Hadronic contribution to the running of $\sin^2\theta_W$ from lattice QCD

Konstantin Ottnad

Institut für Kernphysik, Johannes Gutenberg-Universität Mainz

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Outline

1. Theory and lattice setup
2. Quark connected contribution
3. Quark disconnected contribution
4. Error budget on full hadronic contribution

Disclaimer: Results are still very preliminary!

Lattice calculation of the running of $\sin^2\theta_W$

Master formula for the hadronic contribution to the running of $\sin^2\theta_W$

$$\Delta^{\gamma Z}_{\rm had}\sin^2\theta_W(Q^2) = -\frac{e^2}{\sin^2\theta_W}\int_0^\infty dt\, C^{\gamma Z}_{\rm 2pt}(t)\underbrace{\left[t^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qt}{2}\right)\right]}_{\equiv K(t,Q)}, \qquad Q = Q_0\,. \qquad (1)$$

F. Jegerlehner, Nuovo Cim. 034C, 31 (2001)
F. Jegerlehner, Z. Phys. C32 (1986)

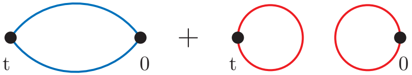
The 2pt-function $C^{\gamma Z}_{\rm 2pt}(t) = -\frac{1}{3}\sum_k\sum_{\vec{x}}\langle 0|\, J^\gamma_k(x) J^Z_k(0)\,|0\rangle$ is defined from vector currents:

- The el-mag. current $J^\gamma_k(x) = \frac{2}{3}\bar{u}(x)\gamma_k u(x) - \frac{1}{3}\bar{d}(x)\gamma_k d(x) - \frac{1}{3}\bar{s}(x)\gamma_k s(x) + \frac{2}{3}\bar{c}(x)\gamma_k s(c)$
- The vector coupling of the Z to quark fields $J^Z_\mu(x) = J^3_\mu(x) - \sin^2\theta_W J^\gamma_\mu(x)$,

  where $J^3_\mu(x) = \frac{1}{4}\bar{u}\gamma_\mu u(x) - \frac{1}{4}\bar{d}\gamma_\mu d(x) - \frac{1}{4}\bar{s}\gamma_\mu s(x) + \frac{1}{4}\bar{c}(x)\gamma_k s(c)$.

The kernel function $K(t,Q)$ can be evaluated analytically at all t, Q.

$C^{\gamma Z}_{\rm 2pt}(t) = C^{\gamma Z}_{\rm con}(t) + C^{\gamma Z}_{\rm disc}(t)$ involves quark-line connected and disconnected diagrams



which can be calculated non-perturbatively on the lattice:

- Can compute $C^{\gamma Z}_{\rm con}(t)$ to high statistical precision
- Disconnected diagrams are **noisy and very expensive**
- $C^{\gamma Z}_{\rm con}(t)$ can be integrated over the entire lattice volume; $C^{\gamma Z}_{\rm disc}(t)$ requires a cutoff $t_{\rm cut}$.

Strategy to estimate disconnected contribution:

- Integrate up to some cutoff $t_{\rm cut}$
- Obtain upper bound on remainder using theory input ...

$\Rightarrow$ **Statistical and systematic error can be "tuned" by appropriate choice of $t_{\rm cut}$.**

Asymptotic behavior of $C^{\gamma Z}_{\mathrm{disc}}(t)$

The isovector (isoscalar) channel opens at $2M_\pi$ ($3M\pi$), leading to the expectation

$$C^{\gamma Z}(t) \to \left(\frac{1}{2} - \sin^2\theta_W\right) C^{\rho\rho}(t), \quad \text{for } t \to \infty, \tag{2}$$

where $C^{\rho\rho}(t)$ is the (purely connected) isovector part. Neglecting charm quarks, we have:

$$\frac{C^{\gamma Z}_{\mathrm{disc}}(t)}{C^{\rho\rho}(t)} = \frac{C^{\gamma Z}(t) - \left(\frac{1}{2} - \sin^2\theta_W\right) C^{\rho\rho}(t)}{C^{\rho\rho}(t)} + \frac{1}{9}\sin^2\theta_W - \left(\frac{1}{6} - \frac{2}{9}\sin^2\theta_W\right)\frac{C^{s}_{\mathrm{con}}(t)}{C^{l}_{\mathrm{con}}(t)}, \tag{3}$$

where $C^{f}_{\mathrm{con}}(t)$ denote **connected** correlation functions of individual quark flavors. Now:

