Lattice results for a_μ^{HVP}

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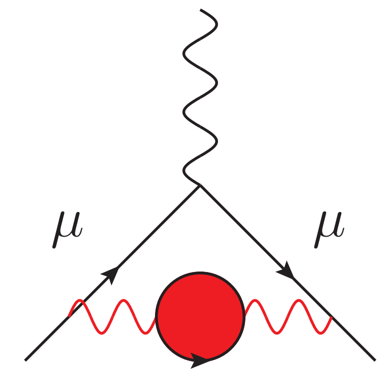
a_μ as a stringent test of the SM

	$a_\mu\,[10^{-11}]$	$\Delta a_\mu\,[10^{-11}]$
experiment	116 592 089.	63.
QED $\mathcal{O}(\alpha)$	116 140 973.21	0.03
QED $\mathcal{O}(\alpha^2)$	413 217.63	0.01
QED $\mathcal{O}(\alpha^3)$	30 141.90	0.00
QED $\mathcal{O}(\alpha^4)$	381.01	0.02
QED $\mathcal{O}(\alpha^5)$	5.09	0.01
QED total	116 584 718.95	0.04
electroweak, total	153.6	1.0
HVP (LO) [Hagiwara et al. 11]	6 949.	43.
HVP (NLO) [Hagiwara et al. 11]	−98.	1.
HLbL [Jegerlehner-Nyffeler 09]	116.	40.
HVP (NNLO) [Kurz, Liu, Marquard, Steinhauser 14]	12.4	0.1
HLbL (NLO) [GC, Hoferichter, Nyffeler, Passera, Stoffer 14]	3.	2.
theory	116 591 855.	59.



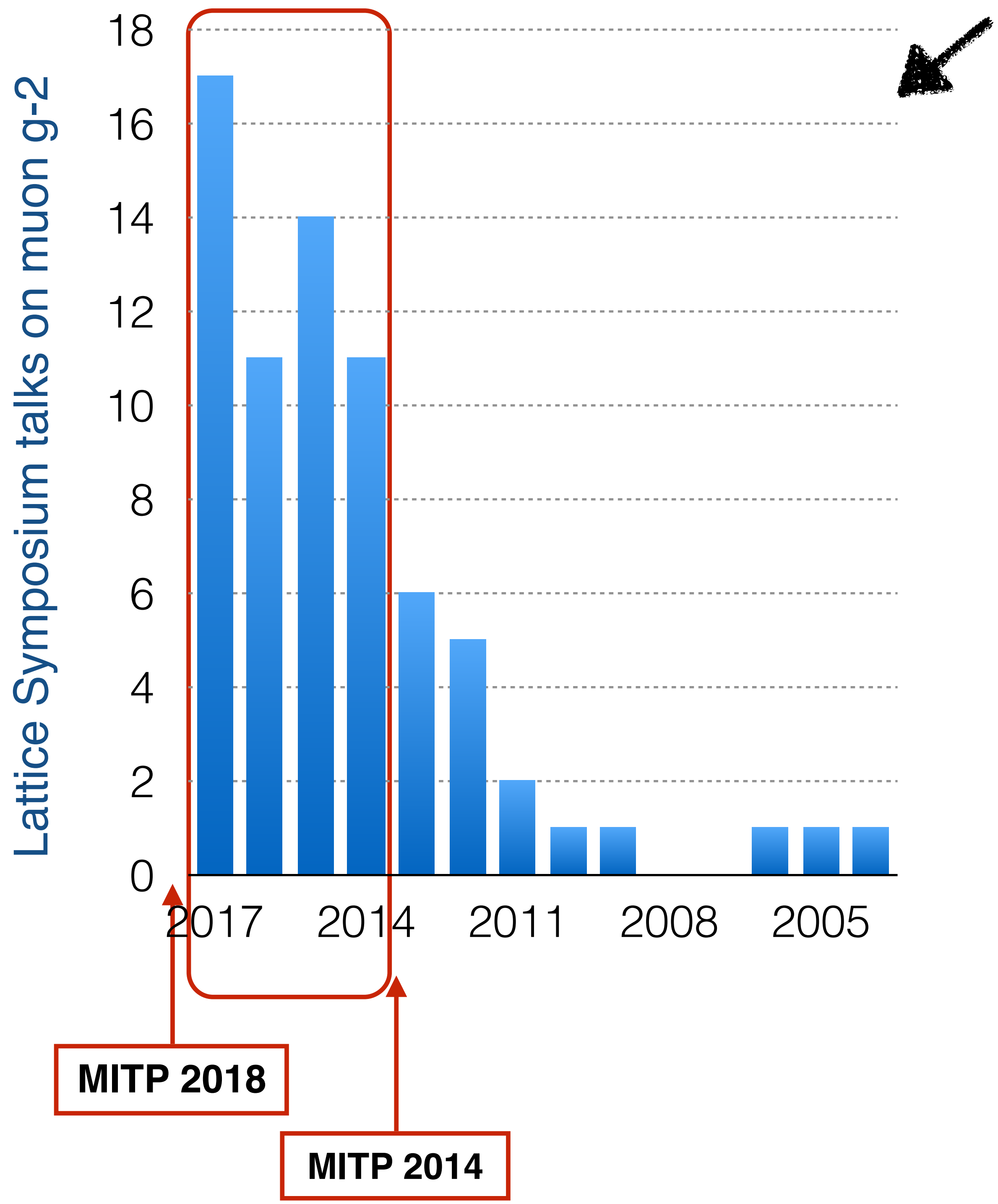
- Current TH estimate affected by
  - ➡ the experimental uncertainties;
  - ➡ perturbation theory/models

- Lattice QCD estimate —> for a final cross-check of the SM result and to keep up with the planned experimental improvements

$$a_\mu^{HVP} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left\{ \int_{m_\pi^2}^{E_{cut}^2} ds \frac{\boxed{R_{\text{had}}^{\text{data}}}(s)\hat{K}(s)}{s^2} + \int_{\boxed{E_{cut}^2}}^{\infty} ds \frac{R_{\text{had}}^{\text{pQCD}}(s)\hat{K}(s)}{s^2} \right\}$$

- HVP leading order: **largest uncertainty!** (around 50% of total th. error)
- **Lattice QCD provides a way to compute this contribution in a model-independent way**

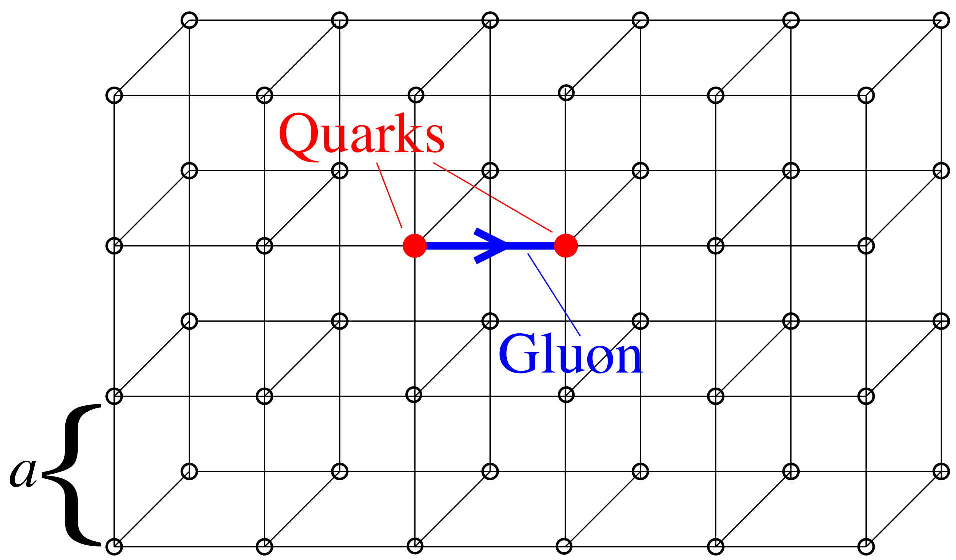
Activity on muon g-2 in the lattice community

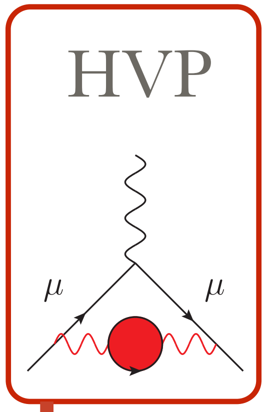


- **HVP** from the lattice:
  - ➡ **[T. Blum Phys.Rev.Lett. 91 (2003) 052001]**
  - ➡ RBC/UKQCD, Mainz U.[CLS], FNAL/HPQCD/MILC, BMW, ABGP, Regensburg U. PACS-CS … **[g-2 WS@KEK, 12-14 Feb. 2018]**
- **HLbL** from the lattice
  - ➡ RBC **[arXiv:1610.04603, arXiv:1705.01067]**, Mainz U. **[arXiv:1712.00421,arXiv:1801.04238].**

- HVP from the lattice+experiment (R-ratio data):
  - ➡ Bernecker&Meyer **[arXiv:1107.4388]**
  - ➡ ETM, MILC, RBC/UKQCD …
- HVP from the lattice+experiment (space-like data)
  - ➡ MUonE **[see Massimo's talk for intro…]**

Non - perturbative computation of a_μ





- Recipe for lattice QCD computation :

1. Generate ensembles of field configurations using *Monte Carlo*
2. Average over a set of configurations:
   - ➡ Compute correlation function of fields, extract Euclidean matrix elements or amplitude
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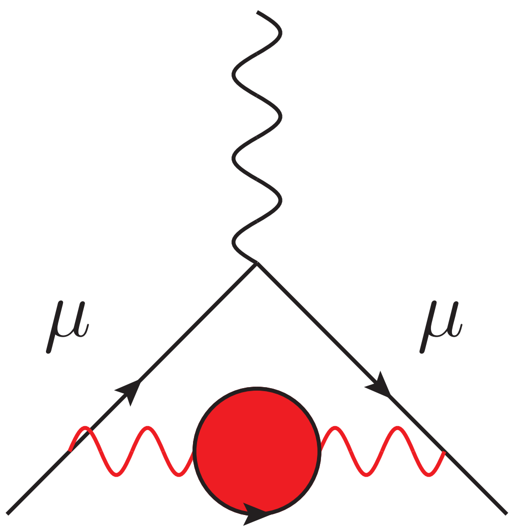
The leading hadronic contribution - HVP

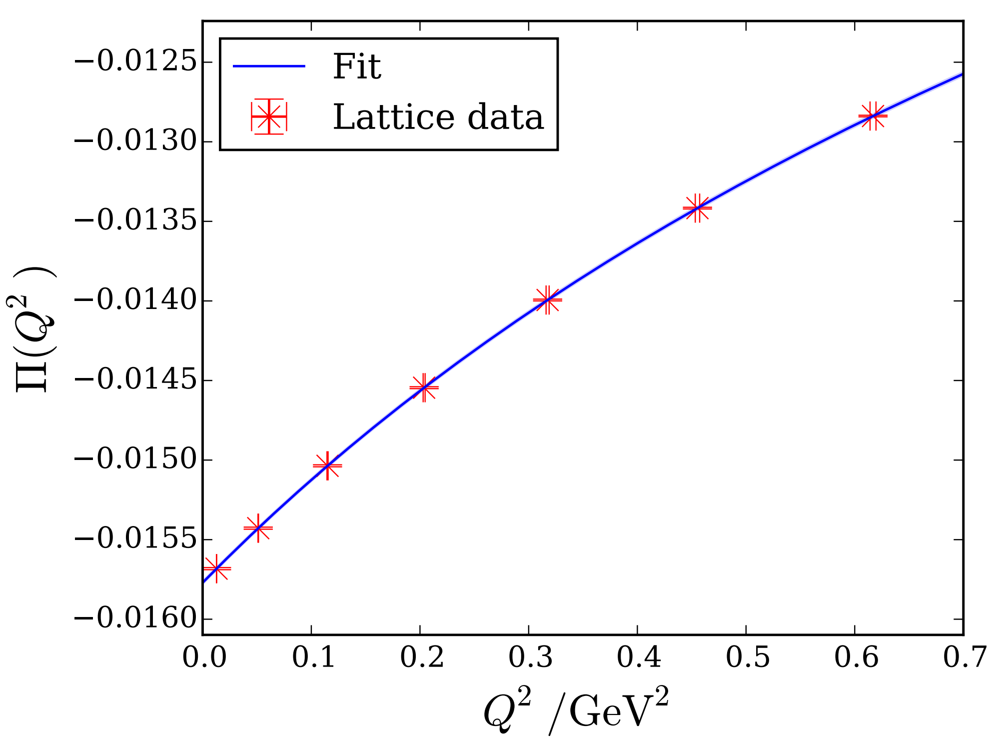


Vacuum polarisation inserted in the photon propagator

$$\boxed{a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 \boxed{f(Q^2)} \times \boxed{\hat{\Pi}(Q^2)}}$$

$$\boxed{f(Q^2) = m_\mu^2 Q^2 Z^3(Q^2) \frac{1 - Q^2 Z(Q^2)}{1 + m_\mu^2 Q^2 Z^2(Q^2)}}$$

$$Z(Q^2) = \frac{\sqrt{Q^4 + 4m_\mu^2 Q^2} - Q^2}{2m_\mu^2 Q^2}$$



$$\boxed{\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)}$$

separate contribution for: u/d,s,c

$$\Pi_{\mu\nu}(Q) = \boxed{\sum_f Q_f^2} \sum_x e^{iQx} \langle J_\mu^f(x) J_\nu^f(0) \rangle$$

$$\Pi_{\mu\nu}(Q) = (Q_\mu Q_\nu - g_{\mu\nu} Q^2) \Pi(Q^2)$$

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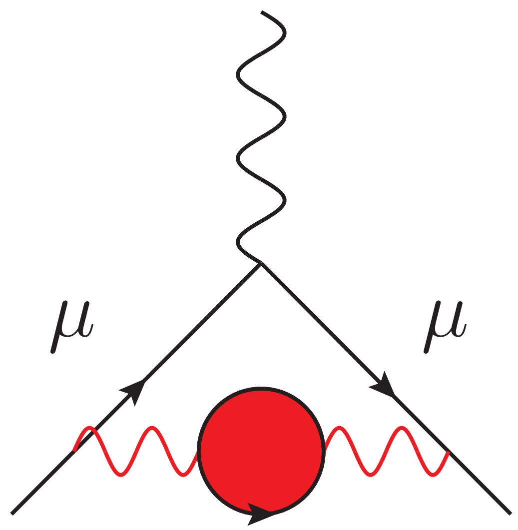

