Recent developments in NNLOJET and applications towards higher orders

High Time for Higher Orders: From Amplitudes to Phenomenology

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OUTLINE

- Antenna subtraction method up to NNLO
- Higgs production at N3LO using q_T subtraction
- The projection-to-Born method

Parton Level Event Generator using Antenna Subtraction

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* NNLOJET: A multiprocess parton level event generator at O(alpha_s^3)*
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X. Chen, J. Cruz-Martinez, J. Currie, R. Gauld, A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, M. Höfer, A. Huss, I. Majer, T. Morgan, J. Niehues, J. Pires, D. Walker

✔	$pp \to H + 1$ jet (ggF)	NNLO	1408.5325, 1607.08817, 1805.00736, 1805.05916
✔	$pp \to H + 2$ jet (VBF)	NNLO	1802.02445
✔	$pp \to Z/\gamma^*/W^\pm + 1$ jet	NNLO	1507.02850, 1605.04295, 1708.00008, 1712.07543
✔	$pp \to 2$ jets	NNLO	1310.3993, 1611.01460, 1705.10271, 1804.05663
✔	$ep \to e + 2$ jets	NNLO	1606.03991, 1703.05977, 1804.05663, 1807.02529
✔	$e + e- \to 3$ jets	NNLO	0710.0346, 0711.4711, 1709.01097
✔	$pp \to H$ (ggF)	N3LO	(approx.) 1807.11501
✔	$ep \to e + 1$ jet	N3LO	1803.09973
	$\cdots$		all process @NNLO or above

Infrared Subtraction up to NNLO

- Structure of parton level $pp \to X+$Jet up to NNLO (subtraction approach):

$$\hat{\sigma}_{LO}^{X+J} = \int_{d\Phi_{X+1}} d\hat{\sigma}_{LO}^{B,X+J}$$

$$\hat{\sigma}_{NLO}^{X+J} = \int_{d\Phi_{X+2}} (d\hat{\sigma}_{NLO}^{R,X+J} - d\hat{\sigma}_{NLO}^{S,X+J}) + \int_{d\Phi_{X+1}} (d\hat{\sigma}_{NLO}^{V,X+J} - d\hat{\sigma}_{NLO}^{T,X+J})$$

$$\hat{\sigma}_{NNLO}^{X+J} = \int_{d\Phi_{X+3}} (d\hat{\sigma}_{NNLO}^{RR,X+J} - d\hat{\sigma}_{NNLO}^{S,X+J}) + \int_{d\Phi_{X+2}} (d\hat{\sigma}_{NNLO}^{RV,X+J} - d\hat{\sigma}_{NNLO}^{T,X+J}) + \int_{d\Phi_{X+1}} (d\hat{\sigma}_{NNLO}^{VV,X+J} - d\hat{\sigma}_{NNLO}^{U,X+J})$$

Infrared Subtraction up to NNLO

- Structure of parton level $pp \to X+$Jet up to NNLO (subtraction approach):

$$\hat{\sigma}^{X+J}_{LO} = \int_{d\Phi_{X+1}} d\hat{\sigma}^{B,X+J}_{LO}$$

$$\hat{\sigma}^{X+J}_{NLO} = \int_{d\Phi_{X+2}} (d\hat{\sigma}^{R,X+J}_{NLO} - d\hat{\sigma}^{S,X+J}_{NLO}) + \int_{d\Phi_{X+1}} (d\hat{\sigma}^{V,X+J}_{NLO} - d\hat{\sigma}^{T,X+J}_{NLO})$$

$$\hat{\sigma}^{X+J}_{NNLO} = \int_{d\Phi_{X+3}} (d\hat{\sigma}^{RR,X+J}_{NNLO} - d\hat{\sigma}^{S,X+J}_{NNLO}) + \int_{d\Phi_{X+2}} (d\hat{\sigma}^{RV,X+J}_{NNLO} - d\hat{\sigma}^{T,X+J}_{NNLO}) + \int_{d\Phi_{X+1}} (d\hat{\sigma}^{VV,X+J}_{NNLO} - d\hat{\sigma}^{U,X+J}_{NNLO})$$

- Consistency requirement:

$$0 = \int_{d\Phi_{X+1}} d\hat{\sigma}^{T,X+J}_{NLO} + \int_{d\Phi_{X+2}} d\hat{\sigma}^{S,H+J}_{NLO}$$

$$0 = \int_{d\Phi_{X+3}} d\hat{\sigma}^{S,X+J}_{NNLO} + \int_{d\Phi_{X+2}} d\hat{\sigma}^{T,X+J}_{NNLO} + \int_{d\Phi_{X+1}} d\hat{\sigma}^{U,X+J}_{NNLO}$$

Infrared Subtraction up to NNLO

- Structure of parton level $pp \to X$+Jet up to NNLO (subtraction approach):

$$\hat{\sigma}^{X+J}_{LO} = \int_{d\Phi_{X+1}} d\hat{\sigma}^{B,X+J}_{LO}$$

$$\hat{\sigma}^{X+J}_{NLO} = \int_{d\Phi_{X+2}} (d\hat{\sigma}^{R,X+J}_{NLO} - d\hat{\sigma}^{S,X+J}_{NLO}) + \int_{d\Phi_{X+1}} (d\hat{\sigma}^{V,X+J}_{NLO} - d\hat{\sigma}^{T,X+J}_{NLO})$$

$$\hat{\sigma}^{X+J}_{NNLO} = \int_{d\Phi_{X+3}} (d\hat{\sigma}^{RR,X+J}_{NNLO} - d\hat{\sigma}^{S,X+J}_{NNLO}) + \int_{d\Phi_{X+2}} (d\hat{\sigma}^{RV,X+J}_{NNLO} - d\hat{\sigma}^{T,X+J}_{NNLO}) + \int_{d\Phi_{X+1}} (d\hat{\sigma}^{VV,X+J}_{NNLO} - d\hat{\sigma}^{U,X+J}_{NNLO})$$

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$$0 = \int_{d\Phi_{X+3}} d\hat{\sigma}^{S,X+J}_{NNLO} + \int_{d\Phi_{X+2}} d\hat{\sigma}^{T,X+J}_{NNLO} + \int_{d\Phi_{X+1}} d\hat{\sigma}^{U,X+J}_{NNLO}$$

- Subtraction terms mimic the divergent behaviour of matrix elements
- Each bracket is IR divergent until apply Jet algorithm $X + 1, 2, 3 \to X + J$
- The construction of red terms depends on different subtraction schemes

Atenna factorisation at NLO

- Exploit universal factorisation properties in IR limits

$$\underbrace{|\mathcal{A}^0_{m+1}(\ldots,i,j,k,\ldots)|^2}_{\text{colour-ordered amplitude}} \xrightarrow{j \text{ unresolved}} \underbrace{X^0_3(i,j,k)}_{\text{antenna function}} \underbrace{|\mathcal{A}^0_m(\ldots,\widetilde{I},\widetilde{K},\ldots)|^2}_{\text{reduced ME}}$$

+ mapping $\{p_i, p_j, p_k\} \to \{\widetilde{p}_I, \widetilde{p}_K\}$

- Captures multiple limits and smoothly interpolates between them

limit	$X^0_3(i,j,k)$	mapping
$p_j \to 0$	$\frac{2s_{ik}}{s_{ij}s_{jk}}$	$\widetilde{p}_I \to p_i,\ \widetilde{p}_K \to p_k$
$p_j \parallel p_i$	$\frac{1}{s_{ij}} P_{ij}(z)$	$\widetilde{p}_I \to (p_i + p_j),\ \widetilde{p}_K \to p_k$
$p_j \parallel p_k$	$\frac{1}{s_{jk}} P_{kj}(z)$	$\widetilde{p}_I \to p_i,\ \widetilde{p}_K \to (p_j + p_k)$

- Remove $\cdots$ for the simplist case to define X^0_3

$$X^0_3(i,j,k) \sim \frac{|\mathcal{A}^0_3(i,j,k)|^2}{|\mathcal{A}^0_2(\tilde{I},\tilde{K})|^2}$$

  - $|\mathcal{A}^0_2(\tilde{I},\tilde{K})|^2$ is form factor with no knowledge of mapping
  - $X^0_3(i,j,k)$ doesn't have angular correlation of collinear limits (spin averaged)

