Higgs boson plus gluon amplitudes at one-loop

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Prospect 2020

- Characterization of the Higgs boson will be a primary focus of Run 3 and High Luminosity LHC.
- The Higgs boson is a completely new type of particle;
- It is the only known fundamental scalar particle;
- It is the only boson that couples to itself;
- After heroic efforts by the LHC collaborations, current knowledge is at the ~20% level.

Guaranteed deliverables

- Mass of Higgs
- Total Width of Higgs
- Couplings of Higgs to all? particles
- (Bounds on) off-diagonal couplings
- Trilinear coupling of Higgs
- Higgs invisible width, Higgs untagged width

H^0 $J = 0$

Mass $m = 125.09 \pm 0.24$ GeV
Full width $\Gamma < 0.013$ GeV, CL = 95%

H^0 Signal Strengths in Different Channels

See Listings for the latest unpublished results.

Combined Final States $= 1.10 \pm 0.11$
$W W^* = 1.08^{+0.18}_{-0.16}$
$Z Z^* = 1.29^{+0.26}_{-0.23}$
$\gamma\gamma = 1.16 \pm 0.18$
$b\bar{b} = 0.82 \pm 0.30$ (S = 1.1)
$\mu^+\mu^- = 0.1 \pm 2.5$
$\tau^+\tau^- = 1.12 \pm 0.23$
$Z\gamma < 9.5$, CL = 95%
$t\bar{t}H^0$ Production $= 2.3^{+0.7}_{-0.6}$

PDG-May 2017

$$\mathcal{V}(\phi^\dagger\phi) = \lambda\,(\phi^\dagger\phi)^2 - \mu^2\phi^\dagger\phi\,.$$

$$\mathcal{L}_{\rm Higgs} = \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}M_h h^2 - \lambda_3\left(\frac{M_h^2}{2v}\right)h^3 - \lambda_4\left(\frac{M_h^2}{8v^2}\right)h^4$$

SM: $\lambda_3 = 1, \lambda_4 = 1$

Higgs Signal Strengths- a snapshot

$\mu_{\gamma\gamma}$	1.14 ± 0.14
μ_{ZZ^*}	1.17 ± 0.23
μ_{WW^*}	0.99 ± 0.15
$\mu_{b\bar{b}}$	0.98 ± 0.20
$\mu_{\tau\tau}$	1.09 ± 0.23
$\mu_{c\bar{c}}$	< 104
$\mu_{\mu\mu}$	< 2.8
μ_{ee}	$< 4 \times 10^5$

$\mathrm{BR}(t \to ch)$	$\leq 2.2 \times 10^{-3}$
$\mathrm{BR}(t \to uh)$	$\leq 2.4 \times 10^{-3}$
$\mathrm{BR}(h \to \tau\mu)$	$\leq 2.5 \times 10^{-3}$
$\mathrm{BR}(h \to \tau e)$	$\leq 6.1 \times 10^{-3}$
$\mathrm{BR}(h \to \mu e)$	$\leq 3.4 \times 10^{-4}$

μ_{ggF}	1.03 ± 0.14
μ_{VBF}	1.18 ± 0.23
μ_{Wh}	0.89 ± 0.38
μ_{Zh}	0.79 ± 0.36
$\mu_{t\bar{t}h}$	1.56 ± 0.42

$\mathrm{BR}(h \to \mathrm{inv})$	≤ 0.28
$\mathrm{BR}(h \to \mathrm{untagged})$	≤ 0.34

Compiled by Yossi Nir, (private communication)

Beyond the $m_t \longrightarrow \infty$ limit

- Set of references

[1] J. M. Lindert, K. Kudashkin, K. Melnikov and C. Wever, *Higgs bosons with large transverse momentum at the LHC*, *Phys. Lett.* **B782** (2018) 210 [1801.08226].

[2] S. P. Jones, M. Kerner and G. Luisoni, *Next-to-Leading-Order QCD Corrections to Higgs Boson Plus Jet Production with Full Top-Quark Mass Dependence*, *Phys. Rev. Lett.* **120** (2018) 162001 [1802.00349].

[3] V. Del Duca, W. Kilgore, C. Oleari, C. Schmidt and D. Zeppenfeld, *Higgs + 2 jets via gluon fusion*, *Phys. Rev. Lett.* **87** (2001) 122001 [hep-ph/0105129].

[4] V. Del Duca, W. Kilgore, C. Oleari, C. Schmidt and D. Zeppenfeld, *Gluon fusion contributions to H + 2 jet production*, *Nucl. Phys.* **B616** (2001) 367 [hep-ph/0108030].

[5] F. Campanario and M. Kubocz, *Higgs boson production in association with three jets via gluon fusion at the LHC: Gluonic contributions*, *Phys. Rev.* **D88** (2013) 054021 [1306.1830].

[6] N. Greiner, S. Hche, G. Luisoni, M. Schnherr, J.-C. Winter and V. Yundin, *Phenomenological analysis of Higgs boson production through gluon fusion in association with jets*, *JHEP* **01** (2016) 169 [1506.01016].

[7] R. V. Harlander, T. Neumann, K. J. Ozeren and M. Wiesemann, *Top-mass effects in differential Higgs production through gluon fusion at order α_s^4*, *JHEP* **08** (2012) 139 [1206.0157].

- All of these authors have used (at least) one-loop results in the full theory
- We want to revisit the one-loop calculations to see if simple analytic forms can be obtained.
- Analytic forms may offer the possibility of faster, more stable code.
- Numerical results can act as a crutch, to aid in getting accurate analytic results.

