 sp[k_1,k_2] - 2 d m^4 sp[k_1,k_2] + 8 m^6 sp[k_1,k_2] - 4 d m^6 sp[k_1,k_2] - 32 t\, sp[k_1,k_2] + \\
&4 d t\, sp[k_1,k_2] + 24 m^2 t\, sp[k_1,k_2] - 40 m^4 t\, sp[k_1,k_2] + 12 d m^4 t\, sp[k_1,k_2] - 32 t^2 sp[k_1,k_2] + 2 d t^2 sp[k_1,k_2] + 56 m^2 t^2 sp[k_1,k_2] - \\
&12 d m^2 t^2 sp[k_1,k_2] - 24 t^3 sp[k_1,k_2] + 4 d t^3 sp[k_1,k_2] - 56\, sp[k_1,k_2]^2 + 26 d\, sp[k_1,k_2]^2 - 3 d^2 sp[k_1,k_2]^2 + 48 m^2 sp[k_1,k_2]^2 - \\
&12 d m^2 sp[k_1,k_2]^2 + 32 m^4 sp[k_1,k_2]^2 - 12 d m^4 sp[k_1,k_2]^2 - 16 t\, sp[k_1,k_2]^2 - 64 m^2 t\, sp[k_1,k_2]^2 + 24 d m^2 t\, sp[k_1,k_2]^2 + 32 t^2 sp[k_1,k_2]^2 - \\
&12 d t^2 sp[k_1,k_2]^2 - 24 m^2 sp[k_1,k_2]\, sp[k_1,p_1] + 14 d m^2 sp[k_1,k_2]\, sp[k_1,p_1] - 3 d^2 m^2 sp[k_1,k_2]\, sp[k_1,p_1] + 32 m^4 sp[k_1,k_2]\, sp[k_1,p_1] - \\
&8 d m^4 sp[k_1,k_2]\, sp[k_1,p_1] + 56 t\, sp[k_1,k_2]\, sp[k_1,p_1] - 26 d t\, sp[k_1,k_2]\, sp[k_1,p_1] + 3 d^2 t\, sp[k_1,k_2]\, sp[k_1,p_1] - 64 m^2 t\, sp[k_1,k_2]\, sp[k_1,p_1] + \\
&16 d m^2 t\, sp[k_1,k_2]\, sp[k_1,p_1] + 32 t^2 sp[k_1,k_2]\, sp[k_1,p_1] - 8 d t^2 sp[k_1,k_2]\, sp[k_1,p_1] - 56\, sp[k_1,k_2]\, sp[k_1,p_2] + 26 d\, sp[k_1,k_2]\, sp[k_1,p_2] - \\
&3 d^2 sp[k_1,k_2]\, sp[k_1,p_2] + 24 m^2 sp[k_1,k_2]\, sp[k_1,p_2] + \langle\langle 1169 \rangle\rangle + 24 d m^2 sp[k_1,p_1]^2 \mu_{2,2} + 80 t\, sp[k_1,p_1]^2 \mu_{2,2} - 24 d t\, sp[k_1,p_1]^2 \mu_{2,2} + \\
&16 m^2 sp[k_1,p_2]\, \mu_{2,2} - 12 d m^2 sp[k_1,p_2]\, \mu_{2,2} - 32 m^4 sp[k_1,p_2]\, \mu_{2,2} - 32 t\, sp[k_1,p_2]\, \mu_{2,2} + 12 d t\, sp[k_1,p_2]\, \mu_{2,2} + 48 m^2 t\, sp[k_1,p_2]\, \mu_{2,2} - \\
&16 t^2 sp[k_1,p_2]\, \mu_{2,2} + 80\, sp[k_1,p_1]\, sp[k_1,p_2]\, \mu_{2,2} - 24 d\, sp[k_1,p_1]\, sp[k_1,p_2]\, \mu_{2,2} - 128 m^2 sp[k_1,p_1]\, sp[k_1,p_2]\, \mu_{2,2} + \\
&36 d m^2 sp[k_1,p_1]\, sp[k_1,p_2]\, \mu_{2,2} + 128 t\, sp[k_1,p_1]\, sp[k_1,p_2]\, \mu_{2,2} - 36 d t\, sp[k_1,p_1]\, sp[k_1,p_2]\, \mu_{2,2} + 48\, sp[k_1,p_2]^2 \mu_{2,2} - 12 d\, sp[k_1,p_2]^2 \mu_{2,2} - \\
&48 m^2 sp[k_1,p_2]^2 \mu_{2,2} + 12 d m^2 sp[k_1,p_2]^2 \mu_{2,2} + 48 t\, sp[k_1,p_2]^2 \mu_{2,2} - 12 d t\, sp[k_1,p_2]^2 \mu_{2,2} - 16 m^2 sp[k_1,p_3]\, \mu_{2,2} - 32 m^4 sp[k_1,p_3]\, \mu_{2,2} + \\
&64 m^2 t\, sp[k_1,p_3]\, \mu_{2,2} - 32 t^2 sp[k_1,p_3]\, \mu_{2,2} - 80\, sp[k_1,p_1]\, sp[k_1,p_3]\, \mu_{2,2} + 24 d\, sp[k_1,p_1]\, sp[k_1,p_3]\, \mu_{2,2} - 32 m^2 sp[k_1,p_1]\, sp[k_1,p_3]\, \mu_{2,2} + \\
&12 d m^2 sp[k_1,p_1]\, sp[k_1,p_3]\, \mu_{2,2} + 32 t\, sp[k_1,p_1]\, sp[k_1,p_3]\, \mu_{2,2} - 12 d t\, sp[k_1,p_1]\, sp[k_1,p_3]\, \mu_{2,2} + 16\, sp[k_1,p_2]\, sp[k_1,p_3]\, \mu_{2,2} + \\
&32 m^2 sp[k_1,p_2]\, sp[k_1,p_3]\, \mu_{2,2} - 12 d m^2 sp[k_1,p_2]\, sp[k_1,p_3]\, \mu_{2,2} - 32 t\, sp[k_1,p_2]\, sp[k_1,p_3]\, \mu_{2,2} + 12 d t\, sp[k_1,p_2]\, sp[k_1,p_3]\, \mu_{2,2} - \\
&32\, sp[k_1,p_3]^2 \mu_{2,2} + 12 d\, sp[k_1,p_3]^2 \mu_{2,2} + 40 \mu_{1,1}\mu_{2,2} - 12 d\, \mu_{1,1}\mu_{2,2} - 24 m^2 \mu_{1,1}\mu_{2,2} - 24 m^4 \mu_{1,1}\mu_{2,2} + 48 m^2 t\, \mu_{1,1}\mu_{2,2} - 24 t^2 \mu_{1,1}\mu_{2,2}\big)
\end{aligned}
$$

- 69 Feynman diagrams identified
- 10 genuine 2 loop 4-point functions appear

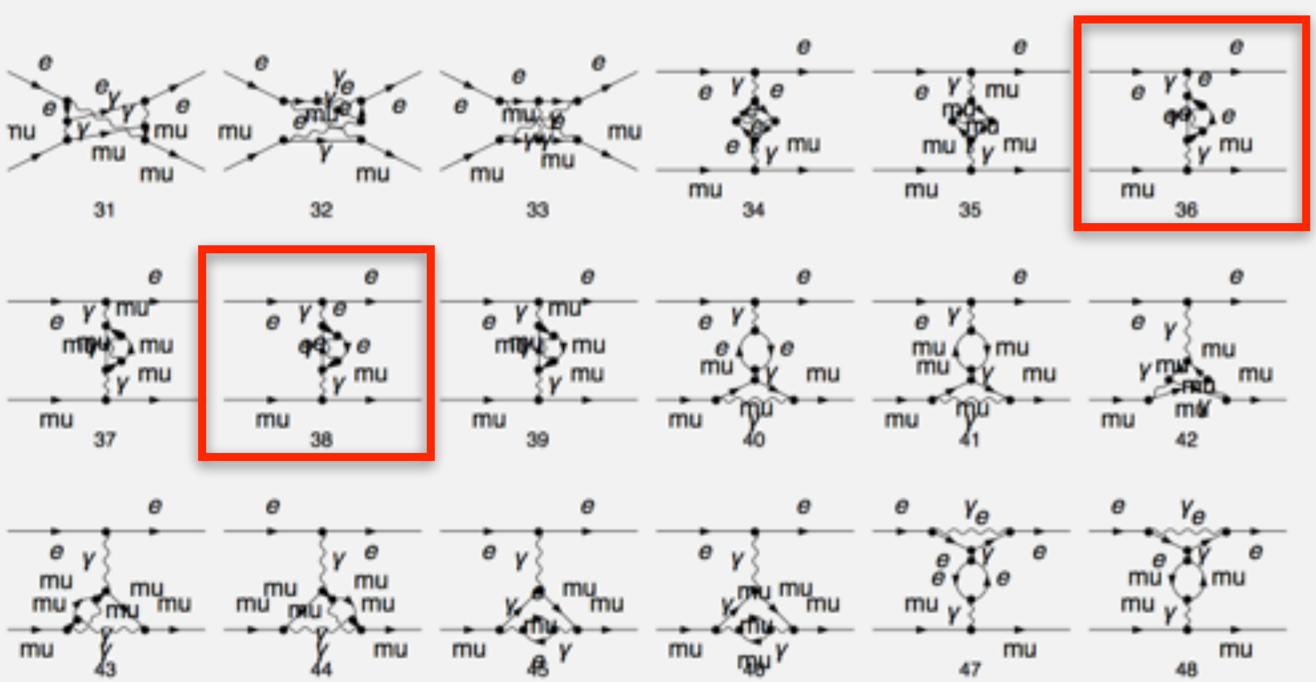
Input: rank 4 numerator with 289 monomials

AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]



$$-4\,i\,\Big(8 m^2 \mathrm{sp}[k_1,k_2] - 2 d m^2 \mathrm{sp}[k_1,k_2] - 8 m^4 \mathrm{sp}[k_1,k_2] - 2 d m^4 \mathrm{sp}[k_1,k_2] + 8 m^6 \mathrm{sp}[k_1,k_2] - 4 d m^6 \mathrm{sp}[k_1,k_2] - 32 t\, \mathrm{sp}[k_1,k_2] +$$
$$4 d t\, \mathrm{sp}[k_1,k_2] + 24 m^2 t\, \mathrm{sp}[k_1,k_2] - 40 m^4 t\, \mathrm{sp}[k_1,k_2] + 12 d m^4 t\, \mathrm{sp}[k_1,k_2] - 32 t^2 \mathrm{sp}[k_1,k_2] + 2 d t^2 \mathrm{sp}[k_1,k_2] + 56 m^2 t^2 \mathrm{sp}[k_1,k_2] -$$
$$12 d m^2 t^2 \mathrm{sp}[k_1,k_2] - 24 t^3 \mathrm{sp}[k_1,k_2] + 4 d t^3 \mathrm{sp}[k_1,k_2] - 56\, \mathrm{sp}[k_1,k_2]^2 + 26 d\, \mathrm{sp}[k_1,k_2]^2 - 3 d^2 \mathrm{sp}[k_1,k_2]^2 + 48 m^2 \mathrm{sp}[k_1,k_2]^2 -$$
$$12 d m^2 \mathrm{sp}[k_1,k_2]^2 + 32 m^4 \mathrm{sp}[k_1,k_2]^2 - 12 d m^4 \mathrm{sp}[k_1,k_2]^2 - 16 t\, \mathrm{sp}[k_1,k_2]^2 - 64 m^2 t\, \mathrm{sp}[k_1,k_2]^2 + 24 d m^2 t\, \mathrm{sp}[k_1,k_2]^2 + 32 t^2 \mathrm{sp}[k_1,k_2]^2 -$$
$$12 d t^2 \mathrm{sp}[k_1,k_2]^2 - 24 m^2 \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_1] + 14 d m^2 \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_1] - 3 d^2 m^2 \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_1] + 32 m^4 \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_1] -$$
$$8 d m^4 \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_1] + 56 t\, \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_1] - 26 d t\, \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_1] + 3 d^2 t\, \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_1] - 64 m^2 t\, \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_1] +$$
$$16 d m^2 t\, \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_1] + 32 t^2 \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_1] - 8 d t^2 \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_1] - 56\, \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_2] + 26 d\, \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_2] -$$
$$3 d^2 \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_2] + 24 m^2 \mathrm{sp}[k_1,k_2]\, \mathrm{sp}[k_1,p_2] + \cdots 1169 \cdots + 24 d m^2 \mathrm{sp}[k_1,p_1]^2 \mu_{2,2} + 80 t\, \mathrm{sp}[k_1,p_1]^2 \mu_{2,2} - 24 d t\, \mathrm{sp}[k_1,p_1]^2 \mu_{2,2} +$$
$$16 m^2 \mathrm{sp}[k_1,p_2]\, \mu_{2,2} - 12 d m^2 \mathrm{sp}[k_1,p_2]\, \mu_{2,2} - 32 m^4 \mathrm{sp}[k_1,p_2]\, \mu_{2,2} - 32 t\, \mathrm{sp}[k_1,p_2]\, \mu_{2,2} + 12 d t\, \mathrm{sp}[k_1,p_2]\, \mu_{2,2} + 48 m^2 t\, \mathrm{sp}[k_1,p_2]\, \mu_{2,2} -$$
$$16 t^2 \mathrm{sp}[k_1,p_2]\, \mu_{2,2} + 80\, \mathrm{sp}[k_1,p_1]\, \mathrm{sp}[k_1,p_2]\, \mu_{2,2} - 24 d\, \mathrm{sp}[k_1,p_1]\, \mathrm{sp}[k_1,p_2]\, \mu_{2,2} - 128 m^2 \mathrm{sp}[k_1,p_1]\, \mathrm{sp}[k_1,p_2]\, \mu_{2,2} +$$
$$36 d m^2 \mathrm{sp}[k_1,p_1]\, \mathrm{sp}[k_1,p_2]\, \mu_{2,2} + 128 t\, \mathrm{sp}[k_1,p_1]\, \mathrm{sp}[k_1,p_2]\, \mu_{2,2} - 36 d t\, \mathrm{sp}[k_1,p_1]\, \mathrm{sp}[k_1,p_2]\, \mu_{2,2} + 48\, \mathrm{sp}[k_1,p_2]^2 \mu_{2,2} - 12 d\, \mathrm{sp}[k_1,p_2]^2 \mu_{2,2} -$$
$$48 m^2 \mathrm{sp}[k_1,p_2]^2 \mu_{2,2} + 12 d m^2 \mathrm{sp}[k_1,p_2]^2 \mu_{2,2} + 48 t\, \mathrm{sp}[k_1,p_2]^2 \mu_{2,2} - 12 d t\, \mathrm{sp}[k_1,p_2]^2 \mu_{2,2} - 16 m^2 \mathrm{sp}[k_1,p_3]\, \mu_{2,2} - 32 m^4 \mathrm{sp}[k_1,p_3]\, \mu_{2,2} +$$
$$64 m^2 t\, \mathrm{sp}[k_1,p_3]\, \mu_{2,2} - 32 t^2 \mathrm{sp}[k_1,p_3]\, \mu_{2,2} - 80\, \mathrm{sp}[k_1,p_1]\, \mathrm{sp}[k_1,p_3]\, \mu_{2,2} + 24 d\, \mathrm{sp}[k_1,p_1]\, \mathrm{sp}[k_1,p_3]\, \mu_{2,2} - 32 m^2 \mathrm{sp}[k_1,p_1]\, \mathrm{sp}[k_1,p_3]\, \mu_{2,2} +$$
$$12 d m^2 \mathrm{sp}[k_1,p_1]\, \mathrm{sp}[k_1,p_3]\, \mu_{2,2} + 32 t\, \mathrm{sp}[k_1,p_1]\, \mathrm{sp}[k_1,p_3]\, \mu_{2,2} - 12 d t\, \mathrm{sp}[k_1,p_1]\, \mathrm{sp}[k_1,p_3]\, \mu_{2,2} + 16\, \mathrm{sp}[k_1,p_2]\, \mathrm{sp}[k_1,p_3]\, \mu_{2,2} +$$
$$32 m^2 \mathrm{sp}[k_1,p_2]\, \mathrm{sp}[k_1,p_3]\, \mu_{2,2} - 12 d m^2 \mathrm{sp}[k_1,p_2]\, \mathrm{sp}[k_1,p_3]\, \mu_{2,2} - 32 t\, \mathrm{sp}[k_1,p_2]\, \mathrm{sp}[k_1,p_3]\, \mu_{2,2} + 12 d t\, \mathrm{sp}[k_1,p_2]\, \mathrm{sp}[k_1,p_3]\, \mu_{2,2} -$$
$$32\, \mathrm{sp}[k_1,p_3]^2 \mu_{2,2} + 12 d\, \mathrm{sp}[k_1,p_3]^2 \mu_{2,2} + 40 \mu_{1,1}\mu_{2,2} - 12 d \mu_{1,1}\mu_{2,2} - 24 m^2 \mu_{1,1}\mu_{2,2} - 24 m^4 \mu_{1,1}\mu_{2,2} + 48 m^2 t\, \mu_{1,1}\mu_{2,2} - 24 t^2 \mu_{1,1}\mu_{2,2}\Big)$$