- The first term vanishes for large t due to Eq. (2)
- The last term $\sim C^{s}_{\mathrm{con}}(t)/C^{l}_{\mathrm{con}}(t)$ is exponentially suppressed.

$$\text{Therefore,} \quad C^{\gamma Z}_{\mathrm{disc}}(t) \to \frac{\sin^2\theta_W}{9} C^{\rho\rho}(t), \quad \text{for } t \to \infty. \tag{4}$$

$\Rightarrow$ **Estimate remainder of $\Delta^{\gamma Z}_{\mathrm{had,disc}}\sin^2\theta_W(Q^2)$ by replacing $C^{\gamma Z}_{\mathrm{disc}}(t) \to \frac{\sin^2\theta_W}{9} C^{\rho\rho}(t)$.**

Lattice Setup

- Our lattice simulations use $2+1$ **dynamical flavors** of (Wilson, Clover-improved) sea-quarks.

  $\rightarrow$ Degenerate light quarks; valence charm quarks

- Simulations are performed at **finite lattice spacing** a.

  $\rightarrow$ Need different values of $a = 0.050 \ldots 0.086\,\mathrm{fm}$ for continuum extrapolation

- Simulations mostly employ **unphysical quark masses** ($M_\pi = 200 \ldots 340\,\mathrm{MeV}$).

  $\rightarrow$ Need to perform chiral extrapolation.

  $\rightarrow$ Ensemble with physical quarks in production, no disconnected diagrams yet.

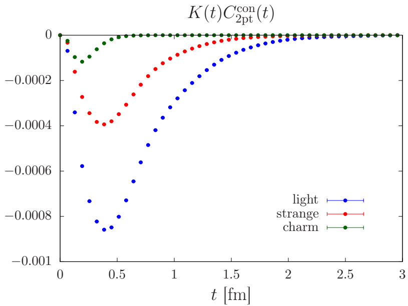
- Quark connected diagrams receive contributions from u, d, s and c quarks.

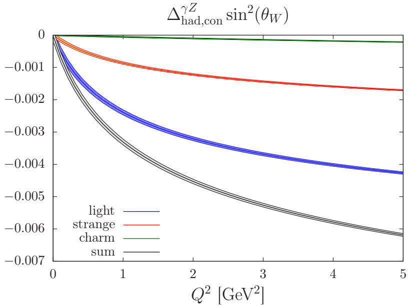
- For quark disconnected diagrams we neglect the charm contribution.

- Results shown in this talk are for a single, "average" ensemble:

$$M_\pi = 280\,\mathrm{MeV},\ a = 0.064\,\mathrm{fm},\ T \cdot L^3 = 128 \cdot 48^3.$$

Connected contribution





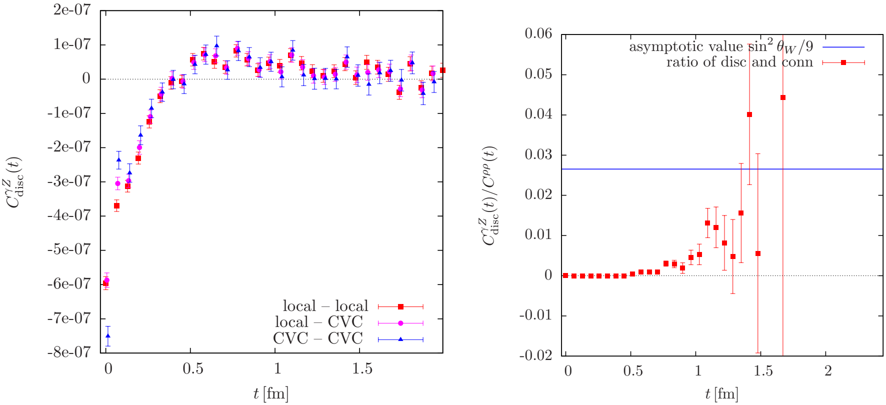
Connected data and plots courtesy of A. Gerardin

- Integrand peaked at small distances.
- Individual connected contributions to $\Delta_{\mathrm{had}}^{\gamma Z}\sin^2\theta_W(Q^2 = 4\,\mathrm{GeV}^2)$:

light	strange	charm
-0.004001(44) (69.4%)	-0.001581(20) (27.4%)	-0.000178(13) (3.1%)

