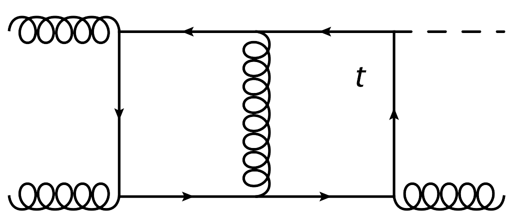
Mass effects in loop-induced processes at NLO

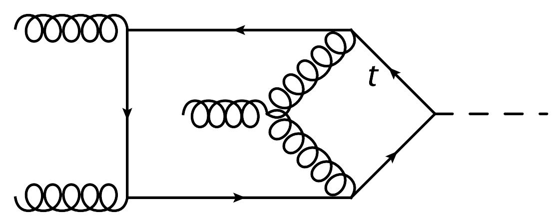
Matthias Kerner (MPP)
Chris Wever (KIT)

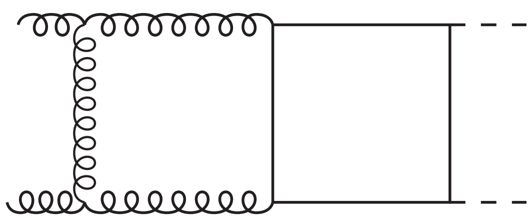
AMPHE 2018
Mainz, 21. Aug. 2018

Introduction

NLO corrections to loop-induced
2$\rightarrow$2 processes, e.g. HJ or HH production







Problems:

- $m_t \rightarrow \infty$ limit often not valid
- many scales (e.g. m_H, m_T, s, t)
- IBP reduction challenging
- large #masters/sector
- elliptic integrals

Overview:

1. Introduction: HH production
   numerical calculation vs. approximations
2. Numerical calculation of HJ and HH production
3. High energy expansions in HJ production

1. Introduction: HH production

numerical calculation vs. approximations

Analytic results

AA / jj - production via top-quark loop [Becchetti, Bonciani 17]

only planar integrals calculated:

- alphabet containing square roots
- mostly GPLs
- up to 2-fold integrals at weights 3,4

---

see also [Primo, Tancredi 16]

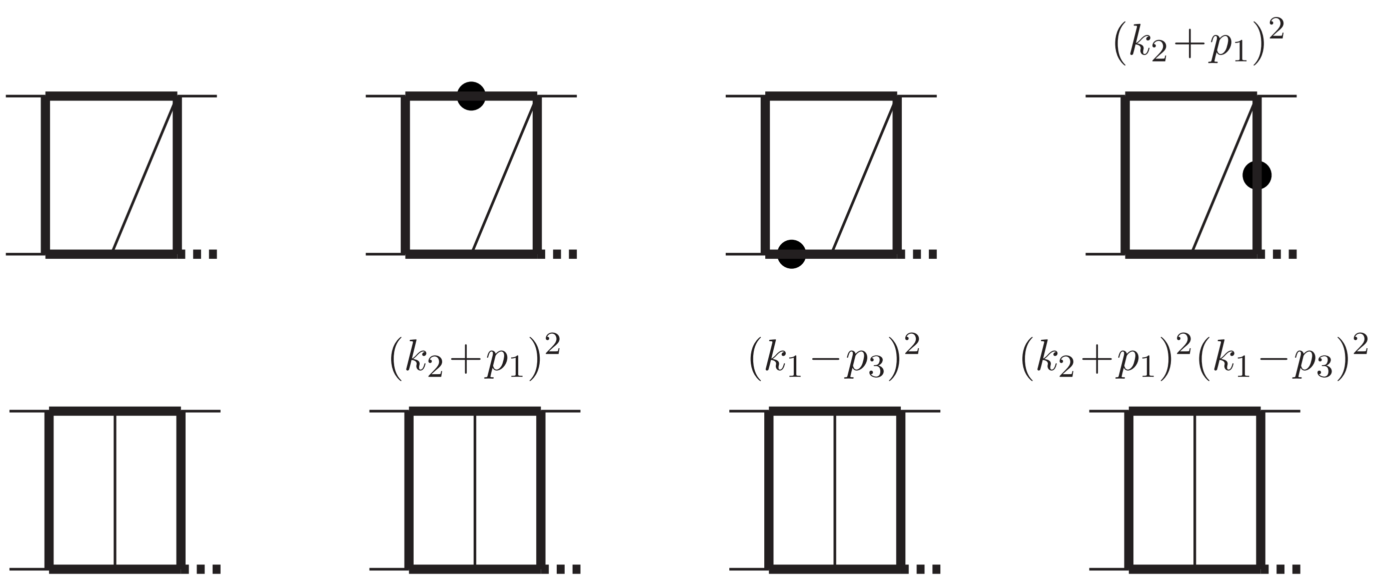
HJ production [Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov 16]

most planar integrals can be expressed in terms of

- alphabet with 3 variables, 49 letters, many square roots
- log, Li_2 up to weight 2
- 1-fold integrals at weights 3,4

2 sectors contain elliptic functions

can be expressed as 2- and 3-fold iterated integrals with elliptic kernel

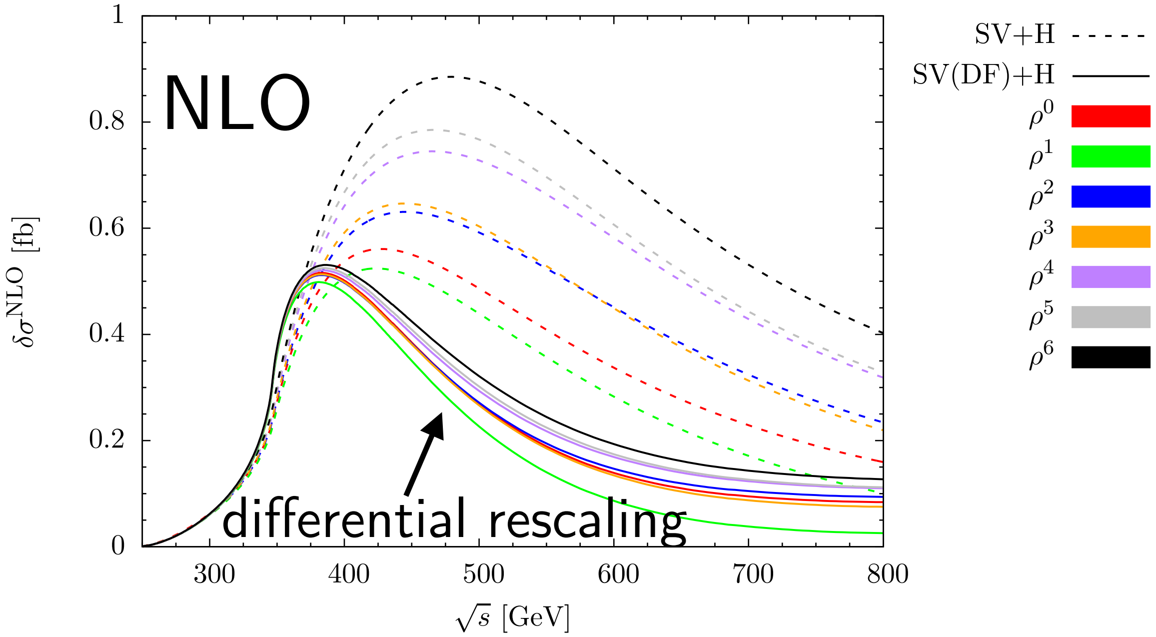


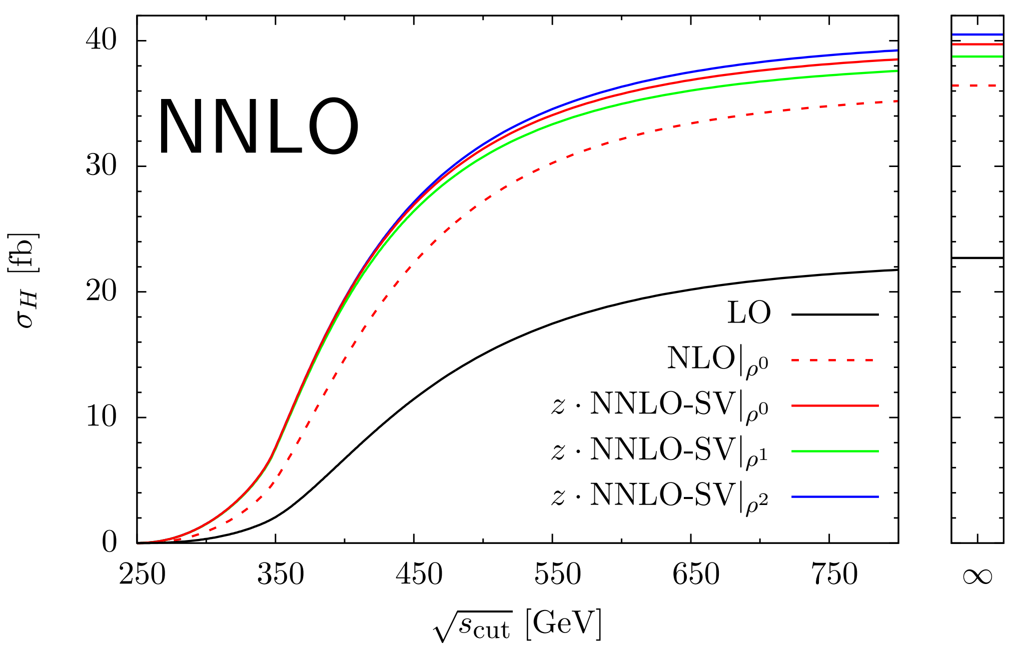
so far no non-planar results

Large mass expansion

HH: expansion in $1/m_t$

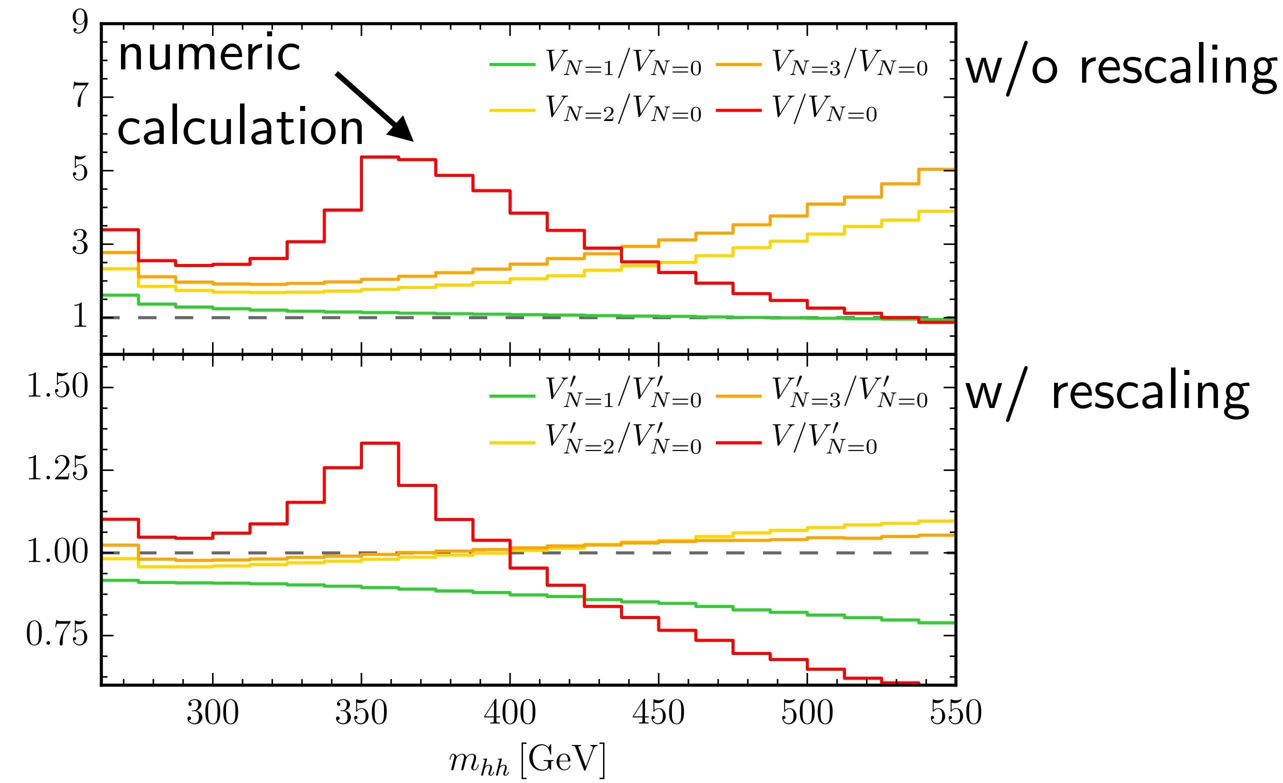
[Grigo, Hoff, Steinhauser 15]





rescaling can improve convergence:

$$d\sigma(m_t) \approx d\sigma(m_t \to \infty)\frac{d\sigma_{LO}(m_t)}{\sigma_{LO}(m_t \to \infty)}$$



[Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke 16]

expansion improves convergence only for $m_{HH} < 2m_t$

HJ: [Harlander, Neumann, Ozeren, Wiesemann 12][Neumann, Wiesemann 14] [Frederix, Frixione, Vryonidou, Wiesemann 16] [Neumann, Williams 16]

ZZ: [Campbell, Ellis, Czakon, Kirchner 16] [Caola, Dowling, Melnikov, Röntsch, Tancredi 16]

ZH: [Hasselhuhn, Luthe, Steinhauser, 16]

