NNLO soft function for top pair production at small q_T

Sebastian Sapeta

IFJ PAN Kraków

In collaboration with René Ángeles-Martinez and Michał Czakon

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National Science Centre Poland

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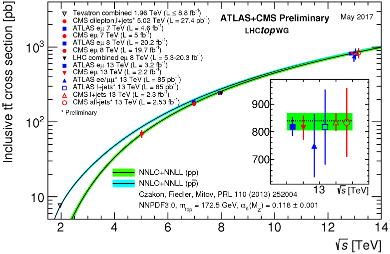
Top pair production: the status of QCD calculations

- A single *complete* NNLO result for total and differential cross section obtained with STRIPPER methodology [Czakon, Fiedler, Mitov '13; Czakon, Heymes, Mitov '16]



- Flavour off-diagonal channels at NNLO from q_T subtraction [Bonciani, Catani, Grazzini, Sargsyan, Torre '15]
- Approximate NNLO [Broggio, Papanastasiou, Signer '14] and N^{3}LO [Kidonakis '14]
- Soft and small-mass resummation at NNLL [Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, Yang '18]
- Small-q_T resummation at NNLL [Li, Li, Shao, Yang, Zhu '13; Catani, Grazzini, Torre '14]

The q_T slicing method

[Catani, Grazzini '07, '15]

$$p + p \to F\,(q_T) + X$$

$$\sigma^{F}_{\mathrm{N}^{m}\mathrm{LO}} = \int_0^{q_{T,\mathrm{cut}}} dq_T\,\frac{d\sigma^{F}_{\mathrm{N}^{m}\mathrm{LO}}}{dq_T} + \int_{q_{T,\mathrm{cut}}}^{\infty} dq_T\,\frac{d\sigma^{F}_{\mathrm{N}^{m}\mathrm{LO}}}{dq_T}$$

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enough to know in
small-q_T approximation

known

Soft Collinear Effective Theory (SCET)

$$\text{SCET} \simeq \text{QCD}\Big|_{\text{IR limit}}$$

- Hard degrees of freedom are integrated out into Wilson coefficients, which are then used to adjust new couplings of the (effective) theory.

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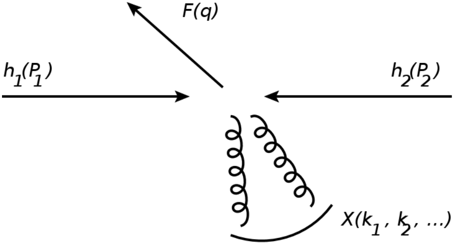
The new fields decouple in the Lagrangian

$$\mathcal{L}_{\mathrm{SCET}} = \mathcal{L}_c + \mathcal{L}_{\bar{c}} + \mathcal{L}_s$$

- The separation of fields in the Lagrangian into collinear, anti-collinear and soft sectors, facilitates proofs of factorization theorems

Small-q_T factorization in SCET



where $F = H, Z, W, ZZ, WW, t\bar{t}, \ldots$

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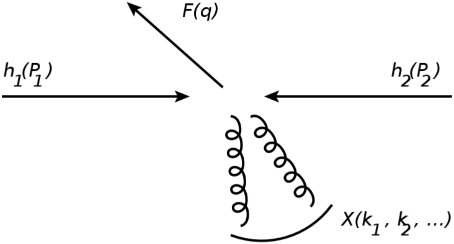

where $F = H, Z, W, ZZ, WW, t\bar{t}, \ldots$

$$\frac{d\sigma^F}{d\Phi} = \mathcal{B}_1 \otimes \mathcal{B}_2 \otimes \mathcal{H} \otimes \mathcal{S} + \mathcal{O}\left(\frac{q_T^2}{q^2}\right)$$

Small-q_T factorization in SCET

Gluons' momenta in light-cone coordinates

$$k_i^\mu = \left(k_i^+, k_i^-, \boldsymbol{k}_i^\perp\right) \qquad \text{where} \qquad k^\pm = k^0 \pm k^3$$

Expansion parameter

$$\lambda = \frac{q_T^2}{q^2} \ll 1$$

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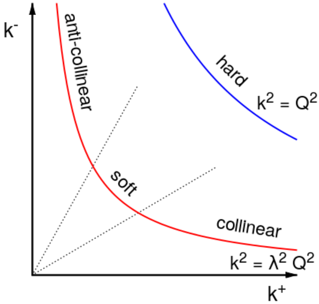
Regions

collinear	$k_i^\mu \sim (1, \lambda^2, \lambda)\, Q^2$	$\mathcal{B}_1$
anti-collinear	$k_i^\mu \sim (\lambda^2, 1, \lambda)\, Q^2$	$\mathcal{B}_2$
hard	$k_i^\mu \sim (1, 1, 1)\, Q^2$	$\mathcal{H}$
soft	$k_i^\mu \sim (\lambda, \lambda, \lambda)\, Q^2$	$\mathcal{S}$