Atenna factorisation at NNLO

- Unresolved IR limits at NNLO[1]:

$$|\mathcal{A}^0_{m+2}(\cdots,i,j,k,l,\cdots)|^2 \xrightarrow{j,k \text{ unresolved}} X^0_4(i,j,k,l)|\mathcal{A}^0_m(\cdots,\tilde{I},\tilde{L},\cdots)|^2$$

$$|\mathcal{A}^1_{m+1}(\cdots,i,j,k,\cdots)|^2 \xrightarrow{j \text{ unresolved}} X^0_3(i,j,k)|\mathcal{A}^1_m(\cdots,\tilde{I},\tilde{K},\cdots)|^2 + X^1_3(i,j,k)|\mathcal{A}^0_m(\cdots,\tilde{I},\tilde{K},\cdots)|^2$$

- New Antenna functions at NNLO:

$$X^0_4(i,j,k,l) \sim \frac{|\mathcal{A}^0_4(i,j,k,l)|^2}{|\mathcal{A}^0_2(\tilde{I},\tilde{L})|^2}$$

$$X^1_3(i,j,k) \sim \frac{|\mathcal{A}^1_3(i,j,k)|^2}{|\mathcal{A}^0_2(\tilde{I},\tilde{K})|^2} - X^0_3(i,j,k)\frac{|\mathcal{A}^1_2(\tilde{I},\tilde{K})|^2}{|\mathcal{A}^0_2(\tilde{I},\tilde{K})|^2}$$

- With multiple build-in IR limits

$$X^0_4(i,j,k,l) : j,k \to 0 \;\&\; (i||j||k) \;\&\; (j||k||l) \;\&\; (i||j),(k||l) \;\&\; (i||j),k \to 0 \;\&\; (k||l),j \to 0 \;\&\; \text{single-unresolved}$$

$$X^1_3(i,j,k) : j \to 0 \;\&\; (j||k) \;\&\; (i||j)$$

[1] Notation $|\mathcal{A}^1|^2 = \mathcal{A}^1\mathcal{A}^{0\dagger} + \mathcal{A}^{0\dagger}\mathcal{A}^1$

Momentum mapping in antenna subtraction

- Need mapped momentum set in reduced matrix elements
- Need to know the phase space volume for integrated antenna functions
- Two resolved hard radiators $\tilde{I}, \tilde{K}$ could undergo (FF), (IF) or (II) mapping
- Momentum mapping example (Initial-Initial case):$\{p_{\hat{i}}, p_{\mathcal{J}}, p_{\hat{m}}\} \to \{p_{\hat{I}}, p_{\hat{M}}\}$

$$p^{\mu}_{\hat{I}} = \hat{x}_i p^{\mu}_i, \qquad \hat{x}_i = \left(\frac{s_{i\mathcal{J}m}(s_{im} - s_{m\mathcal{J}})}{s_{im}(s_{im} - s_{i\mathcal{J}})}\right)^{\frac{1}{2}}$$

$$p^{\mu}_{\hat{M}} = \hat{x}_m p^{\mu}_m, \qquad \hat{x}_m = \left(\frac{s_{i\mathcal{J}m}(s_{im} - s_{i\mathcal{J}})}{s_{im}(s_{im} - s_{m\mathcal{J}})}\right)^{\frac{1}{2}}$$

$$\tilde{p}^{\mu}_n = p^{\mu}_n - \frac{2p_n \cdot (q + \tilde{q})}{(q + \tilde{q})^2}(q^{\mu} + \tilde{q}^{\mu}) + \frac{2p_n \cdot q}{q^2}\tilde{q}^{\mu}$$

where $n \neq \mathcal{J}, \qquad q^{\mu} = p^{\mu}_i + p^{\mu}_m - p^{\mu}_{\mathcal{J}}, \qquad \tilde{q}^{\mu} = p^{\mu}_{\hat{I}} + p^{\mu}_{\hat{M}}$

- $p^{\mu}_{\mathcal{J}} = p^{\mu}_j$ (NLO), $\quad p^{\mu}_{\mathcal{J}} = p^{\mu}_j + p^{\mu}_k$ (NNLO), $\quad p^{\mu}_{\mathcal{J}} = p^{\mu}_j + p^{\mu}_k + p^{\mu}_l$ (N3LO)

Antenna subtraction at NNLO

- Antenna function form physical matrix elements (2005)

$$A,\ \widetilde{A},\ B,\ C\ \sim\ \gamma^* \to q\bar{q}\ + \text{partons (quark-antiquark pair)}$$
$$D,\ E,\ \widetilde{E}\ \sim\ \widetilde{\mathcal{X}} \to \tilde{g}\ + \text{partons (quark-gluon pair)}$$
$$F,\ G,\ \widetilde{G},\ H\ \sim\ H \to \text{partons (gluon-gluon pair)}$$

A.Gehrmann-De Ridder, T.Gehrmann, N.Glover, 05

- Complete set of Antenna tool box (NNLO)

$$p^\mu\text{mapping}\ \otimes\quad \text{topology}\quad \otimes\ \text{parton ID}$$
$$[FF, IF, II] \otimes [X_3^0, X_4^0, X_3^1] \otimes [A \sim H]$$

  - All antenna functions are analytically integrated (2012)
    - Final-Final $\mathcal{X}_3^0$, $\mathcal{X}_4^0$ and $\mathcal{X}_3^1$ Gehrmann-De Ridder, Gehrmann, Glover (05) (e^+e^- collider: ILC, CEPC)
    - Initial-Final $\mathcal{X}_3^0$, $\mathcal{X}_4^0$ and $\mathcal{X}_3^1$ Daleo, Gehrmann, Gehrmann-De Ridder, Luisoni, Maitre (06,09,12) (ep collider: HERA, EIC)
    - Initial-Initial $\mathcal{X}_3^0$, $\mathcal{X}_4^0$ and $\mathcal{X}_3^1$ Boughezal, Daleo, Gehrmann-De Ridder, Gehrmann, Maitre, et al (10,11,12) (pp collider: LHC, SPPC, FCC)

Antenna subtraction framework

$$d\hat{\sigma}^{S}_{NNLO} = \boxed{d\hat{\sigma}^{S,a}} + \boxed{d\hat{\sigma}^{S,b_1}} + \boxed{d\hat{\sigma}^{S,b_2}} + \boxed{d\hat{\sigma}^{S,c}} + \boxed{d\hat{\sigma}^{S,d}}$$

$$d\hat{\sigma}^{T}_{NNLO} = \boxed{d\hat{\sigma}^{T,a}} + \boxed{d\hat{\sigma}^{T,b_1}} + \boxed{d\hat{\sigma}^{T,b_2}} + \boxed{d\hat{\sigma}^{T,c}}$$

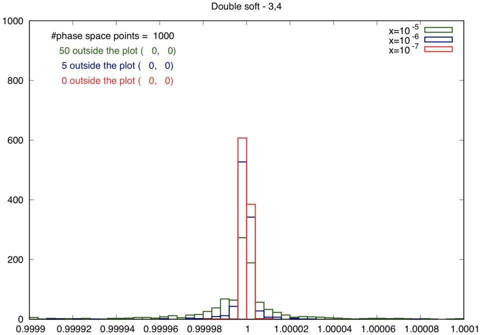
$$d\hat{\sigma}^{U}_{NNLO} = \boxed{d\hat{\sigma}^{U,a}} + \boxed{d\hat{\sigma}^{U,b}} + \boxed{d\hat{\sigma}^{U,c}}$$

- Test structure of unresolved IR limits

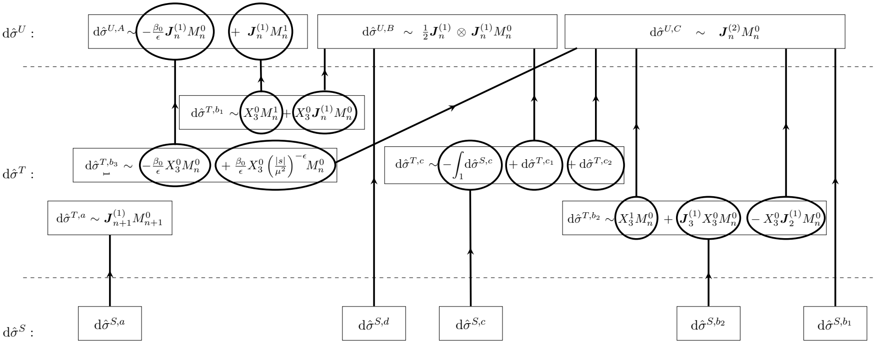
$$R = \frac{d\hat{\sigma}^{RR,RV}_{NNLO}}{d\hat{\sigma}^{S,T}_{NNLO}} \sim 1$$

- Horizontal axis $\sim R$ (expect at 1)
- Vertical axis $\sim$ number of P.S. points
- Control IR divergence by x $(3,4 \to 0)$:

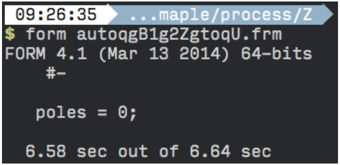
$$x = \frac{s_{34} + s_{45} + s_{35} + s_{4H} + s_{3H}}{s}$$



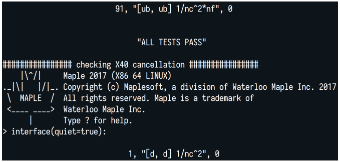
Antenna subtraction framework



- Analytical check of poles for (VV-U):



- Analytical check of cross-layer terms



OUTLINE

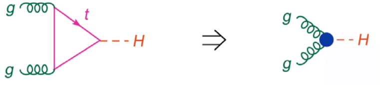
- Antenna subtraction method up to NNLO

- Higgs production at N3LO using q_T subtraction

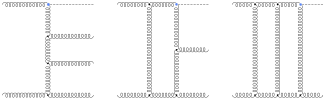
- The projection-to-Born method

Higgs + Jet and Higgs p_T distributions at medium p_T

- Use effective interaction for ggH vertex in large top mass limit (Higgs production @ LO $\rightarrow$ only $\delta(p_T)$ contribution):



- The state-of-the-art FO predictions for medium Higgs p_T region are @ NNLOEFT (same framework of H+J @ NNLOEFT no jet algorithm but with small Higgs p_T^{cut})



- One of the first NNLO processes done with three different subtraction schemes
  - Antenna subtraction. XC, Gehrmann, Glover et. al. [1408.5325], [1604.04085], [1607.08817]
  - Sector Improved Decomposition subtraction. Boughezal, Caola, Melnikov, Petriello, Schulze et. al. [1302.6216], [1504.07922], [1508.02684]
  - N-jettiness subtraction. Boughezal, Focke, Giele, Liu, Petriello et. al. [1505.03893]

Higgs pT distributions at small pT

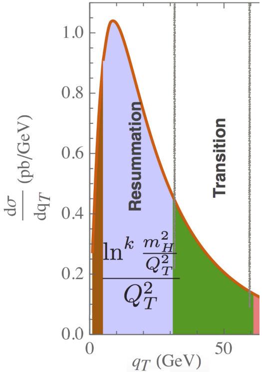
- Higgs production scale is $\mathcal{O}(M_H)$ but the scale at 1 GeV p_T is 10^{-2} different
- Large log terms $\ln^k(M_H^2/p_T^2)/p_T^2$ dominant at small p_T (singular terms $d\sigma^s$)
- Non-singular contribution $d\sigma^n = d\sigma^f \ominus d\sigma^s$ is unphysical
- Resum log divergence in $d\sigma^r$ at small p_T
- Match non-singluar and resummed contribution for physical p_T distributions:

$$d\sigma^f \ominus d\sigma^s \oplus d\sigma^r$$

  - $d\sigma^s$ and $d\sigma^r$ depends on resummation scheme
  - Many choices for $\ominus$, $\oplus$ and transition region
- This talk focus on $d\sigma^n$, theoretically one would except

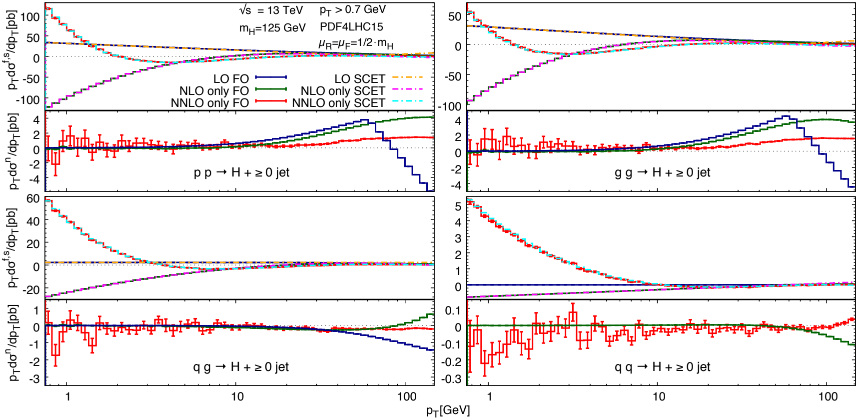
$$d\sigma^f - d\sigma^s \xrightarrow{p_T \to 0} 0$$

  - P.S. integration of $d\sigma^f$ at small p_T has large numerical cancellations from asympotic tri-soft, quard-collinear etc.
  - Reality needs high numerical stability and careful validation



Validation of singular behaviour at small pT

- Compare asympotic divergent behaviour from log terms between $d\sigma^f$ and $d\sigma^s$



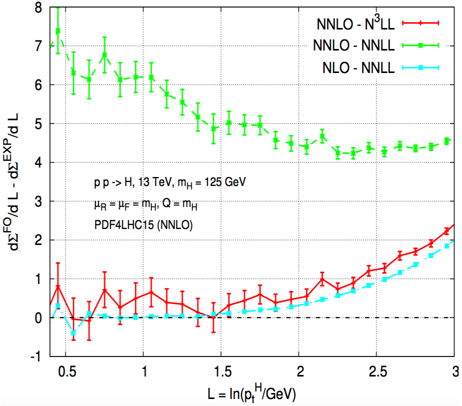
$d\sigma^s$ from SCET

XC, T. Gehrmann, N. Glover, A. Huss, Y. Li, D. Neill, M. Schulze, I. Stewart, H.X. Zhu [1805.00736]

- Calculate $d\sigma^n$ for $p_T \geq 0.7$ GeV with or without p_T reweighting
- Excellent agreement between $p_T d\sigma^f$ and $p_T d\sigma^s$ within numerical error ($\sim 1\%$)

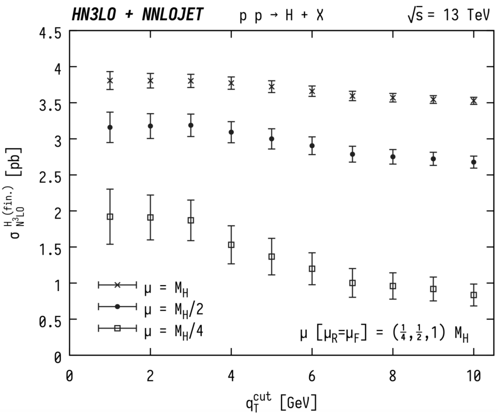
Validation of singular behaviour at small pT

- Compare asympotic divergent behaviour from log terms between $d\sigma^f$ and $d\sigma^s$



$d\sigma^s$ from RadISH

W. Bizoń, XC, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, A. Huss, P. Monni, E. Re, L. Rottoli, P. Torrielli [1805.05916]



$d\sigma^s$ from HN3LO

XC, L. Cieri, T. Gehrmann, N. Glover, A. Huss [1807.11501]

- Excellent agreement between $d\sigma^{f,\mathrm{inc.}}$ and $d\sigma^{s,\mathrm{inc.}}$
- Accumulated σ^n stabilized at < 3 GeV

q_T subtraction at N^{3}LO

XC, L. Cieri, T. Gehrmann, N. Glover, A. Huss [1807.11501]

- q_T subtraction for Higgs production at general F.O. has the following structure:

$$d\sigma^H_{N^nLO} = \mathcal{H}^H_{N^nLO} \otimes d\sigma^H_{LO}\Big|_{\delta(p_T)} + \left[d\sigma^{H+J}_{N^{n-1}LO} - d\sigma^{H;s}_{N^nLO}\right]_{p_T > p_T^{cut}}$$

- In principle, $\delta(p_T)$ contains form factor of Higgs and integrated $d\sigma^{H;s}_{N^nLO}$
- Design $d\sigma^{H;s}_{N^nLO} \to \Sigma^H_{N^nLO} \otimes d\sigma^H_{LO}$ that $\delta(p_T)$ has the resummation form:

G. Bozzi, S. Catani et. al. [hep-ph/0508068]; S. Catani and M. Grazzini [hep-ph/0703012]; S. Catani, L. Cieri et. al. [1311.1654]

$$\left(\Sigma^H_{gg\leftarrow a_1a_2}\left(\frac{p_T^2}{M_H^2};\frac{M_H^2}{\hat{s}};\alpha_s\right) + \mathcal{H}^H_{gg\leftarrow a_1a_2}\left(\frac{M_H^2}{\hat{s}};\alpha_s\right)\right) \otimes d\sigma^H_{LO} = \frac{M_H^2}{s}\int \frac{b}{2}db$$

$$\times J_0(bp_T) S_c(M_H,b) \prod_{i=1,2}\int_{x_i}^1 \frac{dz_i}{z_i} f_{a_i/h_i}(z_i,b) \otimes d\hat{\sigma}^{H;(0)}_{gg} \otimes [H^H C_1 C_2]_{gg\leftarrow a_1a_2}$$

$$S_c(M_H,b) = \exp\left\{-\int_{b_0^2/b^2}^{M_H^2}\frac{dq^2}{q^2}\left[A_g(\alpha_s(q^2))\ln\frac{M_H^2}{q^2} + B_g(\alpha_s(q^2))\right]\right\}$$

$$[H^H C_1 C_2]_{gg\leftarrow ab} = H^H_g(\alpha_s)[C_{ga}(z_1;\alpha_s)C_{gb}(z_2;\alpha_s) + G_{ga}(z_1;\alpha_s)G_{gb}(z_2;\alpha_s)]$$

q_T subtraction at N^{3}LO

XC, L. Cieri, T. Gehrmann, N. Glover, A. Huss [1807.11501]

- The factorisation of $\mathcal{H}^H_{N^nLO} \otimes d\sigma^H_{LO}$ depends on resummation scheme choice
- Above formulae is invariant under the following scheme transformation:

$$H^H_g(\alpha_s) \to H^H_g(\alpha_s)[h(\alpha_s)]^{-1}$$

$$A_g(\alpha_s) \to A_g(\alpha_s)$$

$$B_g(\alpha_s) \to B_g(\alpha_s) - \beta(\alpha_s)\frac{d\ln h(\alpha_s)}{d\ln\alpha_s}$$

$$C(G)_{ga}(z;\alpha_s) \to C(G)_{ga}(z;\alpha_s)[h(\alpha_s)]^{1/2}$$

- Above ingredients can be expressed in series expasion of α_s
- Exact formulae from SCET, CSS or hard resummation schemes are transferable
- Collect results from different schemes and transfrom into hard scheme
    - All analytical formulae known for NNLO Higgs production
    - For N^{3}LO Higgs production, we only know some of the ingredients
      $A^{(3)}_g$ →(SCET) T. Becher, M. Neubert [1405.4827]
      $B^{(3)}_g$ → (SCET, CSS) Y. Li, H.X. Zhu [1604.01404]; A.A. Vladimirov [1610.05791]
      $\tilde{H}^{H;(3)}_g = H^{H;(3)}_g - [H^{H;(3)}_g]_{\delta^{p_T}_{(2)}}$ →(CSS) S. Catani, L. Cieri et. al. [1311.1654]

q_T subtraction at N^{3}LO

XC, L. Cieri, T. Gehrmann, N. Glover, A. Huss [1807.11501]

- The currently unknown pieces are inside $\mathcal{H}^{H}_{gg\leftarrow ab}(z;\alpha_s)$ with following structure:

$$\delta_{ga}\delta_{gb}\delta(1-z)[H_g^{H;(3)}]_{\delta^{p_T}_{(2)}} + \delta_{ga}C^{(3)}_{gb}(z) + \delta_{gb}C^{(3)}_{ga}(z) + \left(G^{(1)}_{ga}\otimes G^{(2)}_{gb}\right)(z) + \left(G^{(2)}_{ga}\otimes G^{(1)}_{gb}\right)(z) \rightarrow C_{N3}\delta_{ga}\delta_{gb}\delta(1-z)$$

- Use C_{N3} to approximate the unknown pieces
  - C_{N3} is process dependent but independent of scale choices
  - C_{N3} contains exact unknown pieces propotional to $\delta(1-z)$

q_T subtraction at N^{3}LO

XC, L. Cieri, T. Gehrmann, N. Glover, A. Huss [1807.11501]

- The currently unknown pieces are inside $\mathcal{H}^H_{gg\leftarrow ab}(z;\alpha_s)$ with following structure:

$$\delta_{ga}\delta_{gb}\delta(1-z)[H_g^{H;(3)}]_{\delta^{p_T}_{(2)}} + \delta_{ga}C^{(3)}_{gb}(z) + \delta_{gb}C^{(3)}_{ga}(z)$$
$$+\left(G^{(1)}_{ga}\otimes G^{(2)}_{gb}\right)(z) + \left(G^{(2)}_{ga}\otimes G^{(1)}_{gb}\right)(z) \rightarrow C_{N3}\delta_{ga}\delta_{gb}\delta(1-z)$$

- Use C_{N3} to approximate the unknown pieces
  - C_{N3} is process dependent but independent of scale choices
  - C_{N3} contains exact unknown pieces propotional to $\delta(1-z)$
- C_{N3} can be numerically determined using following strategy (N^{3}LO exclusive):

$$C_{N3}\otimes\sigma^H_{LO} = \sigma^H_{N^3LO} - \tilde{\mathcal{H}}^H_{N^3LO}\otimes\sigma^H_{LO}\Big|_{\delta(p_T)} - \left[d\sigma^{H+J}_{NNLO} - d\sigma^{H;s}_{N^3LO}\right]_{p_T>p_T^{cut}}$$

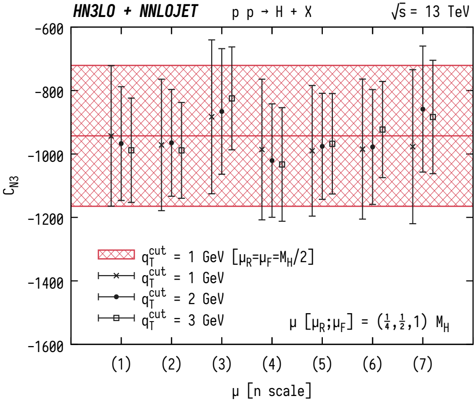
  - Terms in black are available from previous discussions
  - $\sigma^H_{N^3LO}$ is taken from Higgs total cross section at N^{3}LO using ihixs 2

B. Mistlberger [1802.00833]; F. Dulat, A. Lazopoulos and B. Mistlberger [1802.00827]

Extraction of C_{N3}

XC, L. Cieri, T. Gehrmann, N. Glover, A. Huss [1807.11501]

- Numerical abstraction of C_{N3} using newly developed package HN3LO:
  - $\sqrt{s} = 13$TeV, $M_H = 125$ GeV
  - PDF4LHC15,$\alpha_s(M_z) = 0.118$
  - Central scale $\mu_R = \mu_F = M_H/2$
  - With 7-point scale variaitons
  - $p_T^{cut} = 1, 2, 3, 4, 5 \cdots$ GeV
- C_{N3} independent of scale choices
- C_{N3} independent of p_T^{cut} at 1, 2, 3 GeV
- Benchmark value of C_{N3} is recommended at central scale

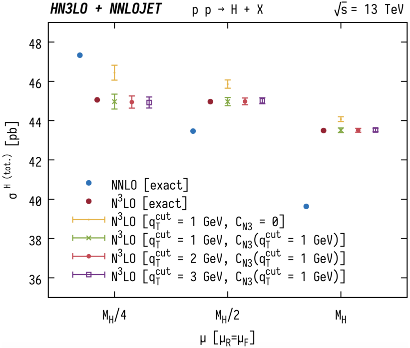


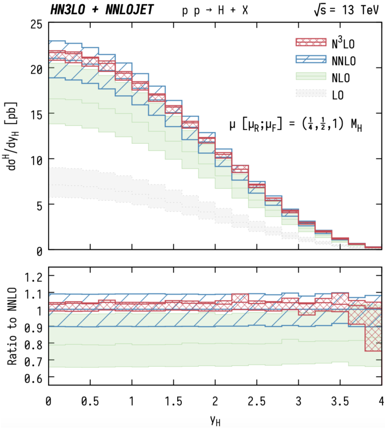
$$C_{N3} = -942 \pm 222$$

N^3LO Higgs total cross section and rapidity distribution

XC, L. Cieri, T. Gehrmann, N. Glover, A. Huss [1807.11501]

- With C_{N3} approximation, the $\sigma^H_{N^3LO}$ and $d\sigma^H_{N^3LO}/dy^H$ distributions are:



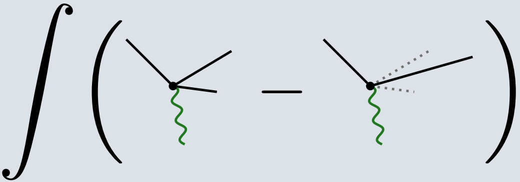


- Total XS agree with exact results at level of 0.2%
- y^H distribution take uncertainties from p_T^{cut}, 7-scales and C_{N3} uncertainty
- Uncertainty reduction $> 50\%$, flat k factor (~ 1.04 central) same as total XS

