The muon $g-2$ and $\alpha(M_Z^2)$: the hadronic part

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New updated and expanded edition
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(693 pages on one number to 8 digits)

Outline of Talk:

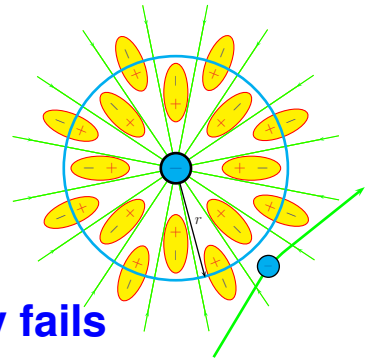
- Introduction
- $g-2$ Status: Theory vs experiment – do we see New Physics?
- Hadronic Vacuum Polarization (HVP) – Data & Status
- Evaluation of $\alpha(M_Z^2)$
- $\Delta\alpha^{\rm had}$ Adler function controlled: Euclidean split trick
- Issues in standard data based time-like approach
- A problem in DR for HVP and a first direct measurement of $\Pi'_\gamma(s)$
- Effective field theory: the Resonance Lagrangian Approach
- HVP from lattice QCD
- Alternative method: measure space-like $\alpha_{\rm QED,eff}(t) \to a_\mu^{\rm had}$
- Conclusion

1. Introduction

Non-perturbative hadronic effects in electroweak precision observables, main effect via

effective fine-structure "constant" $\alpha(E)$

(charge screening by vacuum polarization)



Of particular interest:

$$\alpha(M_Z) \text{ and } a_\mu \equiv (g-2)_\mu/2 \Leftrightarrow \alpha(m_\mu)$$

- **electroweak effects (leptons etc.) calculable in perturbation theory**
- **strong interaction effects (hadrons/quarks etc.) perturbation theory fails**

$\Longrightarrow$ **Dispersion integrals over** e^+e^-**–data**

encoded in

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

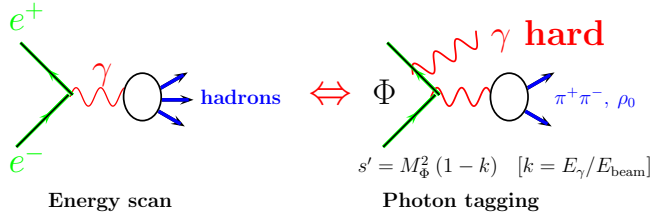
Errors of data $\Longrightarrow$ **theoretical uncertainties !!!**

The art of getting precise results from non-precision measurements !

The challenge for precision experiments on $\sigma(e^+e^- \to hadrons)$

σ_{hadronic} **via scan or radiative return:**

NSK, KLOE, BABAR, BESIII...



Muon $g-2$ to go

5 Numbers to establish the "g-2 Test"

(that is, 5 that have relevant uncertainties to keep watch on)

$$a_\mu(\text{New Physics}) \equiv a_\mu(\text{Expt}) - a_\mu(\text{SM})$$

Discussion today

$$a_\mu(\text{Expt}) = \frac{\omega_a/\tilde{\omega}_p}{\mu_\mu/\mu_p - \omega_a/\tilde{\omega}_p}$$

Expression in BNL PRD

Essentially experimental; limited at 120 ppb by μ_μ/μ_p

- $a_\mu(SM) = a_\mu(QED) + a_\mu(Weak) + a_\mu(HVP) + a_\mu(Had\ HO) + a_\mu(HLbL)$

✓ ✓ ✓

Discussion today

Goals:		
	Δa_μ(Expt)	~ 140 ppb
	Δa_μ(SM)	< 220 ppb

slide from D. Hertzog

2. Muon $g-2$ Status: Theory vs experiment

Given the CODATA/PDG recommended value of α the theory confronts experiment as follows: see Marc Knecht's Talk

Standard model theory and experiment comparison

Contribution	Value$\times 10^{10}$	Error$\times 10^{10}$	Reference
QED incl. 4-loops + 5-loops	11 658 471.886	0.003	Aoyama et al 12,Laporta 17
Hadronic LO vacuum polarization	689.46	3.25	
Hadronic light–by–light	10.34	2.88	
Hadronic HO vacuum polarization	-8.70	0.06	
Weak to 2-loops	15.36	0.11	Gnendiger et al 13
Theory	11 659 178.3	3.5	–
Experiment	11 659 209.1	6.3	BNL 04
The. - Exp. 4.3 standard deviations	-30.6	7.2	–

Standard model theory and experiment comparison [in units 10^{-10}]. What represents the 4 σ deviation: ❒ new physics? ❒ a statistical fluctuation? ❒ underestimating uncertainties (experimental, theoretical)?

❖do experiments measure what theoreticians calculate?

- a “New Physics” interpretation of the persisting 3 to 4 σ requires relatively strongly coupled states in the range below about 250 GeV.

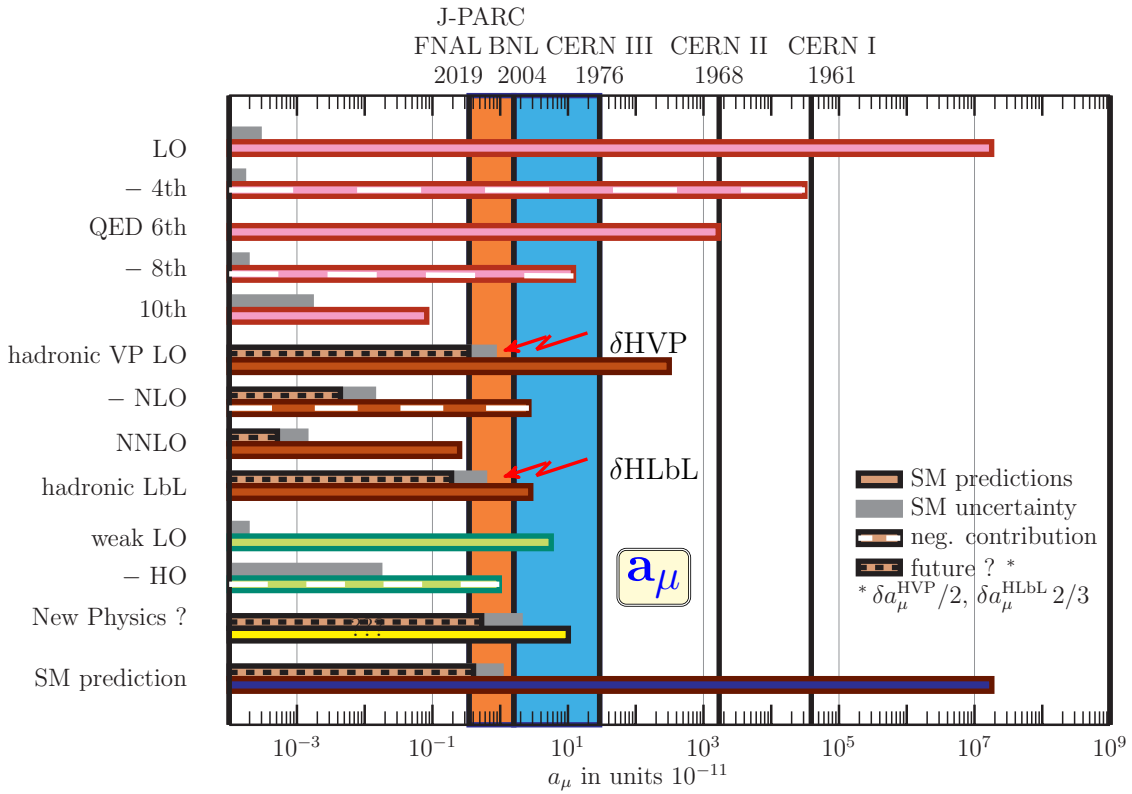
- Search bounds from LEP, Tevatron and specifically from the LHC already have ruled out a variety of Beyond the Standard Model (BSM) scenarios, so much hat standard motivations of SUSY/GUT extensions seem to fall in disgrace.

- There is no doubt that performing doable improvements on both the theory and the experimental side allows one to substantially sharpen (or diminish) the apparent gap between theory and experiment.

- ☼ Or is it unaccounted for real photon radiation effects?
  Do experiments measure what theory calculates?

- At the present/future level of precision a_μ depends on all physics incorporated in the SM: electromagnetic, weak, and strong interaction effects and beyond that all possible new physics we are hunting for.

here we are and hope to go:



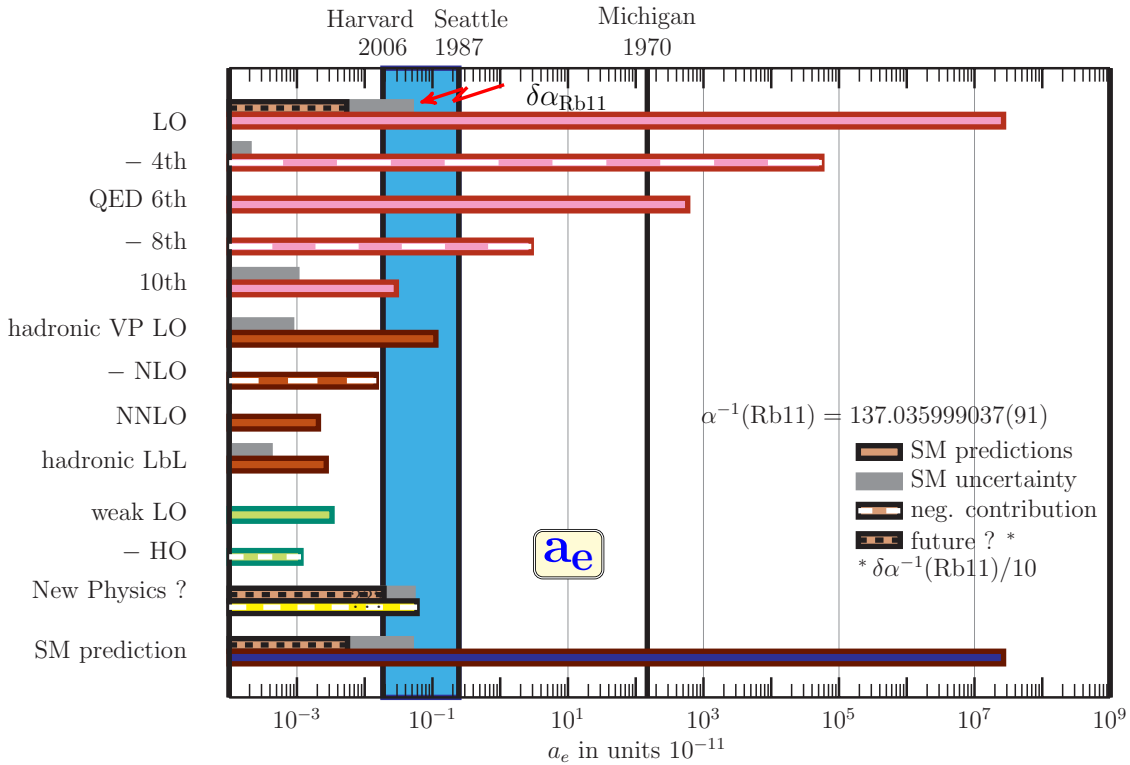
Past and future $g-2$ experiments testing various contributions.

