Analytical Calculation for Energy-Energy Correlation in QCD

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High time for Higher Orders
Mainz Institute for Theoretical Physics
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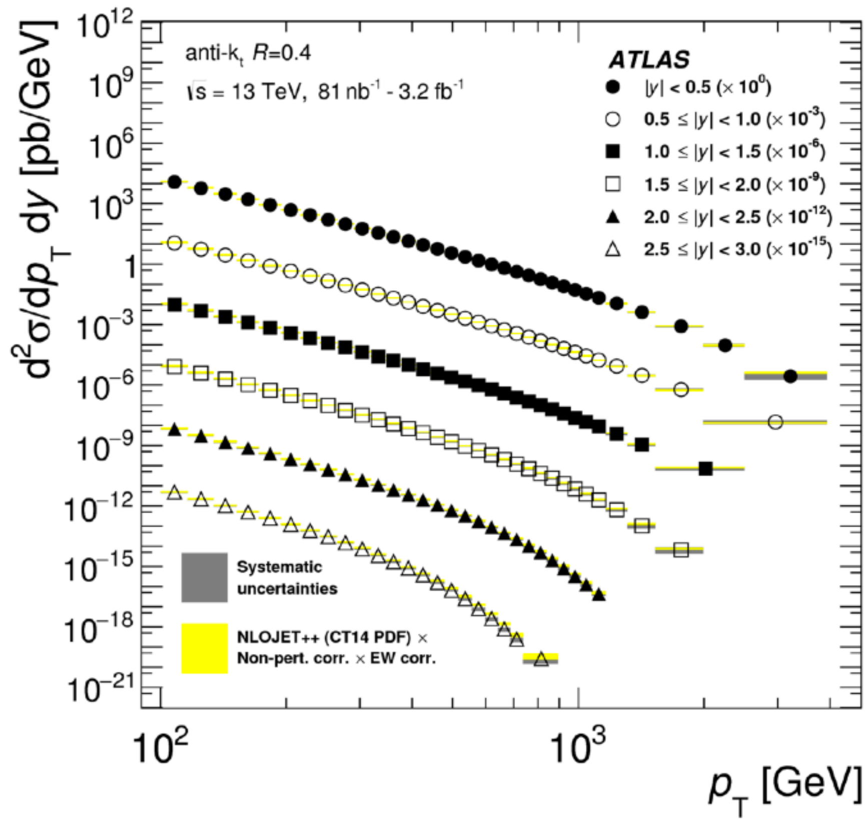
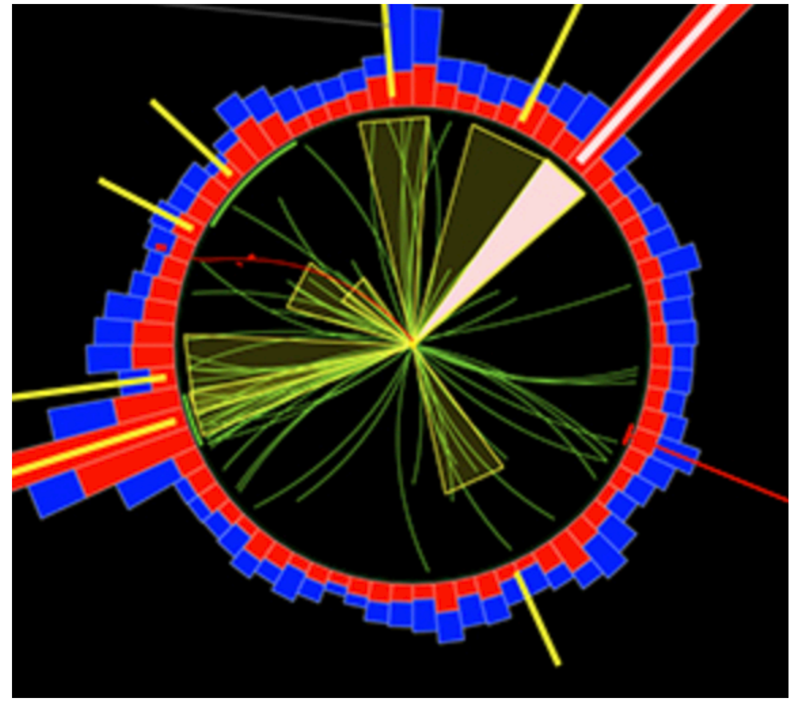
Plan of the talk

- Why Energy-Energy Correlation (EEC) is special
- The analytical calculation of EEC at the NLO in QCD
- Some interesting aspects of the results

Motivation





- High precision jet data over large energy range
- Probe QCD dynamics from weak to strong coupling

For a recent review **Larkoski, Moult, Nachmann, 2017**

Can we have analytical understanding of structure of jet?

Not just at LL etc., but for the whole perturbative spectrum

Easier to study this problem in e+e- collider first

Energy-Energy Correlation (EEC)

- Energy correlation of two calorimeter detector with angle χ, and sum over orientation

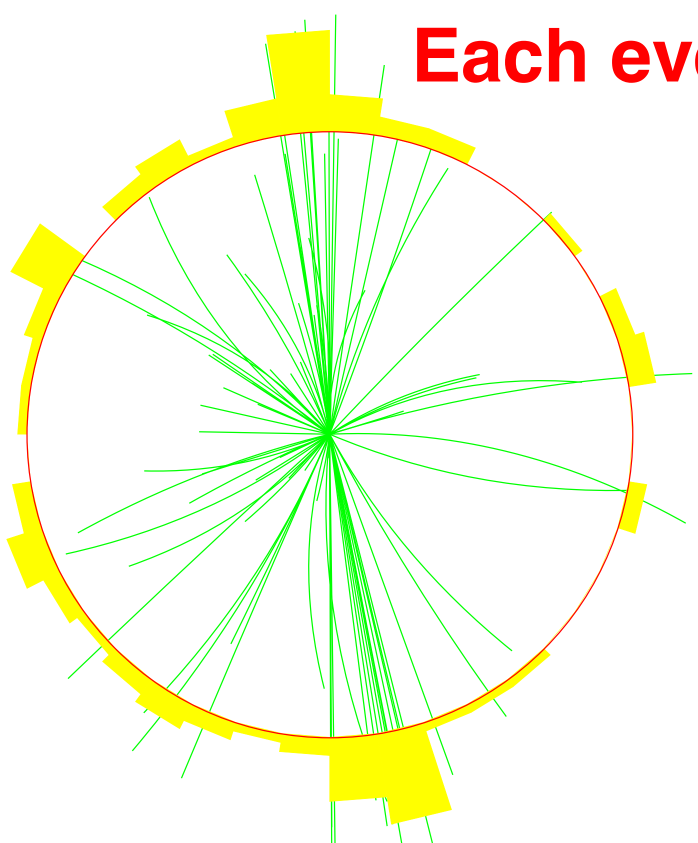
Basham, Brown, Ellis, Love, 1978

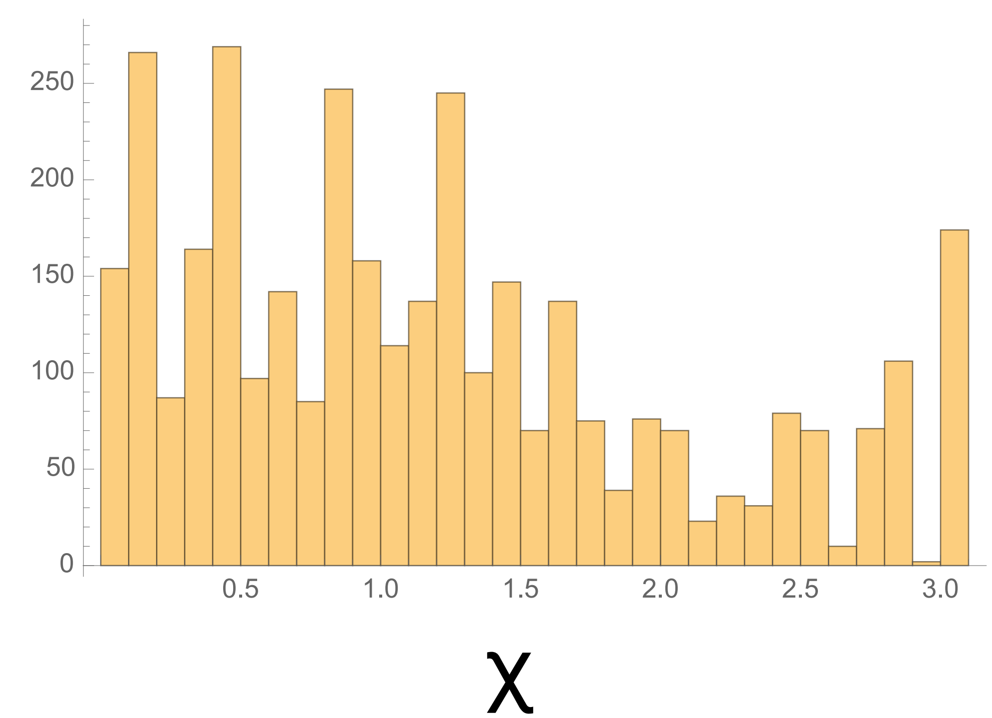
$$\frac{1}{\sigma}\frac{d\Sigma_{\mathrm{EEC}}(\chi)}{d\cos\chi} = \frac{1}{\Delta\chi N_{\mathrm{events}}}\sum_{N_{\mathrm{events}}}\sum_{ij}\frac{E_i E_j}{E^2}$$

