Nested soft-collinear subtractions

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HIGH TIME FOR HIGHER ORDERS

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Handling IR singularities at NNLO

- SLICING
  - qT [Catani, Grazzini '07] [S. Sapeta]
  - N-jettiness [Gaunt *et al* '15; Boughezal *et al* '15] [H.X. Zhu, I. Moult]
- SUBTRACTION
  - Antenna [Gehrmann-de Ridder, Gehrmann, Glover '05, …] [A. Huss, X. Chen]
  - STRIPPER [Czakon '10, '11]
  - Projection-to-Born [Cacciari *et al* '15] [T. Melia]
  - CoLoRFulNNLO [Somogyi, Trócsányi, Del Duca '05, …]

  } "ESTABLISHED"

  - **Nested soft-collinear [Caola, Melnikov, R.R. '17]**
  - Geometric [Herzog '18] [F. Herzog]
  - Local analytic sector [Magnea *et al* '18] [P. Torrielli]

  } "NEW WAVE"

Improving NNLO subtractions

<u>Goal:</u> Replicate success of NLO subtraction methods (FKS/CS).

A “better” subtraction scheme should:

- Be fully local
  - *avoid large numerical cancellations in intermediate steps.*
- Have a minimal structure displaying a clear origin of physical singularities
  - *easier for others to implement.*
- Have explicit, analytic cancellation of poles
  - *control over singular structures.*
- Allow four-dimensional evaluation of amplitudes
  - *improved numerical efficiency.*
- Be process-independent.
- Be flexible
  - *allow freedom in phase-space parametrization/mapping.*

Nested soft-collinear subtraction

[Caola, Melnikov, R.R. ‘17]

- Extension of FKS subtraction to NNLO.
- Independent subtraction of soft and collinear divergences **(color coherence).**
- Use of sectors (as in STRIPPER) to separate overlapping ***collinear*** singularities.

  [Czakon ‘10, ‘11]

  - Natural splitting by rapidity.

- Fully local. ✓
- Clear physical origin of singularities (soft & collinear). ✓
- **Recombination** of sectors leading to simplifications in integrated subtraction terms.
  - Final IR structure very transparent.
  - Explicit (not yet fully analytic) pole cancellation (independent of matrix element). ✓
- Allows four-dimensional evaluation of matrix elements. ✓
- Process-independent *in principle* – details only worked out for color singlet hadroproduction & color singlet decay. ✓
- Not tied to phase space parametrization (currently using STRIPPER parametrization of angular phase space). ✓

Current status and outline

- Color singlet production:
  - ✓ Corrections to $q\bar{q} \to V$ (e.g. DY, *VH*, *VV*,...)
  - ✓ Corrections to $gg \to V$ (e.g. *H*, *HH*, ...)
- Color singlet decay:
  - ✓ Corrections to $V \to q\bar{q}$ (e.g. $H \to b\bar{b}$)
- Extension to initial & final states with color conceptually straightforward.
- Discuss corrections to $q\bar{q} \to V + ng$
  - Most complicated singular structure.

NNLO subtraction scheme

Aim to replicate FKS subtraction strategy *as far as possible:*

- Explicit (ideally analytical) cancellation of poles in each *kinematic structure*, *before* numerical implementation.
- Numerical implementation of finite result only: four-dimensional matrix elements.
- Finite terms:
  - lower multiplicity structures (i.e. LO-like or NLO-like) convoluted with functions (related to A-P splitting functions), and
  - regulated double-real term.

FKS subtraction at NLO: Notation

Consider real-real correction to color singlet production

$$q(p_1)\bar{q}(p_2) \to V + g(p_4) + g(p_5):$$

$$\mathrm{d}\sigma^{\mathrm{RR}} = \frac{1}{2s}\int [\mathrm{d}g_4][\mathrm{d}g_5] F_{LM}(1,2,4,5)$$

$$F_{LM}(1,2,4,5) = \mathrm{dLips}_V\, |\mathcal{M}(1,2,4,5,V)|^2\, \mathcal{F}_{\mathrm{kin}}(1,2,4,5,V)$$

Lorentz-inv. Phase space for V **(incl. delta-fn)**

Matrix-element sq.

IR-safe observable

$$[\mathrm{d}g_i] = \frac{\mathrm{d}^{d-1}p_i}{(2\pi)^d 2E_i}\theta(\sqrt{s}/2 - E_i)$$

Integration in partonic CoM frame

NNLO: Real-real Corrections

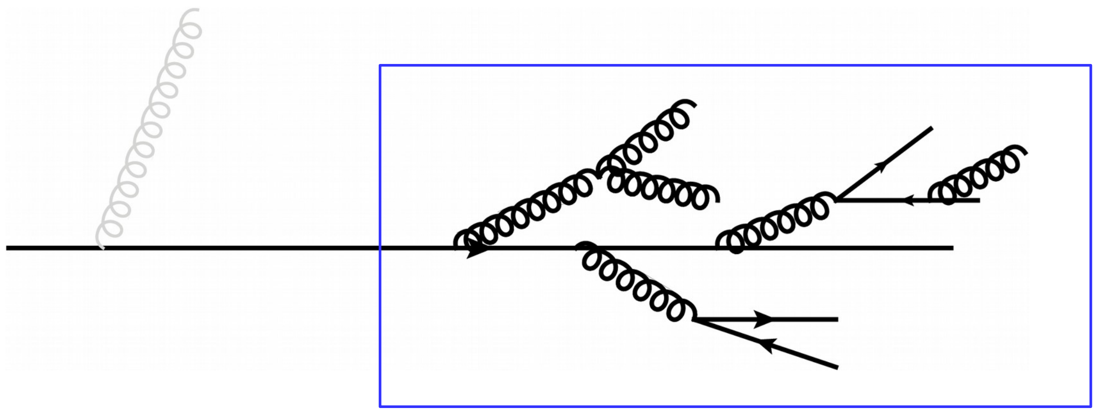
IR singularities from

- g_4 or g_5 → soft.
- g_4 or g_5 → collinear to initial state partons.
- g_4 or g_5 → collinear to each other.
- g_4 and g_5 collinear to same initial state parton (triple collinear limit).
- Combination of the above – can approach **each limit in different ways!**

Separating the singularities is the name of the game!

