Muon $g-2$: Theory Status

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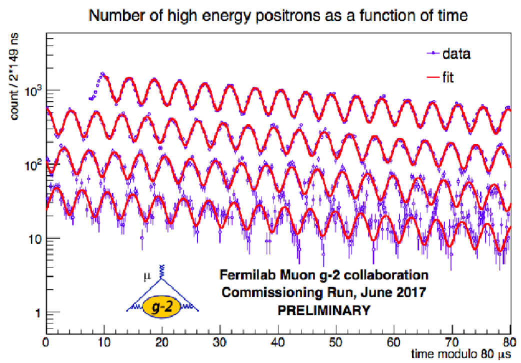
Evaluation of the Leading Hadronic Contribution to a_μ – MITP Mainz, February 19 - 23, 2018



OUTLINE

- I. Motivations
- II. Progress in QED calculations
- III. Improvements in the evaluation of HVP
- IV. Improvements in the evaluation of HLxL
- V. Summary - Conclusion

I. Motivations

At present, there is a discrepancy, of $\gtrsim 3.5\sigma$, between the measured value of the anomalous magnetic moment of the muon

$$a_\mu^{\text{exp}} = 116\,592\,089(63) \cdot 10^{-11} \quad [0.54ppm]$$

G. W. Bennett et al, Phys Rev D 73, 072003 (2006)

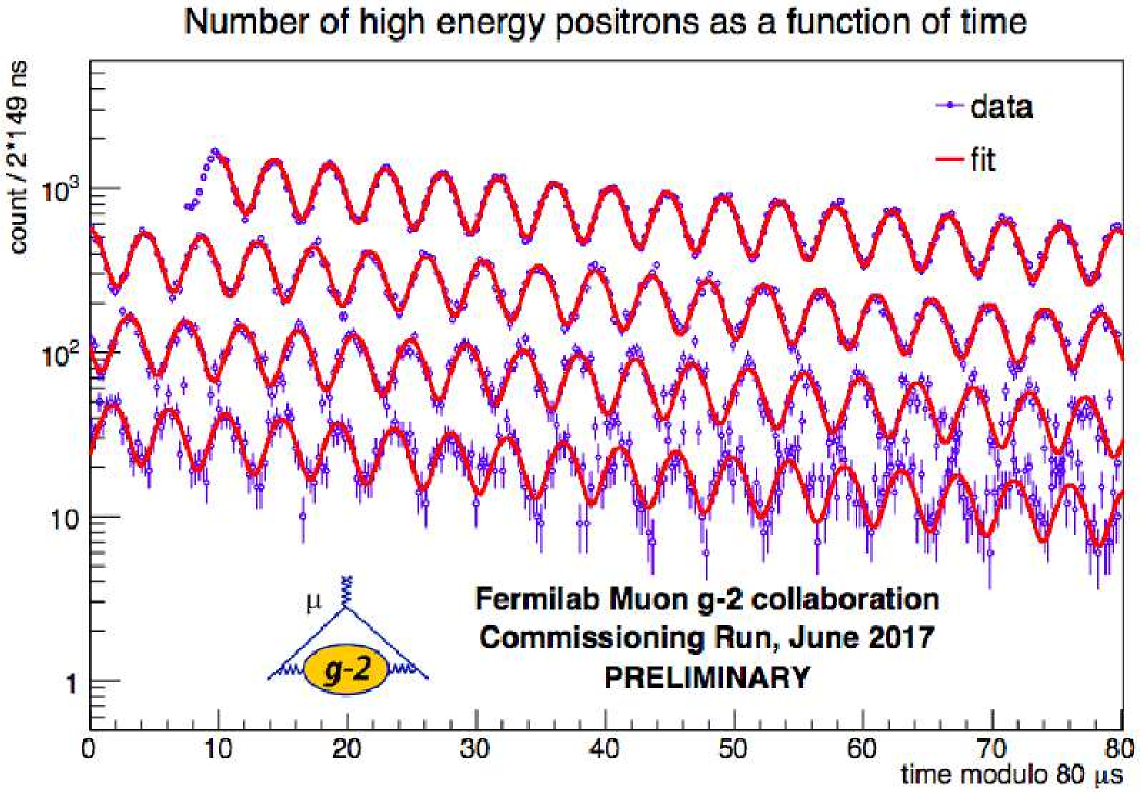
and its value as evaluated within the standard model

This situation results, on the one hand, from the outcome of the BNL-E821 experiment and, on the other hand, from the evaluation of rather subtle quantum effects

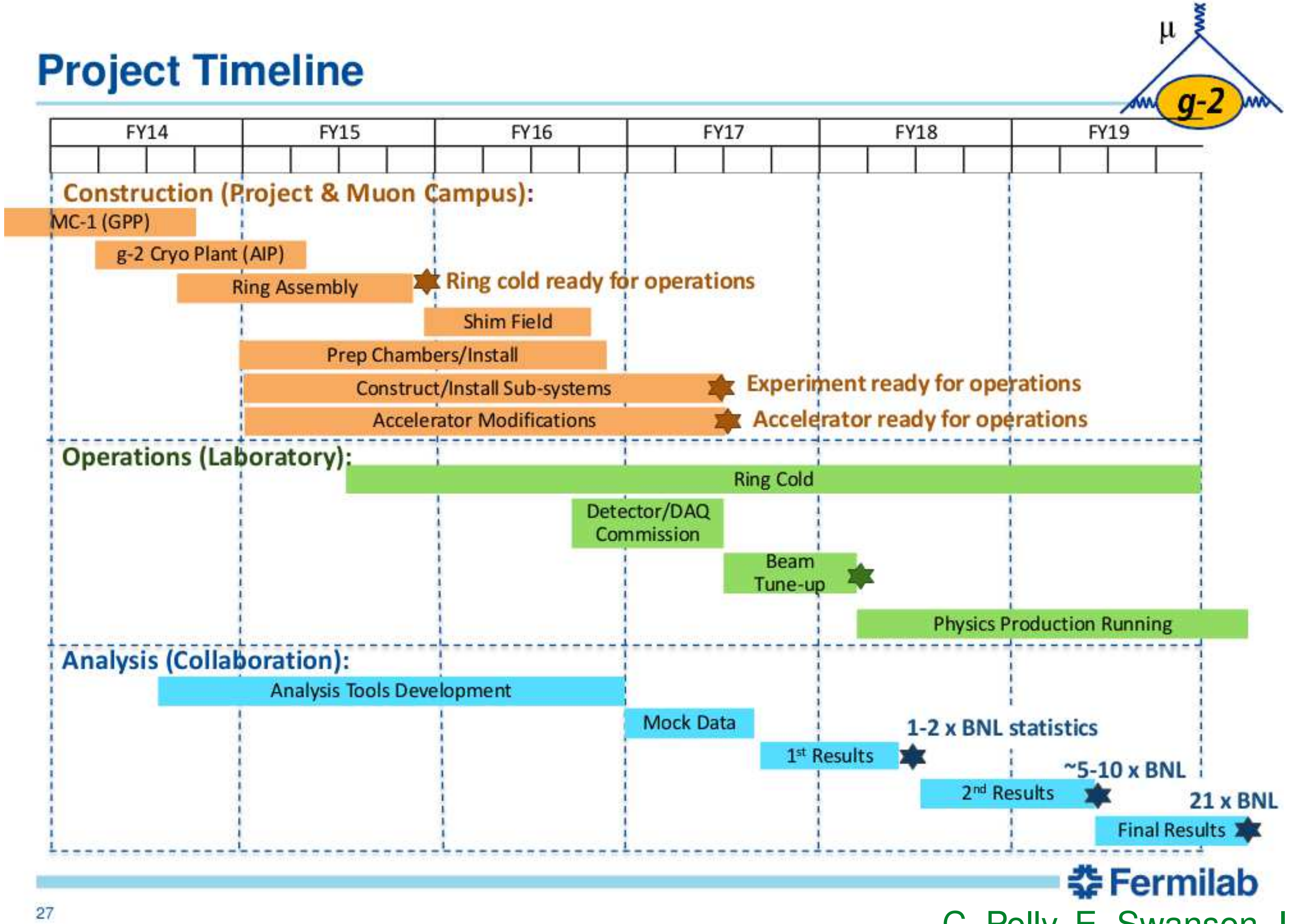
On the experimental side, the confirmation (or not!) of the Brookhaven measurement by the FNAL E989 experiment is around the corner... [and should also be checked later on by the E34 experiment at J-PARC]

