



Towards higher logarithmic resummation of non-global observables

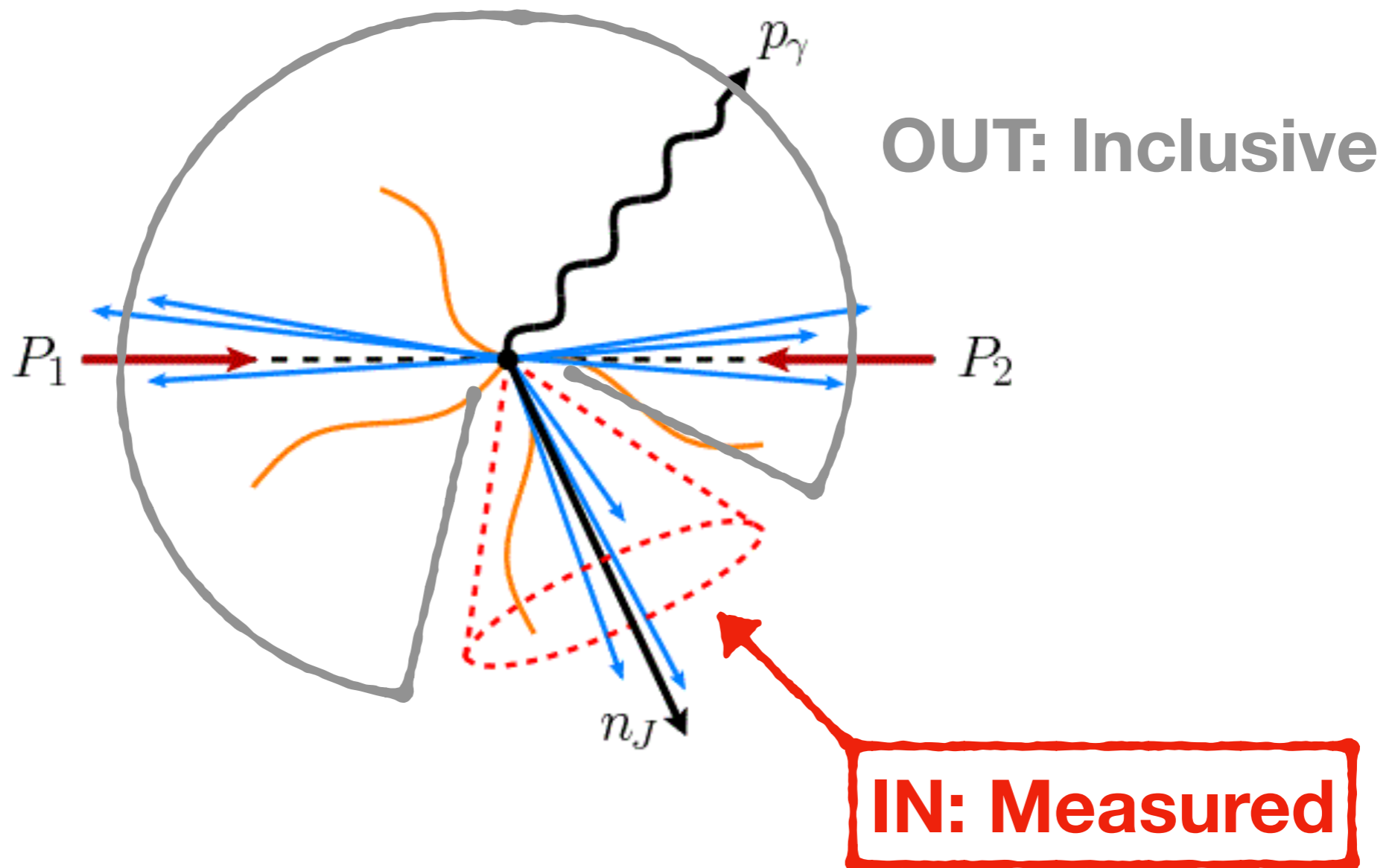
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High Time for Higher Orders

MITP 24.08.2018

A brief review

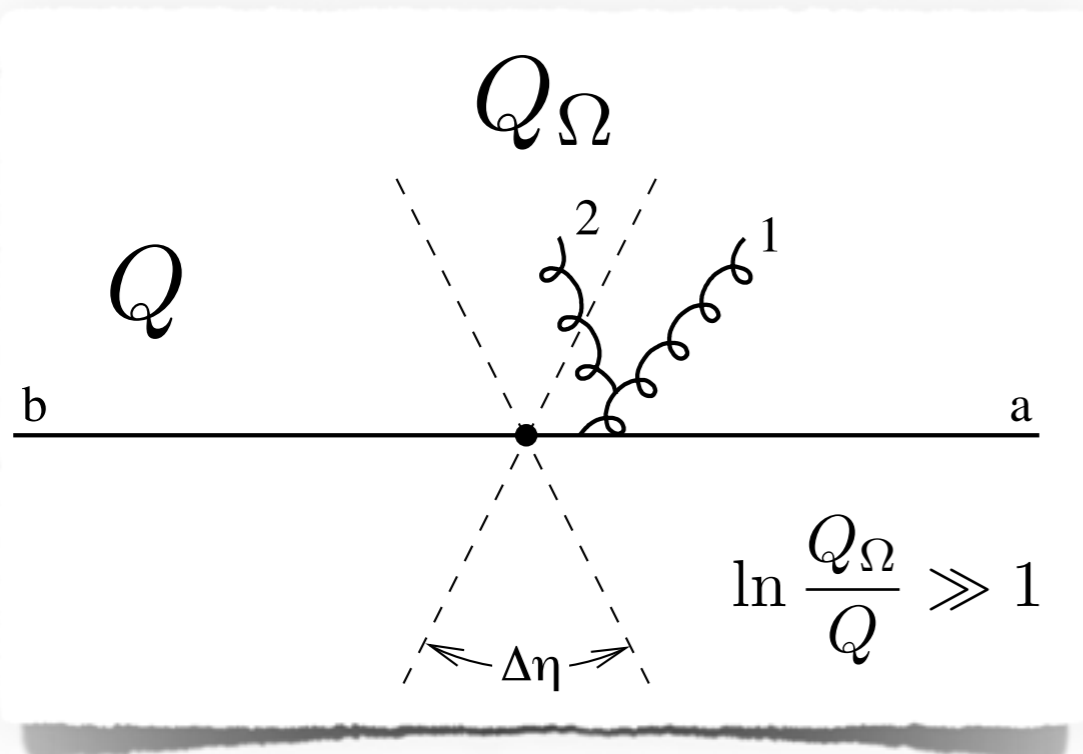
Non-global observable: Observables which are insensitive to emissions into certain regions of phase space



Non-Global Logs

- Non-global observables involve additional NGLs **not captured** by the usual resummation formula
- Exponentiating soft anomalous dimension only resum part of logs

$$\exp \left[-4 C_F \Delta\eta \int_{\alpha(Q_\Omega)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)} \frac{\alpha}{2\pi} \right] = 1 + 4 \frac{\alpha_s}{2\pi} C_F \Delta\eta \ln \frac{Q_\Omega}{Q} + \left(\frac{\alpha_s}{2\pi} \right)^2 \left(8 C_F^2 \Delta\eta^2 - \frac{22}{3} C_F C_A \Delta\eta + \frac{8}{3} C_F T_F n_f \Delta\eta \right) \ln^2 \frac{Q_\Omega}{Q}$$



Non-global logs:

$$\left(\frac{\alpha_s}{2\pi} \right)^2 C_F C_A \left[-\frac{2\pi^2}{3} + 4 \text{Li}_2 (e^{-2\Delta\eta}) \right] \ln^2 \frac{Q_\Omega}{Q}$$

(Dasgupta & Salam 2002)

LL resummation for non-global observables

- The leading logarithms arise from configuration in which the emitted gluons are strongly ordered

$$E_1 \gg E_2 \gg \dots \gg E_m$$

- In the large- N_c limit, multi-gluon emission amplitudes become simple:

$$N_C^m g_s^{2m} \sum_{(1, \dots, m)} \frac{p_a \cdot p_b}{(p_a \cdot p_1)(p_1 \cdot p_2) \dots (p_m \cdot p_b)}$$

- Dasgupta-Salam shower

$$S(\alpha_s L) \simeq \exp\left(-C_F C_A \frac{\pi^2}{3} \left(\frac{1 + (at)^2}{1 + (bt)^c}\right) t^2\right) \quad \begin{array}{l} a = 0.85 C_A, \quad b = 0.86 C_A \\ c = 1.33 \end{array}$$

(Dasgupta & Salam 2001)

- Banfi-Marchesini-Smye equation

$$\partial_{\hat{L}} G_{kl}(\hat{L}) = \int \frac{d\Omega(n_j)}{4\pi} W_{kl}^j \left[\Theta_{\text{in}}^{n\bar{n}}(j) G_{kj}(\hat{L}) G_{jl}(\hat{L}) - G_{kl}(\hat{L}) \right]$$