Low mass expansion

can describe

→ b-quark contributions

→ t-quark contributions at large s,t

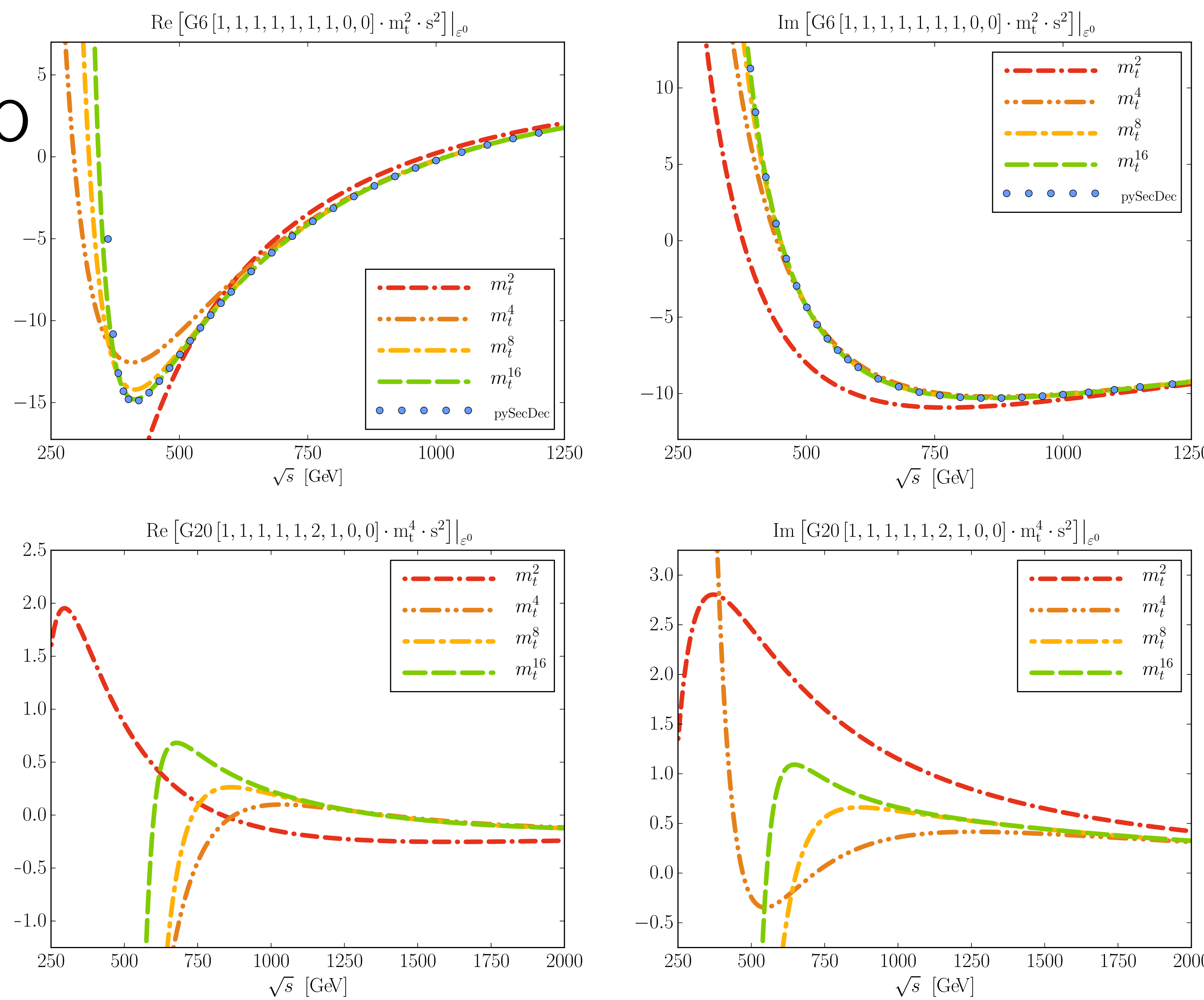
HJ production → Chris

HH production

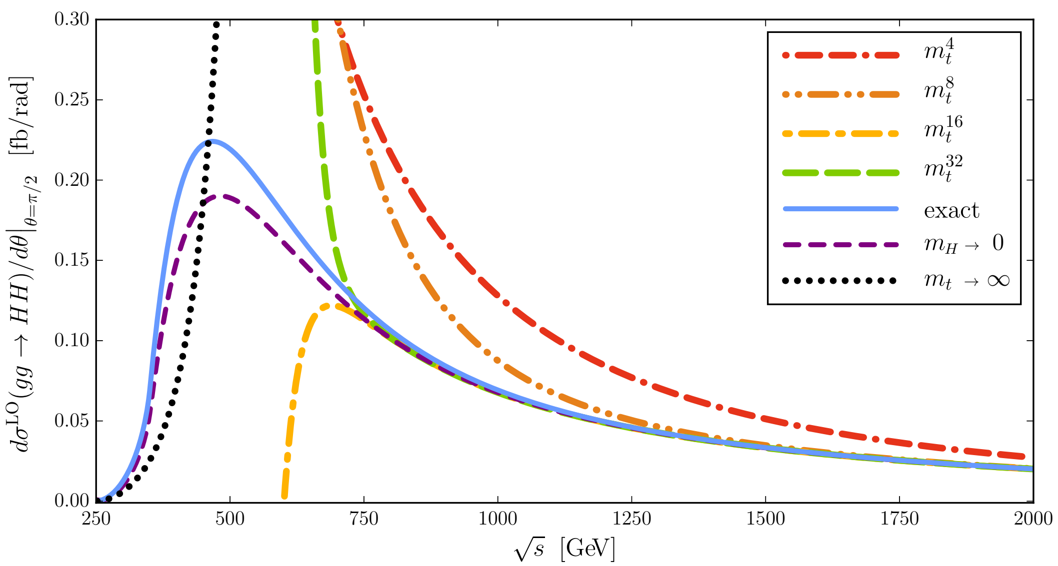
[Davies, Mishima, Steinhauser, Wellmann 18]

many expansion terms required

NLO



LO



Padé approximation & threshold behavior

HH production [Gröber, Maier, Rauh 17]

Form factors F(z) approximated by

$$[F(z) - \mathrm{thr}(1-z)]\,(1 + a_R z) \simeq [n/m](\omega(z))$$

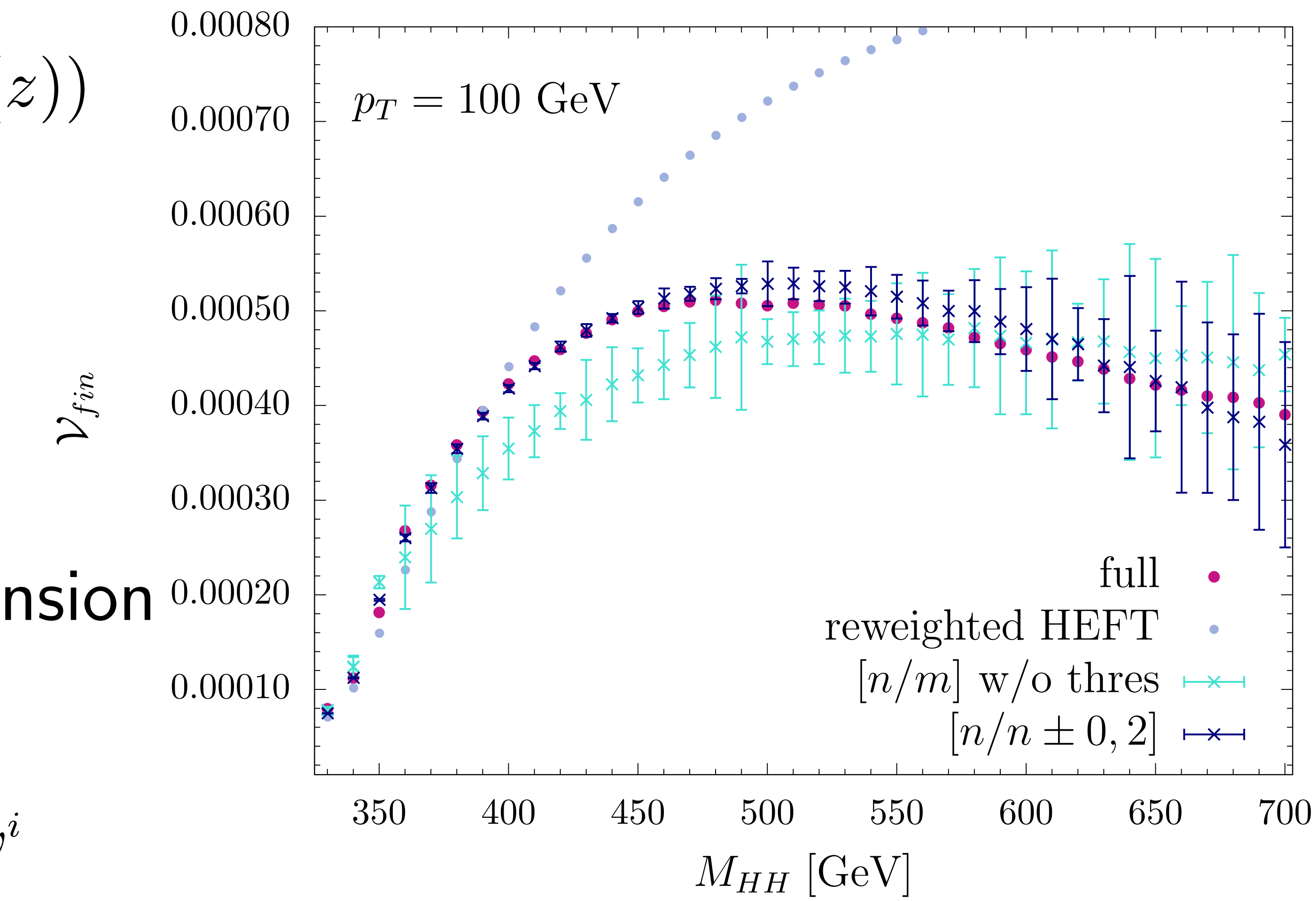
with

- $z = \dfrac{s + i0}{4m_t^2}$ $\qquad z = \dfrac{4\omega}{(1+\omega)^2}$
- $\mathrm{thr}(1-z)$

  non-analytic terms of threshold expansion obtained using EFT approach

  $\rightarrow$ PNRQCD, SCET
- Padé approximation $[n/m](\omega) = \dfrac{\sum_{i=0}^{n} a_i \omega^i}{1 + \sum_{j=1}^{m} b_j \omega^j}$

  coefficients fixed by expansion in $1/m_t^2$ and threshold z=1



ZZ production: Padé approximation [Campbell, Ellis, Czakon, Kirchner 16]

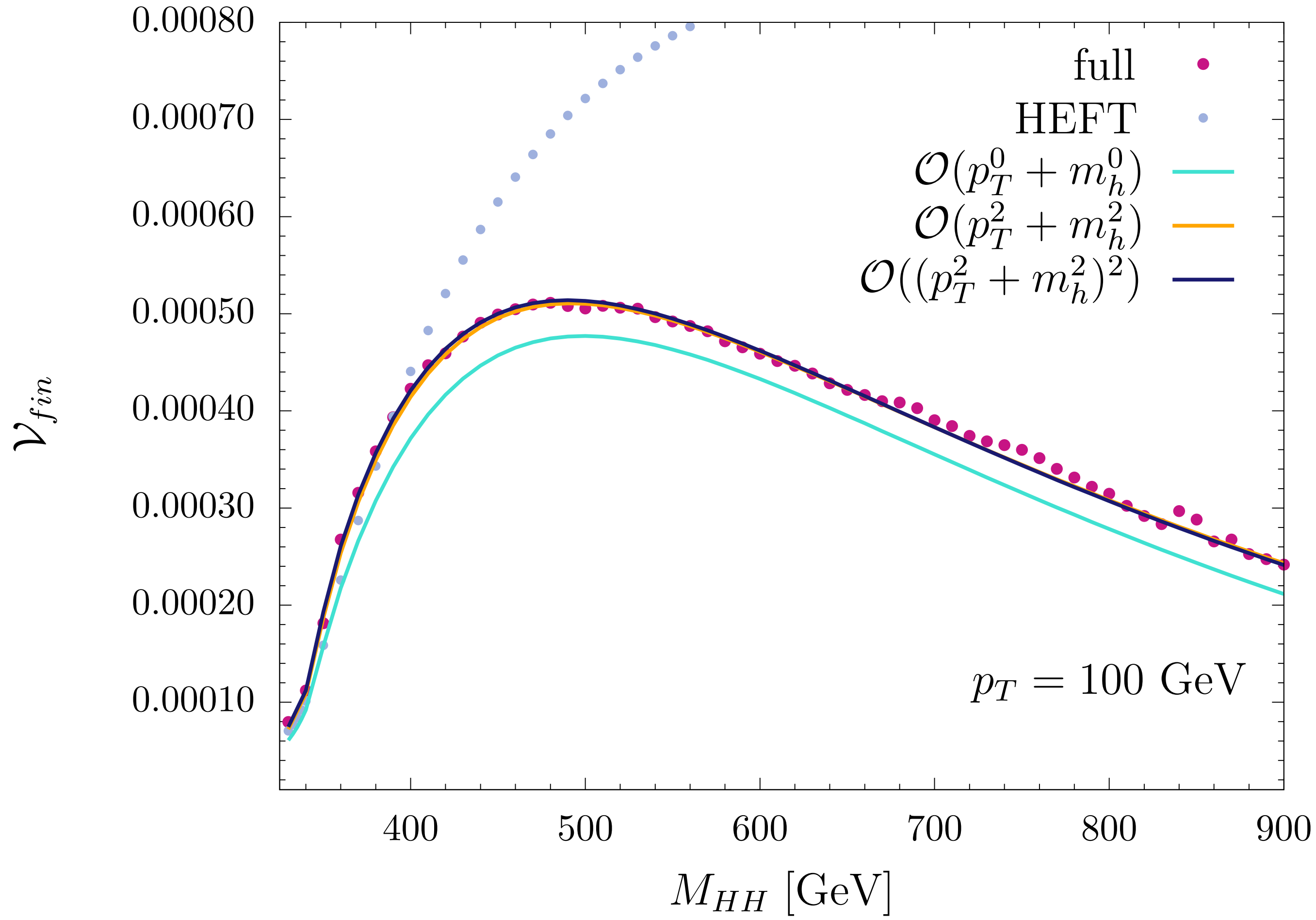
Expansion in E_T

HH production [Bonciani, Degrassi, Giardino, Gröber 18]

$$p_T^2 + m_H^2 \leq \frac{\hat{s}}{4}$$

$\rightarrow$ expand in $p_T^2 + m_H^2$

$\rightarrow$ solve remaining dependence on $\hat{s}$ and m_t



2. Numeric calculations of HJ and HH production

HH production

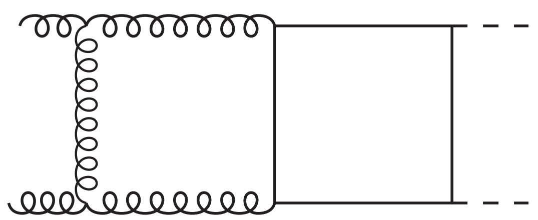
[Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke 16]
PRL 117 (2016) 012001 [1604.06447]
JHEP 1610 (2016) 107 [1608.04798]

HJ production

[Jones, MK, Luisoni 18]
PRL 120 (2018) 162001 [1802.00349]

Computational Method

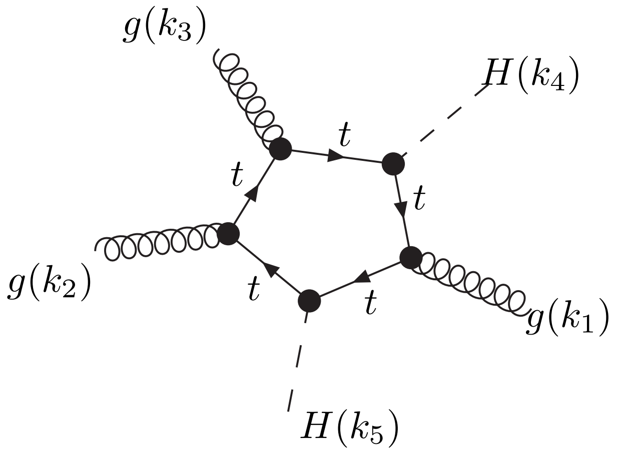
Method for calculating virtual amplitude:



1. Form factor decomposition
2. Integral reduction
3. Sector decomposition
4. Numerical integration of loop integrals
   using Quasi Monte Carlo algorithm
5. Generate histograms of virtual contribution
   using unweighted LO events for phase-space sampling

Combine with real radiation at histogram level

$\rightarrow$ 1-loop 5-point amplitudes

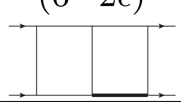
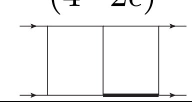
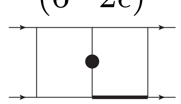
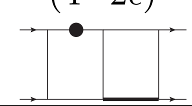


Integral Reduction

IBP reduction obtained using Reduze 2 [von Manteuffel, Studerus 12]
with modifications to

- specify list of required integrals
  $\rightarrow$ consider only equations containing these integrals
- change order of solving the system of equations,
  sorting the equations by number of unreduced integrals

Preferred Masters: (quasi-)finite integrals [von Manteuffel, Panzer, Schabinger 14]

Integral	run time	rel. error	Integral	run time	rel. error
$(6-2\epsilon)$  (s,t)	280 s	1.00×10^{-3}	$(4-2\epsilon)$  (s,t)	214135 s	8.29×10^{-3}
$(6-2\epsilon)$  (s,t)	294 s	1.21×10^{-3}	$(4-2\epsilon)$  (s,t)	3484378 s	30.9

[von Manteuffel, Schabinger 17]

HH:

- fix m_H=125 GeV, m_t=173 GeV
- only reduction of planar sectors achieved
  $\rightarrow$ non-planar tensor integrals evaluated directly with SecDec

HJ:

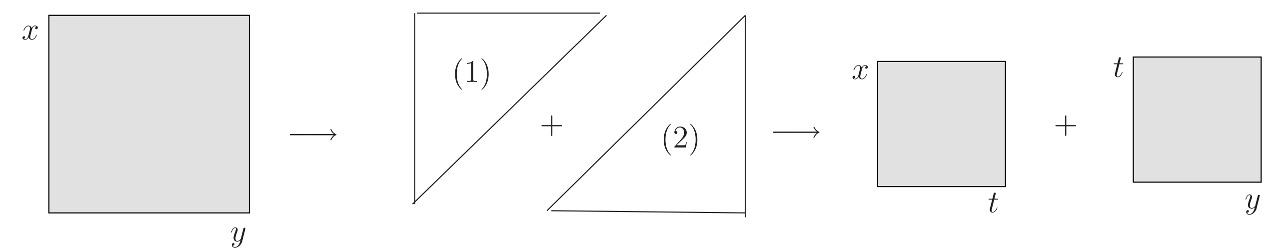
full reduction done twice:

1. $m_H^2/m_t^2 = 12/23$ fixed
   used in amplitude calculation
2. full dependence on m_H, m_t
   reduction available, but not used

Loop Integrals — Sector Decomposition

Numerical evaluation of loop integrals with SecDec

[Borowka, Heinrich, Jahn, Jones, MK, Schlenk, Zirke]



- Sector decomposition [Binoth, Heinrich `00]
  factorizes overlapping singularities
- Subtraction of poles & expansion in ϵ
- Contour deformation [Soper 00; Binoth et al. 05, Nagy, Soper 06, Borowka et al. 12]
  analytic continuation from Euclidean to physical region

$\rightarrow$ finite integrals at each order in ϵ

$\rightarrow$ numerical integration possible

SecDec 3 used for calculation of HJ and HH production

new version: pySecDec
- implementation using python and Form
- modular structure
- generates libraries that
  $\rightarrow$ can be directly linked to amplitude code
- handling of non-logarithmic poles improved
- better symmetry finder
- ...
- coming soon: QMC integration

Loop Integrals — Numerical Integration

after sector decomposition and expansion in $\boldsymbol{\varepsilon}$:
amplitude written in terms of $\mathcal{O}(10\,\mathrm{k})$ finite integrals

- all integrals evaluated using Quasi-Monte-Carlo integration
  - generating vector
    - constructed component-by-component [Nuyens 07]
    - minimizing worst-case error
    - for fixed lattice sizes
  - $\mathcal{O}(n^{-1})$ scaling of integration error
- dynamically set n for each integral, minimizing

$$T = \sum_{\text{integral}\, i} t_i + \lambda \left(\sigma^2 - \sum_i \sigma_i^2 \right) \qquad \sigma_i = c_i \cdot t_i^{-e}$$

$\sigma_i =$ error estimate (including coefficients in amplitude)
$\lambda =$ Lagrange multiplier $\qquad \sigma =$ precision goal

- parallelization on gpu
- avoid reevaluation of integrals for different orders in ε and form factors

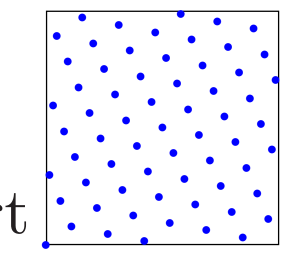
compute only once

$$F^a = \sum_i \left[\left(\sum_j C^a_{i,j} \varepsilon^j \right) \cdot \left(\sum_k I_{i,k} \varepsilon^k \right) \right] = \frac{C^a_{1,-2} I_{1,0} + C^a_{1,-1} I_{1,-1} + \ldots}{\varepsilon^2} + \frac{C^a_{1,-1} I_{1,0} + \ldots}{\varepsilon^1} + \ldots$$

QMC rank-1 lattice rule

$$I = \int \mathrm{d}\vec{x} f(\vec{x}) \approx I_k = \frac{1}{n} \sum_{i=1}^{n} f(\vec{x}_{i,k})$$

$$\vec{x}_{i,k} = \left\{ \frac{i \cdot \vec{g}}{n} + \vec{\Delta}_k \right\}$$



$\{\ldots\} =$ fractional part
$\vec{g} =$ generating vector
$\vec{\Delta}_k =$ randomized shift

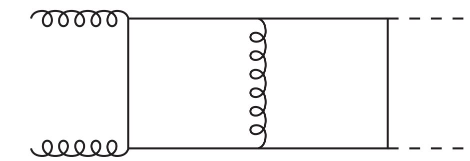
m different estimates $I_1 \ldots I_m$
$\rightarrow$ error estimate

[Li, Wang, Yan, Zhao 16]
Review: [Dick, Kuo, Sloan]

HH Amplitude Evaluation — Example

$\sqrt{s} = 327.25\,\mathrm{GeV},\ \sqrt{-t} = 170.05\,\mathrm{GeV},\ M^2 = s/4$

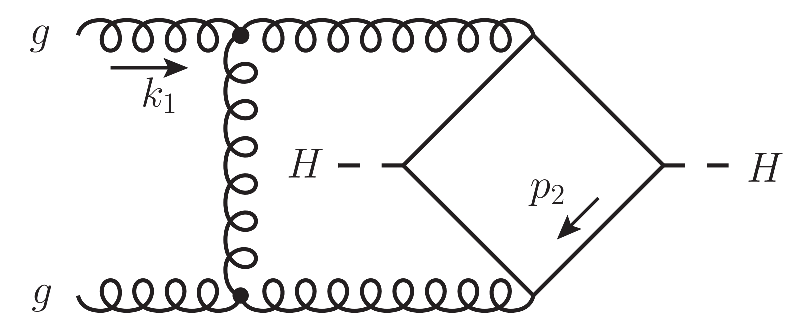
contributing integrals:



integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2k2p2_ord0	(0.0929, -0.224)	(6.32e-05, 5.93e-05)	235.412
N3_111111100_1_ord0	(-0.0282, 0.179)	(8.01e-05, 9.18e-05)	265.896
N3_111111100_k1p2k1p2_ord0	(0.0245, 0.0888)	(5.06e-05, 5.31e-05)	282.794
N3_111111100_k1p2_ord0	(-0.00692, -0.108)	(3.05e-05, 3.05e-05)	433.342

≈ 700 integrals

$$I(s,t,m_t^2,m_h^2) = -\left(\frac{\mu^2}{M^2}\right)^{2\varepsilon} \Gamma(3+2\epsilon) M^{-4} \left(\frac{A_{-2}}{\epsilon^2} + \frac{A_{-1}}{\epsilon^1} + A_0 + \mathcal{O}(\epsilon)\right)$$



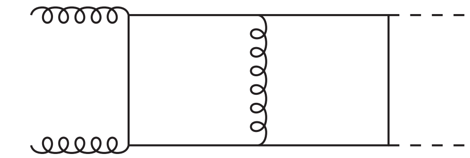
sector decomposition

sector	integral value	error	time [s]	#points
5	(-1.34e-03, 2.00e-07)	(2.38e-07, 2.69e-07)	0.255	1310420
6	(-1.58e-03, -9.23e-05)	(7.44e-07, 5.34e-07)	0.266	1310420
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

HH Amplitude Evaluation — Example

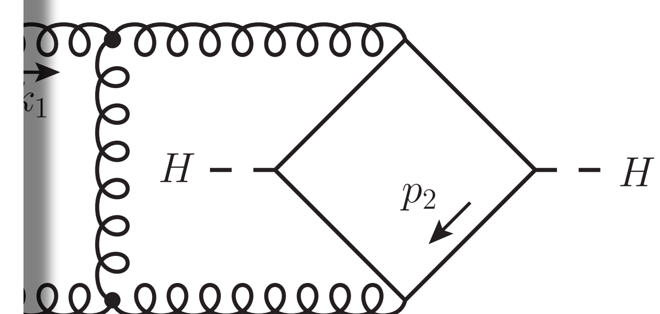
$$\sqrt{s} = 327.25\,\mathrm{GeV},\ \sqrt{-t} = 170.05\,\mathrm{GeV},\ M^2 = s/4$$

contributing integrals:

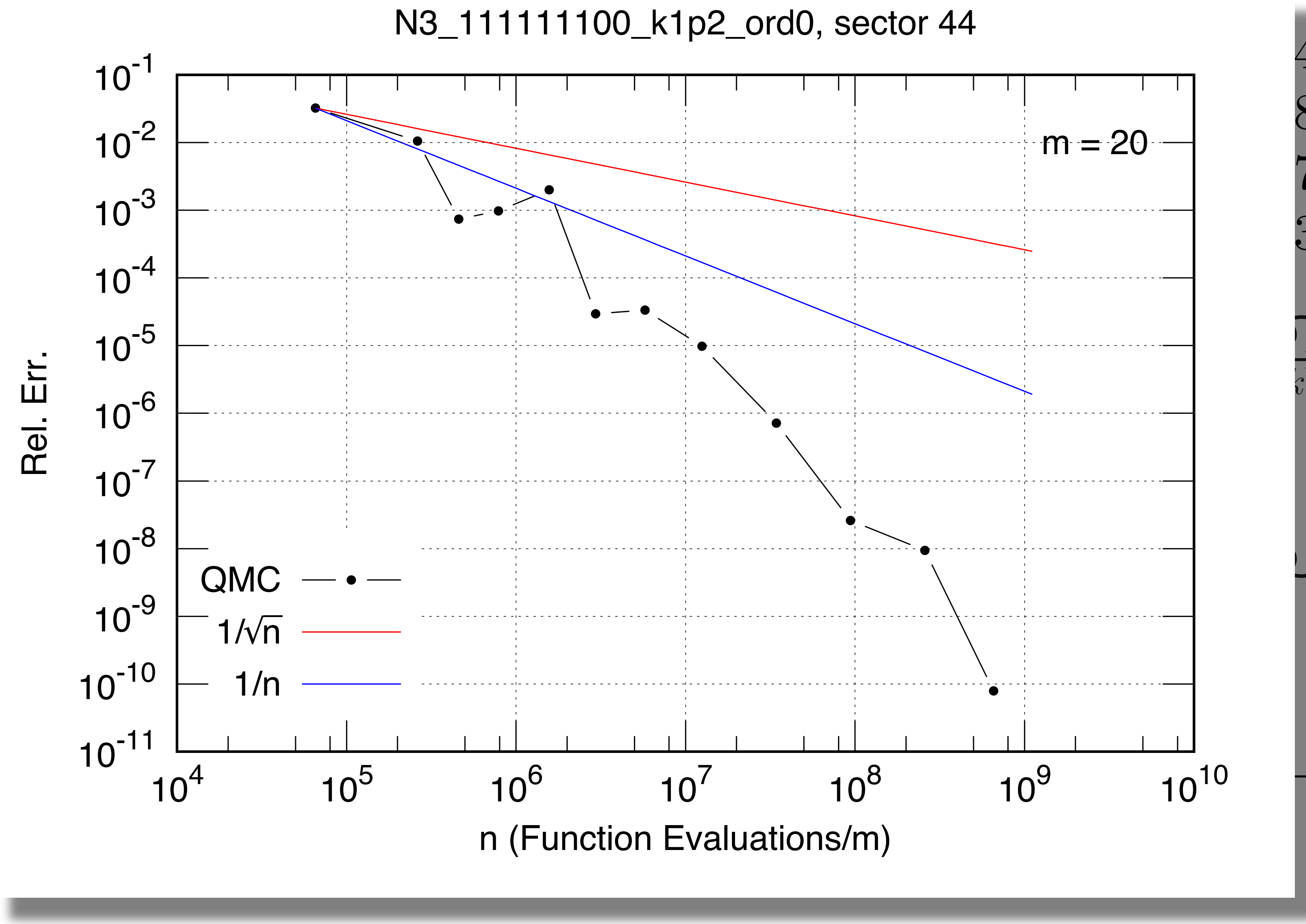


integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2			412
N3_111111100_1_or			896
N3_111111100_k1p2			794
N3_111111100_k1p2			342

≈ 700 integrals



$$I(s,t,m_t^2,m_h^2) = - ($$



sector	in			
5	(-1.34e-0			
6	(-1.58e-03			
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

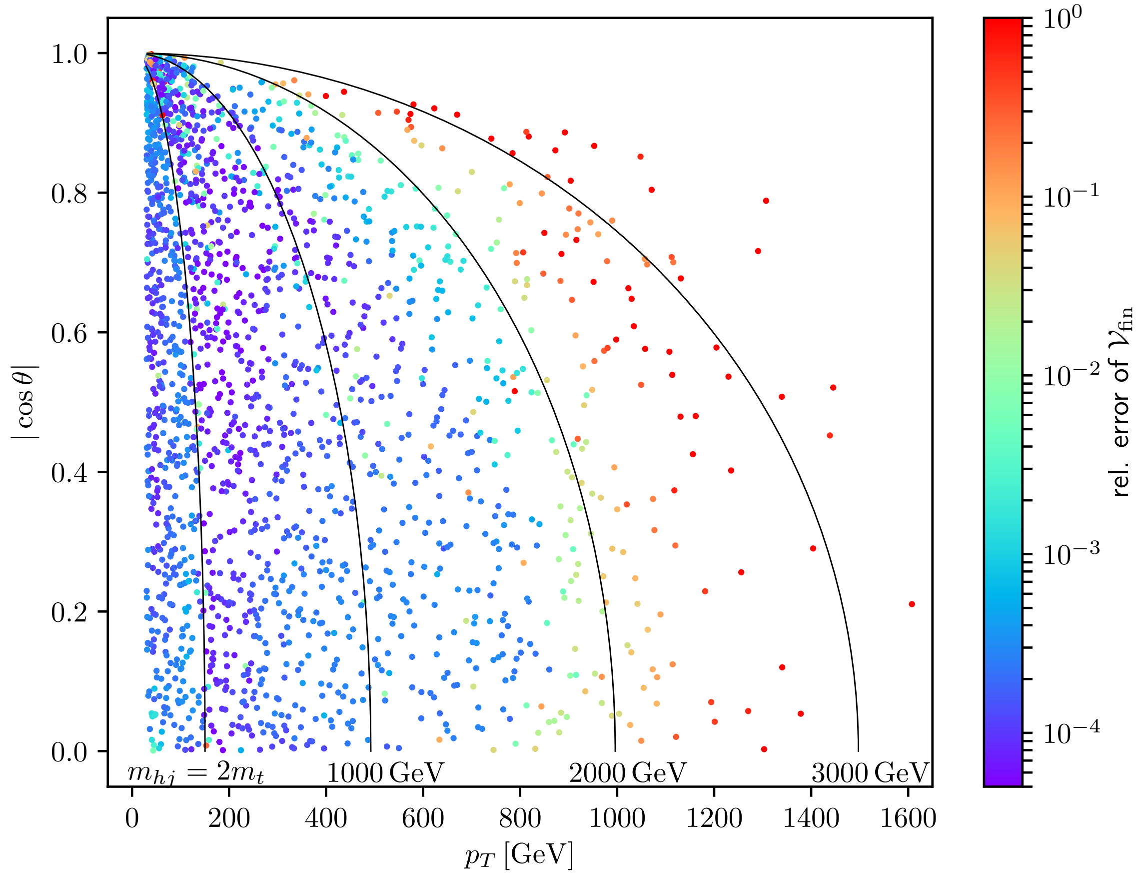
HJ Numerical Stability & Run Time

numerical evaluation of virtual amplitude:

- precision goal: 0.5% for each form factor
- wall-clock limit: 2d GPU-time (Tesla K20X GPUs)

accuracy reached for $|\mathcal{M}|^2$:

- better than per-mill
  for most points below $m_{hj} = 1.5\,\mathrm{TeV}$
- region $m_{hj} \gtrsim 2\,\mathrm{TeV}$ numerically challenging



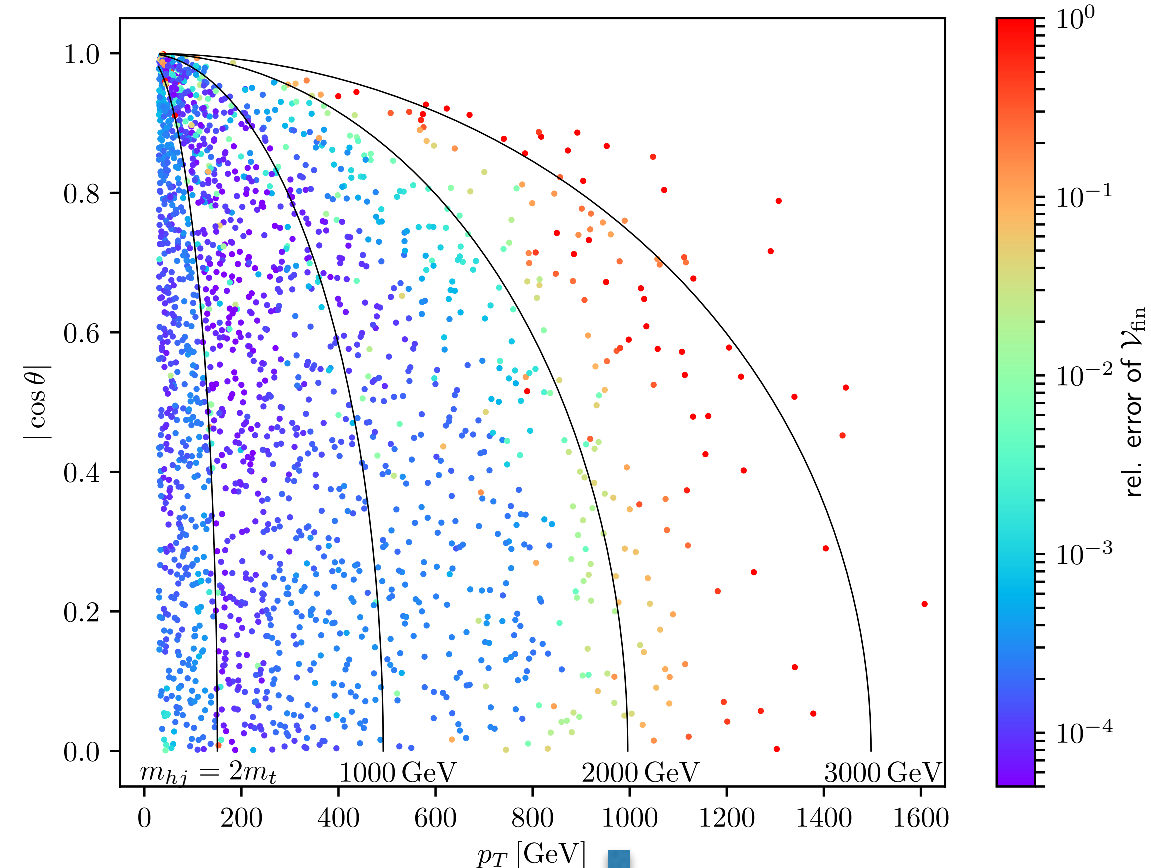
HJ Numerical Stability & Run Time

numerical evaluation of virtual amplitude:

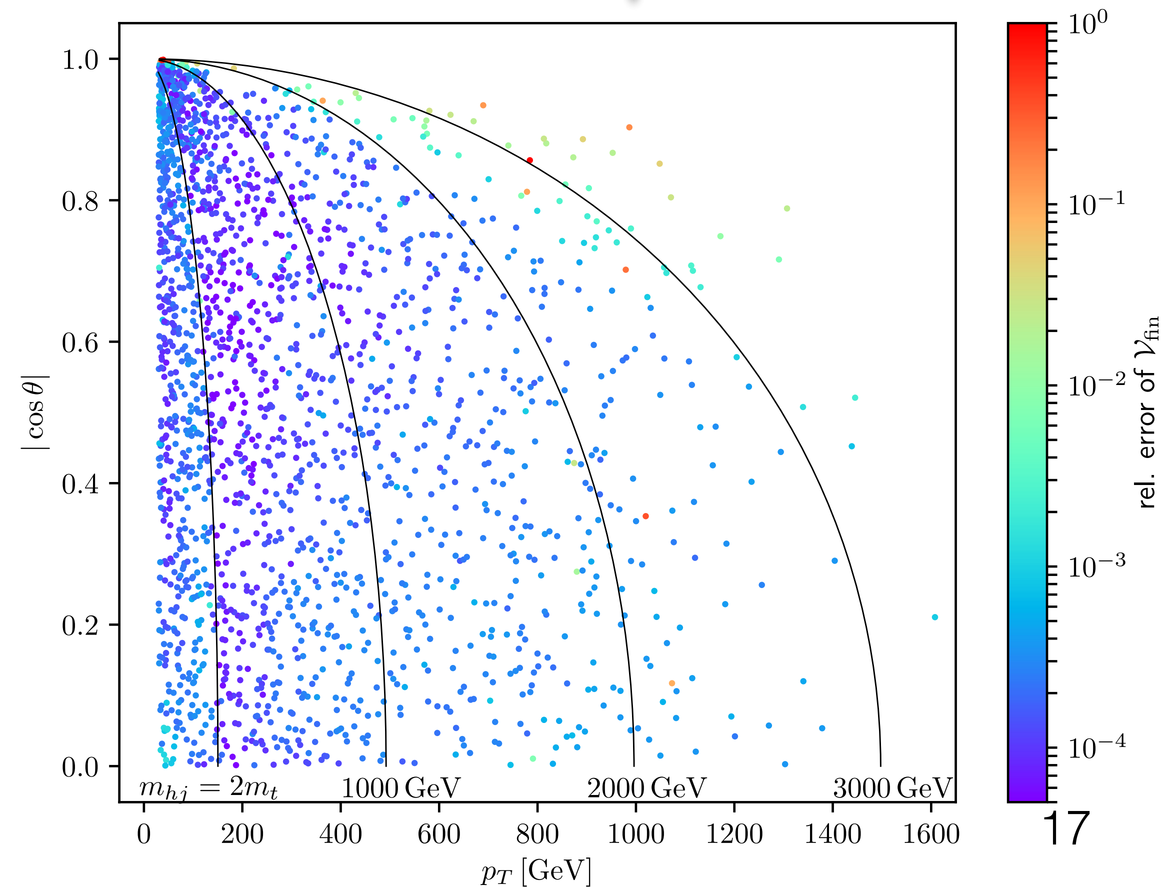
- precision goal: 0.5% for each form factor
- wall-clock limit: 2d GPU-time (Tesla K20X GPUs)

accuracy reached for $|\mathcal{M}|^2$:

- better than per-mill for most points below $m_{hj} = 1.5\,\mathrm{TeV}$
- region $m_{hj} \gtrsim 2\,\mathrm{TeV}$ numerically challenging



improved basis choice



- use finite integrals with $\mathrm{exponent}(\mathcal{F}) = -1$
  $\rightarrow$ possibly better convergence
- avoid poles in sectors with large #prop
- prefer basis with simple, factorizing denom.

➡ reduced median runtime 15h $\rightarrow$ <2h

➡ reduced size of code for coefficients

➡ avoid spurious poles & cancellations

Phase-Space Integration

Evaluation of virtual amplitude very slow
$\rightarrow$ good sampling of phase space required

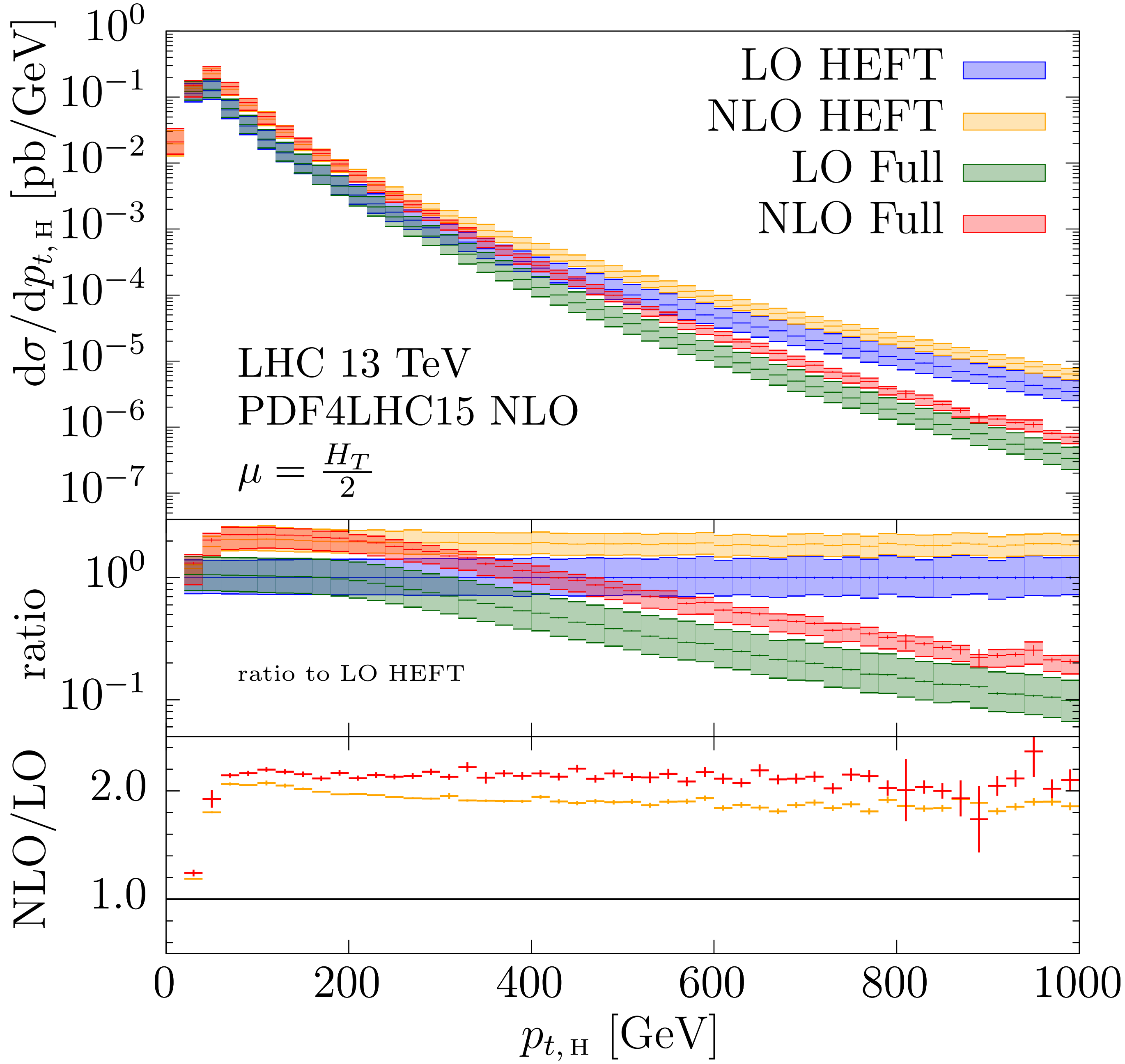
Phase-space integration of virtual corrections:

- generate unweighted events based on differential LO cross section
  $\rightarrow$ nearly perfect importance sampling for evaluating total cross section
- for HJ: include additional p_T-dependent reweighing factor
  enhances number of events in tail of distribution, reducing their weight

Only $\mathcal{O}(1\,k)$ virtual amplitude results required

HJ Results — p_T of Higgs boson

mass effects compared to HEFT



HEFT and full theory predict different scaling of $d\sigma/dp_T^2$

$\sim p_T^{-2}$ in HEFT

$\sim p_T^{-4}$ in full theory

[Caola, Forte, Marzani, Muselli, Vita, 15,16]

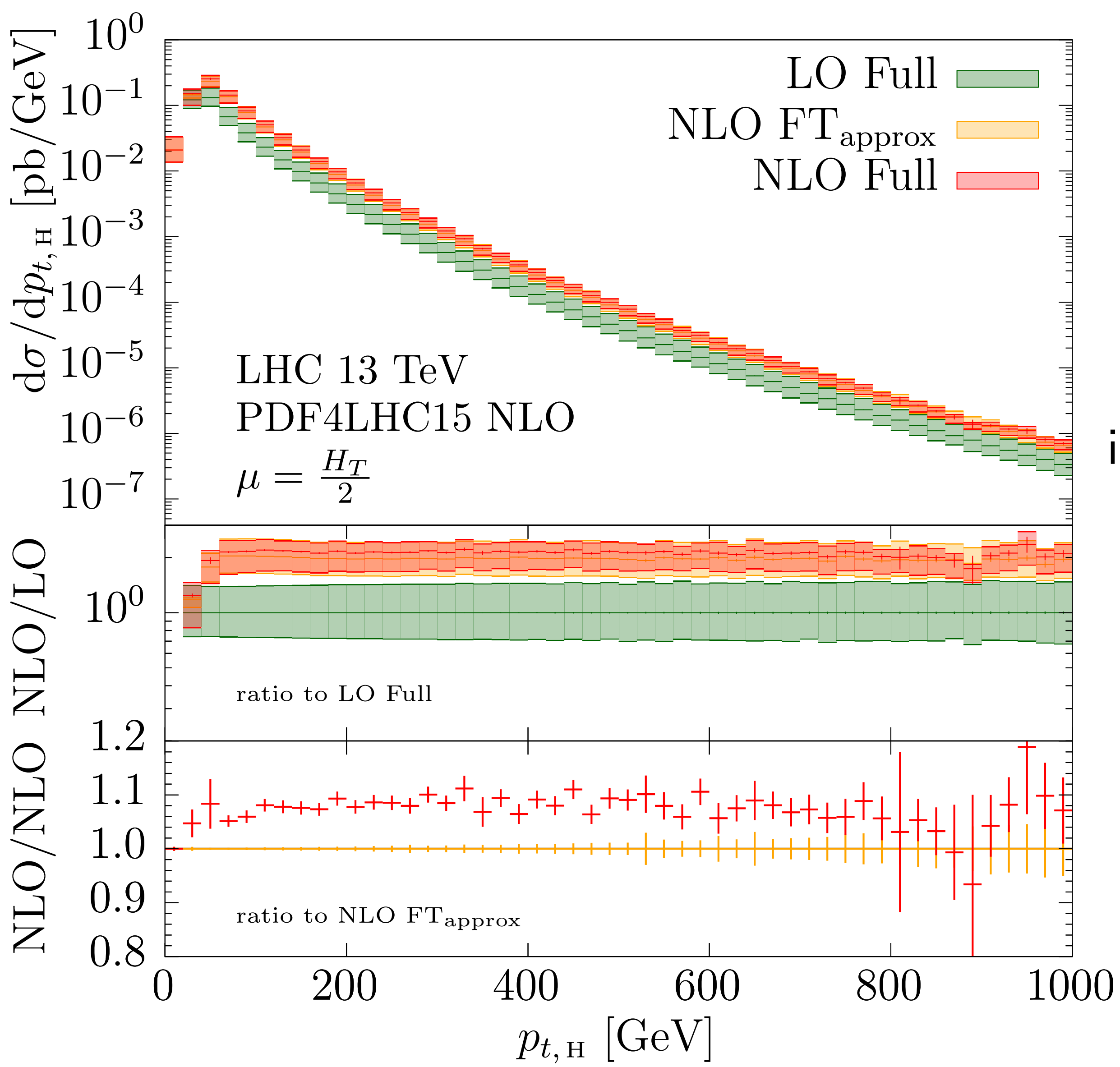
confirmed at NLO

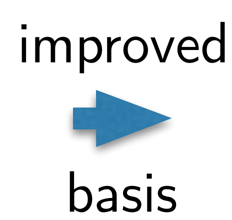
nearly constant K-factor in full theory

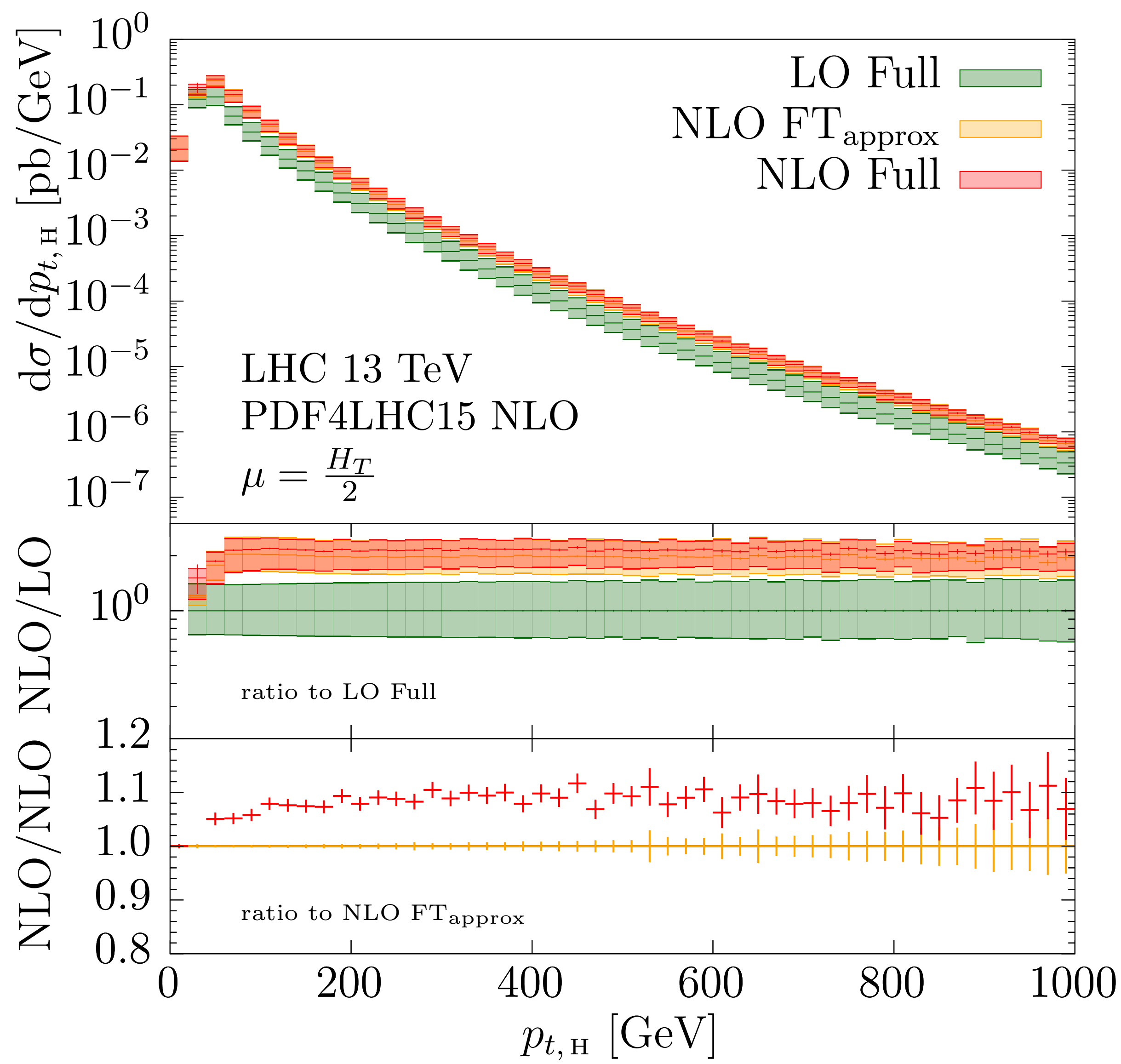
HJ Results — p_T of Higgs boson

mass effects compared to FT_{approx}

- full mt dependence in real radiation
- virtual correction in HEFT, rescaled by $\mathrm{B}(m_t)/\mathrm{B}(m_t \to \infty)$