Scalar one-loop Integrals

Passarino-Veltman type notation
but, Bjorken-Drell metric

$$
\begin{aligned}
d_0 &= l^2 - m^2 + i\varepsilon \\
d_1 &= (l + p_1)^2 - m^2 + i\varepsilon = (l + q_1)^2 - m^2 + i\varepsilon \\
d_{12} &= (l + p_1 + p_2)^2 - m^2 + i\varepsilon = (l + q_2)^2 - m^2 + i\varepsilon \\
d_{123} &= (l + p_1 + p_2 + p_3)^2 - m^2 + i\varepsilon = (l + q_3)^2 - m^2 + i\varepsilon \\
d_{1234} &= (l + p_1 + p_2 + p_3 + p_4)^2 - m^2 + i\varepsilon = (l + q_4)^2 - m^2 + i\varepsilon \\
d_{12345} &= (l + p_1 + p_2 + p_3 + p_4 + p_5)^2 - m^2 + i\varepsilon = (l + q_5)^2 - m^2 + i\varepsilon
\end{aligned}
$$

$$
\begin{aligned}
C_0(p_1, p_2; m) &= \frac{1}{i\pi^2} \int d^4 l \frac{1}{d_0 d_1 d_{12}} \\
D_0(p_1, p_2, p_3; m) &= \frac{1}{i\pi^2} \int d^4 l \frac{1}{d_0 d_1 d_{12} d_{123}} \\
E_0(p_1, p_2, p_3, p_4; m) &= \frac{1}{i\pi^2} \int d^4 l \frac{1}{d_0 d_1 d_{12} d_{123} d_{1234}} \\
F_0(p_1, p_2, p_3, p_4, p_5; m) &= \frac{1}{i\pi^2} \int d^4 l \frac{1}{d_0 d_1 d_{12} d_{123} d_{1234} d_{12345}}
\end{aligned}
$$

Melrose, 1965
van Neerven, Vermaseren 1984

Hexagon to Pentagons

Simple application of Schouten Identity

$$l^\mu \epsilon(p_1,p_2,p_3,p_4) = l\cdot p_1\, \epsilon(\mu,p_2,p_3,p_4) + l\cdot p_2\, \epsilon(p_1,\mu,p_3,p_4) + l\cdot p_3\, \epsilon(p_1,p_2,\mu,p_4) + l\cdot p_4\, \epsilon(p_1,p_2,p_3,\mu)$$

$$F_0(p_1,p_2,p_3,p_4,p_5;m) = \sum_{i=1}^{6} c^{(i)}_{12345} F^{(i)}_0 \,.$$

$$\begin{aligned}
c^{(1)}_{12345} &= +\mathrm{tr}_5\{2\,3\,4\,5\}/\mathrm{tr}_5\{1\,2\,3\,4\,5\,6\} \\
c^{(2)}_{12345} &= -\mathrm{tr}_5\{(1+2)\,3\,4\,5\}/\mathrm{tr}_5\{1\,2\,3\,4\,5\,6\} \\
c^{(3)}_{12345} &= +\mathrm{tr}_5\{1\,(2+3)\,4\,5\}/\mathrm{tr}_5\{1\,2\,3\,4\,5\,6\} \\
c^{(4)}_{12345} &= -\mathrm{tr}_5\{1\,2\,(3+4)\,5\}/\mathrm{tr}_5\{1\,2\,3\,4\,5\,6\} \\
c^{(5)}_{12345} &= +\mathrm{tr}_5\{1\,2\,3\,(4+5)\}/\mathrm{tr}_5\{1\,2\,3\,4\,5\,6\} \\
c^{(6)}_{12345} &= -\mathrm{tr}_5\{1\,2\,3\,4\}/\mathrm{tr}_5\{1\,2\,3\,4\,5\,6\}
\end{aligned}$$

$$\begin{aligned}
F^{(1)}_0 &\equiv E_{(1)} = E_0(p_2,p_3,p_4,p_5;m) \\
F^{(2)}_0 &\equiv E_{(2)} = E_0(p_{12},p_3,p_4,p_5;m) \\
F^{(3)}_0 &\equiv E_{(3)} = E_0(p_1,p_{23},p_4,p_5;m) \\
F^{(4)}_0 &\equiv E_{(4)} = E_0(p_1,p_2,p_{34},p_5;m) \\
F^{(5)}_0 &\equiv E_{(5)} = E_0(p_1,p_2,p_3,p_{45};m) \\
F^{(6)}_0 &\equiv E_{(6)} = E_0(p_1,p_2,p_3,p_4;m)
\end{aligned}$$

$$\mathrm{tr}_5\{(1+2)\,3\,4\,5\} \equiv \mathrm{tr}(\gamma_5\,(\not p_1 + \not p_2)\,\not p_3\,\not p_4\,\not p_5)\,.$$

$$\sum_{i=1}^{6} c^{(i)}_{12345} = 0\,.$$

Trace notation

In order to obtain compact expressions for the coefficients of the scalar integrals, we define the following traces of γ-matrices.

$$\begin{aligned}
\mathrm{tr}_5\{1\,2\ldots n\} &= \mathrm{tr}\{\gamma_5\, \not{p}_1\, \not{p}_2 \ldots \not{p}_n\} \\
\mathrm{tr}_+\{1\,2\ldots n\} &= \mathrm{tr}\{\gamma_R\, \not{p}_1\, \not{p}_2 \ldots \not{p}_n\} \\
\mathrm{tr}_-\{1\,2\ldots n\} &= \mathrm{tr}\{\gamma_L\, \not{p}_1\, \not{p}_2 \ldots \not{p}_n\} \\
\mathrm{tr}_5\{1\,2\ldots n\} &\equiv \mathrm{tr}_+\{1\,2\ldots n\} - \mathrm{tr}_-\{1\,2\ldots n\}\,,
\end{aligned} \tag{B.1}$$

with $\gamma_{R/L} = (1 \pm \gamma_5)/2$. For the special case of lightlike vectors we have that

$$\begin{aligned}
\mathrm{tr}_+\{1\,2\,3\ldots n\} &= [1\,2]\,\langle 2\,3\rangle\,[3\,4]\ldots\langle n\,1\rangle \\
\mathrm{tr}_-\{1\,2\,3\ldots n\} &= \langle 1\,2\rangle\,[2\,3]\,\langle 3\,4\rangle\ldots[n\,1]\,.
\end{aligned} \tag{B.2}$$