- 69 Feynman diagrams identified
- 10 genuine 2 loop 4-point functions appear

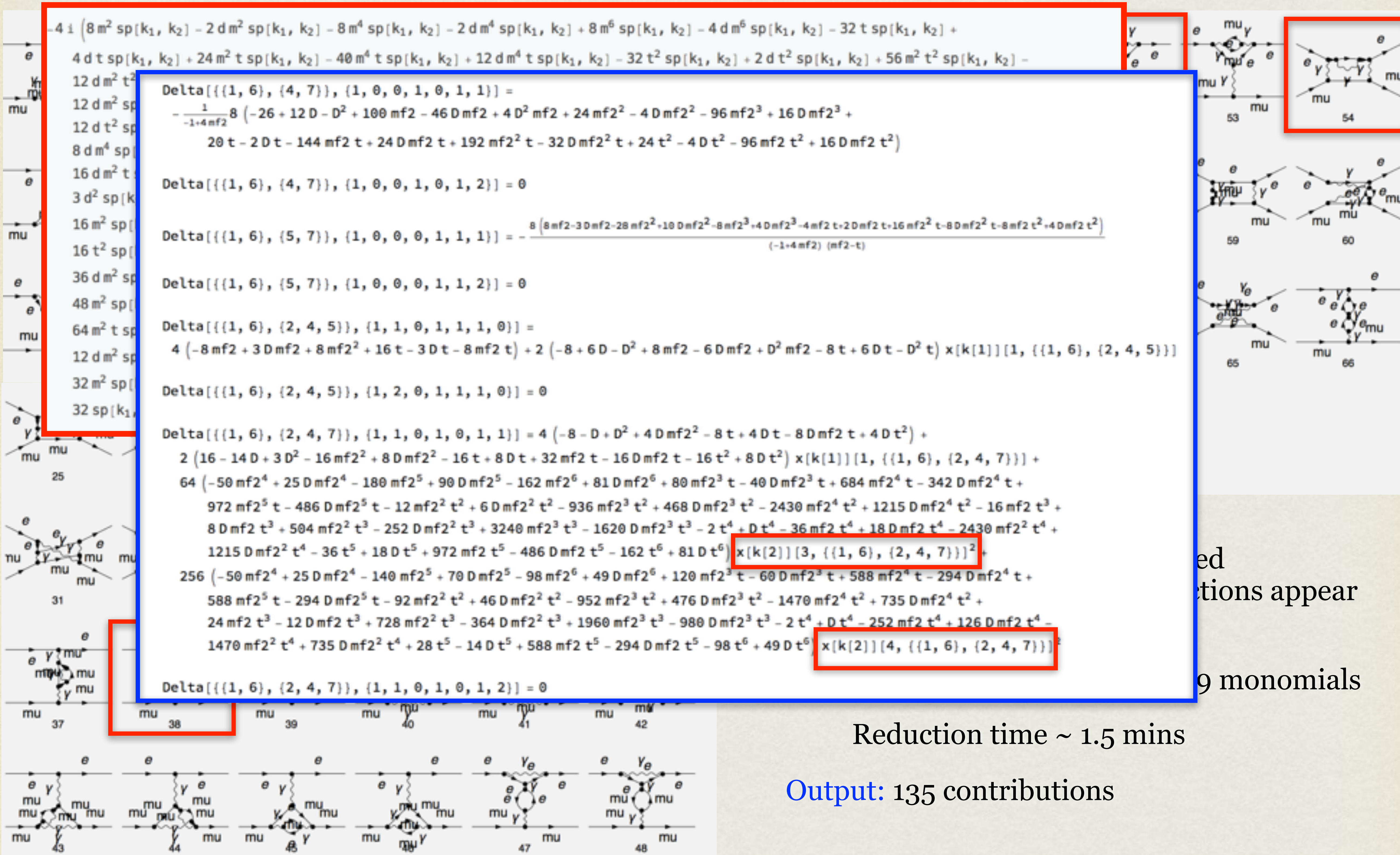
Input: rank 4 numerator with 289 monomials

Reduction time ~ 1.5 mins

Output: 135 contributions

AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]



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tions appear

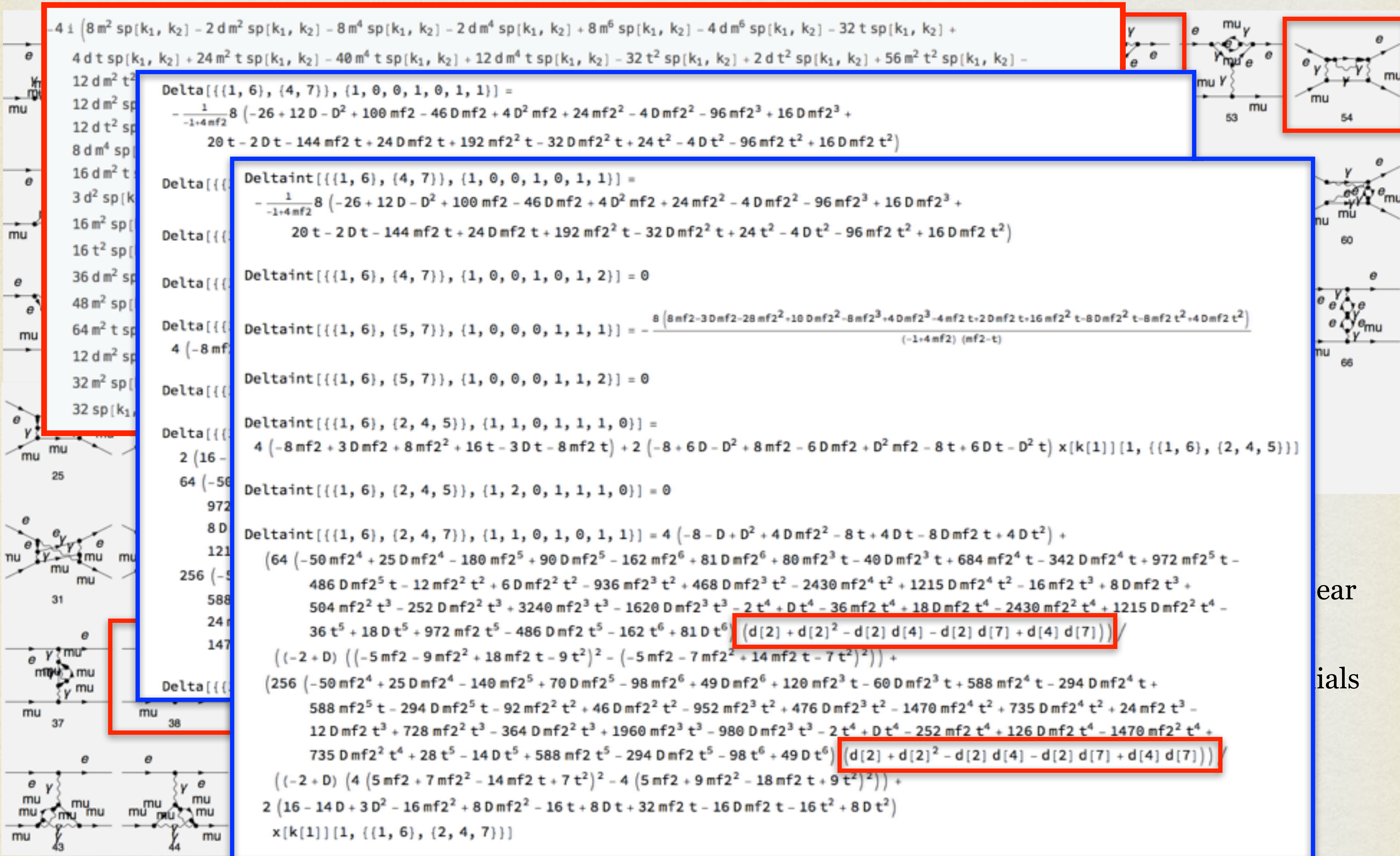
9 monomials

Reduction time ~ 1.5 mins

Output: 135 contributions

AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]