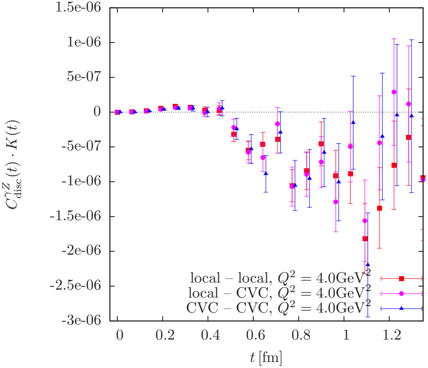
- Final error dominated by scale setting (included in plot)
- Overall precision on $\Delta_{\mathrm{had,con}}^{\gamma Z}\sin^2\theta_W(Q^2 = 4\,\mathrm{GeV}^2) = -0.005760(53)$ **better than 1%**!

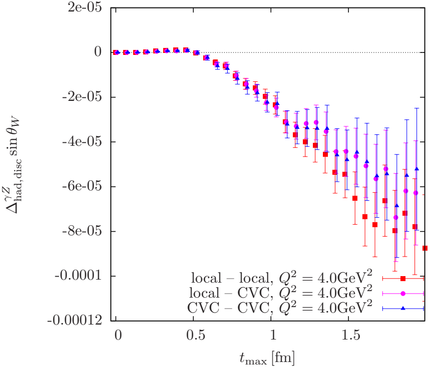
Disconnected contribution



- Two-point function crosses zero as expected → integrand flips sign.
- Agreement between correlators built from local and point-split (CVC) operators.
- Signal seems to approach asymptotic value for $t = 1\ldots2\,\mathrm{fm}$.

Disconnected contribution

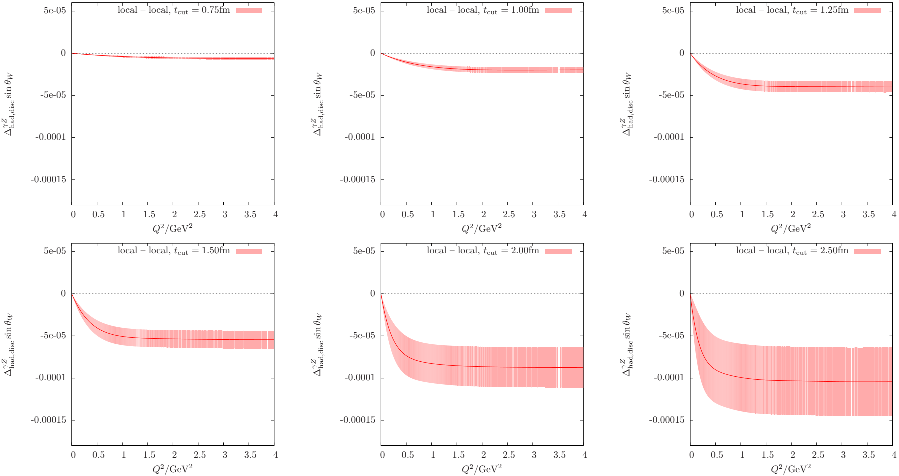




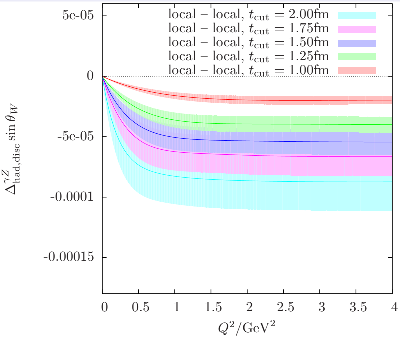
- For small distances the integrand is positive
- At distances $t \gtrsim 1.5\,\mathrm{fm}$ the signal of the two-point function is lost
- However, the noise is correlated...