$$\frac{d\Sigma}{d\cos\chi} = \sum_{i,j}\int\frac{E_i E_j}{Q^2}\delta(\vec{n}_i\cdot\vec{n}_j - \cos\chi)d\sigma$$

$$z = \frac{1-\cos\chi}{2}$$

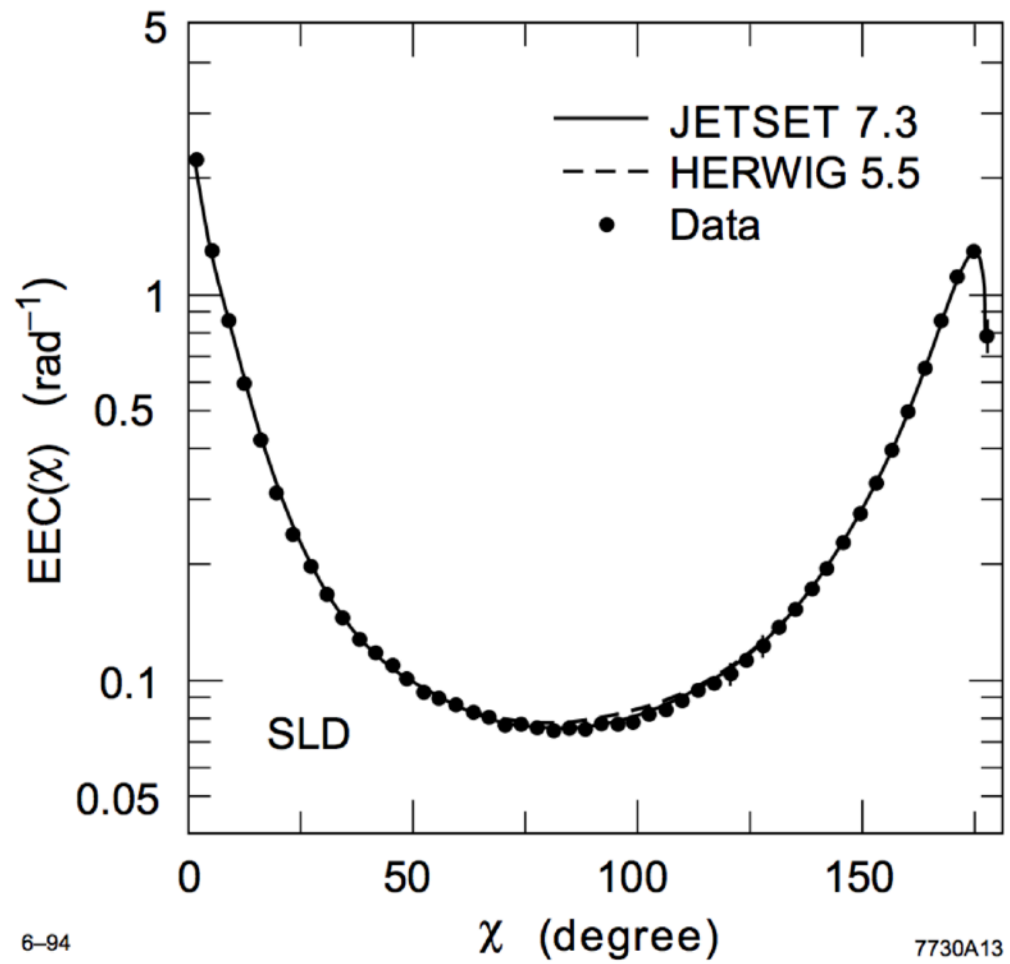
Each event gives a distribution!

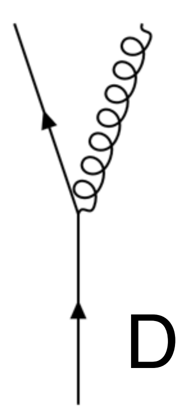


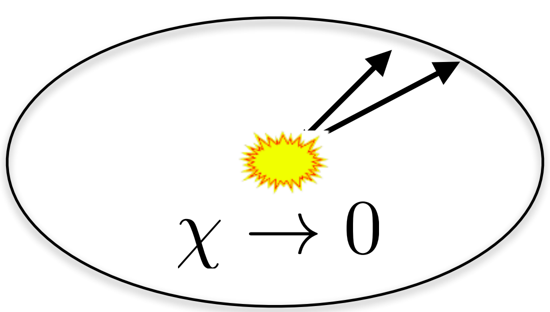


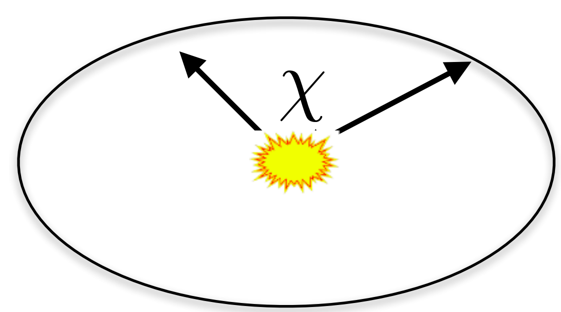
An interesting distribution with two ends

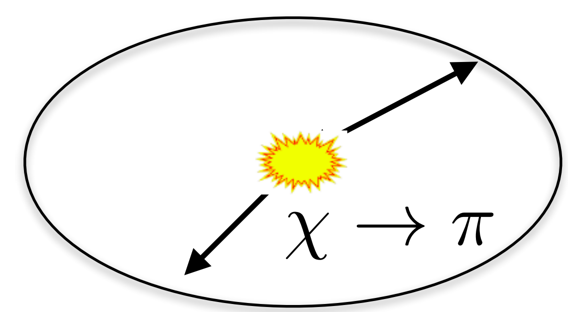
- Only the EEC distribution for an ensemble can be perturbatively computed











Collinear limit
DGLAP evolution

Rigid math structure
DGLAP meet Sudakov

back-to-back
Sudakov double log

Some history for the Numerical calculation of EEC

$$\int \sin^{2+m}\chi \cos^n \chi \, d\Sigma(\chi)\, d\cos\chi =$$

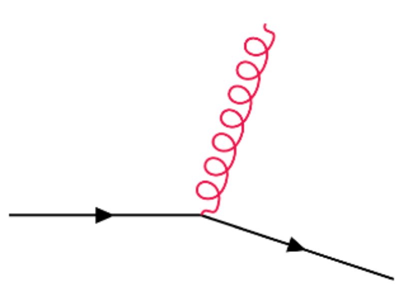
$$+\frac{\alpha_s}{2\pi} A^{(m,n)} + C_F \left(C_A B^{(m,n)}_{C_A} + C_F B^{(m,n)}_{C_F} + T_F N_F B^{(m,n)}_{T_F} \right) + \mathcal{O}(\alpha_s^3)$$

Comparison of different computations of the $B^{(m,n)}_{C_A}$ coefficients

m	n	N	G	S	C
0	0	50.82 ± 0.05	50.54 ± 0.03	50.72 ± 0.02	46.4 ± 0.2
1	0	35.76 ± 0.04	35.53 ± 0.02	35.64 ± 0.02	32.09 ± 0.06
2	0	28.94 ± 0.03	28.75 ± 0.02	28.82 ± 0.02	25.73 ± 0.04
3	0	24.92 ± 0.03	24.75 ± 0.02	24.80 ± 0.02	22.03 ± 0.04
4	0	22.20 ± 0.03	22.05 ± 0.02	22.09 ± 0.02	19.54 ± 0.04
5	0	20.21 ± 0.03	20.07 ± 0.02	20.10 ± 0.02	17.74 ± 0.03
0	1	-6.468 ± 0.006	-6.50 ± 0.01	-6.455 ± 0.005	-6.0 ± 0.15
1	1	-2.356 ± 0.004	-2.365 ± 0.009	-2.344 ± 0.003	-2.15 ± 0.03
2	1	-1.189 ± 0.003	-1.194 ± 0.008	-1.177 ± 0.003	-1.06 ± 0.02
3	1	-0.714 ± 0.003	-0.718 ± 0.007	-0.702 ± 0.003	-0.62 ± 0.01
4	1	-0.478 ± 0.003	-0.479 ± 0.007	-0.466 ± 0.003	-0.41 ± 0.01
5	1	-0.344 ± 0.003	-0.344 ± 0.006	-0.331 ± 0.003	-0.28 ± 0.01