In the case of lightlike vectors, the traces with γ_5 can be written as differences of spinor strings,

$$\mathrm{tr}(\gamma_5\, \not{p}_1\, \not{p}_2\, \not{p}_3\, \not{p}_4) = \big([1\,2]\,\langle 2\,3\rangle\,[3\,4]\,\langle 4\,1\rangle - \langle 1\,2\rangle\,[2\,3]\,\langle 3\,4\rangle\,[4\,1]\big) \tag{B.3}$$

$$\mathrm{tr}(\gamma_5\, \not{p}_1\, \not{p}_2\, \not{p}_3\, \not{p}_4\, \not{p}_5\, \not{p}_6) = \big([1\,2]\,\langle 2\,3\rangle\,[3\,4]\,\langle 4\,5\rangle\,[5\,6]\,\langle 6\,1\rangle - \langle 1\,2\rangle\,[2\,3]\,\langle 3\,4\rangle\,[4\,5]\,\langle 5\,6\rangle\,[6\,1]\big) \tag{B.4}$$

In the case where external vectors are not light-like, (e.g. in our case the Higgs momentum p_6), the spinor expressions must be modified using momentum conservation, e.g. Eq. (4.6) for the five gluon case.

Pentagon to boxes

- In four dimensions, reduction of a scalar pentagon to the sum of the five boxes obtained by removing one propagator.

$$E_0(w^2-4\Delta_4 m^2) = E^{(1)}\,[2\Delta_4 - w\cdot(v_1+v_2+v_3+v_4)] + E_0^{(2)}\, v_1\cdot w + E_0^{(3)}\, v_2\cdot w + E_0^{(4)}\, v_3\cdot w + E_0^{(5)}\, v_4\cdot w\,,$$

where

$$v_1^\mu = \epsilon^{\mu,q_2,q_3,q_4},\quad v_2^\mu = \epsilon^{q_1,\mu,q_3,q_4},\quad v_3^\mu = \epsilon^{q_1,q_2,\mu,q_4},\quad v_4^\mu = \epsilon^{q_1,q_2,q_3,\mu},$$
$$w^\mu = r_1 v_1^\mu + r_2 v_2^\mu + r_3 v_3^\mu + r_4 v_4^\mu\,.$$

Preamble gg → H

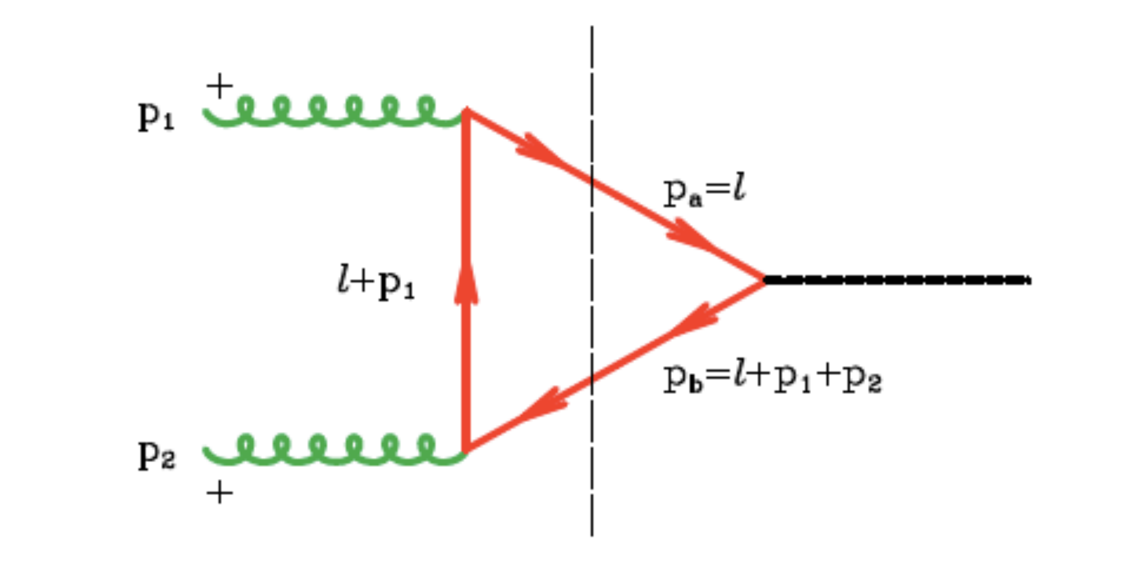
- The very first calculations of Higgs —> gg (Wilczek,1977) and gg-> Higgs (Georgi et al, 1978) emphasized the role of the process in counting the number of heavy constituents that couple to the Higgs boson.
- Our calculation of this process relies heavily on the work of Bern and Morgan (hep/ph9511336) who calculated the processes gggg and ggggg analytically with a massive loop of fermions using unitarity techniques.