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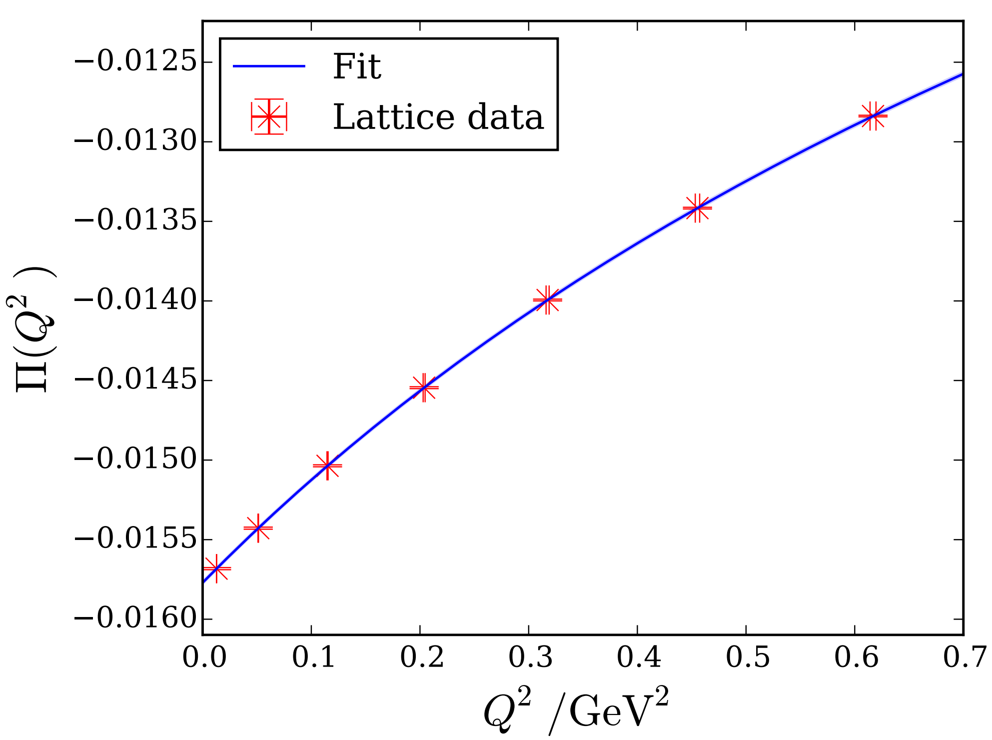
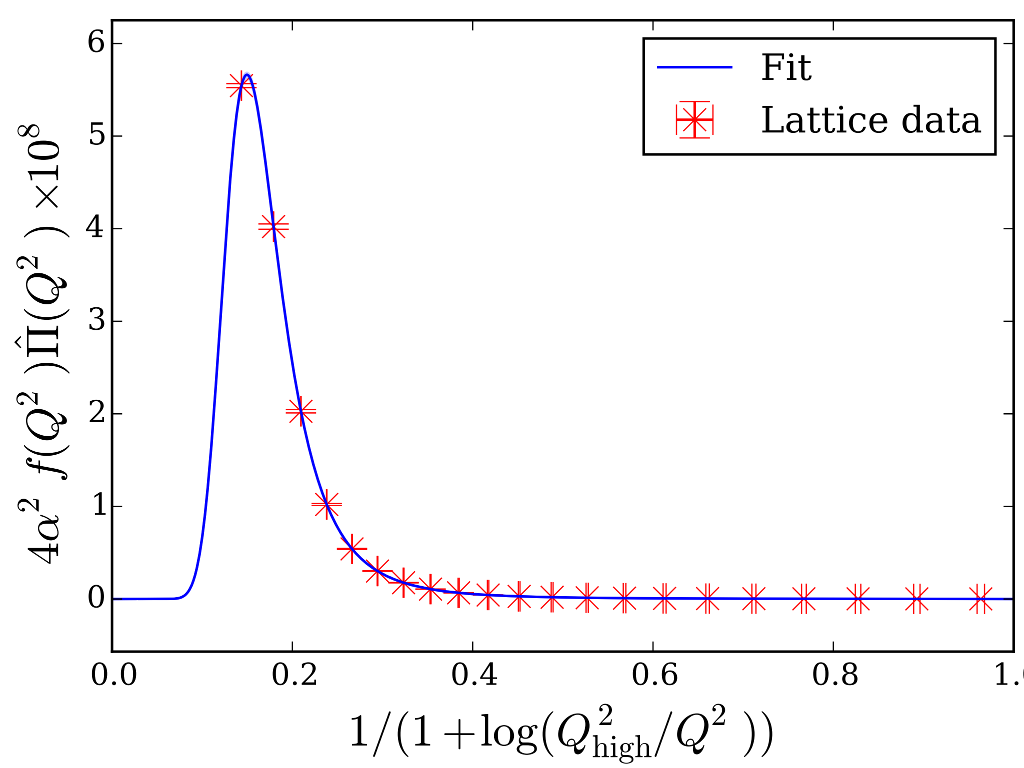
JHEP 1604 (2016) 063 [T.Blum, P.A.Boyle, L. Del Debbio, R.J. Hudspith,T. Izubuchi, A.Juettner, C.Lehner, R. Lewis, K. Maltman, M.K.M., A. Portelli, M.Spraggs]

strange quark HVP, RBC-UKQCD '16





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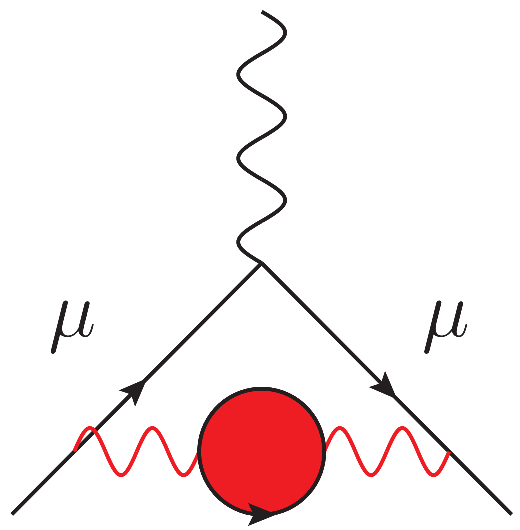

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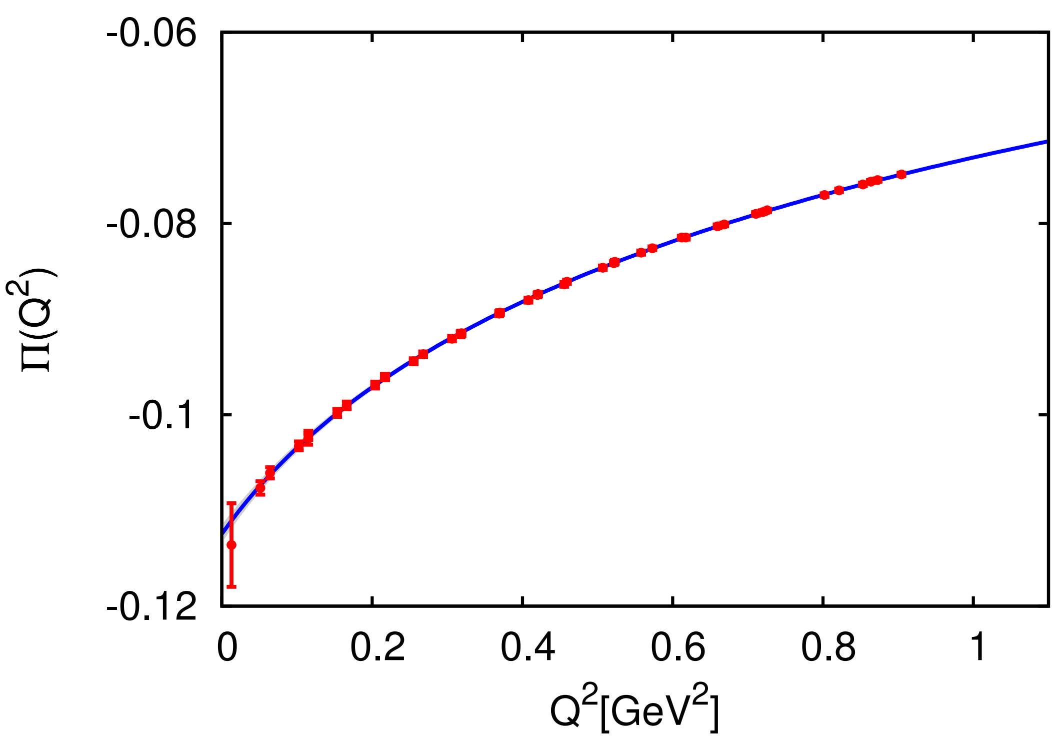
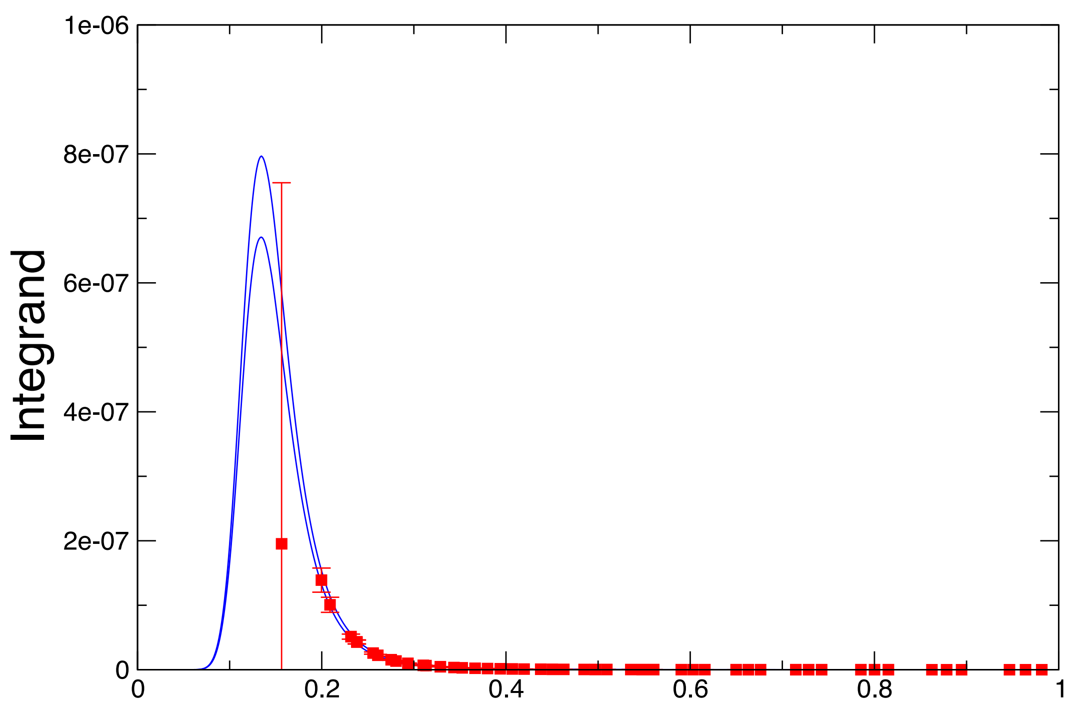
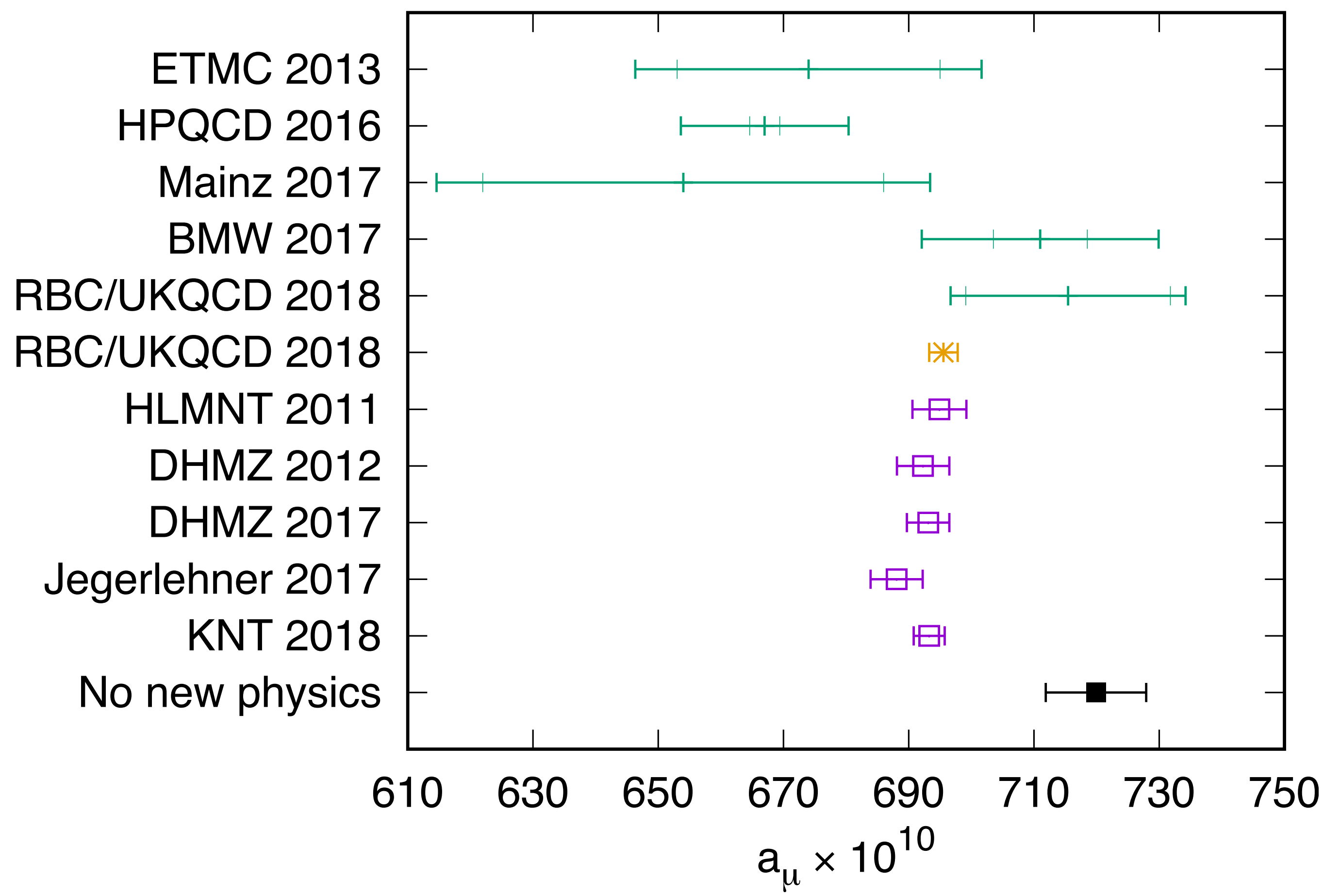
Phys. Rev. D85 (2012) [P.A.Boyle, L. Del Debbio, E.Kerrane, J.Zanotti]

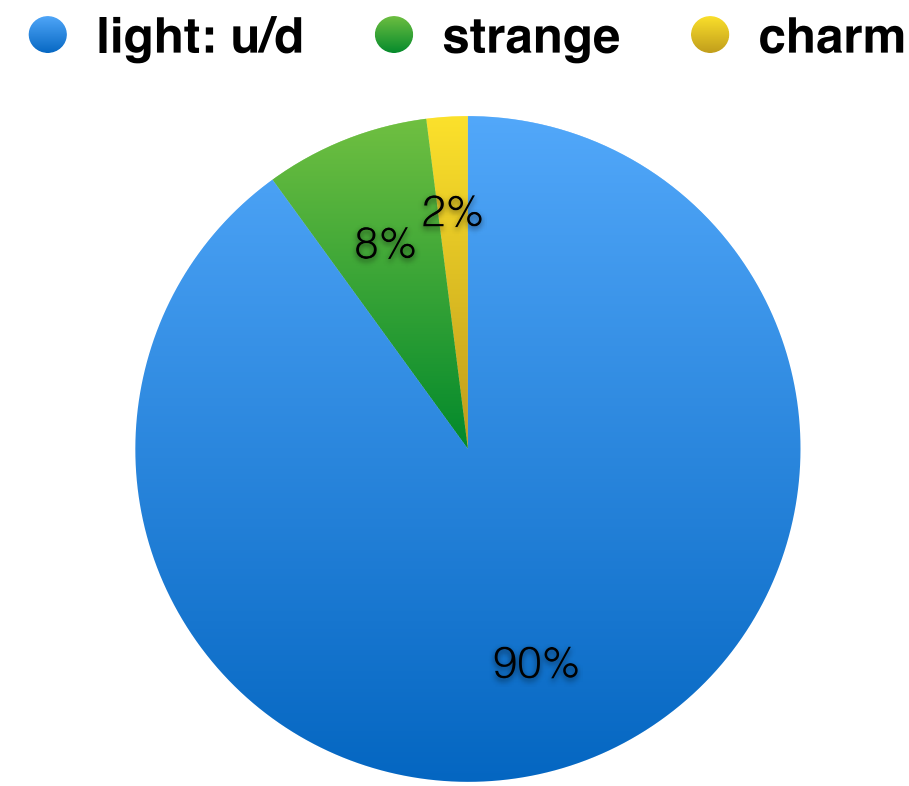
light quark HVP, RBC-UKQCD '12





Summary: HVP from the lattice/R-ratios





[**HPQCD: arXiv:1601.03071, Mainz: arXiv:1705.01775, BMW: arXiv:1711.04980, RBC/UKQCD: arXiv:1801.07224**]

Summary by C. Lehner @ KEK g-2 WS 2018

1. Generate ensembles of field configurations using *Monte Carlo*
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3. Extrapolate to continuum, infinite volume, physical quark masses (now directly accessible)