- **C—Clay, Ellis, 95**
- **S—Catani, Seymour, 96**
- **G—Glover, Sutton, 95**
- **N—Kunszt, Nason, Marchesini, Webber, 89**

Summarize in “QCD” by **Nason et al., 1996**



Large angle soft radiation contribute to full spectrum, not just the end point

NNLO results now available **Del Duca, Duhr, Kardos, Somogyi, and Trocsanyi, 2016; Tulipant, Kardos, Somogyi, 2017**

Analytical event shape

- Very few analytical fixed-order predictions

Observable	Full analytic result at LO	Full analytic result at NLO
C-parameter	No[1]	No
Thrust	Yes [De Rujula et al., 1978]	No
Heavy jet mass	Yes[2]	No
EEC	Yes [Basham et al., 1978]	Yes [THIS WORK]

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(3)}}{\mathrm{d}C} = \frac{\alpha_s}{2\pi} C_F \int_{x_2^-(C)}^{x_2^+(C)} \mathrm{d}x$$

Integral representation for C parameter@LO

$$\times \frac{6x\left[C\left(x^3+(x-2)^2\right)-6(1-x)(1+x^2)\right]}{C(C+6)^2(x-6/(C+6))\sqrt{(6/(C+6)-x)(x_2^+-x)(x-x_2^-)x}}$$

Ellis, Ross, Terrano, 1981

EEC as integral of four-point correlation function

$$\Sigma(\chi) = \sigma^{-1} \int d\Omega_{\boldsymbol{n}_1} d\Omega_{\boldsymbol{n}_2} \delta(\boldsymbol{n}_1 \cdot \boldsymbol{n}_2 - \cos\chi)$$
$$\times \sum_X \int d^4 x e^{iQx} \langle 0|O^\dagger(x)\mathcal{E}(\boldsymbol{n}_1)\mathcal{E}(\boldsymbol{n}_2)|X\rangle\langle X|O(0)|0\rangle$$

Korchemsky,Sterman; Belitsky,Korchemsky,Sterman; Hofman,Maldacena

- O(x): operator that create QCD radiation. e.g. e+e- to jets $O^\mu(x) = \bar{\psi}\gamma^\mu\psi(x)$
- **Energy flow operator** **Sveshnikov,Tkachov; Korchemsky,Oderda,Sterman; Bauer, Fleming, Lee, Sterman**

$$\mathcal{E}(\boldsymbol{n})|X\rangle = \sum_a E_a \delta^{(2)}(\Omega_{\boldsymbol{p}_a} - \Omega_{\boldsymbol{n}})|X\rangle \qquad \mathcal{E}(\boldsymbol{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\boldsymbol{n})$$

- EEC now expressed as four-point **wightman correlation**
- Known to high orders in N=4 SYM in **Euclidean region**
- Tour de force calculation for EEC in N=4 SYM from four-point correlation
  - Compute the Mellin amplitude, for which analytic continuation is easier
  - Then Inverse the Mellin transformation to momentum space

Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 2013

EEC@NLO in N=4 SYM

$$\Sigma_{\mathcal{N}=4}(z) = \frac{1}{4z^2(1-z)}\left(aF_1(z) + a^2[(1-z)F_2(z) + F_3(z)]\right), \quad a = \frac{g^2_{\rm YM}N}{4\pi^2}$$

$$z = \frac{1-\cos\chi}{2}$$

$$F_1(z) = -\ln(1-z),$$

$$F_2(z) = 4\sqrt{z}\left[\mathrm{Li}_2(-\sqrt{z}) - \mathrm{Li}_2(\sqrt{z}) + \frac{\ln z}{2}\ln\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\right] + (1+z)\left[2\mathrm{Li}_2(z) + \ln^2(1-z)\right] + 2\ln(1-z)\ln\left(\frac{z}{1-z}\right) + z\frac{\pi^2}{3},$$

$$F_3(z) = \frac{1}{4}\left\{(1-z)(1+2z)\left[\ln^2\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\ln\left(\frac{1-z}{z}\right) - 8\mathrm{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}-1}\right) - 8\mathrm{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}+1}\right)\right] - 4(z-4)\mathrm{Li}_3(z) + 6(3+3z-4z^2)\mathrm{Li}_3\left(\frac{z}{z-1}\right) - 2z(1+4z)\zeta_3 + 2\left[2(2z^2-z-2)\ln(1-z) + (3-4z)z\ln z\right]\mathrm{Li}_2(z) + \frac{1}{3}\ln^2(1-z)\left[4(3z^2-2z-1)\ln(1-z) + 3(3-4z)z\ln z\right] + \frac{\pi^2}{3}\left[2z^2\ln z - (2z^2+z-2)\ln(1-z)\right]\right\}.$$

- Simple and compact results
- Only polylogarithms, no elliptic function
- Symmetry in √ z to -√ z
- Uniform transcendentality violated

Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 2013

- Simplicity of EEC in N=4 SYM strongly encourage calculation in QCD

Plan of the talk

- Why Energy-Energy Correlation (EEC) is special
- The analytical calculation of EEC at the NLO in QCD
- Some interesting aspects of the results

Can we compute EEC in QCD?

- The Mellin space approach not realistic for QCD
- Four-point correlation and the Mellin amplitudes not known in QCD
- Doing the inverse Mellin integral is too difficult
- We pursue a more canonical approach: calculating the scattering amplitudes, and do the energy weighted phase space integrals

NLO virtual corrections

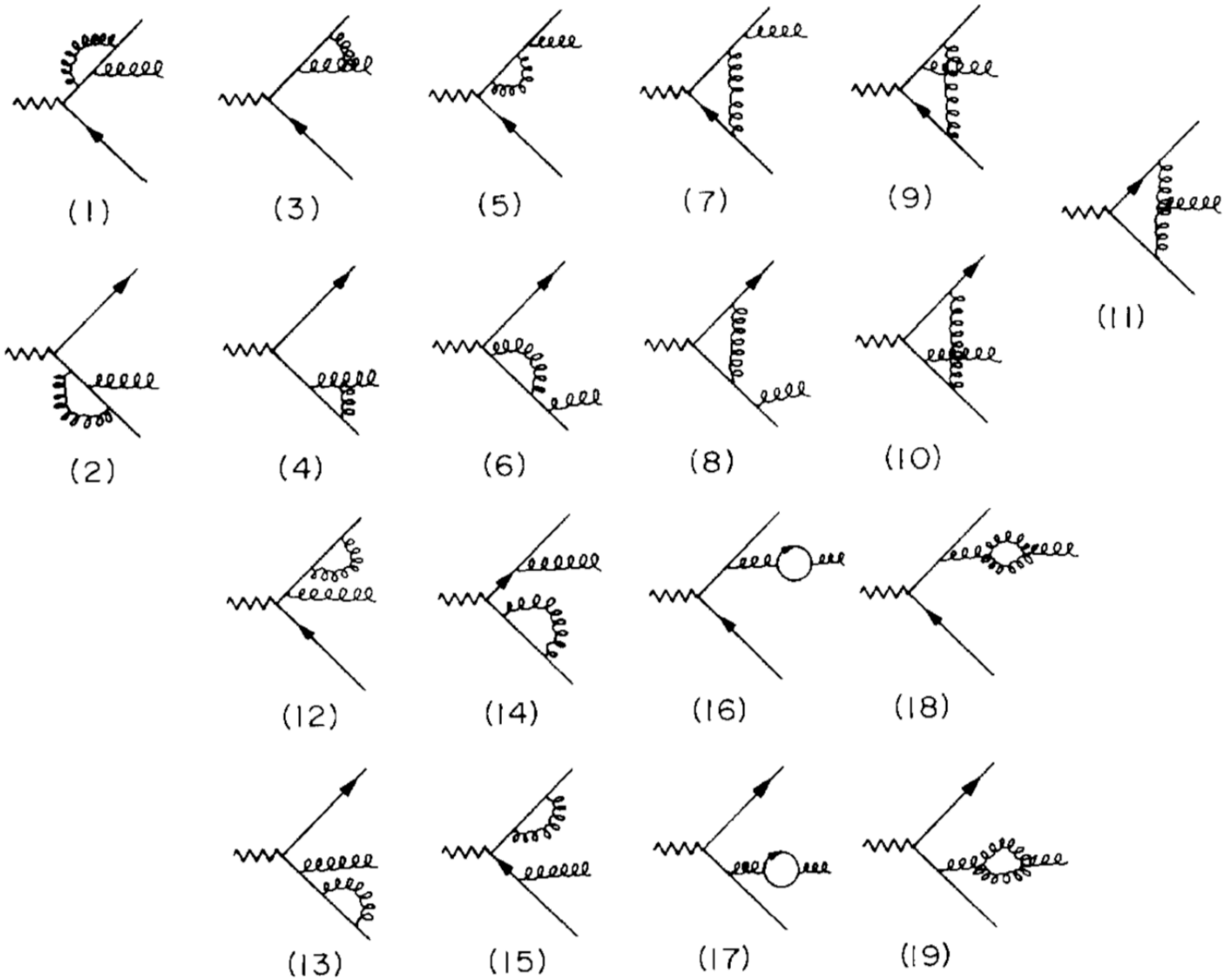


Figure from Ellis, Ross, Terrano, 1981

- One-loop amplitudes known in compact form
- Can be integrated directly in 4 dimension. No phase space singularity