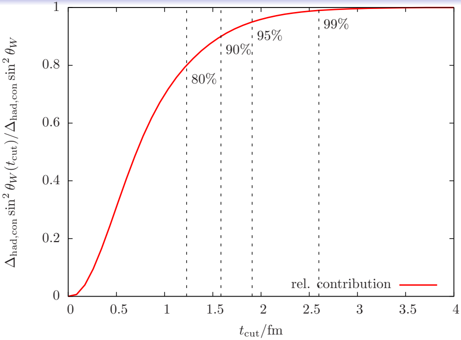
$\Rightarrow$ **Can integrate much further without losing the signal (even up to $t \approx 2.5\,\mathrm{fm}$)**

Q^2-dependence for different value of t_{cut}



- Disconnected contribution seems to saturate between for $t_{\mathrm{cut}} \gtrsim 2\,\mathrm{fm}$.





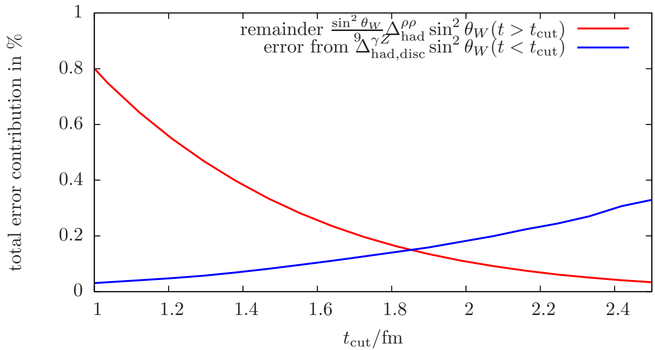
- Disconnected part $\gtrsim 0.0001$ contributes $\sim 2\%$ to overall signal at $Q^2 = 4\,\mathrm{GeV}^2$.
- Connected contribution becomes negligible ($< 0.1\%$) for $t_{\rm cut} > 3\,\mathrm{fm}$.
- Upper bound for remainder of disc contribution is suppressed by $\frac{\sin^2\theta_W}{9} \approx 0.027$:

$$\Delta^{\gamma Z}_{\rm had,disc} \sin^2\theta_W(t > t_{\rm cut}) \leq \frac{\sin^2\theta_W}{9} \Delta^{\rho\rho}_{\rm had} \sin^2\theta_W(t > t_{\rm cut}).$$

$\Rightarrow$ Disc contribution becomes negligible for $t_{\rm cut} > 2\,\mathrm{fm}$.

How to choose an "optimal" $t_{\rm cut}$ value?

Error budget



- "Sweet spot" around $t_{\text{cut}} \approx 2\,\text{fm}$.
- Total error from disconnected part is of $\mathcal{O}(0.2\%)$.
- However, taking $\frac{\sin^2\theta_W}{9}\Delta^{\rho\rho}_{\text{had}}\sin^2\theta_W$ as an error may be too conservative...

$\Rightarrow$ **Total error on given ensemble is $\sim 1\%$ (dominated by connected part!)**

However, this estimate has to be taken with a grain of salt:

- Still need to perform chiral and continuum extrapolation.
- Calculations at smaller values of M_π, a are more expensive.
- If the ratio of conn. vs. disc. changes, also the error estimate will be affected.
- Conn. and disc. data are correlated; this might change the final error.

Possible strategy to further improve the error:

- Split up the full two-point function:

$$C_{2\mathrm{pt}}^{\gamma Z}(t) = \frac{1}{4} C^{\mathrm{rest}}(t) + \left(\frac{9}{20} - \sin^2\theta_W\right) C^{\gamma\gamma}(t),$$

$$C^{\mathrm{rest}}(t) = \frac{2}{15}\left(C_{\mathrm{con}}^{s} - C_{\mathrm{con}}^{c}\right) - \frac{1}{5} C_{\mathrm{disc}}^{l+\frac{2}{3}s,\,l-s}.$$

- Estimate $C^{\gamma\gamma}(t)$ using e^+e^- data
- Compute $C^{\mathrm{rest}}(t)$ on the lattice (**no** $C_{\mathrm{con}}^{l}(t)$ **contribution!**)

$\rightarrow$ **Need careful error analysis, including correlations.**

Summary and outlook

So far:

- **Full, direct lattice calculation of $\Delta^{\gamma Z}_{\mathrm{had}} \sin^2\theta_W$ possible with error of $\sim 1\%$.**
- Error dominated by the quark-connected part (actually: scale setting)

TODO:

- Investigate hybrid approach using e^+e^- data for $C^{\gamma\gamma}(t)$.
- Add more pion masses and lattice spacings for disconnected part.
- Perform chiral and continuum extrapolation.
- Add the ensemble at the physical point (at least for connected part)