Renormalons and the Top quark mass measurement

Paolo Nason

Work done in collaboration with
Silvia Ferrario Ravasio and Carlo Oleari

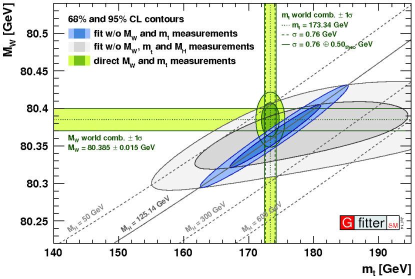
CERN and INFN, sez. di Milano Bicocca

Mainz, August 22 2018

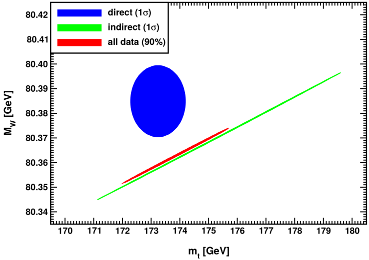
Outline

- 10 recicled slides on top mass (quickly)
- 1 reminder slide on renormalons
- What we computed and why
- Some results
- Understanding results
- Some puzzles

Top and precision physics



From PDG:



$$\Delta G_\mu / G_\mu = 5 \cdot 10^{-7}; \quad \Delta M_Z / M_Z = 2 \cdot 10^{-5};$$

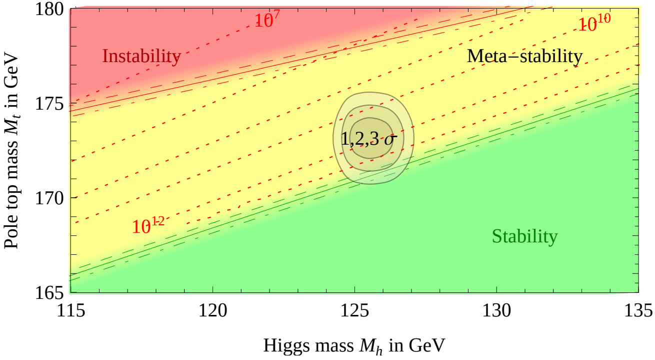
$$\Delta\alpha(M_Z)/\alpha(M_Z) = \begin{cases} 1 \cdot 10^{-4} (\text{Davier et al.; PDG}) \\ 3.3 \cdot 10^{-4} (\text{Burkhardt, Pietrzyk}) \end{cases}$$

Now that M_H is known, tight constraint on M_W-m_t, (depending on how aggressive is the error on $\alpha(M_Z)$).

But: precision on M_W is more important now ...

Top and vacuum stability

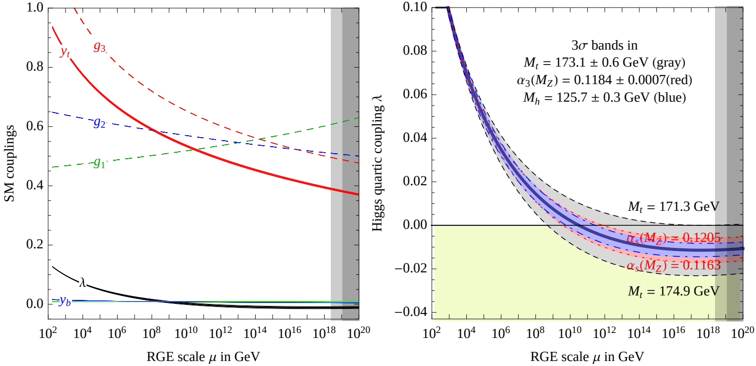
Degrassi et al. 2012



With current value of M_t and M_H the vacuum is metastable.
No indication of new physics up to the Plank scale from this.

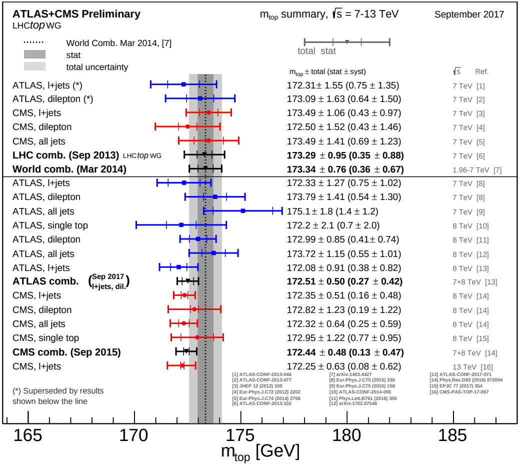
Top and vacuum stability

Degrassi et al. 2012



The quartic coupling λ_H becomes tiny at very high field values, and may turn negative, leading to vacuum instability.
M_t as low as 171 GeV leads to $\lambda_H \to 0$ at the Plank scale.