Top pair production at small-q_T through NNLO

$$\frac{d\sigma^{\text{NNLO}}}{dq_T\, dy\, dM\, d\cos\theta} = \sum_{i,\bar{i}} \mathcal{B}_{i/h_1} \otimes \mathcal{B}_{\bar{i}/h_2} \otimes \text{Tr}\left[\mathcal{H}_{i\bar{i}} \otimes \mathcal{S}_{i\bar{i}}\right]$$

where

q_T, y, M : transverse momentum, rapidity, mass of top quark pair

θ : scattering angle of the top quark in $t\bar{t}$ rest frame

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$\mathcal{B}$ - known up to NNLO [Gehrmann, Lübbert, Yang '12, '14]

$\mathcal{H}$ - known up to NNLO [Czakon '08; Baernreuther, Czakon, Fiedler '13]

$\mathcal{S}$ - known up to NLO in small-q_T limit [Li, Li, Shao, Yan, Zhu '13; Catani, Grazzini, Torre '14] (and up to NNLO in the threshold limit [Ferroglia, Pecjak, Yang '12; Wang, Xu, Yang and Zhu '18])

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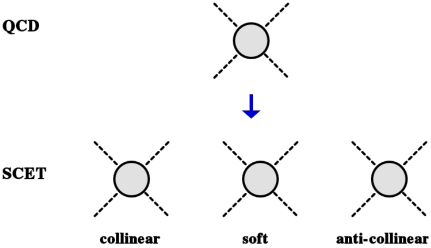
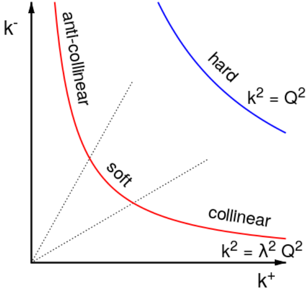
Calculating the missing NNLO correction to the soft function in the small-q_T limit, $\mathcal{S}$, is the aim of this phase of our work.

Rapidity divergences and analytic regulator





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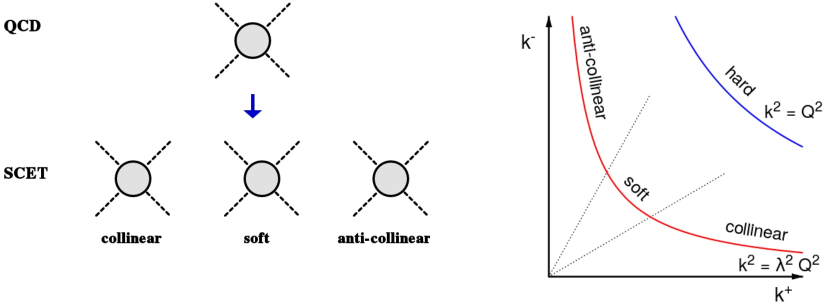

Modification of the measure [Becher, Bell '12]

$$\int d^d k\, \delta^+(k^2) \to \int d^d k \left(\frac{\nu}{k_+}\right)^{\alpha} \delta^+(k^2)$$

- The regulator is necessary at intermediate steps of the calculation.
- Rapidity divergences do not appear in QCD, hence, the complete SCET result has to stay finite in the limit $\alpha \to 0$.

Kinematics and notation

$$
\begin{aligned}
q(p_1) + \bar{q}(p_2) &\;\rightarrow\; t(p_3) + \bar{t}(p_4) + \textstyle\sum_i g(k_i) \\
g(p_1) + \bar{g}(p_2) &\;\rightarrow\; t(p_3) + \bar{t}(p_4) + \textstyle\sum_i g(k_i)
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- Invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4)^2$$

$$t_1 = (p_1 - p_3)^2 - m_t^2 \qquad u_1 = (p_1 - p_4) - m_t^2$$

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- Small-q_T limit

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- Momenta

$$n = (1, 0, 0, 1)\,, \qquad \bar{n} = (1, 0, 0, -1)$$

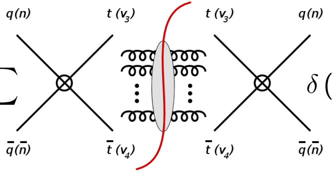
$$k_i^\mu = (n \cdot k_i)\,\frac{\bar{n}^\mu}{2} + (\bar{n} \cdot k_i)\,\frac{n^\mu}{2} + k_{i\perp}^\mu$$

$$p_1^\mu = m_t n\,, \qquad p_2^\mu = m_t \bar{n}\,, \qquad p_{3,4}^\mu = m_t v_{3,4}^\mu + \lambda_{3,4}^\mu$$

Soft function

- Represents corrections coming from exchanges of real, soft gluons, whose transverse momenta sum up to a fixed value q_T.

$$S_{\text{bare}}(q_T, v_3, v_4) \propto \sum$$



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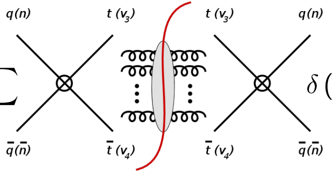