Wilczek,1977
Georgi, Glashow, Machacek, Nanopoulos, 1977

Preamble: Higgs to 2 gluons

- LHS=

$$G_2^{\text{tree}}(a,1^+,2^+,b) = \frac{[1\,2]}{\langle 1\,2\rangle}\frac{\bar{u}(p_a)\gamma_R(\not{l}+m)u(p_b)}{(s_{a1}-m^2)},$$



- RHS=

$$H_0^{\text{tree}} = m\bar{u}(p_b)u(p_a)\,.$$

- Sewn together

$$m^2\frac{[1\,2]}{\langle 1\,2\rangle}\frac{\text{Tr}\{\gamma_R(\not{p}_b+m)(\not{p}_a+m)\}}{(s_{a1}-m^2)} = m^2\frac{[1\,2]}{\langle 1\,2\rangle}\frac{(2p_a\cdot p_b+2m^2)}{(s_{a1}-m^2)} = m^2\frac{[1\,2]}{\langle 1\,2\rangle}\frac{(4m^2-M_h^2)}{(s_{a1}-m^2)}\,.$$

- Full result =

$$A_2(1_g^+,2_g^+;H) = 2m^2\frac{[1\,2]}{\langle 1\,2\rangle}\Big[(4m^2-M_h^2)C_0(p_1,p_2;m)+2\Big]\,.$$

1) Essential ingredient for simple result is the simplicity of tree-level inputs
2) Rational part is provided by the mass dependence, so it can be inserted at the end

Tree level ingredients for higher n

- All + helicity amplitude, for all n from BCFW

$$G_n(a,1^+,2^+,\ldots,n^+,b) = m\frac{\bar{u}(a)\gamma_R u(b)\ [1|\prod_{j=1}^{n-2}\{\not{p}_{a\ldots j}\not{p}_{j+1}+(s_{a1\ldots j}-m^2)\}|n]}{(s_{a1}-m^2)(s_{a12}-m^2)\ldots(s_{a1\ldots(n-1)}-m^2)\ \langle 12\rangle\langle 23\rangle\ldots\langle n-1|n\rangle}.$$

- For n=3,4

$$G_3(a,1^+,2^+,3^+,b) = m\frac{\bar{u}(a)\gamma_R u(b)\ [1|\left(\not{p}_{a1}\not{p}_2+(s_{a1}-m^2)\right)|3]}{(s_{a1}-m^2)(s_{a12}-m^2)\ \langle 1\,2\rangle\langle 2\,3\rangle}.$$

$$G_4(a,1^+,2^+,3^+,4^+,b) = m\frac{\bar{u}(a)\gamma_R u(b)\ [1|\left(\not{p}_{a1}\not{p}_2+(s_{a1}-m^2)\right)\left(\not{p}_{a12}\not{p}_3+(s_{a12}-m^2)\right)|4]}{(s_{a1}-m^2)(s_{a12}-m^2)(s_{a123}-m^2)\ \langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle}.$$

- With one negative helicity (not used so far)

$$\begin{aligned}A(\underline{1}^a,3^-,4^+,\ldots,n^+,\overline{2}^b) = &-\frac{i\langle 3|1|2|3\rangle\left(\langle\underline{1}^a3\rangle[\overline{2}^b|1+2|3\rangle-\langle\overline{2}^b3\rangle[\underline{1}^a|1+2|3\rangle\right)}{s_{12}\langle 34\rangle\ldots\langle n-1|n\rangle\langle 3|1|2|n\rangle}\\ &-\sum_{k=4}^{n-1}\frac{im\langle 3|\not{p}_1\not{p}_{3\ldots k}|3\rangle\left(\langle\underline{1}^a\overline{2}^b\rangle\langle 3|\not{p}_1\not{p}_{3\ldots k}|3\rangle+\langle\underline{1}^a3\rangle\langle\overline{2}^b3\rangle s_{3\ldots k}\right)}{s_{3\ldots k}(s_{13\ldots k}-m^2)\ldots(s_{13\ldots(n-1)}-m^2)\langle 34\rangle\ldots\langle k-1|k\rangle\langle 3|\not{p}_1\not{p}_{3\ldots k}|k\rangle}\\ &\times\frac{\langle 3|\not{p}_{3\ldots k}\prod_{j=k}^{n-2}\{\not{p}_{13\ldots j}\not{p}_{j+1}+(s_{13\ldots j}-m^2)\}|n]}{\langle 3|\not{p}_1\not{p}_{3\ldots k}|k+1\rangle\langle k+1|k+2\rangle\ldots\langle n-1|n\rangle}\end{aligned}$$

Ochirov, 1802.06730

notation: Arkani-Hamed, Huang, Huang 1709.04891

Projection of H+ng amplitudes

- For the all positive amplitudes we need to project with the same factor $M_0^\dagger = m\,\bar{u}(b)\gamma_R u(a)$

$$M_0^\dagger\, H_0(a;H,b) = m^2\left(4m^2 - M_h^2\right)$$

$$M_0^\dagger\, H_1(a,1_g^+;H,b) = m^2\left(4m^2 - M_h^2\right)\frac{1}{\langle 1\,q\rangle}\left\{\frac{[1|a|q\rangle}{[1|a|1\rangle} - \frac{[1|b|q\rangle}{[1|b|1\rangle}\right\} = m^2\left(4m^2 - M_h^2\right)\frac{[1|ab|1]}{[1|a|1\rangle\,[1|b|1\rangle}$$

$$M_0^\dagger\, H_2(a,1_g^+,2_g^+;H,b) = 2m^2\left(4m^2 - M_h^2\right)\frac{1}{\langle 1\,2\rangle}\left\{\frac{[2|\not{b}(\not{b}-\not{a})|1]}{[1|a|1\rangle\,[2|b|2\rangle}\right.$$

$$\left. - m^2\,[2\,1]\left(\frac{1}{[2|b|2\rangle\,((b-p_1-p_2)^2 - m^2)} + \frac{1}{[1|a|1\rangle\,[2|b|2\rangle} - \frac{1}{[1|a|1\rangle\,((a+p_1+p_2)^2 - m^2)}\right)\right\}$$

