# strange electromagnetic form factors of the nucleon from lattice QCD

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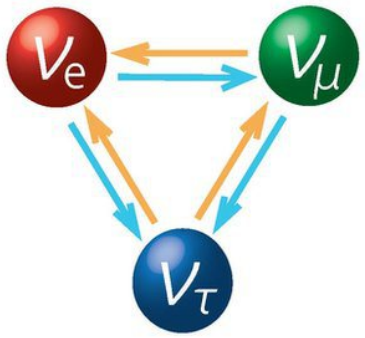
# table of contents

# experimental influence of the form factors

- measurements of the weak mixing angle $\Theta_W$
- neutrino scattering: $\nu N$
  - neutrino oscillation
  - mass hierarchy




- measurements of CKM matrix elements
  - CP-violation in the quark sector of the SM

# expectation values

- Monte Carlo Methods

$$\langle O \rangle \approx \frac{1}{N_{\text{cfg}}} \sum_{i=1}^{N_{\text{cfg}}} O[D[U_i]^{-1}, U_i]$$

- generation of ensembles

$$dP(U) = \frac{1}{Z} e^{-S_{QCD}^{Lat.}[U]} \mathcal{D}[U]$$

- **C**oordinated **L**attice **S**imulations (CLS)
  - $N_f = 2 + 1$ O(a)-improved Wilson fermions
  - open boundary conditions
  - $a \in \{0.049, 0.064, 0.076, 0.086\}$ fm, $m_\pi \in [200, 360]$ MeV

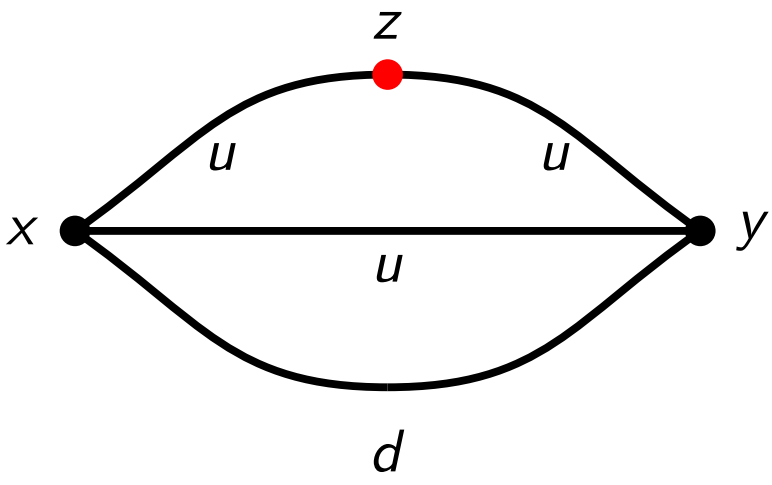
# three-point functions

$$\langle O \rangle = \left\langle N(\vec{y}, y_0) J^q_\mu(\vec{z}, z_0) \bar{N}(\vec{x}, x_0) \right\rangle$$

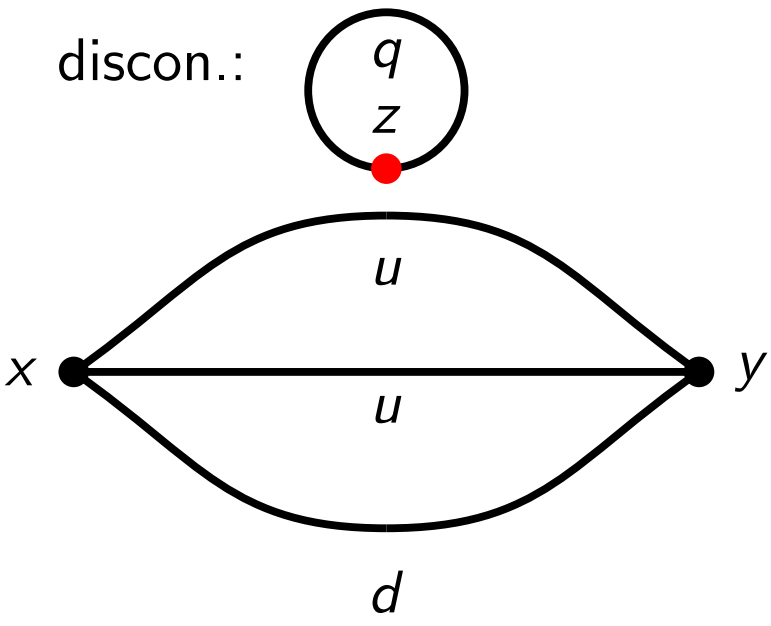
- current operator

$$J^q_\mu(\vec{z}, z_0) = \begin{cases} V^q_\mu(\vec{z}, z_0) = \bar{q}(\vec{z}, z_0)\gamma_\mu q(\vec{z}, z_0) \\ A^q_\mu(\vec{z}, z_0) = \bar{q}(\vec{z}, z_0)\gamma_5\gamma_\mu q(\vec{z}, z_0) \end{cases}$$

connected:




discon.:



# matrix elements

- parameterized by form factors

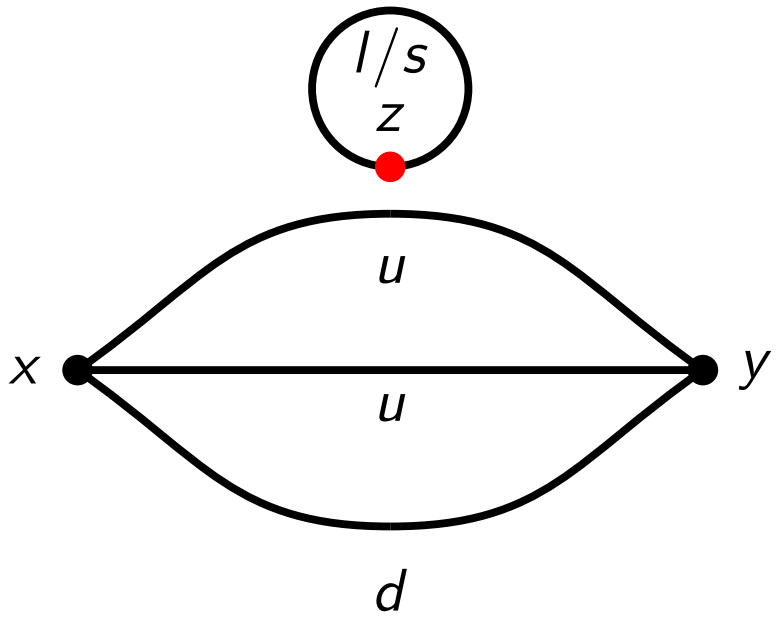
$$\left\langle N, \vec{k}, s \left| V_\mu(x) \right| N, \vec{k}', s' \right\rangle = \bar{u}^s(\vec{k}) \left( \gamma_\mu F_1(Q^2) + \mathrm{i}\sigma_{\mu\nu} \frac{q^\nu}{2m} F_2(Q^2) \right) u^{s'}(\vec{k}') e^{\mathrm{i}q\cdot x}$$

