Neutron Lifetime – Axial Coupling Connection

τ_n → ← g_A

(Implications for the neutron lifetime puzzle)

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The Neutron Lifetime Puzzle

τ_n^{beam} =888.0(2.0)s τ_n^{trap}=879.4(6)s

Differ by 4σ

Both Right?

Fornal-Grinstein Solution BR(n→dark particles) ~ 1%

But Neutron Decay Asymmetry g_A determination

has 5σ Problem

$g_A^{pre2002}$=1.2637(21) $g_A^{post2002}$=1.2755(11)

PerkeoII (2013)

****** UCNA (2017)***

Also solves τ_n problem

Results

- 1. τ_n, g_A, V_{ud} Master Formula

  $$\tau_n (1+3g_A^2)|V_{ud}|^2=4908.6(1.9)\text{sec}$$

  (Future Goals: Theory & Experiment)

- 2. $\tau_n - g_A$ Connection: $|V_{ud}|=0.97420(10)(18)$

  Hardy & Towner

  $\tau_n (1+3g_A^2)=5172.0(1.1)\text{sec}$ Connection

- 3. $\tau_n^{favored}=879.4(6)\text{sec}$ $g_A^{favored}=1.2755(11)$

  implies τ_n^{beam} wrong!

- 4. BR(any exotic n decay)< 0.27% (95% C.L.)

<u>Some References</u>

<u>Refs</u>: ***<u>A. Sirlin RMP 50, 573 (1978)</u>*** + earlier work

WJM & A. Sirlin, PRL <u>56,</u> 22 (1986); ibid <u>96,</u> 032002 (2006)

A. Czarnecki, WJM, A. Sirlin, PRD <u>70,</u> 093006 (2004);

<u>The classic</u>: D. Wilkinson, NP A377, 474 (1982).

<u>Reviews</u>: "*The neutron. Its properties and basic interactions*"

Hartmut Abele Prog.Part.Nucl.Phys. 60 (2008) 1.

"*The Neutron and Its Role in Cosmology and Particle Physics*"

Dirk Dubbers, Michael G. Schmidt RMP. 83 (2011) 1111.

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1. τ_n , g_A, V_{ud} & CKM Unitarity

$SU(2)_L x U(1)_Y$ Standard Model Electroweak Loop Corrections to $\mu \rightarrow e \nu_e \nu_\mu$ and $n \rightarrow p e \nu_e$ ***both Infinite*** but renormalized using ($G_F^0 \rightarrow G_\mu$) Quark mixing divergences absorbed in $V^0_{ud} \rightarrow V_{ud}$ maintaining Unitarity

The CKM Quark Mixing Matrix:

$$V^{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

3x3 Unitary Matrix

Unitarity $\rightarrow |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1$ **etc.**

Any "Apparent" Deviation from 1 Implies "New Physics" at the tree or quantum loop level

Muon Decay $\Gamma_0(\mu\rightarrow e\nu\nu)=F(m_e^2/m_\mu^2)G^0{}_F{}^2 m_\mu^5/192\pi^3= 1/\tau_\mu^0$

Neutron Decay $\Gamma_0(n\rightarrow pe\nu)=\mathbf{f}G^0{}_F{}^2|V^0{}_{ud}|^2 m_e^5(1+3g_A{}^2)/2\pi^3=1/\tau_n^0$

$F(x)=1-8x+8x^3-x^4-12x^2\ln x$ Phase Space Factor

f=1.6887(1) phase space factor, including Fermi function proton recoil, finite nucleon size… Uncertainty O(few x10^{-5})

Other Small Effects: Weak Magnetism, **Induced Pseudoscalar** etc.

g_A and τ_n **important for: *CKM Unitarity***, solar neutrino flux, reactor neutrino flux, ***primordial abundances*** ΔN_ν, spin content of proton, Goldberger-Treiman/***Muon Capture***, Bjorken Sum Rule, lattice benchmark $\mathbf{g_A}$ **&** $\mathbf{g_p}$…

Must be precisely determined!

±0.01% Outstanding/Appropriate Goals for τ_n and $g_A{}^2$

Electroweak Radiative Corrections to Muon Decay

Virtual One Loop Corrections + Inclusive Bremsstrahlung

Absorb Ultraviolet divergences and some finite parts in

$G^0_F = g_0^2/4\sqrt{2}m^2_{W0} \rightarrow G_\mu$

$\tau_\mu^{-1} = \Gamma(\mu^+ \rightarrow e^+ \nu_e \nu_\mu (\gamma)) \equiv F(m_e^2/m_\mu^2) G_\mu^2 m_\mu^5 [1+RC]/192\pi^3$

$RC = \alpha/2\pi(25/4-\pi^2)(1+\alpha/\pi[2/3\ln(m_\mu/m_e)-3.7)\ldots]$ Fermi Th.