Color coherence

- On-shell, gauge-invariant QCD scattering amplitudes : color coherence.
- Soft gluon cannot resolve details of later splittings; only sees total color charge.



⇒ Soft and collinear emissions can be treated **independently**:

- Regularize soft singularities first, then collinear singularities.
- No need for energy-angle ordering – energies and angles can be independently parametrized.

Treatment of real-real singularities

- **Step 1:** Limit operators.
  - NLO-like: $S_i A = \lim_{E_i \to 0} A \qquad C_{ij} A = \lim_{\rho_{ij} \to 0} A. \qquad (\rho_{ij} = 1 - \cos\theta_{ij})$
  - NNLO like:

$$\mathbb{S} A = \lim_{E_4, E_5 \to 0} A, \text{ at fixed } E_5/E_4,$$

$$\mathbb{C}_i A = \lim_{\rho_{4i}, \rho_{5i} \to 0} A, \text{ with non vanishing } \rho_{4i}/\rho_{5i}, \rho_{45}/\rho_{4i}, \rho_{45}/\rho_{5i},$$

- **Step 2:** Order gluon energies $E_4 > E_5$.

$$2\,\mathrm{s} \cdot \mathrm{d}\sigma^{\mathrm{RR}} = \int [\mathrm{d}g_4][\mathrm{d}g_5]\theta(E_4 - E_5) F_{LM}(1,2,4) \equiv \langle F_{LM}(1,2,4,5) \rangle.$$

  - Gluon energies bounded by $\sqrt{s}/2$
  - Energies defined in CoM frame.
  - Soft singularities: either double soft or g_5 soft.

Soft singularities

- **Step 3:** Regulate the soft singularities:

$$\big\langle F_{LM}(1,2,4,5)\big\rangle = \big\langle \not{S} F_{LM}(1,2,4,5)\big\rangle + \big\langle S_5(I-\not{S})F_{LM}(1,2,4,5)\big\rangle + \big\langle (I-S_5)(I-\not{S})F_{LM}(1,2,4,5)\big\rangle.$$

- First term: both g_4 and g_5 soft.
- Second term: g_5 soft, soft singularities in g_4 are regulated.
- Third term: regulated against all soft singularities,
- All three terms contain **(potentially overlapping)** collinear singularities.

Phase-space partitioning

- **Step 4:** Introduce **phase-space partitions**

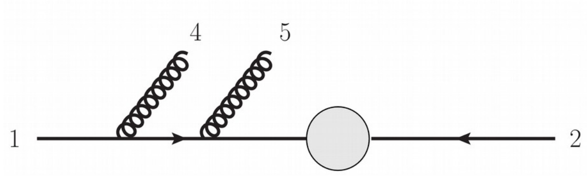
$$1 = w^{14,15} + w^{24,25} + w^{14,25} + w^{15,24}.$$

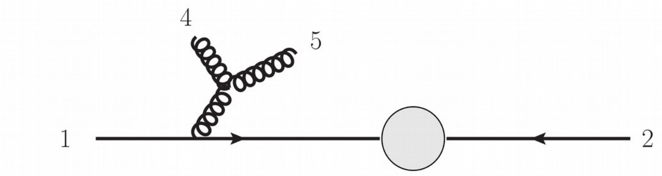
with

$$C_{42} w^{14,15} = C_{52} w^{14,15} = 0 \quad \Rightarrow \quad w^{14,15} \text{ contains } C_{41},\ C_{51},\ C_{45}$$

$$C_{41} w^{24,25} = C_{51} w^{24,25} = 0 \qquad\quad w^{24,25} \text{ contains } C_{42},\ C_{52},\ C_{45}$$

Triple collinear
partition



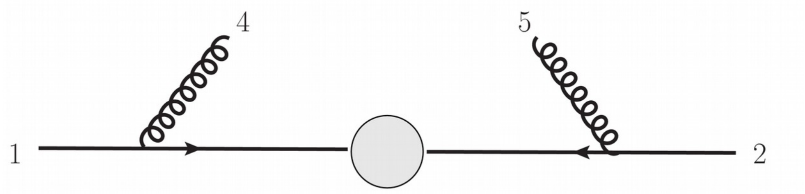


and

$$C_{42} w^{14,25} = C_{51} w^{14,25} = C_{45} w^{14,25} = 0 \quad \Rightarrow \quad w^{14,25} \text{ contains } C_{41},\ C_{52}$$

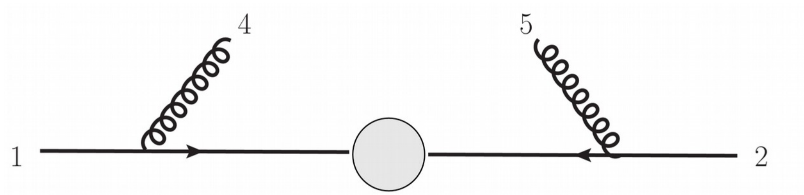
$$C_{41} w^{15,24} = C_{52} w^{15,24} = C_{45} w^{15,24} = 0 \qquad\quad w^{15,24} \text{ contains } C_{42},\ C_{51}$$