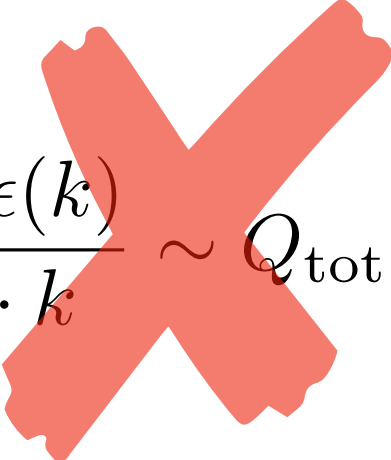
(Banfi, Marchesini & Smye 2002)

Some recent progress

- **Dressed gluon expansion** Larkoski, Moult & Neill '15 '16
- **Color density matrix** Caron-Huot '15
- **Multi-Wilson-line theory in SCET** Becher, Neubert, Rothen & DYS '15 '16
- **Soft (Glauber) gluon evolution at amplitude level, finite N_c** Martínez, Angelis, Forshaw, Plätzer & Seymour '18
- **Reduced density matrix** Neill & Vaidya '18

Soft radiations inside Non-global observables

- Non-global observables: soft radiation resolves the colors and directions of individual energetic partons.

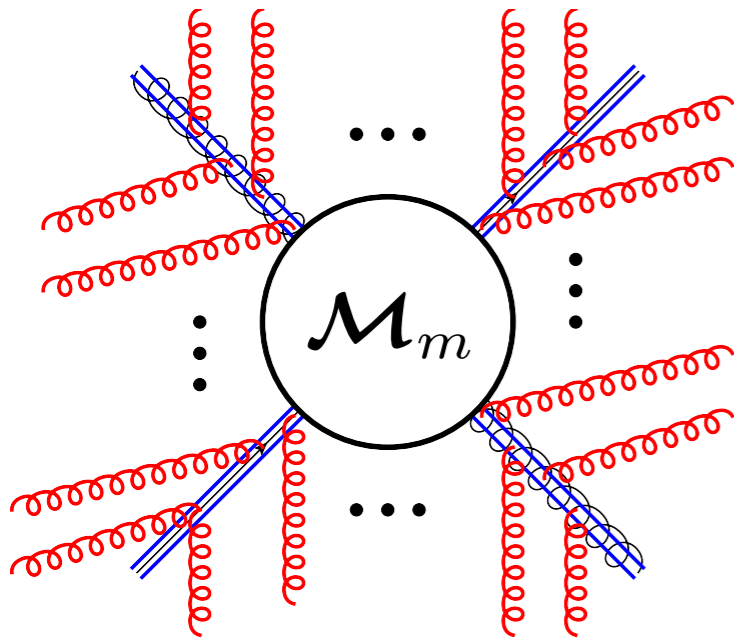
$$\sum_i Q_i \frac{p_i \cdot \epsilon(k)}{p_i \cdot k} \sim Q_{\text{tot}} \frac{n \cdot \epsilon}{n \cdot k}$$


- For a wide-angle jet, the energetic particles are not collinear.
- For a narrow-angle jets, we find that small-angle soft radiation plays an important role. Resolves directions of individual energetic partons!

Becher, Neubert, Rothen & DYS '15

Factorization

- The operator for the emission from an amplitude with m hard partons



hard scattering amplitude with m particles
(vector in color space)

$$\mathcal{S}_1(n_1) \mathcal{S}_2(n_2) \dots \mathcal{S}_m(n_m) |\mathcal{M}_m(\{\underline{p}\})\rangle$$

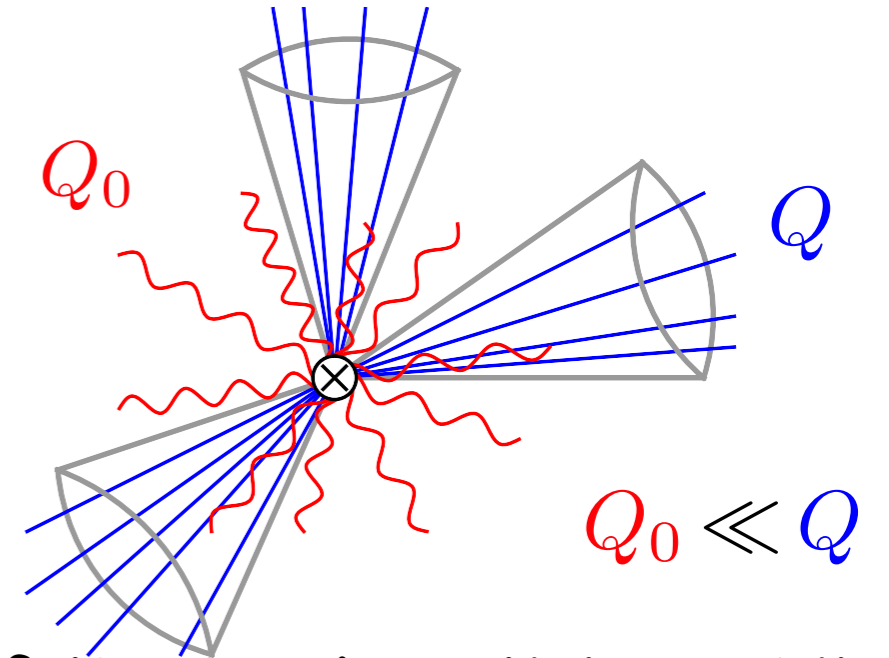
soft Wilson lines along the directions of the
energetic particles (color matrices)

$$\mathcal{S}_i(n_i) = \mathbf{P} \exp \left(ig_s \int_0^\infty ds n_i \cdot A_s^a(sn_i) \mathbf{T}_i^a \right)$$

Factorization for jet cross section

- For k jets process at lepton collider

$$d\sigma(Q, Q_0) = \sum_{m=k}^{\infty} \langle \mathcal{H}_m(\{\underline{n}\}, Q, \mu) \otimes \mathcal{S}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$



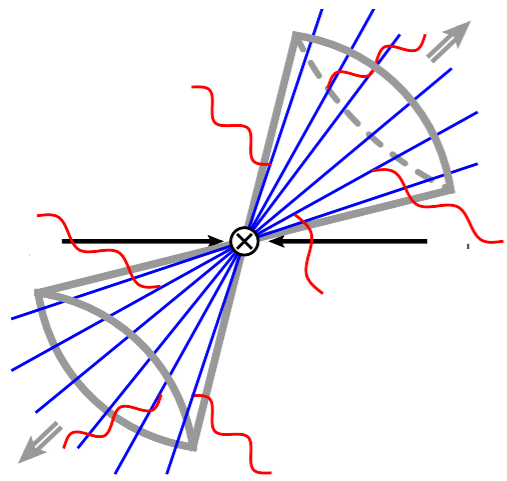
- Hard function: integrating over the energies of the hard particles, while keeping their direction fixed

$$\mathcal{H}_m(\{\underline{n}\}, Q, \mu) = \frac{1}{2Q^2} \sum_{\text{spins}} \prod_{i=1}^m \int \frac{dE_i E_i^{d-3}}{(2\pi)^{d-2}} |\mathcal{M}_m(\{\underline{p}\})\rangle \langle \mathcal{M}_m(\{\underline{p}\})| (2\pi)^d \delta\left(Q - \sum_{i=1}^m E_i\right) \delta^{(d-1)}(\vec{p}_{\text{tot}}) \Theta_{\text{in}}(\{\underline{p}\})$$

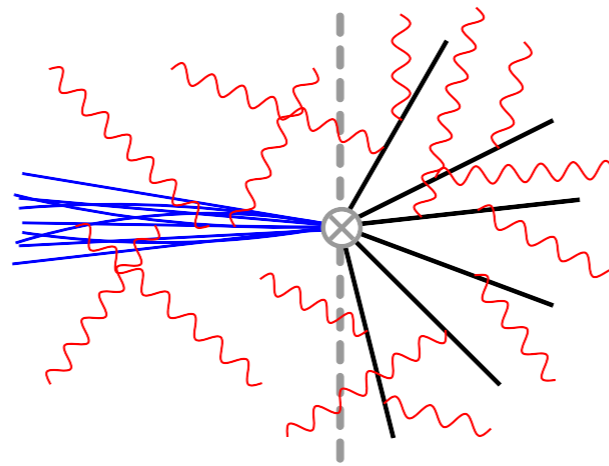
- Soft function

$$\mathcal{S}_m(\{\underline{n}\}, Q_0, \mu) = \sum_{X_s} \langle 0 | \mathcal{S}_1^\dagger(n_1) \dots \mathcal{S}_m^\dagger(n_m) | X_s \rangle \langle X_s | \mathcal{S}_1(n_1) \dots \mathcal{S}_m(n_m) | 0 \rangle \theta(Q_0 - E_{\text{out}})$$