Successful commissioning run during Summer 2017...

June 2017 ~ 700,000 positrons (~2 weeks)



... and result for a_μ with statistical sample of size comparable to the one collected by BNL-E821 should be available by the end of 2018!



C. Polly, E. Swanson, ICHEP, Aug. 2016

If running on schedule [which seems, so far, to be the case], FNAL-E989 will measure a_μ with precision improved by a factor of ~ 4 within a couple of years...

By the end of ~ 2020, a_μ may well be the only single observable of the standard model [besides neutrino masses] showing a deviation from its measured value by more than 5σ!

The possibility to end up with such a far-reaching conclusion not only requires a clean experimental measurement [E34 at J-PARC may later on cross-check the FNAL-E989 result], but also puts high requirements on the theoretical evaluation of a_μ within the standard model

- provide, whenever possible, cross-checks for existing calculations

- improve precision, especially in the hadronic contributions, which are the most delicate to evaluate, and which at present dominate the theoretical uncertainties

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- provide, whenever possible, cross-checks for existing calculations

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Progress has been made in both directions during the last two years or so...

II. Progress in QED calculations

QED provides more than 99.99% of the standard-model value of a_μ

General structure of the perturbative series

$$a_\ell{}^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

Issue: computing high perturbative orders

$\longrightarrow$ cf. Stefano Laporta's talk

Situation in ~ 2016

- Expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

J. Schwinger, Phys. Rev. 73, 416L (1948)
C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)
A. Petermann, Helv. Phys. Acta 30, 407 (1957)
H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)
A. Petermann, Phys. Rev. 105, 1931 (1955)
H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)
M. Passera, Phys. Rev. D 75, 013002 (2007)
S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)
S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

$\longrightarrow$ no uncertainties in $A_1^{(2)}$, $A_1^{(4)}$, $A_1^{(6)}$

$\longrightarrow$ precision on $A_2^{(4)}$, $A_2^{(6)}$, $A_3^{(6)}$ only limited by precision in $m_\ell / m_{\ell'}$ (not relevant for a_μ at present and future precisions)

- Values of $C_\ell^{(8)}$ and $C_\ell^{(10)}$ computed numerically

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); D 85, 093013 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012); Phys. Rev. D 91, 033006 (2015) [Err. D 96, 019901 (2017)]; Phys. Rev. D 97, 036001 (2018)

$\longrightarrow$ for all practical purposes, a_μ^{QED} has no uncertainties

Situation in ~ 2016

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

	$\ell = e$	$\ell = \mu$
$C_\ell^{(2)}$	0.5	0.5
$C_\ell^{(4)}$	$-0.328\,478\,444\,00\ldots$	$0.765\,857\,425(17)$
$C_\ell^{(6)}$	$1.181\,234\,017\ldots$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.9096(20)$	$130.879\,6(63)$
$C_\ell^{(10)}$	$9.16(58)$	$753.29(1.04)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots\cdot 10^{-3}$	$5.39\ldots\cdot 10^{-6}$	$1.25\ldots\cdot 10^{-8}$	$2.91\ldots\cdot 10^{-11}$	$6.76\ldots\cdot 10^{-14}$

order $(\alpha/\pi)^4$: 891 diagrams

$$A_1^{(8)} = -1.912\,98(84)$$

$$A_2^{(8)}(m_e/m_\mu) = 9.161\,970\,703(373)\cdot 10^{-4} \quad A_2^{(8)}(m_e/m_\tau) = 7.429\,24(118)\cdot 10^{-6}$$

$$A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) = 7.468\,7(28)\cdot 10^{-7}$$

$$A_2^{(8)}(m_\mu/m_e) = 132.685\,2(60) \quad A_2^{(8)}(m_\mu/m_\tau) = 0.042\,34(12)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\,72(4)$$

Independent check of mass-dependent contributions

A. Kataev, Phys. Rev. D 86, 013019 (2012)
A. Kurz, T. Liu, P. Marquard, M. Steinhauser, Nucl. Phys. B 879, 1 (2014)
A. Kurz et al., Phys. Rev. D 92, 073019 (2015); Phys. Rev. D 93, 053017 (2016)

Agreement at the level of accuracy required by present (and future) experiments for a_μ

Important cross-check, since $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \sim a_\mu^{\text{QED}}(\alpha^4)$

Cross-check for $C_\mu^{(10)}$? $C_\mu^{(10)}(\alpha/\pi)^5 \sim 0.5 \cdot 10^{-10}$ $\Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10} \longrightarrow \sim 1.6 \cdot 10^{-10}$

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A. Kurz et al., Phys. Rev. D 92, 073019 (2015); Phys. Rev. D 93, 053017 (2016)

semi-analytic computation of $A_1^{(8)}$! $A_1^{(8)} = -1.912\,245\,764\,926\,4\ldots$

S. Laporta, Phys. Lett. B 772, 232 (2017) [arXiv:1704.06996 [hep-ph]]

Main order α^4 contribution to a_e!