New Physics $\stackrel{?}{=}$ deviation $(a_\mu^{\rm exp} - a_\mu^{\rm the})/a_\mu^{\rm exp}$.

Limiting theory precision: hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL)

*** digging deeper and deeper ***

same status for the electron:



Status and sensitivity of the a_e experiments testing various contributions. The error is dominated by the uncertainty of $\alpha(\mathrm{Rb11})$ from atomic interferometry. No "New Physics" $\stackrel{?}{=}$ deviation $(a_e^{\mathrm{exp}} - a_e^{\mathrm{the}})/a_\mu^{\mathrm{exp}}$. The blue band illustrates the improvement by the Harvard experiment. Note the very different sensitivities to non-QED contributions in comparison with a_μ.

*** still is and remains a QED test mainly ***

Experiment now:

$$a_\mu^{\rm exp} = (11\,659\,209.1 \pm 5.4 \pm 3.3[6.3]) \times 10^{-10}$$ BNL updated

To come – :

New muon $g-2$ experiments at Fermilab and J-PARC: improve error by factor 4

$\Rightarrow$new muon $g-2$ experiment: $\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm the} = 8\,\sigma$ theory as today

Reduction of hadronic VP uncertainty by factor 2, same HLbL $\Rightarrow \quad \Delta a_\mu = 12\,\sigma$

That's what we hope to achieve!

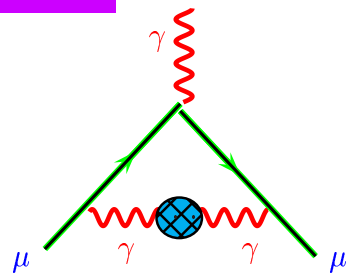
Key problem: limited accuracy of HVP! How to safely reduce and crosscheck?

- New lattice QCD now starts to have impact
- Ongoing improvement on R measurements
- New alternative methods: the workshops main theme here

❒ R-data Evaluation of $a_\mu^{\rm had}$

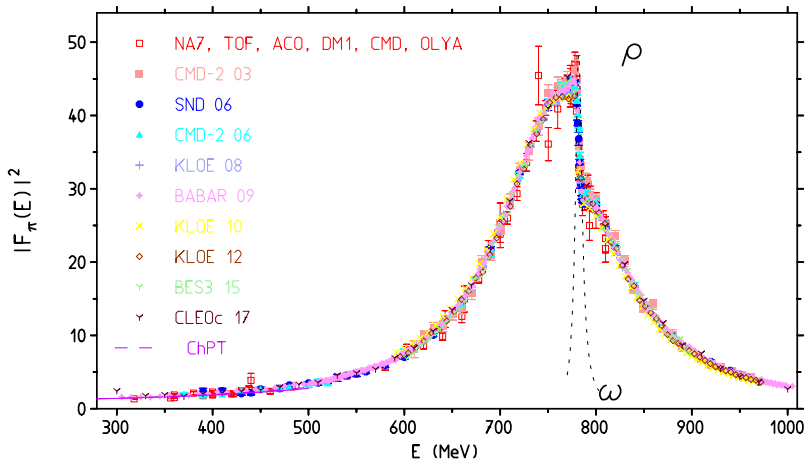
Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via **Dispersion Relation** (DR):

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E_{\rm cut}^2} ds \frac{R_\gamma^{\rm data}(s)\ \hat{K}(s)}{s^2} + \int_{E_{\rm cut}^2}^{\infty} ds \frac{R_\gamma^{\rm pQCD}(s)\ \hat{K}(s)}{s^2} \right)$$

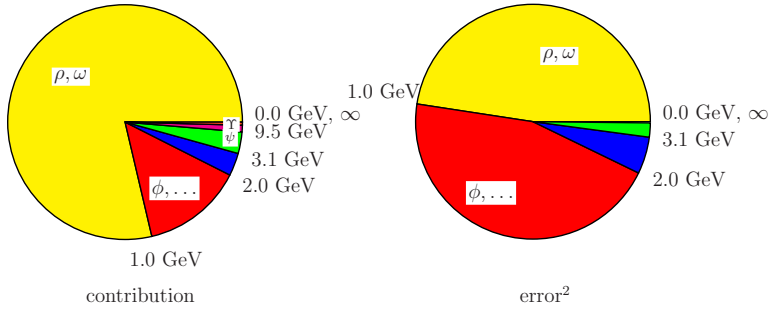


- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

Data: NSK, KLOE, BaBar, BES3, CLEOc



$$a_\mu^{\rm had(1)} = (686.99 \pm 4.21)[687.19 \pm 3.48]\ 10^{-10}$$
$$e^+e^-\text{–data based [incl. } \tau\text{]}$$



Issues in standard data based time-like approach

❶ How to combine a pretty large number of data-sets to a truly reliable R-function: true uncertainty? How much just taken from pQCD? Choosing/selecting data-sets? Bare vs physical cross sections, how reliable is VP subtraction?

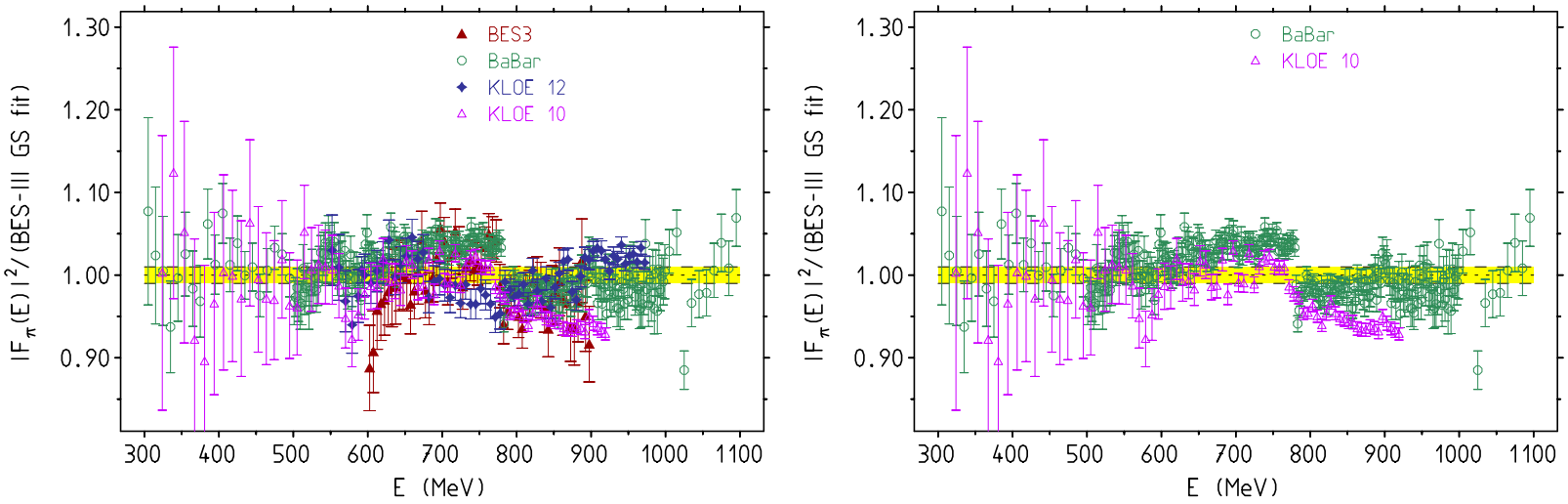
❷ Radiative corrections specifically for ISR method, sQED issues etc. The ISR method requires one order in α more precise RC calculation relative to SCAN method, at least full 2–loop Bhabha and/or $e^+e^- \to \mu^+\mu^-$ as well as ISR–FSR interference in $\pi^+\pi^-$ channel. What about RC to other more complicated channels (see e.g. F.J.&Karol Kołodziej 2017)? see Hernryk Czyż's Talk What about disentangling 30 channels and recombining them in the 1 to 2 GeV region (quantum interference, missing parts, double counting issues)?

❸ What precisely do we need in the DR? The 1PI "blob", which is not a measurable quantity. Need undressing from QED effects, photon VP subtraction, FSR modeling, $\rho^0 - \gamma$ mixing? Do we do this at sufficient precision?

❹ Non-convergence of Dyson series for OZI suppressed narrow resonances

In addition:

❒ Data compatibility



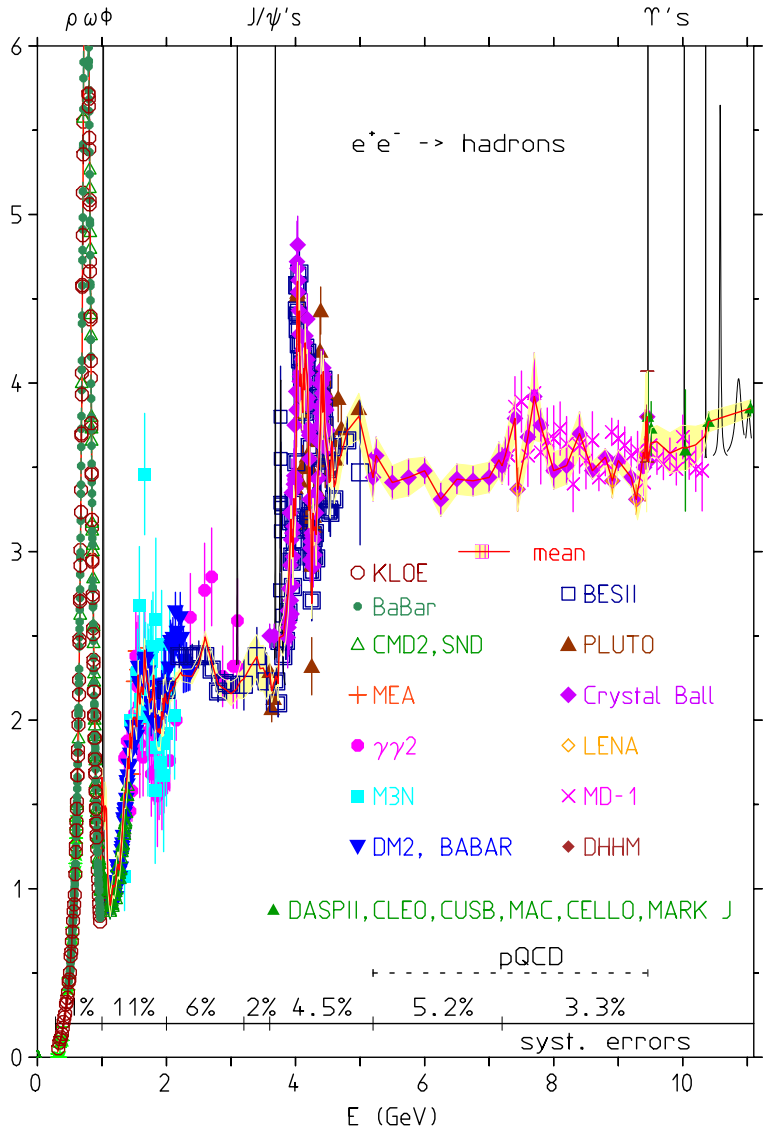
Recent BES-III vs BaBar and KLOE: dominant $\pi\pi$ channel still could be better

Recent results:

- $\pi^+\pi^-$ from BES-III, CMD-3 and CLEOc
- $\pi^+\pi^-\pi^0$ from Belle
- K^+K^- from CMD-3 and SND
- $\omega\pi^0 \to \pi^0\pi^0\gamma$ from SND
- $K_S K^\pm\pi^0\pi^\mp$, $K_S K^\pm\pi^\mp\eta$, $\pi^+\pi^-\pi^0\pi^0$, $K_S K_L\pi^0$, $K_S K_L\eta$, $K_S K_L\pi^0\pi^0$ from BaBar

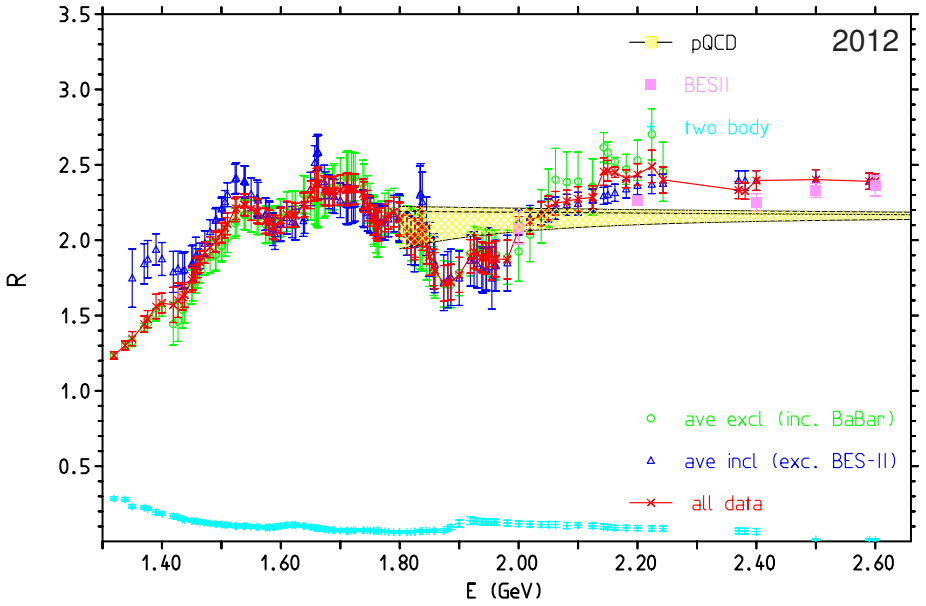
see Simon Eidelman's Talk

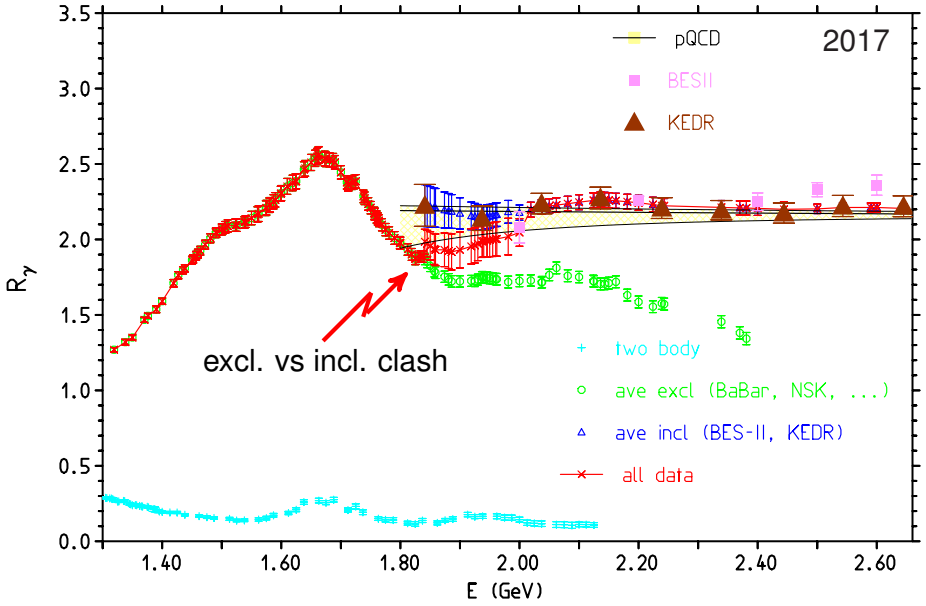
Energy range	$a_\mu^{\rm had}[\%](\text{error})\times 10^{10}$	rel. err.	abs
ρ,ω ($E<1$ GeV)	540.98 [78.6](2.80)	0.5 %	50
1 GeV $< E <$ 2 GeV	96.49 [14.0](2.54)	2.6 %	41
2 GeV $< E < \infty$ incl pQCD	51.09 [7.4](1.10)	2.2 %	7
total	688.65 [100.0](3.94)	0.6 %	100



❒ Still an issue in HVP

❒ region 1.2 to 2 GeV data; test-ground exclusive vs inclusive R measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty

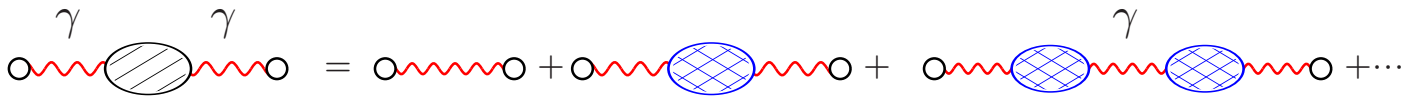




● illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR. Why point at 1.84 GeV so high?

❒ A problem in DR for HVP?

Full photon propagator Dyson resummation of 1PI part (blue blob)



$$
\begin{aligned}
\mathrm{i}\, D'_\gamma(q^2) &\equiv \frac{-\mathrm{i}}{q^2} + \frac{-\mathrm{i}}{q^2}\,(-\mathrm{i}\Pi_\gamma)\,\frac{-\mathrm{i}}{q^2} + \frac{-\mathrm{i}}{q^2}\,(-\mathrm{i}\Pi_\gamma)\,\frac{-\mathrm{i}}{q^2}\,(-\mathrm{i}\Pi_\gamma)\,\frac{-\mathrm{i}}{q^2} + \cdots \\
&= \frac{-\mathrm{i}}{q^2}\left\{1 + \left(\frac{-\Pi_\gamma}{q^2}\right) + \left(\frac{-\Pi_\gamma}{q^2}\right)^2 + \cdots\right\} \\
&= \frac{-\mathrm{i}}{q^2}\left\{\frac{1}{1+\frac{\Pi_\gamma}{q^2}}\right\} = \frac{-\mathrm{i}}{q^2+\Pi_\gamma(q^2)} = \frac{-\mathrm{i}}{q^2}\,\frac{1}{1+\Pi'_\gamma(q^2)}\,.
\end{aligned}
$$

Including external e.m. coupling

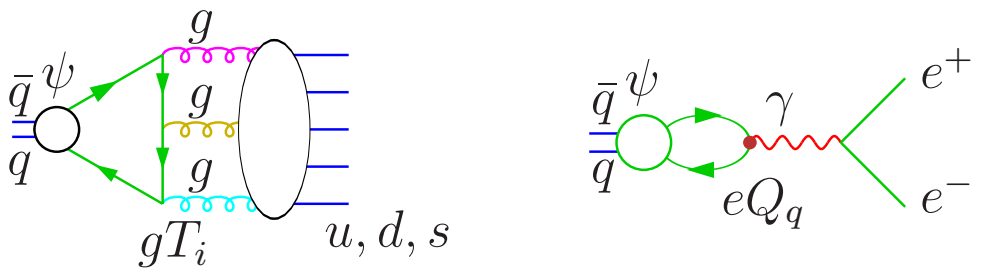
$$\mathrm{i}\, e^2\, D'_\gamma(q^2) = \frac{-\mathrm{i}}{q^2}\,\frac{e^2}{1+\Pi'_\gamma(q^2)}$$

Effective charge

$$\frac{e^2}{1+\Pi'_\gamma(s)} = \frac{e^2}{1-\Delta\alpha(s)} = e^2(s)$$

Usually, $\Delta\alpha(s)$ is a correction i.e $\Delta\alpha(s) \ll 1$ and the Dyson series converges well.

Exceptions: narrow OZI suppressed resonances (below $q\bar{q}$-thresholds)



Γ_{ee} not much smaller than $\Gamma_{\rm QCD}$ (i.e strong decays): $J/\psi, \psi_2, \Upsilon_1, \Upsilon_2, \Upsilon_3$

Note: imaginary parts from narrow resonances, $\mathrm{Im}\,\Pi'(s)) = \frac{\alpha}{3} R(s) = \frac{3}{\alpha}\frac{\Gamma_{ee}}{\Gamma}$ at peak, are sharp spikes and are obtained correctly only by appropriately high resolution scans. For example,

$$\boxed{|1-\Pi'(s)|^2 - (\alpha/\alpha(s))^2 = (\mathrm{Im}\,\Pi'(s))^2}$$

at $\sqrt{s} = M_R$ is given by

ρ	1.23	$\times 10^{-3}$
ω	2.76	$\times 10^{-3}$
ϕ	1.56	$\times 10^{-2}$
J/ψ	594.81	
ψ_2	9.58	
ψ_3	2.66	$\times 10^{-4}$
Υ_1	104.26	
Υ_2	30.51	
Υ_3	55.58	

- What is measured in an experiment is the full propagator, corresponding to

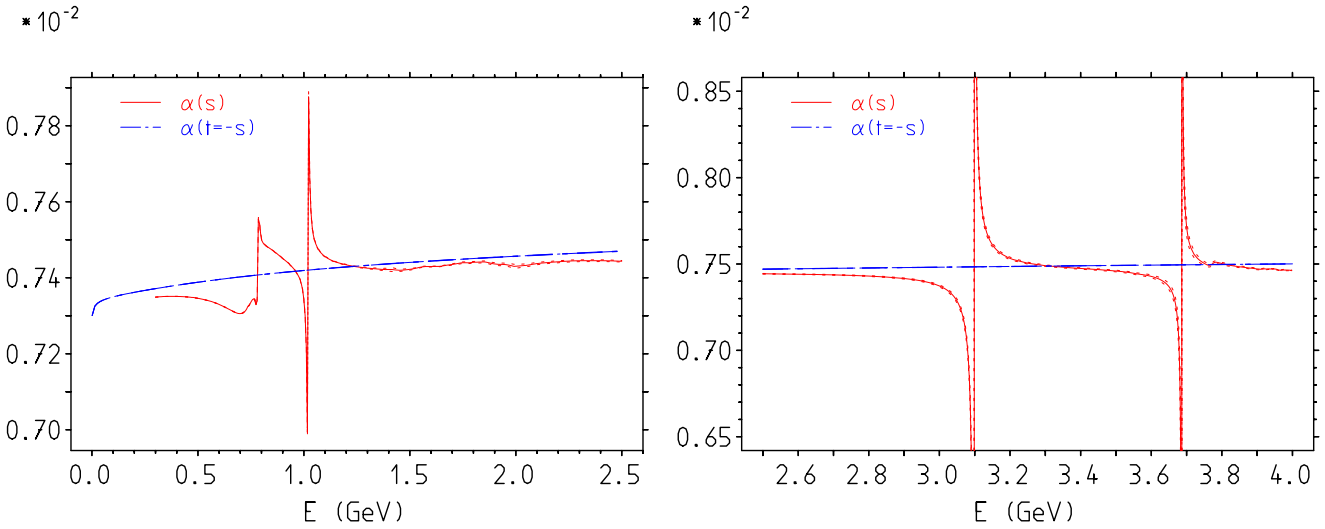
$$\boxed{\frac{1}{1-x}\ ;\quad x \ \text{ irreducible part}}$$

Object required in the DR:

$$R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}\ R^{\text{undressed}} = R^{\text{observed}} * |(1-x)|^2$$

VP subtraction is iterative procedure: does not converge!