Real corrections

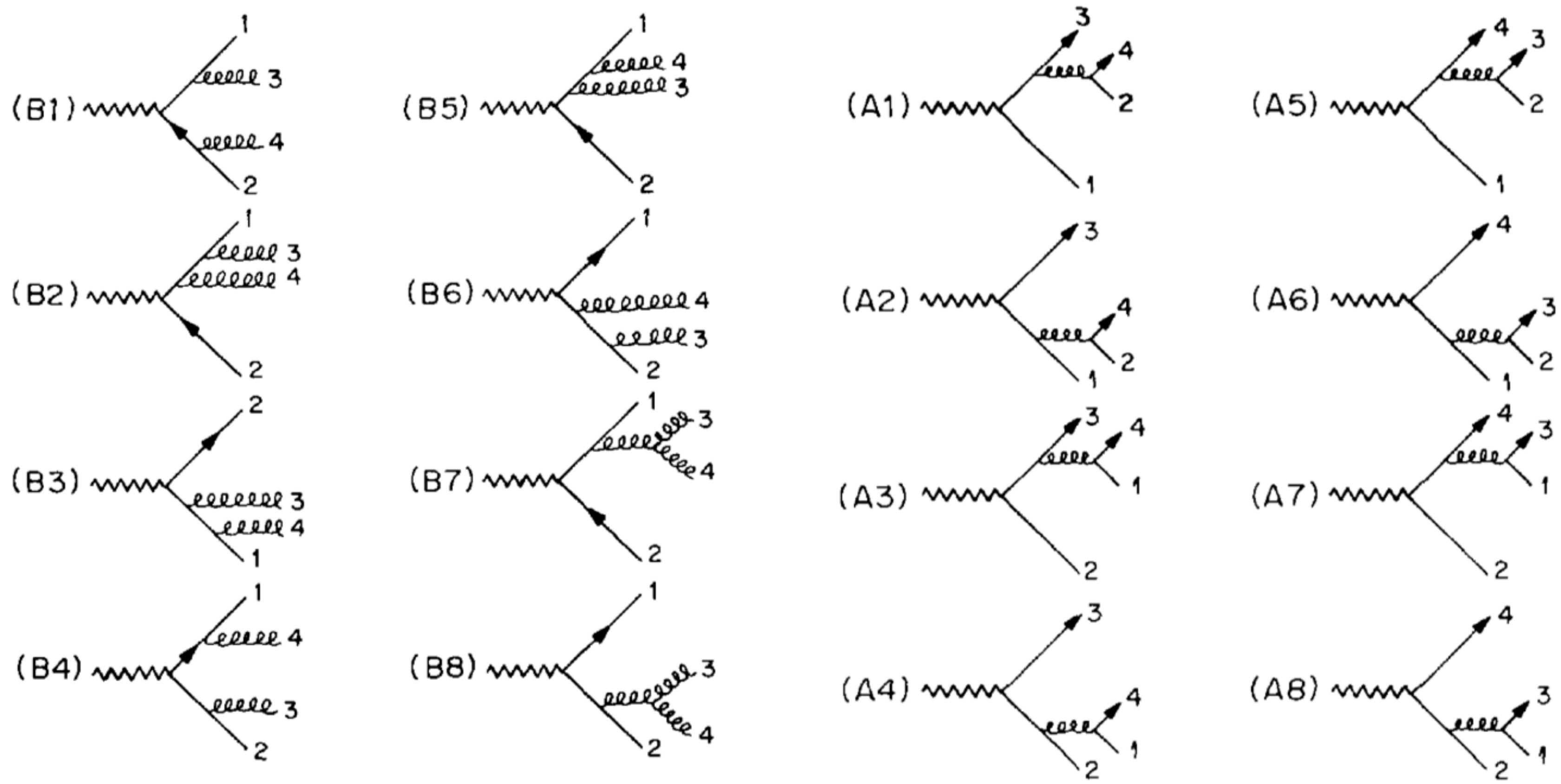


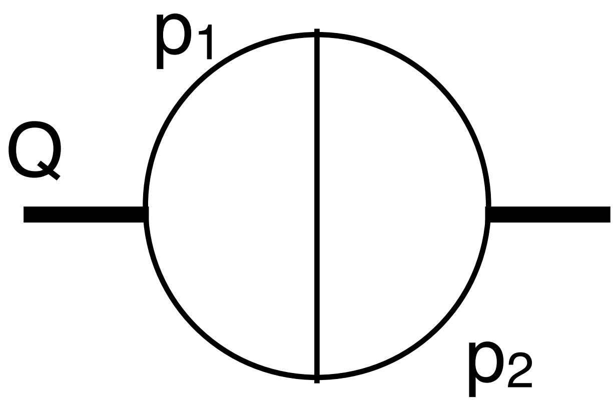
Figure from Ellis, Ross, Terrano, 1981

$$\delta_+(p^2) \to \left[\frac{1}{p^2}\right]_{\rm cut}$$

- Standard techniques:
  - Reverse unitarity **[Anastasiou, Melnikov]**
  - IBP equations **[Chetyrkin, Tkachov; Tkachov]**
  - Differential equations **[Kotikov; Gehrmann, Remiddi; Henn; …]**
- Would be almost trivial if not for the quartic measurement function

$$\delta(\cos\chi - \cos\theta_{12}) = p_1 \cdot Q p_2 \cdot Q \delta((1-\cos\chi) p_1 \cdot Q p_2 \cdot Q - Q^2 p_1 \cdot p_2)$$

IBP equation: One-loop example



Measuring the energy correlation of p_1 and p_2

$$\delta(\cos\chi - \cos\theta_{12}) = p_1 \cdot Q p_2 \cdot Q \delta((1-\cos\chi) p_1 \cdot Q p_2 \cdot Q - Q^2 p_1 \cdot p_2)$$

$$I(\{a_i\}) = \int \frac{d^d p_1 d^d p_2}{[p_1^2]_c^{a_1} [p_2^2]_c^{a_2} [(Q-p_1-p_2)^2]_c^{a_3} [(Q-p_1)^2]^{a_4} [(Q-p_2)^2]^{a_5} [2z(p_1\cdot Q)(p_2 \cdot Q) - Q^2(p_1\cdot p_2)]_c^{a_6}}$$

IBP equation from $p_2^\mu \dfrac{\partial}{\partial p_1^\mu}$

$$-\frac{1}{2}\mathbf{6}_+ \mathbf{2}_-^2 a_6 z - \mathbf{6}_+\mathbf{2}_- a_6 z + \mathbf{5}_-\mathbf{6}_+\mathbf{2}_- a_6 z - \frac{1}{2}\mathbf{5}_-^2 \mathbf{6}_+ a_6 z - \frac{1}{2}\mathbf{6}_+ a_6 z + \mathbf{5}_-\mathbf{6}_+ a_6 z - \mathbf{3}_+\mathbf{2}_- a_3$$
$$+\mathbf{4}_+\mathbf{2}_- a_4 + \mathbf{6}_+\mathbf{2}_- a_6 - \mathbf{3}_-\mathbf{1}_+ a_1 + \mathbf{4}_-\mathbf{1}_+ a_1 + \mathbf{5}_-\mathbf{1}_+ a_1 - \mathbf{1}_+ a_1 + \mathbf{4}_-\mathbf{3}_+ a_3 - a_3 - \mathbf{3}_-\mathbf{4}_+ a_4 + a_4 = 0$$

To terminate the IBP reduction, crucial to have another equation, which is nothing but that the measurement function is nonlinear combination of ordinary propagator

$$\int \frac{d^d p_1 d^d p_2 (2z(p_1\cdot Q)(p_2\cdot Q) - Q^2(p_1\cdot p_2))}{[p_1^2]_c^{a_1} [p_2^2]_c^{a_2} [(Q-p_1-p_2)^2]_c^{a_3} [(Q-p_1)^2]_c^{a_4} [(Q-p_2)^2]_c^{a_5} [2z(p_1\cdot Q)(p_2 \cdot Q) - Q^2(p_1\cdot p_2)]_c^{a_6}}$$
$$= I(a_1, a_2, a_3, a_4, a_5, a_6 - 1)$$

$$-(\mathbf{1}_- - \mathbf{4}_- + 1)(\mathbf{2}_- - \mathbf{5}_- + 1) z + \mathbf{3}_- - \mathbf{4}_- - \mathbf{5}_- + 2\,\mathbf{6}_- + 1 = 0$$