Cross-check for $C_e^{(10)}$? $C_e^{(10)}(\alpha/\pi)^5 \sim 5\cdot 10^{-13}$ $\Delta a_e^{\rm exp} = 2.8\cdot 10^{-13} \longrightarrow ?$

Present situation

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

	$\ell = e$	$\ell = \mu$
$C_\ell^{(2)}$	0.5	0.5
$C_\ell^{(4)}$	$-0.328\,478\,444\,00\ldots$	$0.765\,857\,425(17)$
$C_\ell^{(6)}$	$1.181\,234\,017\ldots$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.911\,321\,390\ldots$	$130.878\,0(61)$
$C_\ell^{(10)}$	$6.595(223)$	$750.72(93)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots\cdot 10^{-3}$	$5.39\ldots\cdot 10^{-6}$	$1.25\ldots\cdot 10^{-8}$	$2.91\ldots\cdot 10^{-11}$	$6.76\ldots\cdot 10^{-14}$

A few comments about the QED contributions

- $a_e^{\mathrm{QED}} = 1\,159\,652\,180.277(00)_{\alpha^4}(15)_{\alpha^5}(720)_{\alpha(Rb11)} \cdot 10^{-12}$

$$a_e^{\mathrm{exp}} - a_e^{\mathrm{QED}} = 0.434(772) \cdot 10^{-12}$$

T. Aoyama et al., Phys. Rev. D 91, 033006 (2015)

S. Laporta, Phys. Lett. B 772, 232 (2017)

- Uncertainties on the coefficients $C_\mu^{(2n)}$ not relevant for a_μ at the present (and future) level of precision

$$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13} \qquad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13}$$

$$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13} \qquad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.7 \cdot 10^{-13} \qquad \Delta a_\mu^{\mathrm{exp}} = 6.3 \cdot 10^{-10}$$

- Order $\mathcal{O}(\alpha^4)$ and even order $\mathcal{O}(\alpha^5)$ relevant for a_μ at the present (and future) level of precision

$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 3.8 \cdot 10^{-9} \qquad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.5 \cdot 10^{-10}$$

- Drastic increase with n in the coefficients $C_\mu^{(2n)}$ [$\pi^2 \ln(m_\mu/m_e) \sim 50$!]
- Estimate of $\mathcal{O}(\alpha^6)$ contributions with these enhancement factors

$$\delta a_\mu \sim A_2^{(6)}(m_\mu/m_e; \mathrm{LxL}) \left[\frac{2}{3}\ln\frac{m_\mu}{m_e} - \frac{5}{9}\right]^3 \cdot 10 \left(\frac{\alpha}{\pi}\right)^6 \sim 0.6 \cdot 10^4 \cdot \left(\frac{\alpha}{\pi}\right)^6 \sim 1 \cdot 10^{-12}$$

- No sign of substantial contribution to a_μ from higher order QED

III. Improvement in the evaluation of HVP

- $e^+e^- \to \mathrm{hadrons}$
- lattice QCD
- other approaches

III. Improvement in the evaluation of HVP

- $e^+e^- \to \mathrm{hadrons} \longrightarrow$ cf. talks by F. Jegerlehner and S. Eidelman
- lattice QCD $\longrightarrow$ cf. talk by M. Krstić Marinković
- other approaches

HVP from $e^+e^- \to \mathrm{hadrons}$

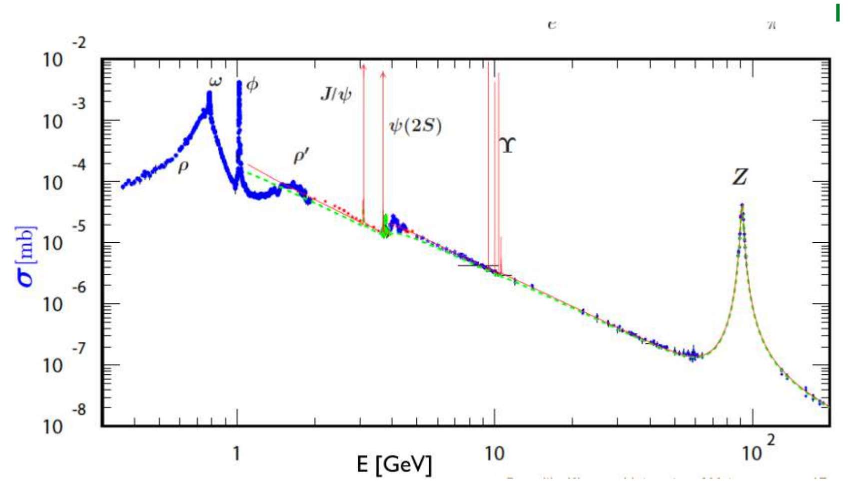
Can be expressed as (optical theorem)

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K(t) R^{\text{had}}(t) \qquad K(t) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)\frac{t}{m_\ell^2}}$$

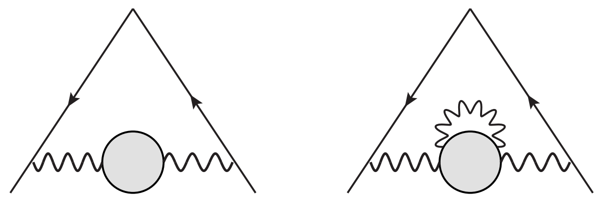
C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)

L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)

M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)



$\longrightarrow$



Note that some order $\mathcal{O}(\alpha^3)$ corrections included

- exchange of virtual photons between final state hadrons
- some radiative exclusive modes, e.g. $\pi^0\gamma$, $\eta\gamma$

$$a_\mu^{\pi^0\gamma}(600\,\mathrm{MeV} - 1030\,\mathrm{MeV}) = 4.4(1.9)\cdot 10^{-10}$$

New determinations from $e^+e^- \to \text{hadrons}$

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$ situation in 2011

$692.3(4.2)$ M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)

$694.9(4.3)$ K. Hagiwara et al., J. Phys. G 38, 085003 (2011) $\sim 0.6\%$

$690.75(4.72)$ F. Jegerlehner, R. Szafron, Eur. Phys. J. C 71, 1632 (2011)

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$\Downarrow$

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$ situation today

$693.1(3.4)$	M. Davier et al., Eur. Phys. J. C 77, 827 (2017)	
$693.27(2.46)$	A. Keshavarzi et al., arXiv:1802.02995 [hep-ph]	$\sim 0.4\%$
$688.07(4.14)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]	

Some tension remains

Experiment	$a_\mu^{\text{HVP}-\text{LO}\,2\pi}(600 - 900\ \text{MeV})$
BaBar(09)	376.7(2.7)
KLOE(comb)	366.7(2.2)
BESIII(15)	368.2(4.2)
SND(04)	371.7(5.0)
CMD-2(comb)	372.4 (3.0)

A. Anastasi et al. [KLOE-2], arXiv:1711.03085 [hep-ex]

New determinations from $e^+e^- \to \text{hadrons}$

Higher-order corrections

$a_\mu^{\text{HVP-NLO}} \cdot 10^{10}$

$-9.84(7)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$-9.93(7)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]
$-9.82(4)$	A. Keshavarzi et al., arXiv:1802.02995 [hep-ph]

$a_\mu^{\text{HVP-NNLO}} \cdot 10^{10}$

$1.24(1)$	A. Kurz et al., Phys. Lett. B 734, 144 (2014)
$1.22(1)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]

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$1.24(1)$	A. Kurz et al., Phys. Lett. B 734, 144 (2014)
$1.22(1)$	F. Jegerlehner, arXiv:1705.00263 [hep-ph]

Possibilities to cross-check these results ?