OUTLINE

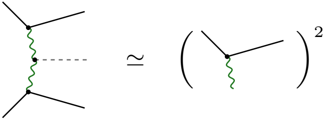
- Antenna subtraction method up to NNLO
- Higgs production at N3LO using q_T subtraction
- The projection-to-Born method

Projection-to-Born Method



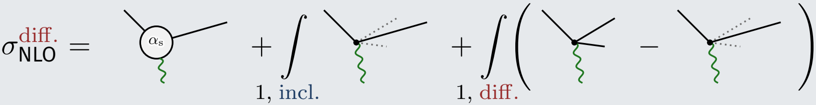
The Projection-to-Born Method

first introduced: **VBF @ NNLO** [Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15]



VBF$\simeq$ (DIS)^2

1. take the *inclusive* calculation (real emissions integrated out analytically)
2. make it fully differential (set up as local subtraction)



DIS structure function @ NLO

DIS 2 jet @ LO

Born kinematics: $Q^2 = -q^2 > 0, \quad x = \frac{-Q^2}{2P \cdot q} \quad \Rightarrow p^\mu_{\mathrm{in}} = xP^\mu, \quad p^\mu_{\mathrm{out}} = xP^\mu - q^\mu$

The Projection-to-Born Method

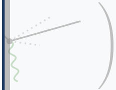
first introduced: **VBF @ NNLO** [Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15]

$$\left.\begin{array}{lll} & \text{inclusive } X & @\ \mathrm{N^{n}LO} \\ + & X + \text{jet} & @\ \mathrm{N^{n\text{-}1}LO} \end{array}\right\} \rightsquigarrow X\ @\ \mathrm{N^{n}LO}$$

1. beyond NNLO
   ↪ DIS @ N^{3}LO
2. beyond DIS kinematics
   ↪ colour-singlet production in pp
3. connection to Antenna subtraction

1. take t... ...lly)
2. make ...

$\sigma^{\text{diff.}}_{\text{NLO}} =$



DIS structure function @ NLO

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DIS 2 jet @ NNLO

[Currie, Gehrmann, Niehues '16]
[Currie, Gehrmann, AH, Niehues '17]
CC: [Niehues, Walker '18]

Projection-to-Born

$\oplus$

[Cacciari, et al. '15]

DIS structure function @ N^{3}LO

[Moch, Vermaseren, Vogt '05]

=

DIS fully differential @ N^{3}LO

[Currie, Gehrmann, Glover, AH, Niehues, Vogt. '18]

- precise probe to resolve the inner structure of the proton (α_s & PDF)
- first step towards fully differential VBF $\simeq$ (DIS)^2

The Projection-to-Born Method — DIS @ N^{3}LO

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^{\mathsf{NNLO}}_{\mathsf{DIS}\,2j}}{\mathrm{d}\mathcal{O}} = & \int_{\Phi_4} \Big(\mathrm{d}\sigma^{RR}_{\mathsf{DIS}\,2j}\; J(\mathcal{O}_4) \\
& \qquad - \mathrm{d}\sigma^{S,a}_{\mathsf{DIS}\,2j}\; J(\mathcal{O}_3) \qquad - \mathrm{d}\sigma^{S,b}_{\mathsf{DIS}\,2j}\; J(\mathcal{O}_2) \Big) \\
& + \int_{\Phi_3} \Big(\mathrm{d}\sigma^{RV}_{\mathsf{DIS}\,2j}\; J(\mathcal{O}_3) \\
& \qquad - \mathrm{d}\sigma^{T,a}_{\mathsf{DIS}\,2j}\; J(\mathcal{O}_3) \qquad - \mathrm{d}\sigma^{T,b}_{\mathsf{DIS}\,2j}\; J(\mathcal{O}_2) \Big) \\
& + \int_{\Phi_2} \Big(\mathrm{d}\sigma^{VV}_{\mathsf{DIS}\,2j}\; J(\mathcal{O}_2) \qquad - \mathrm{d}\sigma^{U}_{\mathsf{DIS}\,2j}\; J(\mathcal{O}_2) \Big)
\end{aligned}
$$

* subtraction terms add up to zero

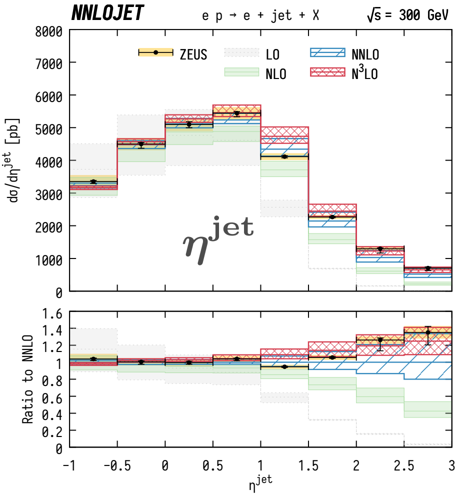
The Projection-to-Born Method — DIS @ N^{3}LO

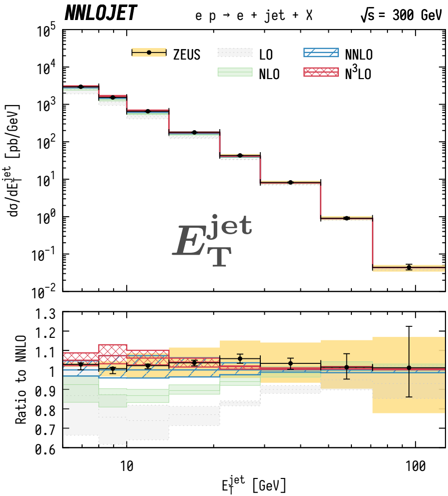
$$
\begin{aligned}
\frac{\mathrm{d}\sigma^{\text{N}^3\text{LO}}_{\text{DIS }1j}}{\mathrm{d}\mathcal{O}} = & \int_{\Phi_4} \Big(\mathrm{d}\sigma^{RRR}_{\text{DIS }1j} \big(J(\mathcal{O}_4) - J(\mathcal{O}_\text{B})\big) \\
& \qquad - \mathrm{d}\sigma^{S,a}_{\text{DIS }2j} \big(J(\mathcal{O}_3) - J(\mathcal{O}_\text{B})\big) - \mathrm{d}\sigma^{S,b}_{\text{DIS }2j} \big(J(\mathcal{O}_2) - J(\mathcal{O}_\text{B})\big)\Big) \\
& + \int_{\Phi_3} \Big(\mathrm{d}\sigma^{RRV}_{\text{DIS }1j} \big(J(\mathcal{O}_3) - J(\mathcal{O}_\text{B})\big) \\
& \qquad - \mathrm{d}\sigma^{T,a}_{\text{DIS }2j} \big(J(\mathcal{O}_3) - J(\mathcal{O}_\text{B})\big) - \mathrm{d}\sigma^{T,b}_{\text{DIS }2j} \big(J(\mathcal{O}_2) - J(\mathcal{O}_\text{B})\big)\Big) \\
& + \int_{\Phi_2} \Big(\mathrm{d}\sigma^{RVV}_{\text{DIS }1j} \big(J(\mathcal{O}_2) - J(\mathcal{O}_\text{B})\big) - \mathrm{d}\sigma^{U}_{\text{DIS }2j} \big(J(\mathcal{O}_2) - J(\mathcal{O}_\text{B})\big)\Big) \\
& + \frac{\mathrm{d}\sigma^{\text{N}^3\text{LO, incl.}}_{\text{DIS }1j}}{\mathrm{d}\mathcal{O}_\text{B}}
\end{aligned}
$$

$$
\leftarrow \quad \mathrm{d}\sigma^{VVV}_{\text{DIS }1j} + \sum_{n=1}^{3} \int_{\Phi_{n+1}} \mathrm{d}\sigma^{R^n V^{(3-n)}}_{\text{DIS }1j}
$$

* subtraction terms add up to zero

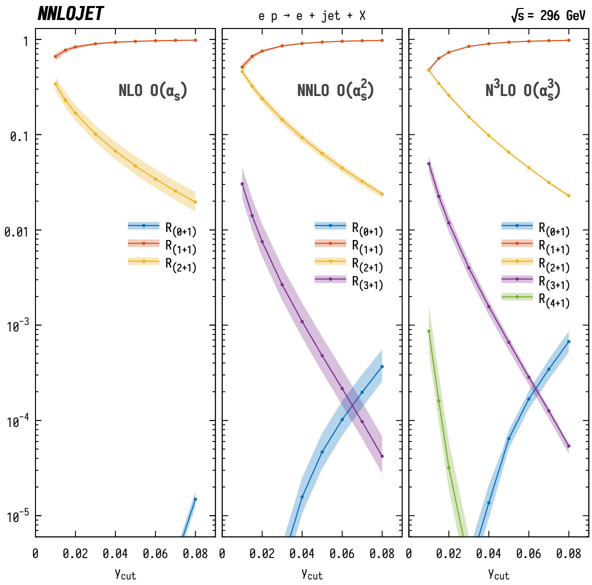
Differential distributions at N^{3}LO





- for the first time: *overlapping* scale bands agreement with data
- reduction of scale uncertainties

Jet Rates



Jet rates:

$$R_{(n+1)} = N_{(n+1)}/N_{\text{tot}}$$

JADE algorithm
↪ cluster partons if:

$$\frac{2E_iE_j(1-\cos\theta_{ij})}{W^2} < y_{\text{cut}}$$

Beyond DIS kinematics — colour-singlet production in pp

H + jet @ N^{n-1}LO

Projection-to-Born

$\oplus$

$\mathrm{d}\sigma/\mathrm{d}y_{\mathrm{H}}$ @ N^{n}LO

= H fully differential @ N^{n}LO ?