Workshop on hadronic vacuum polarization contributions to muon g-2

February 12-14, 2018
KEK, Tsukuba, Japan

- Home
- Program
- Registration
- Accommodation
- Access
- g-2 theory initiative
- Committees
- Web Streaming
- Participant List
- Links
- Contacts

About

The muon g-2 is arguably one of the most important observables in contemporary particle physics. The long-standing anomaly at the level of more than 3 standard deviations between the experimental value and the Standard Model (SM) prediction of the muon g-2 may indicate the existence of new physics beyond the SM, which has attracted lots of physicists. The level of the significance may become even higher in the near

2nd Workshop of the **Muon g-2 Theory Initiative** [previous: June 2017 @Fermilab]

Highlights from KEK Feb. 2018: g-2 WS

Error budget for $a_\mu^{ud,conn.}$

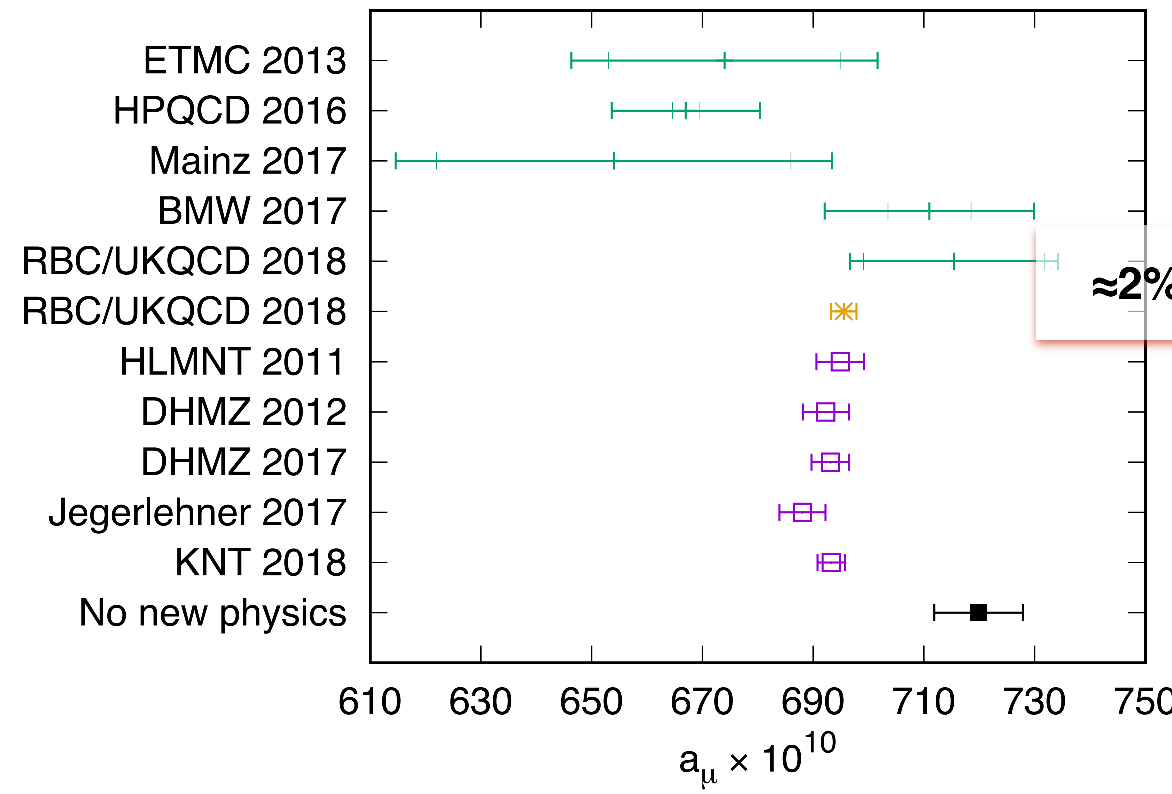
✦ Shown for isospin-limit quantity ➡ include QED & isospin-breaking errors only in total LO HVP contribution

	$a_\mu^{ud,HVP}$ (%) HPQCD + RV 1601.03071	$a_\mu^{ud,HVP}$ (%) Fermilab-HPQCD-MILC *Preliminary*
Statistics + 2pt fit	0.4	0.3
Finite-volume & discretization corrections	0.7	0.2
$\pi\pi$ states ($t > t^*$)	0.5	0.2
Continuum ($a \to 0$) extrapolation	0.2	0.1
Current renormalization (Z_V)	0.2	0.1
Chiral (m_l) extrapolation/interpolation	0.4	0.1
Pion mass ($M_{\pi,5}$) uncertainty	–	0.1
Sea (m_s) adjustment	0.2	0.1
Experimental M_ρ	–	0.1
Lattice-spacing (a^{-1}) uncertainty	<0.05	0.04
Padé approximants	0.4	0.0
Fit total	1.16%	0.52%

1.4% -> 0.52%

R. Van de Water HVP contribution to muon g-2 with (2+1+1) HISQ quarks 19

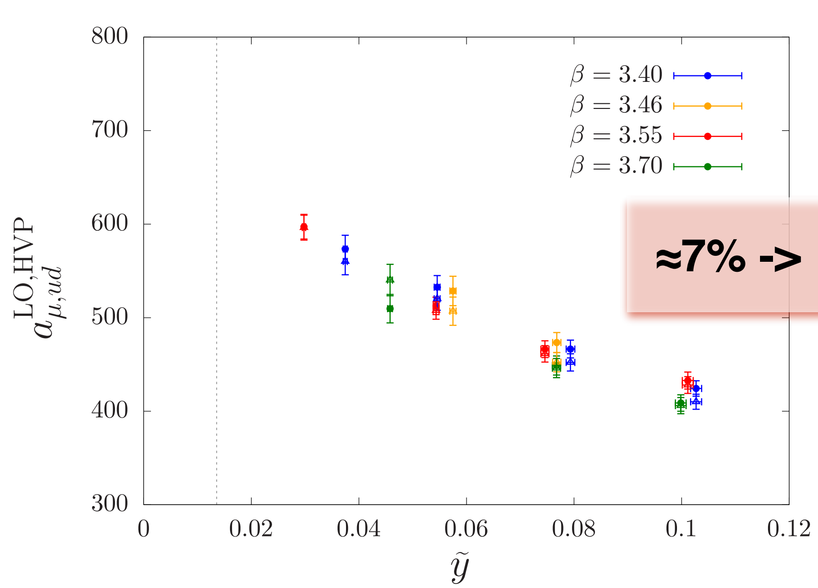
Fermilab/HPQCD/MILC [R.Van de Water@KEK '18]



≈2% -> 0.4%

RBC-UKQCD [C.Lehner@KEK '18]

Pion mass and lattice spacing dependence



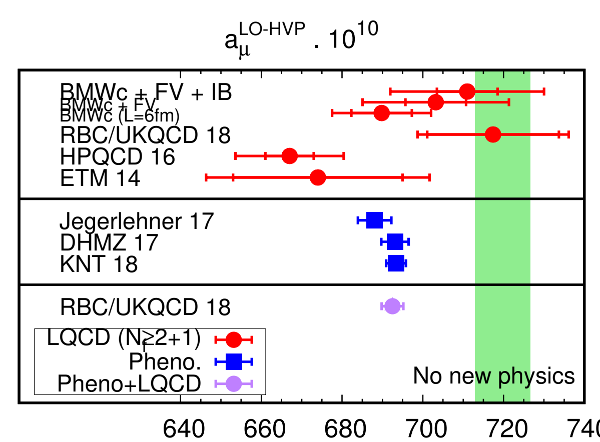
≈7% -> ≈1%

G.M. von Hippel Mainz/CLS lattice results

Mainz/CLS [G.von Hippel@KEK '18]

… AND MANY MORE

Comparison



2.7%

- "No New Physics" scenario: $= (720 \pm 7) \times 10^{-10}$
- BMWc '17 consistent w/ "No new physics" scenario & pheno.
- Total uncertainty of 2.7% is $\sim 6\times$ pheno. error
- BMWc '17 is larger than other $N_f = 2+1+1$ results $\to$ difference w/ HPQCD '16 is $\sim 1.9\sigma$

Laurent Lellouch KEK, 12-14 February 2018

BMW [L. Lellouch@KEK '18]

Nf=2+1+1 (u/d+s+c) HVP

- Five complete computations of a_μ^{HVP}**(u/d+s+c)**
  - ➡ **ETM '15:** [JHEP 1511(2015) 215, arXiv:1505.03283]
  - ➡ **HPQCD: ~1.8%** precision for (u/d+s+c+b) [arXiv:1601.03071]
  - ➡ **recent Mainz:** [arXiv:1705.01775]
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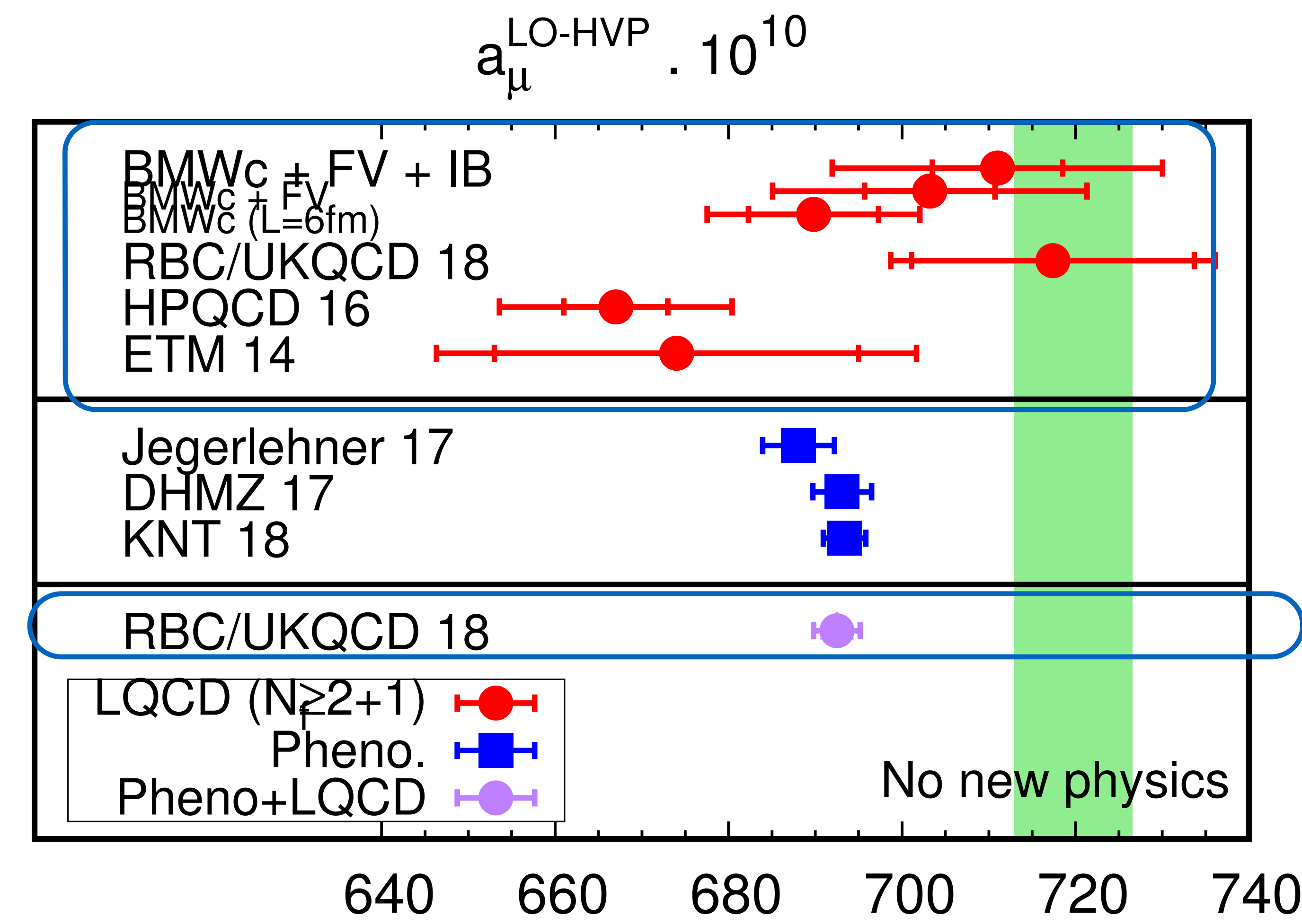
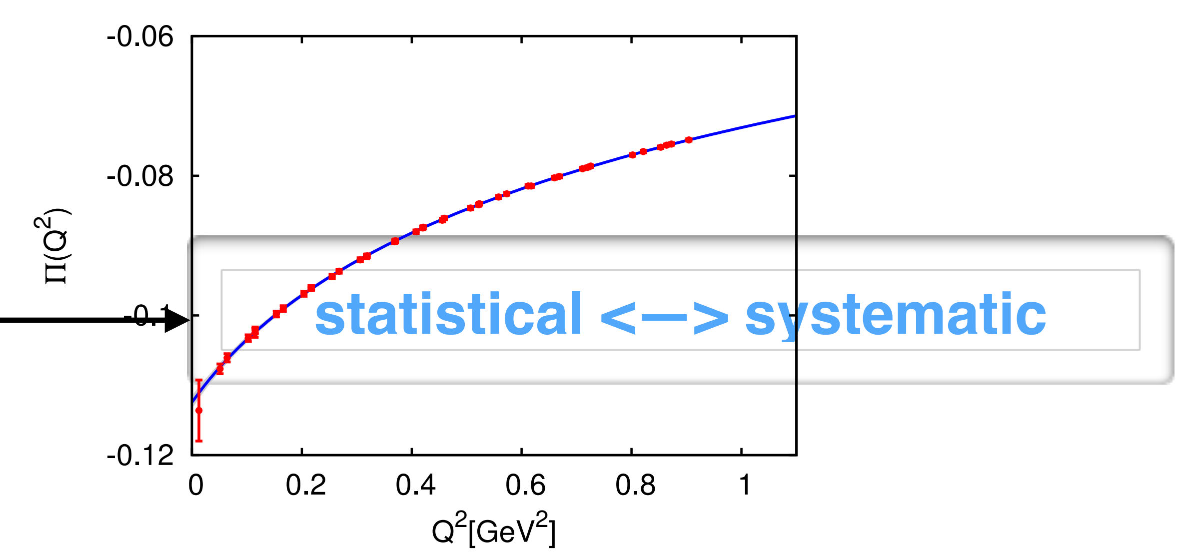
- Understanding the systematics is extremely important

- **Main issues**
  1. deterioration of signal at $\mathbf{Q^2}$ **—> 0**
  2. disconnected diagrams
  3. isospin breaking effects
  4. scale setting error … [arXiv:1705.01775]



Summary by L.Lellouch@ KEK g-2 WS 2018



Time-Momentum Representation (TMR)

- Subtraction prescription, integral representation **[Bernecker & Meyer,Eur Phys J A47 (2011) 148]**

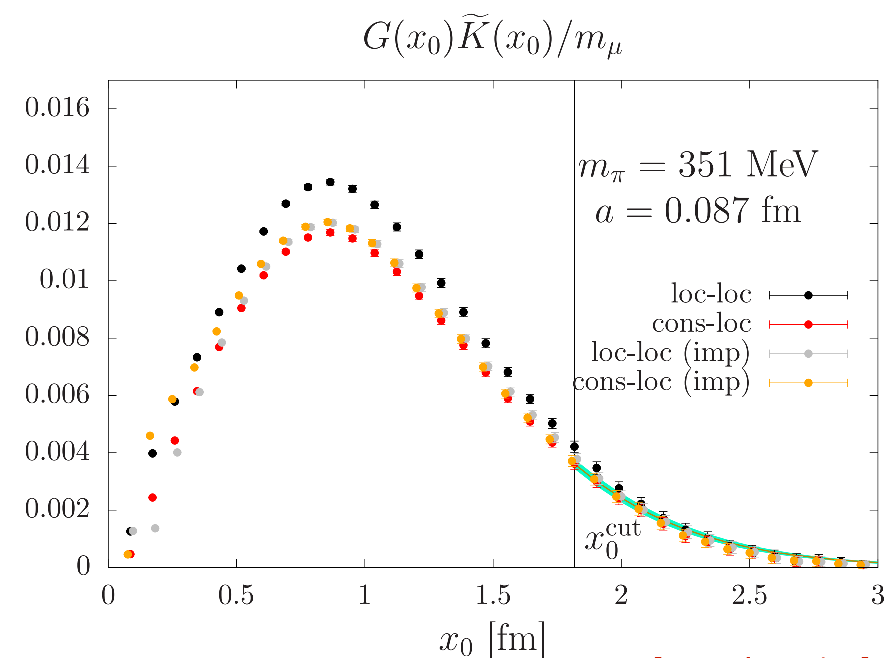
$$\Pi(Q^2) - \Pi(0) = \frac{1}{Q^2}\int_0^\infty dt G(t)[Q^2t^2 - 4\sin^2(\frac{1}{2}Qt)]$$