- FT_{approx} and full theory predict same shape of p_T distribution
- nearly constant increase of ~8% due to top mass in virtual contribution

Grid interpolation (so far only HH)

Calculation of fixed order results:

1. generate unweighted LO events
2. evaluate virtual amplitude at these points
3. obtain histogram of virtual contribution
4. add real radiation (at histogram level)

Problems:

- slow (2h GPU time per phase-space point)
- impractical for
  - combining with parton showers, etc.
  - providing results to other groups

$\rightarrow$ provide results of virtual amplitude together with grid interpolation framework

- use pre-computed amplitude results as input
- obtain interpolated amplitude result for arbitrary phase-space points
- fast & can be interfaced to other codes

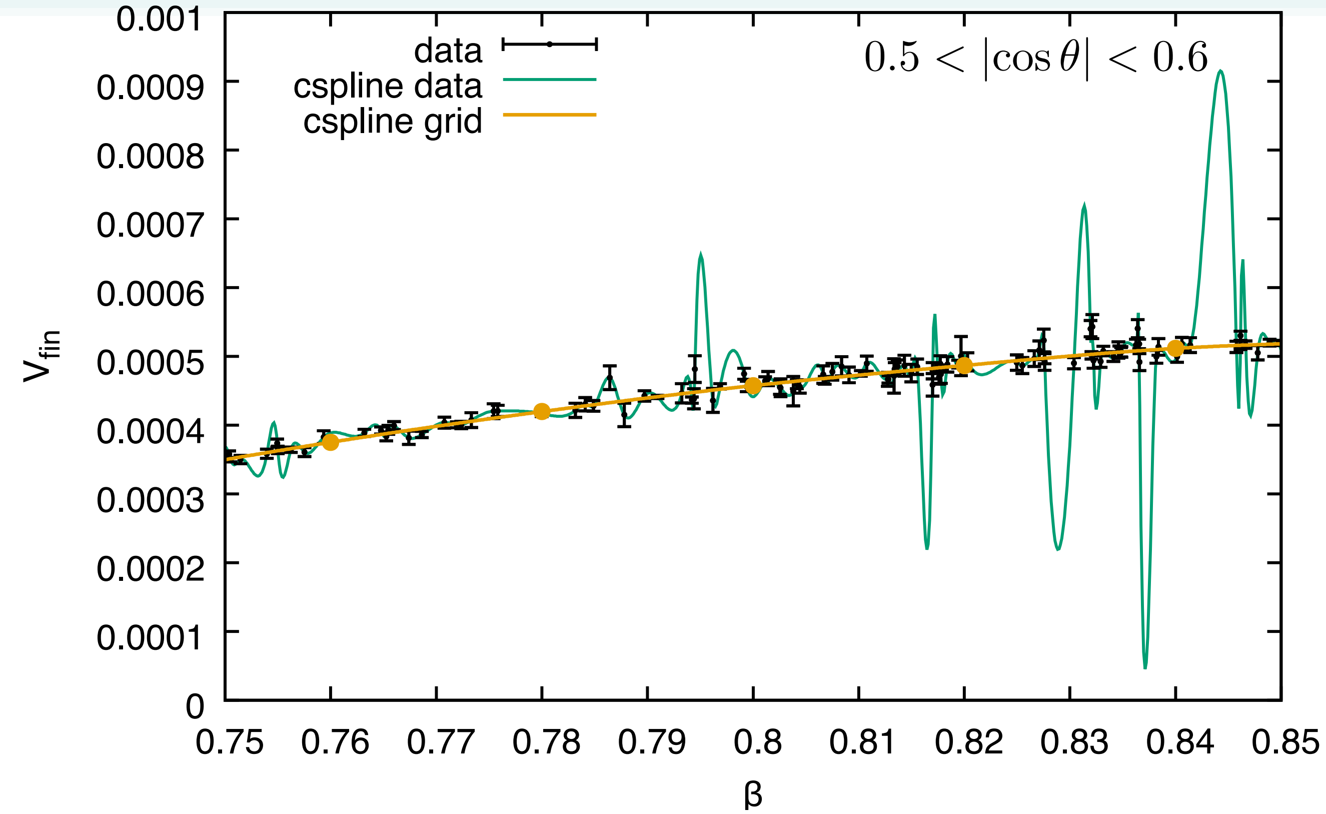
available at github.com/mppmu/hhgrid

Grid interpolation details (so far only HH)

2-dimensional grid interpolation $(\hat{s},\, \hat{t})$

Problems during construction of grid:

- interpolation can enhance numerical uncertainties
- input data not evaluated on equidistant grid points



Details of grid interpolation:

- input parameters $x = f(\beta(\hat{s})), \quad c_\theta = |\cos\theta| = \left|\dfrac{\hat{s} + 2\hat{t} - 2m_H^2}{\hat{s}\beta(\hat{s})}\right|$, with $\beta = \left(1 - \dfrac{4m_H^2}{\hat{s}}\right)^{\frac{1}{2}}$

  $\rightarrow$ nearly uniform distribution of phase space points in $(x, c_\theta) \in [0,1]^2$ if $f(\beta)$ chosen according to cumulative distribution of points in original calculation
- interpolation done in 2 steps:
  1. choose equidistant grid points, estimate result at each grid point with least-square fit to linear function of amplitude results in vicinity
  2. Clough-Tocher interpolation (as implemented in SciPy) to estimate amplitude at arbitrary sampling points

  $\rightarrow$ reduces sensitivity to uncertainties of input-data points

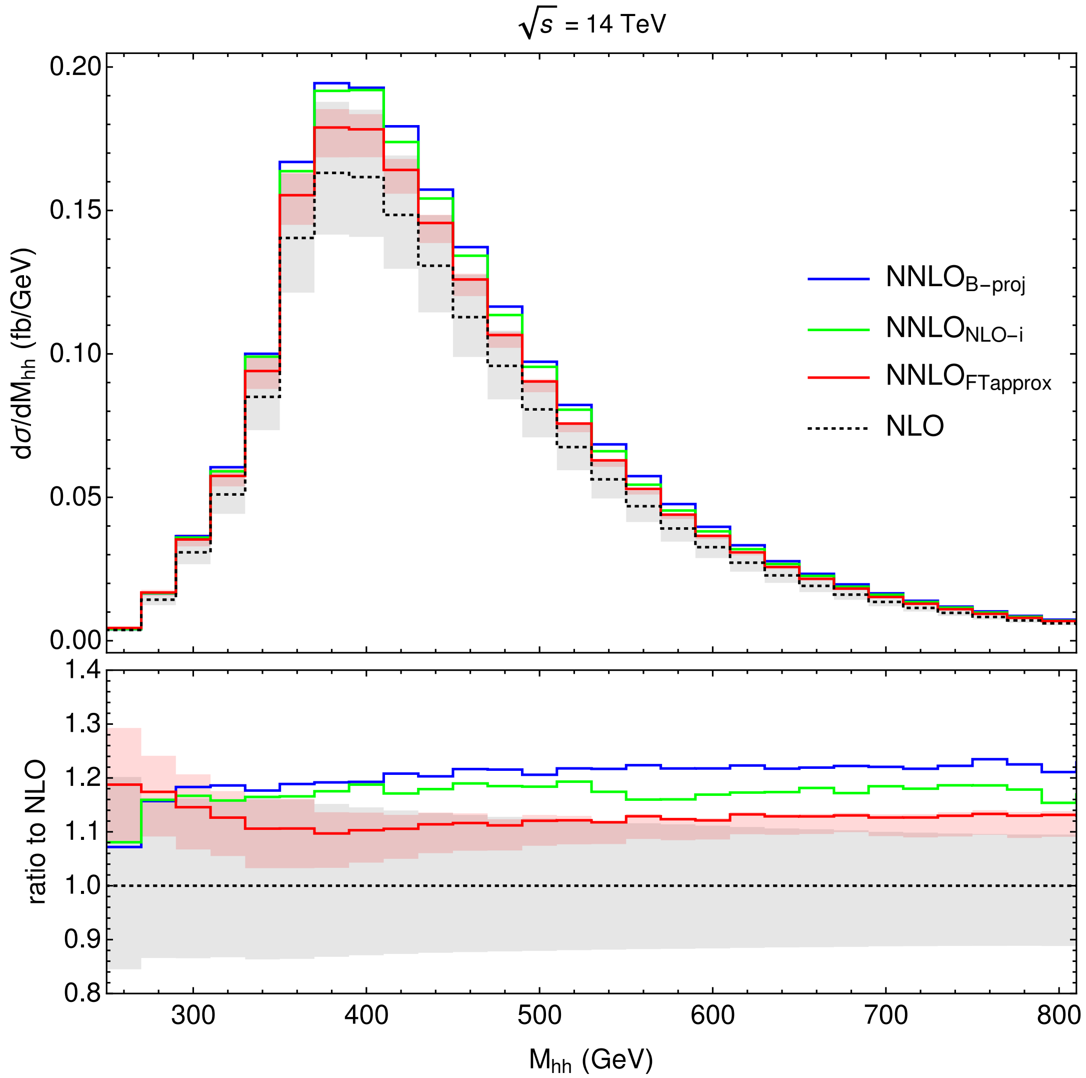
HH — Beyond NLO

combination with parton shower
→ available in PowhegBox-V2
[Heinrich, Jones, MK, Luisoni, Vryonidou 17]
[Jones, Kuttimalai 17]

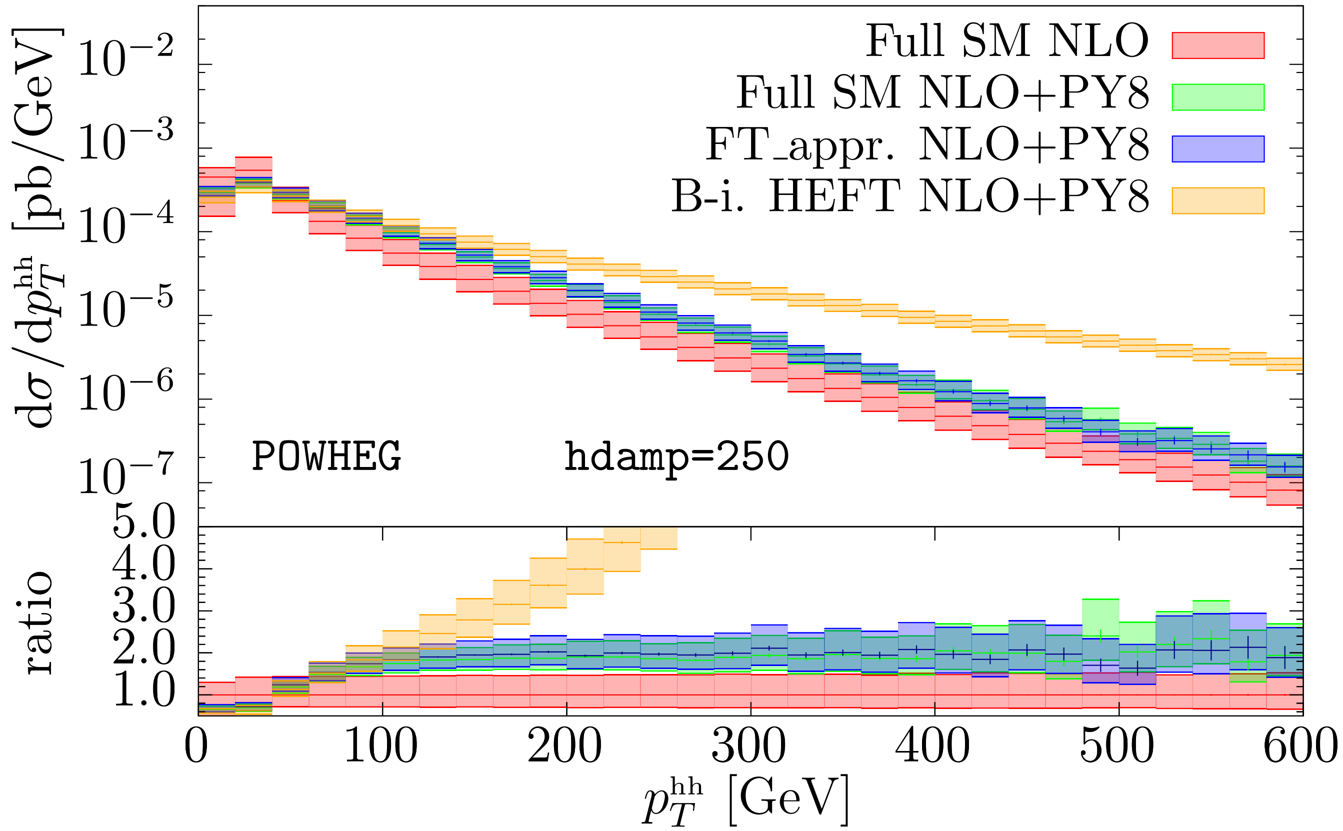
combination with NNLO ($m_t \to \infty$)
→ approx. m_t dependence at NNLO
[Grazzini, Heinrich, Jones, Kallweit, MK, Lindert, Mazzitelli 18]

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO [fb]	$27.78^{+13.8\%}_{-12.8\%}$	$32.88^{+13.5\%}_{-12.5\%}$	$127.7^{+11.5\%}_{-10.4\%}$	$1147^{+10.7\%}_{-9.9\%}$
$\mathrm{NLO_{FTapprox}}$ [fb]	$28.91^{+15.0\%}_{-13.4\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$134.1^{+12.7\%}_{-11.1\%}$	$1220^{+11.9\%}_{-10.6\%}$
$\mathrm{NNLO_{NLO-i}}$ [fb]	$32.69^{+5.3\%}_{-7.7\%}$	$38.66^{+5.3\%}_{-7.7\%}$	$149.3^{+4.8\%}_{-6.7\%}$	$1337^{+4.1\%}_{-5.4\%}$
$\mathrm{NNLO_{B-proj}}$ [fb]	$33.42^{+1.5\%}_{-4.8\%}$	$39.58^{+1.4\%}_{-4.7\%}$	$154.2^{+0.7\%}_{-3.8\%}$	$1406^{+0.5\%}_{-2.8\%}$
$\mathrm{NNLO_{FTapprox}}$ [fb]	$31.05^{+2.2\%}_{-5.0\%}$	$36.69^{+2.1\%}_{-4.9\%}$	$139.9^{+1.3\%}_{-3.9\%}$	$1224^{+0.9\%}_{-3.2\%}$
M_t unc. $\mathrm{NNLO_{FTapprox}}$	±2.6%	±2.7%	±3.4%	±4.6%
$\mathrm{NNLO_{FTapprox}}$/NLO	1.118	1.116	1.096	1.067