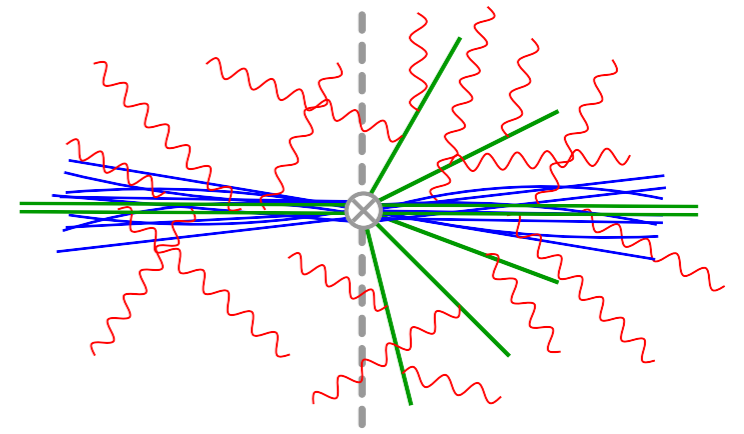
- \otimes indicates integration over the direction of the energetic partons



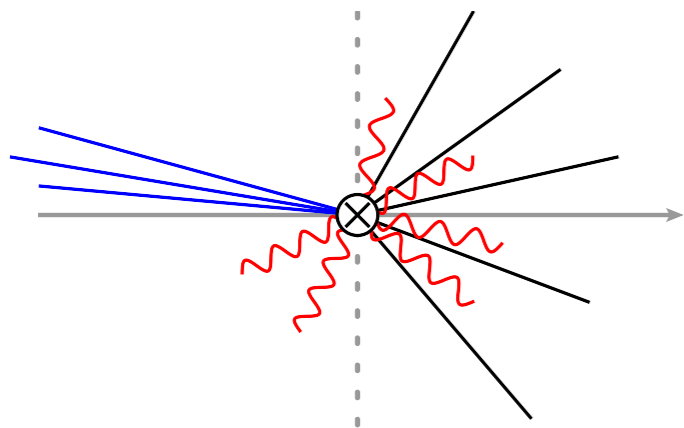
**Jet cross section &
Gap fraction**



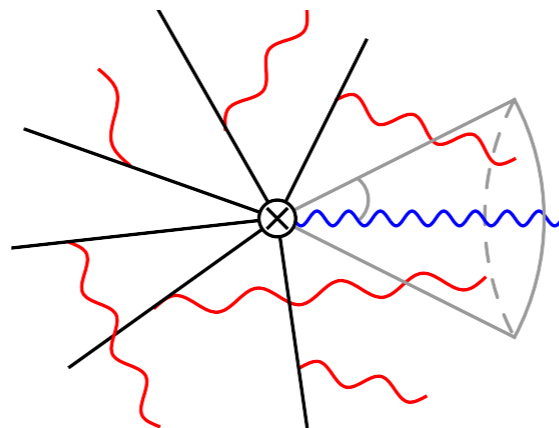
Light-jet mass



Hemisphere soft function



Narrow broadening



Photon isolation

Becher, Neubert, Rothen,
DYS '15 '16

Becher, Pecjak, DYS '16

Becher, Rahn, DYS '17

Balsiger, Becher, DYS, '18

RG evolution & Resummation

Then resummation cross section can be written as

$$d\sigma(Q, Q_0) = \sum_{l=k, m \geq l}^{\infty} \langle \mathcal{H}_l(\{\underline{n}\}, Q, \mu_h) \otimes \mathbf{U}_{lm}(\{\underline{n}\}, \mu_s, \mu_h) \hat{\otimes} \mathcal{S}_m(\{\underline{n}\}, Q_0, \mu_s) \rangle$$

Wilson coefficients fulfill Renormalization Group equation

$$\frac{d}{d \ln \mu} \mathcal{H}_m(Q, \mu) = - \sum_{l=2}^m \mathcal{H}_l(Q, \mu) \Gamma_{lm}^H(Q, \mu)$$

1. Compute \mathcal{H}_m at characteristic high scale $\mu_h \sim Q$
2. Evolve \mathcal{H}_m to the scale of low energy physics $\mu_s \sim Q_0$
3. Compute \mathcal{S}_m at $\mu_s \sim Q_0$

Resum large logarithms $\alpha_s^n \ln^m \frac{Q}{Q_0}$

LL resummation

- LL resummation formula

$$d\sigma_{\text{LL}}(Q, Q_0) = \sum_{m=k}^{\infty} \langle \mathcal{H}_k(\{\underline{n}\}, Q, \mu_h) \otimes \mathbf{U}_{km}(\{\underline{n}\}, \mu_s, \mu_h) \hat{\otimes} \mathbf{1} \rangle$$

One-loop anomalous dimension

$$\mathbf{\Gamma}^{(1)} = \begin{pmatrix} \mathbf{V}_k & \mathbf{R}_k & 0 & 0 & \dots \\ 0 & \mathbf{V}_{k+1} & \mathbf{R}_{k+1} & 0 & \dots \\ 0 & 0 & \mathbf{V}_{k+2} & \mathbf{R}_{k+2} & \dots \\ 0 & 0 & 0 & \mathbf{V}_{k+3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad \begin{aligned} \mathbf{V}_m &= 2 \sum_{(ij)} (\mathbf{T}_{i,L} \cdot \mathbf{T}_{j,L} + \mathbf{T}_{i,R} \cdot \mathbf{T}_{j,R}) \int \frac{d\Omega(n_l)}{4\pi} W_{ij}^l \\ &\quad - 2i\pi \sum_{(ij)} (\mathbf{T}_{i,L} \cdot \mathbf{T}_{j,L} - \mathbf{T}_{i,R} \cdot \mathbf{T}_{j,R}) \Pi_{ij}, \\ \mathbf{R}_m &= -4 \sum_{(ij)} \mathbf{T}_{i,L} \cdot \mathbf{T}_{j,R} W_{ij}^{m+1} \Theta_{\text{in}}(n_{m+1}). \end{aligned}$$

- Collinear divergence regularization

$$W_{ij}^l = \frac{n_i \cdot n_j}{n_i \cdot n_l n_j \cdot n_l} \quad W_{ij}^l \rightarrow W_{ij}^l \theta(n_l \cdot n_i - \lambda^2) \theta(n_l \cdot n_j - \lambda^2)$$

RG evolution = Parton Shower

$$d\sigma_{\text{LL}}(Q, Q_0) = \langle \mathcal{H}_k(t) + \int \frac{d\Omega_1}{4\pi} \mathcal{H}_{k+1}(t) + \int \frac{d\Omega_1}{4\pi} \int \frac{d\Omega_2}{4\pi} \mathcal{H}_{k+2}(t) + \dots \rangle$$

$$\mathcal{H}_k(t) = \mathcal{H}_k(0) e^{t\mathbf{V}_k}$$

$$\mathcal{H}_{k+1}(t) = \int_0^t dt' \mathcal{H}_k(t') \mathbf{R}_k e^{(t-t')\mathbf{V}_{k+1}}$$

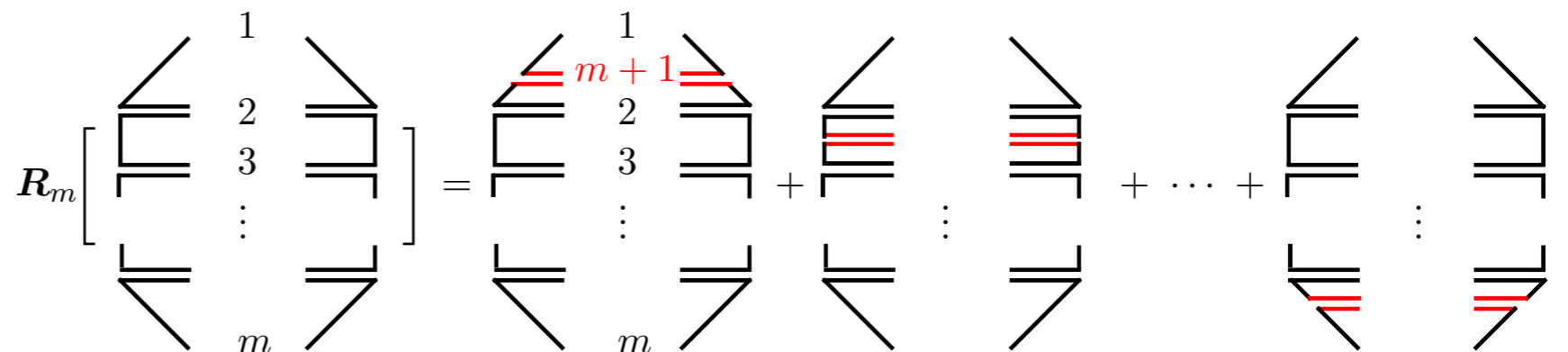
$$t = \int_{\alpha(\mu)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)} \frac{\alpha}{4\pi}$$

$$\mathcal{H}_{k+2}(t) = \int_0^t dt' \mathcal{H}_{k+1}(t') \mathbf{R}_{k+1} e^{(t-t')\mathbf{V}_{k+2}}$$

$$\mathcal{H}_{k+3}(t) = \dots$$

In large N_c limit:

$$\mathbf{T}_i \cdot \mathbf{T}_j \rightarrow -\frac{N_c}{2} \delta_{i,j\pm 1} \mathbf{1}$$



We re-derive Dasgupta-Salam angular dipole shower!!!