Top Mass Measurements



DIRECT MEASUREMENTS

(roughly, from the mass of the system of decay products). The most precise method as of now.

Add: CMS 13 TeV, 172.25±0.08 (stat+JSF) ±0.62 (syst) GeV

Theory issues

- The measurement is performed by reconstructing a top mass peak out of a reconstructed W and a b-jet.
- The reconstructed mass is only loosely related to the top mass (i.e. it cannot be identified with the top mass, for obvious reasons, since it is a colourless system).
- The extracted mass is the mass parameter in the Monte Carlo that yields the best fit to the reconstructed mass distribution.

So:

- in which renormalization scheme is the MC mass parameter? Pole mass? $\overline{\mathrm{MS}}$ mass?
- It has been argued that since MC are Leading-Order, they can't distinguish between Pole and $\overline{\mathrm{MS}}$ mass (the difference is around 10 GeV ...).

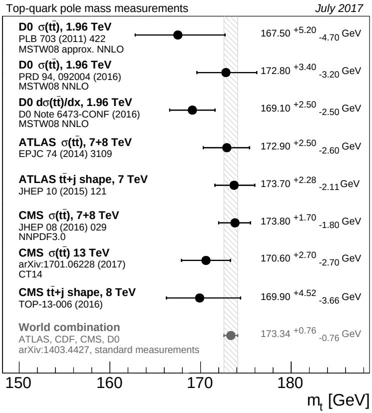
Selected Th. results relevant to top mass measurements

- Narrow width $t\bar{t}$ production and decay at NLO, Bernreuther,Brandenbourg,Si,Uwer 2004, Melnikov,Schulze 2009.
- $l\nu l\nu b\bar{b}$ final states with massive b, Frederix, 2013, Cascioli,Kallweit,Maierhöfer,Pozzorini, 2013.
- NNLO differential top decay, Brucherseifer,Caola,Melnikof 2013.
- NLO+PS in production and decay, Campbell,Ellis,Re,PN
- NNLO production, Czakon,Heymes,Mitov,2015.
- $l\nu l\nu b\bar{b} + \text{jet}$ Bevilacqua,Hartanto,Kraus,Worek 2016.
- Approx. NNLO in production and exact NNLO in decay for $t\bar{t}$. Gao,Papanastasiou 2017.
- Resonance aware formalism for NLO+PS: Ježo,PN 2015;
- Off shell + interference effects+PS, Single top, Frederix,Frixione,Papanastasiou,Prestel,Torielli, 2016
- Off shell + interference effects+PS, $l\nu l\nu b\bar{b}$, Jeo,Lindert,Oleari,Pozzorini,PN, 2016.

Alternative mass-sensitive observables

- Butenschoen,Dehnadi,Hoang,Mateu,Preisser,Stewart,2016 Use boosted top jet mass + SCET.
- Agashe,Franceschini,Kim,Schulze,2016: peak of b-jet energy insensitive to production dynamics.
- Kawabata,Shimizu,Sumino,Yokoya,2014: shape of lepton spectrum. Insensitive to production dynamics and claimed to have reduced sensitivity to strong interaction effects.
- Frixione, Mitov: Selected lepton observables.
- Alioli, Fernandez, Fuster, Irles, Moch, Uwer, Vos ,2013; Bayu etal: M_t from $t\bar{t}j$ kinematics.
- $t\bar{t}$ threshold in $\gamma\gamma$ spectrum (needs very high luminosity), Kawabata,Yokoya,2015

From total cross section and $t\bar{t}j$ kinematics



It is claimed that since higher order calculations (NNLO for total cross section, NLO for $t\bar{t}j$ shape variables) are used in this determination, one is entitle to specify the scheme used for the mass.

In the figure they are quoted as "pole mass measurement".