The time-like vs space-like effective charge

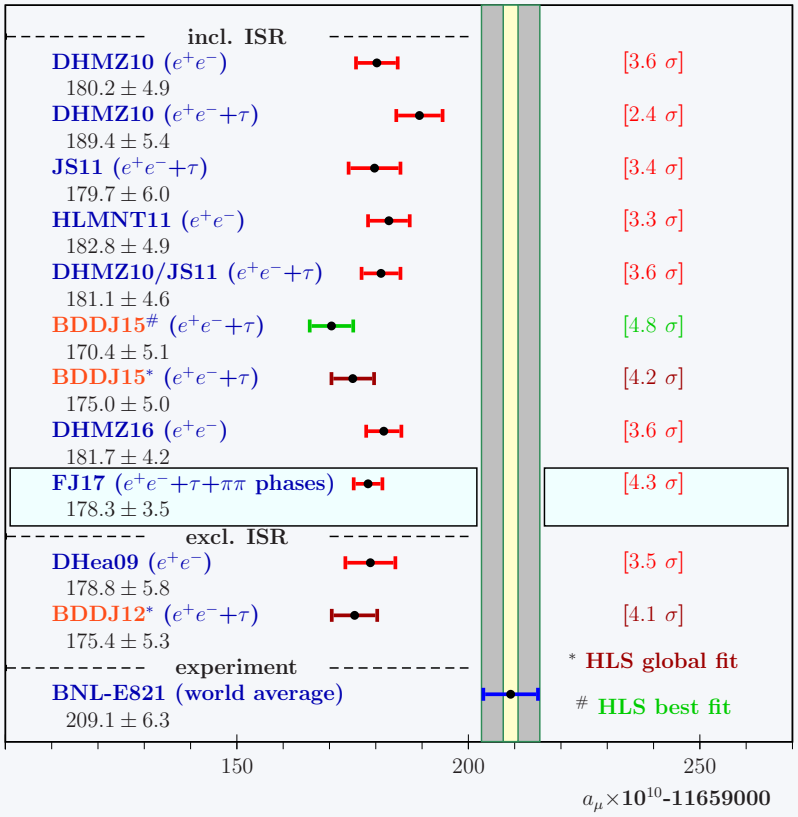


Note that the smooth space-like effective charge agrees rather well with the non-resonant "background" above the Φ (kind of duality)

No proof that this cannot produce non-negligible shifts!

Time-like VP-subtraction cannot be implemented locally near OZI suppressed resonances: $J/\psi, \psi'$ and $\Upsilon_1, \Upsilon_2, \Upsilon_3$

Present leading uncertainty: hard to improve by direct $R(s)$ measurements



Comparison with other Results. Note: results depend on which value is taken for HLbL. JS11 and BDDJ13 includes $116(39) \times 10^{-11}$ [JN], DHea09, DHMZ10, HLMNT11 and BDDJ12 use $105(26) \times 10^{-11}$ [PdRV].

HVP from lattice QCD

The need for ab initio calculation of $a_\mu^{\rm had}$ is well motivated:

– the problems to determine non-perturbative contributions to the muon $g-2$ from experimental data at sufficient precision persists and is not easy to improve,

– a model–independent extension of CHPT to the relevant energies ranges up to 2 GeV is missing while the new experiments E989 FNAL and E34 J-PARC

require an improvement of the hadronic uncertainties by a factor of four.

The hope is that LQCD can deliver estimates of accuracy

$$\delta a_\mu^{\rm HVP}/a_\mu^{\rm HVP} < 0.5\% \ , \quad \delta a_\mu^{\rm HLbL}/a_\mu^{\rm HLbL} \lesssim 10\%$$

in the coming years. see Marina Marinkovic's Talk

Primary object for HVP in LQCD: e.m. current correlator in configuration space

$$\langle J_\mu(\vec{x},t)\, J_\nu(\vec{0},0)\rangle\,, \quad J_\mu = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s + \cdots$$

In principle, a Fourier transform

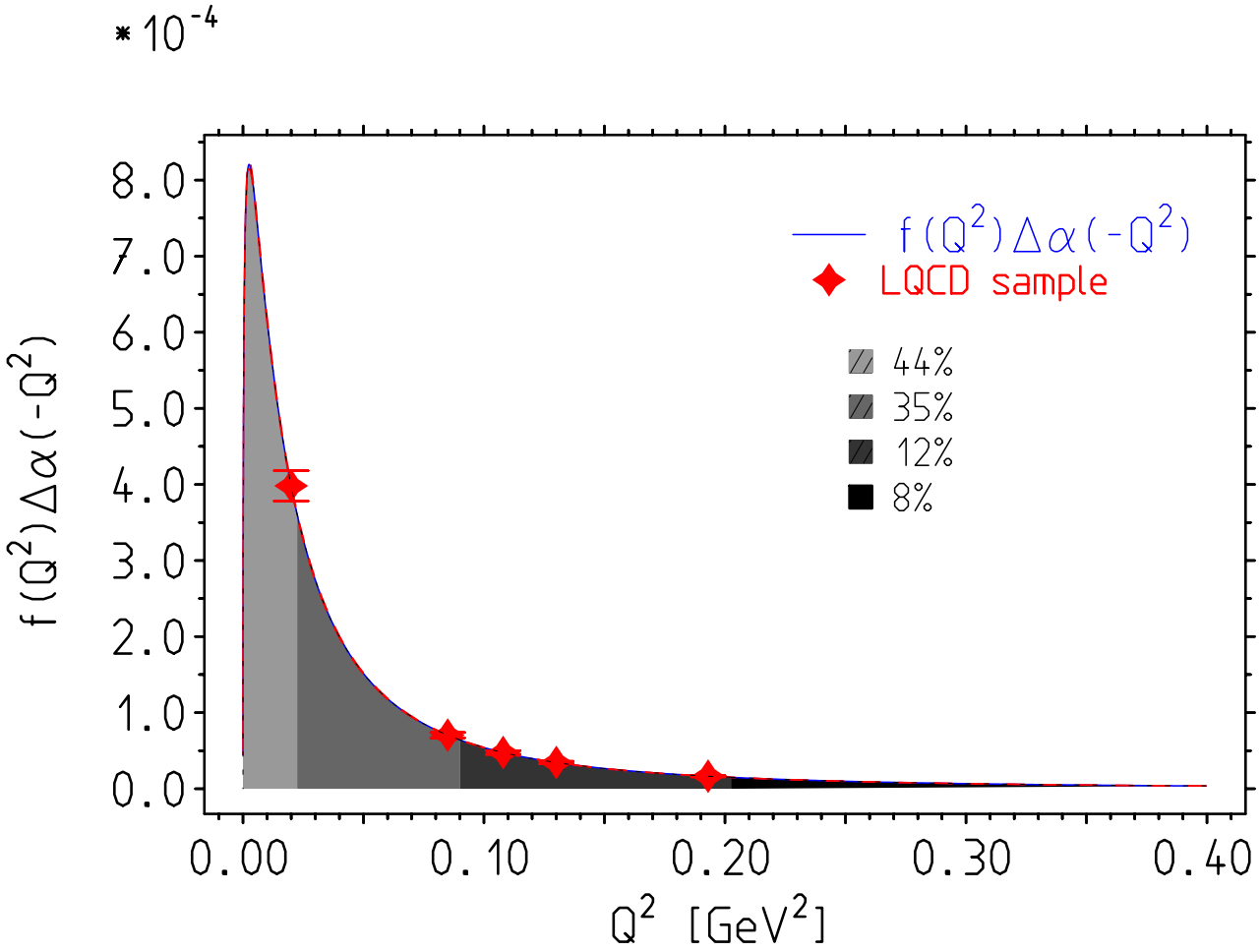
$$\Pi_{\mu\nu}(Q) = \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}\,Qx}\,\langle J_\mu(x)\, J_\nu(0)\rangle = \left(Q_\mu Q_\nu - \delta_{\mu\nu}\, Q^2\right)\,\Pi(Q^2)$$

yields the vacuum polarization function $\Pi(Q^2)$ needed to calculate

$$a_\mu^{\mathrm{HVP}} = 4\alpha^2 \int_0^\infty \mathrm{d}Q^2\, f(Q^2)\,\left\{\Pi(Q^2) - \Pi(0)\right\}$$

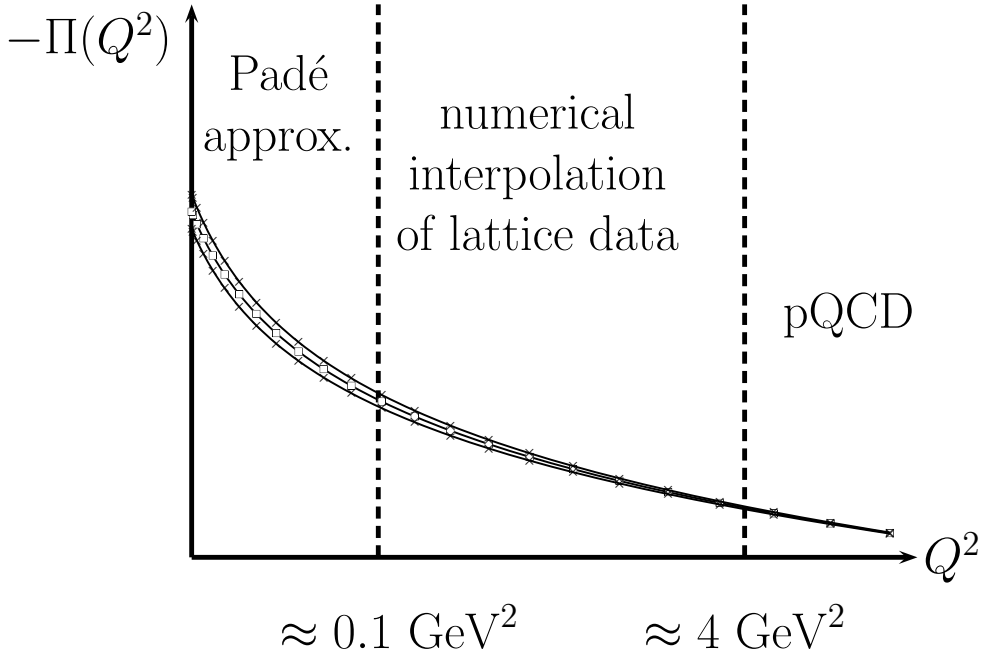
The integration kernel in this representation is

$$f(Q^2) = w(Q^2/m_\mu^2)/Q^2\;; \quad w(r) = \frac{16}{r^2\left(1+\sqrt{1+4/r}\right)^4\,\sqrt{1+4/r}}\,.$$