- Spatially summed vector correlator: $G(t) = -\frac{a^3}{3}\sum_{\vec{x}}\langle J_\mu^{em}(\vec{x},t)J_\nu^{em}(\vec{0},0)\rangle$

- Understanding the large-time behaviour of ***G(t)***

- Applied in recent works by **Mainz, RBC/UKQCD, ETM, Fermilab/HPQCD/MILC, BMW…**

$$G(t) = \begin{cases} G_{data}, & t < t^* \\ G_{fit}, & t > t^* \end{cases}$$



Mainz/CLS **[A.Gerardin@Lat17]**

Dominant sources of errors

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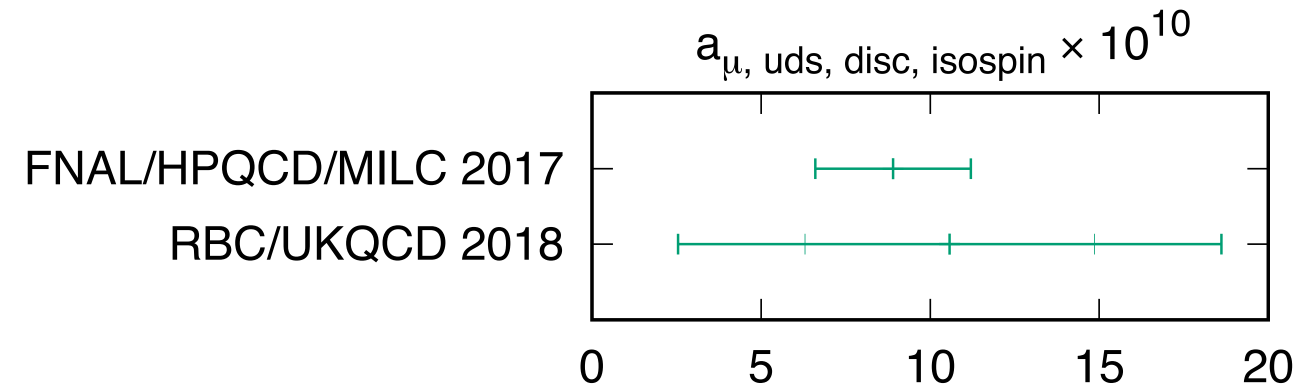

C. Lehner @ KEK g-2 WS 2018

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  1. deterioration of signal at **Q^2 —> 0**
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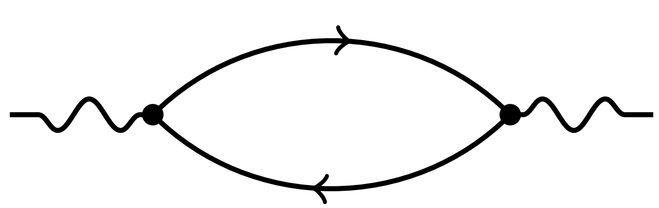
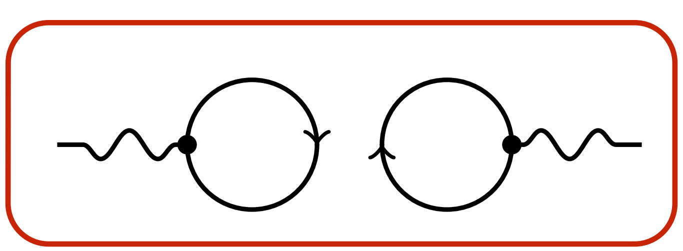
  3. isospin breaking effects
  4. scale setting error … [**arXiv:1705.01775**]

- In addition to this, upper bounds:
  - ➡ Mainz group '14-'17: $\dfrac{a_{\mu\ \ disc.}^{HVP}}{a_{\mu\ \ conn.}^{HVP,light}} < 1\%$
  - ➡ HPQCD&TCD '15: $\dfrac{a_{\mu\ \ disc.}^{HVP}}{a_{\mu\ \ conn.}^{HVP,light}} = -0.14(5)\%$
  - ➡ Bali & Endrodi '15:

$$\frac{\Pi^{disc}}{\Pi^{conn.}} = -(3.6 \pm 4.5) \times 10^{-4} \quad \text{at} \quad Q^2 = 0.03\mathrm{GeV}^2$$





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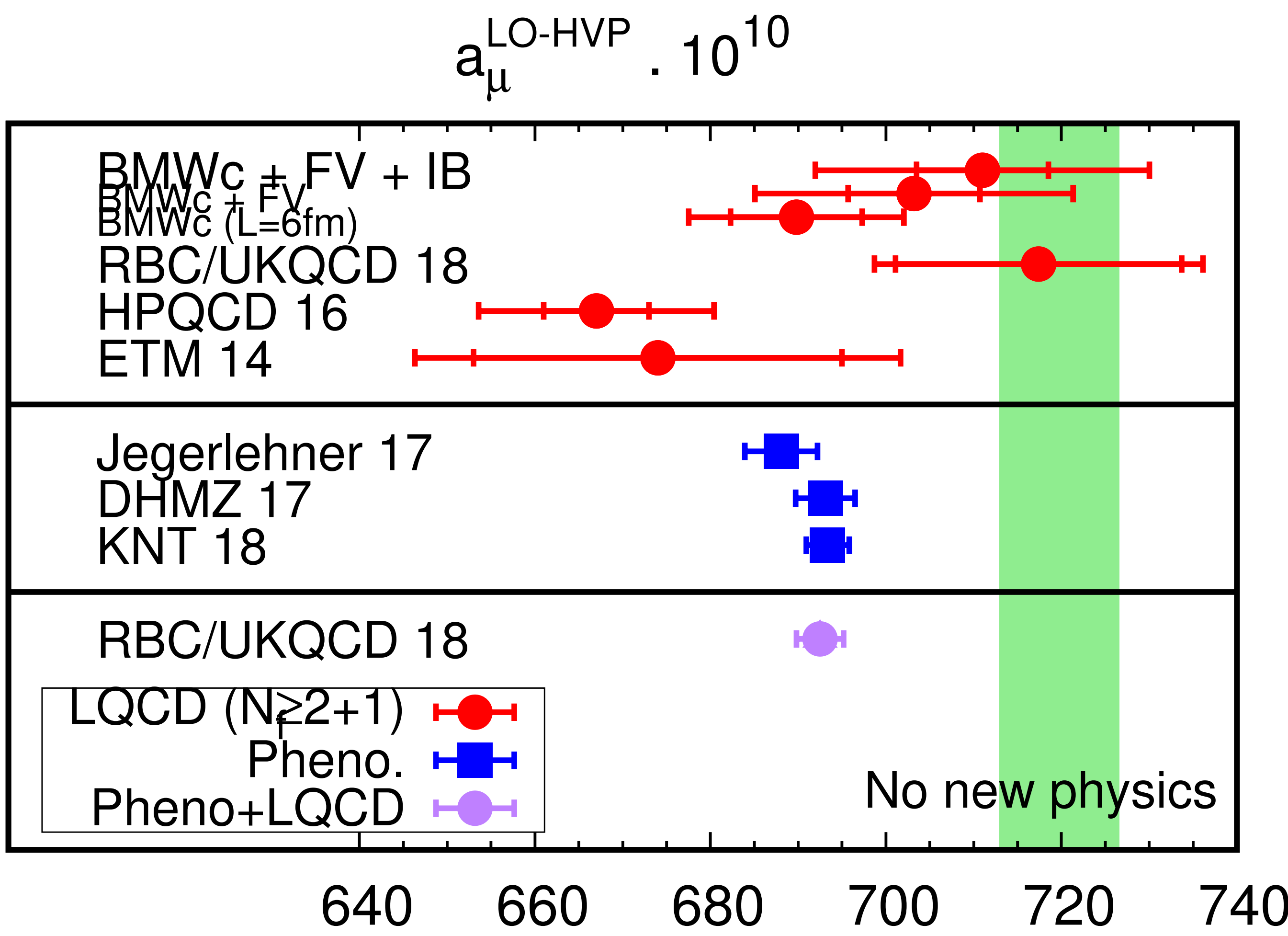

Summary by L.Lellouch@ KEK g-2 WS 2018

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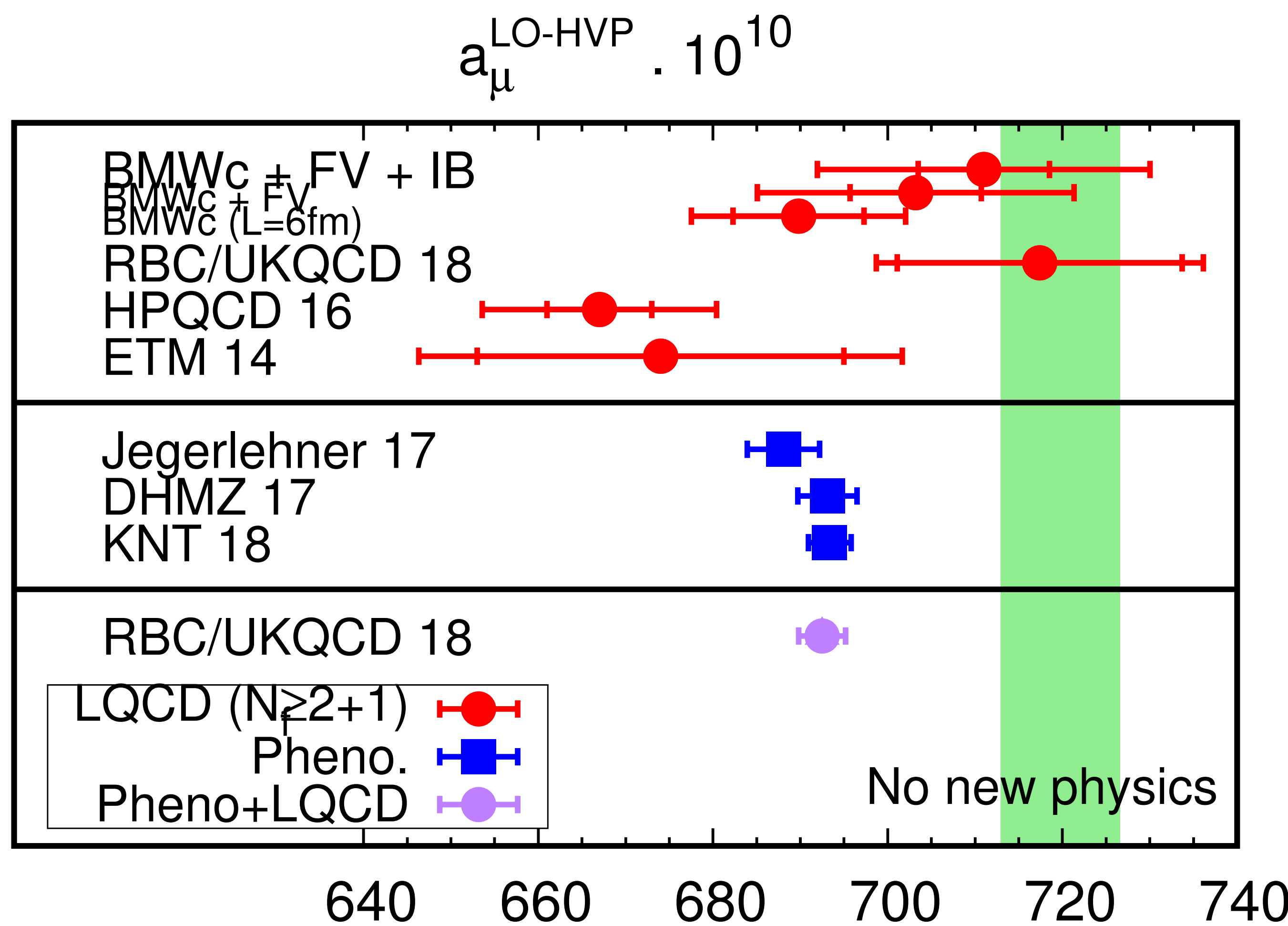

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so far: m_u=m_d and α_{em}=0

- Two ways of treating isospin breaking effects :
  - ➡ treating QED perturbatively
  - ➡ simulating QED+QCD

[work in progress for the HVP (CERN, Mainz, FNAL/MILC, Odense, RBC/UKQCD,R123…)]

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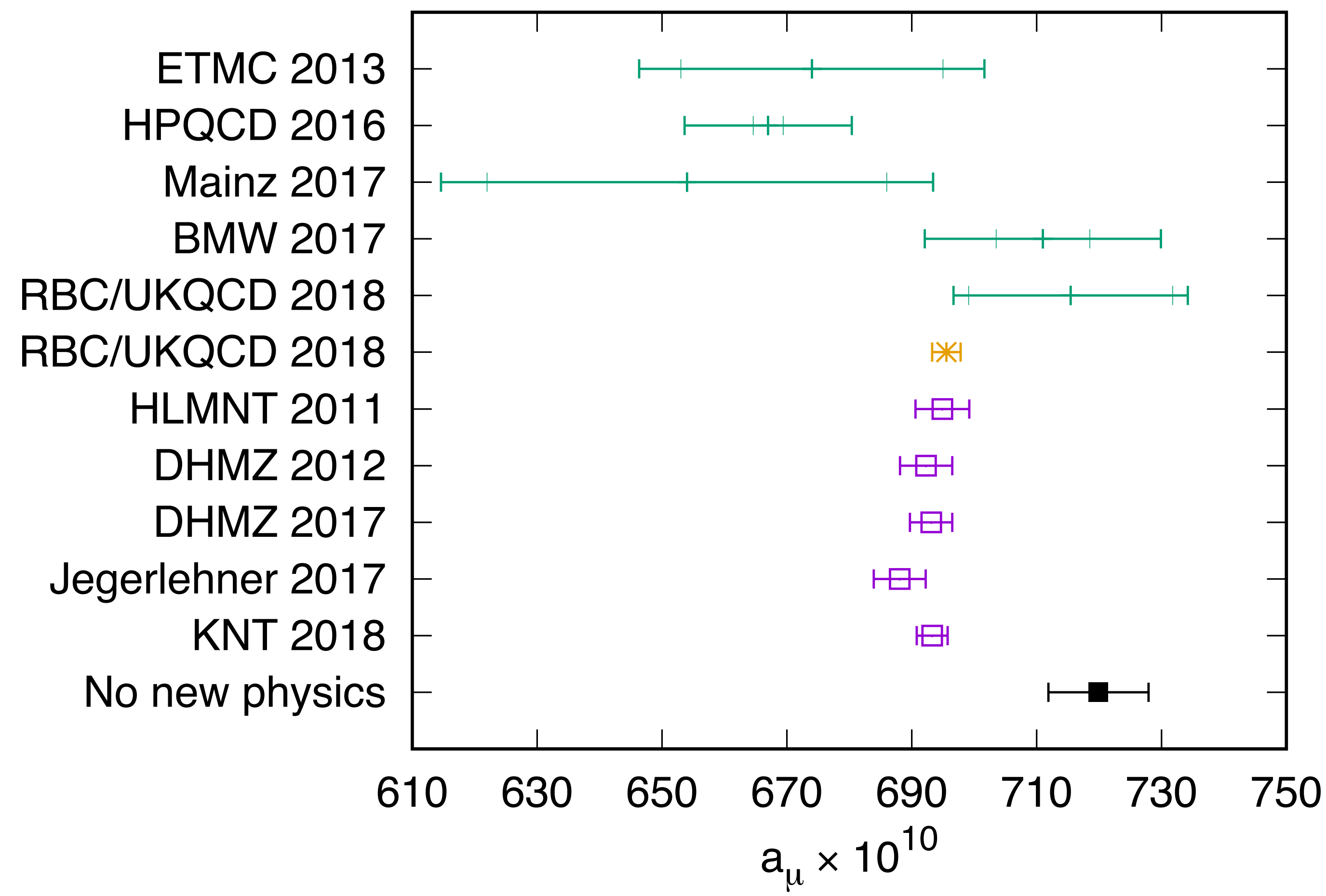