- Note that the results all contain a universal factor of $4m^2 - M_h^2$

n=3

$$A_3(1_g^+, 2_g^+, 3_g^+; H) = m^2 \Bigg[\Bigg\{ \frac{4m^2 - M_h^2}{\langle 1\,2 \rangle \langle 2\,3 \rangle \langle 3\,1 \rangle} \Big[-\frac{1}{2} s_{12} s_{23} D_0(p_1, p_2, p_3; m) - (s_{12} + s_{13}) C_0(p_1, p_{23}; m) \Big] - 2 \frac{s_{12} + s_{13}}{\langle 1\,2 \rangle \langle 2\,3 \rangle \langle 3\,1 \rangle} \Bigg\} + \Bigg\{ 2 \text{ cyclic permutations} \Bigg\} \Bigg] .$$

RKE, Hinchliffe, Soldate, van der Bij, 1988,
Higgs to $\tau\tau$:
a possible signature of intermediate mass Higgs bosons at the SSC (at high energy hadron colliders)

included in MCFM (in a different format) by Neumann and Williams, 2017 (+ many more……)

n=4

Our result for this amplitude is

$$
\begin{aligned}
A_4(1_g^+,2_g^+,3_g^+,4_g^+;H) = m^2\Bigg[\Bigg\{ & \frac{4m^2-M_h^2}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,1\rangle}\Big[-\mathrm{tr}_+\{1\,2\,3\,4\}m^2E_0(p_1,p_2,p_3,p_4;m) \\
& + \frac{1}{2}((s_{12}+s_{13})(s_{24}+s_{34})-s_{14}s_{23})D_0(p_1,p_{23},p_4;m) \\
& + \frac{1}{2}s_{12}s_{23}D_0(p_1,p_2,p_3;m) \\
& + (s_{12}+s_{13}+s_{14})C_0(p_1,p_{234};m)\Big] + 2\frac{s_{12}+s_{13}+s_{14}}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,1\rangle}\Bigg\} \\
& + \Bigg\{3 \text{ cyclic permutations}\Bigg\}\Bigg].
\end{aligned}
$$

n=5

Our result for this amplitude is

$$
\begin{aligned}
A_5(1_g^+,2_g^+,3_g^+,4_g^+,5_g^+;H) = m^2 \Bigg[\Bigg\{ & \frac{(4m^2 - M_h^2)}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,5\rangle \langle 5\,1\rangle} \Big[\sum_{i=1}^{6} e_{(i)} E_{(i)} \\
& - \frac{1}{2} s_{12} s_{23} D_0(p_1, p_2, p_3; m) - \frac{1}{2} \left[(s_{12}+s_{13})(s_{24}+s_{34}) - s_{14}s_{23} \right] D_0(p_1, p_{23}, p_4; m) \\
& - \frac{1}{2} \left[(s_{12}+s_{13}+s_{14})(s_{25}+s_{35}+s_{45}) - s_{15}(s_{23}+s_{24}+s_{34}) \right] D_0(p_1, p_{234}, p_5; m) \\
& - (s_{12}+s_{13}+s_{14}+s_{15})\, C_0(p_1, p_{2345}; m) \Big] - \frac{2(s_{12}+s_{13}+s_{14}+s_{15})}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,5\rangle \langle 5\,1\rangle} \Bigg\} \\
& + \Bigg\{ 4 \text{ cyclic permutations} \Bigg\} \Bigg], \qquad (4.4)
\end{aligned}
$$

Coefficients of pentagons for n=5

$$e_{(1)} = m^2\left[\frac{1}{2}\mathrm{tr}_-\{2\,3\,4\,5\} + \frac{s_{23}s_{34}s_{45}(\mathrm{tr}_-\{2\,6\,5\,1\} + s_{51}s_{12})}{\mathrm{tr}_5\{1\,2\,3\,4\,5\,6\}}\right]$$

$$e_{(2)} = -m^2 s_{45}s_{34}\frac{\mathrm{tr}_-\{5\,1\,2\,3\,(1+2)\,6\}}{\mathrm{tr}_5\{1\,2\,3\,4\,5\,6\}}$$

$$e_{(3)} = -m^2\frac{\mathrm{tr}_+\{5\,4\,(2+3)\,1\}\,\mathrm{tr}_-\{1\,2\,3\,4\,5\,6\}}{\mathrm{tr}_5\{1\,2\,3\,4\,5\,6\}}$$

$$e_{(4)} = -m^2\frac{\mathrm{tr}_+\{1\,2\,(3+4)\,5\}\,\mathrm{tr}_-\{5\,4\,3\,2\,1\,6\}}{\mathrm{tr}_5\{5\,4\,3\,2\,1\,6\}}$$

$$e_{(5)} = -m^2 s_{12}s_{23}\frac{\mathrm{tr}_-\{1\,5\,4\,3\,(4+5)\,6\}}{\mathrm{tr}_5\{5\,4\,3\,2\,1\,6\}}$$

$$e_{(6)} = m^2\left[\frac{1}{2}\mathrm{tr}_-\{4\,3\,2\,1\} + \frac{s_{12}s_{23}s_{34}(\mathrm{tr}_-\{4\,6\,1\,5\} + s_{45}s_{51})}{\mathrm{tr}_5\{5\,4\,3\,2\,1\,6\}}\right]$$

p_6 is the momentum of the Higgs boson, behavior as p_6 goes to zero useful to organize