Double collinear
partition



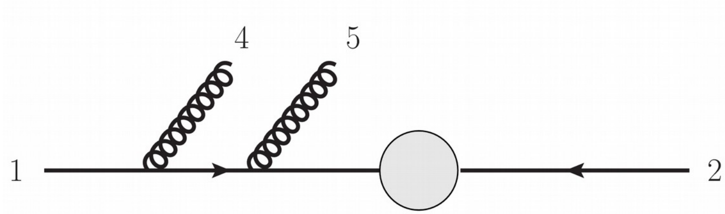
Phase-space partitioning

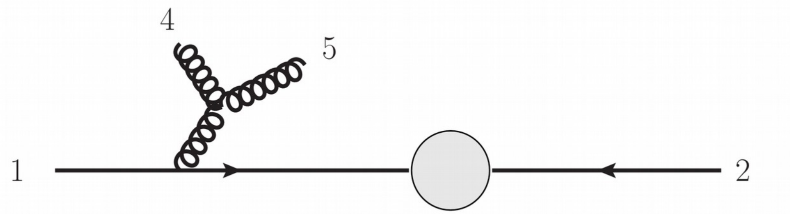
- Double collinear partition – **large** rapidity difference.



~ NLO x NLO → simple

- Triple collinear partition – **small** rapidity difference.





Overlapping singularities **remain** – need one last step to separate these.

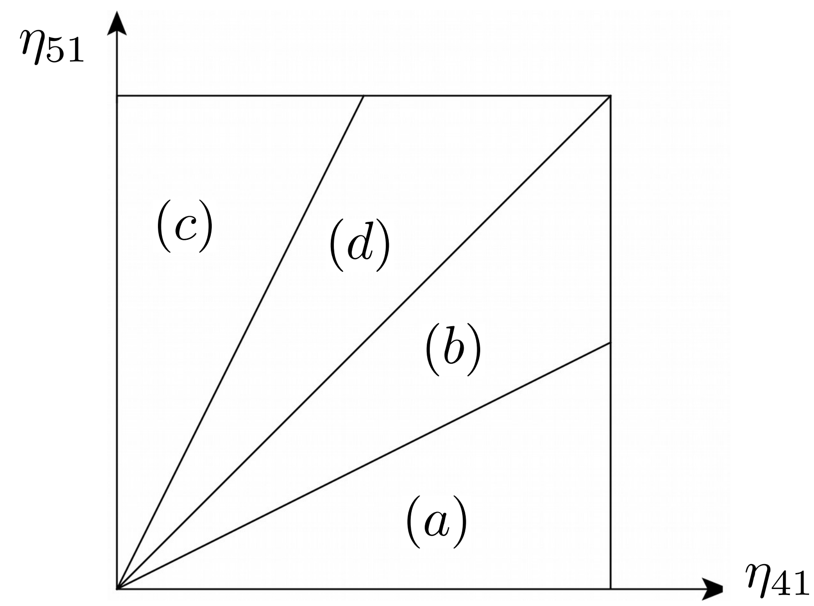
Sector Decomposition

- **Step 5: Sector decomposition:**

- Define angular ordering to separate singularities.

$$\eta_{ij} = \rho_{ij}/2$$

$$1 = \theta\left(\eta_{51} < \frac{\eta_{41}}{2}\right) + \theta\left(\frac{\eta_{41}}{2} < \eta_{51} < \eta_{41}\right) + \theta\left(\eta_{41} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{41} < \eta_{51}\right) \equiv \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}.$$



- Thus the limits are

$$\theta^{(a)} : C_{51} \qquad \theta^{(b)} : C_{45}$$
$$\theta^{(c)} : C_{41} \qquad \theta^{(d)} : C_{45}$$

- Sectors *a,c* and *b,d* same to *4* ↔ *5*, but recall energy ordering.
- Angular phase space parametrization [Czakon '10].

Removing collinear singularities

Then we can write soft-regulated term as

$$\left\langle (I - S_5)(I - \not{S}) F_{LM}(1,2,4,5) \right\rangle = \left\langle F_{LM}^{s_r c_s}(1,2,4,5) \right\rangle + \left\langle F_{LM}^{s_r c_t}(1,2,4,5) \right\rangle + \left\langle F_{LM}^{s_r c_r}(1,2,4,5) \right\rangle,$$

$$\langle F_{LM}^{s_r c_r}(1,2,4,5) \rangle$$

- All singularities removed through iterated subtractions – evaluated in 4-dimensions.
- Only term involving fully-resolved real-real matrix element.

$$\langle F_{LM}^{s_r c_{s,t}}(1,2,4,5) \rangle$$

- Contain (soft-regulated) single and triple collinear singularities.
- Matrix elements of lower multiplicity.
- Partitioning factors and sectors: one collinear singularity in each term.

Treating singular limits

We have four singular subtraction terms:

$$\left\langle \mathbb{S} F_{LM}(1,2,4,5) \right\rangle \quad \left\langle S_5 (I - \mathbb{S}) F_{LM}(1,2,4,5) \right\rangle \quad \left\langle F_{LM}^{s_r c_s}(1,2,4,5) \right\rangle \quad \left\langle F_{LM}^{s_r c_t}(1,2,4,5) \right\rangle$$

We know how to treat them:

- Gluon(s) decouple partially or completely.
- Decouple completely:
  - Integrate over gluonic angles and energy.
- Decouple partially:
  - Integrate over gluonic angles.
  - Integral(s) over energy → integrals over splitting function in *z.*
- Significant analytic simplifications on recombining sectors after integration.
- Integration for first **three** subtraction terms done *analytically*, last one *numerically*.