System of differential EQ

- The IBP reduction with FIRE [Smirnov] requires 3 days on a server equipped with Xeon E5-2965 (18 cores) and 128GB RAM
- After reduction we get 40 master integrals

$$d\vec{f}(z,\epsilon) = \epsilon\left[d\sum_k A_k \ln \alpha_k(z)\right]\vec{f}(z,\epsilon)$$

Canonical form **[Henn, 2013]**

- alphabet (hinted by the N=4 results):

$$\{z, 1-z, x, 1-x, y, 1-y, 1+y\}$$

$$x = \sqrt{z}, \quad y = i\frac{\sqrt{z}}{\sqrt{1-z}}$$

- Once the correct alphabet is identified, convert to canonical form via automatic tool **[Gituliar, Magerya, 2017]**

Boundary constant I

- Scaling behavior in the collinear limit $z \to 0$
- An LO example, pure phase space

$$I = \int d^d p_1 d^d p_2 \delta_+(p_1^2)\delta_+(p_2^2)\delta_+((Q-p_1-p_2)^2)\delta(2z(p_1\cdot Q)(p_2\cdot Q) - Q^2(p_1\cdot p_2))$$

$$\propto \frac{1}{z}\left[(-\ln(1-z)+c_1) + \epsilon\left(-4\mathrm{Li}_2(1-z) + \ln^2(1-z) - 3\ln z\ln(1-z) + c_2\right) + \mathcal{O}(\epsilon^2)\right]$$

- In the $p_1 // p_2$ limit, $p_1 \cdot p_2 \sim \lambda^2 \ll 1$

$$\lim_{\lambda\to 0} I \sim \int ds_{12} ds_{13} ds_{23} \delta(Q^2 - s_{12} - s_{13} - s_{23})\delta(2z(p_1\cdot Q)(p_2\cdot Q) - Q^2 s_{12}/2) \sim \mathcal{O}(1)$$

- Therefore the 1/z pole in I must be spurious ⇒

$$c_1 = 0 \qquad c_2 = 4\zeta_2$$

- Can be systematically generalized beyond LO. Only collinear and hard modes involved

Boundary constant II

- For most of the MIs, boundary constant determined by integrating z and compare with inclusive integral

$$F_{ij}(z,\epsilon) = \int dPS^{(4)} I(\{p\})\delta(2z(p_i\cdot Q)(p_j\cdot Q) - p_i\cdot p_j)$$

In general F could have poles at z=0 and z=1 $F(z,\epsilon) \sim \frac{1}{z^n}\frac{1}{(1-z)^m}$

$$\int dz z^n(1-z)^m F(z,\epsilon) = \int dPS^{(4)} \frac{I(\{p\})}{2(p_i\cdot Q)(p_j\cdot Q)}\left(\frac{p_i\cdot p_j}{2(p_i\cdot Q)(p_j\cdot Q)}\right)^n\left(1-\frac{p_i\cdot p_j}{2(p_i\cdot Q)(p_j\cdot Q)}\right)^m$$

eliminate the poles

Can be reduced to inclusive 4-body phase integral

[Gehrmann-De Ridder, Gehrmann, Heinrich, 2003]

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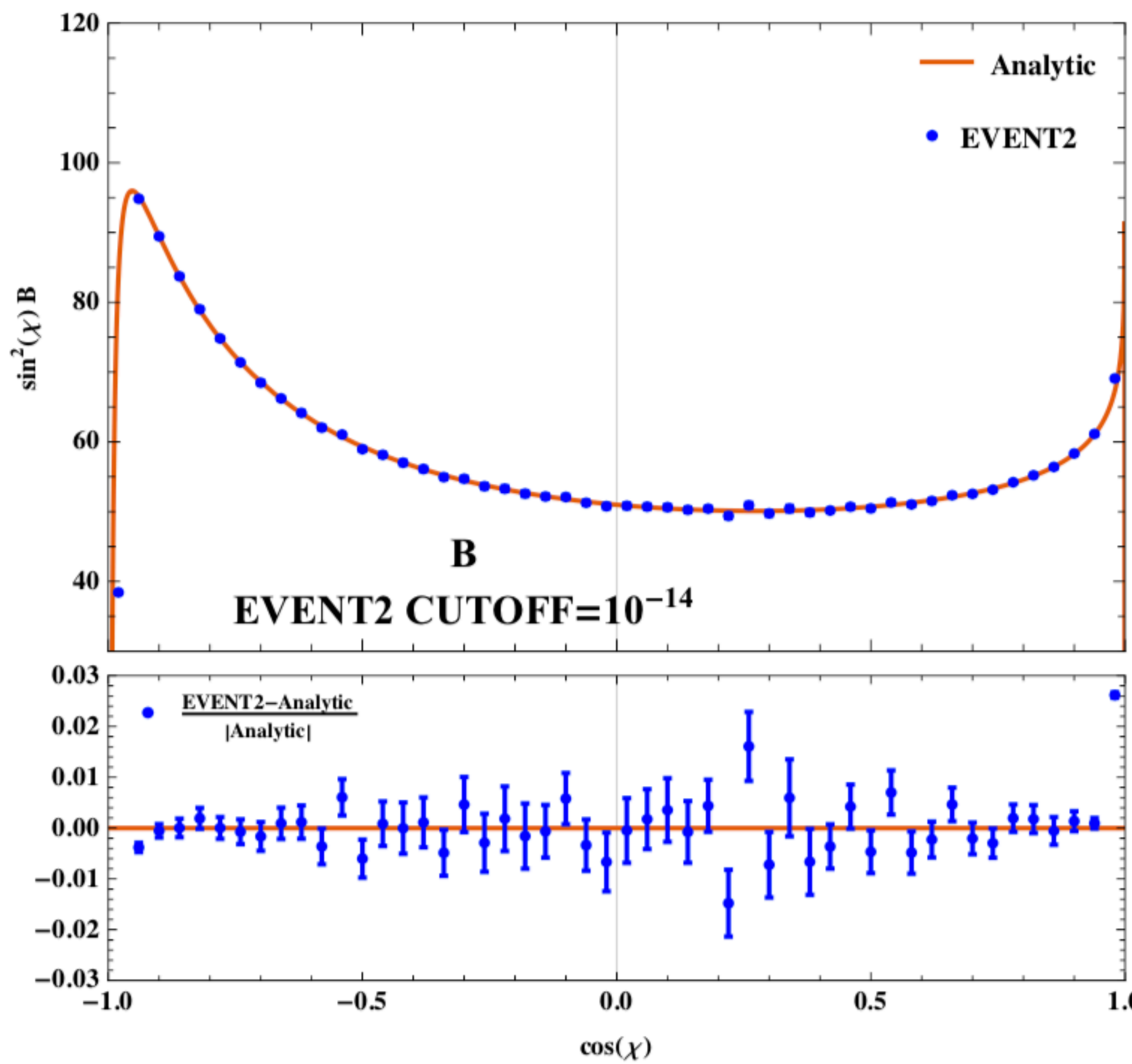
- The analytical calculation of EEC at the NLO in QCD
- Some interesting aspects of the results

Check of the calculation

- After UV renormalized and adding up virtual and real, all IR singularities cancel
- Reproduce the N=4 EEC
- Perfect agreement with EVENT2 **[Catani, Seymour]**



EEC in QCD at NLO

$$\frac{1}{\sigma_0}\frac{d\Sigma}{d\cos\chi}=\frac{\alpha_s(\mu)}{2\pi}A(z)+\left(\frac{\alpha_s(\mu)}{2\pi}\right)^2\left(\beta_0\log\frac{\mu}{Q}A(z)+B(z)\right)+\mathcal{O}(\alpha_s^3)$$

- Color decomposition

$$B=C_F^2B_{\rm lc}+C_F(C_A-2C_F)B_{\rm nlc}+C_FN_fT_fB_{N_f}$$

$$g_1^{(1)}=\log(1-z)\,,\qquad g_2^{(1)}=\log(z)\,,\qquad g_1^{(2)}=2(\mathrm{Li}_2(z)+\zeta_2)+\log^2(1-z)\,,$$

$$g_2^{(2)}=\mathrm{Li}_2(1-z)-\mathrm{Li}_2(z)\,,$$

$$g_3^{(2)}=-2\,\mathrm{Li}_2\left(-\sqrt{z}\right)+2\,\mathrm{Li}_2\left(\sqrt{z}\right)+\log\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)\log(z)\,,\qquad g_4^{(2)}=\zeta_2\,,$$

$$g_1^{(3)}=-6\left[\mathrm{Li}_3\left(-\frac{z}{1-z}\right)-\zeta_3\right]-\log\left(\frac{z}{1-z}\right)\left(2(\mathrm{Li}_2(z)+\zeta_2)+\log^2(1-z)\right)\,,$$

$$g_2^{(3)}=-12\left[\mathrm{Li}_3(z)+\mathrm{Li}_3\left(-\frac{z}{1-z}\right)\right]+6\,\mathrm{Li}_2(z)\log(1-z)+\log^3(1-z)\,,$$

$$g_3^{(3)}=6\log(1-z)\left(\mathrm{Li}_2(z)-\zeta_2\right)-12\,\mathrm{Li}_3(z)+\log^3(1-z)\,,$$

$$g_4^{(3)}=\mathrm{Li}_3\left(-\frac{z}{1-z}\right)-3\,\zeta_2\log(z)+8\,\zeta_3\,,$$

$$g_5^{(3)}=-8\left[\mathrm{Li}_3\left(-\frac{\sqrt{z}}{1-\sqrt{z}}\right)+\mathrm{Li}_3\left(\frac{\sqrt{z}}{1+\sqrt{z}}\right)\right]+2\mathrm{Li}_3\left(-\frac{z}{1-z}\right)$$
$$+4\zeta_2\log(1-z)+\log\left(\frac{1-z}{z}\right)\log^2\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\,.$$