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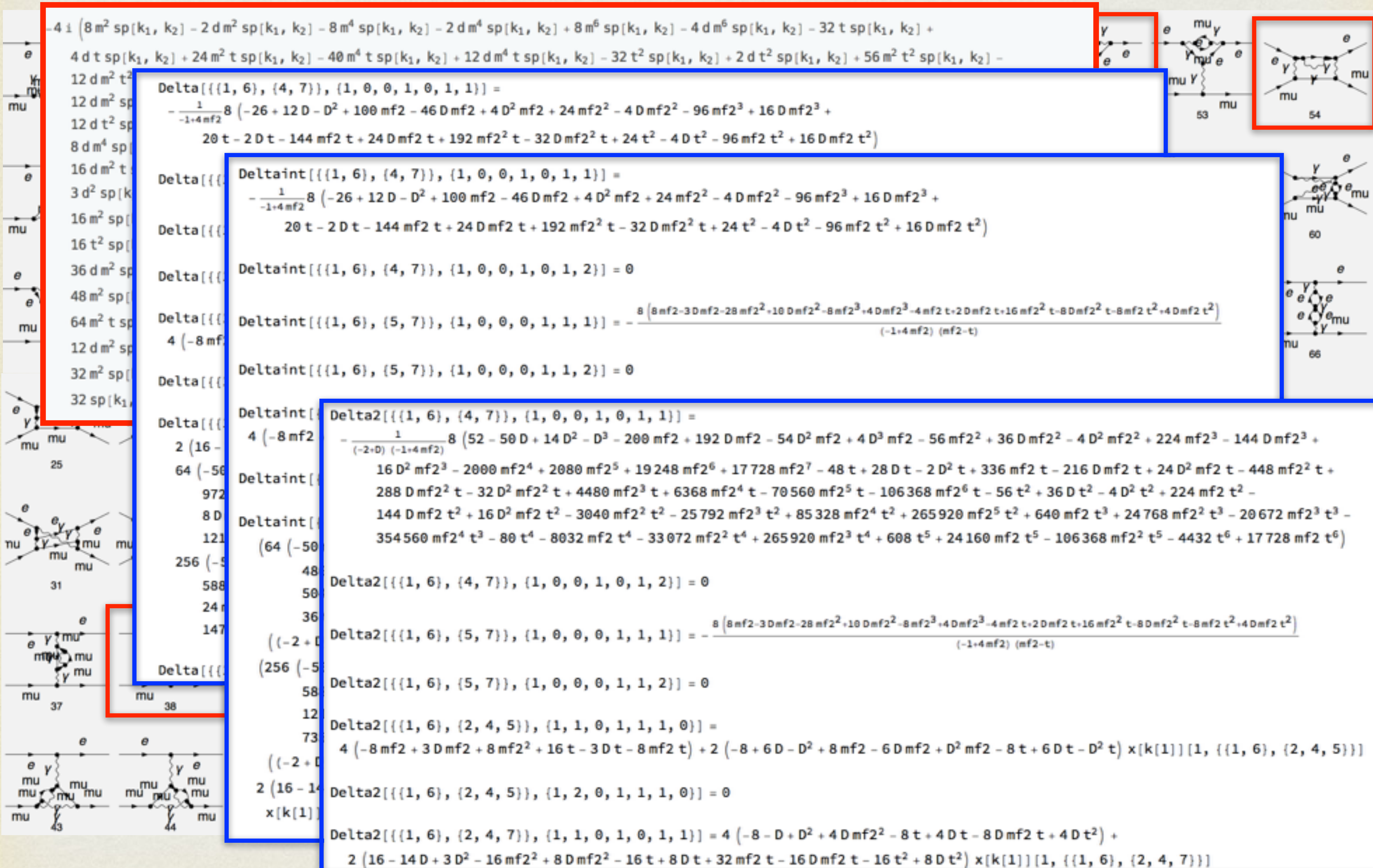
AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, W.J.T. (work in progress)]



AIDA for muon-electron scattering

Interface with IBP generators

[Mastrolia, Primo, Ronca, W.J.T. (work in progress)]

Recall AIDA output

AIDA for muon-electron scattering

Interface with IBP generators

[Mastrolia, Primo, Ronca, W.J.T. (work in progress)]

Recall AIDA output

```
ScalarProducts = {mp[p[1], p[1]] -> 0, mp[p[1], p[2]] -> ex[1]/2,
  mp[p[1], p[3]] -> (mf2 - t - ex[1])/2, mp[p[1], p[4]] -> (-mf2 + t)/2,
  mp[p[2], p[2]] -> 0, mp[p[2], p[3]] -> (-mf2 + t)/2,
  mp[p[2], p[4]] -> (mf2 - t - ex[1])/2, mp[p[3], p[3]] -> mf2,
  mp[p[3], p[4]] -> (-2*mf2 + ex[1])/2, mp[p[4], p[4]] -> mf2}

ParentGraph = {{p[2], e[4], 0, 0, 0}, {p[3], e[5], 0, 0, 0},
  {p[4], e[6], 0, 0, 0}, {p[2], e[1], 0, 0, 0},
  {e[1], e[2], "0", "k[1]", "1"}, {e[2], e[4], "0", "k[2]", "2"},
  {e[2], e[3], "0", "k[1] + k[2]", "3"}, {e[4], e[5], "0", "k[2] + p[2]",
  "4"}, {e[5], e[6], "mf", "k[2] + p[2] + p[3]", "5"},
  {e[1], e[3], "0", "k[1] - p[2] - p[3] - p[4]", "6"},
  {e[3], e[6], "0", "k[2] + p[2] + p[3] + p[4]", "7"}}

listISP = {mp[k[1], p[3]], mp[k[1], p[4]]}

integrals = {INT["emu_2L.g6", {-2, 1, 1, 0, 0, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 0, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 0, 1, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 1, 0, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 1, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 0, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 0, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 1, 1, 0, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 1, 1, 1, 0, 0}],
```

AIDA for muon-electron scattering

Interface with IBP generators

[Mastrolia, Primo, Ronca, W.J.T. (work in progress)]

Recall AIDA output

```
ScalarProducts = {mp[p[1], p[1]] -> 0, mp[p[1], p[2]] ->
  mp[p[1], p[3]] -> (mf2 - t - ex[1]) / 2, mp[p[1], p[4]]
  mp[p[2], p[2]] -> 0, mp[p[2], p[3]] -> (-mf2 + t) / 2,
  mp[p[2], p[4]] -> (mf2 - t - ex[1]) / 2, mp[p[3], p[3]]
  mp[p[3], p[4]] -> (-2*mf2 + ex[1]) / 2, mp[p[4], p[4]]

ParentGraph = {{p[2], e[4], 0, 0, 0}, {p[3], e[5], 0, 0
  {p[4], e[6], 0, 0, 0}, {p[2], e[1], 0, 0, 0},
  {e[1], e[2], "0", "k[1]", "1"}, {e[2], e[4], "0", "k[
  {e[2], e[3], "0", "k[1] + k[2]", "3"}, {e[4], e[5], "
  "4"}, {e[5], e[6], "mf", "k[2] + p[2] + p[3]", "5"},
  {e[1], e[3], "0", "k[1] - p[2] - p[3] - p[4]", "6"},
  {e[3], e[6], "0", "k[2] + p[2] + p[3] + p[4]", "7"}}

listISP = {mp[k[1], p[3]], mp[k[1], p[4]]}

integrals = {INT["emu_2L.g6", {-2, 1, 1, 0, 0, 1, 2, 0,
  INT["emu_2L.g6", {-2, 1, 1, 0, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 0, 1, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 1, 0, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 1, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 0, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 0, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 1, 1, 0, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 1, 1, 1, 0, 0}],
```

```
 INT["emu_2L.g6", {0, 0, 1, 0, 0, 0, 2, 0, 0}], 0,
(-192/(2 - dim) + (96*dim)/(2 - dim) - (16*dim^2)/(2 - dim) +
  (128*mf2^2)/(2 - dim) - (64*dim*mf2^2)/(2 - dim) + (128*t)/(2 - dim) -
  (64*dim*t)/(2 - dim) - (256*mf2*t)/(2 - dim) +
  (128*dim*mf2*t)/(2 - dim) + (128*t^2)/(2 - dim) -
  (64*dim*t^2)/(2 - dim))*INT["emu_2L.g6", {0, 2, 1, 0, 0, 0, 0, 0, 0}],
0, (-32 + 16*dim + 64*mf2^2 + 64*t - 128*mf2*t + 64*t^2)*
 INT["emu_2L.g6", {1, 2, 0, 0, 0, 0, 0, 0, 0}],
(-64/(1 - dim) + (64*dim)/(1 - dim) - (16*dim^2)/(1 - dim) +
  (128*mf2^2)/(1 - dim) - (64*dim*mf2^2)/(1 - dim) + (128*t)/(1 - dim) -
  (64*dim*t)/(1 - dim) - (256*mf2*t)/(1 - dim) +
  (128*dim*mf2*t)/(1 - dim) + (128*t^2)/(1 - dim) -
  (64*dim*t^2)/(1 - dim))*INT["emu_2L.g6", {0, 0, 0, 0, 0, 1, 1, 0, 0}],
0, (64/(1 - dim) - (64*dim)/(1 - dim) + (16*dim^2)/(1 - dim) -
  (128*mf2^2)/(1 - dim) + (64*dim*mf2^2)/(1 - dim) - (128*t)/(1 - dim) +
  (64*dim*t)/(1 - dim) + (256*mf2*t)/(1 - dim) -
  (128*dim*mf2*t)/(1 - dim) - (128*t^2)/(1 - dim) +
  (64*dim*t^2)/(1 - dim))*INT["emu_2L.g6", {0, 0, 0, 1, 0, 1, 0, 0, 0}],
(64/(1 - dim) - (64*dim)/(1 - dim) + (16*dim^2)/(1 - dim) -
  (128*mf2^2)/(1 - dim) + (64*dim*mf2^2)/(1 - dim) - (128*t)/(1 - dim) +
  (64*dim*t)/(1 - dim) + (256*mf2*t)/(1 - dim) -
  (128*dim*mf2*t)/(1 - dim) - (128*t^2)/(1 - dim) +
  (64*dim*t^2)/(1 - dim))*INT["emu_2L.g6", {0, 0, 1, 0, 0, 0, 1, 0, 0}],
0, 0, (-64/(1 - dim) + (64*dim)/(1 - dim) - (16*dim^2)/(1 - dim) +
  (128*mf2^2)/(1 - dim) - (64*dim*mf2^2)/(1 - dim) + (128*t)/(1 - dim) -
  (64*dim*t)/(1 - dim) - (256*mf2*t)/(1 - dim) +
  (128*dim*mf2*t)/(1 - dim) + (128*t^2)/(1 - dim) -
  (64*dim*t^2)/(1 - dim))*INT["emu_2L.g6", {0, 0, 1, 1, 0, 0, 0, 0, 0}],
```