Defines G_μ

Other SM and "**New Physics**" radiative corrections absorbed into G_μ. Eg. Top Mass, Higgs Mass, Technicolor, Susy,W*…

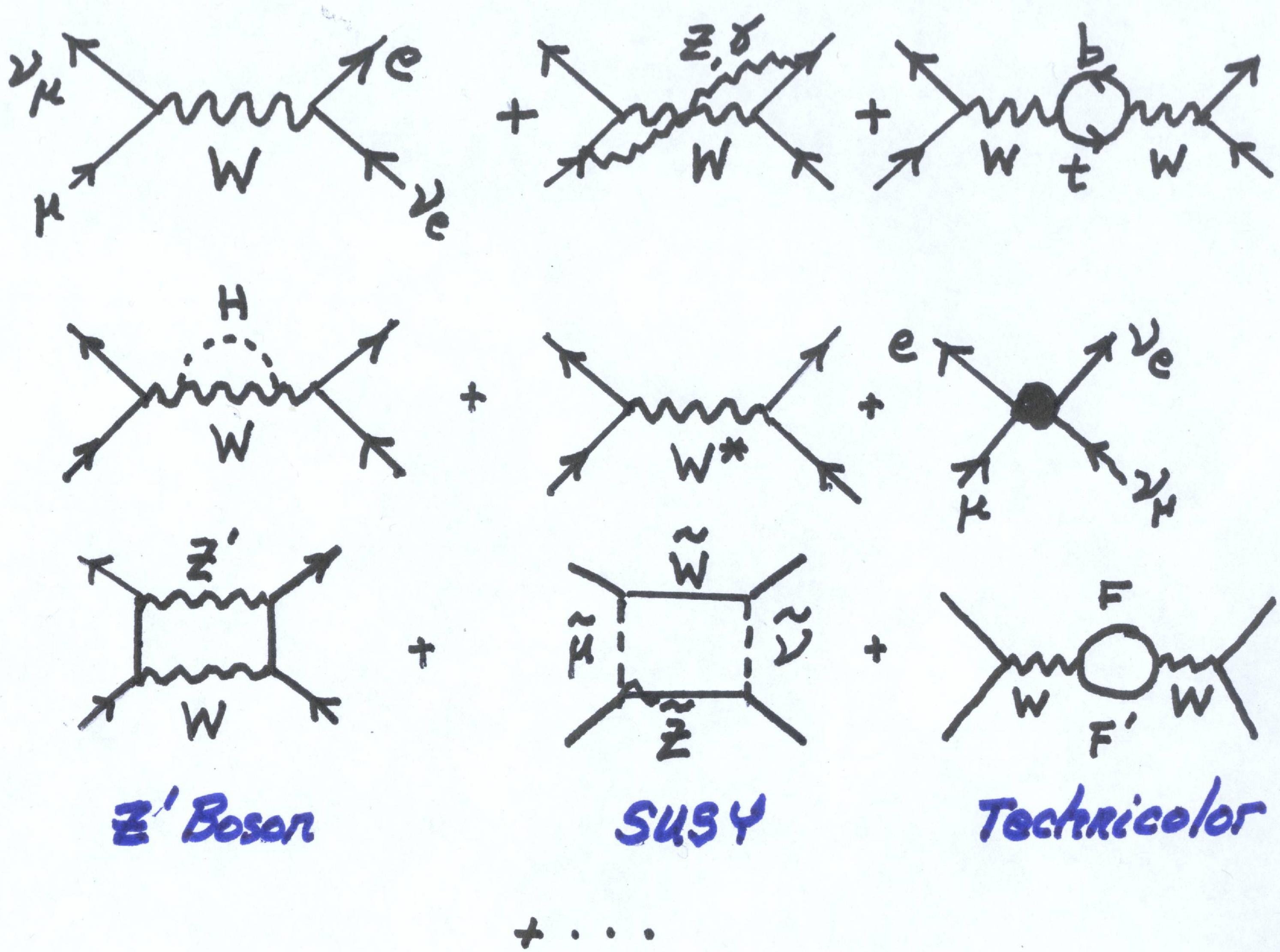
MuLAN experiment at PSI (Complete)

World Ave. $\tau_{\mu+} = 2.1969803(22) \times 10^{-6}$ sec 1ppm!