HVP from the space-like region (from Bhabha or μe scattering)

C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)
G. Abbiendi et al., Eur. Phys. J. C 77, 139 (2017)

- $a_\mu^{\rm HVP} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\rm had}(-\frac{x^2}{1-x} m_\mu^2)$

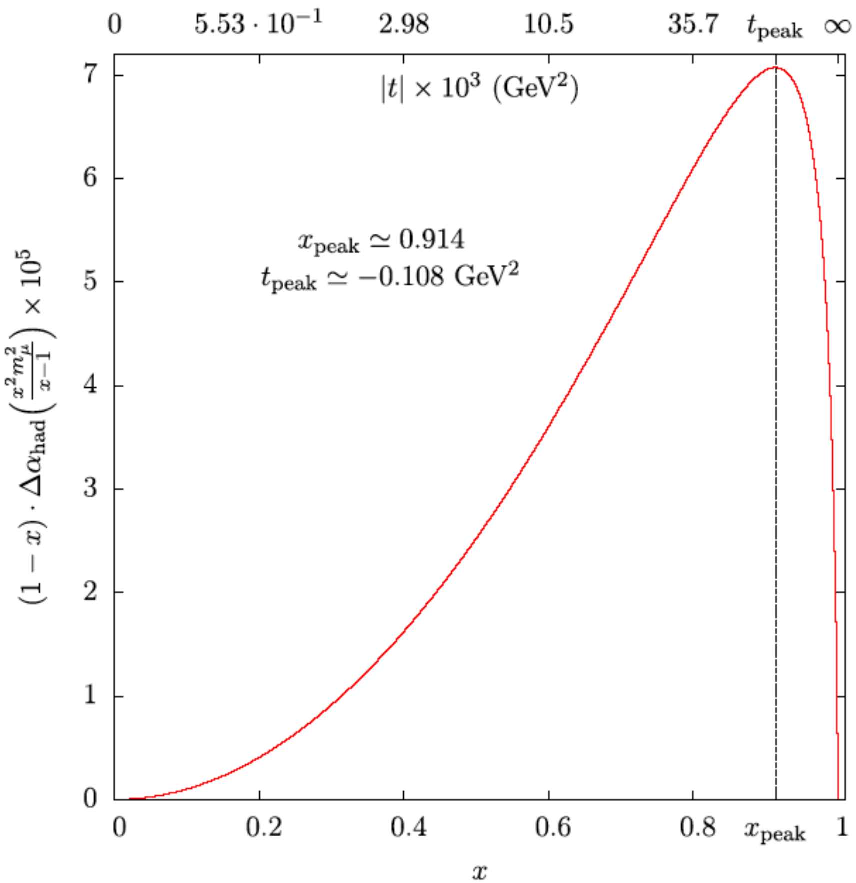
$t = \frac{x^2 m_\mu^2}{x-1}, \ 0 \leq -t < +\infty, \ 0 \leq x < 1$

- $a_\mu^{\rm HVP}$ given by the integral

- measurement of $\Delta\alpha_{\rm had}$ in the space-like region

- contribution at small t enhanced

- a 0.3% error can be achieved in 2y of data taking with 1.3x10^7 $\mu/{\rm s}$ (CERN)



- real challenge: getting the systematics below 10ppm (higher-order corrections,...)

would provide an interesting cross-check of the time-like determinations of HVP

$\longrightarrow$ central topic of this workshop, many talks to come

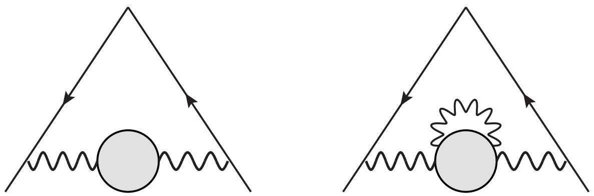
HVP from lattice QCD

Several groups are producing results, e.g. $\longrightarrow$ more in Marina Marinković's talk

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$	Recent lattice results
$654(32)^{+21}_{-23}$	M. Della Morte et al., JHEP 1710, 020 (2017)
$667(6)(12)$	B. Chakraborty et al. [HPQCD], Phys. Rev. D 96, 034516 (2017)
$711.0(7.5)(17.3))$	S. Borsanyi et al. [BMWc], arXiv:1711.04980 [hep-lat]
$715.4(16.3)(9.2))$	T. Blum et al., arXiv:1801.07224 [hep-lat]

Still quite large uncertainties, but steady progress

Comparison with data at the sub-percent level requires to deal with isospin breaking effects (radiative corrections)



(Experimentalists don't live in the theoretician's paradise)

Interesting perspective: combining data and lattice

$$a_\mu^{\text{HVP-LO}} \cdot 10^{10} = 692.5(1.4)(0.5)(0.7)(2.1)$$

T. Blum et al., arXiv:1801.07224 [hep-lat]

HVP from Mellin-Barnes approximants

E. de Rafael, Phys. Lett. B 736, 522 (2014)
E. de Rafael, Phys. Rev. D 96, 014510 (2017)
J. Charles, D. Greynat, E. de Rafael, arXiv:1712.02202 [hep-ph]

Uses the Mellin-Barnes representation

$$a_\mu^{\mathrm{HVP-LO}} = \frac{\alpha}{\pi}\frac{m_\mu^2}{4M_\pi^2}\int_{c_s-i\infty}^{c_s+i\infty} ds \left(\frac{m_\mu^2}{4M_\pi^2}\right)^{-s} \mathcal{F}(s)\mathcal{M}(s) \quad c_s \equiv \mathrm{Re}(s) \in]0,1[$$

$$\mathcal{F}(s) = -\Gamma(3-2s)\Gamma(-3+s)\Gamma(1+s) \qquad \mathcal{M}(s) = \int_{4M_\pi^2}^{\infty} \frac{dt}{t}\left(\frac{t}{4M_\pi^2}\right)^{s-1} \frac{1}{\pi}\mathrm{Im}\Pi(t) \quad \mathrm{Re}()s < 1$$

and the converse mapping theorem, which relates the singularities in the complex s-plane of the integrand to the successive terms of the expansion in powers of $m_\mu^2/(4M_\pi^2)$

$$a_\mu^{\mathrm{HVP-LO}} = \frac{\alpha}{\pi}\frac{m_\mu^2}{4M_\pi^2}\left[\frac{1}{3}\mathcal{M}(0) + \sum_{n\geq 1}\left(\frac{m_\mu^2}{4M_\pi^2}\right)^n \left[c_n\mathcal{M}(-n) + c_n'\mathcal{M}'(-n)\right]\right]$$

Advantages:
- rapid convergence
- $\mathcal{M}(-n) = \frac{(-4M_\pi^2)^{n+1}}{(n+1)!}\partial^{n+1}\Pi(Q^2)/\partial^{n+1}Q^2|_{Q^2=0} \longrightarrow$ easy to obtain from lattice QCD