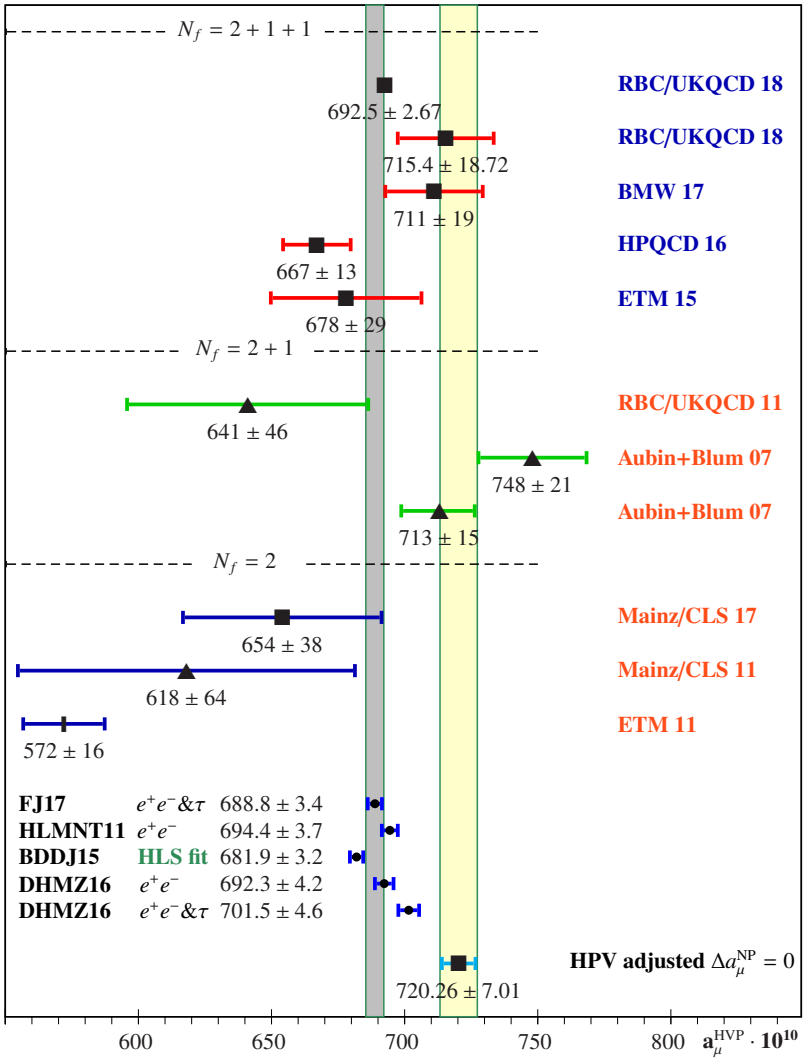
The integrand Q^2. Ranges between $Q_i = 0.00,\ 0.15,\ 0.30,\ 0.45$ and 1.0 GeV and their percent contribution to $a_\mu^{\rm had}$ and the "LQCD sample"

- LQCD lattice in finite box: momenta are quantized $Q_{\rm min} = 2\pi/L$ where L is the lattice box length. $Q_{\rm min} \to 0 \Leftrightarrow L \to \infty$ infinite volume limit
- $Q_{\rm min} = 2\pi/L$ with $m_\pi a L \gtrsim 4$ for $m_\pi \sim 200$ MeV, such that $Q_{\rm min} \sim 314$ MeV
- about 44% of the low x contribution to $a_\mu^{\rm had}$ is not covered by data yet



- lattice data: $Q^2 > (2\pi/L)^2$
- extrapolate to $Q^2 = 0$ via Padé's
- Note need $\Pi(0)$!
- required accuracy: needed LQCD data down to $Q^2_{\rm min} \approx 0.1$ GeV^2

New: RBC/UKQCD 18 use lattice between 0.1 and 4 GeV^2 and R–data for IR and UV tails $\Rightarrow$most precise evaluation so far: $a_\mu^{\rm HVP-LO} = (692.5 \pm 2.7) \times 10^{-10}$
[$a_\mu^{\rm lat} = (232.1 \pm 1.5) \times 10^{-10} \,\hat{=}\, 33.5\%$; $a_\mu^{\rm dat} = (460.4 \pm 2.2) \times 10^{-10}$]



Summary of recent LQCD results for the leading order $a_\mu^{\rm HVP}$, in units 10^{-10}. Labels: ■ marks u, d, s, c, ▲ u, d, s and ❙ u, d contributions. Individual flavor contributions from light (u, d) amount to about 90%, strange about 8% and charm about 2%. Budapest, Marseille, Wuppertal, Brookhaven, Zeuthen, Mainz, Edinburgh, ... The gray vertical band represents my evaluation. The wheat band represents the HVP required such that theory matches the experimental BNL result. The very precise RBC/UKQCD point is obtained by supplementing lattice results by R–data.

3. $\alpha(M_Z^2)$ in precision physics (precision physics limitations)

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

α , G_μ, M_Z **most precise input parameters** $\Rightarrow$ **precision predictions**

$\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

50% non-perturbative

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$$\begin{array}{lcccll}
\frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times & 10^{-9} & \\
\frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times & 10^{-6} & \\
\frac{\delta M_Z}{M_Z} & \sim & 2.4 & \times & 10^{-5} & \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 0.9 \div 1.6 & \times & 10^{-4} & \text{(present : lost } 10^5 \text{ in precision!)} \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 5.3 & \times & 10^{-5} & \text{(ILC requirement)}
\end{array}$$

LEP/SLD: $\sin^2\Theta_{\rm eff} = (1 - g_{Vl}/g_{Al})/4 = 0.23148 \pm$ 0.00017

$\delta\Delta\alpha(M_Z) = 0.00020 \quad \Rightarrow \quad \delta\sin^2\Theta_{\rm eff} =$ 0.00007

affects Higgs mass bounds, precision tests and new physics searches!!!

For pQCD contributions very crucial: precise QCD parameters α_s, m_c, m_b, $m_t \Rightarrow$ Lattice-QCD

❒ Relevance of $\alpha(M_Z^2)$

The Parameters of the Standard Model

– in four fermion and vector boson processes –

in addition QCD coupling α_s, y_t vs. M_t, λ_H vs. M_H etc.

unlike in QED and QCD in SM (SBGT)

parameter interdependence

➠

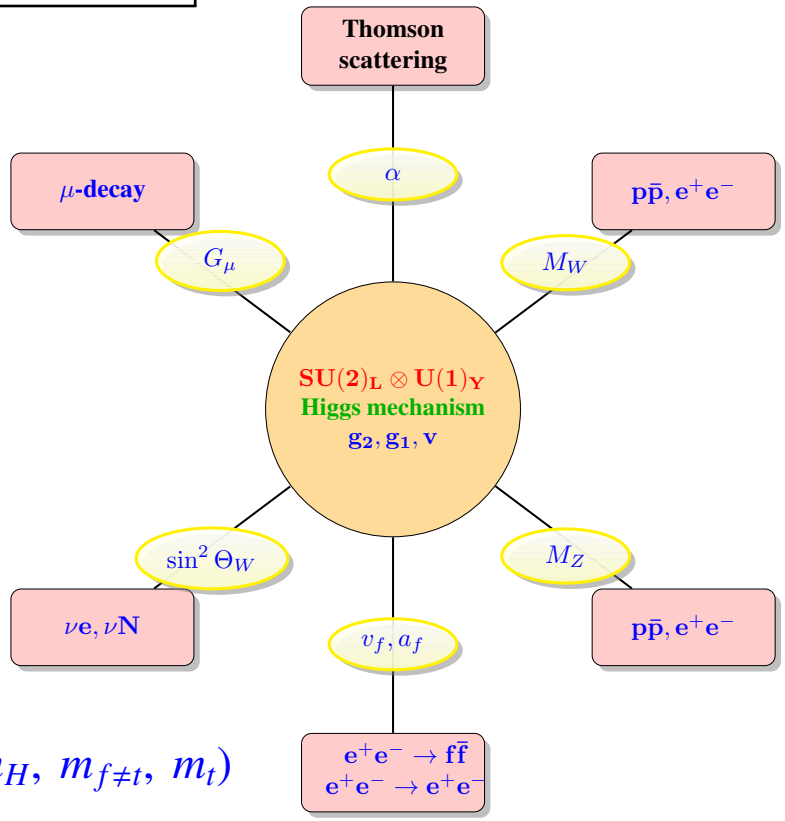
only 3 independent quantities

(besides fermion masses and mixing parameters)

α, G_μ, M_Z $\Rightarrow$ $\alpha_{\rm eff}(M_Z^2)$ $\Rightarrow$large hadronic correction

$$\Downarrow$$

$$\sin^2\Theta_i\,\cos^2\Theta_i = \frac{\pi\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\frac{1}{1-\Delta r_i}\;;\quad \Delta r_i = \Delta r_i(\alpha,\,G_\mu,\,M_Z,m_H,\,m_{f\neq t},\,m_t)$$



parameter relationships between very precisely measurable quantities

precision tests, possible sign of new physics

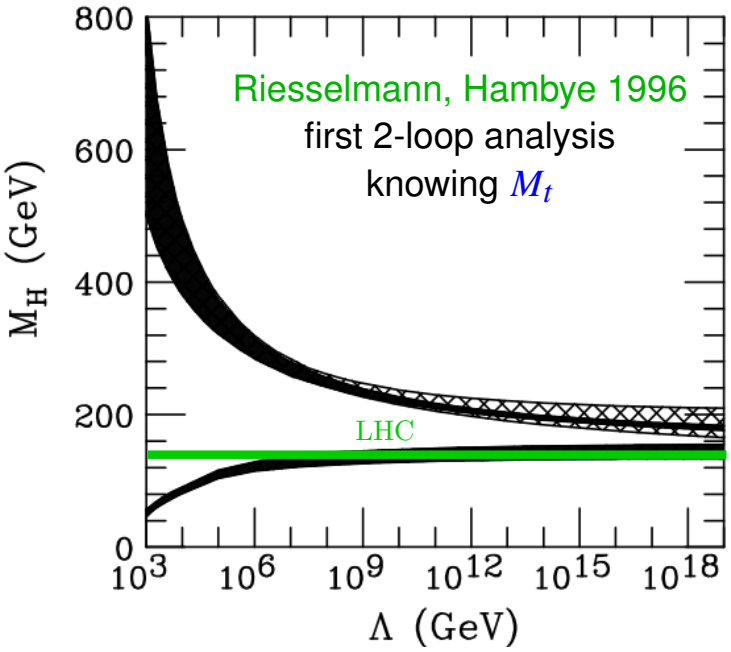
non-perturbative $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ is limiting precision predictions

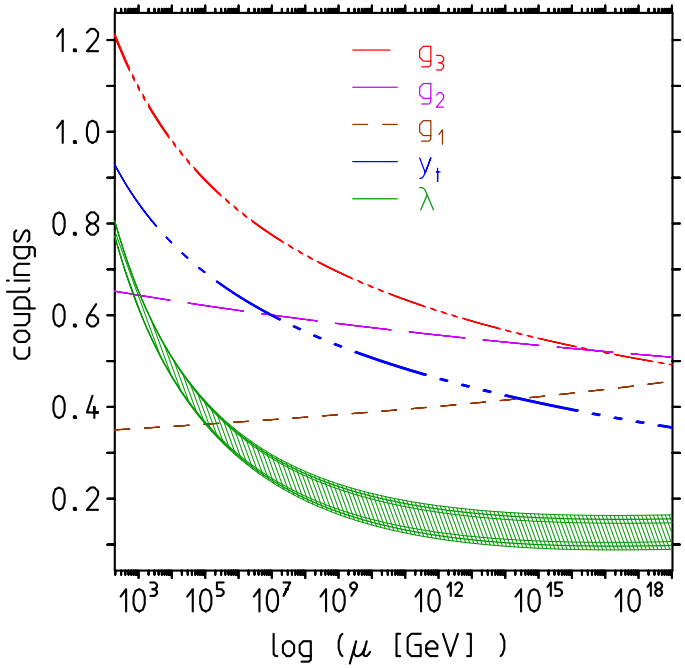
Note: 30 SD disagreement between SM prediction and experiment when subleading corrections are dropped!