Automated resummation for Non-global observables

(Balsiger, Becher, DYS, '18)

$$d\sigma_{\text{LL}}(Q, Q_0) = \sum_{m=k}^{\infty} \langle \mathcal{H}_k(\{\underline{n}\}, Q, \mu_h) \otimes U_{km}(\{\underline{n}\}, \mu_s, \mu_h) \hat{\otimes} \mathbf{1} \rangle$$

- Use Madgraph5_aMC@NLO generator
 - event file with directions and large- N_c color connections of hard partons
 - provides lowest multiplicity hard function for given process
- Run our shower on each event to generate additional partons and write result back into event file
- Analyze events, according to cuts on hard partons, obtain resummed cross section with hard cuts and veto scale

Resummation in isolation cross section

- Experiments use isolation to reduce photon from hard scattering from photons due to hadron decays.

- Experimentalists choose $\sum_{\text{had} \in \mathcal{C}(R)} E_T^{\text{had}} \leq \epsilon_\gamma E_\gamma^T$

- E.g. ATLAS '16 imposes $E_{\text{iso}}^T = 4.8 \text{ GeV} + 0.0042 E_\gamma^T$ on hadronic energy inside cone.

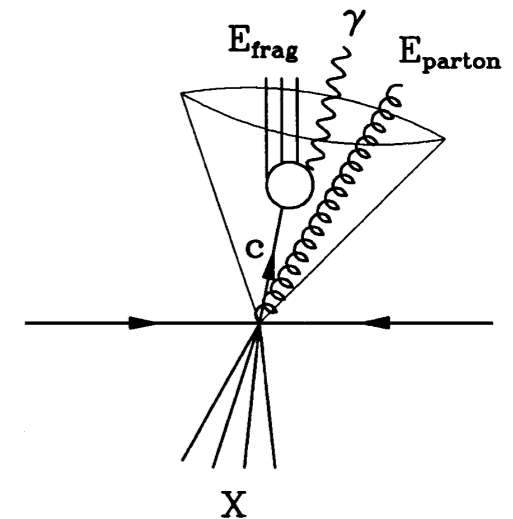
- Smooth isolation Frixione '98 $E_{\text{iso}}(r) = \epsilon_\gamma E_\gamma \left(\frac{1 - \cos r}{1 - \cos R} \right)^n$ $\lim_{r \rightarrow 0} E_{\text{iso}}(r) \rightarrow 0$

- collinear safe; no fragmentation process

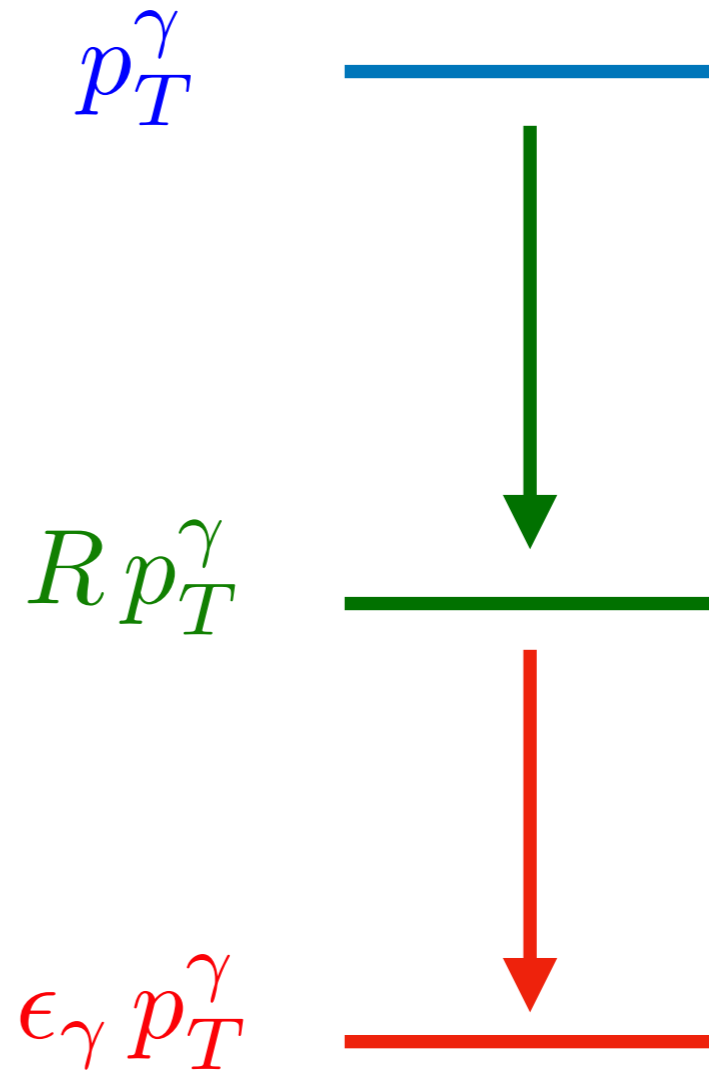
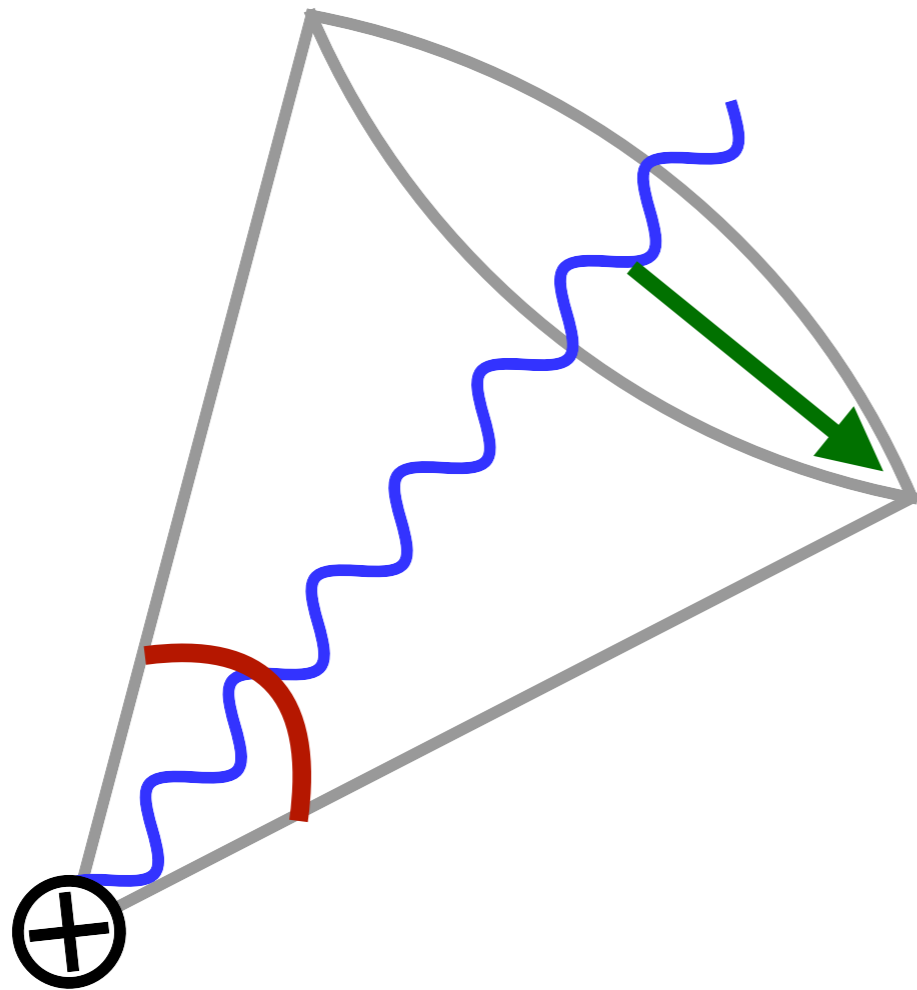
- not applied in experiments now