Summary by C. Lehner @ KEK g-2 WS 2018

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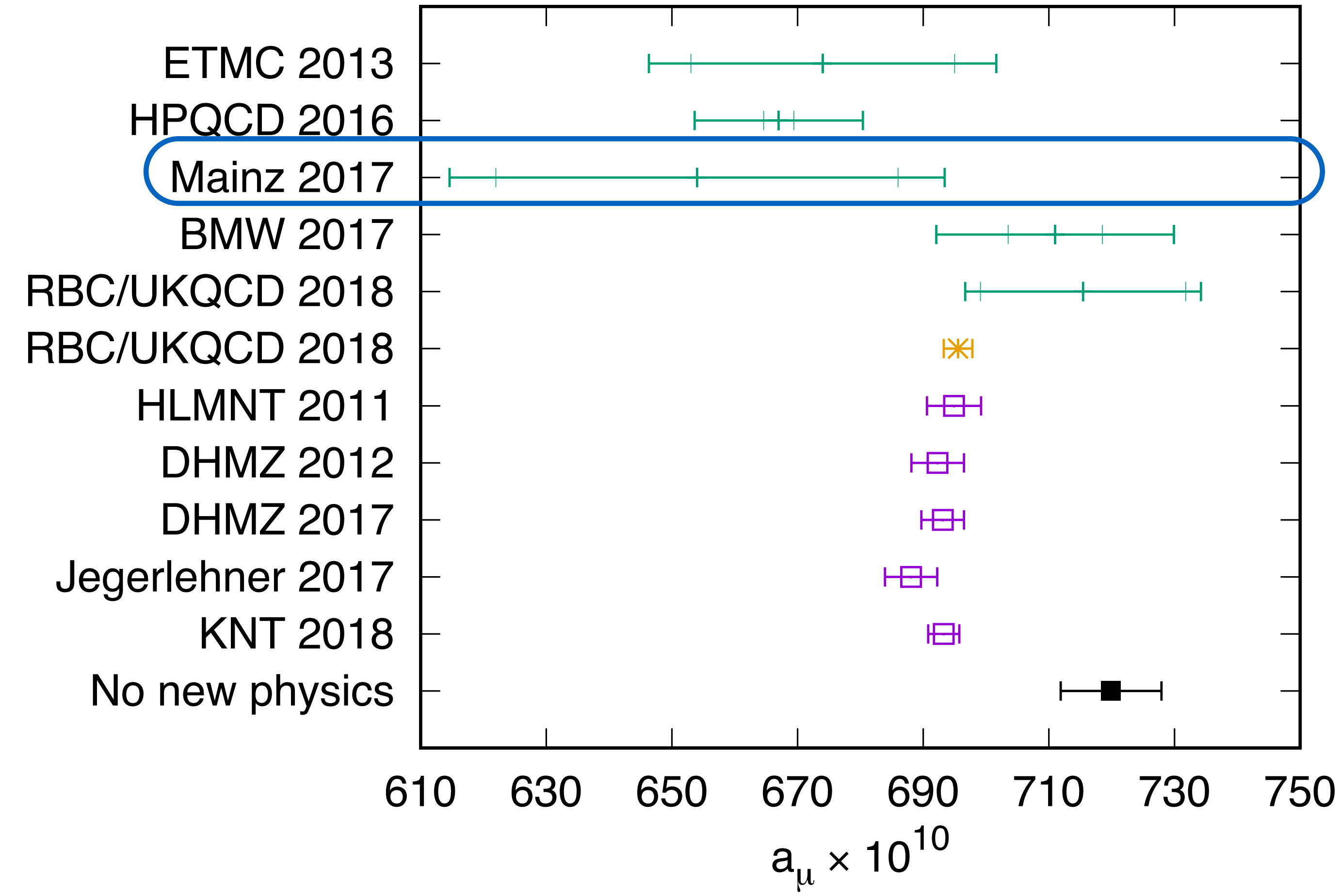

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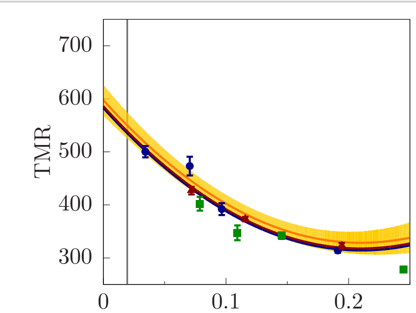
[arXiv:1705.01775]

Mainz/CLS [H.Wittig@Lat17]

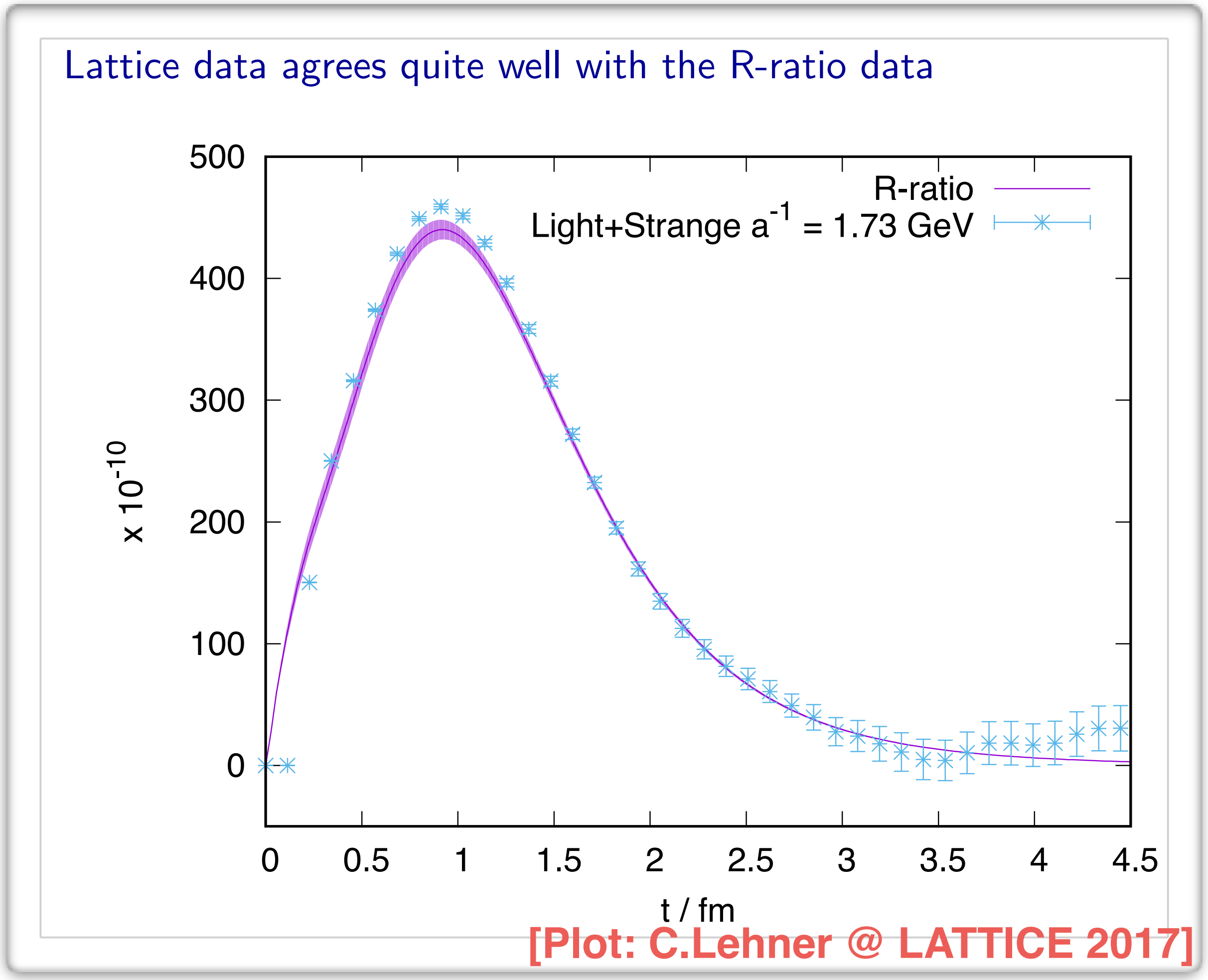
✱ Determine $m_\pi\,(\partial a_\mu^{\mathrm{hvp}}/\partial m_\pi)$ from slope of chiral fit

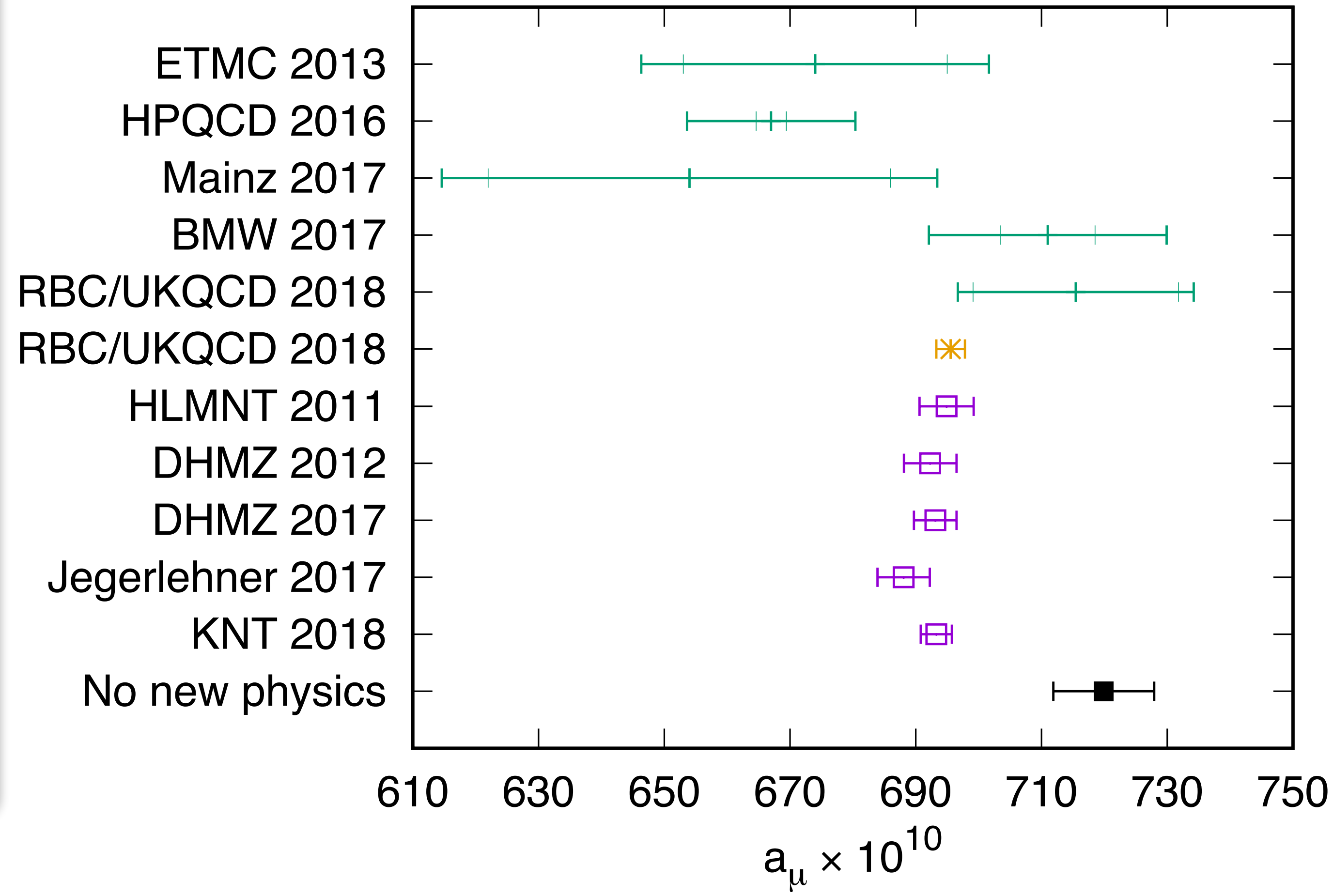
$$\Rightarrow \quad \frac{\delta a_\mu^{\mathrm{hvp}}}{a_\mu^{\mathrm{hvp}}} = \left| \underbrace{\frac{m_\mu}{a_\mu^{\mathrm{hvp}}}\frac{\partial a_\mu^{\mathrm{hvp}}}{\partial m_\mu}}_{\approx\, 1.8} + \underbrace{\frac{m_\pi}{a_\mu^{\mathrm{hvp}}}\frac{\partial a_\mu^{\mathrm{hvp}}}{\partial m_\pi}}_{\approx\, -0.2} \right| \frac{\delta a}{a}$$



$\Rightarrow$ Rather precise determination of lattice spacing a [fm] required!