SM extrapolation up to Planck scale?

After Higgs discovery: Higgs vacuum stability issue!

⇒Need very precise SM parameters: g', g, g_s, y_t, λ





The SM dimensionless couplings in the $\overline{\rm MS}$ scheme as a function of the renormalization scale for $M_H = 124 - 127$ GeV.

- perturbation expansion works up to the Planck scale!
  no Landau pole or other singularities, Higgs potential likely remains stable!
- $U(1)_Y$ screening (IR free), $SU(2)_L$, $SU(3)_c$ antiscreening (UV free): g_1, g_2, g_3

as expected (standard wisdom)

- Top Yukawa y_t and Higgs λ: screening if standalone (IR free, like QED)

as part of SM, transmutation from IR free to UV free

As SM couplings are as they are: QCD dominance in top Yukawa RG requires $g_3 > \frac{3}{4} y_t$, top Yukawa dominance in Higgs RG requires $\lambda < \frac{3(\sqrt{5}-1)}{2} y_t^2$ in the gaugeless ($g_1, g_2 = 0$) limit.
In the focus:

- does Higgs self-coupling stay positive $\lambda > 0$ up to $\Lambda_{\rm Pl}$?
- the key question/problem concerns the size of the top Yukawa coupling y_t
  decides about stability of our world! — [$\lambda = 0$ would be essential singularity!]

Will be decided by:
- more precise input parameters
- better established EW matching conditions

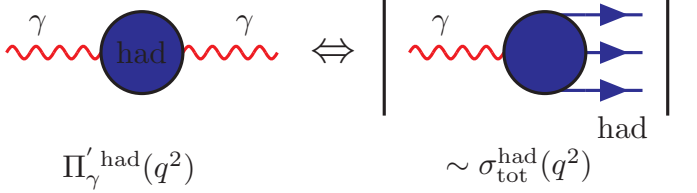
❒ R-data Evaluation of $\alpha(M_Z^2)$

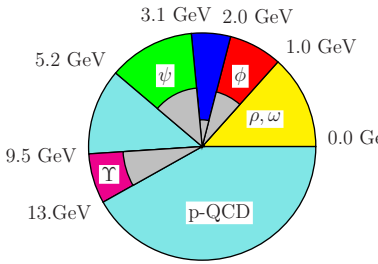
Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\rm had}(s) = -\left(\Pi'_\gamma(s) - \Pi'_\gamma(0)\right)$ can be evaluated in terms of $\sigma(e^+e^- \to \text{hadrons})$ data via dispersion integral:

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha\, s}{3\pi}\left(\fint_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \fint_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$

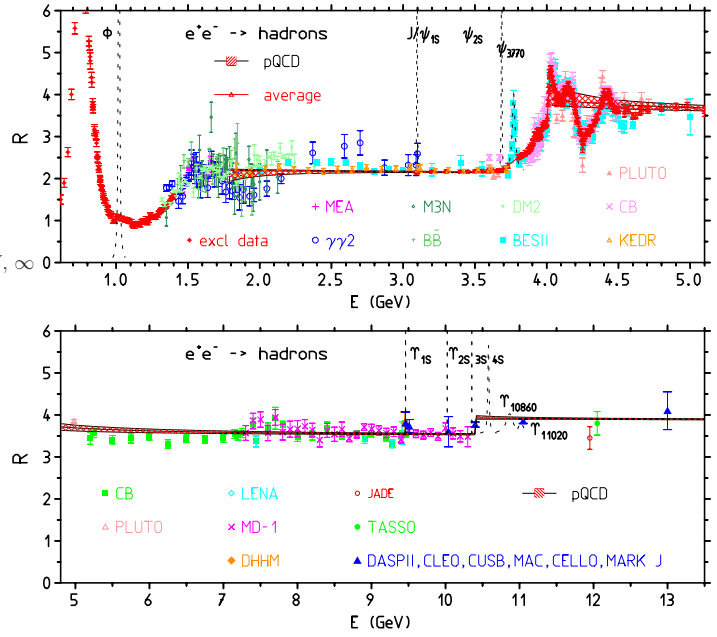
Compilation: FJ 15
Theory = pQCD: Gorishny et al. 91, Chetyrkin et al. 97...09

where $R_\gamma(s) \equiv \dfrac{\sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})}{\frac{4\pi\alpha^2}{3s}}$





hadronic vacuum polarization



$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}\ ;\quad \Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s)$$

Present situation: (after KLOE, BaBar and first BESIII results)

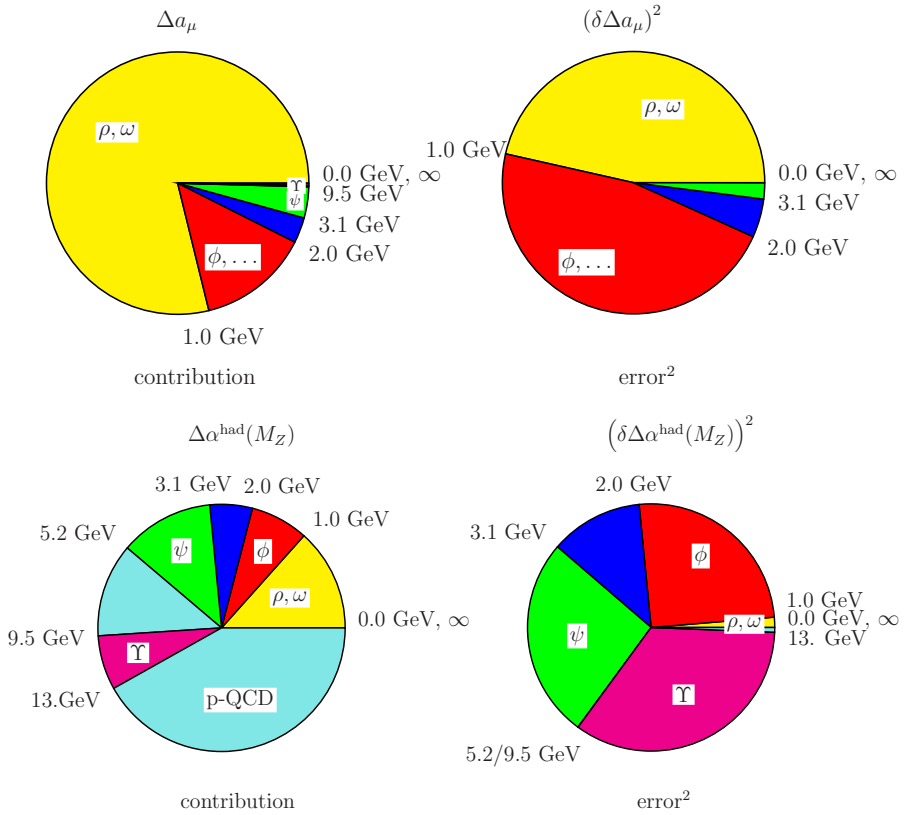
$$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) = 0.027738 \pm 0.000158$$
$$0.027523 \pm 0.000119 \quad \text{Adler}$$
$$\alpha^{-1}(M_Z^2) = 128.919 \pm 0.022$$
$$128.958 \pm 0.016 \quad \text{Adler}$$

$\Delta\alpha_{\rm had}(M_Z^2)$ results from ranges:

for $M_Z = 91.1876$ GeV in units 10^{-4}. 2017 update in terms of e^+e^--data and pQCD. 43% data, 57% perturbative QCD. pQCD is used between 5.2 GeV and 9.5 GeV and above 11.5 GeV.

final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had} \times 10^4$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	33.91 (0.05) (0.18)[0.19]	0.6%	1.4%
ω	(0.42, 0.81)	3.10 (0.04) (0.08)[0.09]	3.0%	0.3%
ϕ	(1.00, 1.04)	4.76 (0.07) (0.11)[0.13]	2.7%	0.7%
J/ψ		12.38 (0.60) (0.67)[0.90]	7.2%	32.1%
Υ		1.30 (0.05) (0.07)[0.09]	6.9%	0.3%
had	(1.05, 2.00)	16.53 (0.06) (0.83)[0.83]	5.0%	27.4%
had	(2.00, 3.20)	15.34 (0.08) (0.61)[0.62]	4.0%	15.2%
had	(3.10, 3.60)	4.98 (0.03) (0.09)[0.10]	1.9%	0.4%
had	(5.20, 5.20)	16.84 (0.12) (0.21)[0.25]	0.0%	2.4%
pQCD	(5.20, 9.46)	33.84 (0.12) (0.25)[0.03]	0.1%	0.0%
had	(9.46,11.50)	11.12 (0.07) (0.69)[0.69]	6.2%	19.2%
pQCD	(11.50,∞)	123.29 (0.00) (0.05)[0.05]	0.0%	0.1%
data	(0.28,11.50)	120.25 (0.63) (1.45)[1.58]	1.0%	0.0%
total		277.38 (0.63) (1.45)[1.58]	0.6%	100.0%

Correlation between different contributions to $a_\mu^{\rm had}$ and $\Delta\alpha^{\rm had\,(5)}$



Contributions from e^+e^- data ranges and form pQCD to $a_\mu^{\rm had}$ and $\Delta\alpha^{\rm had\,(5)}$.

4. Reducing uncertainties via the Euclidean split trick: Adler function controlled pQCD

- experiment side: new more precise measurements of $R(s)$
- future direct measurements Patrick Janot, Luca Trentadue et al
- theory side: $\alpha_{\rm em}(M_Z^2)$ by the "Adler function controlled" approach

$$\alpha(M_Z^2) \quad = \quad \alpha^{\rm data}(-s_0) + \left[\alpha(-M_Z^2) - \alpha(-s_0)\right]^{\rm pQCD} + \left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]^{\rm pQCD}$$

where the space-like $-s_0$ is chosen such that pQCD is well under control for $-s < -s_0$. The monitor to control the applicability of pQCD is the Adler function

$$D(Q^2 = -s) = \frac{3\pi}{\alpha} s \frac{d}{{\rm d}s} \Delta\alpha_{\rm had}(s) = -(12\pi^2)\, s\, \frac{{\rm d}\Pi'_\gamma(s)}{{\rm d}s} = Q^2 \int_{4m_\pi^2}^\infty \frac{R(s)}{(s+Q^2)^2}$$

which also is determined by $R(s)$ and can be evaluated in terms of experimental e^+e^-–data. Perturbative QCD tail: $D(Q^2) \to N_c \sum_f Q_f^2 \,(1 + O(\alpha_s))$ as $Q^2 \to \infty$.