Drawbacks:
- less straightforward to evaluate the log-moments $\mathcal{M}'(-n)$

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E. de Rafael, Phys. Lett. B 736, 522 (2014)
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Application of Ramanujan,s theorem to the MB transform of the HVP function

$$\Gamma(s)\Gamma(1-s)\mathcal{M}(s) = \int_0^\infty \frac{dQ^2}{4M_\pi^2}\left(\frac{Q^2}{4M_\pi^2}\right)^{s-1}\left[\mathcal{M}(0) - \frac{Q^2}{4M_\pi^2}\mathcal{M}(-1) + \left(\frac{Q^2}{4M_\pi^2}\right)^2\mathcal{M}(-2) + \cdots\right]$$

allows to construct a series of approximants

$$\{\mathcal{M}(0), \mathcal{M}(-1), \ldots \mathcal{M}(-N+1)\} \longrightarrow \mathcal{M}_N(s) \longrightarrow \Pi_N(Q^2)$$

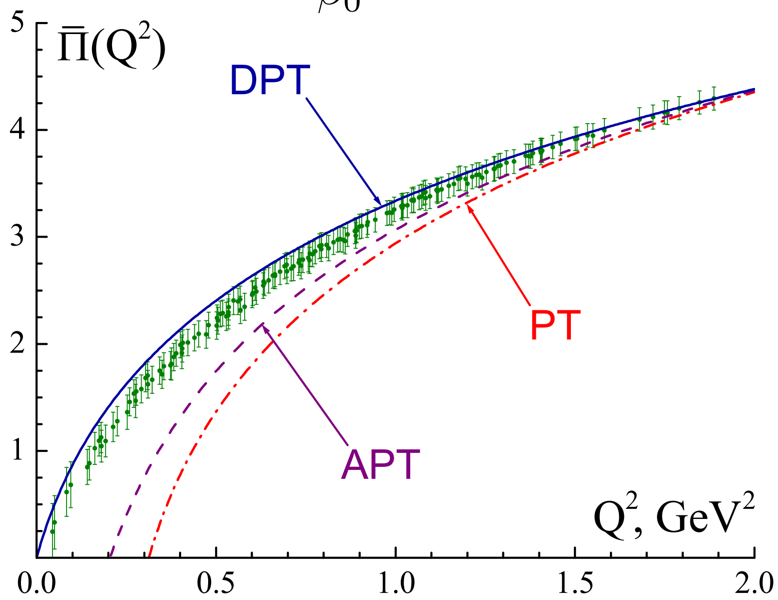
HVP from Dispersively Improved PT

Dispersive representation of the VP function

$$\overline{\Pi}(Q^2) = \overline{\Pi}^{\rm part}(-Q^2) + \int_{4M_\pi^2}^{\infty} \frac{d\sigma}{\sigma}\rho(\sigma)\ln\left(\frac{1+Q^2/4M_\pi^2}{1+Q^2/\sigma}\right)$$

Input: i) QCD asymptotic behaviour, ii) Analyticity properties, iii) Physical threshold at $4M_\pi^2$

$$\rho(\sigma) \longrightarrow \rho^{\rm pert}(\sigma) = \frac{4}{\beta_0}\left[\ln^2(\sigma/\Lambda^2) + \pi^2\right]^{-1} + \cdots$$



A. Nesterenko, J. Phys. G 42, 085004 (2015)

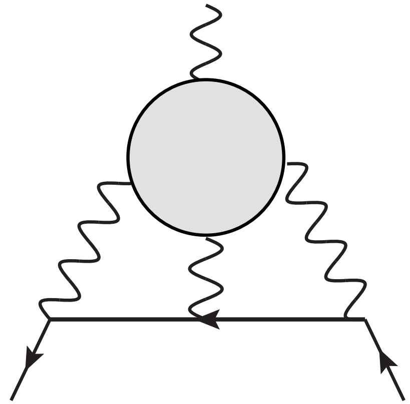
$$a_\mu^{\rm HVP-LO} = 696.1(9.5)\cdot 10^{-10}$$

Systematic uncertainties? $\longrightarrow$ more in Alexander Nestorenko's talk

IV. Improvement in the evaluation of HLxL

Not related, as a whole, to an experimental observable...

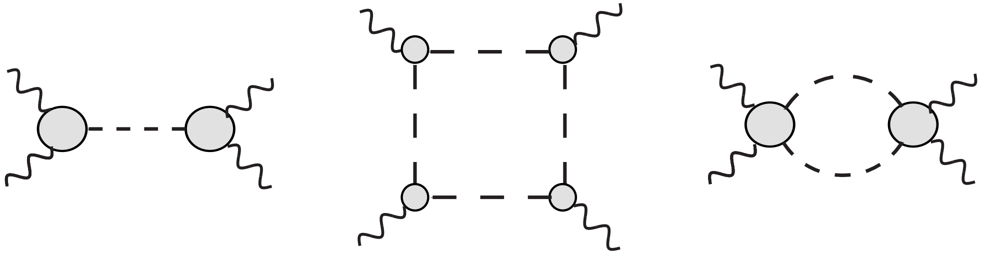
? $\longrightarrow$



Involves a much more complicated object, the fourth-rank vacuum polarization tensor

$$\text{F.T. } \langle 0|T\{VVVV\}|0\rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1,q_2,q_3,q_4) \quad q_1+q_2+q_3+q_4=0$$

Many individual contributions have been identified...

 $+\cdots$

$$\Pi = \Pi^{\pi^0,\eta,\eta' \text{ poles}} + \Pi^{\pi^\pm,K^\pm \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

Π^{residual}: other intermediate states (3π...), high-energy part,...

Present estimates still rely on two model-dependent calculation

$$a_\mu^{\text{HLxL}} = +(8.3 \pm 3.2) \cdot 10^{-10}$$

J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75, 1447 (1995) [Err.-ibid. 75, 3781 (1995)]; Nucl. Phys. B 474, 379 (1995); Nucl. Phys. B 626, 410 (2002)

$$a_\mu^{\text{HLxL}} = +(89.6 \pm 15.4) \cdot 10^{-11}$$

M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75, 790 (1995); Phys. Rev. D **54**, 3137 (1996)
M. Hayakawa, T. Kinoshita, Phys. Rev. D 57, 365 (1998) [Err.-ibid. 66, 019902(E) (2002)

Provide useful hints on the expected sizes of various contributions

Recent overview

J. Bijnens, arXiv:1712.09787 [hep-ph]

units: 10^{-11}

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	–	114 ± 13	99 ± 16
π, K loops	-19 ± 13	-4.5 ± 8.1	–	–	–	-19 ± 19	-19 ± 13
π, K l. + subl. in Nc	–	–	–	0 ± 10	–	–	–
axial vectors	2.5 ± 1.0	1.7 ± 1.7	–	22 ± 5	–	15 ± 10	22 ± 5
scalars	-6.8 ± 2.0	–	–	–	–	-7 ± 7	-7 ± 2
quark loops	21 ± 3	9.7 ± 11.1	–	–	–	2.3	21 ± 3
total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39

BPP: J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75 (1995) 1447 [Erratum-ibid. 75 (1995) 3781]; Nucl. Phys. B 474 (1996) 379; [Erratum-ibid. 626 (2002) 410]

HKS: M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75 (1995) 790; Phys. Rev. D 54 (1996) 3137