Combining R-ratio data + lattice QCD

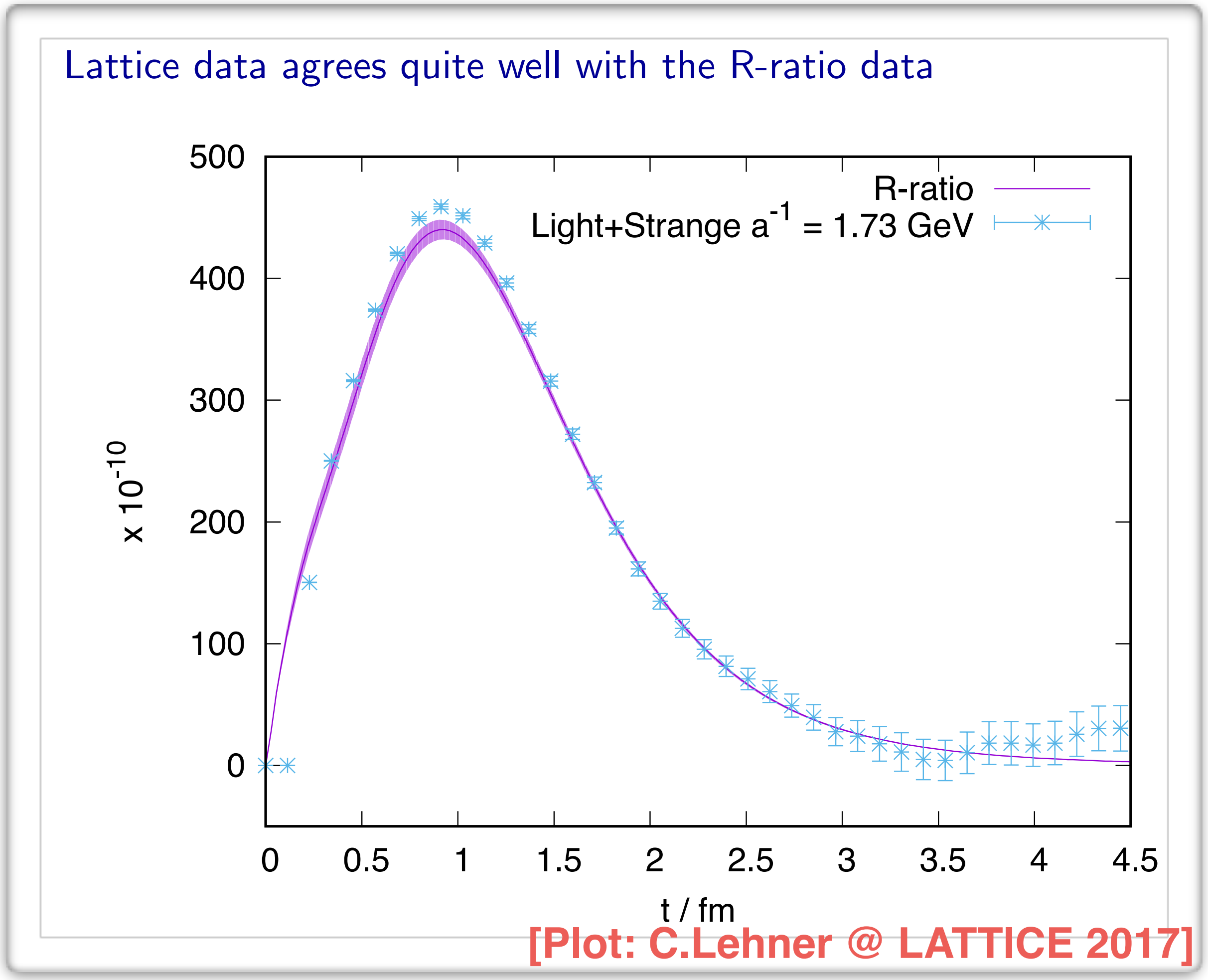




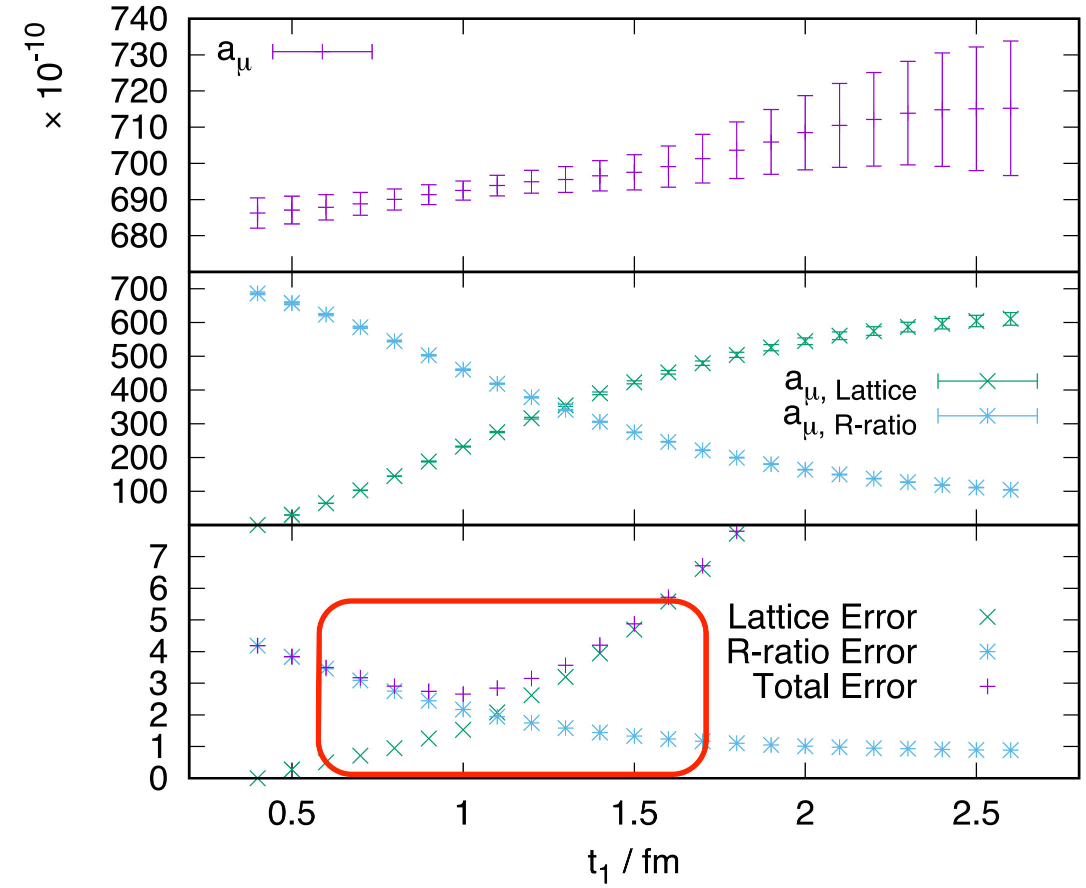
Summary by C. Lehner @ KEK g-2 WS 2018

e^+e^- data, from alphaQED fortran package by F. Jegerlehner

Combining R-ratio data + lattice QCD



Window method with fixed $t_0 = 0.4$ fm



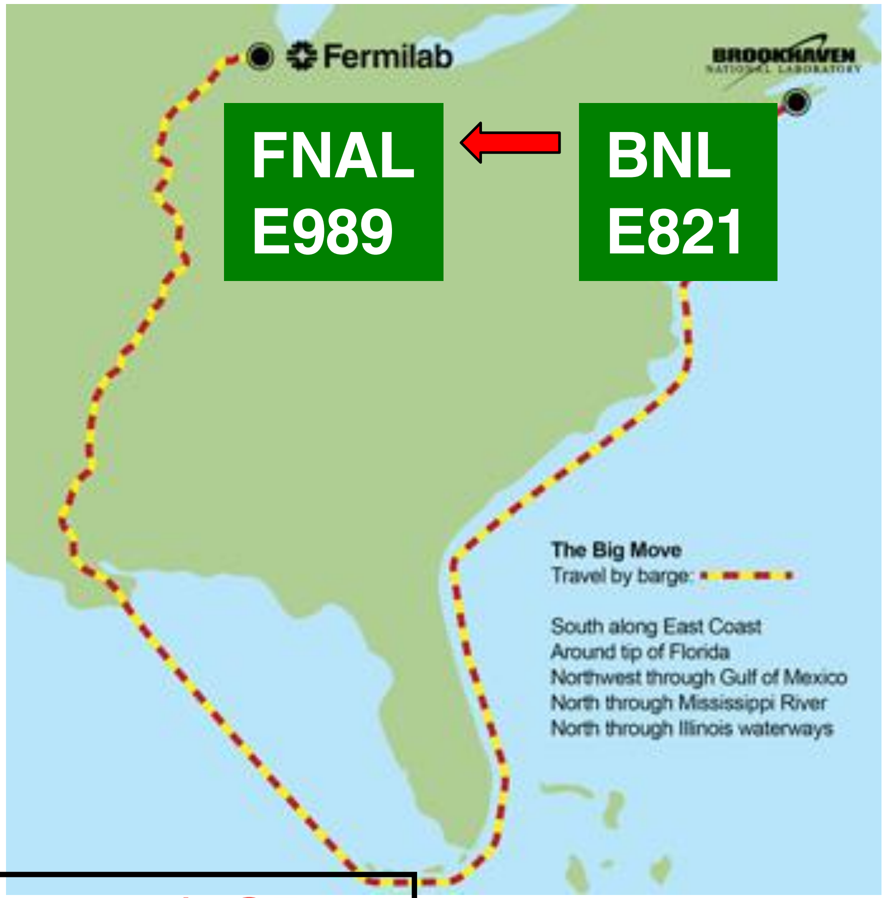
For $t = 1$ fm approximately 50% of uncertainty comes from lattice and 50% of uncertainty comes from the R-ratio. Is there a small slope? More in a few slides!

a_μ from the experiment: FNAL E989

- a_μ^{exp} = 11659208.0(6.3) × 10 $^{-10}$ (0.54ppm) [**BNL, 2006-2008**]

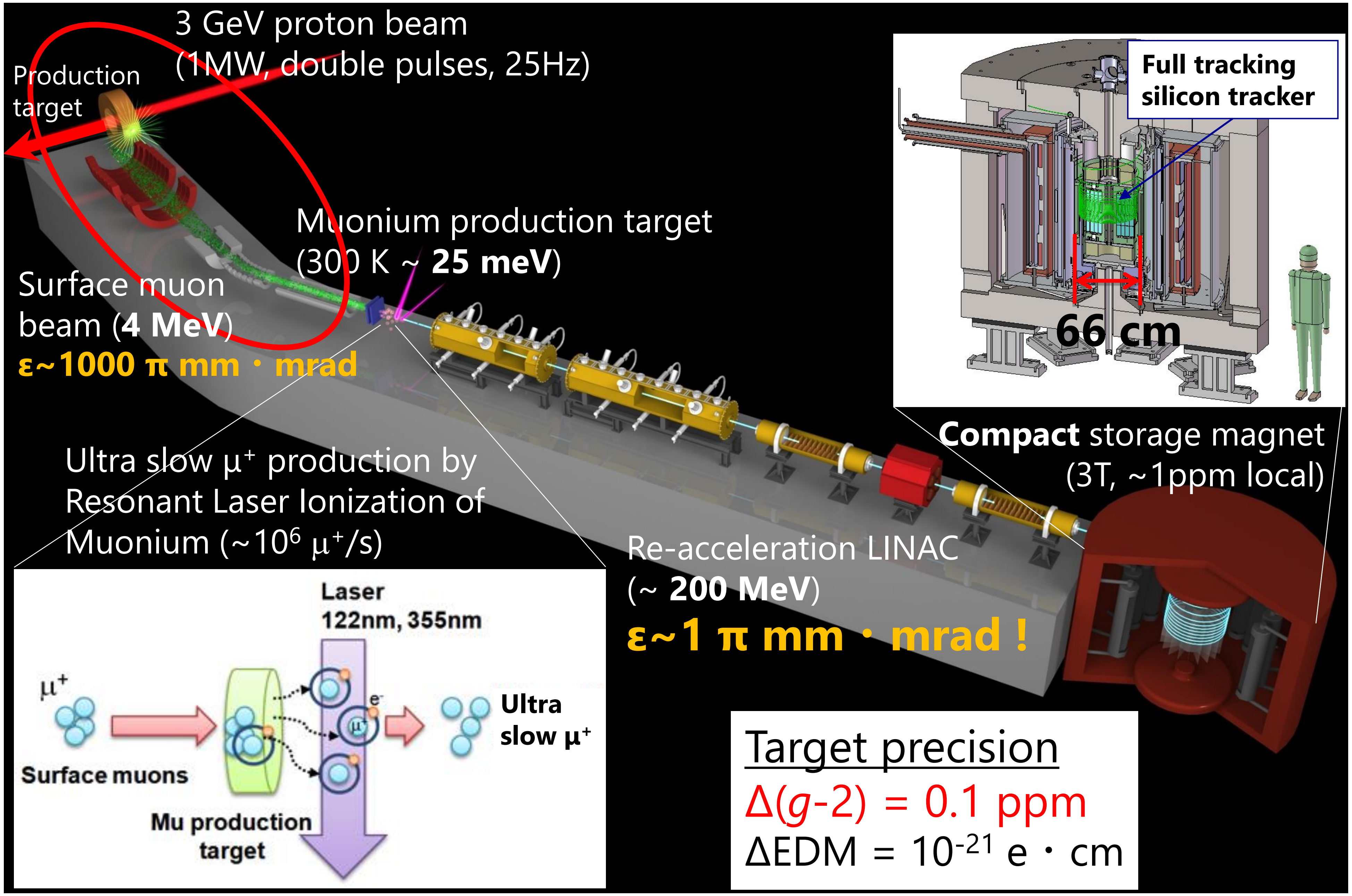
- New experiments (J-PARC, FNAL E989) expected to perform 4× more precise measurement

- Improved precision of the theoretical estimates with dominating uncertainty required



S. Ganguly (@KEK 2018 g-2 WS): first observation of precessing muons (june 2017)! @E989

a_μ from the experiment: J-PARC E34



Target precision
$\Delta(g\text{-}2) = 0.1$ ppm
$\Delta\mathrm{EDM} = 10^{-21}$ e · cm

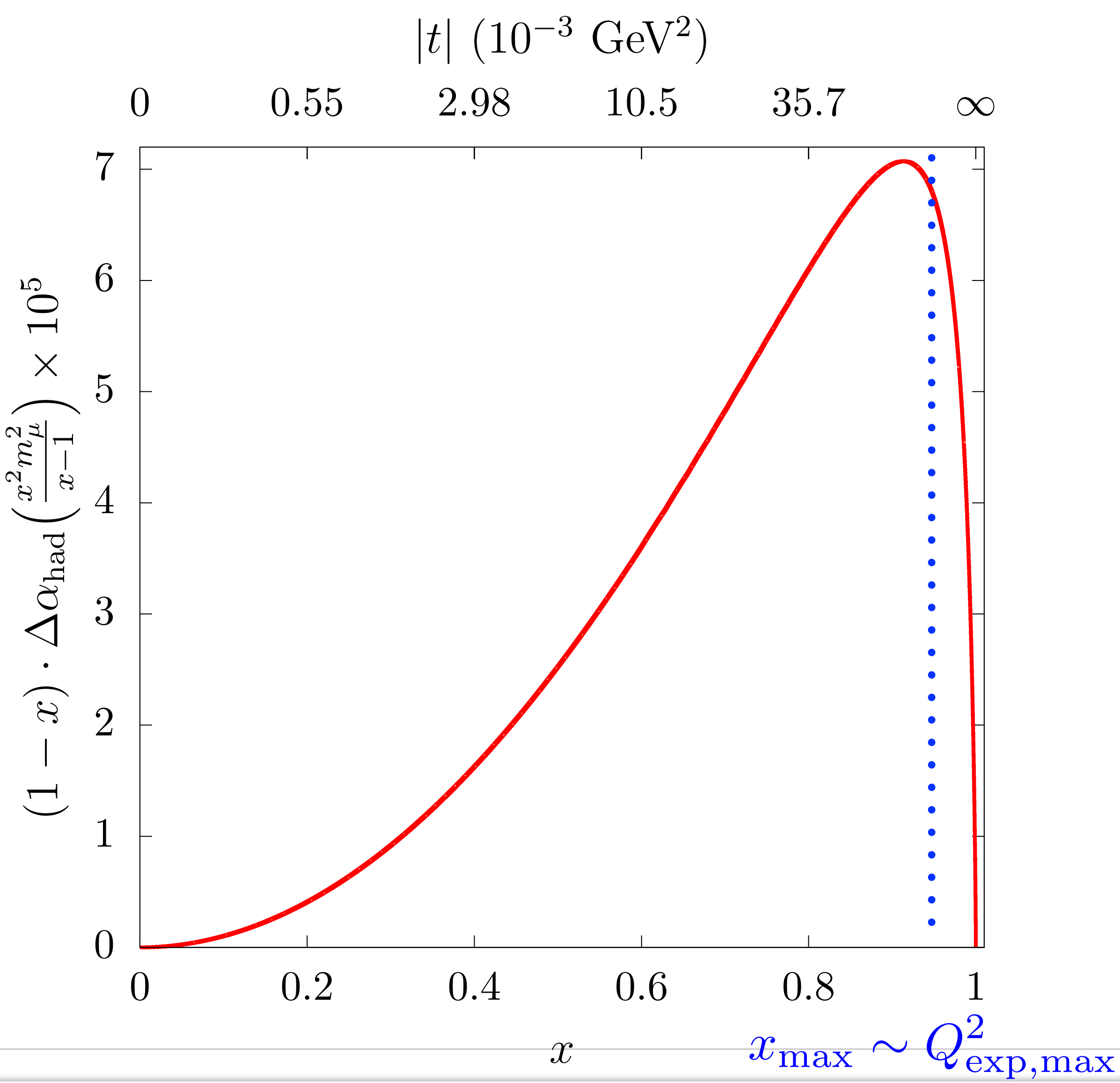
$$\vec{\omega_a} = -\frac{e}{m}\left[a_\mu \vec{B} - \cancel{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}} + \boxed{\frac{\eta_\mu}{2}\left(\beta \times \vec{B}\right.} \cancel{+ \left.\frac{E}{c}\right)}\right]$$

Ultra slow muon beam:
E-term cancels again

T.Yamazaki (@KEK 2018 g-2 WS): muon RF acceleration for the first time 3 weeks ago!