- $\mathrm{d}\sigma/\mathrm{d}y_{\mathrm{H}}$
  - ↪ *analytic* integration over real emissions
  - ↪ lose information on final-state partons
- H fully differential
  - ↪ retain full final-state information
  - ↪ fiducial cuts, jet veto, photon isolation, …

The Projection — colour-singlet production in pp

- real-emission phase space: $\mathrm{d}\Phi_{\mathrm{H}+n}$

$$p_a + p_b \to p_{\mathrm{H}} + k_1 + k_2 + \ldots + k_n$$

- projection to Born: $\mathrm{d}\widetilde{\Phi}_{\mathrm{H}}$

$$\tilde{p}_a + \tilde{p}_b \to \tilde{p}_{\mathrm{H}} \qquad (\tilde{p}_a = \xi_a p_a, \quad \tilde{p}_b = \xi_b p_b)$$

on-shell: $\tilde{p}_{\mathrm{H}}^2 \equiv p_{\mathrm{H}}^2 = M_{\mathrm{H}}^2 \quad \Rightarrow \quad \xi_a\,\xi_b = \dfrac{2p_a p_b - 2(p_a + p_b)k_{1\ldots n} + k_{1\ldots n}^2}{2p_a p_b}$

rapidity: $\tilde{y}_{\mathrm{H}} \equiv y_{\mathrm{H}} \quad \Rightarrow \quad \xi_a/\xi_b = \dfrac{2p_b p_{\mathrm{H}}}{2p_a p_{\mathrm{H}}}$

$\hookrightarrow$ decay products: $p_{\mathrm{H}} \to p_1 + \ldots + p_m \qquad (p_i^\mu \to \tilde{p}_i^\mu = \Lambda^\mu{}_\nu\, p_i^\nu)$

$$\Lambda^\mu{}_\nu(p_{\mathrm{H}}, \tilde{p}_{\mathrm{H}}) = g^\mu{}_\nu - \frac{2(p_{\mathrm{H}} + \tilde{p}_{\mathrm{H}})^\mu (p_{\mathrm{H}} + \tilde{p}_{\mathrm{H}})_\nu}{(p_{\mathrm{H}} + \tilde{p}_{\mathrm{H}})^2} + \frac{2\tilde{p}_{\mathrm{H}}^\mu p_{\mathrm{H},\nu}}{p_{\mathrm{H}}^2}$$

- for $n = 1, 2$: *identical* to the initial–initial antenna mapping

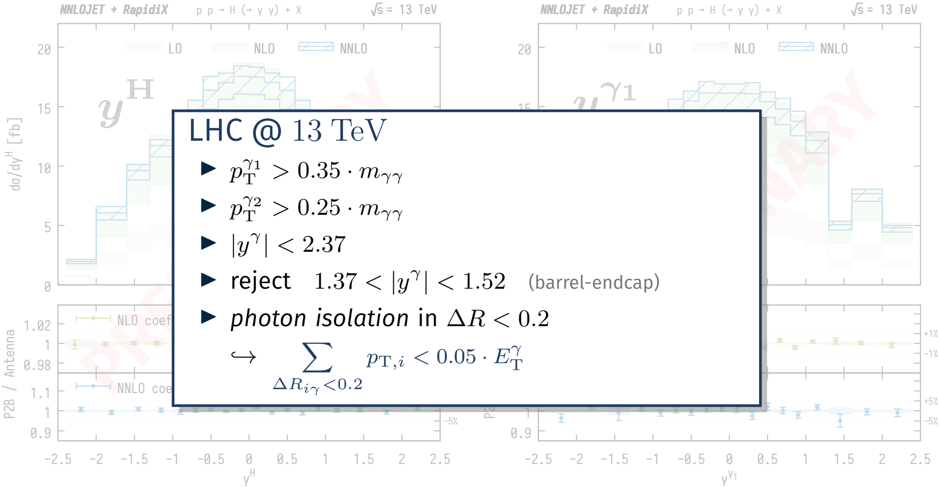
The Projection-to-Born Method — ggH @ N^{3}LO

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_{\text{ggH}}^{\text{N}^3\text{LO}}}{\mathrm{d}\mathcal{O}} = &\int_{\Phi_{\text{H}+3}} \Big(\mathrm{d}\sigma_{\text{ggH}}^{RRR} \big(J(\mathcal{O}_{\text{H}+3}) - J(\mathcal{O}_{\text{H}+3\to\text{B}}) \big) \\
&\qquad - \mathrm{d}\sigma_{\text{H+jet}}^{S,a} \big(J(\mathcal{O}_{\text{H}+2}) - J(\mathcal{O}_{\text{H}+2\to\text{B}}) \big) - \mathrm{d}\sigma_{\text{H+jet}}^{S,b} \big(J(\mathcal{O}_{\text{H}+1}) - J(\mathcal{O}_{\text{H}+1\to\text{B}}) \big) \Big) \\
&+ \int_{\Phi_{\text{H}+2}} \Big(\mathrm{d}\sigma_{\text{ggH}}^{RRV} \big(J(\mathcal{O}_{\text{H}+2}) - J(\mathcal{O}_{\text{H}+2\to\text{B}}) \big) \\
&\qquad - \mathrm{d}\sigma_{\text{H+jet}}^{T,a} \big(J(\mathcal{O}_{\text{H}+2}) - J(\mathcal{O}_{\text{H}+2\to\text{B}}) \big) - \mathrm{d}\sigma_{\text{H+jet}}^{T,b} \big(J(\mathcal{O}_{\text{H}+1}) - J(\mathcal{O}_{\text{H}+1\to\text{B}}) \big) \Big) \\
&+ \int_{\Phi_{\text{H}+1}} \Big(\mathrm{d}\sigma_{\text{ggH}}^{RVV} \big(J(\mathcal{O}_{\text{H}+1}) - J(\mathcal{O}_{\text{H}+1\to\text{B}}) \big) - \mathrm{d}\sigma_{\text{H+jet}}^{U} \big(J(\mathcal{O}_{\text{H}+1}) - J(\mathcal{O}_{\text{H}+1\to\text{B}}) \big) \Big) \\
&+ \frac{\mathrm{d}\sigma_{\text{ggH}}^{\text{N}^3\text{LO, incl.}}}{\mathrm{d}\mathcal{O}_{\text{B}}}
\end{aligned}
$$

* in general: $\mathcal{O}_{\text{H}+3\to\text{B}} \neq \mathcal{O}_{\text{H}+2\to\text{B}} \neq \mathcal{O}_{\text{H}+1\to\text{B}}$

Validation up to NNLO — Antenna vs. P2B

[Chen, Dulat, Gehrmann, Glover, AH, Mistlberger, Pelloni]



LHC @ 13 TeV

- $p_{\mathrm{T}}^{\gamma_1} > 0.35 \cdot m_{\gamma\gamma}$
- $p_{\mathrm{T}}^{\gamma_2} > 0.25 \cdot m_{\gamma\gamma}$
- $|y^{\gamma}| < 2.37$
- reject $1.37 < |y^{\gamma}| < 1.52$ (barrel-endcap)
- *photon isolation* in $\Delta R < 0.2$