Most precise lifetime ever measured gives:

$G_\mu = 1.1663787(6) \times 10^{-5}\,GeV^{-2}$ precise & important

Loop and Tree Level Corrections to Muon Decay



Electroweak Radiative Corrections to Neutron Beta Decay

Include Virtual Corrections + Inclusive Bremsstrahlung

Normalize using G_μ from the muon lifetime

Absorbs Ultraviolet Divergences & some finite parts

$$1/\tau_n = f G_\mu^2 |V_{ud}|^2 m_e^5 (1+3g_A^2)(\underline{1+RC})/2\pi^3$$

f=1.6887(1) (Includes Fermi Function etc. not Rad. Corr.)

RC calculated for (Conserved) Vector Current since it is not renormalized by strong interaction at zero momentum transfer.

Same RC used to define g_A: $[A(g_A)=(1.001)A^{exp}]$

$RC=\alpha/2\pi[\langle g(E_m)\rangle+3\ln(m_Z/m_p)+\ln(m_Z/m_A)+2C+A_{QCD}]$

+ higher order $O(\alpha/\pi)^2$

$g(E_e)$=Universal *Sirlin Function* from Vector Current

A. Sirlin, PRD 164, 1767 (1967).

$\alpha/2\pi$ <g(E_m=1.292579MeV)>=0.015056 long distance loops and brem. averaged over the decay spectrum. Independent of Strong Int. up to O(E_e/m_P)
g(E_e) also applies to Nuclei A. Sirlin (1967) Uncertainty < 10^{-5}

$3\alpha/2\pi \ln(m_Z/m_p)$ short-distance (Vector) log ***not*** renormalized by strong int.
[$\alpha/2\pi$[$\ln(m_Z/m_A)$+2C+A_{QCD}] Induced by axial-current loop
Includes hadronic uncertainty
m_A=1.2GeV long/short distance matching scale (factor 2 m_A unc.)
C=0.8$g_A(\mu_N+\mu_P)$=0.891 (long distance γW Box diagram) WJM&A.Sirlin(1986)
A_{QCD}= $-\alpha_s/\pi(\ln(m_Z/m_A)$+cons)=-0.34 QCD Correction

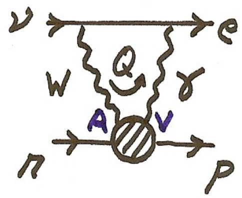
$[\alpha/\pi \ln(m_Z/m)]^n$ leading logs summed via renormalization group, **(+0.0016)**
Next to leading short distance logs ~ **-0.0001,**
and ***$-\alpha^2\ln(m_p/m_e)$=-0.00043*** estimated (for neutron decay)
Czarnecki, WJM, Sirlin (2004) 1+RC=1.0390(8) main unc. from m_A
matching short and long distance γW (VA) Box. Unc*. **±8x10^{-4}**
vs future (±0.1sec goal) τ_n **±1.1x10^{-4} goal.**

*** Note, unc. cancels in neutron vs nuclear beta decays eg V_{ud}**

γW Box Diagram

Weak Axial-Vector Induced Radiative Corrections

AV Loop → V → Superallowed β-decays

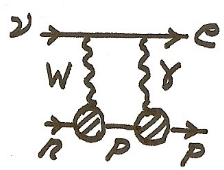


$\longrightarrow \underbrace{\bar{u}\gamma_\mu d}_{\text{Vector}}\,\bar{e}_L\gamma^\mu\nu_L + QCD$

$$RC = \frac{\alpha}{4\pi}\int_0^\infty dQ^2\,\frac{m_W^2}{Q^2+m_W^2}\,F(Q^2)$$

Large Q^2 $\quad F(Q^2) = \frac{1}{Q^2}\left[1 - \frac{\alpha_s(Q^2)}{\pi} + \cdots\right] + O\left(\frac{1}{Q^4}\right)$

Small Q^2 → Nucleon Form Factors



$\rightarrow \frac{\alpha}{2\pi}\left(0.8\,g_A(\mu_p+\mu_n)\right)$

$$\frac{\alpha}{2\pi}\left\{\ln\frac{m_Z}{m_A} + \underbrace{A_g}_{\text{QCD}} + \underbrace{2C}_{\text{Long Distance}}\right\}$$

m_A = matching

<u>2006 Improvement WJM & A. Sirlin</u>

1.) Use large N_{QCD} Interpolator to connect long-short distances

2.) Relate neutron beta decay to Bjorken Sum Rule (N_F=3)

$1-\alpha_s/\pi \rightarrow 1-\alpha_s(Q^2)/\pi-3.583(\alpha_s(Q^2)/\pi)^2-20.212(\alpha_s(Q^2)/\pi)^3$

$-175.7\,(\alpha_s(Q^2)/\pi)^4$ (Baikov,Chetyrkin and Kuhn)

Negligible Effect

The extra QCD corrections lead to a matching between short and long distance corrections at about $Q^2=(0.8\text{GeV})^2$

Very little change in size of RC, but uncertainties reduced by a factor of 2 (perhaps 3)!