Large m_t limit

Our results give the expected result as m_t goes to infinity

$$A_2(1_g^+, 2_g^+; H) = +\frac{2}{3}\frac{M_h^4}{\langle 1\,2\rangle \langle 2\,1\rangle},$$

$$A_3(1_g^+, 2_g^+, 3_g^+; H) = -\frac{2}{3}\frac{M_h^4}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,1\rangle},$$

$$A_4(1_g^+, 2_g^+, 3_g^+, 4_g^+; H) = +\frac{2}{3}\frac{M_h^4}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,1\rangle},$$

$$A_5(1_g^+, 2_g^+, 3_g^+, 4_g^+, 5_g^+; H) = -\frac{2}{3}\frac{M_h^4}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,5\rangle \langle 5\,1\rangle}.$$

$$C_0(p_1, p_2; m) = -\frac{1}{2m^2} - \frac{(p_1^2 + p_2^2 + p_{12}^2)}{24m^4} + O\left(\frac{1}{m^6}\right)$$

$$D_0(p_1, p_2, p_3; m) = \frac{1}{6m^4} + \frac{(s_{23} + s_{12} + p_1^2 + p_2^2 + p_3^2 + p_{123}^2)}{60m^6} + O\left(\frac{1}{m^8}\right)$$

$$E_0(p_1, p_2, p_3, p_4; m) = -\frac{1}{12m^6} + O\left(\frac{1}{m^8}\right)$$

Soft Higgs limit

- The insertion of a soft Higgs boson can be effected by the operator $\frac{m}{v}\frac{d}{dm}$ operating on the amplitude without a Higgs boson
- Not really a viable approximation method;
- May give information about the best way to organize the answer.
- Soft limit established for the 4 gluon amplitude, working on the 5 gluon….

Soft Higgs limit - 4 gluons

- Bern and Morgan result for colour-ordered four-gluon amplitude $A_4(1_g^+,2_g^+,3_g^+,4_g^+)$

$$A_4(1_g^+,2_g^+,3_g^+,4_g^+) = -2\frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}\left[m^4 D_0(p_1,p_2,p_3;m) - \frac{1}{6}\right].$$

- Our result in the limit $p_5 \to 0$

$$\begin{aligned}
&A_4(1_g^+,2_g^+,3_g^+,4_g^+;H) \to -4m^4\frac{[1\,2]\,\langle 2\,3\rangle\,[3\,4]\,\langle 4\,1\rangle}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle} \\
&\times \Big[\frac{1}{2}(D_0(p_1,p_2,p_3;m) + D_0(p_2,p_3,p_4;m) + D_0(p_3,p_4,p_1;m) + D_0(p_4,p_1,p_2;m)) \\
&+ m^2(E_0(p_1,p_2,p_3,p_4;m) + E_0(p_2,p_3,p_4,p_1;m) + E_0(p_3,p_4,p_1,p_2;m) + E_0(p_4,p_1,p_2,p_3;m))\Big] \\
&= -2\frac{[1\,2]\,\langle 2\,3\rangle\,[3\,4]\,\langle 4\,1\rangle}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle}\left[4m^4 D_0(p_1,p_2,p_3;m) + 2m^6\frac{d}{dm^2}D_0(p_1,p_2,p_3;m)\right] \\
&= -2\frac{[1\,2]\,\langle 2\,3\rangle\,[3\,4]\,\langle 4\,1\rangle}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle}2m^2\frac{d}{dm^2}\left[m^4 D_0(p_1,p_2,p_3;m)\right] \qquad (5.10)
\end{aligned}$$

Conclusions

- Compact analytic results for Higgs + 4 parton and Higgs + 5 parton amplitudes.
- Extension to other helicity choices will proceed.
- More systematic methods of simplifying the amplitudes will be most likely needed.

European Strategy

Current European Strategy(2013)

a) Europe should preserve this model (i.e. CERN) in order to keep its leading role, sustaining the success of particle physics and the benefits it brings to the wider society.

b) The European Strategy takes into account the worldwide particle physics landscape and developments in related fields and should continue to do so.

c) Europe's top priority should be the exploitation of the full potential of the LHC, including the high-luminosity upgrade of the machine and detectors with a view to collecting ten times more data than in the initial design, by around 2030. This upgrade programme will also provide further exciting opportunities for the study of flavour physics and the quark-gluon plasma.

d) CERN should undertake design studies for accelerator projects in a global context, with emphasis on proton-proton and electron positron high-energy frontier machines. These design studies should be coupled to a vigorous accelerator R&D programme, including high-field magnets and high-gradient accelerating structures, in collaboration with national institutes, laboratories and universities worldwide.

e) Europe looks forward to a proposal from Japan to discuss a possible participation, (in ILC)

f) CERN should develop a neutrino programme to pave the way for a substantial European role in future long-baseline experiments. Europe should explore the possibility of major participation in leading long-baseline neutrino projects in the US and Japan.

h) Europe should support a diverse, vibrant theoretical physics programme, ranging from abstract to applied topics, in close collaboration with experiments and extending to neighbouring fields such as astroparticle physics and cosmology. Such support should extend also to high-performance computing and software development.

i) Detector R&D programmes should be supported strongly at CERN, national institutes, laboratories and universities. Infrastructure and engineering capabilities for the R&D programme and construction of large detectors, as well as infrastructures for data analysis, data preservation and distributed data-intensive computing should be maintained and further developed.

j) A range of important non-accelerator experiments take place at the overlap of particle and astroparticle physics, such as searches for proton decay, neutrinoless double beta decay and dark matter, and the study of high-energy cosmic-rays. These experiments address fundamental questions beyond the Standard Model of particle physics. The exchange of information between CERN and ApPEC (Astroparticle Physics European Consortium) has progressed since 2006. In the coming years, CERN should seek a closer collaboration with ApPEC on detector R&D with a view to maintaining the community's capability for unique projects in this field.

k) A variety of research lines at the boundary between particle and nuclear physics require dedicated experiments. The CERN Laboratory should maintain its capability to perform unique experiments. CERN should continue to work with NuPECC on topics of mutual interest.

