Subleading power operators and anomalous dimensions

Jian Wang

with M. Beneke, M. Garny, R. Szafron,
[arXiv:1712.04416, 1712.07462, 1808.04742]

High Time for Higher Orders @ MITP

August 24, 2018

Subleading power x-section

The cross section can be expanded in a series of a small variable τ,

$$\sigma(\tau) = C\delta(\tau) + \sum_n \alpha_s^n \left[\frac{\ln^{2n-1}\tau}{\tau} + \underbrace{\ln^{2n-1}\tau}_{NLP} + \cdots \right] \qquad (1)$$

Here τ can be the N-jettiness variable, the threshold variable $1 - M^2/s$, the transverse momentum of a lepton pair q_T, $\cdots$

1. Phenomenology: Useful for NNLO differential calculations in q_T/N-jettiness slicing methods [1612.00450, 1612.02911]
2. Theory: NLP factorization and resummation [1503.05156, 1610.06842, 1804.04665]
3. Amplitude: general structure, soft theorem

Soft-collinear anomalous dimensions at Leading Power

IR divergences of virtual corrections in QCD $\leftrightarrow$ UV divergences of loop diagrams in Soft-Collinear effective theory [0901.0722]
Up to the two-loop level,

$$\boldsymbol{\Gamma} = -\gamma_{\rm cusp}(\alpha_s) \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \ln\left(\frac{-s_{ij}}{\mu^2}\right) + \sum_i \gamma_i(\alpha_s) \qquad (2)$$

IR divergences of virtual corrections in QCD $\leftrightarrow$ UV divergences of loop diagrams in Soft-Collinear effective theory [0901.0722]
Up to the two-loop level,

$$\boldsymbol{\Gamma} = -\gamma_{\text{cusp}}(\alpha_s) \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \ln\left(\frac{-s_{ij}}{\mu^2}\right) + \sum_i \gamma_i(\alpha_s) \tag{2}$$

with $\sqrt{s_{ij}}$ a hard scale.

IR divergences of virtual corrections in QCD $\leftrightarrow$ UV divergences of loop diagrams in Soft-Collinear effective theory [0901.0722]
Up to the two-loop level,

$$\mathbf{\Gamma} = -\gamma_{\rm cusp}(\alpha_s) \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \ln\left(\frac{-s_{ij}}{\mu^2}\right) + \sum_i \gamma_i(\alpha_s) \qquad (2)$$

with $\sqrt{s_{ij}}$ a hard scale.

Next-to-leading power?

Low-Burnett-Kroll Theorem: the relation between the amplitude with a single soft particle and the amplitude without the soft particle

$$\sum_i (-g_s)\mathbf{T}_i \left(\frac{\varepsilon(k)\cdot p_i}{k\cdot p_i} + \frac{\varepsilon_\mu k_\nu J_i^{\mu\nu}}{k\cdot p_i} \right) A_0 \tag{3}$$

with

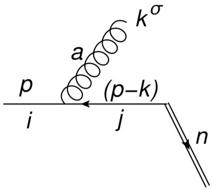
$$J_i^{\mu\nu} = p_i^\mu \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\mu}} + \Sigma_i^{\mu\nu} \tag{4}$$

Not directly related to an observable at high energy experiments. A series of work has been done in this field. [0811.2067, 1010.1860, 1410.6406, 1706.04018]

Subleading power factorization

One difficulty at NLP is the interplay between soft and collinear radiations, which induce the introduction of new elements, such as "radiative jet function". [1503.05156]

$$\int d^d y e^{-i(p-k)y} \langle | \Phi_n(\infty, y) \Psi(y) j_\mu(0) | p \rangle \tag{5}$$



θ-jet/soft function [1804.04665], See I.Moult's talk.

Subleading power operators

$$\mathcal{L}_{\mathrm{SCET}} = \sum_{i=1}^{N} \mathcal{L}_i(\psi_i, \psi_s) + \mathcal{L}_s(\psi_s) \tag{6}$$

The general structure of subleading operators

$$J = \int dt \; C(\{t_{i_k}\}) \, J_s(0) \prod_{i=1}^{N} J_i(t_{i_1}, t_{i_2}, \dots) \tag{7}$$

where

$$J_i(t_{i_1}, t_{i_2}, \dots) = \prod_{k=1}^{n_i} \psi_{i_k}(t_{i_k} n_{i+}) \,, \tag{8}$$

with gauge-invariant collinear "building blocks"

$$\psi_i(t_i n_{i+}) \in \begin{cases} \chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i & \text{collinear quark} \\ \mathcal{A}^\mu_{\perp i}(t_i n_{i+}) \equiv W_i^\dagger [iD^\mu_{\perp i} W_i] & \text{collinear gluon} \end{cases} \tag{9}$$