NNLO

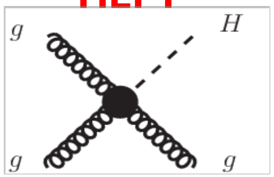
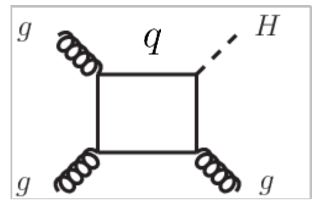


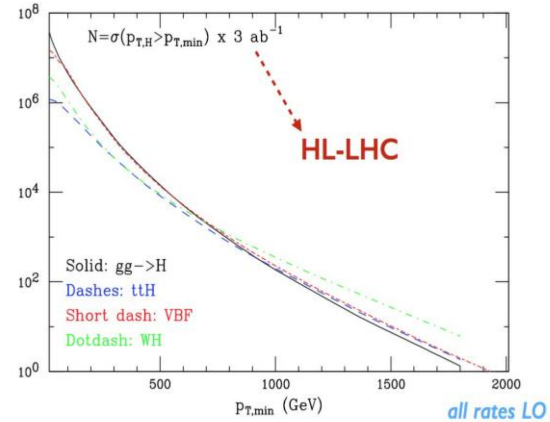
NLO+PS



3. High energy expansions in HJ production

H+j production at LHC

below quark thr.	close to threshold	above quark thr.
HEFT 	increasing $p_{T,H}$	
$m_q \to \infty$	$1 - \frac{4m_q^2}{\hat{s}} \ll 1$	$\frac{4m_q^2}{p_\perp^2} \ll 1$



[Mangano talk at Higgs Couplings 2016]

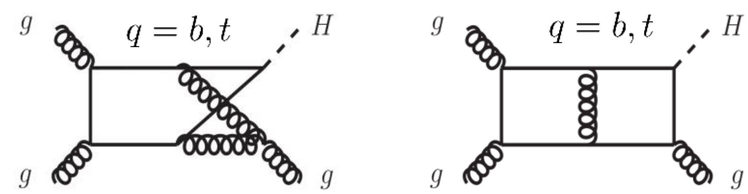
- Computation of bottom contribution starts at 1-loop for moderate $p_{T,H} > 10$ GeV
- Top quark loop resolved at high $p_{T,H} > 350$ GeV

NLO:

- Real corrections can be computed with exact mass dependence (MCFM, Openloops, Recola…)
- New required ingredients are two-loop virtual corrections

Virtual amplitude

- Typical two-loop Feynman diagrams are:



- Project onto form factors

$$\mathcal{A}_{H\to ggg}\,(p_1^{a_1}, p_2^{a_2}, p_3^{a_3}) = f^{a_1 a_2 a_3}\ \epsilon_1^{\mu}\,\epsilon_2^{\nu}\,\epsilon_3^{\rho}\ (F_1\, g^{\mu\nu}\, p_2^{\rho} + F_2\, g^{\mu\rho}\, p_1^{\nu} + F_3\, g^{\nu\rho}\, p_3^{\mu} + F_4\, p_3^{\mu} p_1^{\nu} p_2^{\rho}\,)$$

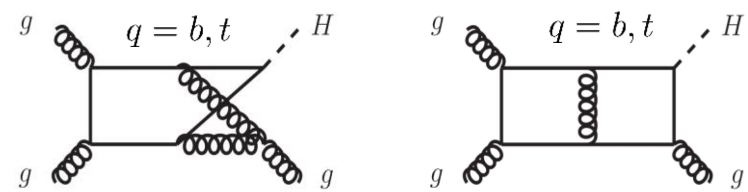
- Reduce with *Integration by parts* (IBP)

$$\mathcal{I}(s) = \sum \mathrm{Rational}(s,d) \times (\mathrm{Master\ Integrals})(s,d)$$

- Exact mass dependence in two-loop Feynman Integrals very difficult and currently out of reach [planar diagrams: Bonciani et al '16]

Virtual amplitude

- Typical two-loop Feynman diagrams are:



- Project onto form factors $$\mathcal{A}_{H\to ggg}\,(p_1^{a_1}, p_2^{a_2}, p_3^{a_3}) = f^{a_1 a_2 a_3}\,\epsilon_1^{\mu}\,\epsilon_2^{\nu}\,\epsilon_3^{\rho}\,(F_1\,g^{\mu\nu}\,p_2^{\rho} + F_2\,g^{\mu\rho}\,p_1^{\nu} + F_3\,g^{\nu\rho}\,p_3^{\mu} + F_4\,p_3^{\mu}p_1^{\nu}p_2^{\rho}\,)$$
- Reduce with *Integration by parts* (IBP) $$\mathcal{I}(s) = \sum \mathrm{Rational}(s,d) \times (\mathrm{Master\ Integrals})(s,d)$$
- Exact mass dependence in two-loop Feynman Integrals very difficult and currently out of reach [planar diagrams: Bonciani et al '16]
- Use expansion approximation

Scale hierarchy below top threshold: $m_b \ll p_\perp, m_h \ll m_t$

Scale hierarchy above top threshold: $m_h \ll 2m_t \ll p_\perp$

Expand in <u>small quark mass</u> approach →

Two-loop *<u>amplitudes</u>* expanded in *<u>quark mass</u>* with differential equation method

[Mueller & Ozturk '15; Melnikov, Tancredi, CW '16, Kudashkin et al '17]

How useful and valid is m_q expansion?

Usefullness:

- Integrals with massive quark loops computed exactly are complicated

$$
\begin{aligned}
&\log\left(x_3x_1^2 - x_1^2 + x_2x_1 - 4x_3x_1 + R_1(x_1)R_2(x_1)R_7(x)\right)\,,\\
&\log\left(-x_2^2 + x_1x_2 - x_1x_3x_2 + 2x_3x_2 + 2x_1x_3 + R_1(x_2)R_2(x_2)R_7(x)\right)\,,\\
&\log\left(-x_3^2x_1^2 + 3x_3x_1^2 + 4x_3^2x_1 - 4x_2x_3x_1 + R_1(x_3)R_5(x)R_6(x)x_1\right)\,,\\
&\log\left(x_3R_1(x_2)R_2(x_2) + x_2R_1(x_3)R_2(x_3)\right)\,,\\
&\log\left(x_1R_1(x_2)R_2(x_2) + x_2R_1(x_1)R_2(x_1)\right)\,,\\
&\log\left(x_1R_1(x_3)R_2(x_3) - R_1(x_1)R_1(x_3)R_5(x)\right)\,,\\
&\log\left(x_3R_1(x_1)R_2(x_1) - R_1(x_1)R_1(x_3)R_5(x)\right)\,,\\
&\log\left(-x_2R_1(x_1)R_2(x_1) + x_3R_1(x_1)R_2(x_1) + x_1R_3(x_3)R_4(x_3)\right)\,,\\
&\log\left(-x_2R_1(x_2)R_2(x_2) + x_3R_1(x_2)R_2(x_2) + x_2R_3(x_3)R_4(x_3)\right)\,,\\
&\log\left(-x_2R_1(x_3)R_2(x_3) + x_1R_1(x_3)R_2(x_3) + x_3R_3(x_1)R_4(x_1)\right)\,,\\
&\log\left(-x_2R_1(x_2)R_2(x_2) + x_1R_1(x_2)R_2(x_2) + x_2R_3(x_1)R_4(x_1)\right)\,,\\
&\log\left(-x_3^2x_1^2 + 3x_3x_1^2 + 4x_3^2x_1 - 3x_2x_3x_1 + R_1(x_1)R_1(x_3)R_5(x)R_7(x)\right)\,,\\
&\log\left(x_2R_1(x_1)R_1(x_3)R_5(x) - x_1x_3R_1(x_2)R_2(x_2)\right)\,,\\
&\log\left(-x_2x_3 + x_1x_3 + R_1(x_2)R_2(x_2)x_3 - R_1(x_1)R_1(x_3)R_5(x)\right).
\end{aligned}
$$

$$
\begin{aligned}
&R_1(x_1) = \sqrt{-x_1}\,,\ R_1(x_3) = \sqrt{-x_3}\,,\ R_1(x_2) = \sqrt{-x_2}\,,\\
&R_2(x_1) = \sqrt{4-x_1}\,,\ R_2(x_3) = \sqrt{4-x_3}\,,\ R_2(x_2) = \sqrt{4-x_2}\,,\\
&R_3(x_1) = \sqrt{x_2-x_1}\,,\ R_3(x_3) = \sqrt{x_2-x_3}\,,\\
&R_4(x_1) = \sqrt{x_2-x_1-4}\,,\ R_4(x_3) = \sqrt{x_2-x_3-4}\,,\\
&R_5(x) = \sqrt{4x_2 + x_1x_3 - 4(x_1+x_3)}\,,\\
&R_6(x) = \sqrt{2x_3(-2x_2 + x_1 + 2x_3) - x_1x_3^2 - x_1}\,,\\
&R_7(x) = \sqrt{2x_1x_3(x_2-x_1) + (x_2-x_1)^2 + (x_1-4)x_1x_3^2}\,.
\end{aligned}
$$

[planar diagrams: Bonciani et al '16]

- Some sectors not known how to express in terms of GPL's anymore plus genuine elliptic sectors
- *Expanding* in small quark mass results in simple *2-dimensional harmonic polylogs*

How useful and valid is m_q expansion?

- Integrals with massive quark loops computed exactly are complicated

Usefullness:

$$
\begin{aligned}
&\log\left(x_3x_1^2 - x_1^2 + x_2x_1 - 4x_3x_1 + R_1(x_1)R_2(x_1)R_7(x)\right),\\
&\log\left(-x_2^2 + x_1x_2 - x_1x_3x_2 + 2x_3x_2 + 2x_1x_3 + R_1(x_2)R_2(x_2)R_7(x)\right),\\
&\log\left(-x_3^2x_1^2 + 3x_3x_1^2 + 4x_3^2x_1 - 4x_2x_3x_1 + R_1(x_3)R_5(x)R_6(x)x_1\right),\\
&\log\left(x_3R_1(x_2)R_2(x_2) + x_2R_1(x_3)R_2(x_3)\right),\\
&\log\left(x_1R_1(x_2)R_2(x_2) + x_2R_1(x_1)R_2(x_1)\right),\\
&\log\left(x_1R_1(x_3)R_2(x_3) - R_1(x_1)R_1(x_3)R_5(x)\right),\\
&\log\left(x_3R_1(x_1)R_2(x_1) - R_1(x_1)R_1(x_3)R_5(x)\right),\\
&\log\left(-x_2R_1(x_1)R_2(x_1) + x_3R_1(x_1)R_2(x_1) + x_1R_3(x_3)R_4(x_3)\right),\\
&\log\left(-x_2R_1(x_2)R_2(x_2) + x_3R_1(x_2)R_2(x_2) + x_2R_3(x_3)R_4(x_3)\right),\\
&\log\left(-x_2R_1(x_3)R_2(x_3) + x_1R_1(x_3)R_2(x_3) + x_3R_3(x_1)R_4(x_1)\right),\\
&\log\left(-x_2R_1(x_2)R_2(x_2) + x_1R_1(x_2)R_2(x_2) + x_2R_3(x_1)R_4(x_1)\right),\\
&\log\left(-x_3^2x_1^2 + 3x_3x_1^2 + 4x_3^2x_1 - 3x_2x_3x_1 + R_1(x_1)R_1(x_3)R_5(x)R_7(x)\right),\\
&\log\left(x_2R_1(x_1)R_1(x_3)R_5(x) - x_1x_3R_1(x_2)R_2(x_2)\right),\\
&\log\left(-x_2x_3 + x_1x_3 + R_1(x_2)R_2(x_2)x_3 - R_1(x_1)R_1(x_3)R_5(x)\right).
\end{aligned}
$$

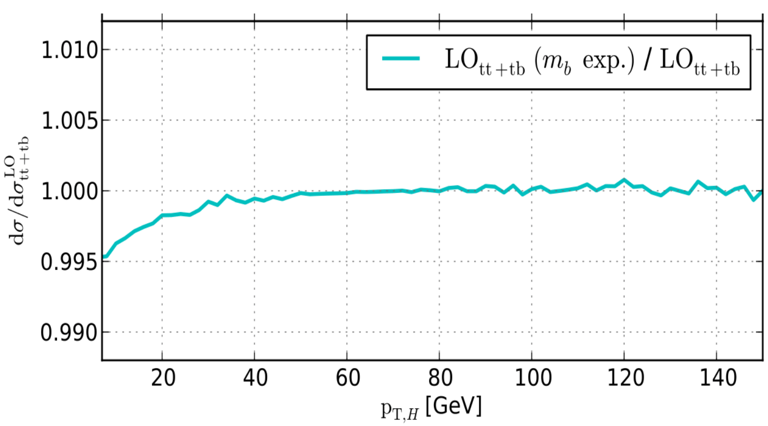
$$
\begin{aligned}
&R_1(x_1) = \sqrt{-x_1}\,,\; R_1(x_3) = \sqrt{-x_3}\,,\; R_1(x_2) = \sqrt{-x_2}\,,\\
&R_2(x_1) = \sqrt{4-x_1}\,,\; R_2(x_3) = \sqrt{4-x_3}\,,\; R_2(x_2) = \sqrt{4-x_2}\,,\\
&R_3(x_1) = \sqrt{x_2-x_1}\,,\; R_3(x_3) = \sqrt{x_2-x_3}\,,\\
&R_4(x_1) = \sqrt{x_2-x_1-4}\,,\; R_4(x_3) = \sqrt{x_2-x_3-4}\,,\\
&R_5(x) = \sqrt{4x_2 + x_1x_3 - 4(x_1+x_3)}\,,\\
&R_6(x) = \sqrt{2x_3(-2x_2 + x_1 + 2x_3) - x_1x_3^2 - x_1}\,,\\
&R_7(x) = \sqrt{2x_1x_3(x_2-x_1) + (x_2-x_1)^2 + (x_1-4)x_1x_3^2}\,.
\end{aligned}
$$

[planar diagrams: Bonciani et al '16]

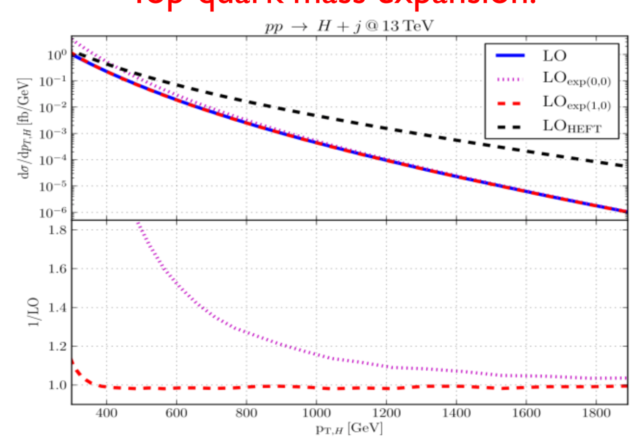
- Some sectors not known how to express in terms of GPL's anymore plus genuine elliptic sectors
- *__Expanding__* in small quark mass results in simple *__2-dimensional harmonic polylogs__*

Validity:

Bottom-quark mass expansion:



Top-quark mass expansion:



IBP reduction difficulties

[Melnikov, Tancredi, CW '16-'17]

- IBP reduction to Master Integrals $\mathcal{I}(s) = \sum \mathrm{Rational}(s,d) \times (\mathrm{Master\ Integrals})(s,d)$

- *Reduction very non-trivial*: we were not able to reduce top non-planar integrals with $t = 7$ denominators with FIRE5/Reduze $\longrightarrow$ coefficients become too large to simplify ~ hundreds of Mb of text

IBP reduction difficulties

[Melnikov, Tancredi, CW '16-'17]

- IBP reduction to Master Integrals $\mathcal{I}(s) = \sum \mathrm{Rational}(s,d) \times (\mathrm{Master\ Integrals})(s,d)$

- *Reduction very non-trivial*: we were not able to reduce top non-planar integrals with $t = 7$ denominators with FIRE5/Reduze $\longrightarrow$ coefficients become too large to simplify ~ hundreds of Mb of text