Last strategy process led to initiation of CERN/European activity in neutrino physics

Prototype cryostats for liquid Argon detectors at CERN

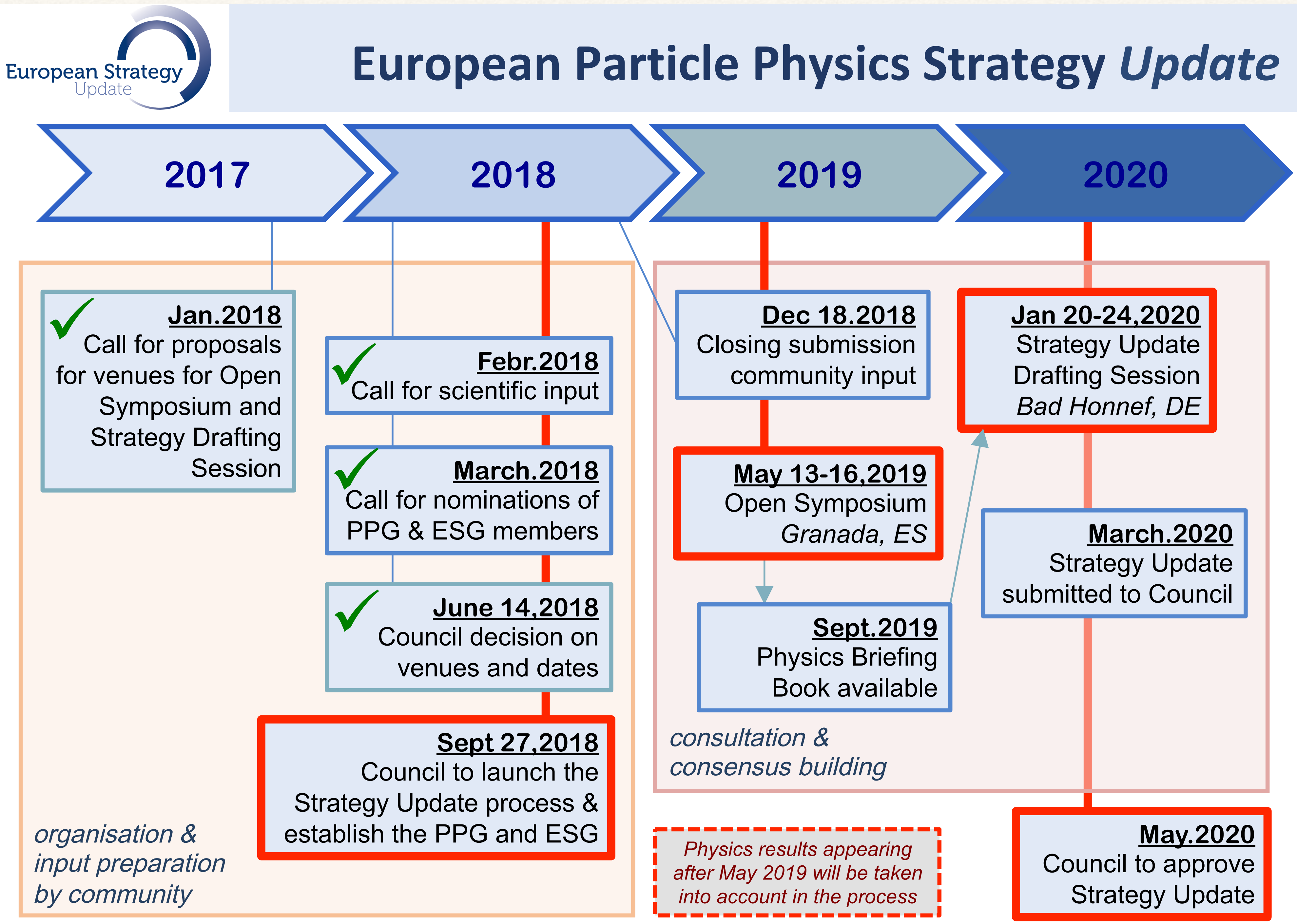
European Strategy Update

- Next update of the European Strategy for Particle Physics.
- Aim to have update of Strategy for May 2020.
    - 7 years since the last update, 2013.
    - End of Run 2 of LHC (12/2018)
    - FCC conceptual design report completed
    - CLIC update
    - Report of Physics beyond Colliders Study Group by end of 2018
    - Japanese decision on ILC should be known by end of 2018.

Update of strategy process 2018-May 2020

- 2018 to early 2019 is a year of preparation, and for generation of ideas.
- "Letting a hundred flowers blossom and a hundred schools of thought contend is the policy for promoting progress in the arts and the sciences"…
- 2019-2020 is to do with fiscal reality, hammering out consensus, uniting the community with common goals.

Countdown to May 2020



Strategy process 2018 to early 2019

- The input is collected by the Physics Preparatory Group, (PPG)
- The PPG organizes the Open Symposium to discuss the proposals.
- The drafting is based on input from the community-collaborations, proposals, national institutes, national roadmaps, individuals.
- The PPG summarizes the input, the discussion and the conclusions in a Briefing book.
- The briefing book constitutes the input for the European Strategy Group(ESG) to draft the update.
- The drafting of the strategy takes place during a dedicated Drafting Session (the conclave the EPPSU process)
- The whole organization is run by the strategy secretariat.
- The strategy update is drafted by the European Strategy Group (ESG).
- All teams are chaired by the Strategy Secretary.