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- eikonal Feynman rules

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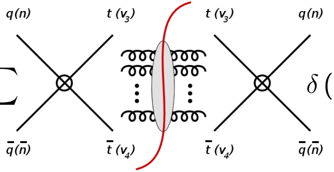

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- external momenta → Wilson Lines (Born kinematics)
- eikonal Feynman rules

$$\boldsymbol{S}_{i\bar{i}} = \sum_{n=0}^{\infty} \boldsymbol{S}_{i\bar{i}}^{(n)} \left(\frac{\alpha_s}{4\pi}\right)^n \qquad \boldsymbol{S}_{i\bar{i}}^{(n)} = \sum_{\{j\}} \boldsymbol{w}_{\{j\}}^{i\bar{i}} I_{\{j\}}$$

colour matrices

phase space integrals

Renormalization

separately divergent

$$\frac{d\sigma}{d\Phi} = \mathcal{B}_1^{(\text{bare})} \otimes \mathcal{B}_2^{(\text{bare})} \otimes \text{Tr}\left[\mathcal{H}^{(\text{bare})} \otimes \mathcal{S}^{(\text{bare})}\right]$$

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$$= Z_B \mathcal{B}_1^{(\mathrm{bare})} \otimes Z_B \mathcal{B}_2^{(\mathrm{bare})} \otimes \mathrm{Tr}\left[\boldsymbol{Z}_H^{\dagger} \mathcal{H}^{(\mathrm{bare})} \boldsymbol{Z}_H \otimes \boldsymbol{Z}_S^{\dagger} \mathcal{S}^{(\mathrm{bare})} \boldsymbol{Z}_S\right]$$

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$$= \mathcal{B}_1(\mu) \otimes \mathcal{B}_2(\mu) \otimes \text{Tr}\left[\mathcal{H}(\mu) \otimes \mathcal{S}(\mu)\right]$$

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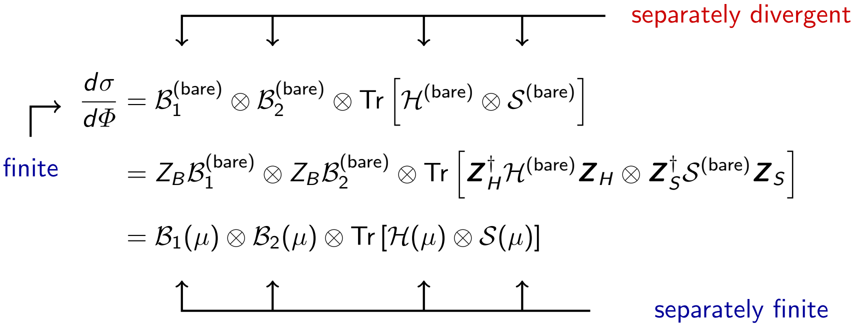

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separately finite

$$\frac{d}{d\mu}\frac{d\sigma}{d\Phi} = 0 \quad \rightarrow \quad \text{Renormalization Group Equations for } \mathcal{B}, \mathcal{H} \text{ and } \mathcal{S}$$

Renormalization

- RG equation

$$\frac{d}{d\ln\mu}\boldsymbol{S}_{i\bar{i}}(\mu) = -\gamma^{s\dagger}_{i\bar{i}}\,\boldsymbol{S}_{i\bar{i}}(\mu) - \boldsymbol{S}_{i\bar{i}}(\mu)\,\gamma^{s}_{i\bar{i}}$$

- Soft anomalous dimension

$$\gamma^{s} = -\boldsymbol{Z}_s^{-1}\frac{d\boldsymbol{Z}_s}{d\ln\mu}$$

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- RG equation

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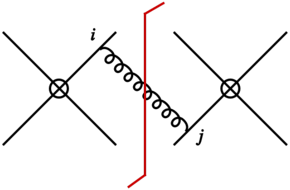
- Soft anomalous dimension

$$\gamma^s = -\boldsymbol{Z}_s^{-1}\frac{d\boldsymbol{Z}_s}{d\ln\mu}$$

Specifically, at the order α_s^2, we get

$$\underbrace{\boldsymbol{S}^{(2)}}_{\text{finite part only}} = \overbrace{\boldsymbol{Z}_s^{\dagger(2)}\boldsymbol{S}^{(0)}_{\text{bare}} + \boldsymbol{S}^{(0)}_{\text{bare}}\boldsymbol{Z}_s^{(2)} + \boldsymbol{Z}_s^{\dagger(1)}\boldsymbol{S}^{(0)}_{\text{bare}}\boldsymbol{Z}_s^{(1)}}^{\text{pole part only}}$$
$$+ \underbrace{\boldsymbol{Z}_s^{\dagger(1)}\boldsymbol{S}^{(1)}_{\text{bare}} + \boldsymbol{S}^{(1)}_{\text{bare}}\boldsymbol{Z}_s^{(1)} + \boldsymbol{S}^{(2)}_{\text{bare}} - \frac{\beta_0}{\epsilon}\,\boldsymbol{S}^{(1)}_{\text{bare}}}_{\text{finite + pole part}}$$