- The "pole mass" attribute is not given to direct measurement.
- In some experimental papers and talks, direct measurements are reported as "Monte Carlo Mass" measurements, often stating that they need some theoretical interpretation.
- "Monte Carlo Mass" measurements are often interpreted as pole mass measurements by theorists. See for example
  - Degrassi etal, 2012 on the EW vacuum stability, adding a further 250 MeV error to direct measurements.
  - Ciuchini etal, 2017 in Global EW fits, adding a further 500 MeV error to direct measurements.
- Theorist have done work in proposing alternative methods to avoid the issues on direct measurements; however, the alternative methods are generally inferior in precision.

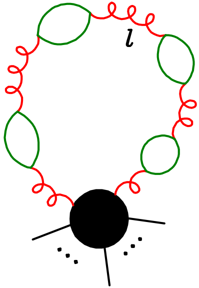
As a result, the most precise experimental results on m_t are left in a limbo, waiting for some illuminating theoretical interpretation that is not in sight.

High-school quiz on top mass measurement

Tick the correct statements:

- ☐ Direct top mass measurements measure the Pole Mass.
- ☐ Direct top mass measurements measure the Monte Carlo Mass.
- ☐ Direct top mass measurements measure the Monte Carlo Mass. but you can pretend that it is the pole mass, just inflate the error a bit.
- ☐ The top is the only SM particle with more than one mass.
- ☐ You should use only leptons to avoid hadronization uncertainty.
- ☐ You should use at least NLO calculations to measure the pole mass.
- ☐ The top pole mass has renormalons, you should stay away from it.
- The MC mass differs from the pole mass by
  ☐ terms of order $m\alpha_S$; ☐ terms of order $\Lambda_{\rm QCD}$; ☐ terms of order $\alpha_S\Gamma_t$.
- The Pole Mass renormalon ambiguity is
  ☐ $\approx$ 1GeV; ☐ $\approx$ 250 MeV; ☐ $\approx$ 200 MeV; ☐ $\approx$ 110 MeV.

All-orders contributions to QCD amplitude of the form



$$\int_0^m \mathrm{d}k^p\, \alpha_s(k^2) = \int_0^m \mathrm{d}k^p \frac{\alpha_s(m^2)}{1 + b_0\alpha_s(m^2)\log\frac{k^2}{m^2}}$$

$$= \alpha_s(m^2) \sum_{n=0}^{\infty} (2b_0\alpha_s(m^2))^n \int_0^m \underbrace{\mathrm{d}k^p \log^n \frac{m}{k}}_{p^n n!} .$$

Asymptotic expansion.

- Minimal term at $n_{\min} \approx \frac{1}{2pb_0\alpha_S(m^2)}$.
- Size of minimal term: $m^p\alpha_s(m^2)\sqrt{2\pi n_{\min}}e^{-n_{\min}} \approx \Lambda^p_{\mathrm{QCD}}$.
- Typical scale dominating at order α_s^{n+1}: $m \exp(-np)$.
- OPE connection; for a short distance process:

$$\int \mathrm{d}^4 l (\text{ph.sp})\, l^2 (\text{gauge inv.}) l^{-2} (\text{gluon prop.}) \propto \Lambda^4 (\text{i.e. a } G^2 \text{ VeV}).$$

Motivation

- Linear (i.e. $p = 1$) renormalons may affect top mass measurements at order Λ (near the present experimental accuracy). Until now, only the top pole mass renormalons has received some attention.
- Several other sources of linear renormalons come into play in top mass measurements (for example, from jet definition). What is their structure, and what is their interplay with the pole mass renormalon?
- There is a temptation to use resummation (typically within SCET) to parametrize linear non perturbative effects in top mass measurement. Is this sound?

Abstract:

The resummed Drell-Yan cross section in the double-logarithmic approximation suffers from infrared renormalons. Their presence was interpreted as an indication for non-perturbative corrections of order $\Lambda_{\rm QCD}/(Q(1-z))$. We find that, once soft gluon emission is accurately taken into account, the leading renormalon divergence is cancelled by higher-order perturbative contributions in the exponent of the resummed cross section.