- *Reduction for complicated t=7 non-planar integrals performed in two steps*:

  1) FORM code reduction: $\mathcal{I}^{\mathrm{NPL}}_{t=7} = \sum c_i \mathrm{MI}^i_{t=7} + \sum d_i \mathcal{I}^i_{t=6}$

  2) Plug reduced integrals into amplitude, expand coefficients c_i, d_i in m_q

  3) Reduce with FIRE/Reduze: $t = 6$ denominator integrals $\mathcal{I}_{t=6}$

- Exact m_q dependence kept at intermediate stages. Algorithm for *solving IBP identities directly expanded in small parameter is still an open problem*

MI with DE method for small m_q (1/2)

- System of partial differential equations (**DE**) in $\boldsymbol{m_q, s, t, m_h^2}$ with IBP relations

$$\frac{\partial}{\partial \tilde{s}_k}\vec{\mathcal{I}}^{MI}(\tilde{s},\epsilon) \overset{\text{IBP}}{=} \overline{\overline{M}}_k(\tilde{s},\epsilon).\vec{\mathcal{I}}^{MI}(\tilde{s},\epsilon)$$

- Interested in m_q expansion of <u>Master integrals</u> I^{MI}

→ expand homogeneous matrix M_k in small m_q

<u>Step 1:</u> solve DE in m_q

- Solve m_q DE with following ansatz

$$\mathcal{I}_i^{MI}(m_q^2, s, t, m_h^2, \epsilon) = \sum_{ijkn} c_{ijkn}(s,t,m_h^2,\epsilon)\left(\frac{m_q^2}{s}\right)^{j-k\epsilon} \log^n\left(\frac{m_q^2}{s}\right)$$

- **<u>Peculiarity</u>**: half-integer powers of (squared) quark mass also in Ansatz, contributing momentum region unknown
- Plug into m_q DE and get constraints on coefficients c_{ijkn}
- c_{i000} is $m_q = 0$ solution (hard region) and has been computed before

[Gehrmann & Remiddi '00, Tausk, Anastasiou et al '99, Argeri et al. '14]

MI with DE method for small m_q (2/2)

- Ansatz $$\mathcal{I}_i^{MI}(m_q^2, s, t, m_h^2, \epsilon) = \sum_{ijkn} c_{ijkn}(s, t, m_h^2, \epsilon) \left(\frac{m_q^2}{s}\right)^{j-k\epsilon} \log^n\left(\frac{m_q^2}{s}\right)$$

Step 2: solve s, t, m_h^2 DE for $c_{ijkn}(s, t, m_h^2)$

- Solution expressed in extensions of usual polylogarithms: *Goncharov Polylogarithms*
- After solving DE for unknown c_{ijkn}, we are left with *unknown boundary constants* that only depend on ε

Step 3: fix ε dependence

- Determination of most boundary constants in ε by *imposing that unphysical cut singularities in solution* vanish
- Other constants in ε fixed by matching solution of DE to Master integrals computed via various methods (Mellin-Barnes, expansion by regions, numerical fits) in a specific point of s, t, m_h^2

Step 4: numerical checks with FIESTA

[A. Smirnov '14]

Constants: Mellin-Barnes method

- Let's say $(m_q^2)^{-1-2\epsilon}$ branch required of $\quad \mathcal{I}^{MI} = c_1 \left(\frac{m_q^2}{s}\right)^{-1-\epsilon} + c_2 \left(\frac{m_q^2}{s}\right)^{-1-2\epsilon} + \mathcal{O}((m_q^2)^0)$

$$\mathcal{I}^{MI} = \int \frac{D^d k D^d l}{((k_1+p_1)^2 - m_q^2)((k_1-p_{23})^2 - m_q^2)(k_2^2 - m_q^2)((k_2+p_1)^2 - m_q^2)((k_1-k_2)^2)^{1+\delta}((k_1-k_2-p_{23})^2)^{1-\delta}}$$

- Mellin-Barnes integration in complex plane $\quad \frac{1}{(x+y)^\lambda} = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \frac{y^z}{x^{z+\lambda}} \frac{\Gamma(-z)\Gamma(\lambda+z)}{\Gamma(\lambda)} \longrightarrow$

- Mellin-Barnes representation in s=u=-1

$$\begin{aligned}\mathcal{I}^{MI} = &- \int_{-i\infty}^{+i\infty} \left(\prod_{i=1}^{4} dz_i\right) (-2-i0)^{-2\epsilon-z_1-z_2-z_3-3} (m_q^2)^{z_1} \Gamma(-z_1)\Gamma(-z_2)\Gamma(z_2+1)\Gamma(-z_3) \\ &\times \frac{\Gamma(-z_4)\Gamma(-\epsilon-z_1-1)\Gamma(z_4-\epsilon)\Gamma(z_3-\delta+1)\Gamma(-2\epsilon-z_1-z_2-2)\Gamma(z_2+z_3+z_4+1)}{\Gamma(1-\delta)\Gamma(\delta+1)\Gamma(\epsilon+1)^2\Gamma(-2\epsilon-2z_1-1)\Gamma(-3\epsilon-z_1-1)\Gamma(-2\epsilon-z_1-1)} \\ &\times \Gamma(2\epsilon+z_1+z_2+z_3+3)\Gamma(-2\epsilon-z_1-z_3+\delta-2)\Gamma(-\epsilon-z_1-z_2-z_3-z_4-1).\end{aligned}$$

- Require the pole at $z_1 = -1-2\epsilon \longrightarrow$ result is coefficient c_2

- After picking up pole, we expand in epsilon and apply Barnes-Lemma's, which reduces the amount of integrations to <u>one</u> (completely automatized steps)

- Fit numerically (integrals converge fastly) the constant or compute analytically by closing contours in complex plane of Mellin-Barnes integration

H+j below top threshold $p_{T,H} \leq 350$ GeV

- Large Sudakov logarithms at very low $p_{T,H} \leq 30$ GeV

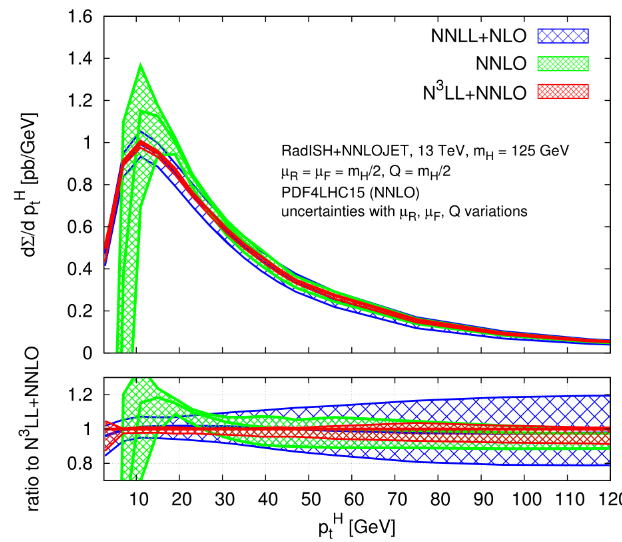
$$\frac{d\sigma}{dp_{T,H}} \sim \exp\{\alpha_s \log^2\left(\frac{p_{T,H}}{m_h}\right) + \alpha_s \log\left(\frac{p_{T,H}}{m_h}\right) + \cdots\}$$

- Resummation reduces scale error: top contribution understood to within few percent error

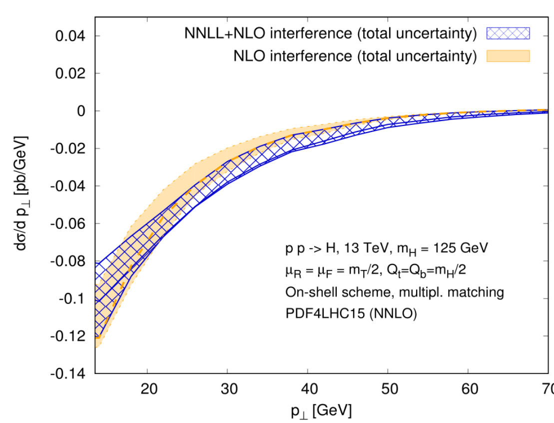
- What about NLO bottom corrections?

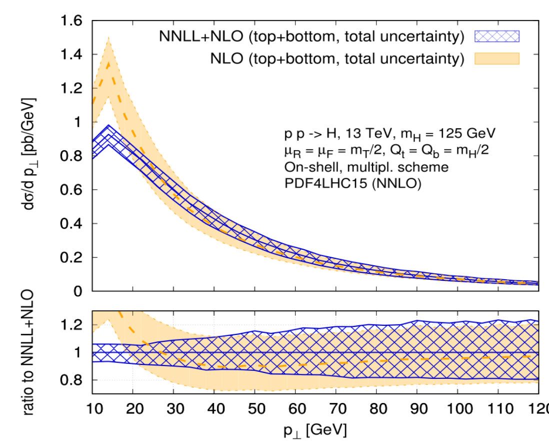
- Top-bottom interference contribution error~20%, translates to ~1-2% error on total

- Largest uncertainty of the top-bottom interference contribution from bottom mass scheme choice



[Bizon, Chen et al., arXiv: 1805.0591]

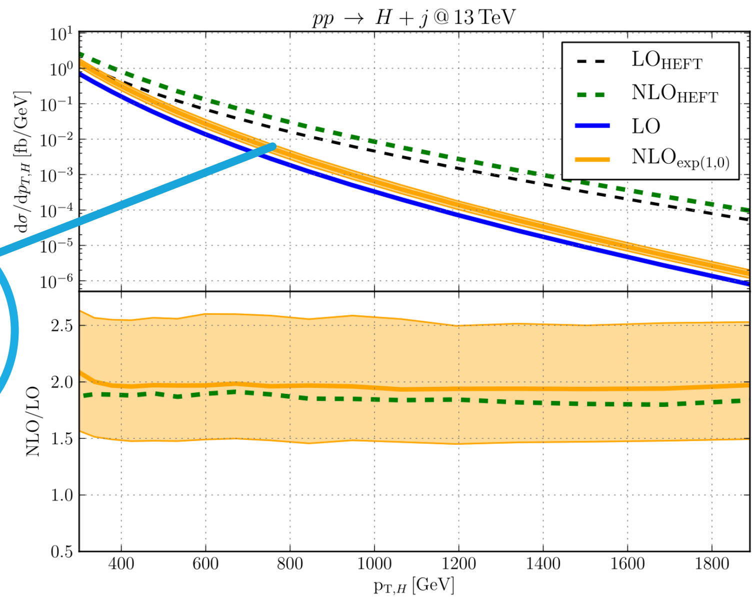




[Caola et al., ArXiv: 1804.07632]

H+j above top threshold $p_{T,H} \geq 350$ GeV

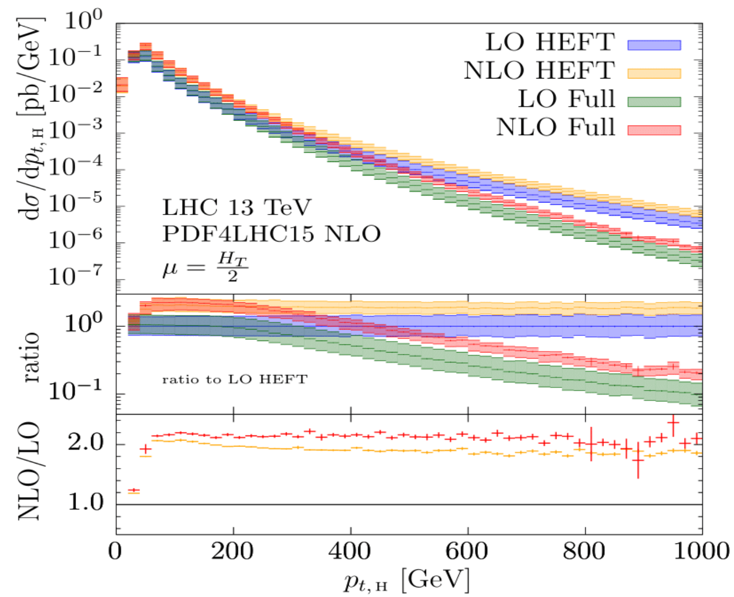
Quark-mass expansion approach:



Expansion in top and Higgs mass

[Kudashkin et al., arXiv: 1801.08226]

Numerical approach:



[Jones et al., arXiv: 1802.00349]

$$\sigma^{\mathrm{theory,NLO}}_{p_{T,H}\geq 450\,\mathrm{GeV}}(gg \to H(\to b\bar{b})) \sim 7\,\mathrm{fb} \pm 20\%$$

$$\sigma^{\mathrm{CMS}}_{p_{T,H}\geq 450\,\mathrm{GeV}}(gg \to H(\to b\bar{b})) \sim 74 \pm 48(\mathrm{stat}) \pm 17(\mathrm{syst})\,\mathrm{fb}$$

- The NLO predictions of expansion and numerical approach agree beautifully
- NLO theory result should be multiplied with $\frac{NNLO_{HEFT}}{NLO_{HEFT}} \sim 1.2$ if proximity of HEFT and SM K-factors postulated to occur at NNLO as well

Summary and Outlook

H+j production at NLO

2 different methods for computing virtual 2-loop amplitude

- Fully numerical with exact mt dependence

→ slow, but grid interpolation can be used
for fast evaluation of virtual amplitude

- Expansion in mt and mb with differential equation method

→ top-bottom interference contribution error ~20%,
translates to ~1-2% error on total