Hybrid method: a_μ^{HVP} from experimental + lattice QCD data

- **MUonE:** estimated precision for the HVP from the **μe** exp. is **0.3%** in **[0,0.14]GeV² after two years of data taking** [see slides by M.Passera, G.Venanzoni Mon. 12h]
- Due to the experimental constraints: region **[0.14, ∞] GeV²** cannot be covered by this exp.
  ➡ complementary to the lattice QCD data



➡ $x_{\rm max} = 0.93$

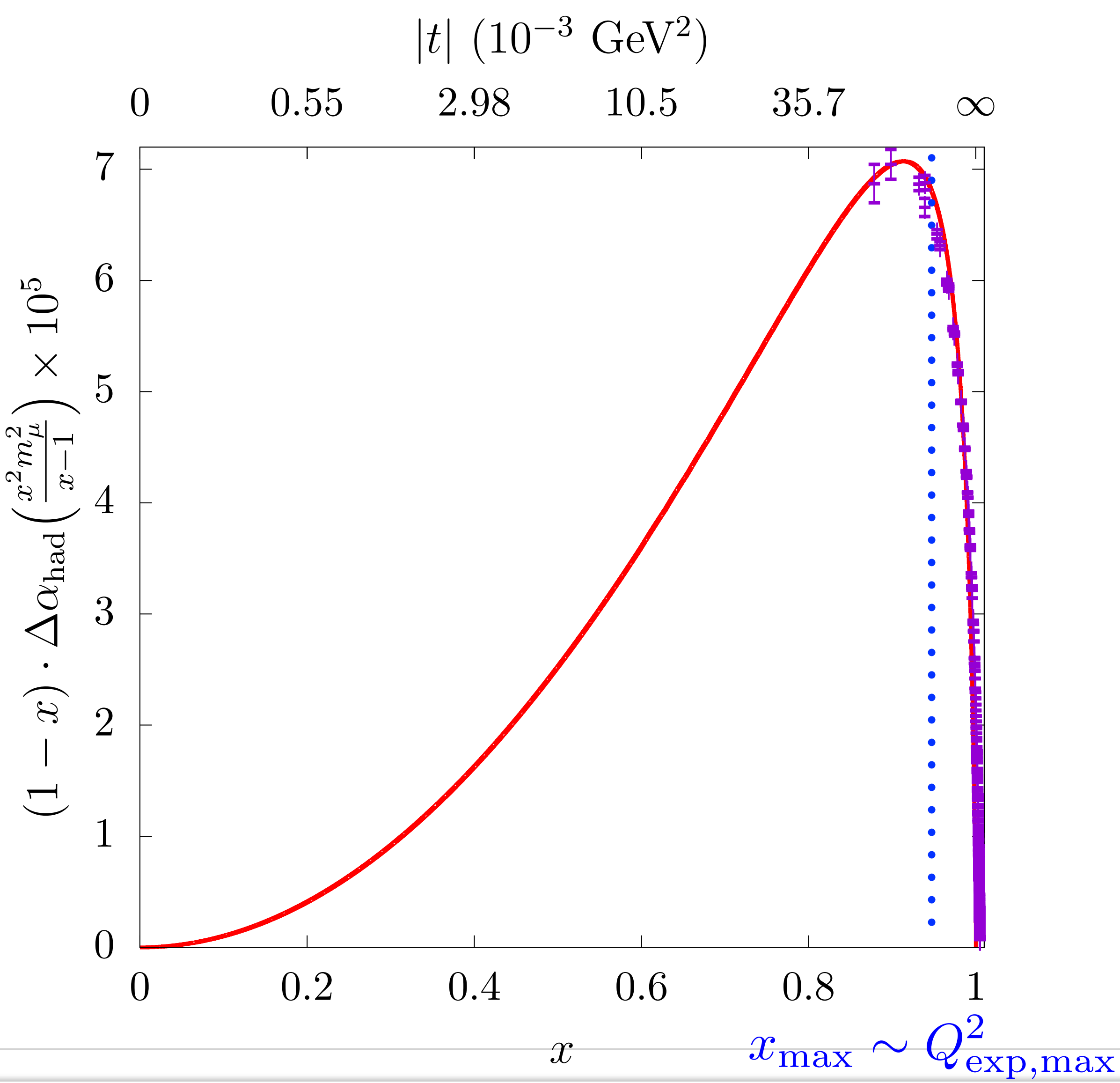
➡ $Q^2 = \dfrac{x^2 m_\mu^2}{1-x}$

➡ $Q^2_{\rm exp,max} = 0.14 {\rm GeV}^2$

Hybrid method: a_μ^{HVP} from experimental + lattice QCD data

- **MUonE:** estimated precision for the HVP from the **μe** exp. is **0.3%** in **[0,0.138]GeV² after two years of data taking** [see slides by M.Passera, Mon. 16.40]

- Due to the experimental constraints: region **[0.14, ∞] GeV²** cannot be covered by this exp.
  ➡ complementary to the lattice QCD data



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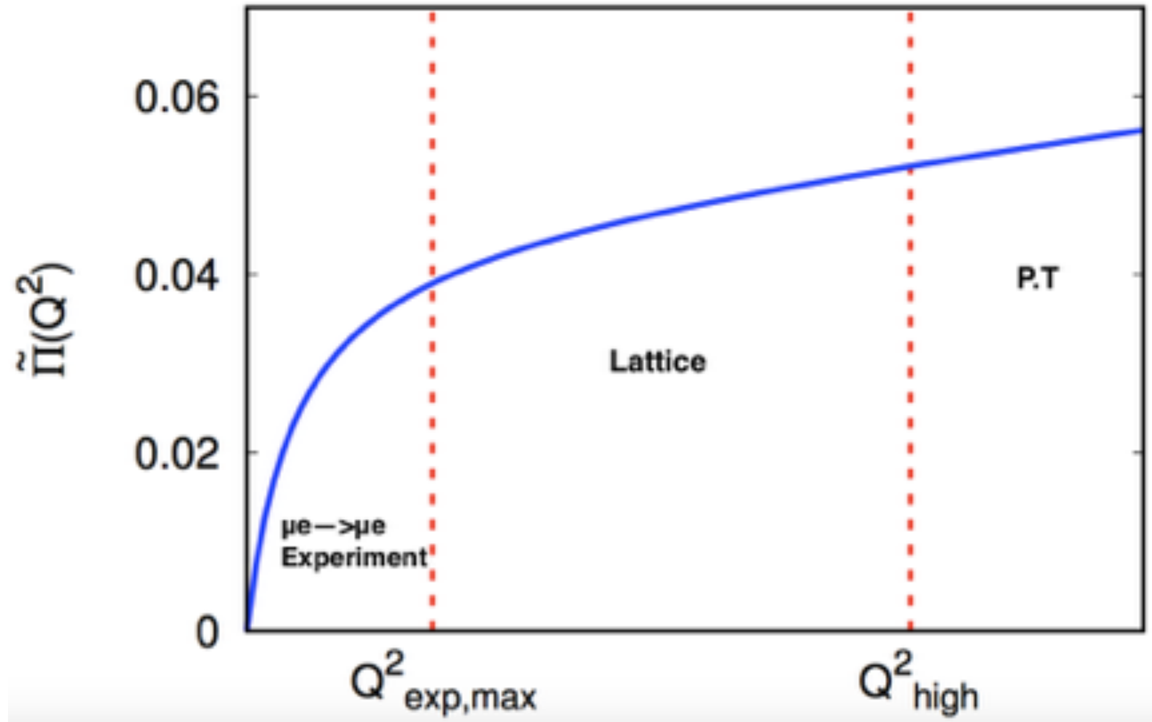
Hybrid method

Phys. Rev. D 90, 074508 (2014), [Golterman,Maltman,Peris]



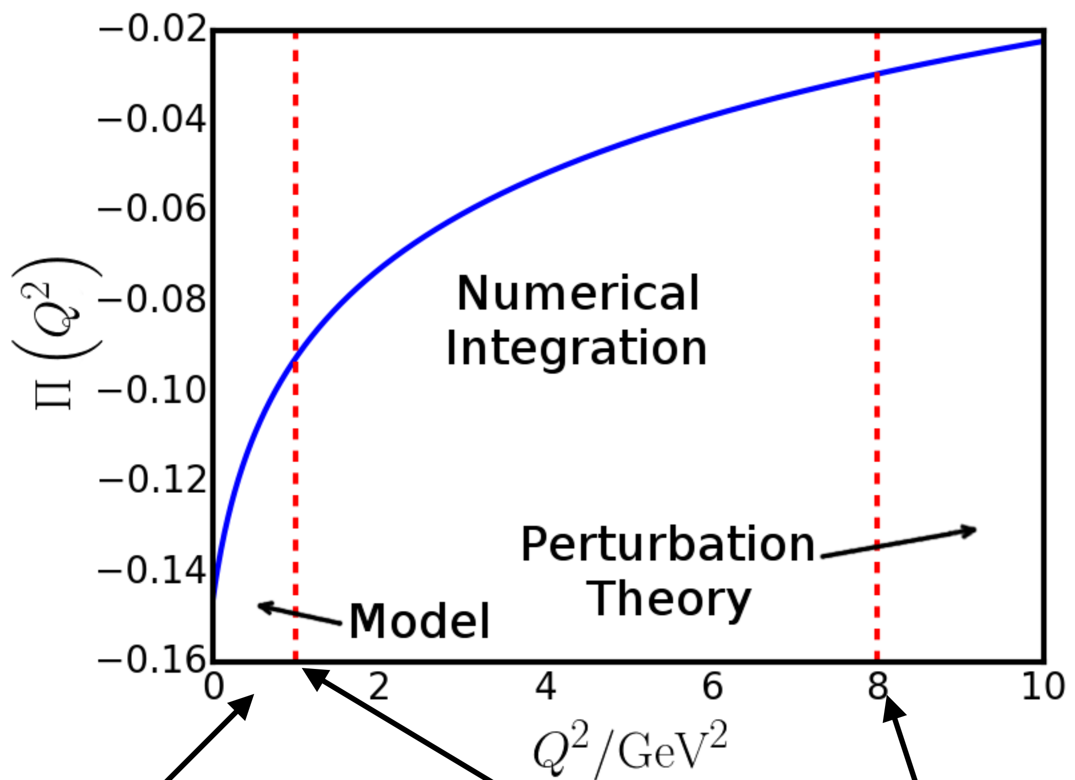
- **Low momentum region**
  - ➡ **Experiment (NLO, NNLO, radiative corrections …)**

- **Vary low and high Q^2 cut**

strategy proposed for the hybrid determination of the total HVP (u+d+s+c+b)

Hybrid method

Phys. Rev. D 90, 074508 (2014), [Golterman,Maltman,Peris]



- Low momentum region
  - ➡ Pade approximants
  - ➡ time moments
  - ➡ conformal polynomials

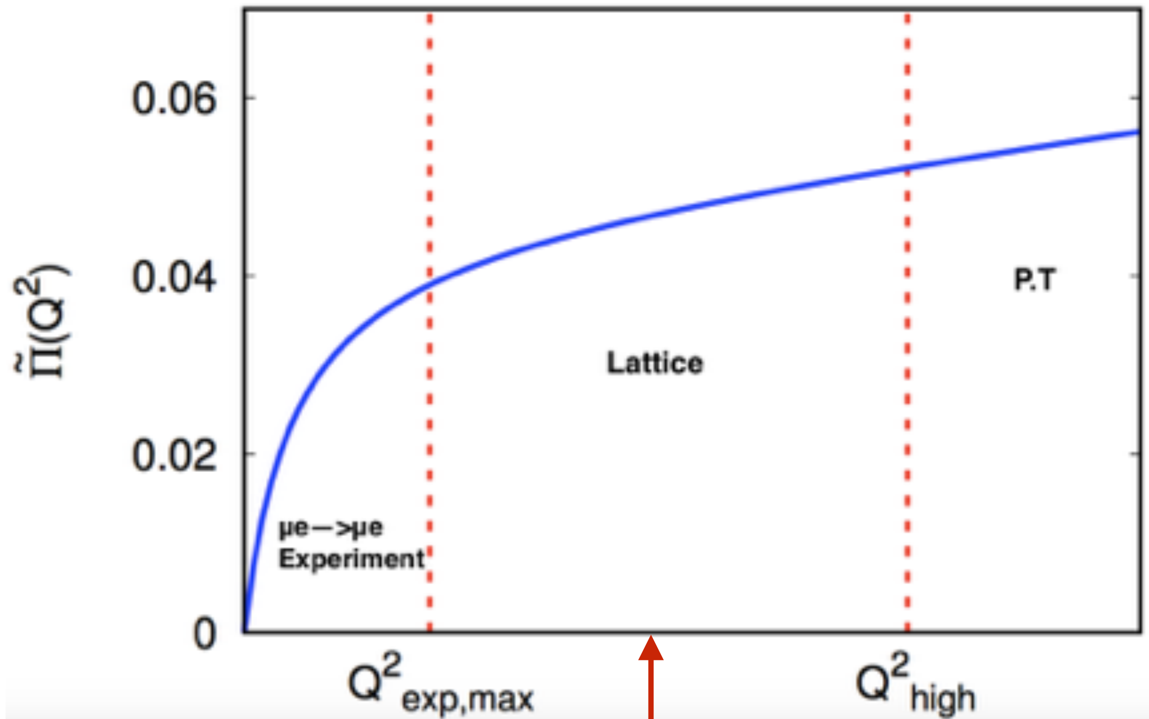
- Vary low and high Q^2 cut

strategy e.g. applied for the strange quark contribution to the HVP [RBC/UKQCD '16]

JHEP 1604 (2016) 063 [T.Blum et al.]

Hybrid method

Phys. Rev. D 90, 074508 (2014), [Golterman,Maltman,Peris]



- **Low momentum region**
  - ➡ **Experiment (NLO, NNLO, radiative corrections …)**

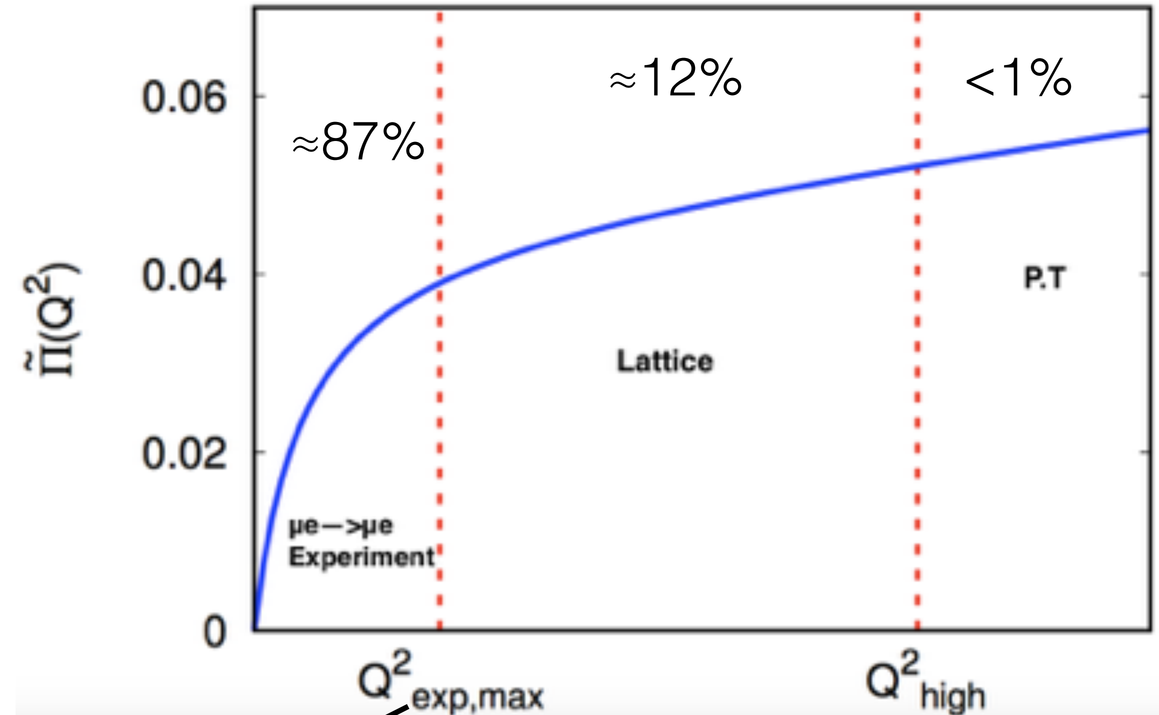
- **Vary low and high Q^2 cut**

- ➡ **continuum limit: a—> 0**
- ➡ **infinite volume limit: V—> ∞**
- ➡ **physical quark masses**
- ➡ **isospin breaking corrections ($m_u \neq m_d$ and $\alpha_{em} \neq 0$)**

strategy proposed for the hybrid determination of the total HVP (u+d+s+c+b)