Their calculation: leading N_f one gluon correction:

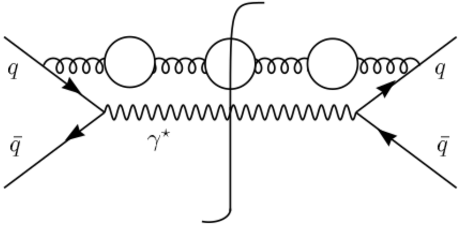
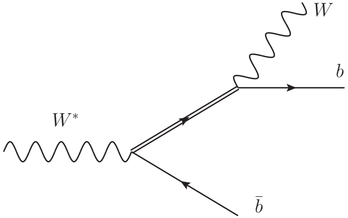


Figure 1: α_s^4-contribution to the partonic Drell-Yan cross section. $\gamma^\star$ represents a photon with invariant mass Q^2 that splits into a lepton pair.

Set up the computation of top mass sensitive observables in leading N_f one gluon correction.

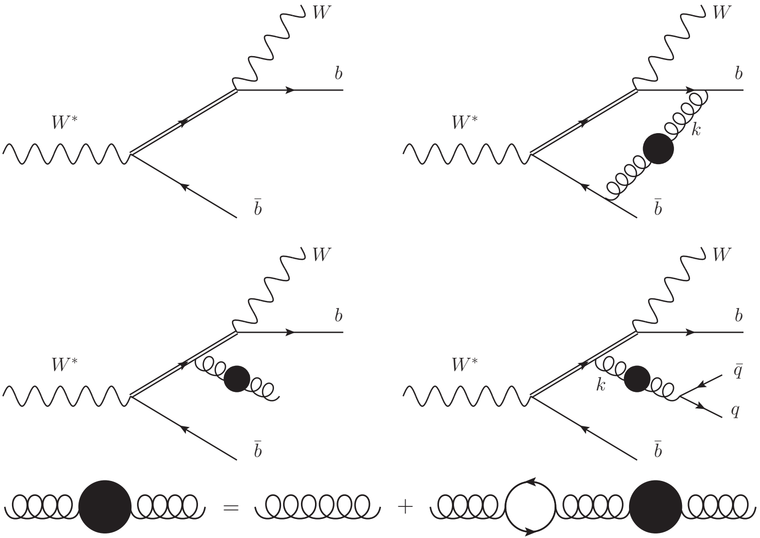
We consider a simplified production framework $W^* \to Wt\bar{b}$:



(i.e. no incoming hadrons). However:

- We consider as our basic observable the mass of the system comprising a b-jet and the W.
- The b is taken massless, the W is taken stable, but the top is taken unstable, with a finite width.

Diagrams up to leading N_f one gluon correction



All-order result

Introducing the notation

- Φ_b, phase space for $Wb\bar{b}$;
- Φ_g, phase space for $Wb\bar{b}g^*$, where g^* is a massive gluon with mass k^2,
- $\Phi_{q\bar{q}}$, phase space for $Wb\bar{b}q\bar{q}$, with $\mathrm{d}\Phi_{q\bar{q}} = \frac{\mathrm{d}k^2}{2\pi}\mathrm{d}\Phi_{g^*}\mathrm{d}\Phi_{\mathrm{dec}}$

the all-order result can be expressed in terms of

- $\sigma_b(\Phi_b)$, the differential cross section for the Born process;
- $\sigma_v(k^2, \Phi_b)$, the virtual correction to the Born process due to the exchange of a gluon of mass k;
- The real cross section $\sigma_{g^*}(k^2, \Phi_{g^*})$, obtained by adding one massive gluon to the Born final state;
- The real cross section $\sigma_{q\bar{q}}(\Phi_{q\bar{q}})$, obtained by adding a $q\bar{q}$ pair, produced by a massless gluon, to the Born final state;

All-order result

Consider a (IR safe) final state observable O. Define:

$$
\begin{aligned}
N^{(0)} &= \left[\int \mathrm{d}\Phi_{\mathrm{b}}\, \sigma_{\mathrm{b}}\right]^{-1}, \quad \langle O\rangle_{\mathrm{b}} = N^{(0)} \int \mathrm{d}\Phi_{\mathrm{b}}\, \sigma_{\mathrm{b}}(\Phi_{\mathrm{b}})\, O(\Phi_{\mathrm{b}})\,, \\
\widetilde{V}(k^2) &= N^{(0)} \int \mathrm{d}\Phi_{\mathrm{b}}\, \sigma_{\mathrm{v}}^{(1)}(k^2, \Phi_{\mathrm{b}}) \left[O(\Phi_{\mathrm{b}}) - \langle O\rangle_{\mathrm{b}}\right], \\
\widetilde{R}(k^2) &= N^{(0)} \int \mathrm{d}\Phi_{g^*}\, \sigma_{g^*}^{(1)}(k^2, \Phi_{g^*}) \left[O(\Phi_{g^*}) - \langle O\rangle_{\mathrm{b}}\right], \\
\widetilde{\Delta}(k^2) &= \frac{1}{2}\frac{3}{\alpha_s T_{\mathrm{F}}}\, k^2\, N^{(0)} \int \mathrm{d}\Phi_{\mathrm{dec}}\, \mathrm{d}\Phi_{g^*}\, \sigma_{q\bar{q}}^{(2)}(\Phi_{q\bar{q}}) \times [O(\Phi_{q\bar{q}}) - O(\Phi_{g^*})]
\end{aligned}
$$