$$\left\langle N, \vec{k}, s \left| A_\mu(x) \right| N, \vec{k}', s' \right\rangle = \bar{u}^s(\vec{k}) \left( \gamma_\mu \gamma_5 G_\mathrm{A}(Q^2) + \gamma_5 \frac{q_\mu}{2m} G_\mathrm{P}(Q^2) \right) u^{s'}(\vec{k}') e^{\mathrm{i}q\cdot x}$$

$$G_\mathrm{E}(Q^2) = F_1(Q^2) + \frac{Q^2}{4m^2} F_2(Q^2)$$
$$G_\mathrm{M}(Q^2) = F_1(Q^2) + F_2(Q^2)$$

# isolated disconnected contributions

- light/strange quark




$$\Rightarrow \begin{cases} G_{\mathrm{E}}^{l/s}(Q^2), G_{\mathrm{M}}^{l/s}(Q^2) \\ G_{\mathrm{A}}^{l/s}(Q^2), G_{\mathrm{P}}^{l/s}(Q^2) \end{cases}$$

- quark loop $\Rightarrow$ most challenging part

# quark loop

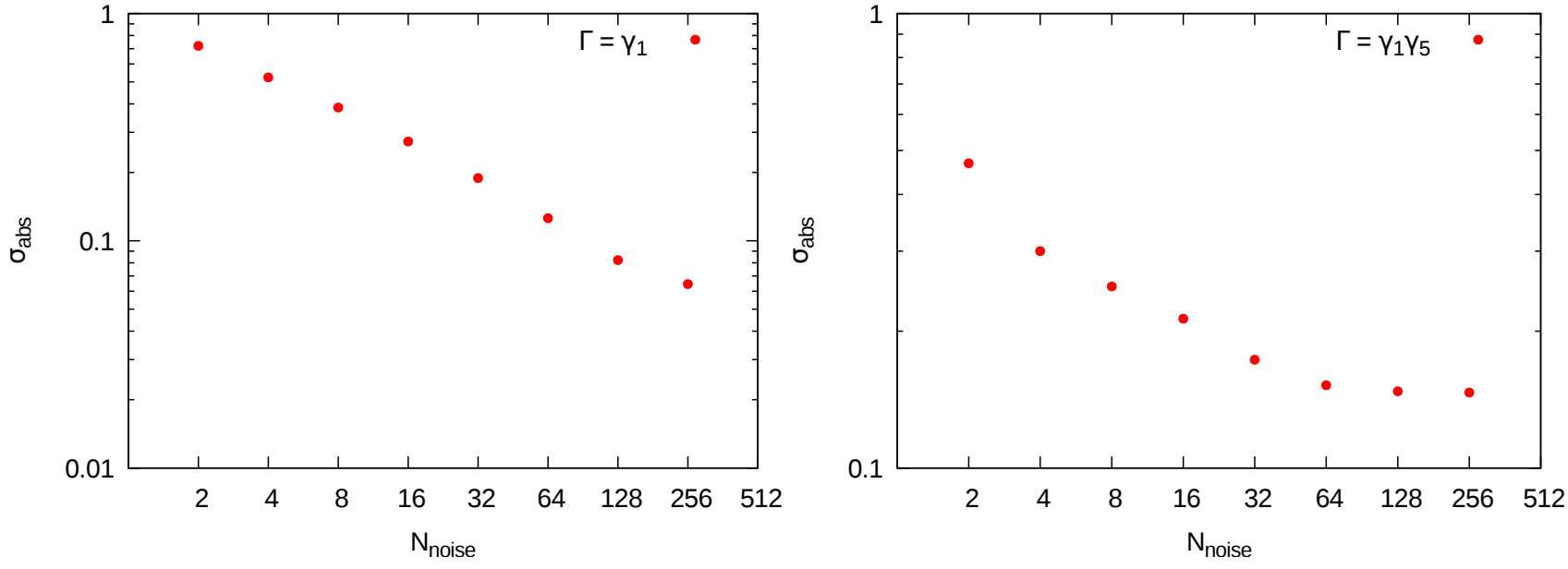
$$L_\Gamma^{l/s}(\vec{q}, z_0) = -\sum_{\vec{z}\in\Lambda} e^{i\vec{q}\cdot\vec{z}}\ \mathrm{tr}\left[S^{l/s}(\vec{z}, z_0; \vec{z}, z_0)\ \Gamma\right] \quad ,$$

- estimated stochastically with noise vectors
  1. $\left\langle \eta^{(i)}(x)^a_\alpha \right\rangle_\eta = 0$
  2. $\left\langle \eta^{(i)}(x)^a_\alpha \eta^{(i)\dagger}(y)^b_\beta \right\rangle_\eta = \delta(x-y)\delta^{ab}\delta_{\alpha\beta}$

$$\mathrm{tr}\left[S^{l/s}(z,z)\ \Gamma\right] = \left\langle \eta^{(i)\dagger}(z)\cdot\Gamma s^{(i)}(z)\right\rangle_\eta \quad , \quad D(y,x)s^{(i)}(x) = \eta^{(i)}(y)$$

- stochastic noise **and** gauge noise!