Soft function at NLO



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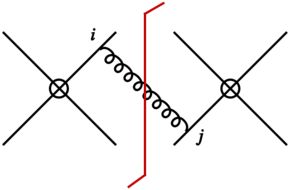


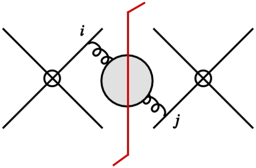
- Known in analytic form
  [Li, Li, Shao, Yan, Zhu '13; Catani, Grazzini, Torre '13]

$$
\begin{aligned}
\boldsymbol{S}^{(1)}_{i\bar{i}} = {} & 4L_\perp \left(2\boldsymbol{w}^{13}_{i\bar{i}} \ln \frac{-t_1}{m_t M} + 2\boldsymbol{w}^{23}_{i\bar{i}} \ln \frac{-u_1}{m_t M} + \boldsymbol{w}^{33}_{i\bar{i}} \right) \\
& - 4 \left(\boldsymbol{w}^{13}_{i\bar{i}} + \boldsymbol{w}^{23}_{i\bar{i}} \right) \mathrm{Li}_2 \left(1 - \frac{t_1 u_1}{m_t^2 M^2} \right) + 4\boldsymbol{w}^{33}_{i\bar{i}} \ln \frac{t_1 u_1}{m_t^2 M^2} \\
& - 2\boldsymbol{w}^{34}_{i\bar{i}} \frac{1+\beta_t^2}{\beta_t} \left[L_\perp \ln x_s - \mathrm{Li}_2 \left(-x_s \, \mathrm{tg}^2 \frac{\theta}{2} \right) + \mathrm{Li}_2 \left(-\frac{1}{x_s} \, \mathrm{tg}^2 \frac{\theta}{2} \right) \right. \\
& \left. + 4 \ln x_s \ln \cos \frac{\theta}{2} \right], \qquad \text{where} \qquad L_\perp = \ln \frac{x_T^2 \mu^2}{4e^{-2\gamma_E}}
\end{aligned}
$$

Soft function at NNLO

Three distinct groups of diagrams:

- Bubble



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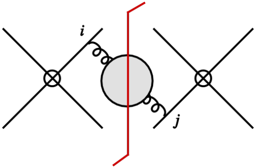
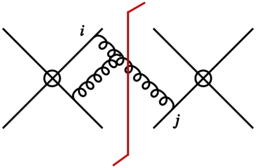


- Single-cut



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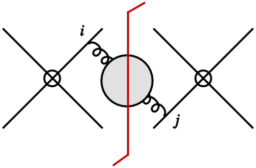

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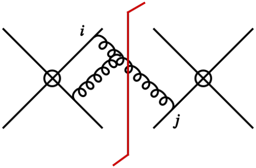

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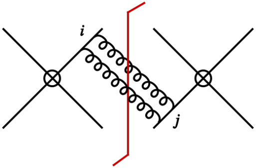
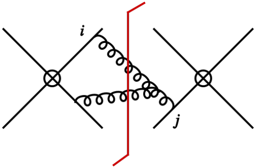
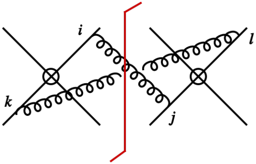






+ ...

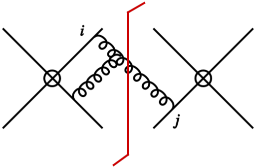
Soft function at NNLO

Three distinct groups of diagrams:

- Bubble

  DIFFERENTIAL EQUATIONS

- Single-cut



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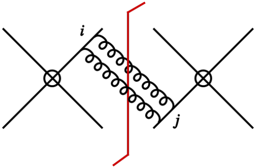

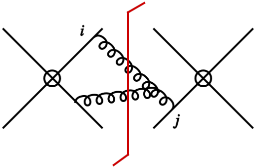

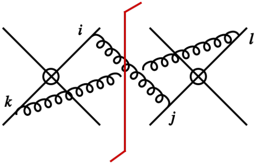

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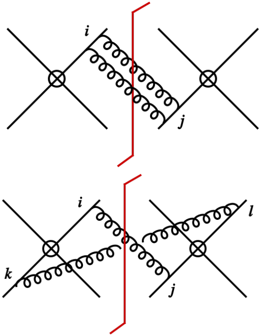