$\langle O\rangle_{\mathrm{b}} + \widetilde{V}(k^2) + \widetilde{R}(k^2)$ is the average value of O in a theory with a massive gluon with mass k^2, accurate to order α_s.

Notice: $\widetilde{V}(k^2) + \widetilde{R}(k^2)$ has a finite limit for $k^2 \to 0$, while each contribution is log divergent.

defining $\widetilde{T}(k^2) = \widetilde{V}(k^2) + \widetilde{R}(k^2) + \widetilde{\Delta}\left(k^2\right)$ our final result is

$$\langle O \rangle = \langle O \rangle_{\mathrm{b}} - \frac{3\pi}{\alpha_s T_{\mathrm{F}}} \int_0^\infty \frac{\mathrm{d}k}{\pi} \frac{\mathrm{d}}{\mathrm{d}k} \left[\widetilde{T}(k^2) \right] \mathrm{Im} \left\{ \log \left[1 + \Pi\left(k^2, \mu^2\right) - \Pi_{\mathrm{ct}} \right] \right\},$$

where

$$\Pi\left(k^2, \mu^2\right) - \Pi_{\mathrm{ct}} = \alpha_s b_0 \left(\log \frac{k^2}{\mu^2} = \frac{5}{3} - i\pi \right), \qquad b_0 = -\frac{4 N_{\mathrm{F}} T_{\mathrm{F}}}{12\pi}$$

So

$$\mathrm{Im} \left\{ \log \left[1 + \Pi\left(k^2, \tilde{\mu}^2\right) - \Pi_{\mathrm{ct}} \right] \right\} = -\mathrm{atan} \left(\frac{\alpha_s \pi b_0}{1 + \alpha_s b_0 log \frac{k^2}{\tilde{\mu}^2}} \right)$$

that essentially exhibits the same Landau pole discussed earlier.
If we thus have:

$$\widetilde{T}(k^2) = a + b\, k + \mathcal{O}(k^2) \tag{1}$$

we have a linear renormalon in our result.

Comments

- In order to get our results, we need $\lim_{k^2 \to \infty} \widetilde{T}(k^2) = 0\,.$

  This happens if we use the Pole Mass Scheme for m_t.
- The need to include the Δ term has a long story:
  - Seymour,P.N. 1995, I.R. renormalons in e^+e^- event shapes.
  - Dokshitzer,Lucenti,Marchesini,Salam, 1997-1998 Milan factor
- We compute $T(k^2)$ numerically. The $k^2 \to 0$ limit implies the cancellation of two large logs in V and R. However, the precise value at $k^2 = 0$ can also be computed directly by standard means (which we do).

Changing the mass scheme

The relation of the pole mass as a function of the $\overline{\mathrm{MS}}$ mass in the large N_{F} approximation is well known (Beneke, 1999)

$$
\begin{aligned}
m &= \bar{m}[1 + R_f(\alpha_s, \mu, \bar{m}) + R_d(\alpha_s, \mu, \bar{m})], \\
R_f &= -\frac{3\pi}{\alpha_s T_F} \int_0^\infty \frac{\mathrm{d}k}{\pi} \frac{\mathrm{d}r_f(k^2)}{\mathrm{d}k} \mathrm{atan} \frac{-\alpha_s \pi b_0}{1 + \alpha_s b_0 \log \frac{k^2}{\tilde{\mu}^2}} \\
r_f(k^2) &= -\alpha_s \frac{C_F}{2} \frac{k}{m} \,.
\end{aligned} \tag{2}
$$