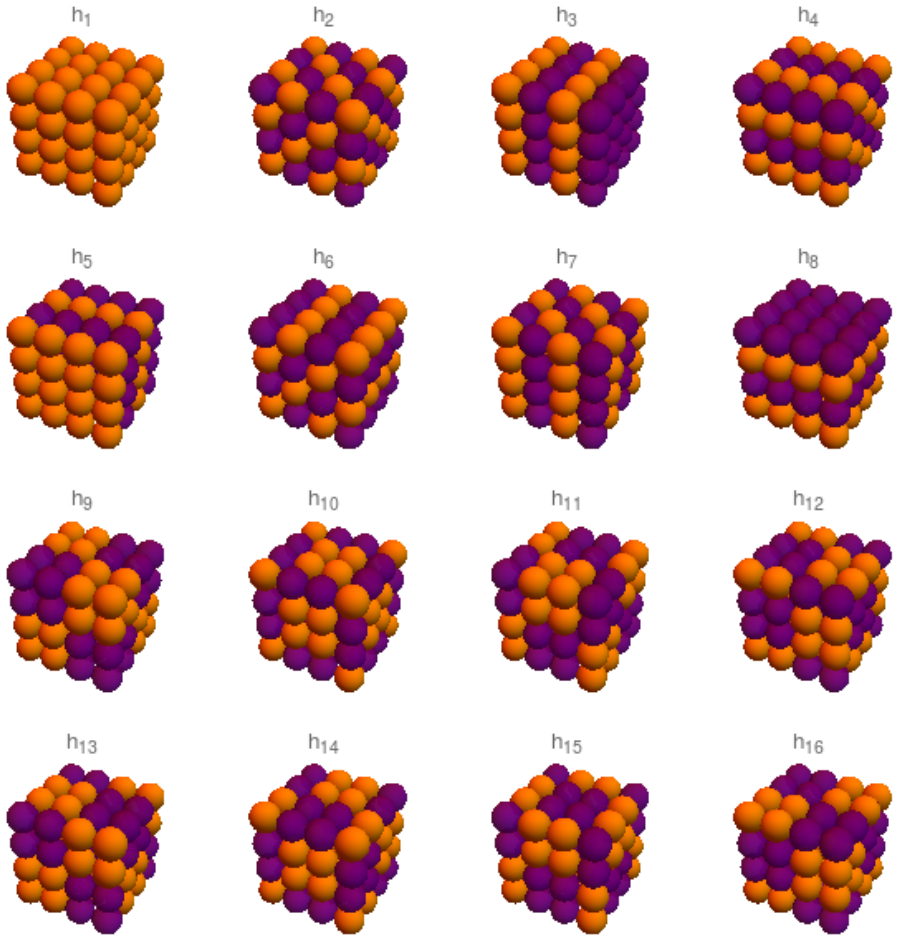
# stochastic noise vs. gauge noise




- "hit" gauge noise with axial vector loops
- no saturation for vector loops!

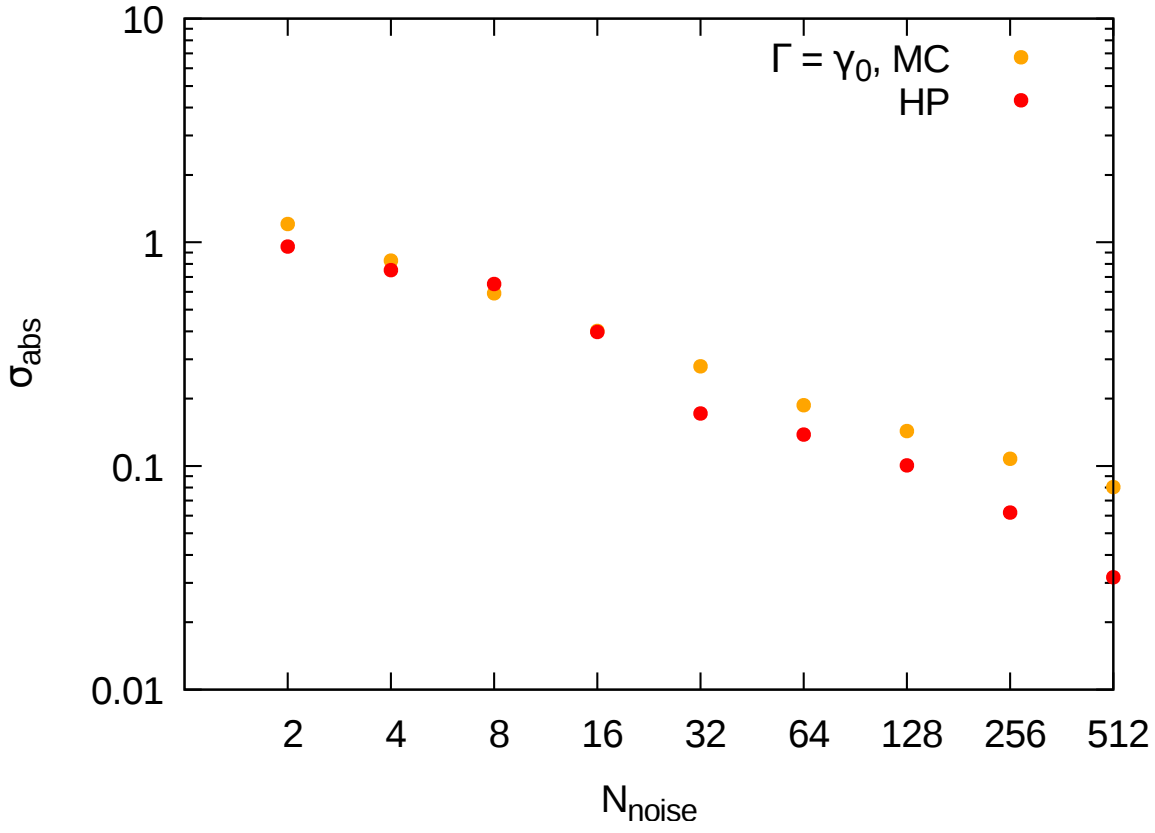
# hierarchical probing

$\eta_n \rightarrow h_n \odot \eta$ [Stathopoulos et al., arXiv:1302.4018v1]