Integrated double soft term

$$\left\langle \mathbb{S} F_{LM}(1,2,4,5)\right\rangle = F_{LM}(1,2)\int [\mathrm{d}g_4][\mathrm{d}g_4]\theta(E_4-E_5)\mathrm{Eik}_2(1,2,4,5)$$

- Computed recently [Caola, Delto, Frellesvig, Melnikov ‘18]
- Relatively simple result.

$$\begin{aligned}
\mathbb{S}_{ij}^{(q\bar{q})} = {}& (2E_{\max})^{-4\epsilon}\left[\frac{1}{8\pi^2}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\right]^2 \Bigg\{ -\frac{1}{3\epsilon^3}+\frac{1}{\epsilon^2}\left[\frac{2}{3}\ln(s^2)-\frac{4}{3}\ln 2\right.\\
&\left.+\frac{13}{18}\right]+\frac{1}{\epsilon}\left[-\frac{4}{3}\mathrm{Li}_2(c^2)-\frac{2}{3}\ln^2(s^2)+\ln(s^2)\left(\frac{8}{3}\ln 2-\frac{13}{9}\right)+\frac{\pi^2}{9}\right.\\
&\left.+\frac{4}{3}\ln^2 2+\frac{35}{9}\ln 2-\frac{125}{54}\right]-\frac{8}{3}\mathrm{Ci}_3(2\delta)-\frac{2}{3\tan(\delta)}\mathrm{Si}_2(2\delta)-\frac{4}{3}\mathrm{Li}_3(c^2)\\
&-\frac{8}{3}\mathrm{Li}_3(s^2)+\mathrm{Li}_2(c^2)\left[\frac{29}{9}-\frac{8}{3}\ln 2\right]+\frac{4}{9}\ln^3(s^2)+\ln^2(s^2)\left[-\frac{4}{3}\ln(c^2)\right.\\
&\left.-\frac{8}{3}\ln 2+\frac{13}{9}\right]+\ln(s^2)\left[-\frac{8}{3}\ln^2 2-\frac{70}{9}\ln 2+\frac{2}{9}\pi^2+\frac{107}{27}\right]+9\zeta_3\\
&+\frac{2\pi^2}{3}\ln 2-\frac{8}{9}\ln^3 2-\frac{23}{108}\pi^2-\frac{35}{9}\ln^2 2-\frac{223}{27}\ln 2+\frac{601}{162}+\mathcal{O}(\epsilon)\Bigg\}.
\end{aligned}$$

$$c=\cos\delta;\quad s=\sin\delta$$

$$\delta=\theta/2$$

Treating singular limits

After integration: subtraction terms written as lower multiplicity terms:

- LO-like:

  $\langle F_{LM}(z\cdot 1, \bar{z}\cdot 2)\rangle\,,\ \langle F_{LM}(z\cdot 1, 2)\rangle\,,\ \langle F_{LM}(1, z\cdot 2)\rangle\,,\ \langle F_{LM}(1,2)\rangle$

- NLO-real-like (regulated by iterative subtraction):

  $\langle \mathcal{O}_{NLO} F_{LM}(z\cdot 1, 2, 4)\rangle\,,\ \langle \mathcal{O}_{NLO} F_{LM}(1, z\cdot 2, 4)\rangle\,,\ \langle \mathcal{O}_{NLO} F_{LM}(1,2,4)\rangle$

convoluted with splitting functions with explicit singularities

- Pole cancellation within each structure

  (to $1/\epsilon^2$ analytically, $1/\epsilon$ numerically).

Finite remainders

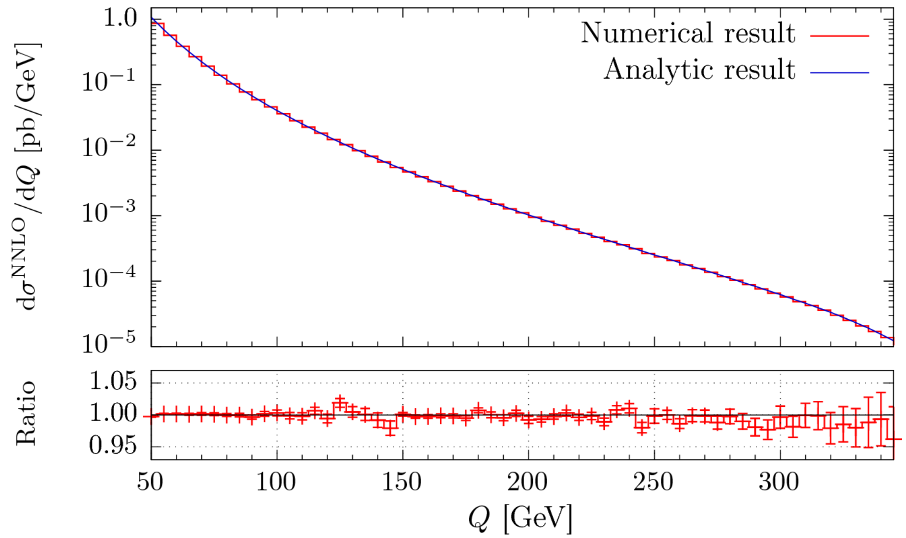
- **Relatively compact** expressions for finite remainders for each *lower-multiplicity structure.*
- Extension of NLO calculation to NNLO:
  - LO and NLO results convoluted with known functions.
  - Nested subtraction for real-real contribution.