AIDA for muon-electron scattering

Interface with IBP generators

[Mastrolia, Primo, Ronca, W.J.T. (work in progress)]

Recall AIDA output

```
ScalarProducts = {mp[p[1], p[1]] -> 0, mp[p[1], p[2]] ->
  mp[p[1], p[3]] -> (mf2 - t - ex[1]) / 2, mp[p[1], p[4]]
  mp[p[2], p[2]] -> 0, mp[p[2], p[3]] -> (-mf2 + t) / 2,
  mp[p[2], p[4]] -> (mf2 - t - ex[1]) / 2, mp[p[3], p[3]]
  mp[p[3], p[4]] -> (-2*mf2 + ex[1]) / 2, mp[p[4], p[4]]

ParentGraph = {{p[2], e[4], 0, 0, 0}, {p[3], e[5], 0, 0
  {p[4], e[6], 0, 0, 0}, {p[2], e[1], 0, 0, 0},
  {e[1], e[2], "0", "k[1]", "1"}, {e[2], e[4], "0", "k[
  {e[2], e[3], "0", "k[1] + k[2]", "3"}, {e[4], e[5], "
   "4"}, {e[5], e[6], "mf", "k[2] + p[2] + p[3]", "5"},
  {e[1], e[3], "0", "k[1] - p[2] - p[3] - p[4]", "6"},
  {e[3], e[6], "0", "k[2] + p[2] + p[3] + p[4]", "7"}}

listISP = {mp[k[1], p[3]], mp[k[1], p[4]]}

integrals = {INT["emu_2L.g6", {-2, 1, 1, 0, 0, 1, 2, 0,
  INT["emu_2L.g6", {-2, 1, 1, 0, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 0, 1, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 1, 0, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 1, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 0, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 0, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 1, 1, 0, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 1, 1, 1, 0, 0}],
```

```
INT["emu_2L.g6", {0, 0, 1, 0, 0, 0, 2, 0, 0}]  0,
(-192/(2 - dim) + (96*dim)/(2 - dim) - (16*dim^2)/(2 - dim) +
  (128*mf2^2)/(2 - dim) - (64*dim*mf2^2)/(2 - dim) + (128*t)/(2 - dim) -
  (64*dim*t)/(2 - dim) - (256*mf2*t)/(2 - dim) +
  (128*dim*mf2*t)/(2 - dim) + (128*t^2)/(2 - dim) -
  (64*dim*t^2)/(2 - dim)) INT["emu_2L.g6", {0, 2, 1, 0, 0, 0, 0, 0, 0}],
0, (-32 + 16*dim + 64*mf2^2 + 64*t - 128*mf2*t + 64*t^2)*
INT["emu_2L.g6", {1, 2, 0, 0, 0, 0, 0, 0, 0}]
(-64/(1 - dim) + (64*dim)/(1 - dim) - (16*dim^2)/(1 - dim) +
  (128*mf2^2)/(1 - dim) - (64*dim*mf2^2)/(1 - dim) + (128*t)/(1 - dim) -
  (64*dim*t)/(1 - dim) - (256*mf2*t)/(1 - dim) +
  (128*dim*mf2*t)/(1 - dim) + (128*t^2)/(1 - dim) -
  (64*dim*t^2)/(1 - dim)) INT["emu_2L.g6", {0, 0, 0, 0, 0, 1, 1, 0, 0}],
0, (64/(1 - dim) - (64*dim)/(1 - dim) + (16*dim^2)/(1 - dim) -
  (128*mf2^2)/(1 - dim) + (64*dim*mf2^2)/(1 - dim) - (128*t)/(1 - dim) +
  (64*dim*t)/(1 - dim) + (256*mf2*t)/(1 - dim) -
  (128*dim*mf2*t)/(1 - dim) - (128*t^2)/(1 - dim) +
  (64*dim*t^2)/(1 - dim)) INT["emu_2L.g6", {0, 0, 0, 1, 0, 1, 0, 0, 0}],
(64/(1 - dim) - (64*dim)/(1 - dim) + (16*dim^2)/(1 - dim) -
  (128*mf2^2)/(1 - dim) + (64*dim*mf2^2)/(1 - dim) - (128*t)/(1 - dim) +
  (64*dim*t)/(1 - dim) + (256*mf2*t)/(1 - dim) -
  (128*dim*mf2*t)/(1 - dim) - (128*t^2)/(1 - dim) +
  (64*dim*t^2)/(1 - dim)) INT["emu_2L.g6", {0, 0, 1, 0, 0, 0, 1, 0, 0}],
0, 0, (-64/(1 - dim) + (64*dim)/(1 - dim) - (16*dim^2)/(1 - dim) +
  (128*mf2^2)/(1 - dim) - (64*dim*mf2^2)/(1 - dim) + (128*t)/(1 - dim) -
  (64*dim*t)/(1 - dim) - (256*mf2*t)/(1 - dim) +
  (128*dim*mf2*t)/(1 - dim) + (128*t^2)/(1 - dim) -
  (64*dim*t^2)/(1 - dim)) INT["emu_2L.g6", {0, 0, 1, 1, 0, 0, 0, 0, 0}],
```

AIDA for muon-electron scattering

Interface with IBP generators

[Mastrolia, Primo, Ronca, W.J.T. (work in progress)]

Reduze files

integralfamilies.yaml

```
integralfamilies:
  - name: emu_2L.g6
    loop_momenta: [k1,k2]
    propagators:
      - [ k1 , 0 ]
      - [ k2 , 0 ]
      - [ k1 + k2 , 0 ]
      - [ k2 + p2 , 0 ]
      - [ k2 + p2 + p3 , mf2 ]
      - [ k1 - p2 - p3 - p4 , 0 ]
      - [ k2 + p2 + p3 + p4 , 0 ]
      - {bilinear: [[k1,p3],0]}
      - {bilinear: [[k1,p4],0]}
```

-:--- integralfamilies.yaml All (1,0) (Fundamental)

kinematics.yaml

```
kinematics:
   incoming_momenta: [p1,p2,p3,p4]
   outgoing_momenta: []
   momentum_conservation: [p1,-p2 - p3 - p4]
   kinematic_invariants:
     - [mf2,2]
     - [t,2]
     - [ex1,2]
   scalarproduct_rules:
     - [[p2,p2],0]
     - [[p2,p3],(-mf2 + t)/2]
     - [[p2,p4],(-ex1 + mf2 - t)/2]
     - [[p3,p3],mf2]
     - [[p3,p4],(ex1 - 2*mf2)/2]
     - [[p4,p4],mf2]
```