We can easily convert our results to the $\overline{\mathrm{MS}}$ scheme:

$$
\langle O \rangle_{\mathrm{b}}(m, m^*) = \langle O \rangle_{\mathrm{b}}(\overline{m}, \overline{m}^*) + \left\{ \frac{\partial \langle O \rangle_{\mathrm{b}}(\overline{m}, \overline{m}^*)}{\partial \overline{m}} (m - \overline{m}) + \mathrm{cc} \right\}
$$

For the leading renormalon this amounts to

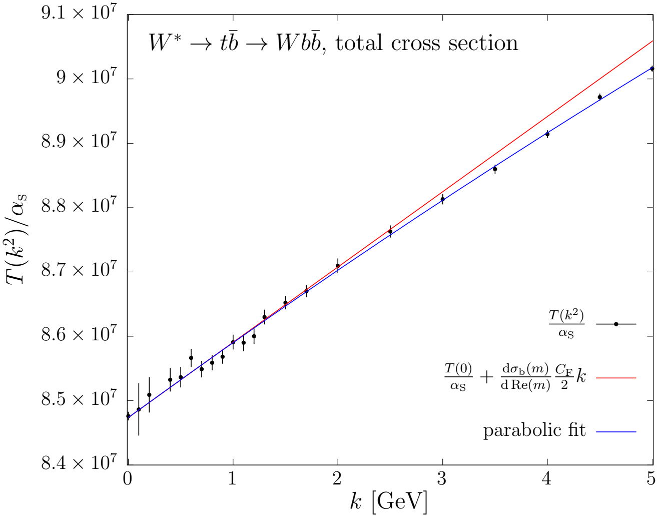
$$
\widetilde{T}(k^2) \to \widetilde{T}(k^2) - \frac{\partial \langle O \rangle_{\mathrm{b}}(\overline{m}, \overline{m}^*)}{\partial \mathrm{Re}(\overline{m})} \frac{C_{\mathrm{F}} \alpha_s}{2} k + \mathcal{O}(k^2) \,.
$$

Selected Results,

some obvious,

some puzzling . . .

Total cross section



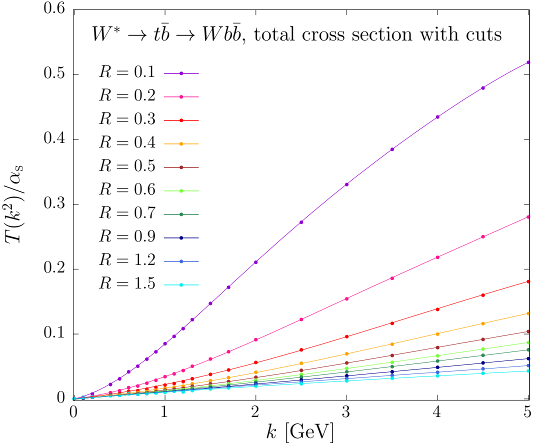
No linear renormalon in $\overline{\mathrm{MS}}$ scheme!

Total cross section

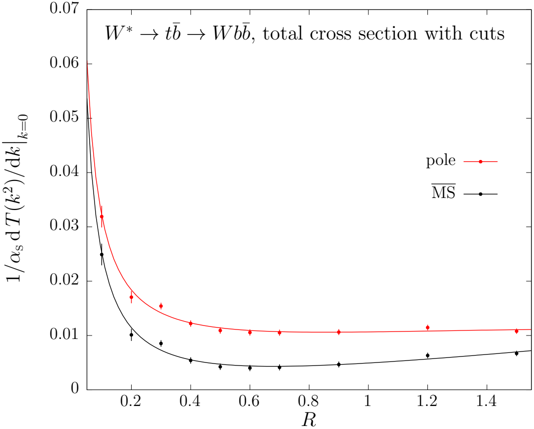
- For $k < \Gamma$: no renormalon in the physics! The top finite width screens the soft sensitivity of the cross section.
  The renormalon is there only if it is present in the mass counterterm; thus, it is not there in the $\overline{\mathrm{MS}}$ scheme.
- What about $k \gg \Gamma$?
  This is the narrow width limit: the cross section factorizes into a production cross section and a partial width.
  The former has no physical renormalons for obviour reasons.
  The latter does not have them (not obvious at all?)

So, the mass from the total σ is free of linear power corrections?