S. Eidelman, F. J., A. Kataev, O. Veretin, Phys. Lett. B **454** (1999) 369

$\Delta\alpha^{\rm had}$ Adler function controlled

✓ **use old idea: Adler function: Monitor for comparing theory and data**

$$D(-s) \doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) = -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}$$

$$\Rightarrow \quad D(Q^2) = Q^2\left(\int_{4m_\pi^2}^{E^2_{\rm cut}} {\rm d}s\, \frac{R(s)^{\rm data}}{(s+Q^2)^2} + \int_{E^2_{\rm cut}}^{\infty} \frac{R^{\rm pQCD}(s)}{(s+Q^2)^2}\,{\rm d}s\right).$$

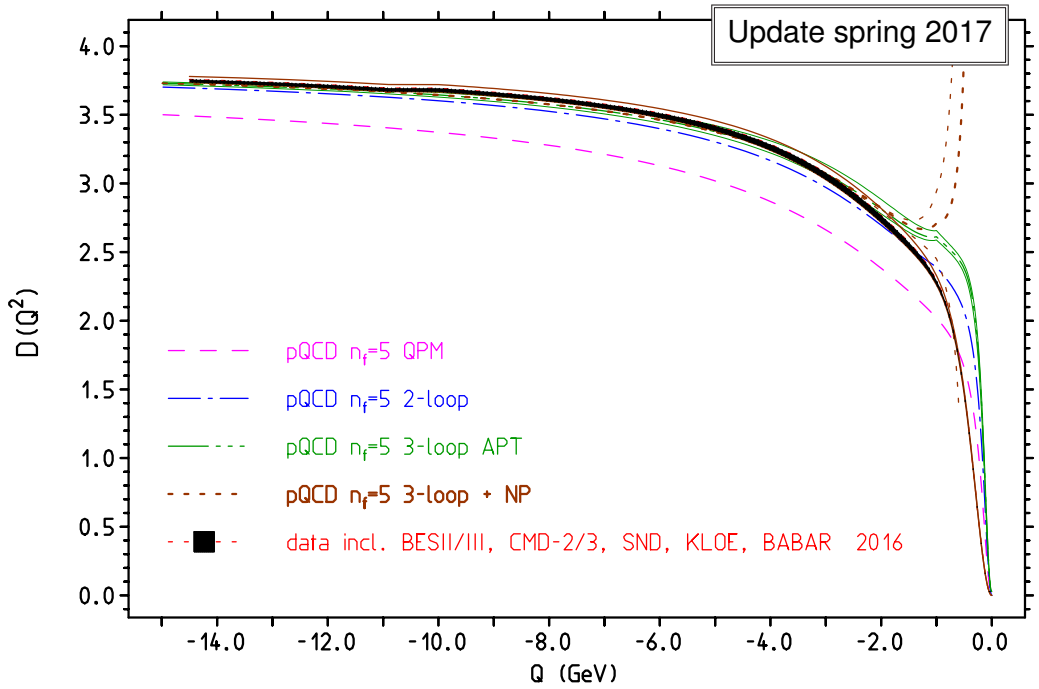
pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in Euclidean region

Conclusion:

- **time-like approach: pQCD works well in "perturbative windows"**
  3.00 - 3.73 GeV, 5.00 - 10.52 GeV and 11.50 - ∞ Kühn,Steinhauser
- **space-like approach: pQCD works well for $\sqrt{Q^2 = -q^2} > 2.0$ GeV (see plot)**

"Experimental" Adler–function versus theory (pQCD + NP)

Error includes statistical + systematic here (in contrast to most R-plots showing statistical errors only)!



(Eidelman, F. J., Kataev, Veretin 98, FJ 08/17 updates)

theory based on results by Chetyrkin, Kühn et al.

$\Rightarrow$ **pQCD works well controlled to predict $D(Q^2)$ down to $s_0 = (2.0\,\text{GeV})^2$; use this to calculate**

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int \mathrm{d}Q'^2 \frac{D(Q'^2)}{Q'^2}$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}$$

and obtain, for $s_0 = (2.0\,\text{GeV})^2$: **(FJ 98/17)**

$$\begin{aligned}
\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} &= 0.006409 \pm 0.000063\\
\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) &= 0.027483 \pm 0.000118\\
\Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.027523 \pm 0.000119
\end{aligned}$$

❖**shift $+0.000008$ from the 5-loop contribution**

❖**error ± 0.000100 added in quadrature form perturbative part**

QCD parameters: ● $\alpha_s(M_Z) = 0.1189(20)$**,**

● $m_c(m_c) = 1.286(13)$ $[M_c = 1.666(17)]$ **GeV** , ● $m_b(m_c) = 4.164(25)$ $[M_b = 4.800(29)]$ **GeV**

based on a complete 3–loop massive QCD analysis Kühn et al 2007

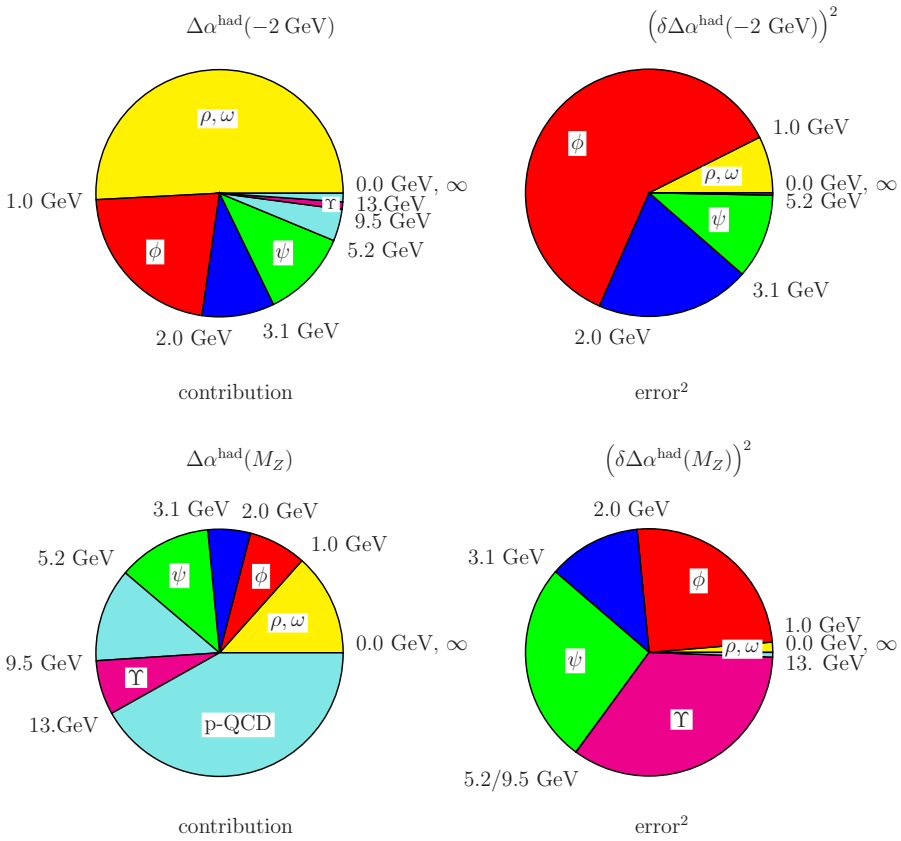
F. J., Nucl. Phys. Proc. Suppl. **181-182** (2008) 135

$\Delta\alpha_{\rm had}(-M_0^2)$ results from ranges:

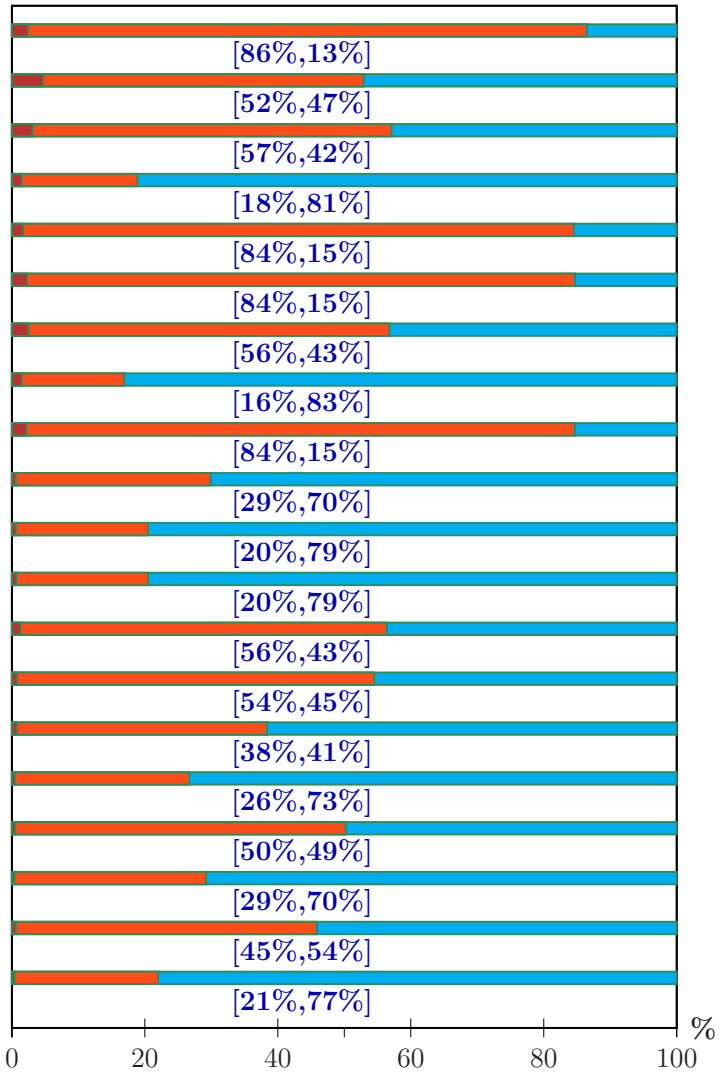
for $M_0 = 2$ GeV in units 10^{-4}. 2015 update in terms of e^+e^--data and pQCD. 94% data, 6% perturbative QCD. pQCD is used between 5.2 GeV and 9.5 GeV and above 11.5 GeV.

final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had}(-M_0^2)\times 10^4$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	29.78 (0.04) (0.16)[0.16]	0.5%	6.6%
ω	(0.42, 0.81)	2.69 (0.03) (0.07)[0.08]	3.0%	1.6%
ϕ	(1.00, 1.04)	3.78 (0.05) (0.09)[0.10]	2.7%	2.6%
J/ψ		3.21 (0.15) (0.15)[0.21]	6.7%	11.4%
Υ		0.05 (0.00) (0.00)[0.00]	6.8%	0.0%
had	(1.05, 2.00)	10.36 (0.04) (0.49)[0.49]	4.8%	61.2%
had	(2.00, 3.20)	6.06 (0.03) (0.25)[0.25]	4.2%	16.1%
had	(3.10, 3.60)	1.31 (0.01) (0.02)[0.03]	1.9%	0.2%
had	(5.20, 5.20)	2.90 (0.02) (0.02)[0.03]	0.0%	0.2%
pQCD	(5.20, 9.46)	2.66 (0.02) (0.02)[0.00]	0.1%	0.0%
had	(9.46,11.50)	0.39 (0.00) (0.02)[0.02]	5.7%	0.1%
pQCD	(11.50,∞)	0.90 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,11.50)	60.53 (0.18) (0.61)[0.63]	1.0%	0.0%
total		64.09 (0.18) (0.61)[0.63]	1.0%	100.0%

Of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ 22% data, 78% pQCD!