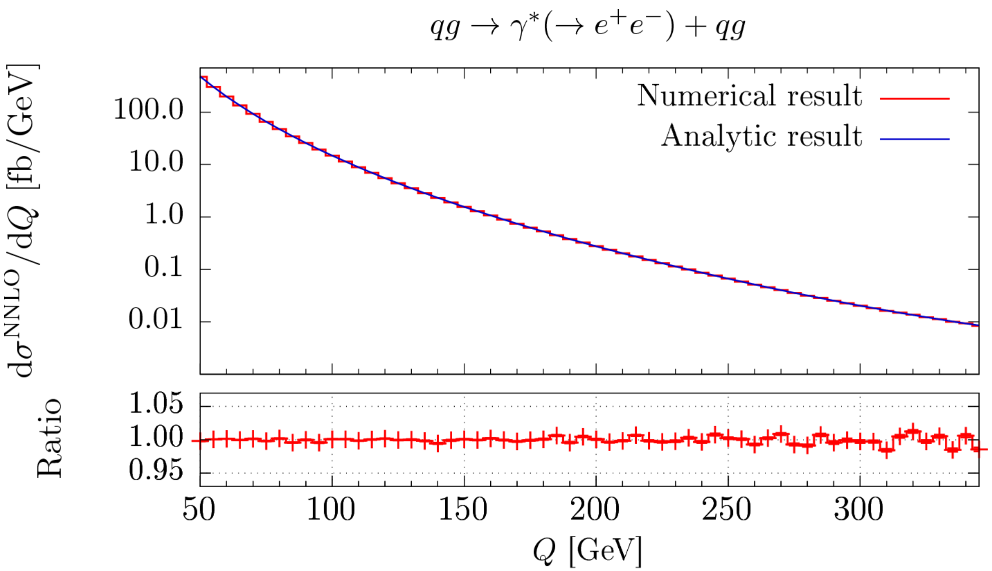
$$
\begin{aligned}
&\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}}_{F_{LM}(z\cdot 1,2)}(\mu^2 = s) = \\
&\left[\frac{\alpha_s(\mu)}{2\pi}\right]^2 \int\limits_0^1 \mathrm{d}z \Bigg\{ C_F^2 \Bigg[8\tilde{\mathcal{D}}_3(z) + 4\tilde{\mathcal{D}}_1(z)(1+\ln 2) + 4\tilde{\mathcal{D}}_0(z)\left[\frac{\pi^2}{3}\ln 2 + 4\zeta_3\right] \\
&+ \frac{5z-7}{2} + \frac{5-11z}{2}\ln z + (1-3z)\ln 2\ln z + \ln(1-z)\left[\frac{3}{2}z - (5+11z)\ln z\right] \\
&\quad + 2(1-3z)\mathrm{Li}_2(1-z) \\
&+ (1-z)\Bigg[\frac{4}{3}\pi^2 + \frac{7}{2}\ln^2 2 - 2\ln^2(1-z) + \ln 2\big[4\ln(1-z) - 6\big] + \ln^2 z \\
&\quad + \mathrm{Li}_2(1-z)\Bigg] + (1+z)\Bigg[-\frac{\pi^2}{3}\ln z - \frac{7}{4}\ln^2 2\ln z - 2\ln 2\ln(1-z)\ln z \\
&\quad + 4\ln^2(1-z)\ln z - \frac{\ln^3 z}{3} + \big[4\ln(1-z) - 2\ln 2\big]\mathrm{Li}_2(1-z)\Bigg] \\
&\quad + \left[\frac{1+z^2}{1-z}\right]\ln(1-z)\big[3\mathrm{Li}_2(1-z) - 2\ln^2 z\big] - \frac{5-3z^2}{1-z}\mathrm{Li}_3(1-z) \\
&\quad + \frac{\ln z}{(1-z)}\left[12\ln(1-z) - \frac{3-5z^2}{2}\ln^2(1-z) - \frac{7+z^2}{2}\ln 2\ln z\right]\Bigg] \\
&\quad + C_A C_F\Bigg[-\frac{22}{3}\tilde{\mathcal{D}}_2(z) + \left(\frac{134}{9} - \frac{2}{3}\pi^2\right)\tilde{\mathcal{D}}_1(z) + \Bigg[-\frac{802}{27} + \frac{11}{18}\pi^2 \\
&\quad + (2\pi^2 - 1)\frac{\ln 2}{3} + 11\ln^2 2 + 16\zeta_3\Bigg]\tilde{\mathcal{D}}_0(z) + \frac{37-28z}{9} + \frac{1-4z}{3}\ln 2 \\
&\quad - \left(\frac{61}{9} + \frac{161}{18}z\right)\ln(1-z) + (1+z)\ln(1-z)\left[\frac{\pi^2}{3} - \frac{22}{3}\ln 2\right] \\
&\quad - (1-z)\left[\frac{\pi^2}{6} + \mathrm{Li}_2(1-z)\right] - \frac{2+11z^2}{3(1-z)}\ln 2\ln z - \frac{1+z^2}{1-z}\mathrm{Li}_2(1-z)\times \\
&\quad \times\big[2\ln 2 + 3\ln(1-z)\big]\Bigg] + R_+^{(\epsilon)}\mathcal{D}_0(z) + R^{(\epsilon)}(z)\Bigg\}\left\langle\frac{F_{LM}(z\cdot 1,2)}{z}\right\rangle.
\end{aligned}
$$

Proof-of-principle

- Extensively tested in DY production against analytic results [Hamberg, Matsuura, van Neerven '91]:
  - All channels relevant for DY.
  - *NNLO corrections* to cross section agree at < 1 permille.
  - *NNLO corrections* show permille to percent agreement across 5 orders of magnitude in virtuality of vector boson Q.
  - Also in channels which are numerically negligible.