LP:

$$J_i^{A0}(t_i) = \psi_i(t_i n_{i+}) \,. \quad (10)$$

NLP $[O(\lambda), O(\lambda^2)]$:

- $i\partial_\perp \quad \rightarrow J^{A1} = i\partial_\perp J^{A0}$
- $in_- D_s \equiv in_-\partial + g_s n_- A_s \quad \rightarrow$ eliminated by E.o.M
- more building blocks $\rightarrow J^{B1} = \psi_{i_1}(t_{i_1} n_{i+})\psi_{i_2}(t_{i_2} n_{i+})$
- new building blocks, e.g., $n_-\mathcal{A} \quad \rightarrow$ eliminated by E.o.M
- pure soft sector J_s, e.g., $q \sim O(\lambda^3), F_s^{\mu\nu} \sim O(\lambda^4)$, not needed at NLP
- time-ordered product operators

$$J_i^{T1}(t_i) = i \int d^4x \, \mathbf{T}\left\{ J_i^{A0}(t_i), \mathcal{L}_i^{(1)}(x) \right\} \quad (11)$$

LBK Theorem

Draw diagrams on the blackboard.

LBK Theorem

Draw diagrams on the blackboard.

We reproduce LBK theorem with two time-ordered products

$$\int d^4x \mathbf{T}\{J^{A0}, \mathcal{L}^{(2)}(x)\}, \int d^4x \mathbf{T}\{J^{A1}, \mathcal{L}^{(1)}(x)\}$$

Draw diagrams on the blackboard.

We reproduce LBK theorem with two time-ordered products

$$\int d^4x \mathbf{T}\{J^{A0}, \mathcal{L}^{(2)}(x)\}, \int d^4x \mathbf{T}\{J^{A1}, \mathcal{L}^{(1)}(x)\}$$

The LBK theorem has also been proven in the framework of label-SCET, [1412.3108]. different operators, different propagators, different vertices

The two formulations of SCET recover the LBK formula in rather different ways.

Anomalous dimensions

With the definition $\mathbf{\Gamma} \equiv -\left(\frac{d}{d\ln\mu}\mathbf{Z}\right)\mathbf{Z}^{-1}$,

$$\Gamma_{PQ}(x,y) =$$

$$\delta_{PQ}\delta(x-y)\left[-\gamma_{\rm cusp}(\alpha_s)\sum_{i<j}\sum_{k,l}\mathbf{T}_{i_k}\cdot\mathbf{T}_{j_l}\ln\left(\frac{-s_{ij}x_{i_k}x_{j_l}}{\mu^2}\right)+\sum_i\sum_k\gamma_{i_k}(\alpha_s)\right]$$

$$+2\sum_i\delta^{[i]}(x-y)\gamma^i_{PQ}(x,y)+2\sum_{i<j}\delta(x-y)\gamma^{ij}_{PQ}(y)\,, \qquad (12)$$

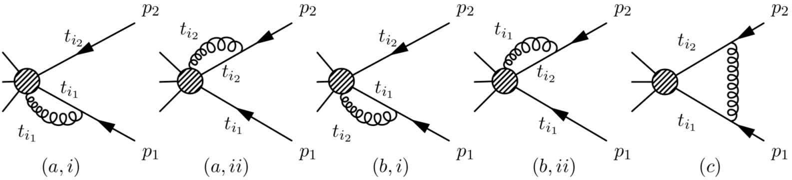
In the calculation, we have used offshellness to regularize the IR poles, and found that they cancel between the soft and collinear contributions.