✓ Good agreement of both calculations

- Top mass effects increase cross section by ~9%

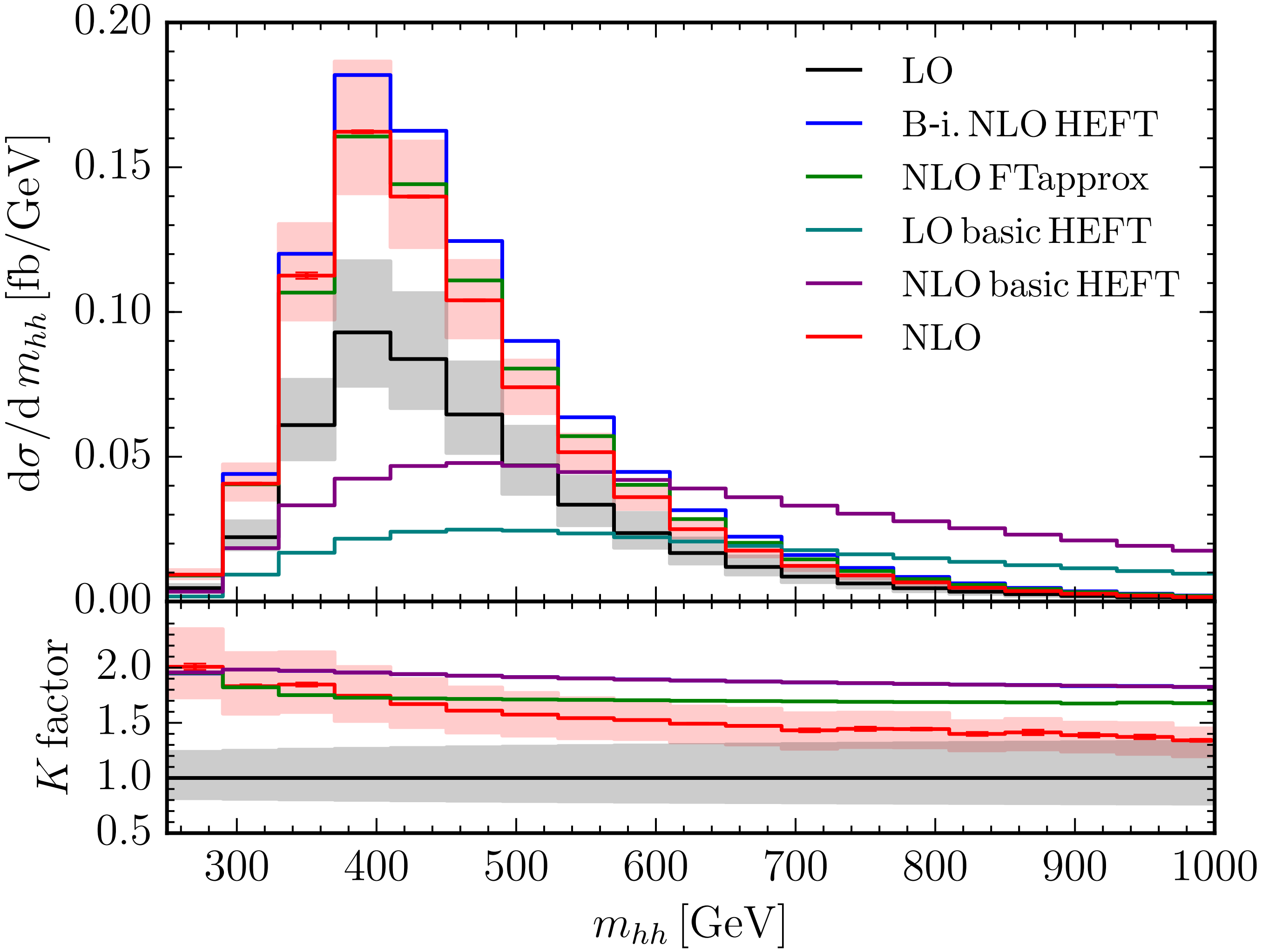
HH production at NLO

- Calculated using numerical approach
- Combined with parton shower & NNLO HEFT

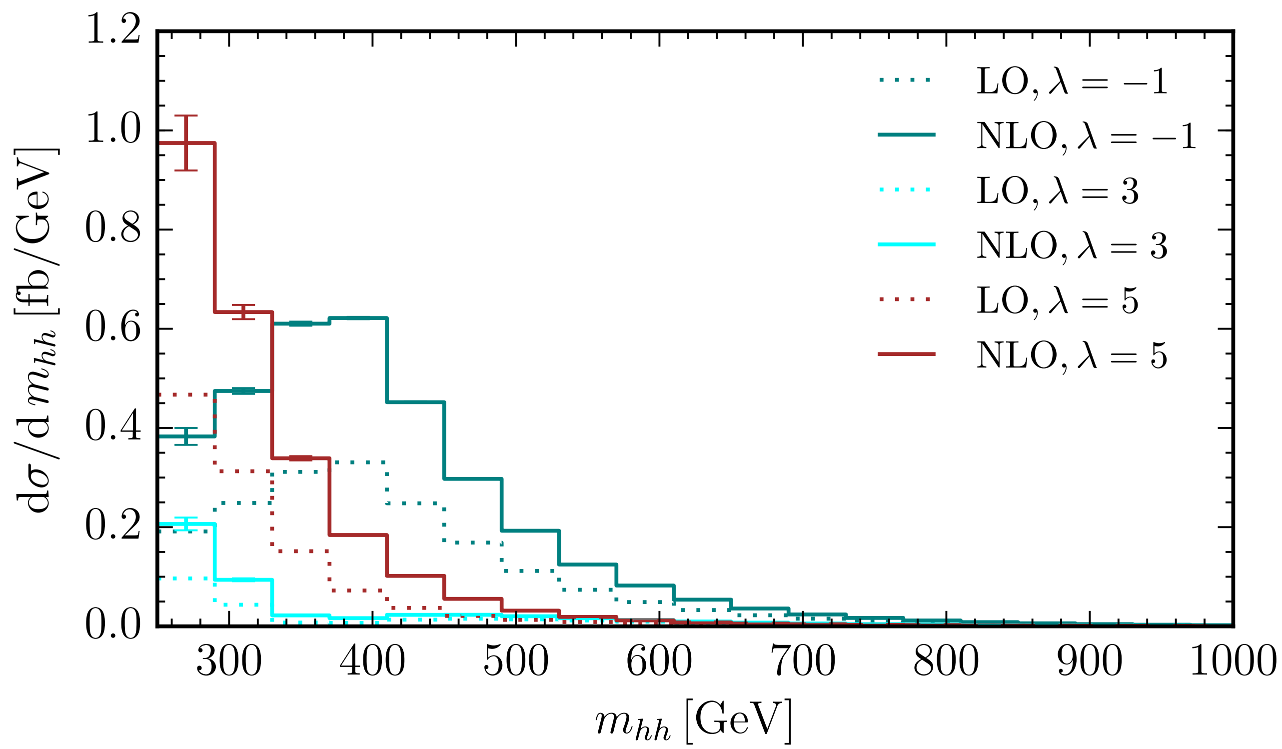
Backup

HH — Results

[Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Zirke `16]

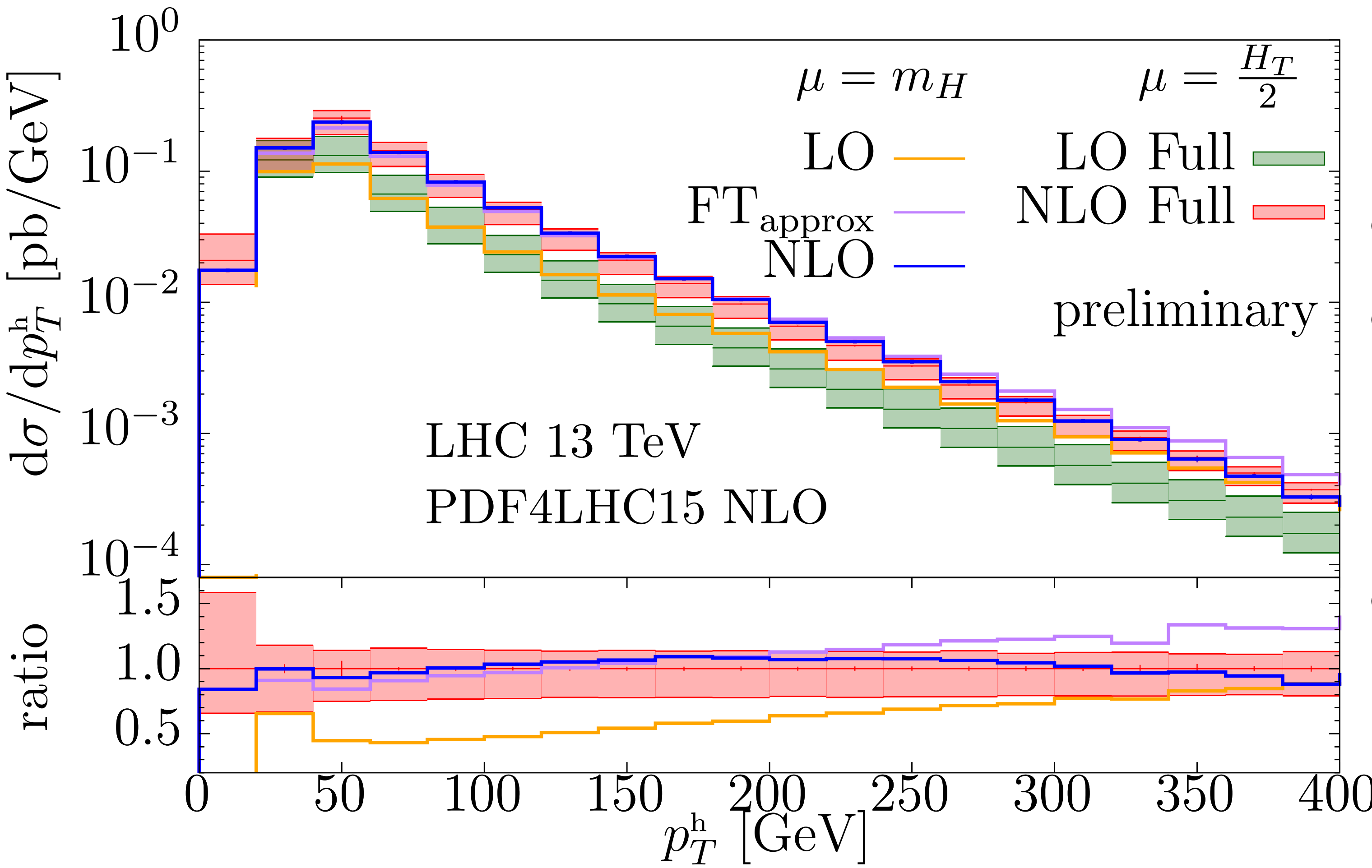


	14 TeV	100 TeV
LO	$19.85^{+27.6\%}_{-20.5\%}$	$731.3^{+20.9\%}_{-15.9\%}$
B.i. HEFT	$38.32^{+18.1\%}_{-14.9\%}$	$1511^{+16.0\%}_{-13.0\%}$
FT approx	$34.26^{+14.7\%}_{-13.2\%}$	$1220^{+11.9\%}_{-10.7\%}$
NLO full	$32.91^{+13.6\%}_{-12.6\%}$	$1149^{+10.8\%}_{-10.0\%}$



HJ Results — Different scale choices

comparison of central scales $H_T/2$ and m_H

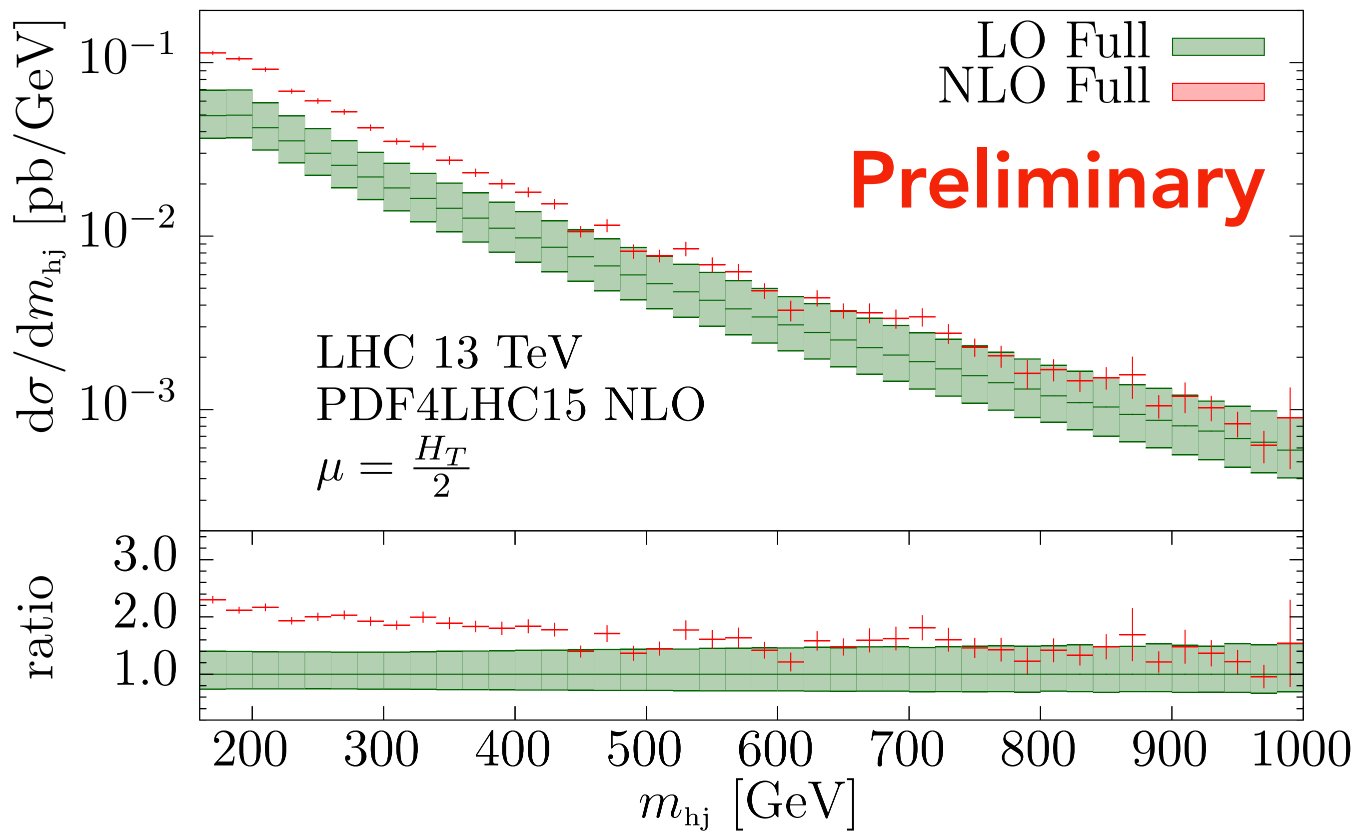


choosing $\mu_R, \mu_F = m_H$ leads to

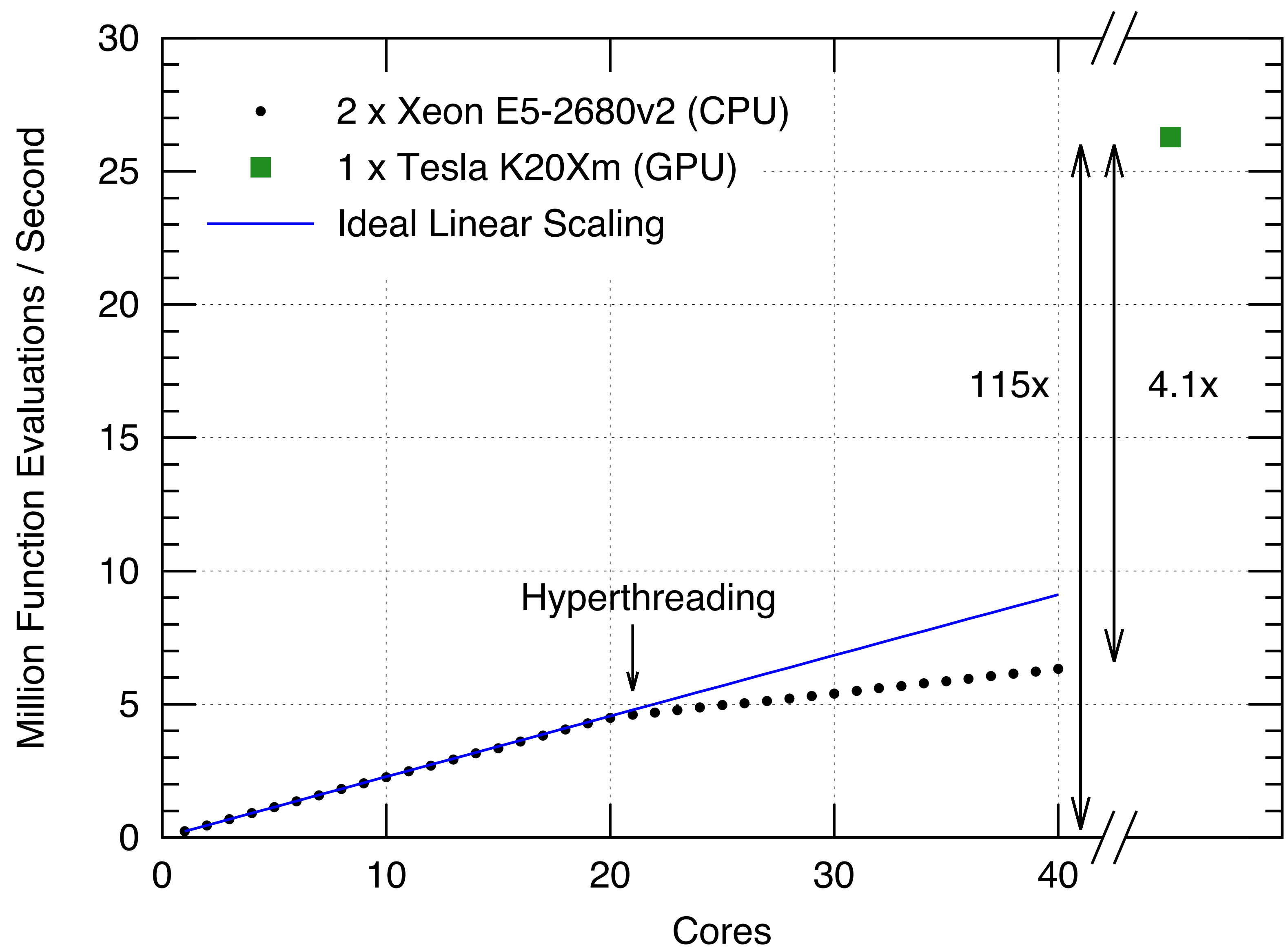
- different shape of LO distribution
- FT$_{\text{approx}}$
  - good agreement at low p_T
  - overestimates the tail
- full result in very good agreement with results with $\mu_R, \mu_F = H_T/2$

$\rightarrow$ top-quark mass effects only small for $\mu_R, \mu_F = H_T/2$!

HJ Results — invariant mass



CPU vs. GPU



plot:
Stephen Jones