-:--- kinematics.yaml All (1,0) (Fundamental)

jobs_IBPs.yaml

```
jobs:
  - setup_sector_mappings:
      allow_general_crossings: false
  - reduce_sectors:
      sector_selection:
        select_recursively:
          - [emu_2L.g6,127]
      identities:
        ibp:
          - { r: [t,7], s: [0,3] }
        lorentz:
          - { r: [t,7], s: [0,3] }
        sector_symmetries:
          - { r: [t,7], s: [0,3] }
      solutions:
        requested_solutions:
          - { r: [t,8], s: [0,3] }
      reduzer_options:
        num_equations_per_subjob: 1
      num_seeds_for_identities_auxjobs: 1000
      delete_temporary_files: true
```

jobs_reduction_basis.yaml

```
jobs:
   - select_reductions:
       input_file: "myintegrals"
       output_file: "tmp/myintegrals.tmp"
       notfound_file: "myintegrals.tmp.masters"
       conditional: false
   - reduce_files:
       equation_files: ["tmp/myintegrals.tmp"]
       output_file: "myintegrals.sol"
   - export:
       input_file: "myintegrals.sol"
       output_file: "myintegrals.sol.mma"
       output_format: "mma"
```

myintegrals

```
{
INT[emu_2L.g6, {-2, 1, 1, 0, 0, 1, 2, 0, 0}],
INT[emu_2L.g6, {-2, 1, 1, 0, 1, 1, 1, 0, 0}],
INT[emu_2L.g6, {-2, 1, 1, 0, 1, 1, 2, 0, 0}],
INT[emu_2L.g6, {-2, 1, 1, 1, 0, 1, 2, 0, 0}],
INT[emu_2L.g6, {-2, 1, 1, 1, 1, 1, 1, 0, 0}],
INT[emu_2L.g6, {-1, 0, 1, 0, 1, 1, 1, 0, 0}],
INT[emu_2L.g6, {-1, 0, 1, 1, 0, 1, 1, 0, 0}],
INT[emu_2L.g6, {-1, 0, 1, 1, 1, 1, 0, 0, 0}],
INT[emu_2L.g6, {-1, 0, 1, 1, 1, 1, 1, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 0, 0, 0, 2, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 0, 0, 1, 1, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 0, 0, 1, 2, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 0, 1, 0, 1, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 0, 1, 0, 2, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 0, 1, 1, 0, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 0, 1, 1, 1, 0, -1}],
INT[emu_2L.g6, {-1, 1, 1, 0, 1, 1, 1, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 0, 1, 1, 2, 0, -1}],
INT[emu_2L.g6, {-1, 1, 1, 0, 1, 1, 2, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 0, 0, 1, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 0, 0, 2, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 0, 1, 0, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 0, 1, 1, -1, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 0, 1, 1, 0, -1}],
INT[emu_2L.g6, {-1, 1, 1, 1, 0, 1, 1, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 0, 1, 2, -1, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 0, 1, 2, 0, -1}],
INT[emu_2L.g6, {-1, 1, 1, 1, 0, 1, 2, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 1, 0, 1, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 1, 1, -1, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 1, 1, 0, -1, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 1, 1, 0, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 1, 1, 1, -1, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 1, 1, 1, 0, -1}],
INT[emu_2L.g6, {-1, 1, 1, 1, 1, 1, 1, 0, 0}],
INT[emu_2L.g6, {-1, 1, 1, 1, 1, 1, 2, 0, -1}],
INT[emu_2L.g6, {-1, 1, 1, 1, 1, 1, 2, 0, 0}],
INT[emu_2L.g6, {-1, 2, 1, 0, 0, 0, 1, 0, 0}],
INT[emu_2L.g6, {-1, 2, 1, 1, 0, 0, 0, 0, 0}],
INT[emu_2L.g6, {-1, 2, 1, 1, 0, 0, 1, 0, 0}],
INT[emu_2L.g6, {0, -1, 1, 1, 0, 1, 1, 0, 0}],
INT[emu_2L.g6, {0, -1, 1, 1, 1, 1, 1, 0, 0}],
INT[emu_2L.g6, {0, 0, 0, 0, 0, 1, 1, 0, 0}],
INT[emu_2L.g6, {0, 0, 0, 0, 0, 1, 2, 0, 0}],
INT[emu_2L.g6, {0, 0, 0, 0, 1, 1, 2, 0, 0}],
INT[emu_2L.g6, {0, 0, 0, 1, 0, 1, 0, 0, 0}],
INT[emu_2L.g6, {0, 0, 0, 1, 0, 1, 1, 0, 0}],
INT[emu_2L.g6, {0, 0, 0, 1, 0, 1, 2, 0, 0}],
INT[emu_2L.g6, {0, 0, 0, 1, 1, 1, 1, 0, 0}],
```

AIDA for muon-electron scattering

[Mastrolia, Primo, Ronca, W.J.T. (work in progress)]

Interface with IBP generators

Generate IBPs

```
INT["emu_2L.g6",4,15,5,2,{1,2,1,1,0,0,0,0,-2}] -> 0,

INT["emu_2L.g6",4,23,4,0,{1,1,1,0,1,0,0,0,0}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (3*(4*mf2^2-8*t*mf2-d*(mf2^2-2*t*mf2+t^2)+4*t^2)^(-1)*(-2+d)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (2*(4*mf2^2-8*t*mf2-d*(mf2^2-2*t*mf2+t^2)+4*t^2)^(-1)*(5*t+5*mf2-2*d*(t+mf2))),

INT["emu_2L.g6",4,23,4,1,{1,1,1,0,1,-1,0,0,0}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (-3/4*(4*t-t*d)^(-1)*(-2+d)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (-1/4*(4*t-t*d)^(-1)*(14*t-d*(5*t+3*mf2)+6*mf2)),

INT["emu_2L.g6",4,23,4,1,{1,1,1,0,1,0,0,-1,0}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (3/8*(2*t+2*mf2-d*(t+mf2))*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (-1/8*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)*(6*mf2^2+20*t*mf2-d*(3*mf2^2+8*t*mf2+5*t^2)

INT["emu_2L.g6",4,23,4,1,{1,1,1,0,1,0,0,0,-1}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (-3/8*(2*t+2*mf2-d*(t+mf2))*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (1/8*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)*(6*mf2^2+20*t*mf2-d*(3*mf2^2+8*t*mf2+5*t^2)+
```

AIDA for muon-electron scattering

Interface with IBP generators

[Mastrolia, Primo, Ronca, W.J.T. (work in progress)]

Generate IBPs

```
INT["emu_2L.g6",4,15,5,2,{1,2,1,1,0,0,0,0,-2}] -> 0,

INT["emu_2L.g6",4,23,4,0,{1,1,1,0,1,0,0,0,0}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (3*(4*mf2^2-8*t*mf2-d*(mf2^2-2*t*mf2+t^2)+4*t^2)^(-1)*(-2+d)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (2*(4*mf2^2-8*t*mf2-d*(mf2^2-2*t*mf2+t^2)+4*t^2)^(-1)*(5*t+5*mf2-2*d*(t+mf2))),

INT["emu_2L.g6",4,23,4,1,{1,1,1,0,1,-1,0,0,0}] ->
  INT["emu_2L.g6",3,21,3,1,
      (-3/4*(4*t-t*d)^(-1)*
  INT["emu_2L.g6",3,21,3,0,
      (-1/4*(4*t-t*d)^(-1)*

INT["emu_2L.g6",4,23,4,1,{1
  INT["emu_2L.g6",3,21,3,1,
      (3/8*(2*t+2*mf2-d*(t+mf2))*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (-1/8*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)*(6*mf2^2+20*t*mf2-d*(3*mf2^2+8*t*mf2+5*t^2)

INT["emu_2L.g6",4,23,4,1,{1,1,1,0,1,0,0,0,-1}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (-3/8*(2*t+2*mf2-d*(t+mf2))*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (1/8*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)*(6*mf2^2+20*t*mf2-d*(3*mf2^2+8*t*mf2+5*t^2)+
```