Total cross section with cuts

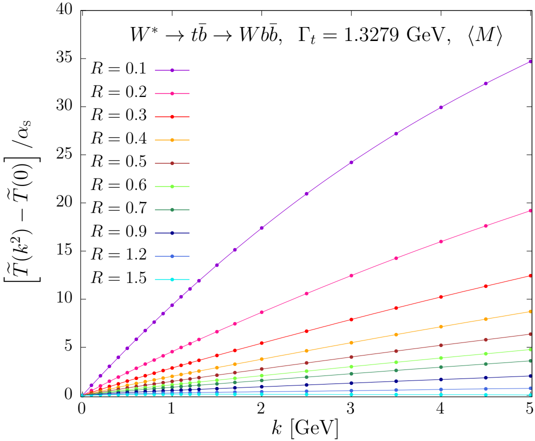


Total cross section with cuts

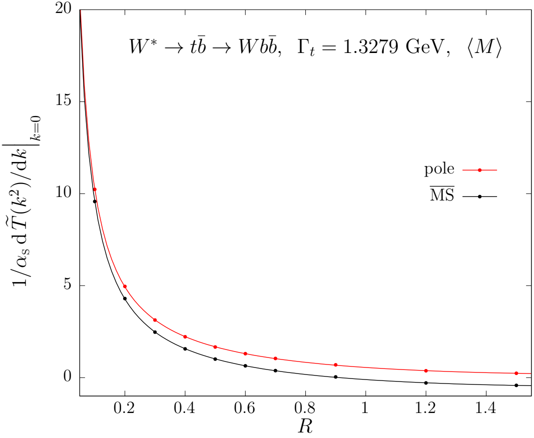


The requirement of a *b* jet spoils this conclusion!

Reconstructed top mass

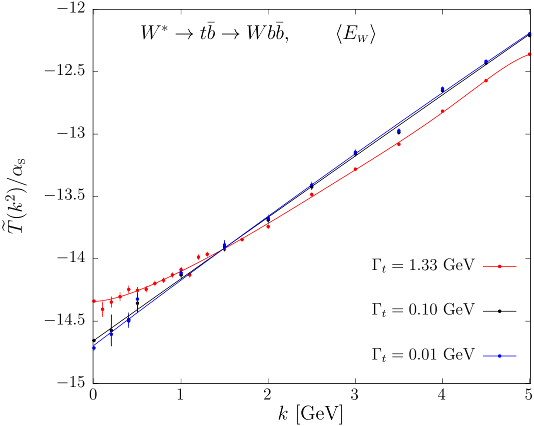


Reconstructed top mass



For large radii, m_{pole} is better!

Choose as mass sensitive observable the average E_W.



For $k \gg \Gamma$, the slope is roughly 0.45. The $\overline{\mathrm{MS}}$ conversion would add -0.067.

Leptonic Observables

It seems that physical linear renormalons are present also in leptonic observables.

But, for $k \ll \Gamma$, the slope of $T(k)$ decreases, approaching 0.067!

So, the top finite width screens the linear renormalons!

Is this an exact statement?

Need better understanding of the theory of soft cancellation.

Back to Old Fashion Perturbation Theory!
(where the KLN theorem comes from!)

Old Fashion Perturbation Theory

The propagator denominators in a Feynmann diagram can be split into an advanced and a retarded part:

$$\frac{i}{k^2 - m^2 + i\epsilon} = \frac{i}{2E_{k,m}} \left[\frac{1}{k^0 - E_{k,m} + i\epsilon} + \frac{1}{-k^0 - E_{k,m} + i\epsilon} \right].$$

The time Fourier transform of the first term vanishes for negative time, while for the second term it vanishes for positive time

$$\begin{aligned} \int \frac{\mathrm{d}k^0}{2\pi} \frac{i \exp(-ik^0 t)}{k^0 - E_{k,m} + i\epsilon} &= \theta(t) \exp(-iE^0_{k,m} t) \\ \int \frac{\mathrm{d}k^0}{2\pi} \frac{i \exp(-ik^0 t)}{-k^0 - E_{k,m} + i\epsilon} &= \theta(-t) \exp(iE^0_{k,m} t) \end{aligned}$$