$$\mathbf{\Gamma} = \begin{pmatrix} \Gamma_{PQ} & \Gamma_{PT(Q')} \\ \Gamma_{T(P')Q} & \Gamma_{T(P')T(Q')} \end{pmatrix} = \begin{pmatrix} \Gamma_{PQ} & 0 \\ \Gamma_{T(P')Q} & \Gamma_{P'Q'} \end{pmatrix} \qquad (13)$$

$$\gamma_{\rm cusp}(\alpha_s) = \frac{\alpha_s}{\pi} \quad \text{and} \quad \gamma_{i_k}(\alpha_s) = \begin{cases} -\dfrac{3\alpha_s C_F}{4\pi} & \text{(q)} \\ 0 & \text{(g)} \end{cases} \qquad (14)$$

$$\gamma^i = \begin{pmatrix} \gamma^i_{PQ} & 0 \\ 0 & \gamma^i_{P'Q'} \end{pmatrix}, \qquad \gamma^{ij} = \begin{pmatrix} 0 & 0 \\ \gamma^{ij}_{T(P')Q} & 0 \end{pmatrix}. \qquad (15)$$

Collinear anomalous dimensions, B1 to B1 with F=2

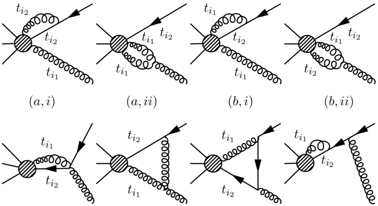


$$\gamma^{i}_{\chi_\alpha\chi_\beta,\chi_\gamma\chi_\delta}(x,y) = \frac{\alpha_s \mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{2\pi}\Bigg\{\delta_{\alpha\gamma}\delta_{\beta\delta}\Bigg(\theta(x-y)\left[\frac{1}{x-y}\right]_+ + \theta(y-x)\left[\frac{1}{y-x}\right]_+ - \theta(x-y)\frac{1-\frac{\bar{x}}{2}}{\bar{y}} - \theta(y-x)\frac{1-\frac{x}{2}}{y}\Bigg) - \frac{1}{4}\left(\sigma_\perp^{\nu\mu}\right)_{\alpha\gamma}\left(\sigma_{\perp\nu\mu}\right)_{\beta\delta}\left(\theta(x-y)\frac{\bar{x}}{\bar{y}} + \theta(y-x)\frac{x}{y}\right)\Bigg\}. \quad (16)$$

$$\gamma^{i}_{\mathcal{A}^\mu \chi_\alpha, \mathcal{A}^\nu \chi_\beta}(x,y) = \frac{\alpha_s \mathbf{T}_{i_1}\cdot \mathbf{T}_{i_2}}{2\pi}\Bigg\{ g_\perp^{\mu\nu}\delta_{\alpha\beta}\Bigg(\theta(x-y)\left[\frac{1}{x-y}\right]_+ + \theta(y-x)\left[\frac{1}{y-x}\right]_+$$
$$- \frac{\theta(x-y)}{\bar{y}}\left(1+\frac{\bar{x}(\bar{x}+\bar{y})}{2x}\right) - \frac{\theta(y-x)}{2y}(\bar{x}+\bar{y})\Bigg)$$
$$+ \frac{1}{4}([\gamma_\perp^\mu,\gamma_\perp^\nu])_{\alpha\beta}(x+y)\bar{x}\left(\frac{\theta(x-y)}{\bar{y}x}+\frac{\theta(y-x)}{y\bar{x}}\right)\Bigg\}$$
$$- \frac{\alpha_s(\mathbf{C_F}+\mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2})}{4\pi}\Bigg\{ g_\perp^{\mu\nu}\delta_{\alpha\beta}\left(\frac{\theta(x-\bar{y})\bar{x}}{yx}(\bar{x}+\bar{y}) + \frac{\theta(\bar{y}-x)}{\bar{y}}(\bar{x}-y)\right)$$
$$+ \frac{1}{2}([\gamma_\perp^\mu,\gamma_\perp^\nu])_{\alpha\beta}\left(\frac{\theta(x-\bar{y})\bar{x}}{yx}(\bar{x}-y-1) + \frac{\theta(\bar{y}-x)}{\bar{y}}(\bar{x}-y)\right)\Bigg\}$$
$$+ \frac{\alpha_s \mathbf{C_F}}{4\pi}\,\bar{x}\,(\gamma_\perp^\mu\gamma_\perp^\nu)_{\alpha\beta}\,, \qquad (17)$$

consistent with previous results [hep-ph/0404217, hep-ph/0508250] and recent work [1806.01278].