Contributions from e^+e^- data ranges and form pQCD to $\Delta\alpha^{(5)}_{\rm had}(-M_0^2)$ vs. $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$.



$[\Delta\alpha^{\rm data}_{\rm had}/\Delta\alpha^{\rm tot}_{\rm had}, \Delta\alpha^{\rm pQCD}_{\rm had}/\Delta\alpha^{\rm tot}_{\rm had}]$ in %

Jegerlehner 1985
Lynn et al. 1985
Burkhardt et al. 1989
Martin, Zeppenfeld 1994
Swartz 1995
Eidelman, Jegerlehner 1995
Burkhardt, Pietrzyk 1995
Adel, Yndurain 1995
Alemany, Davier, Höcker 1997
Kühn, Steinhauser 1998
Davier, Höcker 1998
Erler 1998
Burkhardt, Pietrzyk 2001
Hagiwara et al 2004
Jegerlehner 2006 direct
Jegerlehner 2006 Adler
Hagiwara et al. 2011
Davier et al. 2011
Jegerlehner 2016 direct
Jegerlehner 2016 Adler

- data-driven
- theory-driven
- fifty-fifty
- low energy weighted data

How much pQCD?

Note: the Adler function monitored Euclidean data vs pQCD split approach is only moderately more pQCD-driven, than the time-like approach adopted by Davier et al. and others.

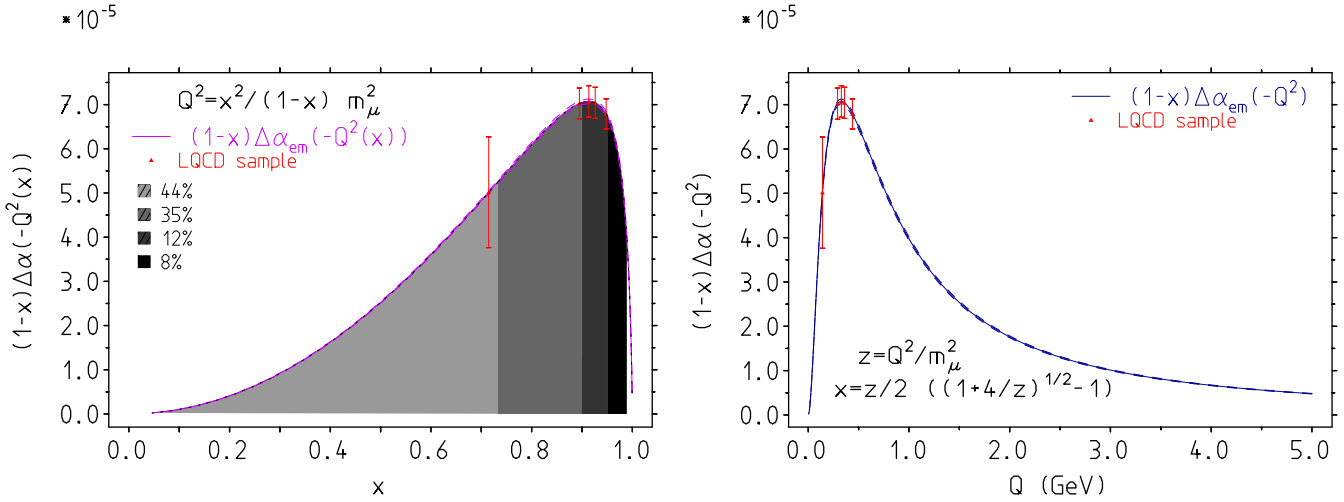
Alternative method: measure space-like $\alpha_{\rm QED,eff}(t)$

Newly proposed recently: [arXiv:1504.02228,1609.08987]
"A new approach to evaluate the leading hadronic corrections to the muon g-2"
Carloni Calame, Passera, Trentadue, Venanzoni 2015; Abbiendi et al. 2016

❒ space-like $\Delta\alpha_{\rm had}(-Q^2) = 1 - \frac{\alpha}{\alpha(-Q^2)} - \Delta\alpha^{\rm lep}(-Q^2)$ determines $a_\mu^{\rm had}$ via

$$a_\mu^{\rm had} = \frac{\alpha}{\pi}\int_0^1 {\rm d}x\,(1-x)\,\Delta\alpha_{\rm had}\left(-Q^2(x)\right)$$

where $Q^2(x) \equiv \frac{x^2}{1-x}m_\mu^2$ is the space–like square momentum–transfer. Also in the Euclidean region the integrand is highly peaked, now around half of the ρ meson mass scale.

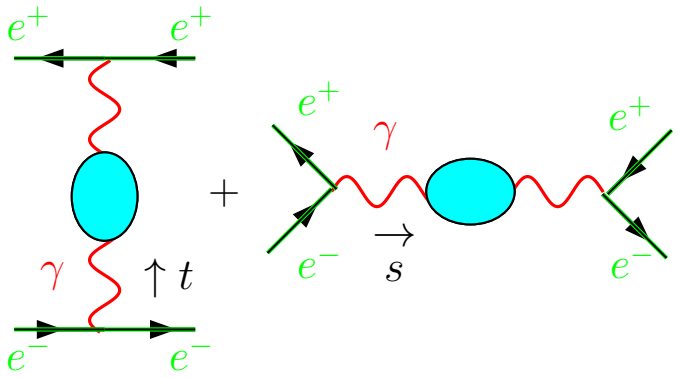


The integrand of $a_\mu^{\rm had}$ integral as functions of x and Q. Strongly peaked at about 330 MeV. Ranges between $Q_i = 0.00,\ 0.15,\ 0.30,\ 0.45$ and 1.0 GeV and their percent contribution to $a_\mu^{\rm had}$.

- measuring directly low energy running $\alpha_{\rm QED}(s)$ in space-like region via
- very different paradigm: no VP subtraction issue!
- no exclusive channel collection
- even 1% level measurement can provide important independent information

Bhabha scattering

$$e^+(p_+)\ e^-(p_-) \to e^+(p'_+)\ e^-(p'_-)$$



VP dressed tree level Bhabha scattering in QED

has two tree level diagrams the t– and the s–channel. With the positive c.m. energy square $s = (p^+ + p^-)^2$ and the negative momentum transfer square

$$t = (p_- - p'_-)^2 = -\frac{1}{2}\,(s - 4m_e^2)\,(1 - \cos\theta)\,,$$

θ the e^- scattering angle, there are two very different scales involved

The VP dressed lowest order cross–section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Theta} = \frac{s}{48\pi}\sum_{ik}|A_{ik}|^2$$

where A_{ik} tree level helicity amplitudes,i,k =L,R left– and right–handed electrons.

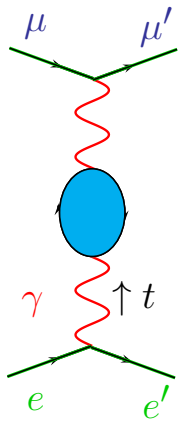
Dressed transition amplitudes: ($m_e \approx 0$)

$$\begin{aligned}
|A_{\mathrm{LL,RR}}|^2 &= \frac{3}{8}(1+\cos\theta)^2\left|\frac{e^2(s)}{s}+\frac{e^2(t)}{t}\right|^2\\
|A_{\mathrm{LR,RL}}|^2 &= \frac{3}{8}(1-\cos\theta)^2\left|\frac{e^2(s)}{s}+\frac{e^2(t)}{t}\right|^2 \quad .
\end{aligned}$$

Preferably one uses small angle Bhabha scattering (small $|t|$) as a normalizing process which is dominated by the t–channel $\sim 1/t$, however, detecting electrons and positrons along the beam axis often has its technical limitations.

Care also is needed concerning the ISR corrections because cuts for the Bhabha process ($e^+e^- \to e^+e^-$) typically are different from the ones applied to $e^+e^- \to$ hadrons. Usually, experiments have included corresponding uncertainties in their systematic errors, if they not have explicitly accounted for all appropriate radiative corrections.

$\mu^- e^-$ scattering $\qquad \mu^-(p_-)\ e^-(q_-) \to \mu^-(p'_-)\ e^-(q'_-)$



Get $a_\mu^{\rm had}$ from $\mu^- e^- \to \mu^- e^-$ process

G. Abbiendi et al. , arXiv:1609.08987 Massimo Passera's Talk

$$\frac{\mathrm{d}\sigma^{\text{unpol.}}_{\mu^-e^-\to\mu^-e^-}}{\mathrm{d}t} = 4\pi\,\alpha(t)^2\,\frac{1}{\lambda(s,m_e^2,m_\mu^2)}\left\{\frac{\left(s-m_\mu^2-m_e^2\right)^2}{t^2}+\frac{s}{t}+\frac{1}{2}\right\}$$

- The primary goal of [arXiv:1504.02228,1609.08987]: determining a_μ^{had} in an alternative way
- $\Pi'_\gamma(Q^2)-\Pi'_\gamma(0) = -\Delta\alpha_{\text{had}}(-Q^2) = \frac{\alpha}{\alpha(-Q^2)} + \Delta\alpha^{\text{lep}}(-Q^2) - 1$

  directly checks lattice QCD data
- My proposal here: determine very accurately

$$\Delta\alpha_{\text{had}}\left(-Q^2\right) \text{ at } Q \approx 2.5 \text{ GeV}$$

by this method (one single number!) as the non-perturbative part of $\Delta\alpha_{\text{had}}\left(M_Z^2\right)$ as in "Adler function" approach.

Conclusions

- Muon $g-2$ theory uncertainty remains the key issue.
- Are presently estimated (essentially agreed) evaluations in terms of R-data reliable?
- Lattice QCD estimates very close to be competitive, tending to larger central values?
- Novel hybrid method lattice + R-data optimized method looks very promising
  D. Bernecker, Harvey B. Meyer 2011, RBC/UKQCD Blum et al. 2018
- In any case on paper $e^-\mu^+ \to e^-\mu^+$ looks to be the ideal process to perform an unambiguous measurement of $\alpha(-Q^2)$, which determines the LO HVP to a_μ
- Radiative corrections much easier than for the time-like hadronic channels
- Key problem: how to control in a fixed target experiment the precision at the 1% level? What range is accessible at what precision?

Likely to be supplemented by lattice QCD results and/or using time-like data

- This experiment G. Abbiendi et al. is absolutely important also as it allows for for direct crosschecks with lattice QCD results and is has completely different systematics. Even a 5% crosscheck would be very helpful to scrutinize the HVP issue, and last bu not least whether the observed deviation is a real BSM effect.

Thanks you for your attention!