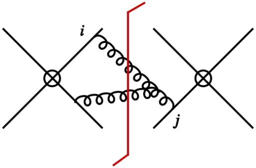

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SECTOR DECOMPOSITION

Double-cut part

$$|\mathcal{M}^{(0)}_{g,g,a_1,\dots}(k,l,p_1,\dots)|^2 \simeq$$

$$\frac{1}{2}\sum_{ijkl}\mathcal{S}_{ij}(k)\mathcal{S}_{kl}(l)\,\langle\mathcal{M}^{(0)}_{a_1,\dots}(p_1,\dots)|\,\{\mathbf{T}_i\cdot\mathbf{T}_j,\mathbf{T}_k\cdot\mathbf{T}_l\}\,|\mathcal{M}^{(0)}_{a_1,\dots}(p_1,\dots)\rangle$$

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$$\boldsymbol{S}^{(2),f\bar{f}}_{i\bar{i}}(q_\perp) = \frac{1}{S_{d-3}}\int d\Omega_{d-3}\,d^dk\,d^dl\;\delta_+(k^2)\,\delta_+(l^2)\;\delta^{(d-2)}(q_\perp-k_\perp-l_\perp)$$

$$\times\,\langle c_I^{i\bar{i}}|\mathcal{M}^{*\,(0)}_{f,\bar{f},a_1,\dots}(k,l,p_1,\dots)\mathcal{M}^{(0)}_{f,\bar{f},a_1,\dots}(k,l,p_1,\dots)|c_J^{i\bar{i}}\rangle$$

Double-cut NNLO integrals

Example:

$$\tilde{I}_{3gv,ij} = \int \frac{d^d k_1\, d^d k_2\ \delta^+(k_1^2)\, \delta^+(k_2^2)\, \delta((k_1+k_2)_T^2 - q_T^2)}{(n\cdot k_1)^\alpha\ (n\cdot k_2)^\alpha\ (n_i\cdot k_1)\ (n_j\cdot (k_1+k_2))\ (k_1+k_2)^2}$$

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- divergent in the limits $\epsilon \rightarrow 0$ and $\alpha \rightarrow 0$
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- complication introduced by $\delta((k_1+k_2)_T^2 - q_T^2)$ which additionally couples gluon's momenta

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To disentangle overlapping singularities and calculate regularized integrals we use the method of sector decomposition [Binoth, Heinrich, '00; Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke '17].

Sector decomposition

Two types of singularities

- Endpoint, *e.g.* soft:

$$\left(k_1^+, k_1^-, k_1^\perp\right) \rightarrow 0$$

Sector decomposition

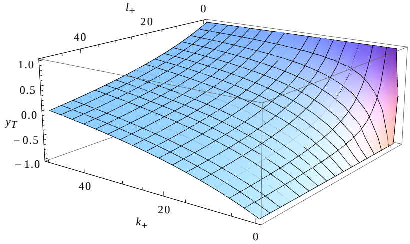
Two types of singularities

- Endpoint, *e.g.* soft:

$$\left(k_1^+, k_1^-, k_1^\perp\right) \to 0$$

- Manifold, *e.g.* collinear

$$k_1 \cdot k_2 \to 0$$



The strategy

Given the integral: $$I_G = \int d^d k_1 d^d k_2 \, \mathcal{I}_G \times \mathcal{W}_G$$

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finite weight

- Analytically integrate 3 out of $2d$ dimensions
- Map the remaining variables to a unit hypercube (split the original integral into a sum if necessary)

$$= \sum_j \int_0^1 \prod_{i=1}^{2d-3} dx_i \left(\mathcal{I}_G \times \mathcal{W}_G \right)_j$$

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$$= \sum_{j,m,n} \sum_{k \in \text{sectors}} \left(\int_0^1 a_{mn} \times \mathcal{W}_G \right) \alpha^m \epsilon^n$$

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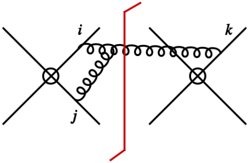
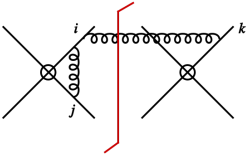
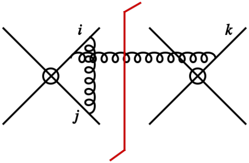
- Numerically integrate series coefficients

$$= \sum_{j,m,n} \sum_{k \in \text{sectors}} c^{jk}_{mn} \, \alpha^m \epsilon^n$$

Single-cut (real-virtual)







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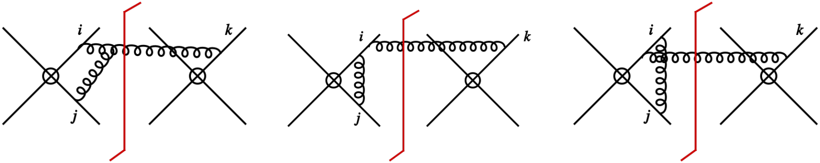

$$S^{(2)}_{\text{1-cut}} = \sum_{ijk} \int d^d l \frac{\delta^+(l^2)\,\delta(l_T - q_T)}{l_+^\alpha\, n_k \cdot l}\, n_k^\mu\, T_k^a\, J^\mu_{ij,a}(l)$$

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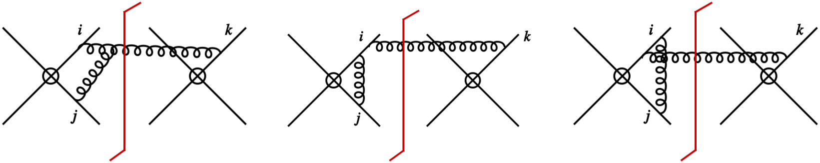

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- The soft current $J^\mu_{ij,a}(l)$ is known up to NLO [Catani, Grazzini '00; Bierenbaum, Czakon, Mitov '12; Czakon, Mitov '18].