- Individual real/virtual depends on HPL with imaginary argument, e.g. Bloch-Wigher function

$$\mathrm{Li}_2(ir)-\mathrm{Li}_2(-ir)-\ln r\ln\frac{1+ir}{1-ir}$$

- Cancel out in the physical results

The leading color coefficient

- Leading color coefficient B_{lc}

$$
\begin{aligned}
&+\frac{122400z^7-244800z^6+157060z^5-31000z^4+2064z^3+72305z^2-143577z+63298}{1440(1-z)z^4} \\
&-\frac{-244800z^9+673200z^8-667280z^7+283140z^6-48122z^5+2716z^4-6201z^3+11309z^2-9329z+3007}{720(1-z)z^5}g_1^{(1)} \\
&-\frac{244800z^8-550800z^7+422480z^6-126900z^5+13052z^4-336z^3+17261z^2-38295z+19938}{720(1-z)z^4}g_2^{(1)} \\
&+\frac{4z^7+10z^6-17z^5+25z^4-96z^3+296z^2-211z+87}{24(1-z)z^5}g_1^{(2)} \\
&+\frac{-40800z^8+61200z^7-28480z^6+4040z^5-320z^4-160z^3+1126z^2-4726z+3323}{120z^5}g_2^{(2)} \\
&-\frac{1-11z}{48z^{7/2}}g_3^{(2)}-\frac{120z^6+60z^5+160z^4-2246z^3+8812z^2-10159z+4193}{120(1-z)z^5}g_4^{(2)} \\
&-2\left(85z^4-170z^3+116z^2-31z+3\right)g_1^{(3)}+\frac{-4z^3+18z^2-21z+5}{6(1-z)z^5}g_2^{(3)}+\frac{z^2+1}{12(1-z)}g_3^{(3)}.
\end{aligned}
$$

- Asymptotic of rational coefficients

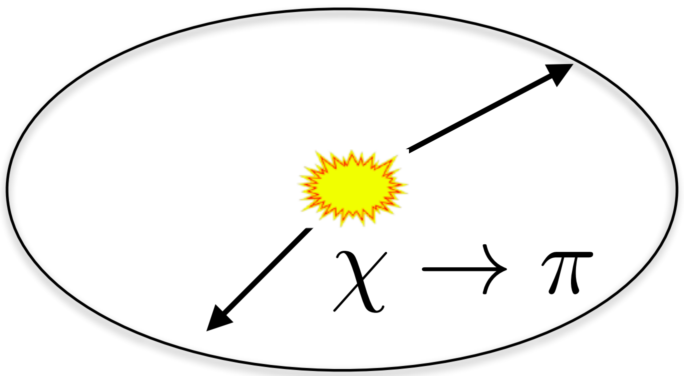
$$
\overset{z\to 0}{\sim}\ \frac{1}{z^5}
$$

$$
\overset{z\to\infty}{\sim}\ z^3
$$

EEC in the back-to-back limit

Leading power

$$
\begin{aligned}
B(z) =& C_F \Bigg\{ \frac{1}{1-z} \Bigg[\frac{1}{2} C_F \log^3(1-z) + \log^2(1-z) \left(\frac{11 C_A}{12} + \frac{9 C_F}{4} - \frac{N_f T_f}{3} \right) \\
&+ \log(1-z) \left(C_A \left(\frac{\zeta_2}{2} - \frac{35}{72} \right) + C_F \left(\zeta_2 + \frac{17}{4} \right) + \frac{N_f T_f}{18} \right) \\
&+ C_A \left(\frac{11\zeta_2}{4} + \frac{3\zeta_3}{2} - \frac{35}{16} \right) + C_F \left(3\zeta_2 - \zeta_3 + \frac{45}{16} \right) + N_f T_f \left(\frac{3}{4} - \zeta_2 \right) \Bigg] \\
&+ \left(\frac{C_A}{2} + C_F \right) \log^3(1-z) + \log^2(1-z) \left(\frac{27 C_A}{8} + \frac{13 C_F}{2} - \frac{N_f T_f}{2} \right) \\
&+ \log(1-z) \left[C_A \left(22\zeta_2 - \frac{2011}{72} \right) + C_F (47 - 19\zeta_2) + N_f T_f \left(\frac{361}{36} - 4\zeta_2 \right) \right] \\
&+ C_A \left(\frac{6347\zeta_2}{80} - 21\zeta_2 \log(2) - \frac{137\zeta_3}{4} - \frac{3305}{72} \right) \\
&+ C_F \left(-\frac{1727\zeta_2}{20} + 42\zeta_2 \log(2) + \frac{121\zeta_3}{2} + \frac{3437}{96} \right) \\
&+ N_f T_f \left(-\frac{1747\zeta_2}{120} + 12\zeta_3 + \frac{2099}{144} \right) \Bigg\} + \mathcal{O}(1-z).
\end{aligned}
$$



Valuable Next-to-Leading Power data, see Ian Moult's talk

- Back-to-back region dominated by soft/collinear emission
- NNLL resummation: **Dokshitzer, Marchesini, Webber, 1998; de Florian, Grazzini, 2002**
- All order factorization in terms of operator matrix element: **Moult, HXZ, 2018**

Collinear limit z ~ 0

- The z -> 0 limit is governed by collinear splitting

$$B(z) = C_F\Bigg\{\frac{1}{z}\Bigg[\log(z)\left(-\frac{107C_A}{120}+\frac{25C_F}{32}+\frac{53N_fT_f}{240}\right)$$
$$+C_A\left(-\frac{25\zeta_2}{12}+\frac{\zeta_3}{2}+\frac{17683}{2700}\right)$$
$$+C_F\left(\frac{43\zeta_2}{12}-\zeta_3-\frac{8263}{1728}\right)-\frac{4913N_fT_f}{3600}\Bigg]+\log(z)\Bigg[C_A\left(\frac{33\zeta_2}{2}-\frac{703439}{25200}\right)$$
$$+C_F\left(\frac{42109}{1200}-21\zeta_2\right)+N_fT_f\left(\frac{86501}{12600}-4\zeta_2\right)\Bigg]$$
$$+C_A\left(\frac{213\zeta_2}{5}-\frac{101\zeta_3}{2}-\frac{26986007}{5292000}\right)+C_F\left(-\frac{1541\zeta_2}{30}+65\zeta_3+\frac{18563}{2700}\right)$$
$$+N_fT_f\left(-\frac{46\zeta_2}{3}+12\zeta_3+\frac{2987627}{330750}\right)\Bigg\}+\mathcal{O}(z).$$

The leading log predicted by jet calculus **[Konish et al., 1978, 79; Richards et al., 1983]**

- **Remarkable cancellation from $1/z^5$ to $1/z$**

z→∞ limit

$$z = \frac{1-\cos\chi}{2}$$

- The z→∞ is unphysical. However, since we have $\Sigma_{\mathrm{EEC}}(z)$ analytically, we can perform analytical continuation

$$A(z) = \frac{C_F}{z^3}\left[2\,\log(-z) - \frac{9}{2}\right] + \mathcal{O}(1/z^4),$$