Strategy Secretariat

Strategy Secretary
Halina Abramowicz (Israel)

ECFA chair
Jorgen D'Hondt(BE)

Laboratory Directors' Group
Leonid Rivkin (CH)

Scientific Policy Committee
Keith Ellis (UK)

Composition of PPG (15-17 people)

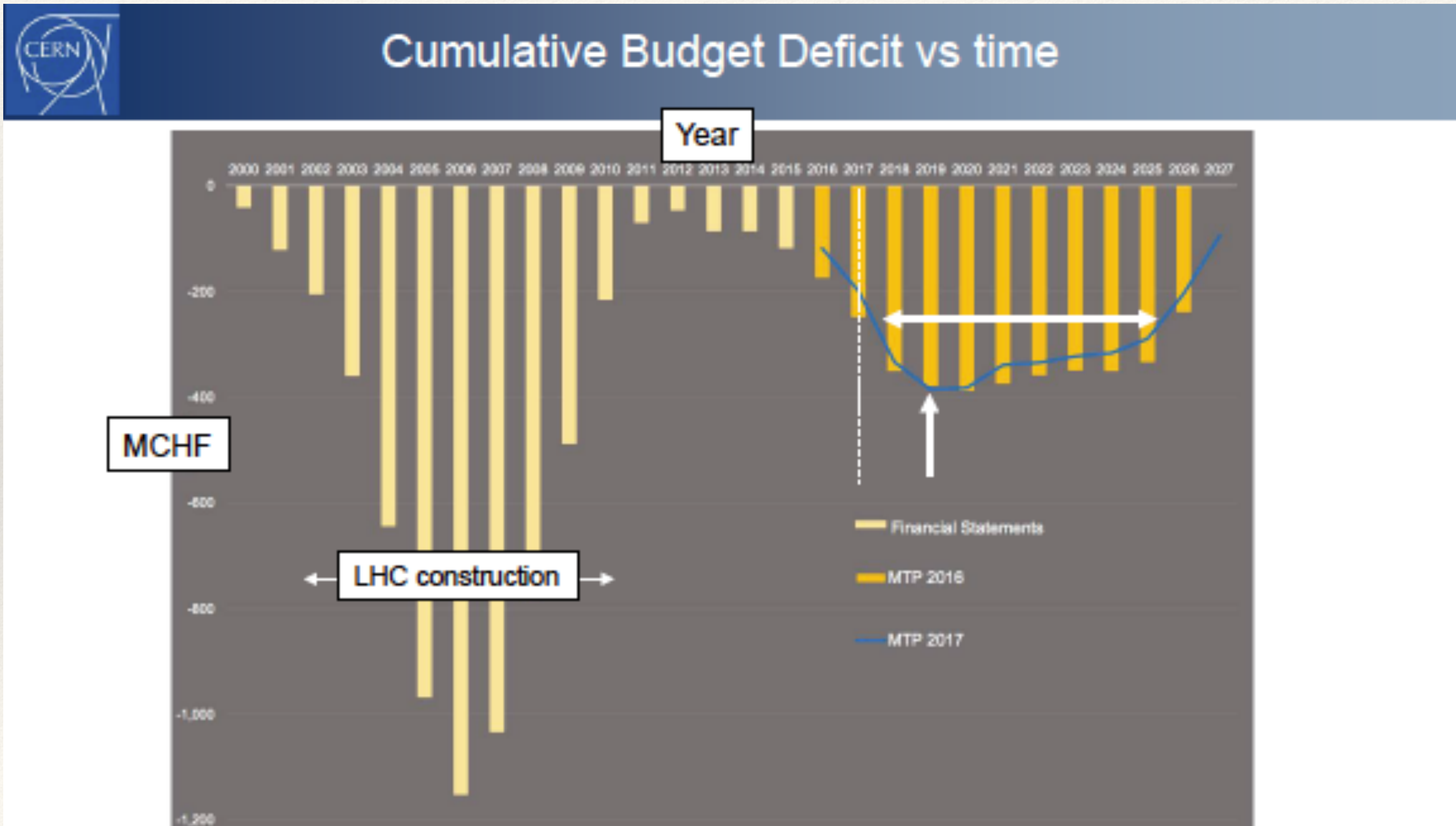
The Strategy Secretary (chair)
- Four members recommended by the SPC
- Four members recommended by ECFA
- SPC chair
- ECFA chair
- Chair of the the European Laboratory Directors Group
- One representative appointed by CERN
- Representatives from Asia (2)
- Representatives from the Americas ≤2

Composition of the ESG (62-64 people)

Members
- The Strategy Secretary (chair)
- One representative appointed by each CERN MS (22)
- One representative appointed by each of the Labs participating in the European Laboratory Directors Group including its Chairperson (9)
- CERN DG
- SPC chair
- ECFA chair

Invitees
- President of CERN Council
- One representative from each AMS and OS (7+3)
- One representative from the European Commission
- Chairs of ApPEC, NuPECC, FALC, ESFRI
- Members of the PPG (17 - Secretariat)

Timeline for Money (at CERN)



Decision for future colliders

- Fabiola Gianotti
- "Current CERN budget has 28M/year for collider R&D"
- "20M FCC and 8M CLIC to cover R&D and continuance of the projects"
- "To start construction before the end of the decade, CLIC needs to produce a TDR by 2026 and the FCC a CDR+."
- "Both would require about 20M per year."
- "Impossible to support both CLIC and FCC at this level."
- "2020 ESPP should give some indications about the next collider."