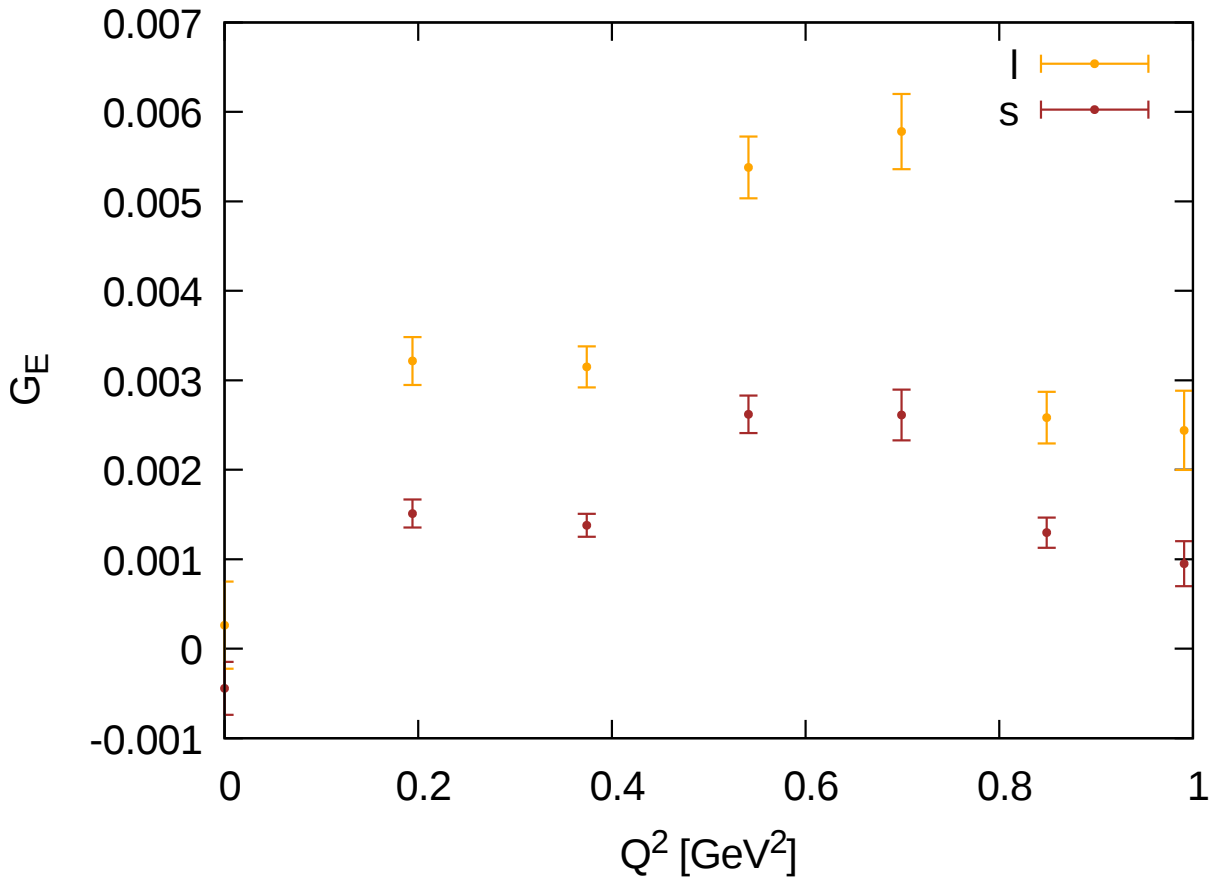


# vector loop

- color complete points

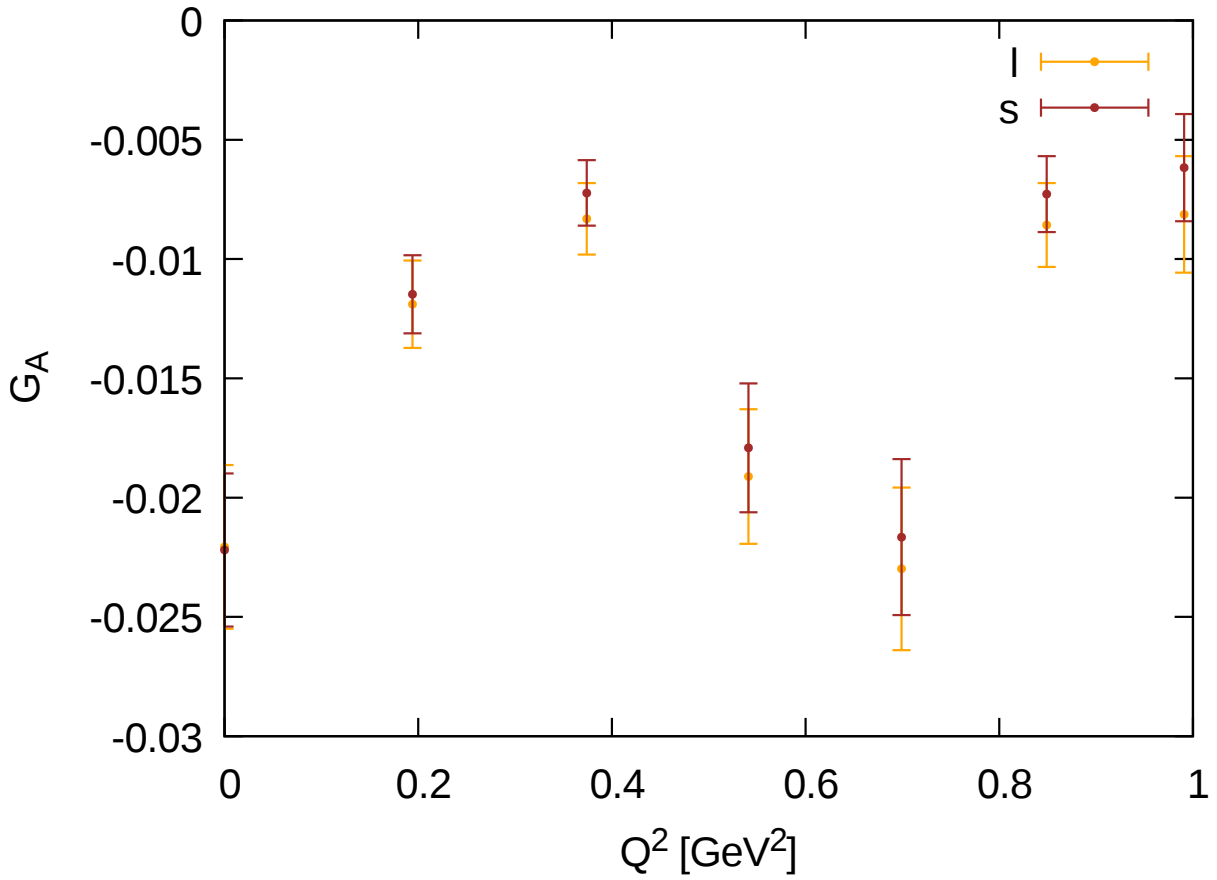


# $G_E^{l/s}$ on H105 ($a = 0.086$fm, $m_\pi = 280$MeV)



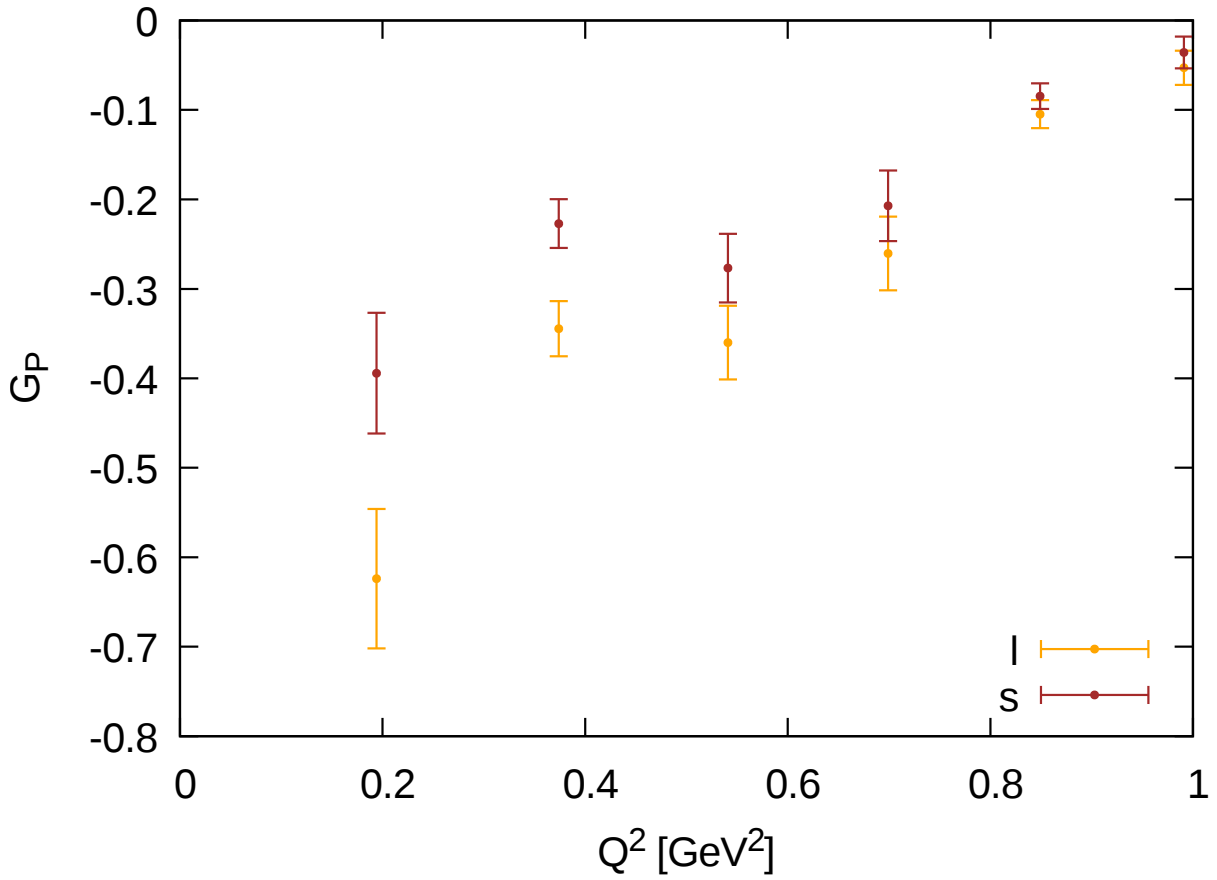