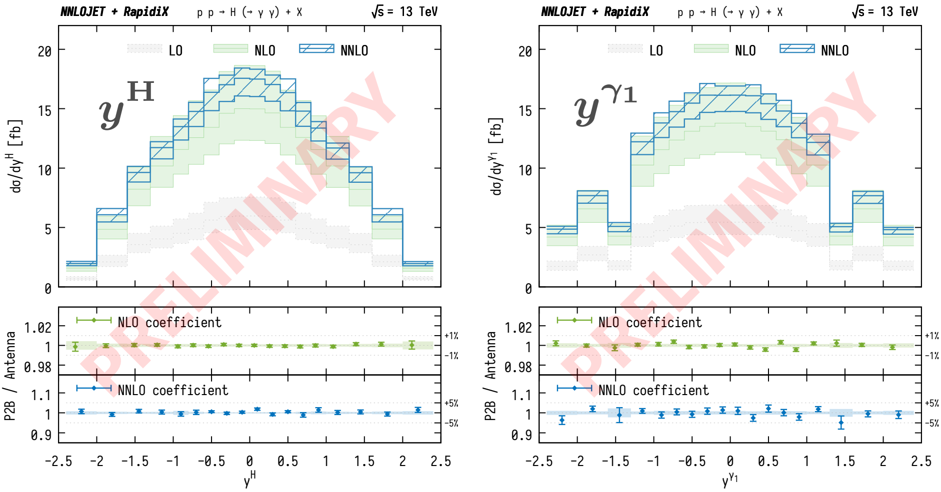
$$\hookrightarrow \sum_{\Delta R_{i\gamma} < 0.2} p_{\mathrm{T},i} < 0.05 \cdot E_{\mathrm{T}}^{\gamma}$$

NLO *coefficient:* $\lesssim 5‰$
NNLO *coefficient:* $\lesssim 5\%$

$\rightsquigarrow$ agreement @ full NNLO: $\lesssim 1‰$

Validation up to NNLO — Antenna vs. P2B

[Chen, Dulat, Gehrmann, Glover, AH, Mistlberger, Pelloni]



NLO *coefficient*: $\lesssim 5‰$

NNLO *coefficient*: $\lesssim 5\%$

$\rightsquigarrow$ agreement @ full NNLO: $\lesssim 1‰$

Projection-to-Born — an "antenna" view

Consider the real-emission subtraction in the antenna subtraction formalism for $\mathrm{H}+0\mathrm{jet}$ (@ LC):

$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H+0jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H+0jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}} \Big\{ \mathtt{A3g0H}(1_\mathrm{g}, 2_\mathrm{g}, 3_\mathrm{g}, \mathrm{H})\, \mathcal{J}(\Phi_{\mathrm{H+1}})$$

$$- F_3^0(1_\mathrm{g}, 2_\mathrm{g}, 3_\mathrm{g})\, \mathtt{A2g0H}(\tilde{1}_\mathrm{g}, \tilde{2}_\mathrm{g}, \mathrm{H})\, \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \Big\}$$

Projection-to-Born — an "antenna" view

Consider the real-emission subtraction in the antenna subtraction formalism for $\mathrm{H}+0\mathrm{jet}$ (@ LC):

$$\int\left\{\mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H}+0\mathrm{jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H}+0\mathrm{jet}}\right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H}+1}\Big\{ \mathtt{A3g0H}(1_{\mathrm{g}},2_{\mathrm{g}},3_{\mathrm{g}},\mathrm{H})\; \mathcal{J}(\Phi_{\mathrm{H}+1}) - F_3^0(1_{\mathrm{g}},2_{\mathrm{g}},3_{\mathrm{g}})\; \mathtt{A2g0H}(\tilde{1}_{\mathrm{g}},\tilde{2}_{\mathrm{g}},\mathrm{H})\; \mathcal{J}(\widetilde{\Phi}_{\mathrm{H}+0})\Big\}$$

Antennae = ratios of *physical* Matrix Elements:

$$F_3^0(i_{\mathrm{g}},j_{\mathrm{g}},k_{\mathrm{g}}) \equiv \frac{\mathtt{A3g0H}(i_{\mathrm{g}},j_{\mathrm{g}},k_{\mathrm{g}},\mathrm{H})}{\mathtt{A2g0H}(\tilde{i}_{\mathrm{g}},\tilde{k}_{\mathrm{g}},\mathrm{H})}$$

Projection-to-Born — an "antenna" view

Consider the real-emission subtraction in the antenna subtraction formalism for $\mathrm{H}+0\mathrm{jet}$ (@ LC):

$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H+0jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H+0jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}} \Big\{ \mathtt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H})\, \mathcal{J}(\Phi_{\mathrm{H+1}}) - F_3^0(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}})\, \mathtt{A2g0H}(\tilde{1}_{\mathrm{g}}, \tilde{2}_{\mathrm{g}}, \mathrm{H})\, \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \Big\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}}\, \mathtt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H}) \left\{ \mathcal{J}(\Phi_{\mathrm{H+1}}) - \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \right\}$$

Projection-to-Born — an "antenna" view

Consider the real-emission subtraction in the antenna subtraction formalism for $\mathrm{H}+0\mathrm{jet}$ (@ LC):

$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H+0jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H+0jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}} \Big\{ \mathtt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\Phi_{\mathrm{H+1}})$$

$$- F_3^0(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}})\ \mathtt{A2g0H}(\tilde{1}_{\mathrm{g}}, \tilde{2}_{\mathrm{g}}, \mathrm{H})\ \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \Big\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}}\ \mathtt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H}) \left\{ \mathcal{J}(\Phi_{\mathrm{H+1}}) - \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \right\}$$

$\Rightarrow$ Simple processes where antenna $\simeq$ real-emission Matrix Element
$\leftrightsquigarrow$ Projection-to-Born

Similarly at NNLO: X_4^0 & $X_3^0 \times X_3^0$ are "projections" of RR ME & NLO(+jet) subtraction term.

$\mathrm{d}\sigma_{\mathrm{N^3LO}}/\mathrm{d}y_{\mathrm{H}} \simeq$ integrated antenna: $\mathcal{X}_5^0$, $\mathcal{X}_4^1$, $\mathcal{X}_3^2$

Summary & Outlook

- Antenna subtraction successfully applied to many important processes:
  ↪ $\mathrm{pp} \to X \; + 0, 1$ jets, $\mathrm{pp} \to$ dijets, $\ldots$

⇒ subtraction set up for: $\mathrm{pp} \to$ "colour neutral" $+ 0, 1, 2$ jets

- **NNLO** corrections important: reduction of scale uncertainties
  ↪ improve theory v.s. data comparison (resolve/reduce tension with data)

- **Antenna** $\oplus$ q_{T} **subtraction**
  ↪ challenge: *numerical stability* of $X + \mathrm{jet}$
  ↪ study of sub-leading power correction
  ↪ straightforward to include exact C_3 once available

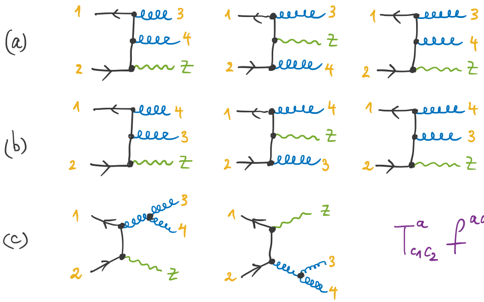
- **Antenna** $\oplus$ **Projection-to-Born**
  ↪ first fully differential $\mathrm{N^3LO}$ prediction: inclusive jets in DIS
  ↪ colour-singlet production in pp (proof-of-concept @ NNLO)
  ↪ differential Higgs @ $\mathrm{N^3LO}$
  ↪ all ingredients in place for differential VBF @ $\mathrm{N^3LO}$

Thank you

Backup Slides

Colour decomposition — Example: $q\,\bar{q} \to g\,g\,Z$

- antenna formalism operates on *colour-ordered* amplitudes:



(a) $\left(T^{a_3}\,T^{a_4}\right)_{c_1 c_2}$

(b) $\left(T^{a_4}\,T^{a_3}\right)_{c_1 c_2}$

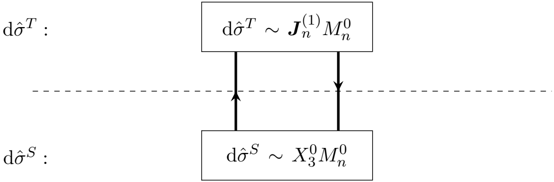
(c) $T^{a}_{c_1 c_2}\, f^{a a_3 a_4} \sim \left[T^{a_3}T^{a_4} - T^{a_4}T^{a_3}\right]_{c_1 c_2}$

$$\Rightarrow \quad \mathcal{M}^0_{q\bar{q}\to ggZ} = \left(T^{a_3}T^{a_4}\right)_{c_1c_2}\, \mathcal{A}_4^0(1_q, 3_g, 4_g, 2_{\bar{q}}, Z) \qquad \longleftrightarrow \text{ "(a)+(c)"}$$

$$+ \left(T^{a_4}T^{a_3}\right)_{c_1c_2}\, \mathcal{A}_4^0(1_q, 4_g, 3_g, 2_{\bar{q}}, Z) \qquad \longleftrightarrow \text{ "(b)−(c)"}$$