KN: M. Knecht, A. Nyffeler, Phys. Rev. D 65 (2002) 073034

MV: K. Melnikov, A. Vainshtein, Phys. Rev. D 70 (2004) 113006

BP: J. Bijnens, J. Prades, Acta Phys. Polon. B 38 (2007) 2819; Phys. Proc. Suppl. 181-182 (2008) 15; Mod. Phys. Lett. A 22 (2007) 767

BdRV: J. Prades, E. de Rafael, A. Vainshtein, arXiv:0901.0306 [hep-ph]

N/NJ: A. Nyffeler, Phys. Rev. D 79, 073012 (2009); F. Jegerlehner, A. Nyffeler, Phys. Rep. (2009)

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N/NJ: A. Nyffeler, Phys. Rev. D 79, 073012 (2009); F. Jegerlehner, A. Nyffeler, Phys. Rep. (2009)

Recent reevaluation of single meson exchanges...

$$a_\mu^{\rm HLxL}(f_1, f_1') = 6.4(2.0) \cdot 10^{-11},\ a_\mu^{\rm HLxL}(f_0, f_0', a_0) = (-1 \text{ to } -4) \cdot 10^{-11},\ a_\mu^{\rm HLxL}(f_2, f_2', a_2, a_2') = 1.1(0.1) \cdot 10^{-11}$$

V. Pauk, M. Vanderhaeghen, Eur. Phys. J C 74, 3008 (2014)

$$a_\mu^{\rm HLxL}(a_1, f_1, f_1') = 7.51(2.71) \cdot 10^{-11}$$ F. Jegerlehner, EPJ Web Conf. 118 (2016)

...and of pion box and $\pi\pi$: $a_\mu^{\pi \text{ box}} + a_\mu^{\pi\pi;\pi \rm LHC} = -24(1) \cdot 10^{-11}$

G. Colangelo et al., Phys. Rev. Lett. 118, 232001 (2017)

See also J. Bijnens, J. Relefors, JHEP 1609, 113 (2016)

HLxL within a dispersive framework

- decompose $\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4)$ into a set of independent invariant functions, free of kinematic singularities and zeros
- write dispersion relations for these functions
- saturate the dispersion relations by lowest one-, two- or more meson states

G. Colangelo et al., JHEP 1409, 091 (2014); JHEP 1509, 074 (2015)

DRs require input for form factors...

G. Colangelo et al., Phys. Lett. B 738, 6 (2014)

either from experiment, or from lattice

A. Nyffeler, Phys. Rev. D 94, 074507 (2016)

A. Feng et al.,, Phys. Rev. D 91, 054504 (2015)

A. Gerardin, H. B. Meyer, A. Nyffeler, Phys. Rev. D 94, 074507 (2016)

Alternative dispersive approaches:
- write dispersion relation directly for the Pauli form factor

V. Pauk, M. Vanderhaeghen, Phys. Rev. D 90, 113012 (2014)

- use the relation between a_μ and the L/T photo-absorption cross-section (Schwinger sum rule)

F. Hagelstein, V. Pascalutsa, arXiv:1710.04571 [hep-ph]

$\longrightarrow$ more in F. Hagelstein's talk

HLxL from lattice QCD

- Several groups involves in computation of HLxL on the lattice

J. Green et al., arXiv:1510.08384 [hep-lat]

T. Blum et al., Phys. Rev. D 93, 014503 (2016); Phys. Rev. Lett 118, 022005 (2017)

N. Asmussen et al., arXiv:1801.04238 [hep-lat]

$$a_\mu^{\mathrm{HLxL}} = 5.35(1.35) \cdot 10^{-10}$$

Summary: updated estimate

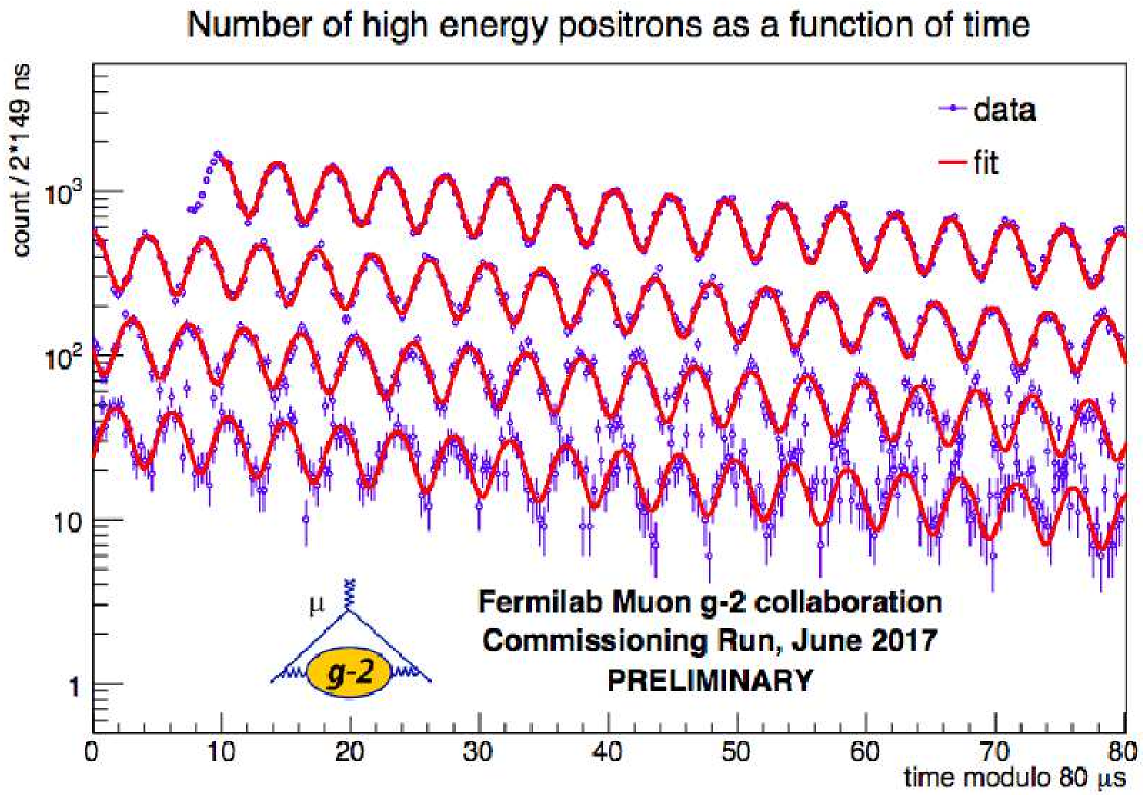
$$a_\mu^{\text{HLxL}} \quad = \quad +(10.3 \pm 2.9) \cdot 10^{-10}$$

F. Jegerlehner, arXiv:1705.00263 [hep-ph]

$\longrightarrow$ Goal: determination of HLxL at ~ 10% with controlled systematics

V. Summary - Conclusion

June 2017 ~ 700,000 positrons (~2 weeks)