- Soft-drop isolation Hall & Thaler '18

- democratic criteria; equivalent to smooth isolation at LO



New scales introduced by isolation



Resummation effects in isolation

- Small cone radius $R \ll 1$

Catani & et. al. '02

Isolation radius	Direct contribution		Fragmentation contribution		Total
	Born	NLO	Born	NLO	NLO
1.0	1764.6	3318.4	265.0	446.7	3765.1
0.7	1764.6	3603.0	265.0	495.0	4098.0
0.4	1764.6	3968.9	265.0	555.6	4524.5
0.1	1764.6	4758.2	265.0	678.9	5431.1
Without isolation	1764.6	3341.1	1724.3	1876.8	5217.9

- NLO results are unphysical as $R = 0.1$!!!
- $\text{Log}(R)$ spoils perturbative convergence
- This log comes from mismatching between inside and outside radiation
- LL resummation has been studied by Catani & et. al. '13
- Higher-order effects are moderate for $R \sim 0.4$

Resummation effects in isolation

- Tight isolation cut $\epsilon_\gamma \ll 1$
- Large logs $\ln \epsilon_\gamma$ from soft gluon radiation inside cone
- Fragmentation process are power suppressed
- At the NLO log term is $\alpha_s R^2 \ln \epsilon_\gamma$ Gordon & Vogelsang '94
- NLO results show no significant infrared sensitivity. Catani & et. al. '02

destabilize the numerical convergence of the perturbative expansion. Nonetheless, owing to the presence of higher powers of $\ln \epsilon_h$ at higher perturbative orders, the actual sensitivity of the cross section to very low values of ϵ_h is ~~probably~~ underestimated in the present NLO calculation.

definitely

- **Non-global observables:** more complicated logarithmic terms will appear beyond NLO

$$R^2 \times \alpha_s^n \ln^n \epsilon_\gamma \ln^{n-1} R$$

Collinear limit and NGLs

(Hatta, Iancu, Mueller, Triantafyllopoulos '17)

- E.g. Inter-jet energy flow @ e^+e^- colliders

- Soft radiations from two Wilson lines (global)

$$\frac{\sigma_{\text{GL}}^{\text{LL}}}{\sigma_0} = \exp[-8 C_F \Delta y t]$$

$$t = \int_{\alpha(Q_0)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)} \frac{\alpha}{4\pi}$$

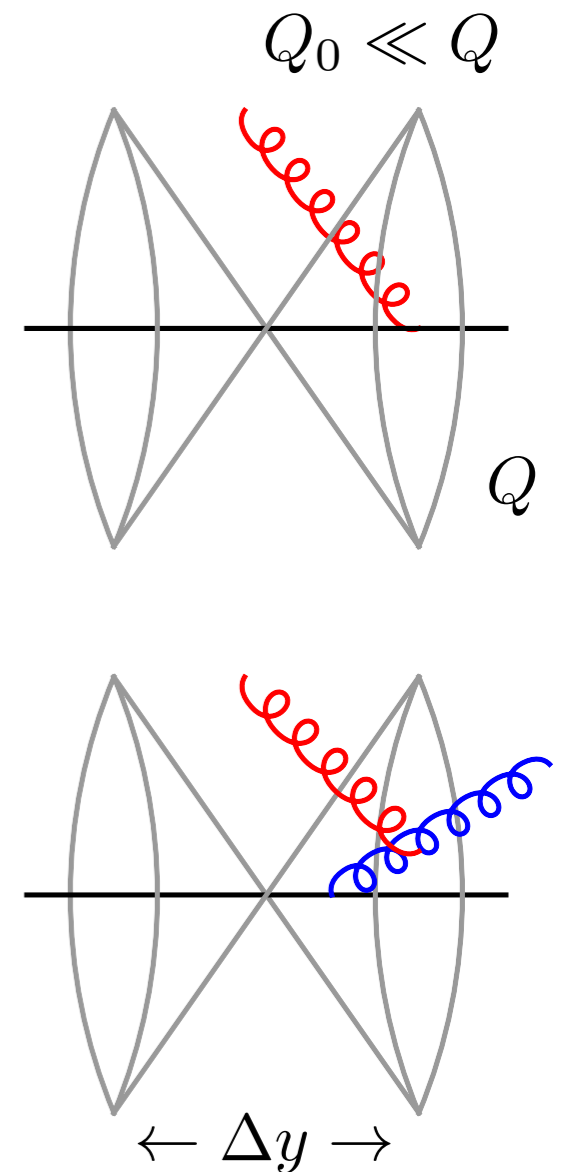
- Leading NGLs at two-loops

$$\frac{\sigma_{\text{NGL}}^{\text{LL}}}{\sigma_0} = 4 C_F C_A \left[-\frac{2\pi^2}{3} + 4 \text{Li}_2(e^{-2\Delta y}) \right] t^2$$

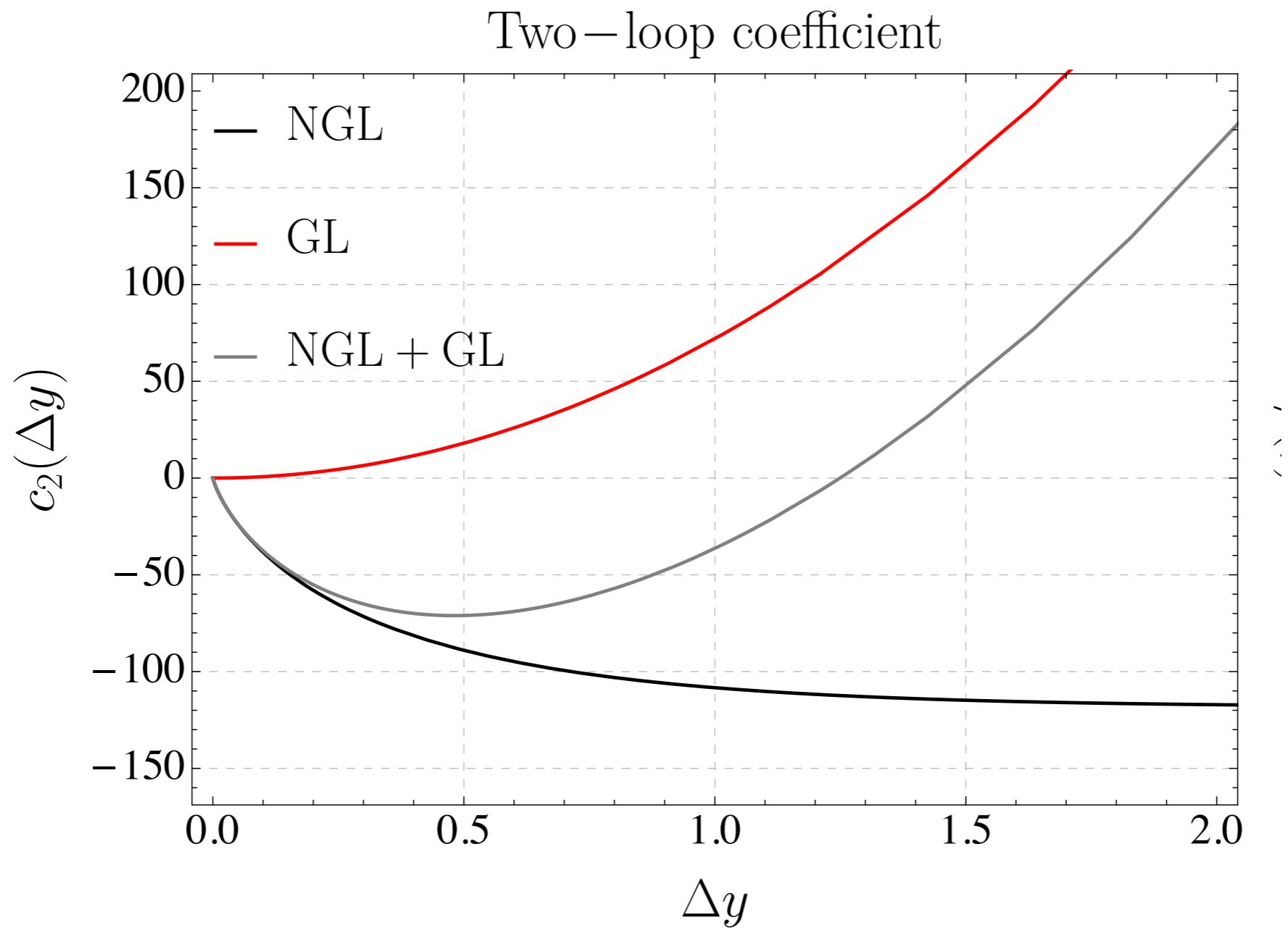
- Narrow gap limit: $\Delta y \rightarrow 0$

$$\frac{\sigma_{\text{NGL}}^{\text{LL}}}{\sigma_0} = 4 C_F C_A \left[8 \Delta y (\ln(2\Delta y) - 1) - 4 \Delta y^2 + \dots \right] t^2$$