➡ Good control of extreme kinematic regions.





Color singlet decay

- NNLO corrections to $V \to q\bar{q}$ can be calculated with identical strategy.
- Integrated subtraction terms much simpler:

Consider collinear limit of $V \to q(p_1)\bar{q}(p_2)g(p_3)$:

$$C_{31}F_{LM}(1,2,3) = \frac{g_{s,b}^2}{E_1 E_3 \rho_{13}} P_{qq}\left(\frac{E_1}{E_1+E_3}\right) F_{LM}(1+3,2)$$

Integrate over the **full phase space** of all final state particles, so write energy integration as:

$$z = E_1/(E_1+E_3)$$

$$\int [dE_1][dE_3] C_{31} F_{LM}(1,2,3) = \left[\int dz (z(1-z))^{-2\epsilon} P_{qq}(z)\right] \times \left[\int [dE_{13}] E_{13}^{-2\epsilon} F_{LM}(1+3,2)\right]$$
$$= \text{const.} \times \langle F_{LM}(1,2)\rangle.$$

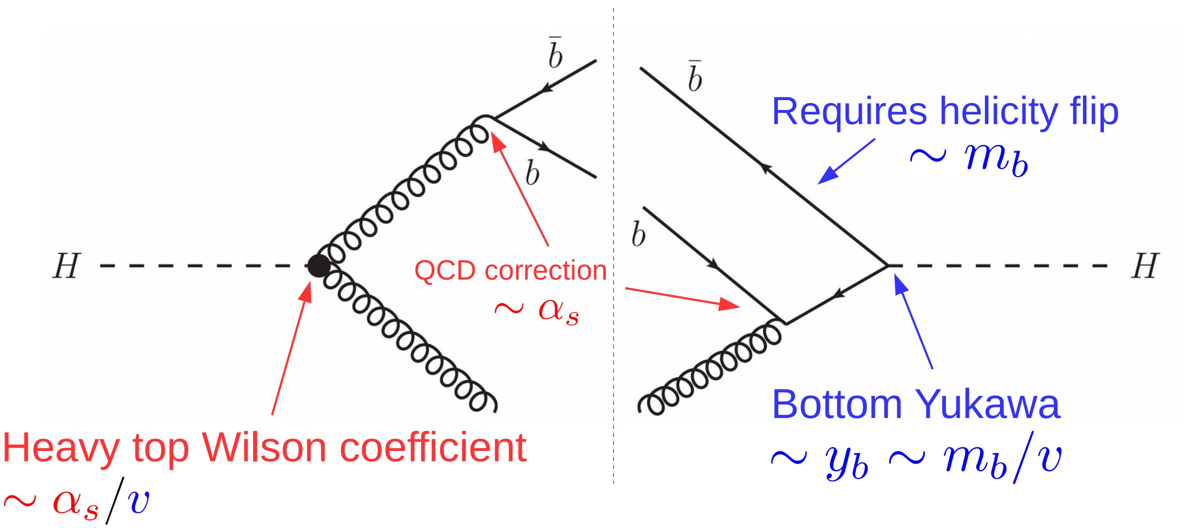
Lower multiplicity terms multiplied by constants rather than splitting functions.

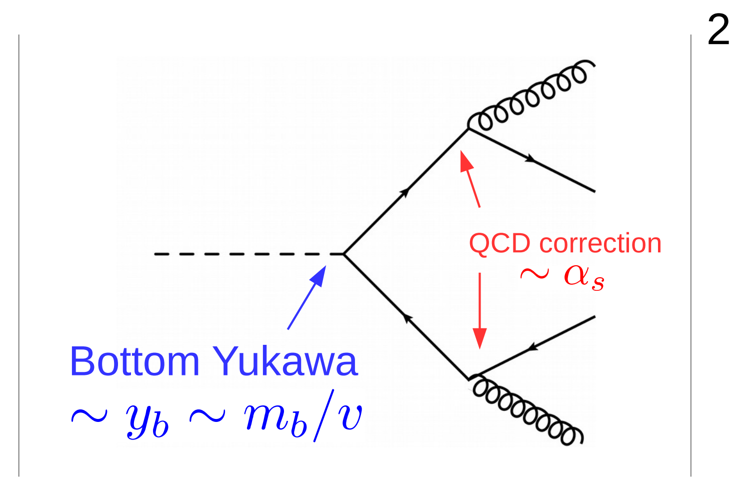
Bottom mass effects in $H \to bb$

- In $H \to bb$ decay, want massless b-quarks but non-zero y_b

$$m_b \ll m_H \Rightarrow \mathrm{d}\sigma \sim y_b^2(A + B\, m_b^2/m_H^2 + \ldots) = A y_b^2$$

- Works at LO & NLO, but not at NNLO – **interference terms.**





Top-loop interference contribution

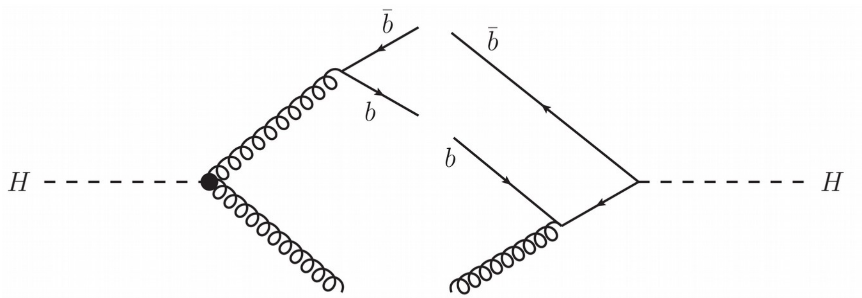
$\sim \alpha_s^2 m_b^2/v^2 \sim \alpha_s^2 y_b^2$

“Regular” contribution squared

$\sim \alpha_s^2 m_b^2/v^2 \sim \alpha_s^2 y_b^2$

Interference contribution has **identical parametric scaling** to other NNLO corrections.