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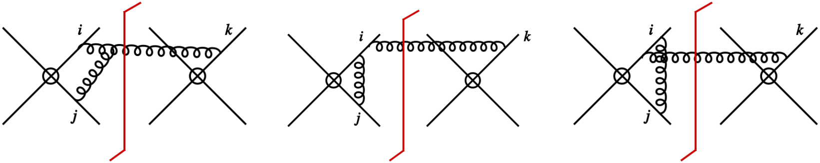

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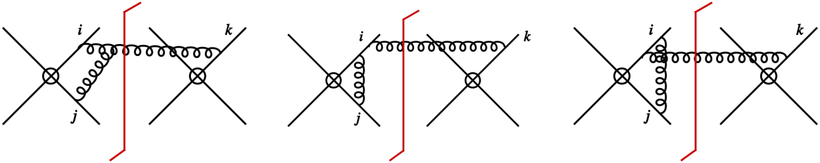

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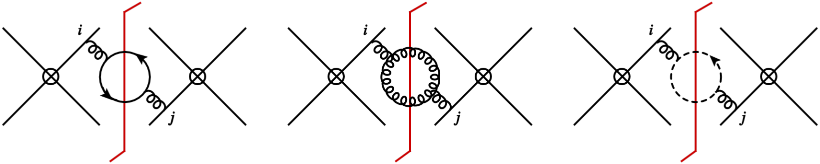
- $S^{(2)}_{\text{1-cut}}$ can be obtained by a relatively simple integration over l^{μ}.
- Single-cut piece of the soft function exhibits both real and imaginary part. The latter when $i \neq j \neq k$, the former, otherwise.

Bubble



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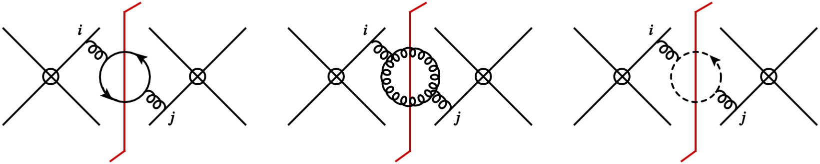
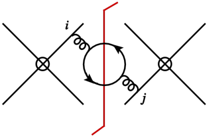
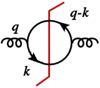


- Solvable analytically: direct cross check of our sector decomposition-based implementation
- Non-trivial tensor structure → challenging numerators
- Laboratory to stress-test sector decomposition-based methodology
- Comparable with n_f part of Renormalization Group prediction

Bubble part of the soft function from differential equations



$$\propto \int \frac{d^d q\, \delta(q_T - 1)\, \theta^+(q^2)\, n_i^\mu n_j^\nu}{q^4\,(n_i \cdot q)\,(n_j \cdot q)} \left(\quad \right)_{\mu\nu}$$



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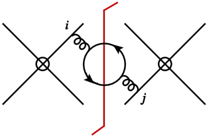

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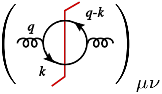
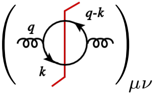


where



$$= \int \frac{d^d k\, N_{\mu\nu}\,\delta^+(k^2)\,\delta^+((q-k)^2)}{(n\cdot k)^\alpha\,(n\cdot(q-k))^\alpha k^2 (q-k)^2}$$

$$= T_{00}\, g^{\mu,\nu} + T_{qq}\, q^\mu q^\nu + T_{nn}\, n^\mu n^\nu + T_{qn}\,(n^\mu q^\nu + q^\mu n^\nu)$$

Bubble part of the soft function from differential equations

- Topology:

$$\int \frac{d^d k\, \delta(k_T - 1)\, \theta(k^2)\, \delta(k^0)}{(n\cdot k)^{a_1+2\alpha}\, (\bar{n}\cdot k)^{a_2}\, (v_3\cdot k)^{a_3}\, (v_4\cdot k)^{a_4}\, (k^2)^{a_0+\epsilon}}$$