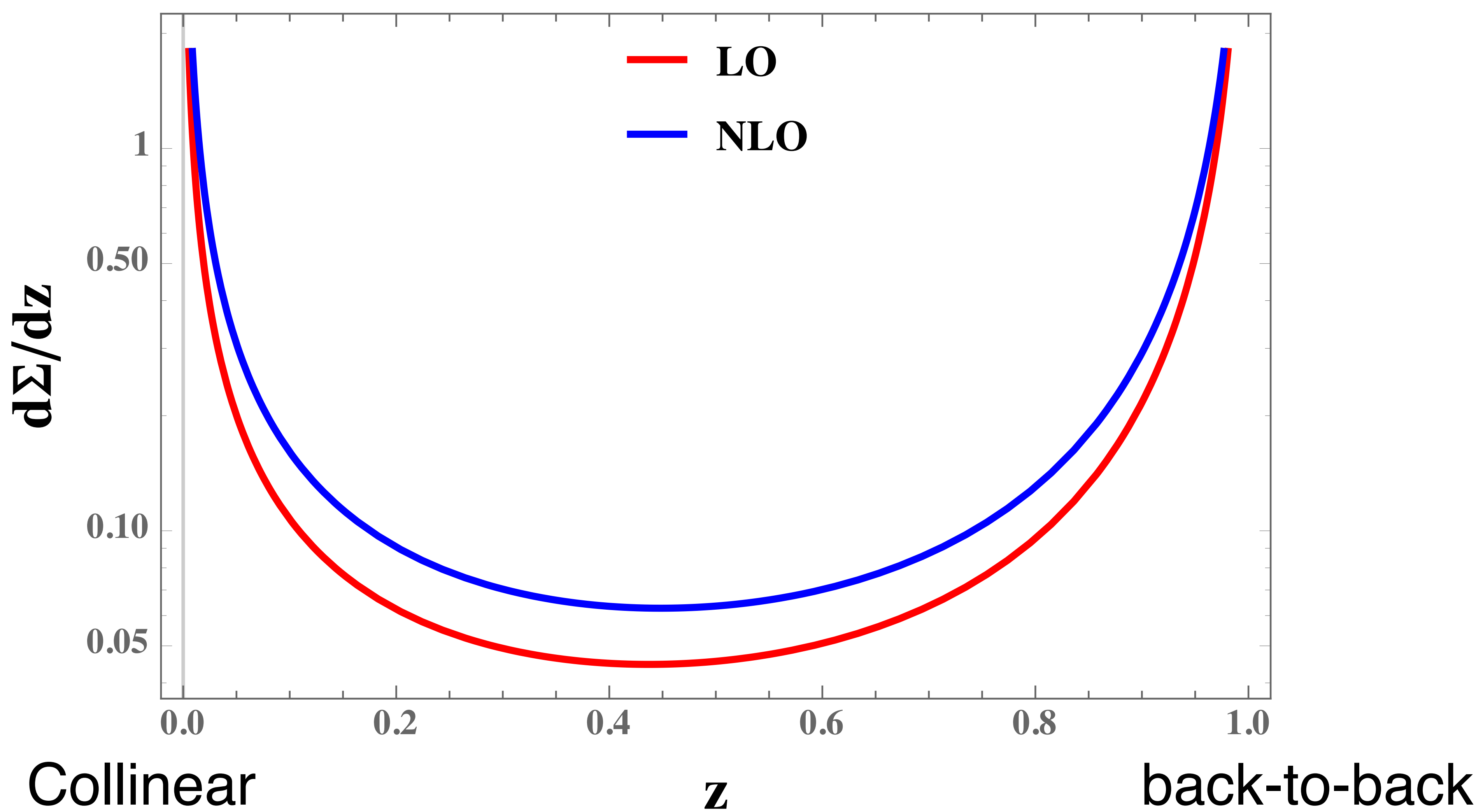
$$B_{\mathrm{lc}}(z) = \frac{1}{z^3}\left[\left(4\,\zeta_2 + \frac{4699}{288}\right)\log(-z) - 8\,\zeta_3 + \frac{991}{84}\,\zeta_2 - \frac{85595}{1728}\right] + \frac{i\,\mathrm{sign}(\mathrm{Im}(z))}{z^3}\left[\frac{11}{8}\,\zeta_2\sqrt{-z} + \pi\left(-\frac{1459}{140}\log(-z) + \frac{466259}{19600}\right)\right] + \mathcal{O}(1/z^{7/2}),$$

$$B_{\mathrm{nlc}}(z) = \frac{1}{z^3}\left[\left(\frac{3}{2}\,\zeta_2 + \frac{473}{72}\right)\log(-z) - \frac{9}{2}\,\zeta_3 + \frac{521}{70}\,\zeta_2 - \frac{32713}{1728}\right] + \frac{i\,\mathrm{sign}(\mathrm{Im}(z))}{z^3}\left[-\frac{2059}{560}\,\zeta_2\sqrt{-z} + \pi\left(-\frac{2407}{420}\log(-z) + \frac{3}{2}\,\zeta_2 + \frac{20518}{1225}\right)\right] + \mathcal{O}(1/z^{7/2}),$$

$$B_{N_f}(z) = \frac{1}{z^3}\left[-\frac{133}{36}\log(-z) - \frac{404}{105}\,\zeta_2 + \frac{51}{4}\right] + \frac{i\,\mathrm{sign}(\mathrm{Im}(z))}{z^3}\left[-\frac{3}{8}\,\zeta_2\sqrt{-z} + \pi\left(\frac{26}{21}\log(-z) - \frac{196003}{88200}\right)\right] + \mathcal{O}(1/z^{7/2}).$$

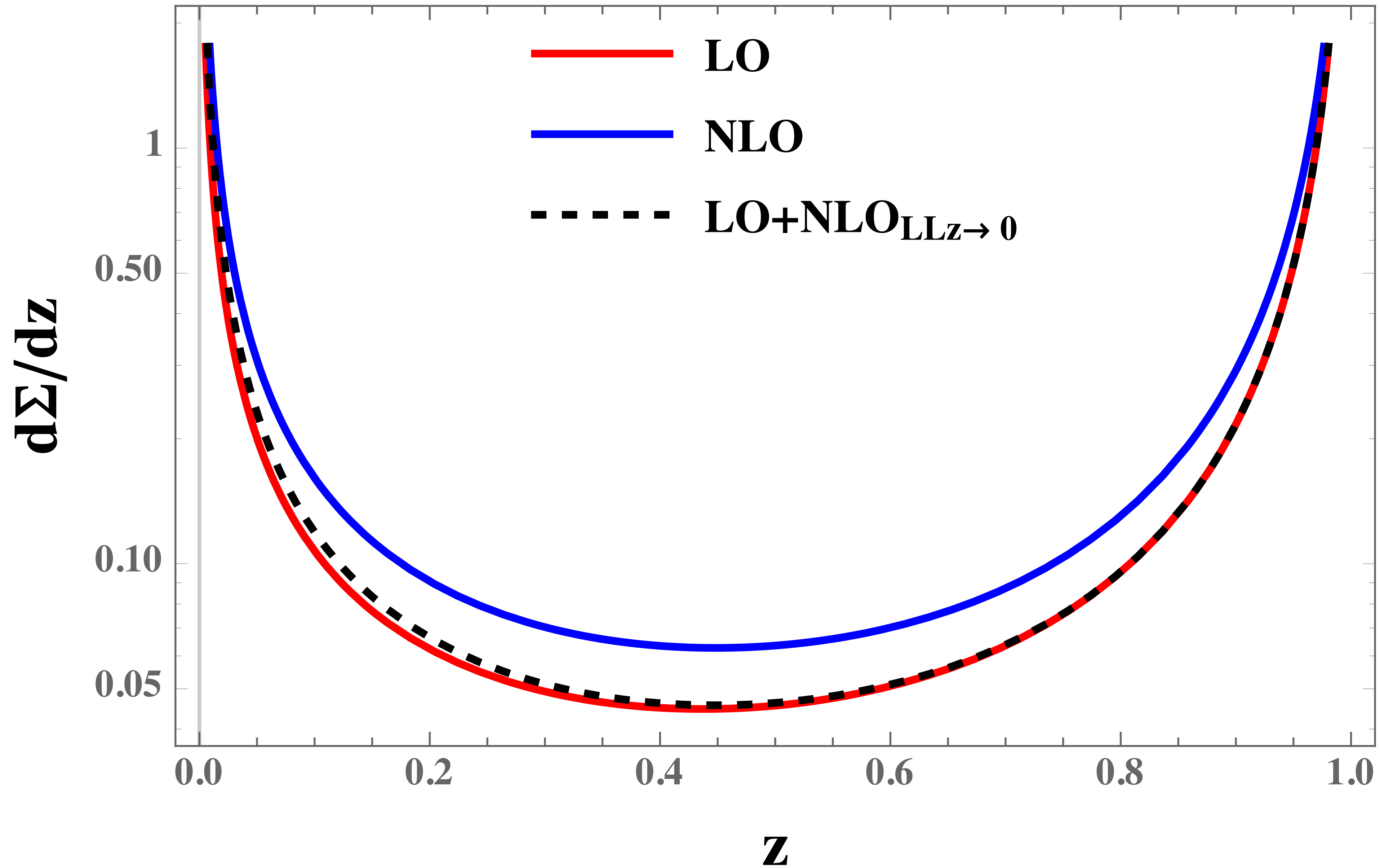
- **Remarkable cancellation from z^3 to $1/z$**
- **z→0 and z→∞ imply very rigid structure for the full perturbative spectrum. Good news for bootstrap**

LO v.s. NLO



- Divergent at both sides, no tail region
- **Important to know how much of the corrections for moderate z comes from the remnant of large logs at z=0 and z=1**