B2 to B2 mixing with F=1



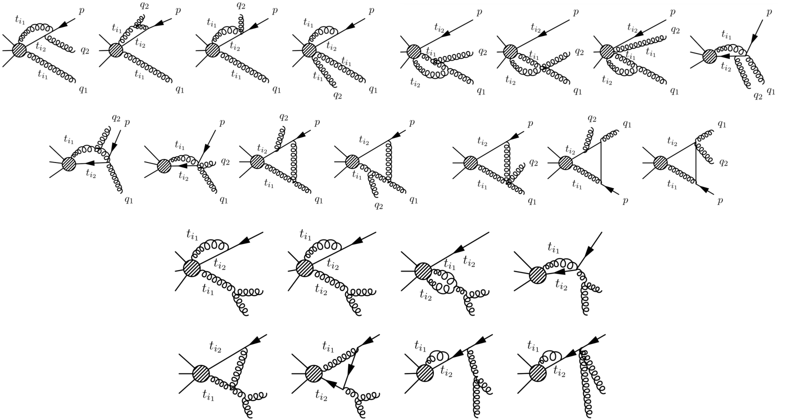
$$\gamma^{i}_{\mathcal{A}^\mu\partial^\nu\chi,\mathcal{A}^\rho\partial^\sigma\chi}(x,y) =$$

$$g_\perp^{\mu\rho} g_\perp^{\nu\sigma} \frac{\alpha_s \mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{2\pi}\left\{\theta(x-y)\left[\frac{1}{x-y}\right]_+ + \theta(y-x)\left[\frac{1}{y-x}\right]_+ \right.$$

$$\left. -\theta(x-y)\frac{\bar{x}+\bar{y}}{\bar{y}^2} - \theta(y-x)\frac{x+2y}{2y^2}\right\}$$

$$+\frac{\alpha_s \mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{8\pi} M^{\mu\nu,\rho\sigma}(x,y) - \frac{\alpha_s(\mathbf{C_F}+\mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2})}{8\pi} N^{\mu\nu,\rho\sigma}(x,y)$$

$$+\frac{\alpha_s \mathbf{C_F}}{8\pi}\frac{\bar{x}}{\bar{y}}(2g_\perp^{\mu\nu} - x\gamma_\perp^\mu\gamma_\perp^\nu)\left(\gamma_\perp^\rho\gamma_\perp^\sigma + \frac{2\bar{y}}{y}g_\perp^{\rho\sigma}\right), \quad (18)$$



$$\gamma^{i}_{\mathcal{A}^{\mu a}\partial^{\nu}\xi,\mathcal{A}^{\sigma d}\mathcal{A}^{\lambda e}\xi}(x,y_1,y_2) = -\frac{\alpha_s}{8\pi}\, I^{\mu\nu\sigma\lambda}_{ade}(x,y_1,y_2)\,. \qquad (19)$$

$$
\begin{aligned}
&I^{\mu\nu\sigma\lambda}_{ade}(x, y_1, y_2)|_{(c,i)_B} \\
&= \bar{x}\Bigg\{\Bigg[- \theta(x-y_2)\theta(\bar{y}_3 - x)\frac{x^2\bar{y}_2 + \bar{x}^2\bar{y}_3 - \bar{y}_2\bar{y}_3}{\bar{y}_2 y_1 \bar{y}_3} \\
&\quad + \theta(y_2 - x)\theta(x)\frac{x^2}{y_2\bar{y}_3} + \theta(\bar{x})\theta(x-\bar{y}_3)\frac{\bar{x}^2}{\bar{y}_2 y_3}\Bigg]\Bigg[\frac{1}{2}g_\perp^{\nu\lambda}\left(\frac{\gamma_\perp^\sigma\gamma_\perp^\mu}{\bar{x}} + \frac{2g_\perp^{\mu\sigma}}{x\bar{x}}(\bar{x}-x)\right) \\
&\quad + \frac{1}{2}g_\perp^{\nu\sigma}\left(\frac{\gamma_\perp^\lambda\gamma_\perp^\mu}{\bar{x}} + \frac{g_\perp^{\mu\lambda}(2\bar{x}-x)}{x\bar{x}}\right) + \frac{1}{2}g_\perp^{\lambda\sigma}\left(\frac{\gamma_\perp^\nu\gamma_\perp^\mu}{\bar{x}} + \frac{2g_\perp^{\mu\nu}(\bar{x}+y_2)}{x\bar{x}}\right) + \frac{x-y_1}{2x\bar{x}}g_\perp^{\mu\lambda}\gamma_\perp^\nu\gamma_\perp^\sigma \\
&\quad - \frac{y_1}{2x\bar{x}}\left(g_\perp^{\nu\lambda}\gamma_\perp^\mu\gamma_\perp^\sigma + g_\perp^{\mu\nu}\gamma_\perp^\lambda\gamma_\perp^\sigma\right) - \frac{y_2}{2x\bar{x}}\left(g_\perp^{\mu\nu}\gamma_\perp^\sigma\gamma_\perp^\lambda + g_\perp^{\nu\sigma}\gamma_\perp^\mu\gamma_\perp^\lambda + g_\perp^{\mu\sigma}\gamma_\perp^\nu\gamma_\perp^\lambda\right)\Bigg] \\
&\quad + \frac{1}{2}\left[\theta(x-y_2)\theta(\bar{y}_3-x)\frac{\bar{x}\bar{y}_3 - x\bar{y}_2}{\bar{y}_2 y_1\bar{y}_3} + \theta(y_2-x)\theta(x)\frac{x}{y_2\bar{y}_3} - \theta(\bar{x})\theta(x-\bar{y}_3)\frac{\bar{x}}{\bar{y}_2 y_3}\right] \\
&\quad \left[- 2g_\perp^{\nu\lambda}g_\perp^{\mu\sigma} + \frac{2y_2}{x}g_\perp^{\mu\nu}g_\perp^{\lambda\sigma} - g_\perp^{\mu\lambda}g_\perp^{\nu\sigma} - g_\perp^{\mu\lambda}\gamma_\perp^\nu\gamma_\perp^\sigma\frac{x-y_1}{\bar{x}}\right]\Bigg\} f^{abe}f^{bcd}t^c + (y_1 d\sigma \leftrightarrow y_2 e\lambda)\,,
\end{aligned}
$$