- $\mathcal{A}_4^0(1_q, 3_\mathrm{g}, 4_\mathrm{g}, 2_{\bar{q}}, \mathrm{Z})$ has $\left\{ \begin{array}{c} 1_q \parallel 3_\mathrm{g} \\ 2_{\bar{q}} \parallel 4_\mathrm{g} \end{array} \right\}$ but *not* $\left\{ \begin{array}{c} 1_q \parallel 4_\mathrm{g} \\ 2_{\bar{q}} \parallel 3_\mathrm{g} \end{array} \right\}$ limit

Antenna subtraction @ NLO — Example: $q\,\bar{q} \to \mathrm{g}\,\mathrm{g}\,\mathrm{Z}$



$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{Z+1jet}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{Z+1jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{Z+2}} \Big\{ \left|\mathcal{A}_4^0(1_q, 3_\mathrm{g}, 4_\mathrm{g}, 2_{\bar{q}}, \mathrm{Z})\right|^2 \mathcal{J}(\Phi_{\mathrm{Z+2}})$$

$$- d_3^0(1_q, 3_\mathrm{g}, 4_\mathrm{g}) \left|\mathcal{A}_3^0(\widetilde{1}_q, \widetilde{(34)}_\mathrm{g}, 2_{\bar{q}}, \mathrm{Z})\right|^2 \mathcal{J}(\widetilde{\Phi}_{\mathrm{Z+1}})$$

$$- d_3^0(2_{\bar{q}}, 4_\mathrm{g}, 3_\mathrm{g}) \left|\mathcal{A}_3^0(1_q, \widetilde{(34)}_\mathrm{g}, \widetilde{2}_{\bar{q}}, \mathrm{Z})\right|^2 \mathcal{J}(\widetilde{\Phi}_{\mathrm{Z+1}}) \Big\} + (3 \leftrightarrow 4)$$

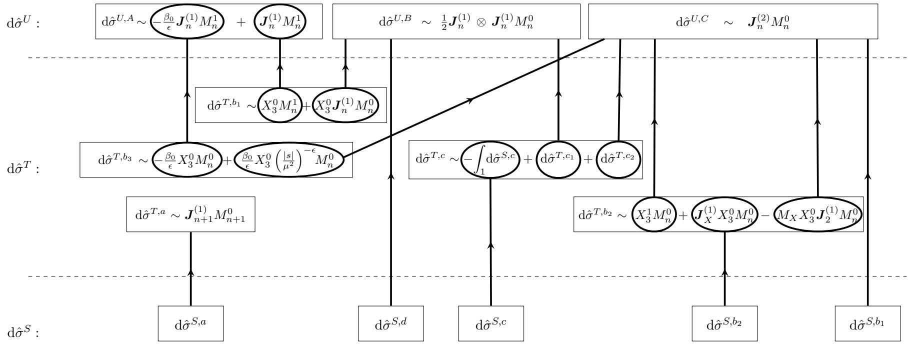
$$\int \left\{ \mathrm{d}\sigma^{\mathrm{V}}_{\mathrm{Z+1jet}} - \mathrm{d}\sigma^{\mathrm{T}}_{\mathrm{Z+1jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{Z+1}} \Big\{ \left|\mathcal{A}_3^1(1_q, 3_\mathrm{g}, 2_{\bar{q}}, \mathrm{Z})\right|^2$$

$$+ \frac{1}{2}\left[\mathcal{D}_3^0(s_{13}) + \mathcal{D}_3^0(s_{23})\right] \left|\mathcal{A}_3^0(1_q, 3_\mathrm{g}, 2_{\bar{q}}, \mathrm{Z})\right|^2 \Big\} \mathcal{J}(\Phi_{\mathrm{Z+1}})$$

Antenna subtraction @ NNLO

[J. Currie , E.W.N. Glover, S. Wells '13]



What about those angular terms?

- Antenna subtraction: $X_n^l \; |\mathcal{A}_m|^2 \quad \leftrightarrow \quad$ spin averaged!
- angular terms in gluon splittings:

$$P_{g\to q\bar{q}} = \frac{2}{s_{ij}} \left[-g^{\mu\nu} + 4z(1-z)\, \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right]$$

$\hookrightarrow$ subtraction non-local in these limits!

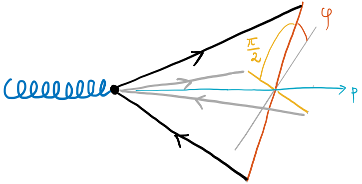
$\hookrightarrow$ vanish upon azimuthal-angle (φ) average ($\Rightarrow$ do not enter $\mathcal{X}$)

sol. 1: supplement angular terms in the subtraction

sol. 2: exploit φ dependence & average in the phase space

$$\mathcal{A}_\mu^* \, \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \, \mathcal{A}_\nu \sim \cos(2\varphi + \varphi_0)$$

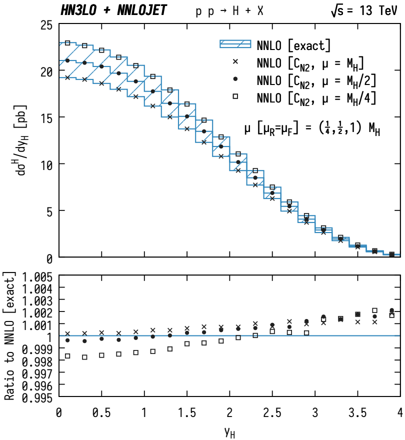
$\Rightarrow$ add φ & $(\varphi + \pi/2)$!



$$\vec{r} \longrightarrow \text{PS}_{\text{gen.}} \longrightarrow \begin{bmatrix} \{p_i, & p_j, & \dots\} \\ \{p_i', & p_j', & \dots\} \end{bmatrix} \xrightarrow{(i\|j)} \begin{bmatrix} \{p_i^{\varphi}, & p_j^{\varphi}, & \dots\} \\ \{p_i^{\varphi+\pi/2}, & p_j^{\varphi+\pi/2}, & \dots\} \end{bmatrix}$$

Validation of y_H with C_{N2}

- Without available fully differential N^{3}LO calculations, one could refer to one order lower and test the C_{N2} approximation against exact NNLO results
  - Three scale results devided by exact NNLO distributions
  - Approximation with C_{N2} deviate from exact NNLO by maximum $\sim 0.2\%$ through out $y_H \subset [0,4]$ for all three scales

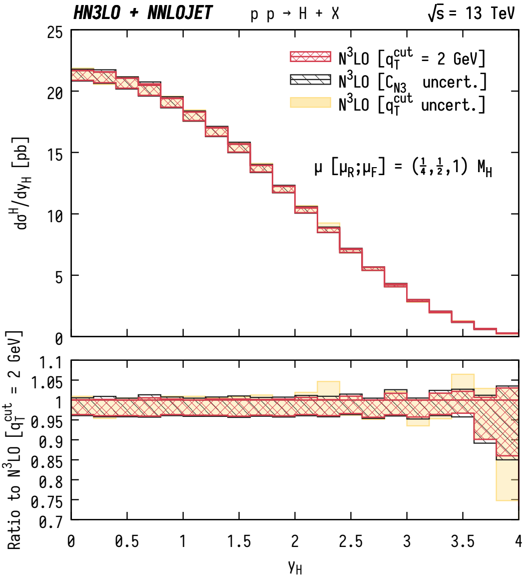


$$C_{N2}\delta_{ga}\delta_{gb}\delta(1-z) \leftarrow \delta_{ga}\delta_{gb}\delta(1-z)[H_g^{H;(2)}]_{\delta_{(1)}^{p_T}}$$

$$+\delta_{ga}C_{gb}^{(2)}(z)+\delta_{gb}C_{ga}^{(2)}(z)+\left(G_{ga}^{(1)}\otimes G_{gb}^{(1)}\right)(z)$$

Uncertainties in N^{3}LO Higgs y^H distribution

- Estimate theoretical uncertainties from:
  - 7-point scale variation
  - p_T^{cut} change to 1 or 2 GeV
  - C_{N3} vairation at $\pm\sigma$
- Theoretical uncertainties at central rapidity are dominant by scale uncertainties
- High rapidity region uncertainties are dominant by p_T^{cut} variation but mainly due to limited numerical statistics



Higgs p_T anatomy at NNLO

- Contribution from fixed order, singular and non-singular contributions to Higgs p_T in ggH EFT