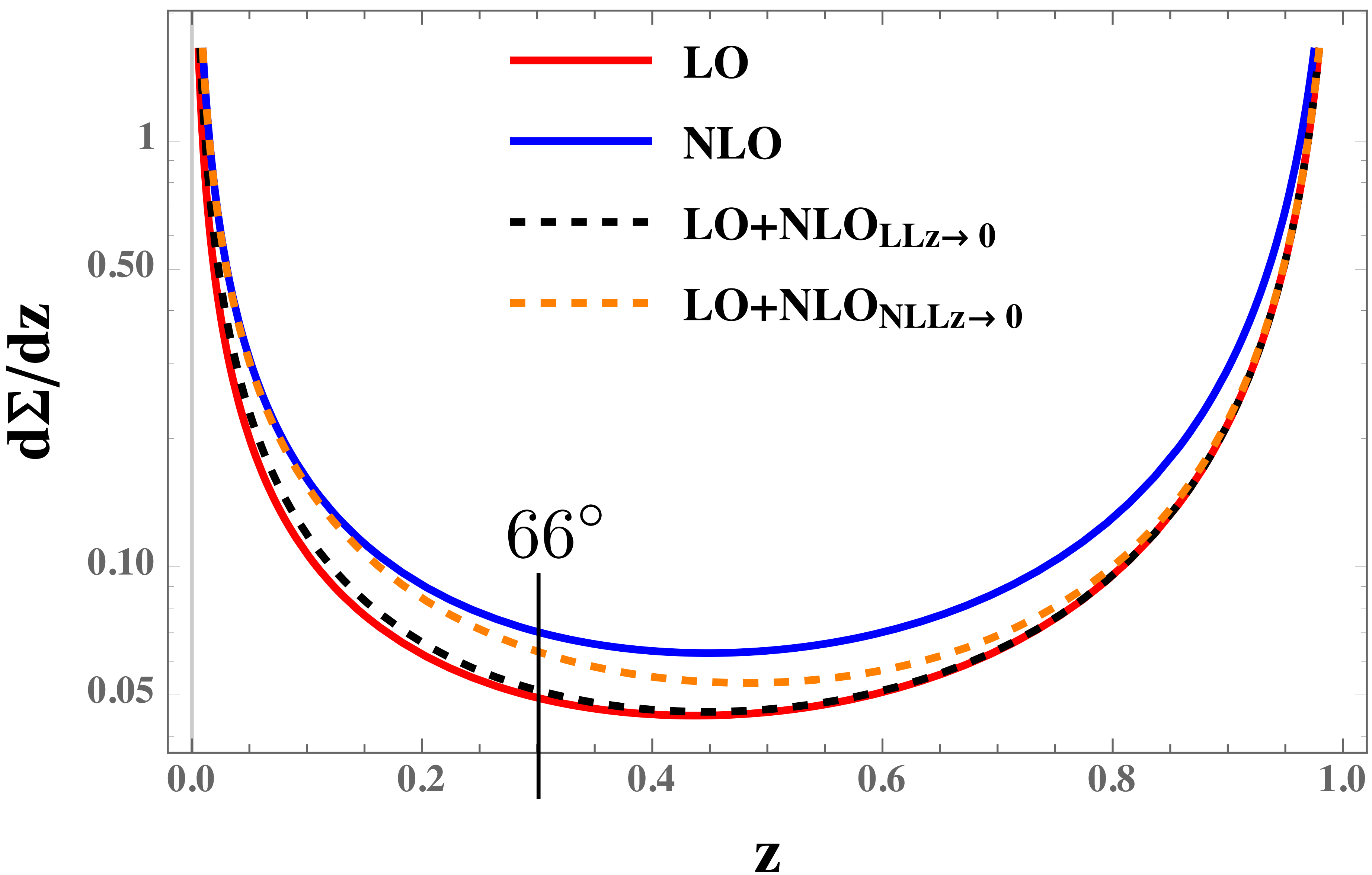
LO + NLO LL approximation



$$\mathrm{NLO}_{\mathrm{LLz0}} = \left(\frac{\alpha_s}{2\pi}\right)^2 \left[-1.44167\frac{\ln z}{z}\right]$$

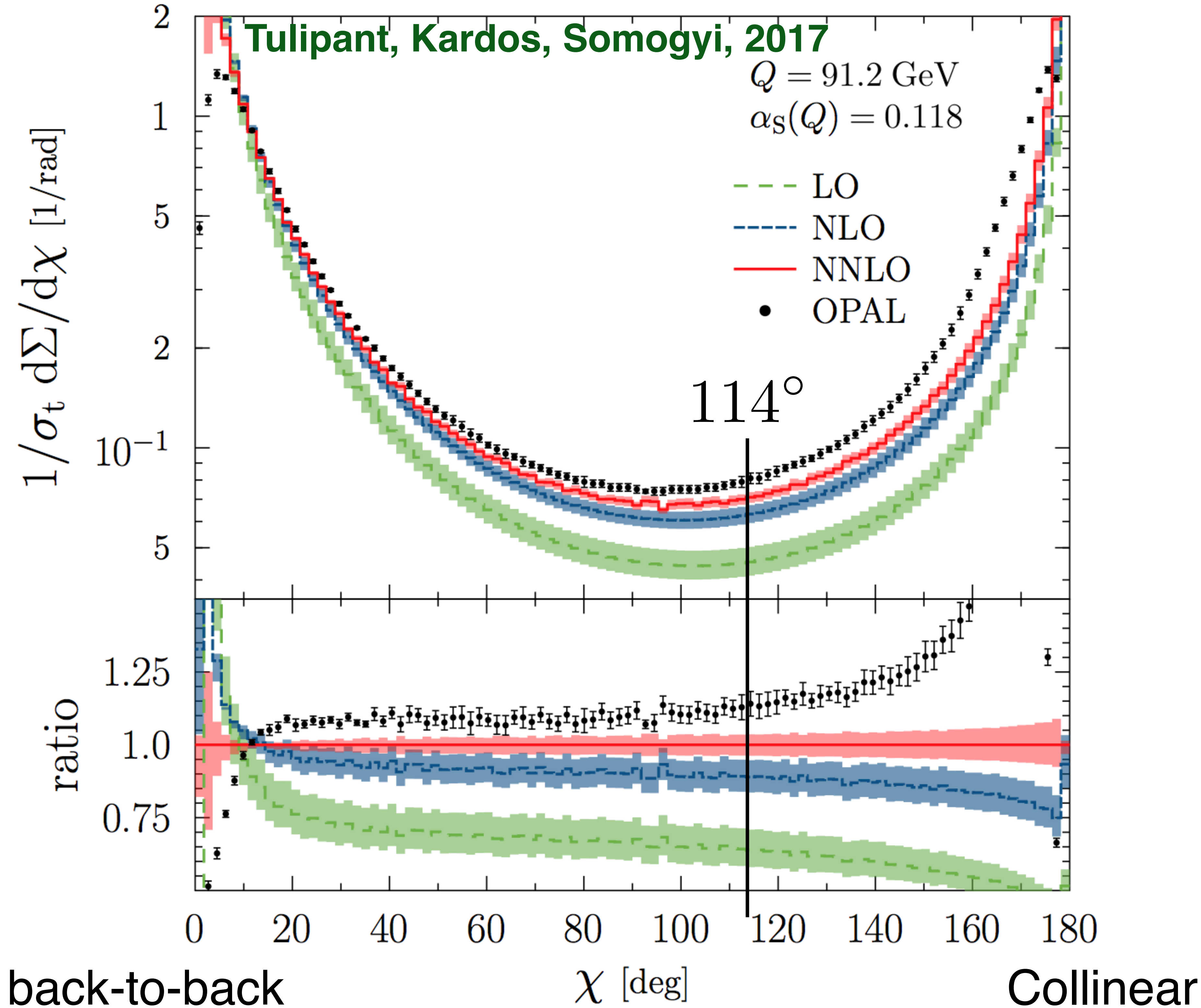
- Adding the Leading log for z=0 at NLO on top of LO: poor approximation

LO + NLO NLL approximation



$$\mathrm{NLO_{LLz0}} = \left(\frac{\alpha_s}{2\pi}\right)^2 \left[-1.44167\frac{\ln z}{z}\right] \qquad \mathrm{NLO_{NLLz0}} = \left(\frac{\alpha_s}{2\pi}\right)^2 \left[-1.44167\frac{\ln z}{z} + \frac{10.1851}{z}\right]$$

- Adding the $\mathrm{NLO_{NLL}}$ term gives much better approximation



- **Will be very interesting to resum log(z) beyond LL**

Conclusion

- First analytical results for event shape in QCD: EEC
- NLO results in terms of classical polylogarithms. Surprising degree of cancellation at $z=0$ and $z=\infty$
- Provide data for resummation at small z ($z=0$) and large z ($z=1$) (power corrections)