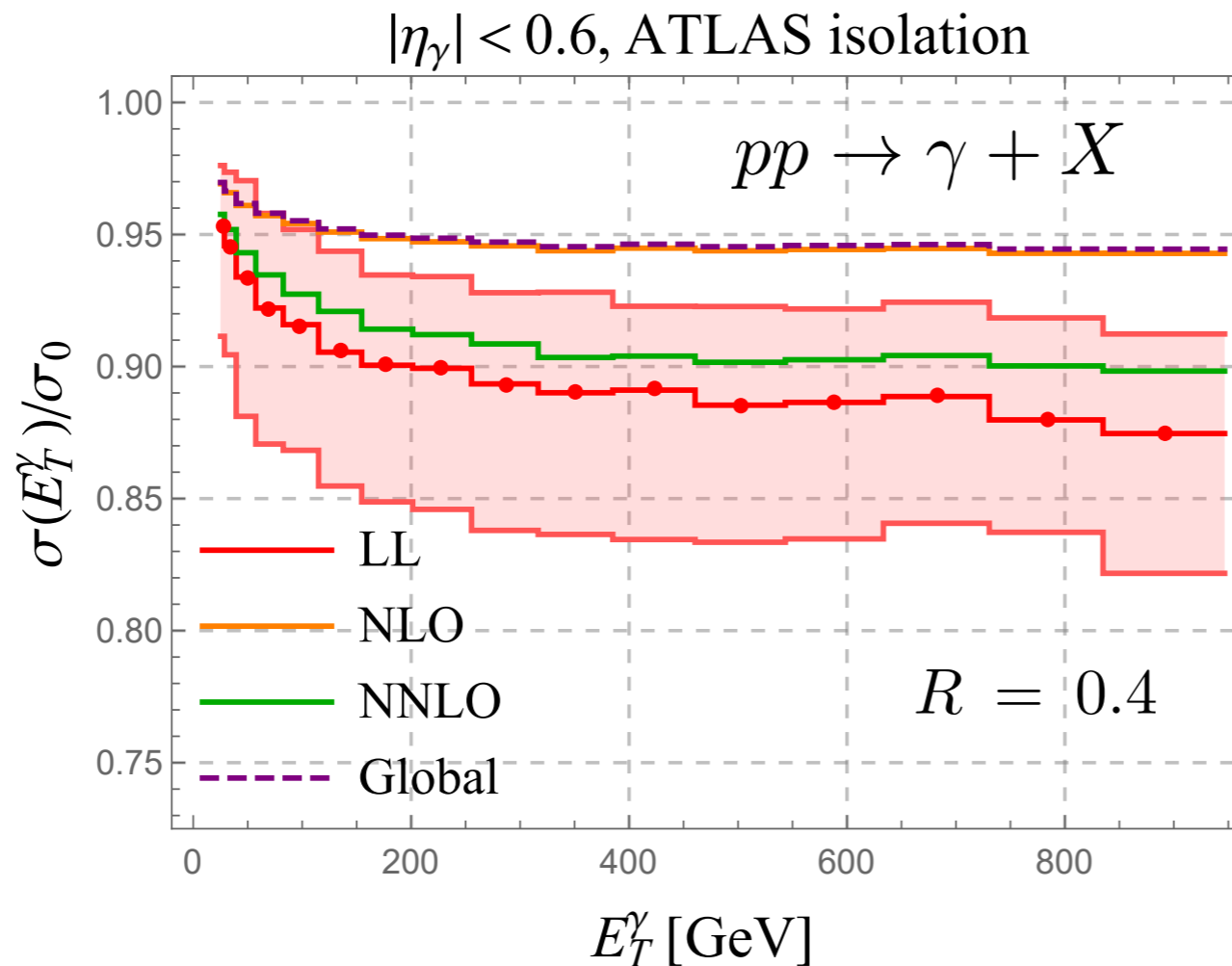
- Collinear enhancement from boundary region



Global vs Non-global Logs



Resummation effects in γ isolation at the LHC



$$1 + \# \alpha_s R^2 \ln \epsilon_\gamma$$

$$+ \# \alpha_s^2 R^2 \ln R \ln^2 \epsilon_\gamma$$

- **NLO**: $\sim 5\%$ reduction, **NNLO** $\sim 10\%$, **resummed** $\sim 12\%$
- NGL dominates over global contribution: naive exponentiation (**dashed**)
- LL result suffers from large scale uncertainties

From LL to NLL: Sub-leading NGLs

- In order to resum sub-leading NGLs, one needs

- ✓ One-loop soft function $\mathcal{S}_m^{(1)}$

- ✓ One-loop hard function $\mathcal{H}_k^{(1)}$ and tree level hard function

$\mathcal{H}_{k+1}^{(1)}$

- Two-loop anomalous dimensions: $\Gamma^{(2)} =$

See Caron-Huot '15 + Herranen '16

$$\begin{pmatrix} v_2 & r_2 & d_2 & 0 & \dots \\ 0 & v_3 & r_3 & d_2 & \dots \\ 0 & 0 & v_4 & r_4 & \dots \\ 0 & 0 & 0 & v_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- Monte-Carlo implementation of all ingredients

Ingredients at LL' level

$$\begin{aligned}
 d\sigma^{\text{LL}'}(Q, Q_0) = & \sum_{m=2}^{\infty} \langle \mathcal{H}_2^{(0)}(\{\underline{n}\}, Q, \mu_h) \otimes \mathbf{U}_{2m}(\{\underline{n}\}, \mu_s, \mu_h) \hat{\otimes} \mathbf{1} \rangle \\
 & + \alpha_s(\mu_h) \sum_{m=2}^{\infty} \langle (\mathcal{H}_2^{(1)}(\{\underline{n}\}, Q, \mu_h)) \otimes \mathbf{U}_{2m}(\{\underline{n}\}, \mu_s, \mu_h) \hat{\otimes} \mathbf{1} \rangle \\
 & + \alpha_s(\mu_h) \sum_{m=3}^{\infty} \langle (\mathcal{H}_3^{(1)}(\{\underline{n}\}, Q, \mu_h)) \otimes \mathbf{U}_{3m}(\{\underline{n}\}, \mu_s, \mu_h) \hat{\otimes} \mathbf{1} \rangle \\
 & + \alpha_s(\mu_s) \sum_{m=2}^{\infty} \langle (\mathcal{H}_2^{(0)}(\{\underline{n}\}, Q, \mu_h)) \otimes \mathbf{U}_{2m}(\{\underline{n}\}, \mu_s, \mu_h) \hat{\otimes} \mathcal{S}_m^{(1)}(\{\underline{n}\}, Q_0, \mu_s) \rangle
 \end{aligned}$$

- One-loop soft function can be implemented by re-weighting the events when they radiate outside jet cone (inside gap)

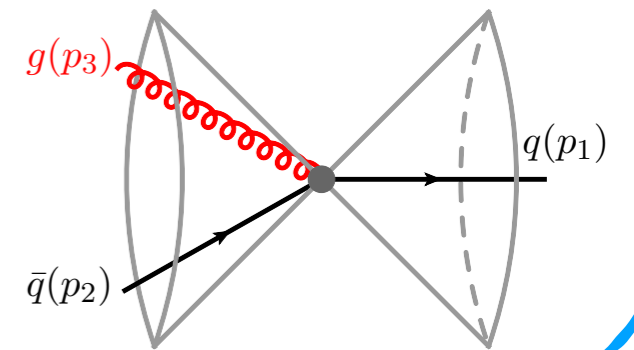
$$S_m^{(1)} = \frac{N_c}{2} \sum_{i,i+1} \int dy \frac{d\phi}{2\pi} \left[-4 \ln \frac{\mu_s}{Q_0} + 4 \ln \frac{2 |\sin \phi|}{f(\phi, y)} \right] \Theta_{\text{out}}(y, \phi)$$

- **One-loop hard function**

$$\mathcal{H}_2^{(1)} = \frac{N_c}{2} \left[-8 \ln^2 \frac{\mu_h}{Q} - 12 \ln \frac{\mu_h}{Q} - 16 + \frac{7}{3} \pi^2 \right]$$

$$\begin{aligned} \mathcal{H}_{3,I}^{(1)} = \frac{N_c}{2} & \left\{ \left[4 \ln^2 \frac{\mu_h}{Q} - 4 \ln \frac{\mu_h}{Q} \ln \left(\frac{\delta^2}{1 + \delta^2} \right) - \frac{\pi^2}{6} + \ln^2 \left(\frac{\delta^2}{1 + \delta^2} \right) \right] \delta(u) \delta(v) \right. \\ & + \left[-\ln \frac{\mu}{Q} F(0, v) + \frac{F(0, v)}{2} \ln \left(\frac{\delta^2}{1 + \delta^2} \right) \right] \delta(u) \left(\frac{1}{v} \right)_+ + \frac{F(0, v)}{2} \delta(u) \left(\frac{\ln v}{v} \right)_+ \\ & + \left[-2 \ln \frac{\mu}{Q} F(u, 0) + \frac{2u^2}{(1 + u)^3} + F(u, 0) \ln \left(\frac{\delta^2}{1 + \delta^2} \right) - 2F(u, 0) \ln(1 + u) \right] \delta(v) \left(\frac{1}{u} \right)_+ \\ & \left. + 2F(u, 0) \delta(v) \left(\frac{\ln u}{u} \right)_+ + F(u, v) \left(\frac{1}{u} \right)_+ \left(\frac{1}{v} \right)_+ \right\} \end{aligned}$$

$$\sin^2 \frac{\theta_2}{2} = \frac{\delta^2 u^2 v}{1 + \delta^2}, \quad \sin^2 \frac{\theta_3}{2} = \frac{\delta^2 v}{1 + \delta^2}.$$