Bottom mass interference



Obvious strategy: factor out one power of m_b and then take $m_b = 0$

BUT:

- Reduced matrix elements have unusual IR behaviour: *subleading power singularities*, e.g. **soft singularities from quarks!**
- $\log(m_b/m_H)$ **don't cancel** between real and virtual interference terms – cannot take massless limit!
- **Cannot** be regulated using flavor-kT algorithm (doesn't regulate soft quark singularity).
- **Cannot** define an inclusive cross section for *H* →*bb* at NNLO with massless *b*-quarks.
- Calculation in double-log approx: **~ 30%** of NNLO corrections to *H* → *bb* decay.
  - ➢ Effect on kinematic distributions?
- Different dependence on bottom Yukawa – **different behavior in BSM models.**

➡ NNLO calculation of *H*→*bb* to massive bottom quarks required.

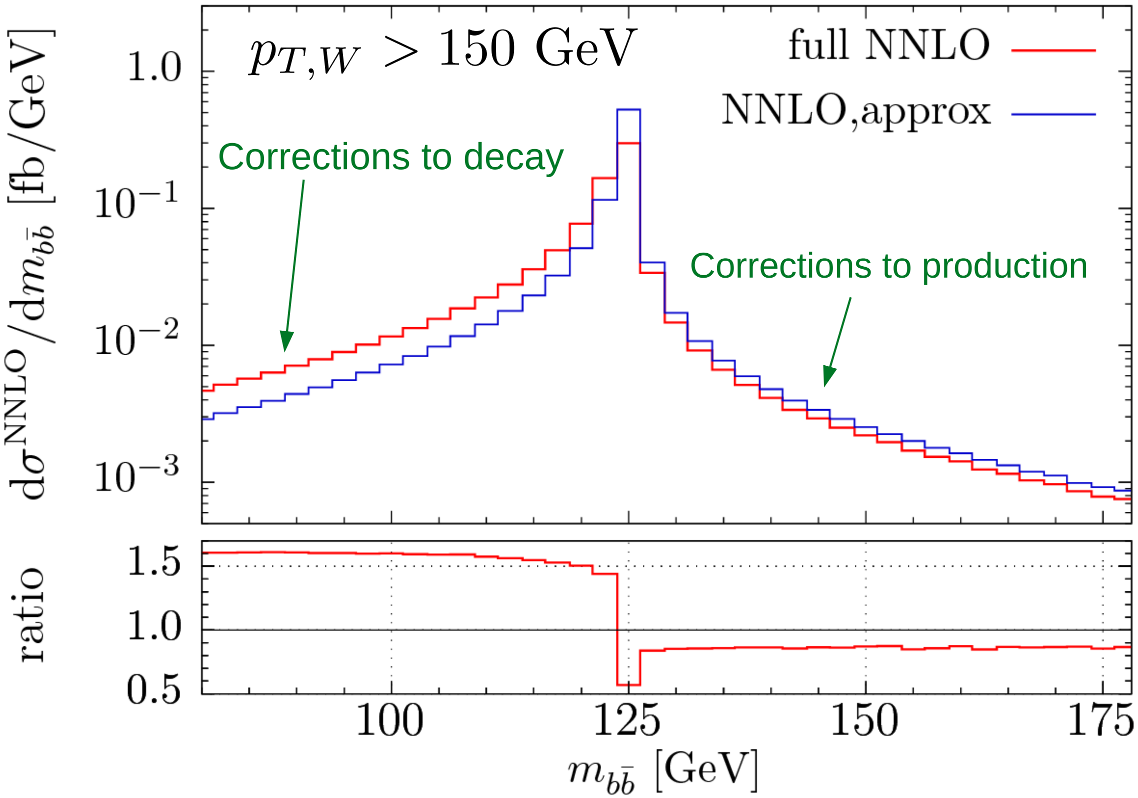
$VH(\to b\bar{b})$ to NNLO in production and decay

[Caola, Luisoni, Melnikov, R.R. '17]

NNLO corrections in production and decay in NWA.