<u>(Both Prescriptions Agree)</u>

1+RC= 1.0390(8)→<u>1.03886(39</u>) for Neutron Beta Decay

Reduction by $1.4\text{x}10^{-4}$ (Same for $0^+ \rightarrow 0^+$ beta decays)

Unc. Reduced to ±$3.9\text{x}10^{-4}$ (about 3x τ_n goal).

RC Error Budget

1) Neglected Two Loop Effects: **±0.0001** conservative

2) Long Distance $\alpha/\pi C \sim \alpha/\pi\ (0.75 g_A(\mu_N+\mu_P)) = 0.0020$

Assumed Uncertainty ±10%→**±0.0002** reasonable?

3) Long-Short Distance Loop Matching: 0.8GeV<Q<1.5GeV

±*100*% → **±0.0003** conservative

Total RC Error **±0.00038** →ΔV_{ud}=**±0.00019**

More Aggressive Analysis →ΔV_{ud}=±0.00013

(1/2 conservative)

➔ only about 2xτ_n goal of ±0.1sec. (well matched)

Superallowed ($0^+ \rightarrow 0^+$) Beta Decays & V_{ud}

RC same as in Neutron Decay but with <$g(E_m)$> averaged Nuclear decay spectrum, C modified by Nucleon-Nucleon Interactions and $+Z\alpha^2 \ln(m_p/m_e)$ corrections (opposite sign from neutron)

$$ft = |V_{ud}|^2 (2984.5\text{s})(1+\text{RC})(1+\text{NP corr.})$$

Nuclear Physics (NP) isospin breaking effects (Hardy & Towner Calculations)

ft values + RC for 13 precisely measured nuclei found to be consistent with CVC: Average $\rightarrow V_{ud}$

Superallowed Nuclear Beta Decays

RC Uncertainty-Same as Neutron Decay

Nuclear Unc. - Significantly Reduced (2006-08)

Nuclear Coulomb Corrections Improved

$$|V_{ud}| = \underline{0.97420}(10)_{Nuc}(18)_{RC}$$

(2016 Hardy and Towner Update)

Neutron Decay Master Relations

1) $|V_{ud}|^2 = \dfrac{4908.6(1.9)\text{sec}}{\tau_n(1+3g_A^2)}$ Unc. Radiative Corrections Same as in Nuclear β Decay

2) $\tau_n = \dfrac{4908.6(1.9)\text{sec}}{|V_{ud}|^2\,(1+3g_A^2)}$ Radiative Corrections Cancel!

3) $(1+3g_A^2) = \dfrac{4908.7(1.9)\text{sec}}{|V_{ud}|^2\,\tau_n}$ Radiative Corrections Cancel!

Current $\Delta|V_{ud}|^2/|V_{ud}|^2 = \pm 0.02\%$ (NP) $\pm 0.04\%$ (RC) Superallowed β

$\Delta\tau_n/\tau_n = \pm 0.07\%$ $\tau_n^{ave} = 879.4(6)$sec.

$\Delta g_A^2/g_A^2 \approx \pm 0.17\%$ *Recent $g_A = 1.2748(13)$ Perkeo II*

$= 1.2772(20)$ UCNA

$= 1.2755(11)$ Ave

±0.01-0.02% Outstanding/Appropriate Goals for τ_n and g_A^2

Neutron Decay ($n \rightarrow pe\nu$) & V_{ud}

$$\tau_n(1+3g_A^2) = \frac{4908.7(1.9)\text{sec}}{|V_{ud}|^2} \quad \textbf{Master Relation}$$

$|V_{ud}|=0.97420(10)(18)$

$$\tau_n(1+3g_A^2) = \underline{5172.0(1.1)\text{sec}}$$

$\tau_n = 879.4(6)\text{s} \rightarrow \quad g_A=1.2756(5)$

$\tau_n = 888.0(2.0)\text{s} \rightarrow \quad g_A=1.2681(17)$

post 2002 PERKEOII + UCNA $g_A=1.2755(11)$

implication either beam lifetime is wrong or post 2002 asymmetry measurements of g_A wrong

Fornal-Grinstein Solution
BR(n➔dark particles) ~ 1%

BR(n➔pev)=0.9999(7) + 1.30(g_A-1.2755)

Total exotic n decay BR < 0.27% at 95%CL

For Fornal-Grinstein Solution to be valid, post 2002 g_A asymmetry values from PERKEOII and UCNA must be wrong (***significant tension***)