Apply IBPS to the integrals

$c(0)$ INT(emu_2L.g6, {0, 0, 1, 0, 1, 1, 0, 0, 0}) + $c(1)$ INT(emu_2L.g6, {0, 0, 1, 1, 0, 1, 0, 0, 0}) +
$c(2)$ INT(emu_2L.g6, {0, 0, 1, 1, 1, 1, 0, 0, 0}) + $c(3)$ INT(emu_2L.g6, {0, 1, 1, 0, 1, 1, 0, 0, 0}) +
$c(4)$ INT(emu_2L.g6, {0, 1, 1, 1, 1, 1, 0, 0, 0}) + $c(5)$ INT(emu_2L.g6, {1, −1, 1, 0, 1, 0, 0, 0, 0}) +
$c(6)$ INT(emu_2L.g6, {1, −1, 1, 1, 1, 0, 1, 0, 0}) + $c(7)$ INT(emu_2L.g6, {1, −1, 1, 1, 1, 1, 0, 0, 0}) +
$c(8)$ INT(emu_2L.g6, {1, 0, 1, 0, 1, 0, 0, 0, 0}) + $c(9)$ INT(emu_2L.g6, {1, 0, 1, 1, 0, 0, 1, 0, 0}) +
$c(10)$ INT(emu_2L.g6, {1, 0, 1, 1, 1, 0, 1, 0, 0}) + $c(11)$ INT(emu_2L.g6, {1, 0, 1, 1, 1, 1, 0, 0, 0})

AIDA for muon-electron scattering

Interface with IBP generators

[Mastrolia, Primo, Ronca, W.J.T. (work in progress)]

Generate IBPs

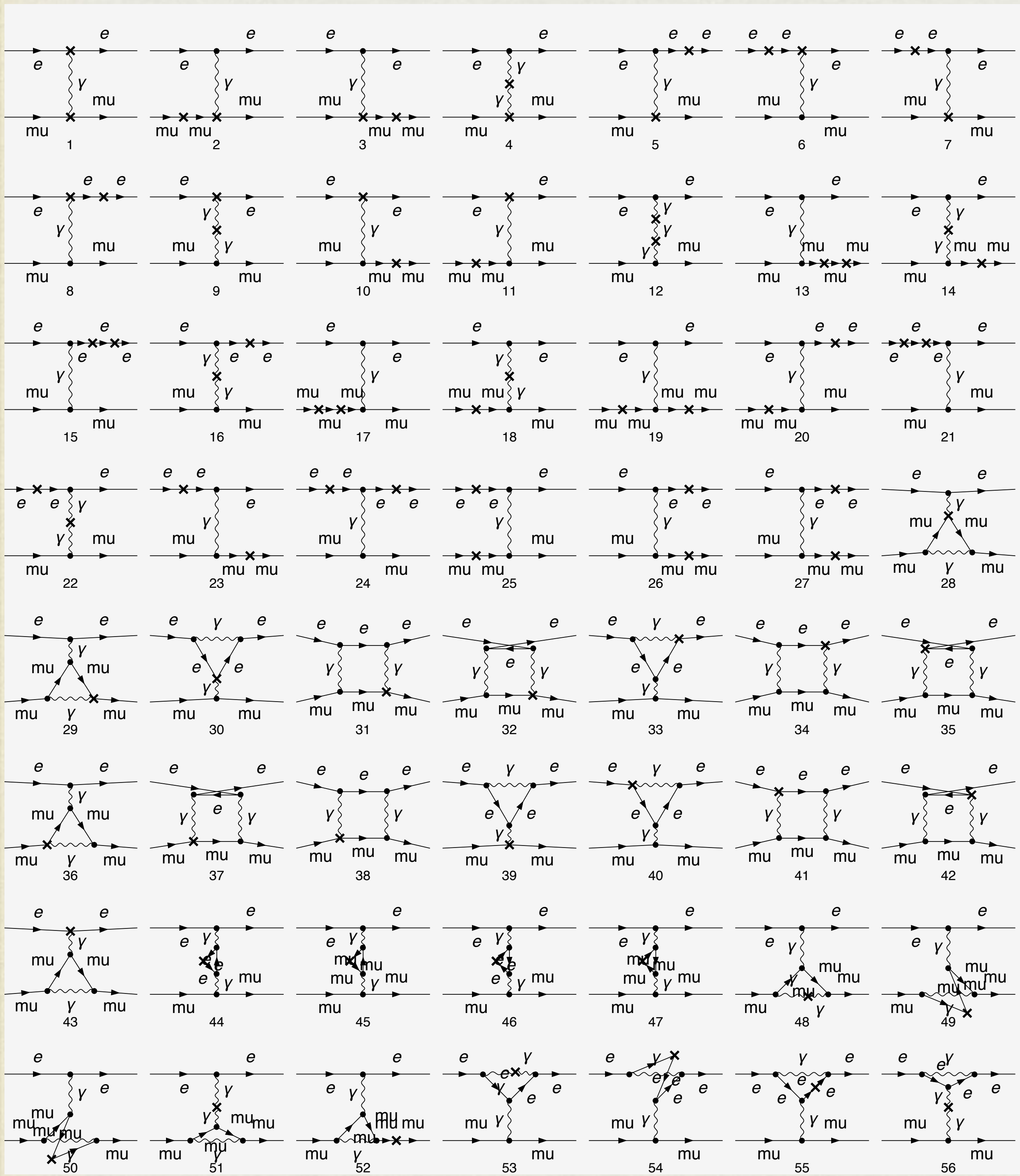
```
INT["emu_2L.g6",4,15,5,2,{1,2,1,1,0,0,0,0,-2}] -> 0,

INT["emu_2L.g6",4,23,4,0,{1,1,1,0,1,0,0,0,0}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (3*(4*mf2^2-8*t*mf2-d*(mf2^2-2*t*mf2+t^2)+4*t^2)^(-1)*(-2+d)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (2*(4*mf2^2-8*t*mf2-d*(mf2^2-2*t*mf2+t^2)+4*t^2)^(-1)*(5*t+5*mf2-2*d*(t+mf2))),

INT["emu_2L.g6",4,23,4,1,{1,1,1,0,1,-1,0,0,0}] ->
  INT["emu_2L.g6",3,21,3,1,
      (-3/4*(4*t-t*d)^(-1)*
  INT["emu_2L.g6",3,21,3,0,
      (-1/4*(4*t-t*d)^(-1)*

INT["emu_2L.g6",4,23,4,1,{1
  INT["emu_2L.g6",3,21,3,1,
      (3/8*(2*t+2*mf2-d*(t+mf2))*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (-1/8*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)*(6*mf2^2+20*t*mf2-d*(3*mf2^2+8*t*mf2+5*t^2)

INT["emu_2L.g6",4,23,4,1,{1,1,1,0,1,0,0,0,-1}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (-3/8*(2*t+2*mf2-d*(t+mf2))*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (1/8*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)*(6*mf2^2+20*t*mf2-d*(3*mf2^2+8*t*mf2+5*t^2)+
```