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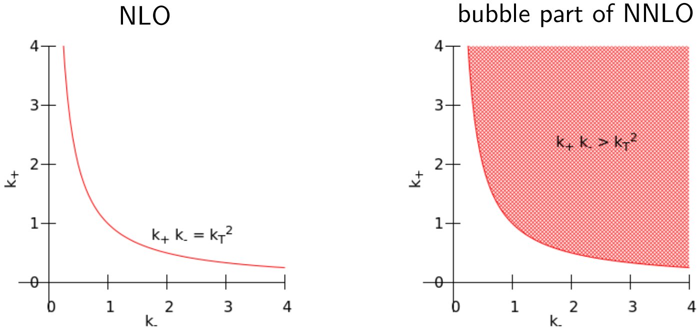

NLO

bubble part of NNLO

Bubble part of the soft function from differential equations

$$1 = \int_0^\infty dm^2 \delta(k^2 - m^2)\,,$$

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$$I = \int_0^\infty dm^2 \delta(k^2 - m^2) \int \frac{d^d k\, \delta(k_T^2 - 1)\, \theta(k^2)\, \theta(k_0)}{(n \cdot k)^{a_1 + 2\alpha} (\bar{n} \cdot k)^{a_2} (v_3 \cdot k)^{a_3} (v_4 \cdot k)^{a_4} (k^2)^{a_0 + \epsilon}}$$
$$= \int_0^\infty \frac{dm^2}{(m^2)^{a_0 + \epsilon}} \int \frac{d^d k\, \delta(k_T^2 - 1)\, \delta(k^2 - m^2) \theta(k_0)}{(n \cdot k)^{a_1 + 2\alpha} (\bar{n} \cdot k)^{a_2} (v_3 \cdot k)^{a_3} (v_4 \cdot k)^{a_4}}\,.$$

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Finally, the topology reads:

$$\int \frac{d^d k}{(n \cdot k)^{a_1+2\alpha}\ (\bar{n} \cdot k)^{a_2}\ (v_3 \cdot k)^{a_3}\ (v_4 \cdot k)^{a_4}\ (k^2 - m^2)^{a_5}\ ((n \cdot k)\, (\bar{n} \cdot k) - m^2 - 1)^{a_6}}$$

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- Identities → reduction → DE → solutions → $\int dm^2$ → $I_{jk}(\beta, \theta)$

Complete Soft Function at NNLO: structure of the result

- In momentum space

$$S^{(2,\text{bare})}(q_T,\beta,\theta) = \frac{1}{q_T^p}\left[S^{(2)}_{\text{bubble}}(\beta,\theta,\epsilon)+S^{(2)}_{\text{1-cut}}(\beta,\theta,\epsilon)+S^{(2)}_{\text{2-cut}}(\beta,\theta,\epsilon)\right]$$

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Fourier Transform

- In position space

$$S^{(2,\text{bare})}(L_\perp, \beta, \theta) = \left[\frac{1}{\epsilon} + L_\perp + L_\perp^2 + \dots\right]$$

$$\times \left[S^{(2)}_{\text{bubble}}(\beta,\theta,\epsilon) + S^{(2)}_{\text{1-cut}}(\beta,\theta,\epsilon) + S^{(2)}_{\text{2-cut}}(\beta,\theta,\epsilon)\right]$$

Complete Soft Function at NNLO: structure of the result

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$$S^{(2,\text{bare})}(q_T,\beta,\theta) = \frac{1}{q_T^p}\left[S^{(2)}_{\text{bubble}}(\beta,\theta,\epsilon)+S^{(2)}_{\text{1-cut}}(\beta,\theta,\epsilon)+S^{(2)}_{\text{2-cut}}(\beta,\theta,\epsilon)\right]$$

Fourier Transform

- In position space

$$S^{(2,\text{bare})}(L_\perp,\beta,\theta) = \left[\frac{1}{\epsilon}+L_\perp+L_\perp^2+\ldots\right]$$

$$\times\left[S^{(2)}_{\text{bubble}}(\beta,\theta,\epsilon)+S^{(2)}_{\text{1-cut}}(\beta,\theta,\epsilon)+S^{(2)}_{\text{2-cut}}(\beta,\theta,\epsilon)\right]$$

$\hookrightarrow$ Momentum-space soft function has to be calculated up to order ϵ.

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- However, we calculate all terms and use the redundant ones for cross checks against Renormalization Group prediction.

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$$\frac{1}{\epsilon^4}\begin{pmatrix} 0.00009\, N_c^{-1} - 0.00009\, N_c & -0.00002\, N_c^2 - 0.00009\, N_c^{-2} + 0.0001 \\ -0.00002\, N_c^2 - 0.00009\, N_c^{-2} + 0.0001 & 0.00008\, N_c^3 - 0.00006\, N_c + 0.00007\, N_c^{-3} - 0.00009\, N_c^{-1} \end{pmatrix}$$