First data successfully taken by FNAL-E989 last Summer

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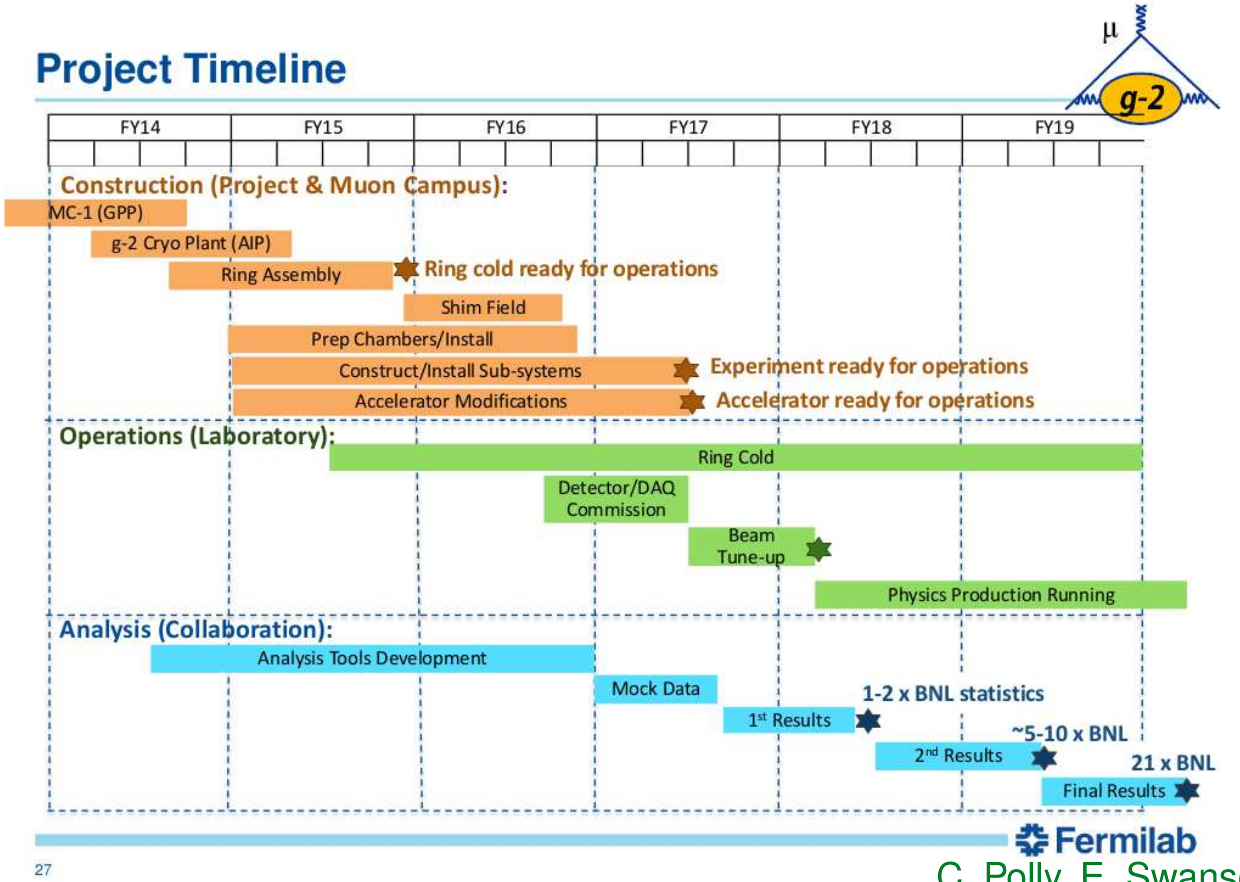
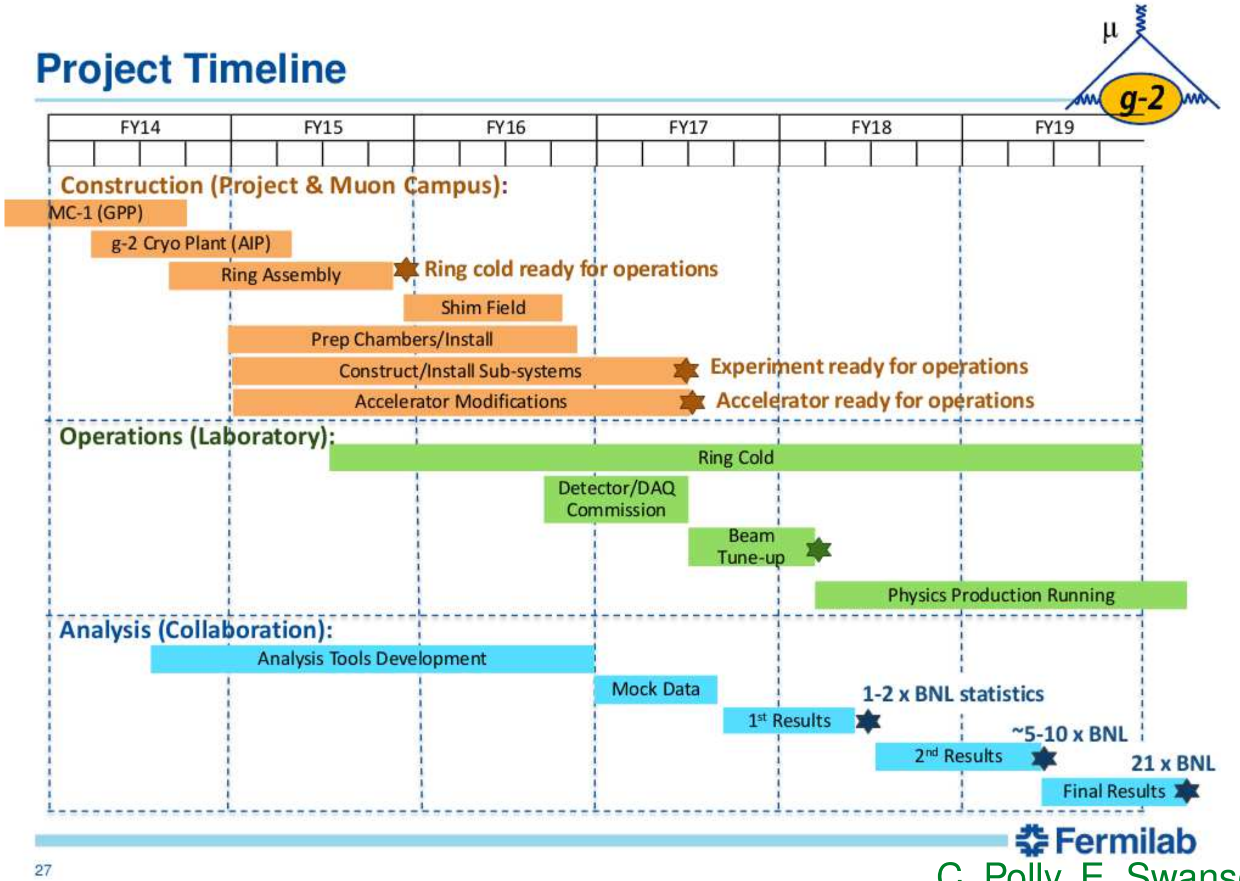


C. Polly, E. Swanson, ICHEP, Aug. 2016

Result with statistics comparable to BNL-E821 might become available this year, and final result expected for 2020

First data successfully taken by FNAL-E989 last Summer



C. Polly, E. Swanson, ICHEP, Aug. 2016

Result with statistics comparable to BNL-E821 might become available this year, and final result expected for 2020

If central value of BNL-E821 confirmed, then a_μ might well be the only single observable showing clear deviation from SM prediction

Experiment E34 at J-PARC in progress (full approval and funding still pending)

Operates under completely different experimental conditions

Important to provide independent cross-check of BNL E821 and FNAL E989 results (even if only at a later stage)

Complete cross-check of α^4 QED contributions...