Collinear anomalous dimensions with F=1

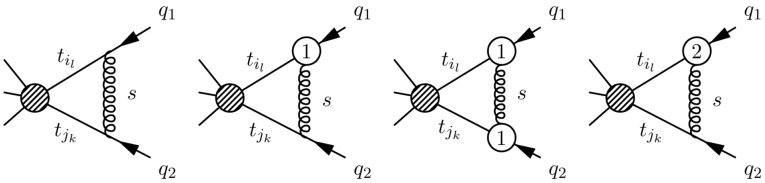
$\gamma^i_{PQ} =$

	$J^{A1}_{\partial\chi}$	$J^{B1}_{\mathcal{A}\chi}$
$J^{A1}_{\partial\chi}$	0	0
$J^{B1}_{\mathcal{A}\chi}$	0	$\gamma^i_{\mathcal{A}\chi,\mathcal{A}\chi}$

$\gamma^i_{PQ} =$

	$J^{A2}_{\partial\partial\chi}$	$J^{B2}_{\mathcal{A}\partial\chi}$	$J^{B2}_{\partial(\mathcal{A}\chi)}$	$J^{C2}_{\mathcal{A}\mathcal{A}\chi}$	$J^{C2}_{\chi\bar{\chi}\chi}$
$J^{A2}_{\partial\partial\chi}$	0	0	0	0	0
$J^{B2}_{\mathcal{A}\partial\chi}$	0	(4.7)	(4.8)	(4.22)	(4.30)
$J^{B2}_{\partial(\mathcal{A}\chi)}$	0	0	(4.9)	0	0
$J^{C2}_{\mathcal{A}\mathcal{A}\chi}$	0	0	0	(4.32)	(4.33)
$J^{C2}_{\chi\bar{\chi}\chi}$	0	0	0	(4.35)	(4.34)

Soft anomalous dimensions

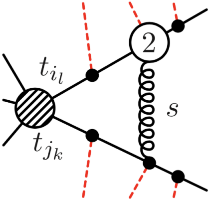


$$\gamma^{ij}_{\left(J^{T1}_{\chi,\xi}\right)_i \left(J^{T1}_{\chi,\xi}\right)_j, \left(J^{A1}_{\partial^\mu \chi}\right)_i \left(J^{A1}_{\partial^\nu \chi}\right)_j} = \frac{2\alpha_s}{\pi} \mathbf{T}_i \cdot \mathbf{T}_j \, G^{\mu\nu}_{ij} \,, \tag{20}$$

$$G^{\mu\nu}_{ij} \equiv \left(g^{\mu\nu} - \frac{n^\nu_{i-} n^\mu_{j-}}{n_{i-} n_{j-}} \right) \frac{1}{(n_{i-} n_{j-}) P_i P_j} \,. \tag{21}$$

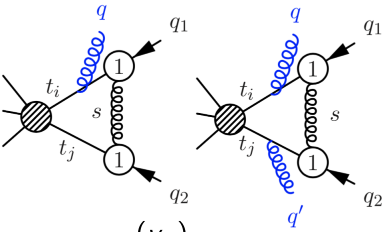
- The single insertions with $\mathcal{L}^{(1)}$ never contribute to the one-loop anomalous dimension matrix to $\mathcal{O}(\lambda^2)$.
- The soft one-loop diagrams within a single collinear direction do not contribute to the anomalous dimension at any power of λ.