Old Fashion Perturbation Theory

also true for unstable particles:

$$\frac{i}{k^2 - m^2 + im\Gamma} = \frac{i}{2E_{k,m,\Gamma}} \left[\frac{1}{k^0 - E_{k,m,\Gamma}} + \frac{1}{-k^0 - E_{k,m,\Gamma}} \right],$$

where

$$E_{k,m,\Gamma} = \sqrt{\underline{k}^2 + m^2 - im\Gamma},$$

so that $E_{k,m,\Gamma}$ has a negative imaginary part. As a consequence, we will also have

$$\begin{aligned} \int \frac{\mathrm{d}k^0}{2\pi} \frac{i \exp(-ik^0 t)}{k^0 - E_{k,m,\Gamma} + i\epsilon} &= \theta(t) \exp(-iE^0_{k,m,\Gamma} t) \\ \int \frac{\mathrm{d}k^0}{2\pi} \frac{i \exp(-ik^0 t)}{-k^0 - E_{k,m,\Gamma} + i\epsilon} &= \theta(-t) \exp(iE^0_{k,m,\Gamma} t) \end{aligned}$$

and both functions will have exponential damping for large positive (negative) time. But the θ functions are there as before.

Old Fashion Perturbation Theory

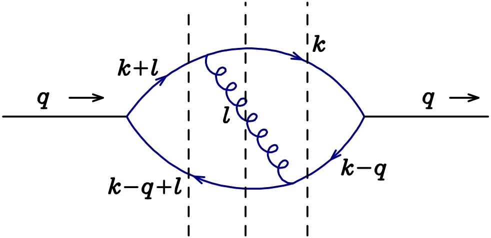
A straightforard manipulation leads to the old fashion perturbation theory rules. In time ordered graphs:

- Split propagators into an advanced and retarded part, and split each Feynmann graph into a sum of time ordered graphs.
- Replace propagators with $1/(2E_{k,m})$
- Put all propagator energies in numerators equal to their on-shell values.
- Include all 3-momentum integrals.
- For each external incoming momentum a line coming from $-\infty$ or going to $+\infty$, carrying momentum and energy with corresponding sign.
- For each intermediate state, include an energy denominator

$$\frac{i}{e - e_i + i\epsilon}$$

  where e is the sum of the incoming energy (from the lines to $-\infty$) and e_i is the sum of the energies of the lines in the intermediate state.

Old Fashion Perturbation Theory



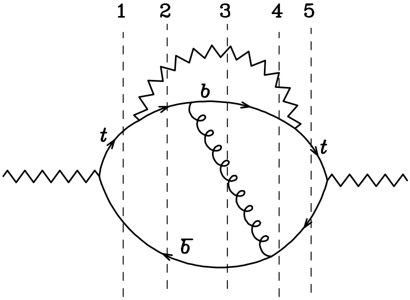
$$\frac{1}{q^0 - E_{k+l} - E_{k-q+l} + i\epsilon} \frac{1}{q^0 - E_k - E_l - E_{k-q+l} + i\epsilon} \frac{1}{q^0 - E_k - E_{k-q} + i\epsilon}$$

Singularities are present only if the momentum integration cannot be displaced in the complex plane away from the poles.
This simply leads to the threshold singularities, and to the Landau conditions for anomalous thresholds.
Away from those, the graph is an analytic function of q^0.

Old Fashion Perturbation Theory

This leads to KLN cancellation of soft singularities.
But it leads to more: the soft sensitivity is the same when q^0 picks up a finite imaginary part!
The energy denominators do not count any more for the soft sensitivity. Only the $\mathrm{d}^3 l/E_l$ counts. Same sensitivity as in Euclidean power counting: $\mathrm{d}^4 l/l^2$. Two more powers of l come from gauge invariance, leading to 4^{th} order power corrections.

back to E_W



Only 2,3,4 cuts should be considered. But:
1 and 5 denominators have opposite imaginary part of order Γ.

$$\mathrm{Im}\left[\frac{1}{E-E_W-E_{b,2}-E_{\bar{b},1}+i\epsilon}\,\frac{1}{E-E_W-E_{b,3}-E_{\bar{b},1}-E_{g,3}+i\epsilon}\,\frac{1}{E-E_W-E_{b,3}-E_{\bar{b},4}+i\epsilon}\right]$$

Analyticity is still there, but the imaginary part of q^0 cannot exceed Γ!.

So: soft sensitivity higher than linear below Γ!

Puzzle: why this does not work also above Γ?

(as is the case for the total cross section) cross section?

Conclusions

Much more to come ...