$G_M$ & more ensembles in progress...

# $G_A^{l/s}$ on H105 ($a = 0.086$fm, $m_\pi = 280$MeV)




renormalization & more ensembles in progress...

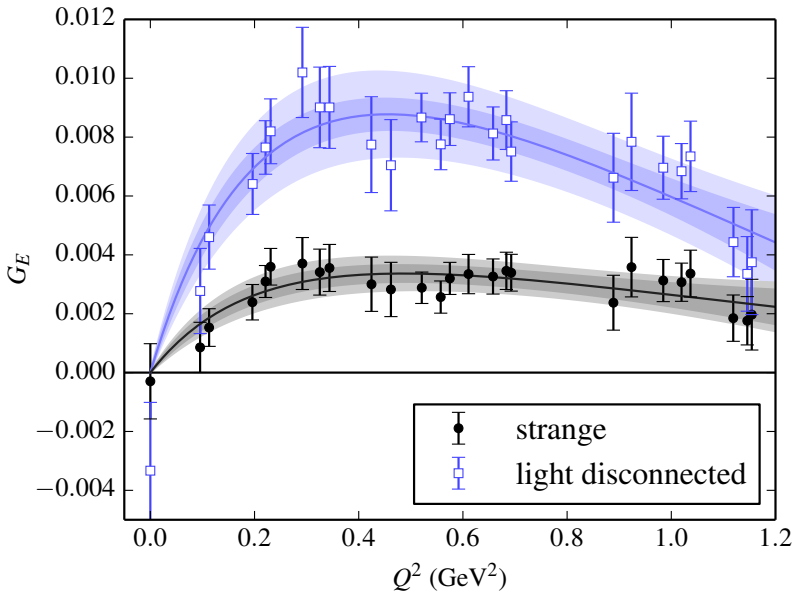
# $G_P^{l/s}$ on H105 ($a = 0.086$fm, $m_\pi = 280$MeV)