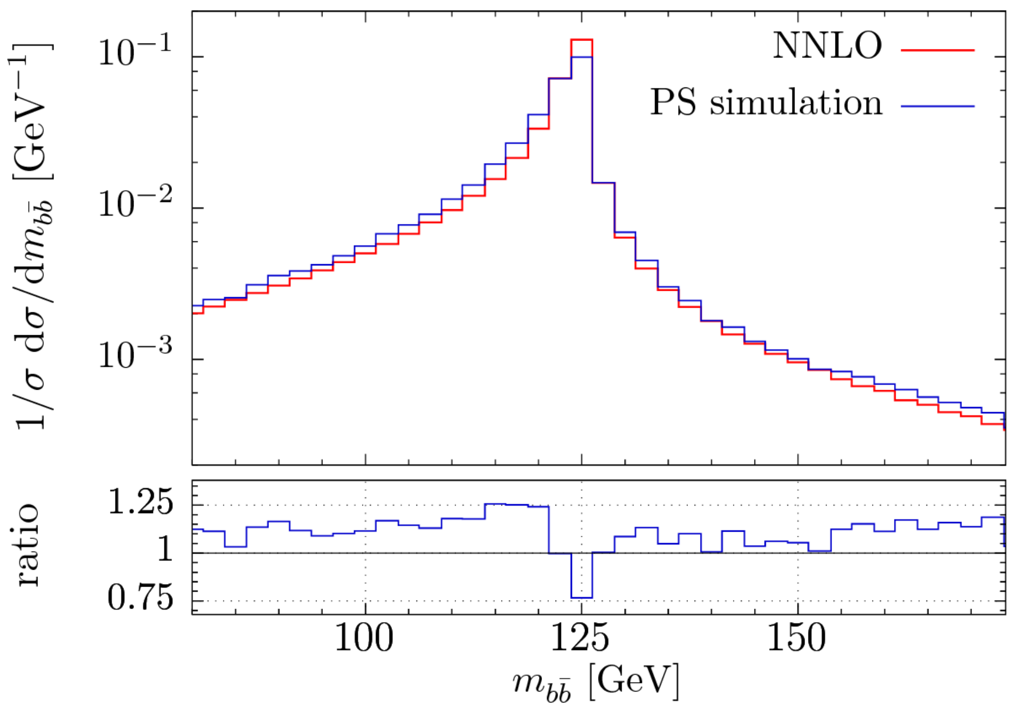
Confirm results of [Ferrera, Somogyi, Tramantano '17]:

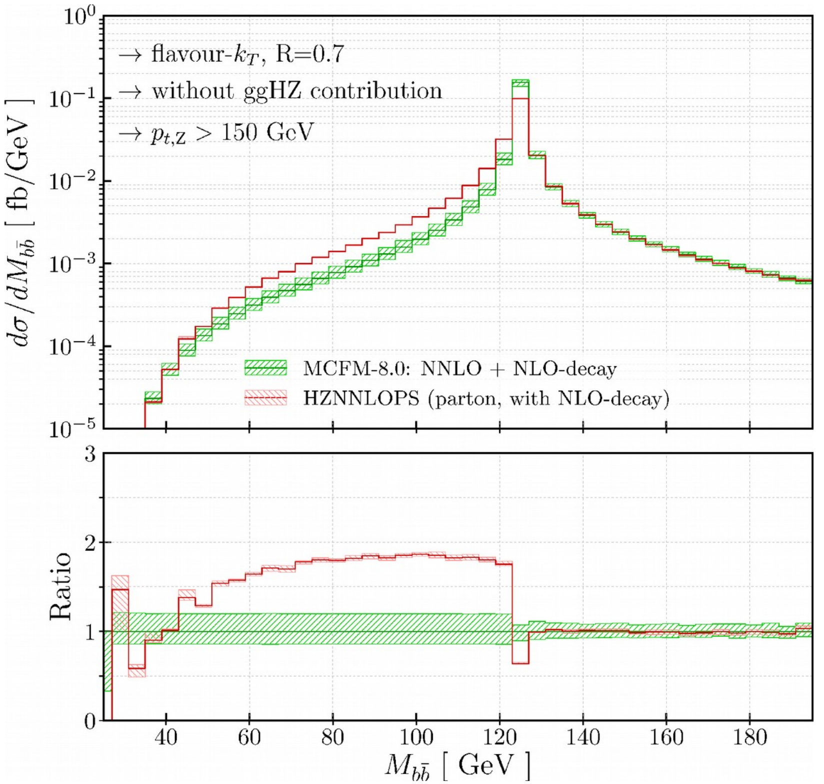
- Large (~60%) at low invariant mass.
- Sharp decrease at Higgs mass.
- ~ 15% depletion at high inv. mass.
- **Expected** as full NNLO includes corrections to decay – reduce inv. mass.



Comparison with parton shower

- Can parton showers capture these effects?
  - 🙂 Reasonable high boost $p_{T,W} > 150$ GeV
  - ☹ Low invariant mass requires **hard** gluon.
- HWJ generator from POWHEG-Box with MiNLO; *H*→*bb* through PYTHIA.
- NNLOPS analysis by [Astill, Bizon, Re, Zanderighi '18]





Extension to colored final states

Use DIS as test process (analytic expression known).

- Analytic form for double soft essential (otherwise require numerical integration at each phase space point).
- Combination of corrections to DY production and $H \to bb$ decay.
- Suggests that extension to **arbitrarily many colored final state particles** conceptually and analytically straightforward.

➡ Compact general analytic formula for NNLO subtraction of arbitrary final state, displaying pole cancellation.

Summary

- Method of handling NNLO subtraction, characterized by decoupling of soft and collinear limits.
- Developed iterative subtraction procedure:
  - Manifestly regulated finite term.
  - Integrated subtraction terms: convolutions of splitting function with explicit poles with lower multiplicity processes.
  - Transparent origin of IR poles.
  - Pole cancellation independent of matrix elements.
- Tested in DY and *W* production *for all partonic channels; H* → *bb* decay
  - Excellent agreement with analytic results in **all partonic channels.**
- Phenomenological application in $VH(\to b\bar{b})$.
- Ongoing work:
  - Remaining channels for color singlet production & color singlet decay.
  - Extension to colored initial-final state.
  - Major obstacle removed: double soft subtraction term known analytically.

THANK YOU!