Hybrid method: from mu-e scattering + lattice QCD data

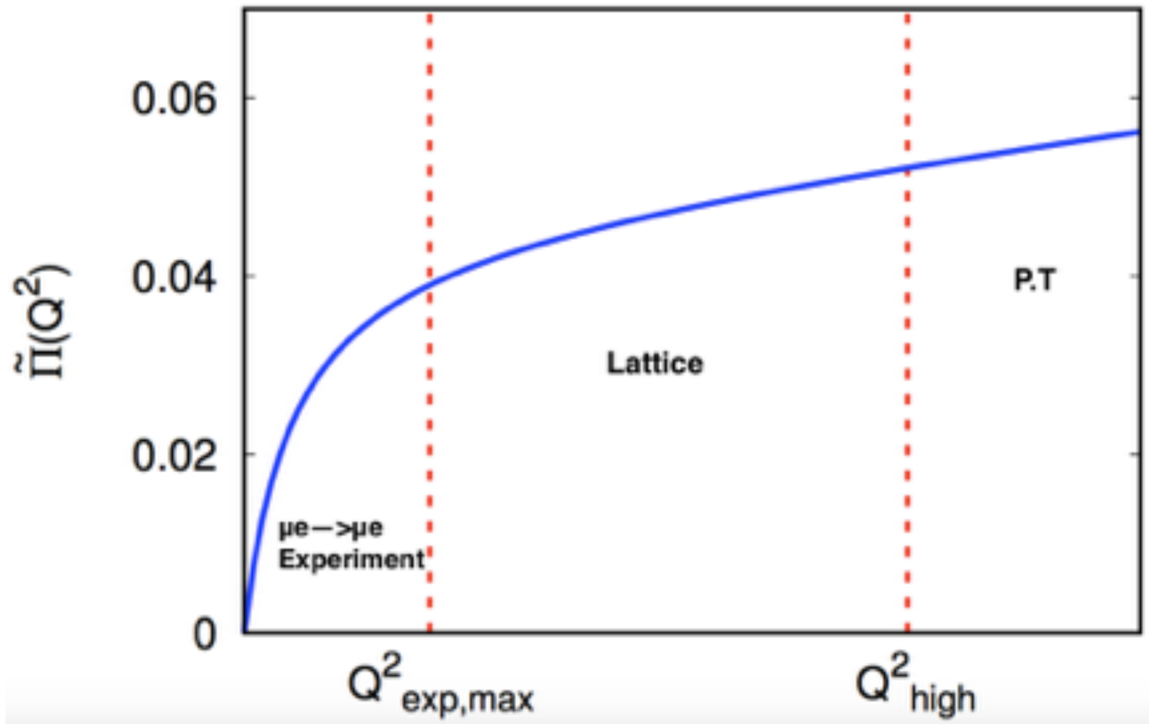
- Nf=2, E5, L/a=32 (CLS), $m_\pi \approx 440$MeV
- Nf=2, F6, L/a=48 (CLS), $m_\pi \approx 311$MeV
- u,d,s,c **[see talk by G. von Hippel, @KEK 2018 g-2 WS]**
- $\Pi(0) = -\frac{\partial \Pi_{12}(Q)}{\partial Q_1 \partial Q_2}|_{Q^2=0}$ **[de Divitiis et al., Phys.Lett. B718 (2012)]**
- Pade [1,1], Pade [2,1] fits to [0,0.5] GeV^2 **[Aubin et al, Phys.Rev. D86 (2012) 054509]**



[1,1]	0.0 GeV^2	0.1 GeV^2	0.2 GeV^2	0.3 GeV^2
stat.	3.6%	2.1%	5.1%	5.1%
syst.	—	0.3% ↘	2.4% ↘	3.6% ↘

- **Vary low $Q^2_{exp,max}$ cut: 0.1, 0.2, 0.3 GeV^2**
- **Assuming relative accuracy 1% under the cut**
- **Pheno. model** [Golterman, Maltman, Peris. Phys.Rev. D88 (2013) no. 11, 114508] **("science fiction" data set: reducing the (diagonal) error by a factor 100)**

Hybrid method: from mu-e scattering + lattice QCD data

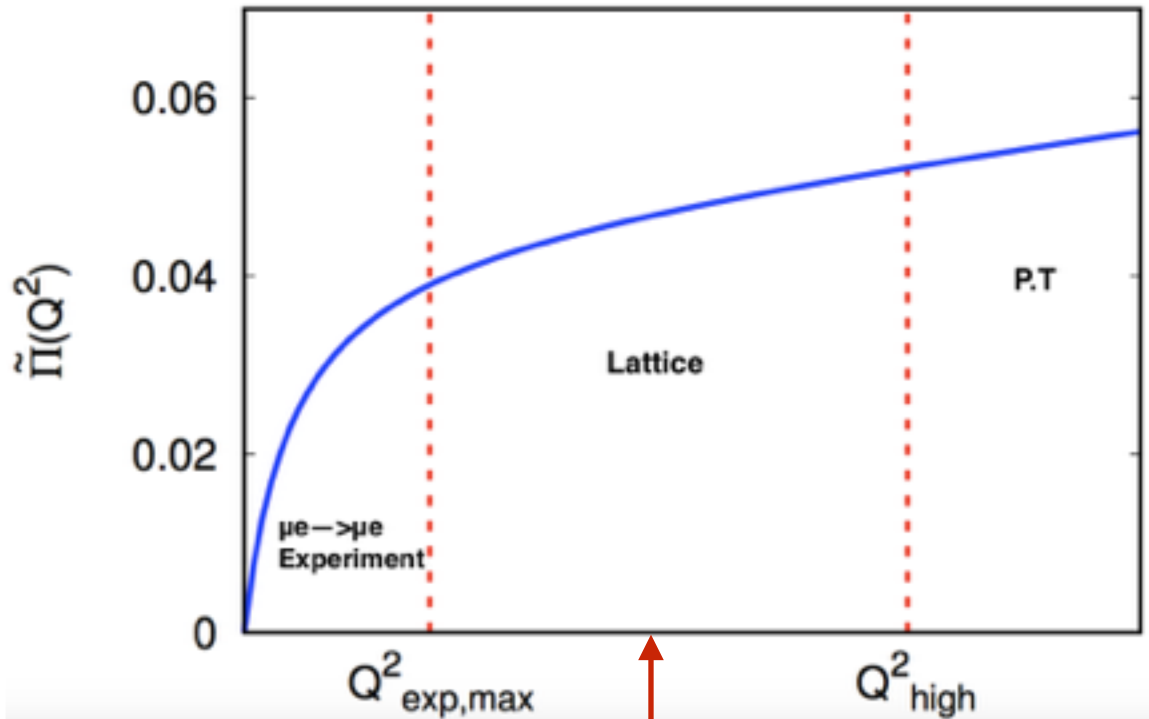


- **Low momentum region**
  - ➡ **Experiment (NLO, NNLO, radiative corrections …)**

- **Vary low and high Q^2 cut**

strategy proposed for the hybrid determination
of the total HVP (u+d+s+c+b)

Hybrid method: from mu-e scattering + lattice QCD data



- **Low momentum region**
  - ➡ **Experiment (NLO, NNLO, radiative corrections …)**

- **Vary low and high Q^2 cut**

- ➡ **continuum limit: $a \to 0$**
- ➡ **infinite volume limit: $V \to \infty$**
- ➡ **physical quark masses**
- ➡ **isospin breaking corrections ($m_u \neq m_d$ and $\alpha_{em} \neq 0$)**

strategy proposed for the hybrid determination of the total HVP (u+d+s+c+b)

Hybrid method: from mu-e scattering + lattice QCD data

- Proposal to do a "hybrid determination" from μ-e scattering and lattice data (+p.t.)
  - ➡ [Eur.Phys.J. C77 (2017) no.3, 139 by Abbiendi et al.] μONe @CERN (?) [see slides by M.Passera, G.Venanzoni Mon. 12h]

- Cross-check experimental $\Pi(Q^2)$ from **[0,0.14]GeV^2** vs. continuum limit from the lattice
- Relevance for lattice QCD determinations of HVP:
  1. "hybrid method" [**Phys. Rev. D 90, 074508 (2014) Golterman,Maltman,Peris**] with experimental+lattice QCD data
     a) to complete the exp. result
     b) to cross-check lattice data
  2. continuum limit of $\Pi(Q^2)$ at fixed **Q^2** (previously extrapolated or measured at **$m_{\pi,phys}$**)
  3. help in choosing the parametrization for $\Pi(Q^2)$ with less FV/cutoff effects
  4. Compare to the slope and curvature for HVP function [**see arXiv:1612.02364**]

1.The HVP integral on a range [$Q^2_{exp,max}$,Q^2_{max}] has continuum&FV limit:

$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_{Q^2_{exp,max}}^{\infty} dQ^2 \; f(Q^2) \times \tilde{\Pi}(Q^2)$$

- ➡ radiative corrections might be relevant (≈ 1‰) **[c.f. slides by C. Carloni Calame from** MUonE Theory kickoff WS **Padova '17]**
- ➡ cutoff effects need to be assessed systematically

Summary & Outlook

- Lattice gives an **independent theory prediction** of hadronic contributions to a_μ
- Lattice goals: for HVP is <1% (<0.5%?) and goal for HLbL is <10%
- u+d+s+c, isospin breaking corrections
- Impressive progress in the past year(s)
- Full control of the systematics is needed
  - ➡ continuum limit, infinite volume limit, isospin breaking corrections are the next challenges

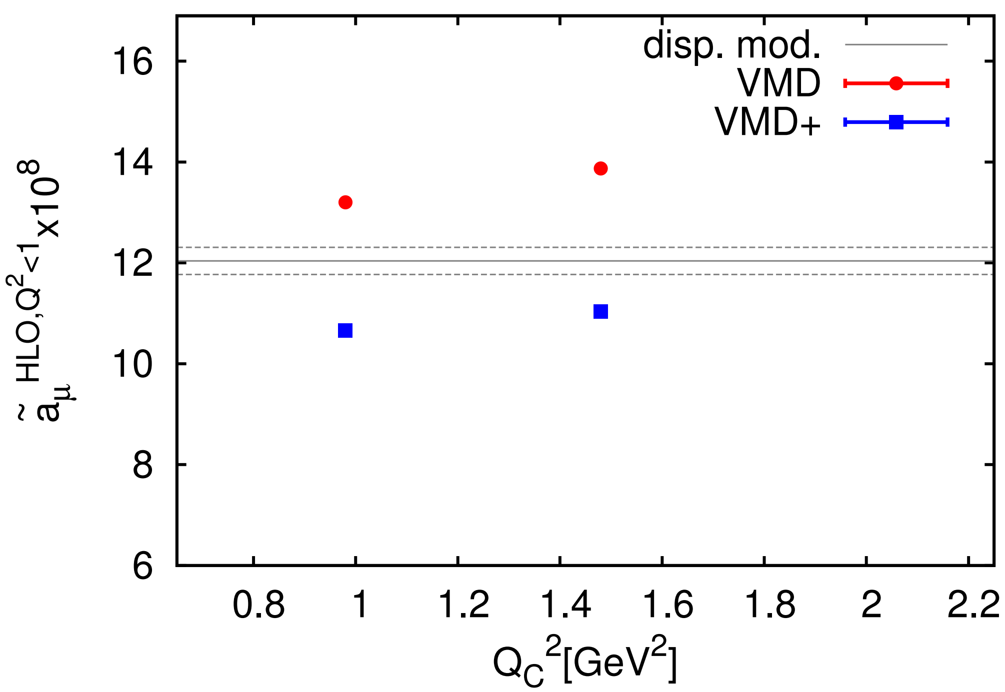
- **Parallel front:** Lattice + experiment
  - ➡ R- ratio + lattice
  - ➡ e-mu scattering data + lattice

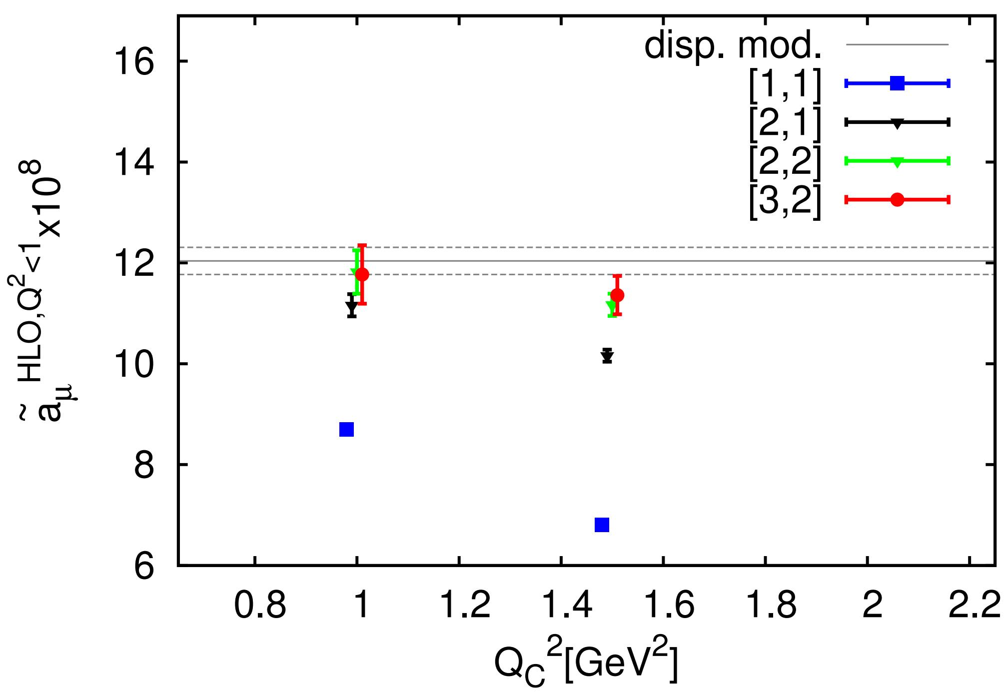
➡ Eventually as lattice data improves, the windows **[0,t*]**, **[$Q^2_{exp,max}$,Q^2_{high}]** can be widened, until we obtain a full lattice result

Thank you!

Phenomenological model of HVP [Golterman, Maltman, Peris '13]

- A method to quantitatively examine the systematics of lattice computations
- Dispersive τ-based $I = 1$ model: $\hat{\Pi}^{I=1}(Q^2) = Q^2 \int_{4m_\pi^2}^{\infty} ds \frac{\rho^{I=1}(s)}{s(s+Q^2)}$
- Fake lattice data for $\Pi(Q^2) - \Pi(0)$ & compared with true answer from model





- Outcome:
  - Fitting until high Q^2 dangerous, unless higher order Padés used
  - Better focus on low-Q^2 region needed

Isospin breaking effects from the Lattice - main issue

- **FV effects** <—— the way the infrared divergence associated with the zero momentum mode of the photon propagator is canceled on the lattice:

  ➡ continuum: $$\alpha_{em} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \cdots$$ ➡ lattice: $$\frac{\alpha_{em}}{V} \sum_k \frac{1}{k^2} \cdots$$

- Gauss law does not allow a non-zero charge to exist in a finite periodic box

- Three ways to deal with IR divergence:

  ➡ **Modify gauge filed**: removing **the global zero-mode**/ **spatial zero mode per timeslice** (**QED$_{TL}$** / **QED$_L$**)
  […, Borsanyi et al., Science 347 (2015), arXiv:1406.4088] [Hayakawa and Uno, Prog. Theor. Phys. 120 (2008) 413]

  ➡ **Massive photon** **[Endres et al., PRL 117 (2016) 7, arXiv:1507.08916]**

  ➡ **C* boundary conditions** (no zero-mode present) **[Lucini et al., JHEP 1602, 076, arXiv:1509.01636]**
  - ✦ **Kronfeld and Wiese, Nucl. Phys. B 357 (1991) 521**
  - ✦ **Wiese, Nucl. Phys. B 375 (1992) 456**
  - ✦ **Polley, Z. Phys. C 59 (1993) 105**