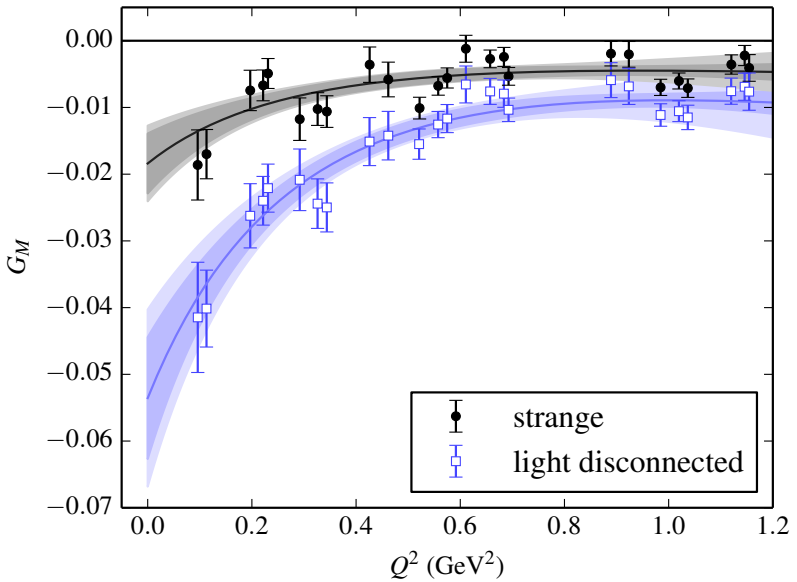



renormalization & more ensembles in progress...

# LHPC

- *High-precision calculation of the strange nucleon electromagnetic form factors*
  Green et al., Phys. Rev. D 92, 031501 (2015)
  - time-dilution → 3D hierarchical probing
  - periodic boundary conditions
  - $a \approx 0.114$fm, $m_\pi \approx 317$MeV



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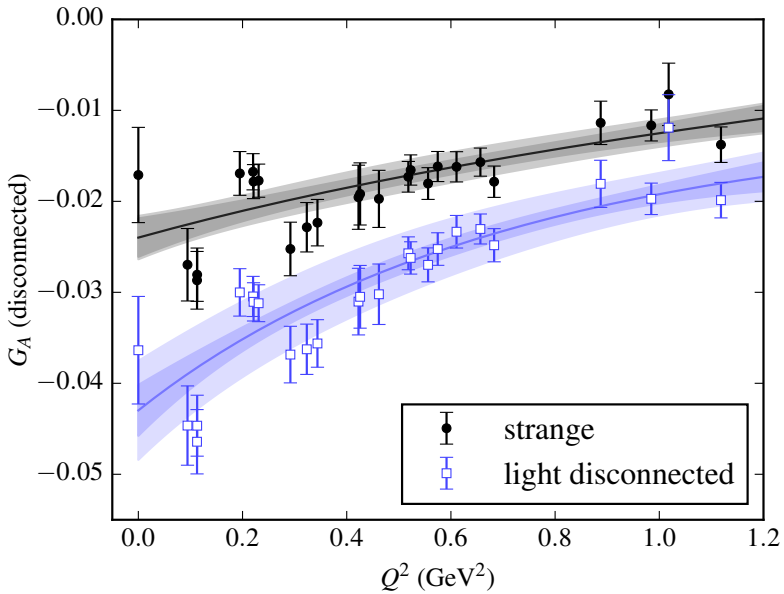


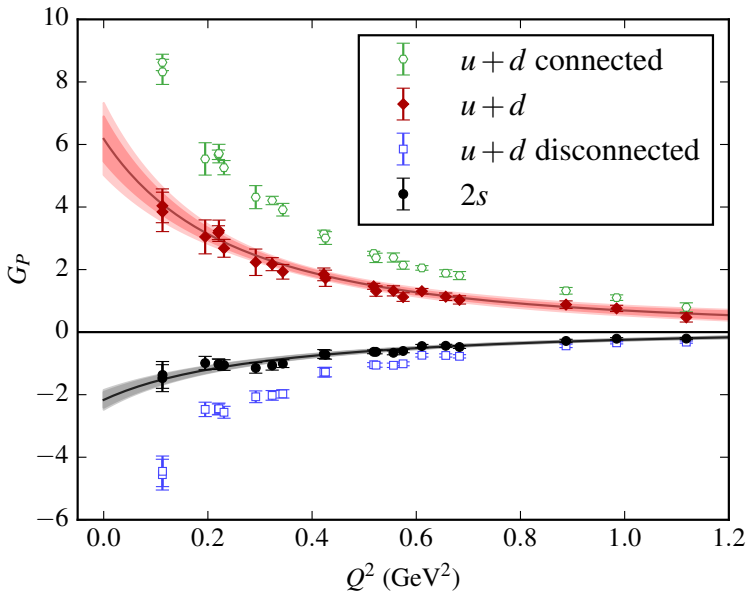

# $\chi$QCD

- *Sea quarks contribution to the nucleon magnetic moment and charge radius at the physical point*
  Sufian et al., Phys. Rev. D 96, 114504 (2017)
  - valence overlap fermions & domain-wall fermion gauge configurations
  - four ensembles: $a \in [0.08, 0.15]$fm, $m_\pi \in [0.135, 0.403]$MeV
  - time-dilution & low-mode averaging (deflation)

$$S(x,y) = S_{low}(x,y) + S_\perp(x,y) \ , \ P_\perp(x,y) = 1 - \sum_k^{N_{ev}} v_k(x) \otimes v_k(y)^\dagger$$

$$S_{low}(x,y) = \frac{1}{V} \sum_k^{N_{ev}} \frac{v_k(x) \otimes v_k(y)^\dagger}{\lambda_k}$$