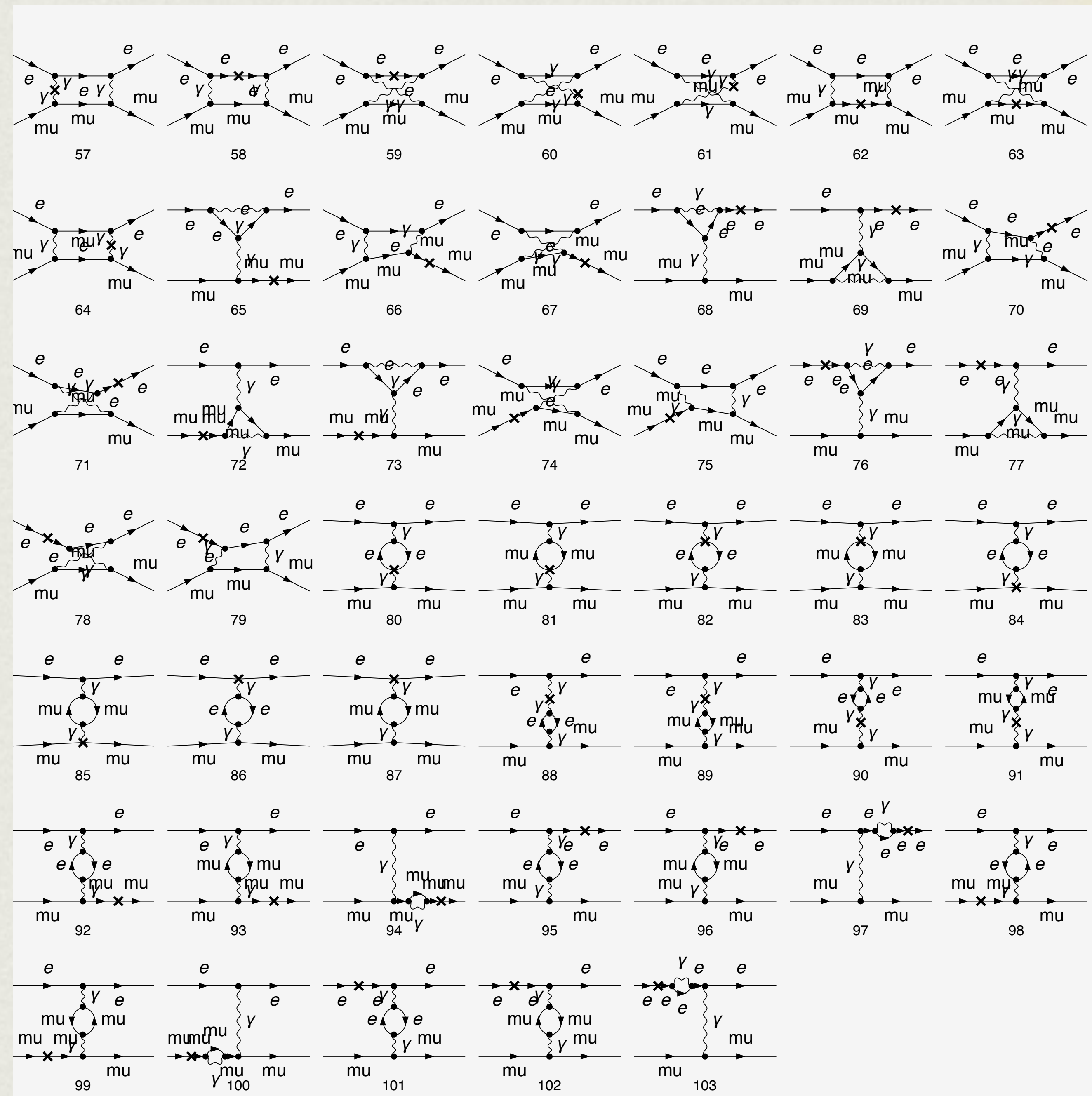
Apply IBPS to the integrals

$$
\begin{aligned}
&c(0)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{0,0,1,0,1,1,0,0,0\}) + c(1)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{0,0,1,1,0,1,0,0,0\}) +\\
&\quad c(2)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{0,0,1,1,1,1,0,0,0\}) + c(3)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{0,1,1,0,1,1,0,0,0\}) +\\
&\quad c(4)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{0,1,1,1,1,1,0,0,0\}) + c(5)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,-1,1,0,1,0,0,0,0\}) +\\
&\quad c(6)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,-1,1,1,1,0,1,0,0\}) + c(7)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,-1,1,1,1,1,0,0,0\}) +\\
&\quad c(8)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,0,1,0,1,0,0,0,0\}) + c(9)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,0,1,1,0,0,1,0,0\}) +\\
&\quad c(10)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,0,1,1,1,0,1,0,0\}) + c(11)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,0,1,1,1,1,0,0,0\})
\end{aligned}
$$

>> PRIMO'S talk

What about renormalisation?





Conclusions/Outlook

Multi-loop scattering amplitudes

- [x] Integrand decomposition methods —> @1 and 2 Loops Automated (AIDA)
- [x] Analytic decomposition for all 2—>2 processes—> Under control
- [x] AIDA's output —> Apply IBPs + evaluation of MIs
- [x] Muon-electron scattering at NNLO is at hand

- [ ] Deal with analytic expressions for 2—>3,4 processes
- [ ] More processes to come in the near future

Scattering Amplitudes in Gauge theories still reserve a lot of surprises.

Conclusions/Outlook

loops

Multi-loop scattering amplitudes

- ☑ Integrand decomposition methods —> @1 and 2 Loops Automated (AIDA)
- ☑ Analytic decomposition for all 2—>2 processes—> Under control
- ☑ AIDA’s output —> Apply IBPs + evaluation of MIs
- ☑ Muon-electron scattering at NNLO is at hand

- ☐ Deal with analytic expressions for 2—>3,4 processes
- ☐ More processes to come in the near future

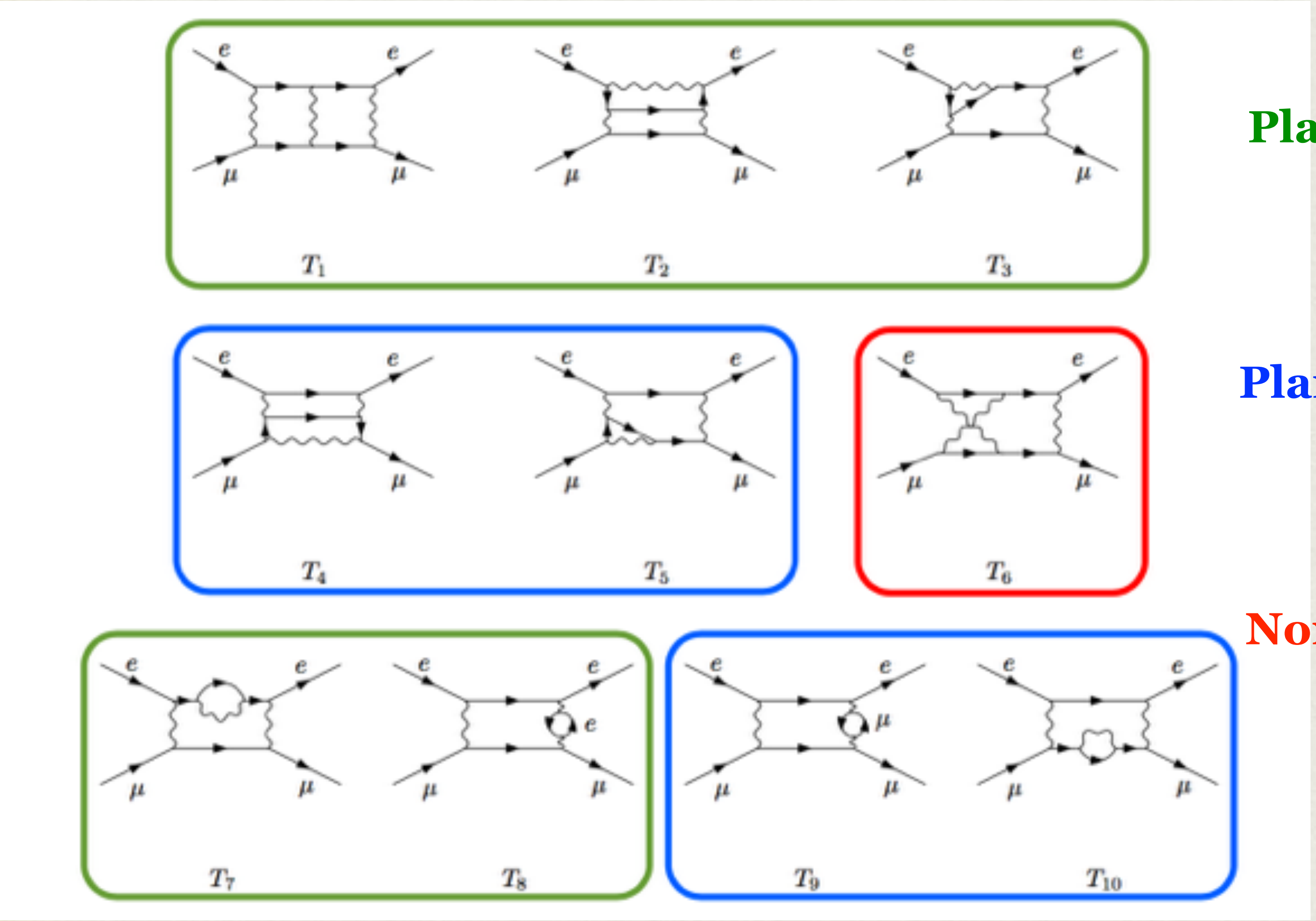
Scattering Amplitudes in Gauge theories still reserve a lot of surprises.

Thanks

Extra slides

Two-loop MIs for muon-electron scattering

[Mastrolia (Muon-electron Scattering: Theory kickoff workshop (2017))]



Planar integrals :: Family 1 (34 MIs)

[Bonciani, Ferroglia, Gehrmann, Maitre, von Manteuffel, Studerus]

Planar integrals :: Family 2 (42 MIs)

[Mastrolia, Passera, Primo, Schubert (2017)]

Non planar integrals :: Family 3 (44 MIs)

[Mastrolia, Primo, Schubert (work in progress)]