I: $x_1 > x_2 > x_3$

- **MC implementation:**

- Phase space slicing method: large log cancellation

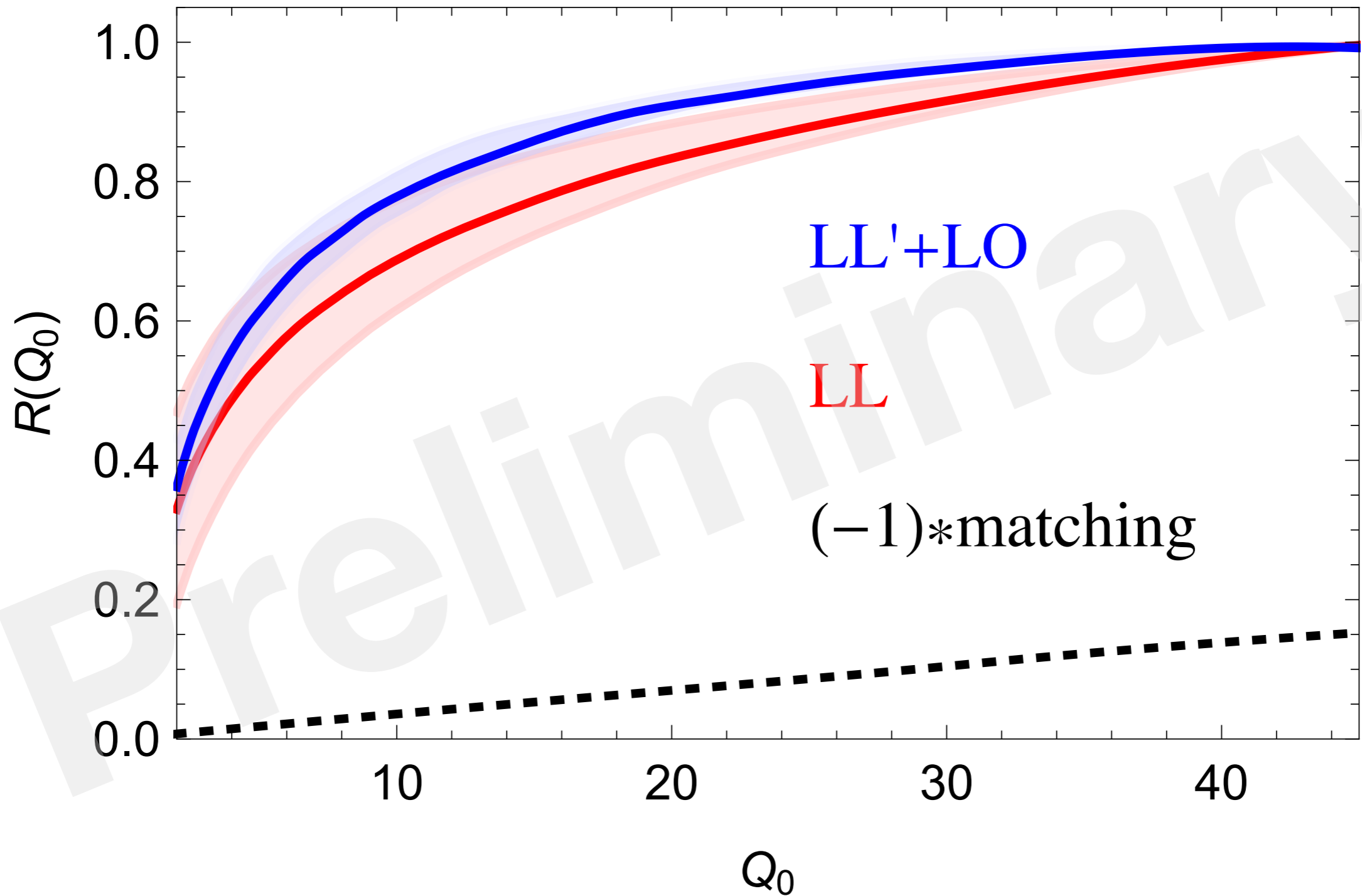
- Numerical extrapolation method: point-by-point shower

- Two independent methods consistent

- Subtraction method? Can hard function be provided by MG5_aMC@NLO?

Inter-jet energy flow at e^+e^- collider

$$\Delta y \sim 1$$



Summary

We extend factorization and resummation in SCET to non-global observables

- have analyzed a variety of such observables
- multi-Wilson line operators are key ingredient

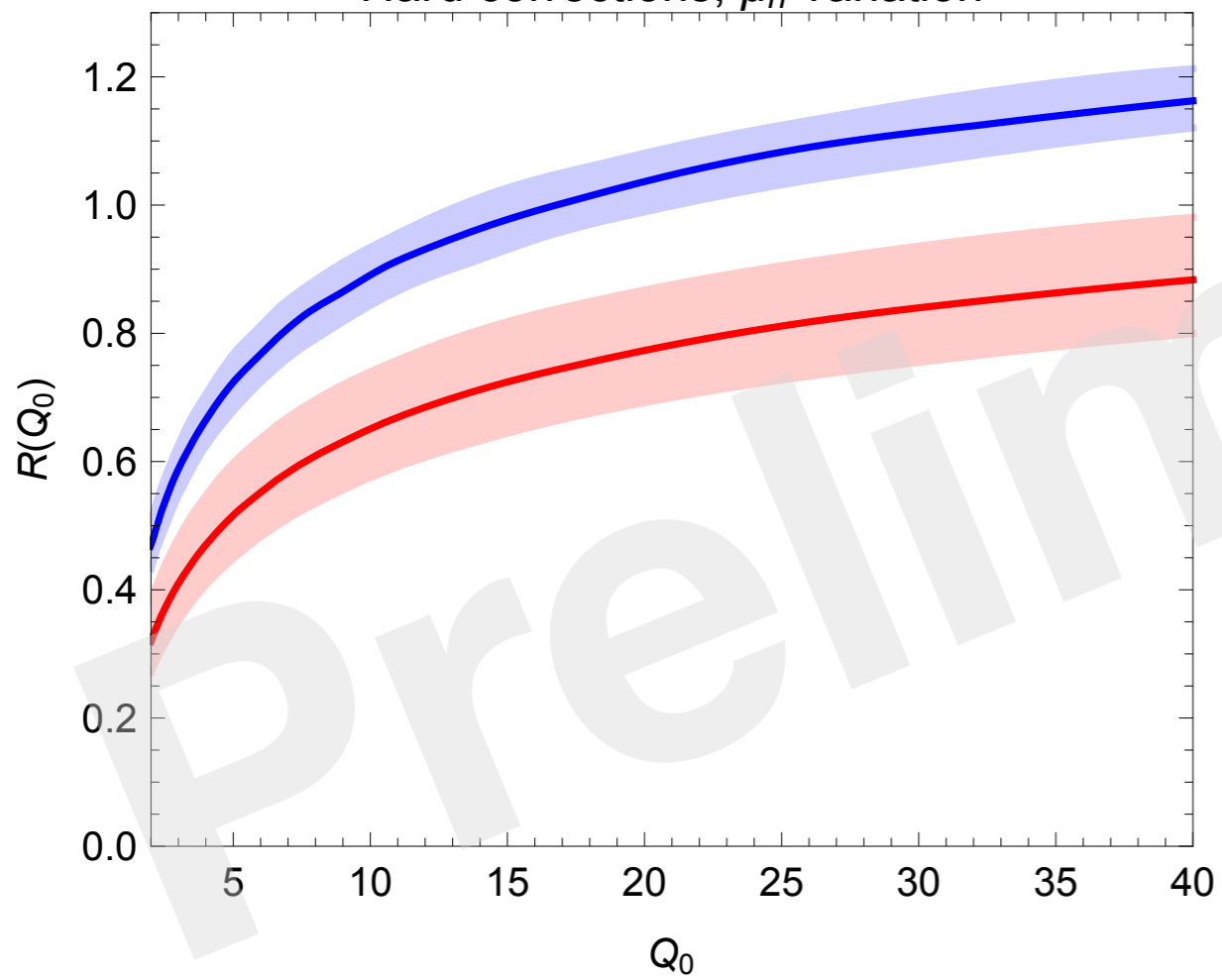
We obtain a parton shower from effective field theory