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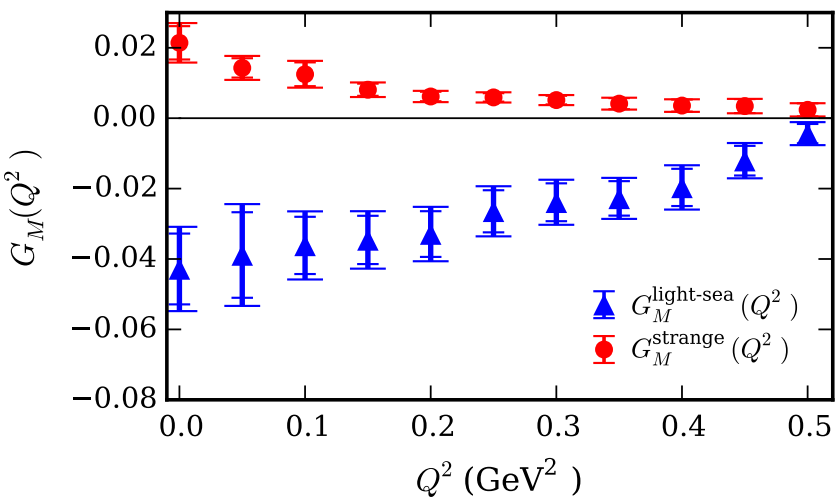
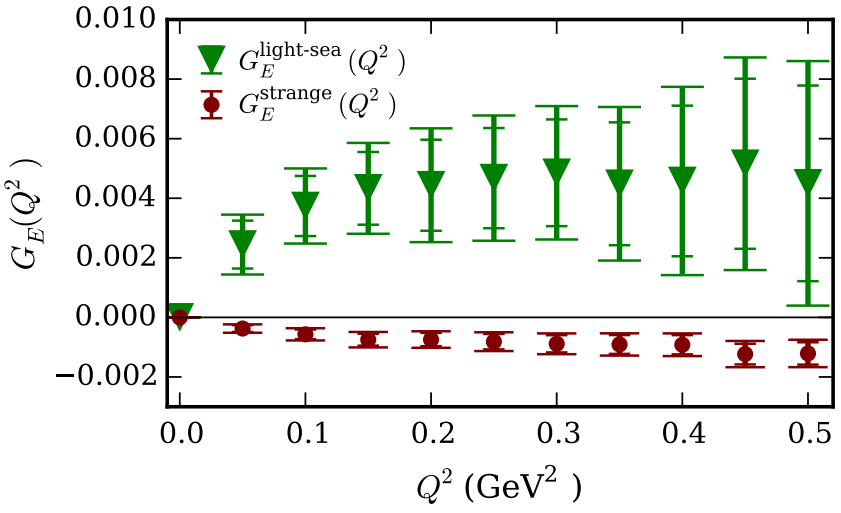


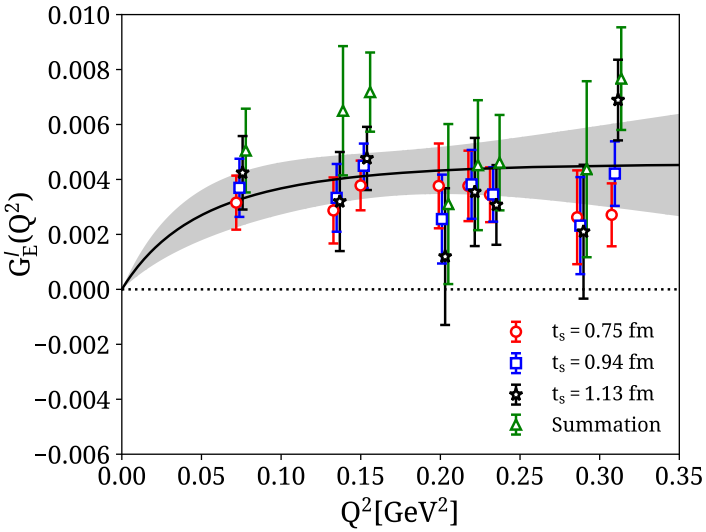
# ETMC

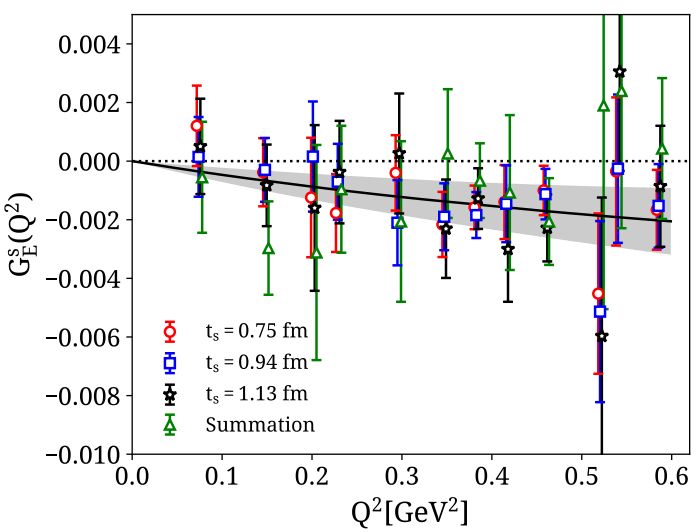
- *Strange nucleon electromagnetic form factors from lattice QCD*
  Alexandrou et al., arXiv:1801.09581 (2018)
  - $N_f = 2$ twisted mass fermions with O(a)-improvement
  - Osterwalder-Seiler strange quarks
  - $a \approx 0.0938$fm, physical pion mass
  - light: deflation, strange: truncated solver method

$$S^l(x,y) = S^l_{low}(x,y) + S^l_{\perp}(x,y)$$

$$\mathrm{tr}\left[S^s(z,z)\,\Gamma\right] = \underbrace{\left\langle \eta^{(i)\dagger}(z)\cdot\Gamma s^{(i)}_{LP}(z)\right\rangle_{\eta}}_{\text{biased estimate}} + \underbrace{\left\langle \eta'^{(i)\dagger}(z)\cdot\Gamma\left(s'^{(i)}_{HP}(z) - s'^{(i)}_{LP}(z)\right)\right\rangle_{\eta'}}_{\text{correction}}$$