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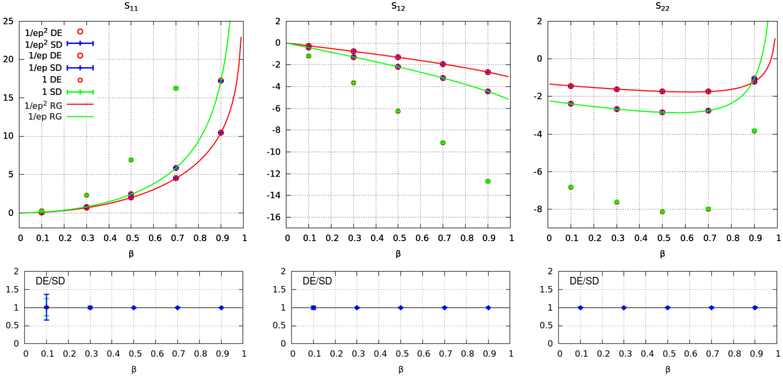
- $\frac{1}{\epsilon^3}$ pole cancels between 1-cut and 2-cut contributions

$$\frac{1}{\epsilon^3}\begin{pmatrix} 0.0004\,N_c^3 - 0.0007\,N_c + 0.0004\,N_c^{-1} & 0.0004\,N_c^2 - 0.0004\,N_c^{-2} - 7.\times 10^{-6} \\ 0.0004\,N_c^2 - 0.0004\,N_c^{-2} - 7.\times 10^{-6} & -0.0004\,N_c^3 - 0.00001\,N_c + 0.0003\,N_c^{-3} + 0.0002\,N_c^{-1} \end{pmatrix}$$

[†] We used $\beta = 0.4$, $\theta = 0.5$.

NNLO, small-q_T soft function for top pair production

Quark bubble contribution ($q\bar{q}$ channel)

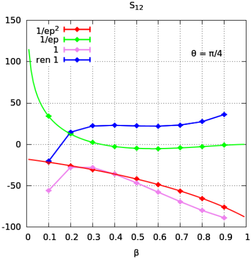


Validation of the framework

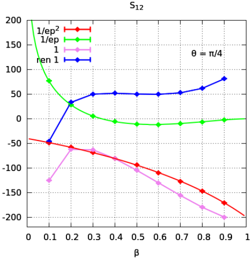
- Perfect agreement of quark bubble results obtained from *differential equations* and *sector decomposition* for all terms in ϵ expansion
- Reproduction of the n_f part of Renormalization Group result

Imaginary part

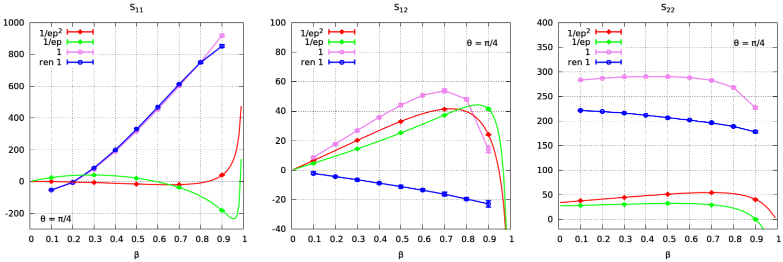
($q\bar{q}$ channel)



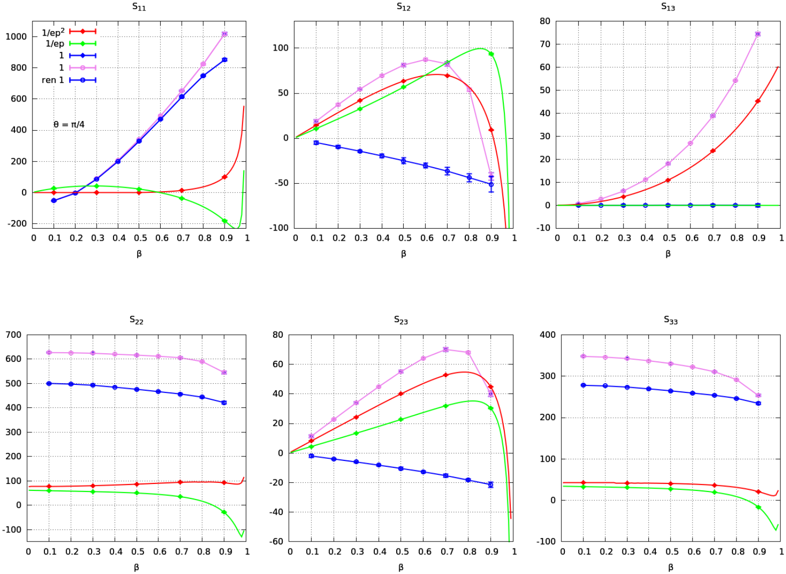
(gg channel)



Real part ($q\bar{q}$ channel)



Real part (gg channel)



Conclusions

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  1. Cancellation of α poles, including ϵ/α, and ϵ poles beyond $1/\epsilon^2$
  2. Perfect agreement with analytic calculation for bubble graphs
  3. RG result for the complete NNLO soft function recovered: real and imaginary part

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- The soft function can now be used to obtain full $t\bar{t}$ cross section at NNLO as well for resummation up to NNLL'

Acknowledgements

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