...including $A_1^{(8)}$, an impressive *tour de force*

QED

Complete cross-check of α^4 QED contributions...

...including $A_1^{(8)}$, an impressive *tour de force*

Next steps?

- $A_2^{(10)}(m_\mu/m_e)$ for a_μ
- $A_1^{(10)}$ for a_e

HVP

New estimates based on 39 measured exclusive channels

Precision below the 0.5% level in relative terms

Some tensions between data remain

$\longrightarrow$ would be interesting to see analysis for the $\pi^+\pi^-$ channel of the data collected at VEPP...)

Possibilities for cross-checks, either from

- Bhabha or $e\mu$ scattering
- Lattice QCD
  - interesting results from several groups
  - statistical error still large; all systematics not yet under full control
  - interesting perspective: combine data-driven evaluations with lattice evaluations
- MB approximants,...

HLxL

Dispersive evaluation $\longrightarrow$ π-box, $\pi\pi$ (π-LHC)

Still missing:

- implementation of short-distance constraints
- estimate for Π^{residual}? Cf. axial vectors (leading in large-N_c) $\to$ 3π channel
- form factors to be provided from data and/or lattice QCD

- Lattice QCD

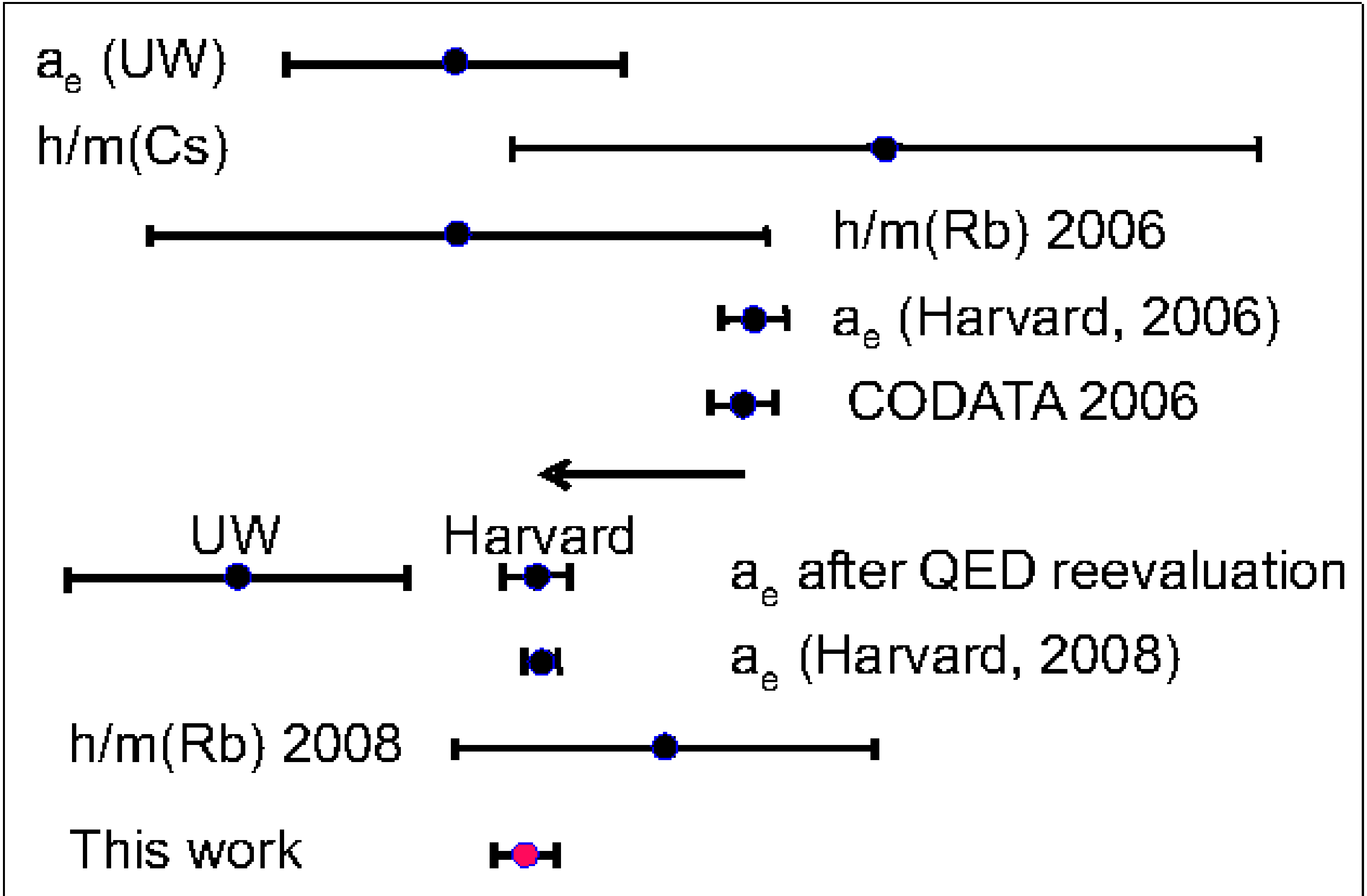
(several groups, different strategies to overcome challenging difficulties)

$\longrightarrow$ Goal: determination of HLxL at ~ 10% with controlled systematics

Crucial to exploit all possibilities to perform necessary cross-checks

Thanks for your attention!

$$
\begin{aligned}
A_1^{(8)} &= -1.434(138) && \text{[Kinoshita and Lindquist (1990)]} \\
&= -1.557(70) && \text{[Kinoshita (1995)]} \\
&= -1.4092(384) && \text{[Kinoshita (1997)]} \\
&= -1.5098(384) && \text{[Kinoshita (2001)]} \\
&= -1.7366(60) && \text{[Kinoshita (2005)]} \\
&= -1.7260(50) && \text{[Kinoshita (2005)]} \\
&= -1.7283(35) && \text{[Kinoshita and Nio, Phys. Rev. D 73, 013003(2006)]} \\
&= -1.9144(35) && \text{[Aoyama et al., Phys. Rev. Lett. 99, 110406 (2007)]} \leftarrow \\
&= -1.9106(20) && \text{[Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)]} \\
&= -1.91298(84) && \text{[Aoyama et al., Phys. Rev. D 91, 033006 (2015)]}
\end{aligned}
$$



a_e (UW)
h/m(Cs)
h/m(Rb) 2006
a_e (Harvard, 2006)
CODATA 2006
UW
Harvard
a_e after QED reevaluation
a_e (Harvard, 2008)
h/m(Rb) 2008
This work