# ETMC


$G_E^l(Q^2)$
$Q^2[GeV^2]$
$t_s$ = 0.75 fm
$t_s$ = 0.94 fm
$t_s$ = 1.13 fm
Summation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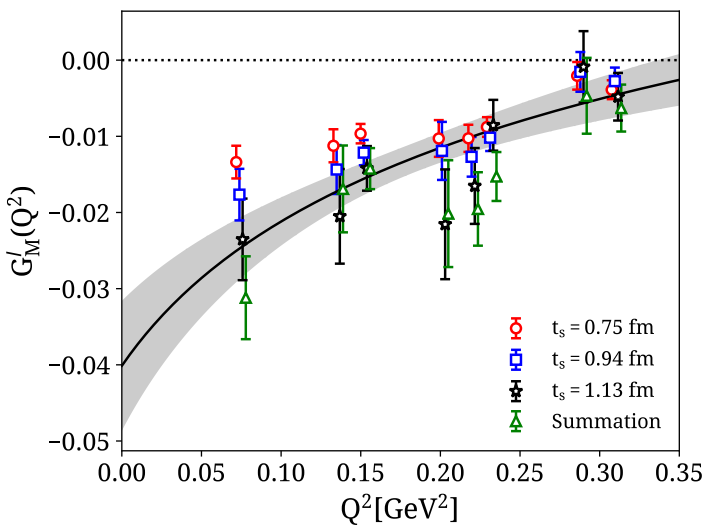
$G_M^l(Q^2)$
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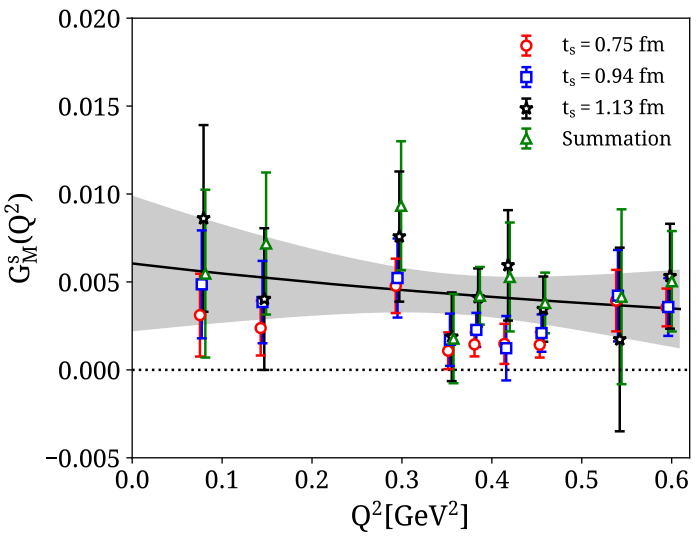
$t_s$ = 1.13 fm
Summation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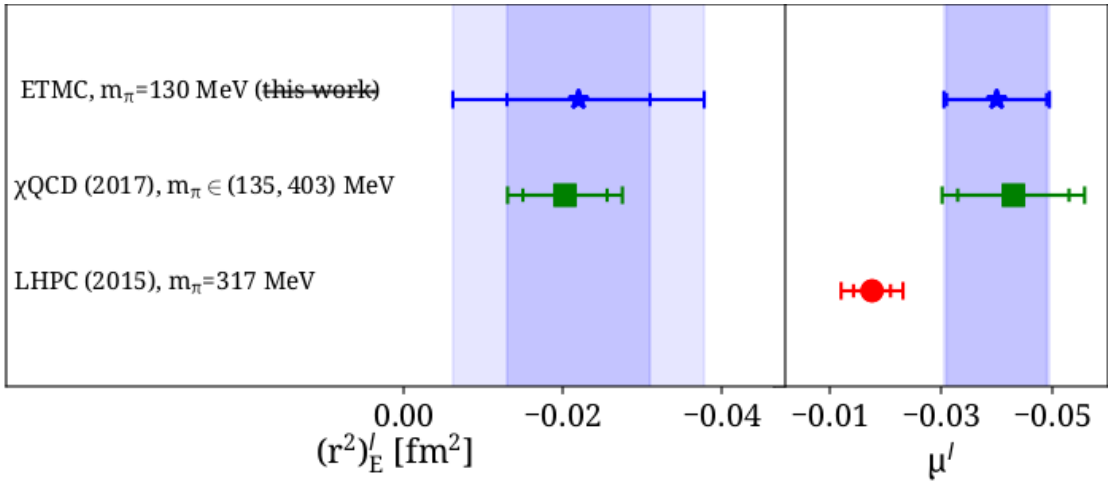
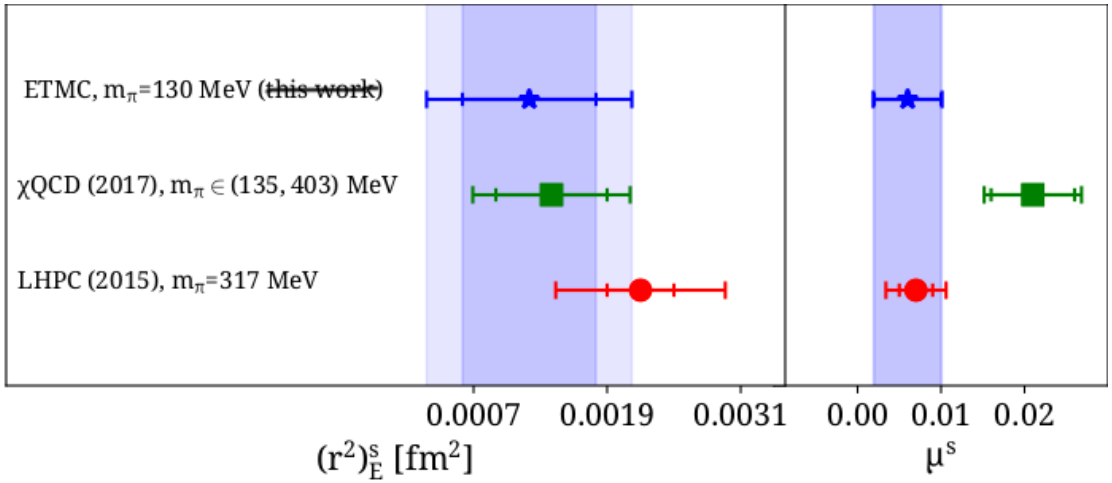
Summation

# charge radius & magnetic moment



Thank you!