- not restricted to leading logarithms or large N_c
- not a General Purpose Parton Shower, but helpful to understand how to extend showers to higher accuracy
- flexible implementation of LL shower using LHE event files
- include one-loop matching coefficients: first important step in sub-leading NGLs resummation

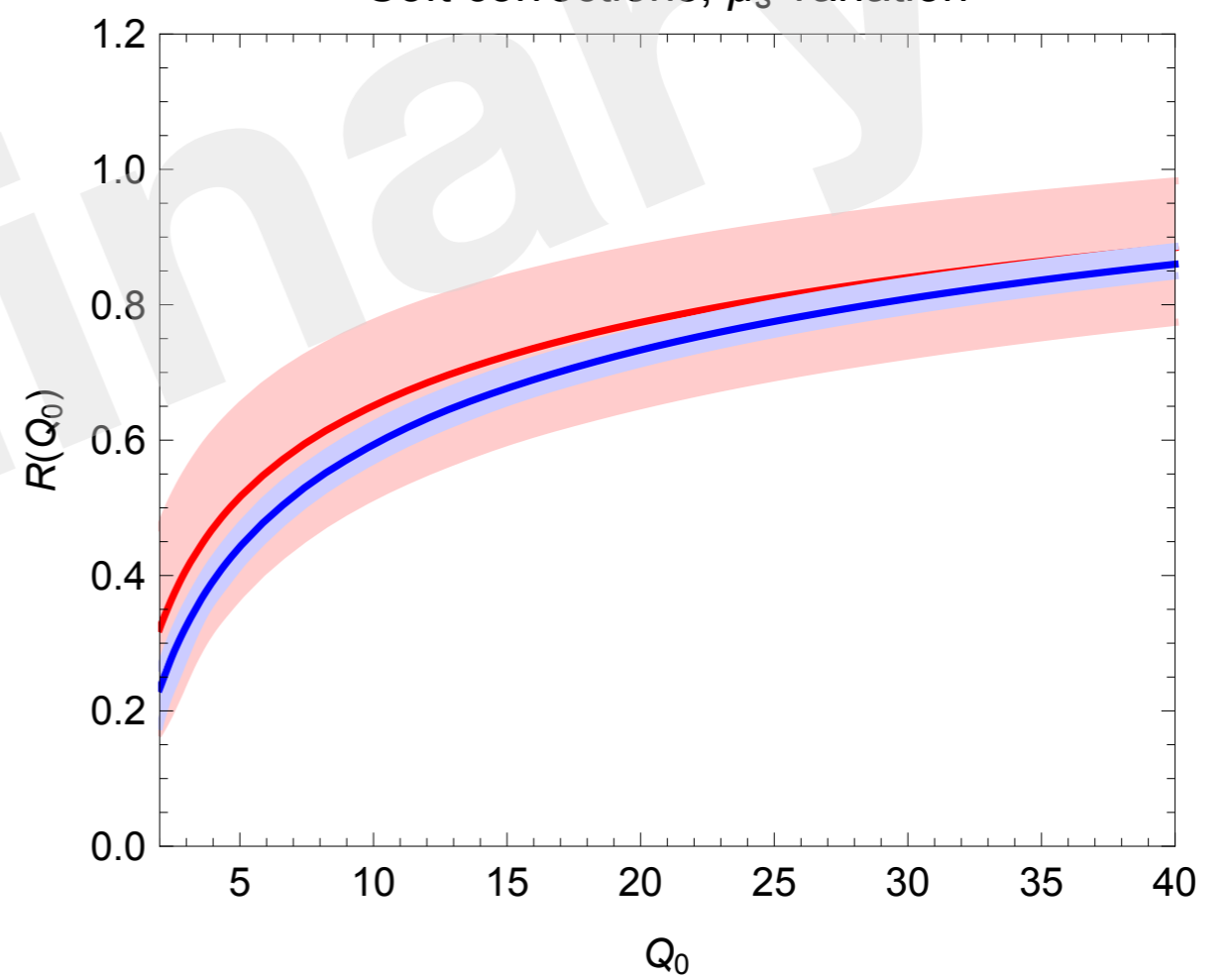
Thank you

Numerical results

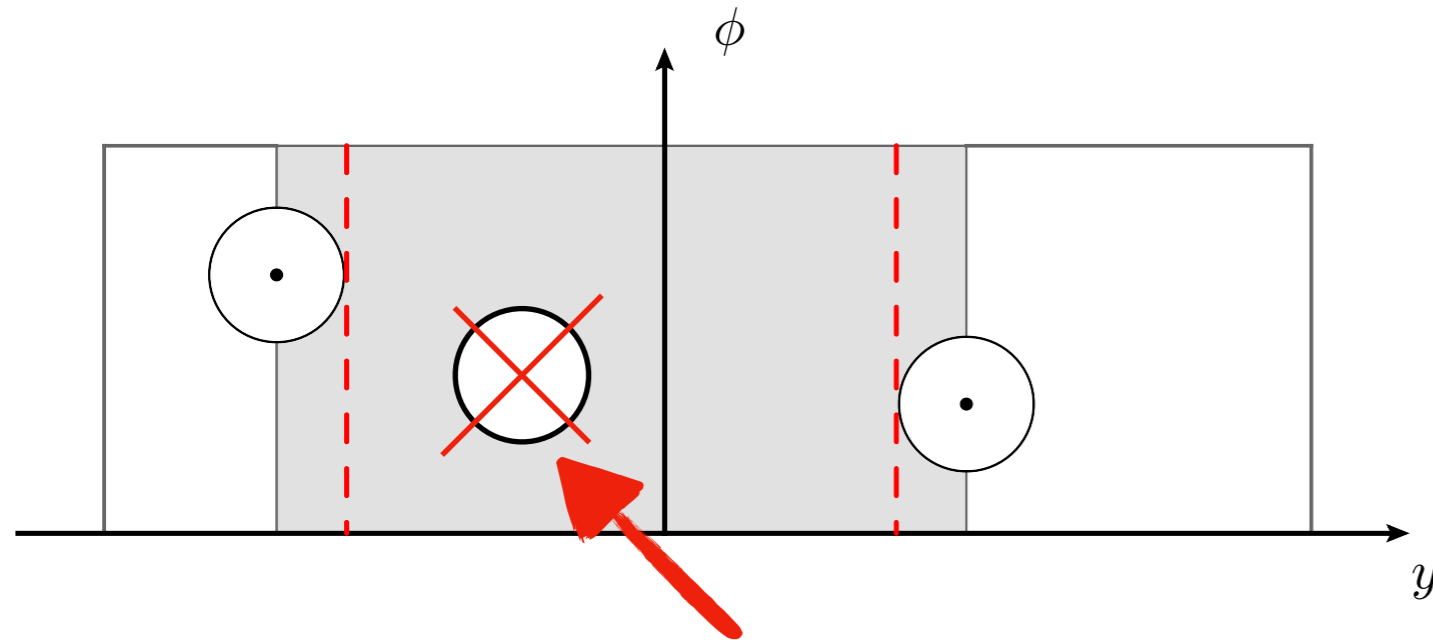
Hard corrections, μ_h variation



Soft corrections, μ_s variation



Gap fraction at the LHC



$$R(\bar{p}_T, Q_0) = \frac{\sigma_{2\text{-jet}}(\bar{p}_T, Q_0)}{\sigma_{2\text{-jet}}(\bar{p}_T, Q_0 = \bar{p}_T)}$$

$$\bar{p}_T = \frac{1}{2} (p_{T,\text{jet1}} + p_{T,\text{jet2}})$$

veto events with $p_{T,\text{jet3}} > Q_0$

- Moderate non-perturbative corrections
- Need soft gluon resummation at small Q_0
- **ATLAS '11 '14** observe MC predictions are not always consistent with data
- GL with full color, NGLs estimated by a K-factor
- Numerical solution of BMS equation but with approximated veto region

Resummation

- Factorization formula to all order is unknown due to Glauber gluons
- We work in LL and large N_c limit

$$\frac{d\sigma(Q_0)}{d\Delta y d\bar{p}_T} = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 f_a(x_1, \mu_f) f_b(x_2, \mu_f) H_2^{ab}(\hat{s}, \Delta y, \bar{p}_T, \mu_h) \langle U_{2m}(\mu_s, \mu_h) \hat{\otimes} 1 \rangle$$

- scale setting $\mu_f = \mu_h = \bar{p}_T$ and $\mu_s = Q_0$
- focus on central jets, no collinear logs