One $\mathcal{L}^{(2)}$ insertion



$$\gamma^{ij}_{T(P,\mathcal{L}_k^{(2)}),Q} = 0 \tag{22}$$

Two $\mathcal{L}^{(1)}$ insertions in two directions



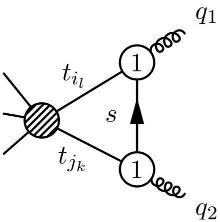
$$\gamma^{ij}_{\left(J^{T1}_{\chi_\alpha,\xi}\right)_i\left(J^{T1}_{\chi_\beta,\xi}\right)_j,\left(J^{B1}_{\mathcal{A}^\mu_b\chi_\gamma}\right)_i\left(J^{A1}_{\partial^\nu\chi_\delta}\right)_j}(y_{i_1})$$

$$= -\frac{\alpha_s}{\pi}\,\mathbf{T}^b_i\,(\mathbf{T}_i\cdot\mathbf{T}_j)\,G^{\lambda\nu}_{ij}\left(\gamma_{\lambda\perp i}\gamma^\mu_{\perp i} + \frac{\gamma^\mu_{\perp i}\gamma_{\lambda\perp i}}{\bar{y}}\right)_{\alpha\gamma}\delta_{\beta\delta}\,, \qquad (23)$$

$$\gamma^{ij}_{\left(J^{T1}_{\chi_\alpha,\xi}\right)_i\left(J^{T1}_{\chi_\beta,\xi}\right)_j,\left(J^{B1}_{\mathcal{A}^\mu_b\chi_\gamma}\right)_i\left(J^{B1}_{\mathcal{A}^\nu_c\chi_\delta}\right)_j}(y_{i_1},y_{j_1})$$

$$= \frac{\alpha_s}{2\pi}\,\mathbf{T}^b_i\mathbf{T}^c_j(\mathbf{T}_i\cdot\mathbf{T}_j)\,G^{ij}_{\lambda\kappa}\left(\gamma^\lambda_{\perp i}\gamma^\mu_{\perp i} + \frac{\gamma^\mu_{\perp i}\gamma^\lambda_{\perp i}}{\bar{y}_{i_1}}\right)_{\alpha\gamma}\left(\gamma^\kappa_{\perp j}\gamma^\nu_{\perp j} + \frac{\gamma^\nu_{\perp j}\gamma^\kappa_{\perp j}}{\bar{y}_{j_1}}\right)_{\beta\delta} \qquad (24)$$

Soft quark interaction



$$\gamma^{ij}_{T(P,\mathcal{L}^{(1)}_{k,\xi q},\mathcal{L}^{(1)}_{l,\xi q}),Q} = 0 \quad (25)$$

$$\gamma^{ij}_{T(P,\mathcal{L}^{(1)}_{k,\xi q}),Q} = 0 \quad (26)$$

The fermion number is conserved up to one-loop and $O(\lambda^2) \rightarrow$ classify the anomalous dimension according to collinear sectors with definite fermion number.

If the quark is massive and light compare to the hard scale, then there are large logarithms $\ln^n m^2/Q^2$. Resummation of such kind of logarithms is possible. [1412.0671, 1709.01092]

Summary

- We list the subleading power operators for a N-jet process in SCET.
- Their anomalous dimensions have been calculated in the cases of fermion number $|F| = 1, 2$.
- The general structure has a similar pattern to the Leading Power result, but contains more information about the operators.
- These results are one important ingredient for a systematic analysis of subleading power factorization and resummation.

- We list the subleading power operators for a N-jet process in SCET.
- Their anomalous dimensions have been calculated in the cases of fermion number $|F| = 1, 2$.
- The general structure has a similar pattern to the Leading Power result, but contains more information about the operators.
- These results are one important ingredient for a systematic analysis of subleading power factorization and resummation.

Thank you !