

$\sin^2\theta_W(Q^2)$, Radiative Corrections & “New Physics”

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“The Weak Mixing Angle, $\sin^2\theta_W$, is a very special parameter that confirmed the Standard Model and continues to constrain and probe New Physics beyond it”



Early History of Electroweak Unification & $\sin^2\theta_W$

Algebra of $SU(2)_L \times U(1)_Y$ mixing **Glashow 1961**

$$A_\mu = B_\mu^0 \cos\theta_W + W_\mu^0 \sin\theta_W \quad \text{photon}$$

$$Z_\mu = W_\mu^0 \cos\theta_W - B_\mu^0 \sin\theta_W \quad \text{Z boson}$$

$$\tan\theta_W = g'/g$$

Spontaneous Symmetry Breaking via Higgs Mechanism

Scalar Doublet \rightarrow Masses **Weinberg 1967**

$$m_W = m_Z \cos\theta_W \quad \text{Higgs Boson!}$$

Renormalizable \rightarrow Finite Predictions **'t Hooft 1971**

Experiment takes over

1973 Weak Neutral Currents Discovered! $\nu_\mu e$
Early Atomic Parity Violation Ambiguous
 $SU(2)_L \times U(1)_Y$ in some peril.

1978 DIS polarized eD at SLAC (E122)
Prescott et al. observe γ -Z interference
Confirm $SU(2)_L \times U(1)_Y$
 $\sin^2\theta_W = 0.22(2)$ lend support to GUTS
Should have received a Nobel Prize

Outline

- 1. Introductory Remarks:** $\sin^2\theta_W$ Status vs SM
Z Pole vs low Q^2 & BSM
- 2. EW Radiative Corrections**
(Running $\sin^2\theta_W(Q^2)$) & Parity Violating Asymmetries
Moller(ee) vs P2 (ep) Loop Effects
- 3. Z' Boson Sensitivity** (extra $U(1)$ ' gauge symmetry)
Heavy Z' up to $\sim 8\text{TeV}$
Light Z' with Very Small "Induced" Couplings
(eg Dark Parity Violation DAVOUDIASL, LEE, WJM)

Precision EW Parameters (status 2008 vs 2014):

<u>Quantity</u>	<u>2008 Value</u>	<u>2014 Value</u>	<u>Comment</u>
α^{-1}	137.035999084(51)	137.035999049(90)	$\alpha^{-1}(a_e)$ vs $\alpha^{-1}(\text{Rb})$
G_μ	$1.16637(1) \times 10^{-5} \text{GeV}^{-2}$	$1.1663787(6) \times 10^{-5} \text{GeV}^{-2}$	τ_{μ^+} PSI
m_Z	91.1875(21) GeV	91.1876(21) GeV	-
* m_t	171.4(2.1) GeV →	<u>173.3(0.8) GeV</u>	FNAL/LHC
* m_H	>114 GeV →	125-126 GeV	
m_W	80.410(32) GeV →	<u>80.385(15) GeV</u>	LEP2/FNAL
$\sin^2\theta_W(m_Z)$	0.23070(26)	0.23070(26)	SLAC A_{LR}
$\sin^2\theta_W(m_Z)$	0.23193(29)	0.23193(29)	CERN $A_{FB}(bb)$
	(3 sigma difference?)		
$\sin^2\theta_W(m_Z)_{ave}$	0.23125(16)	0.23125(16)	Z Pole Ave.

A Beautiful Relation

SU(2)_L × U(1)_Y + Higgs Doublet + Renormalizability

- ***$\sin^2 \theta^0_W = 1 - (m^0_W / m^0_Z)^2 = (e^0 / g^0)^2$ Natural Bare Relation***

**Radiative (Loop) Corrections - Finite & Calculable!
Demonstrated by Bollini, Giambiagi & Sirlin (1972)**

WJM(1974) Thesis: Finite Parts Calculated

but model incomplete: Charm, QCD, 3rd Generation?

time not quite right for full EW Radiative Corrections

Main effect: $\alpha = 1/137 \rightarrow \alpha(m_Z) \sim 1/128$ Large 7% Effect

Later: Large $\alpha m^2_t / m^2_W$ Corrections M. Veltman

Standard Model Predictions Through 2 loops Assuming No New Physics

$$\sin^2\theta_W(m_Z)_{MS} = \pi\alpha/\sqrt{2}m_W^2G_\mu(1-\Delta r(m_Z)_{MS})$$

$$\Delta r(m_Z)_{MS} = 0.0693(2) \rightarrow \sin^2\theta_W(m_Z)_{MS} = \underline{0.23110(9)}$$

$$\sin^2 2\theta_W(m_Z)_{MS} = 2\sqrt{2}\pi\alpha/m_Z^2G_\mu(1-\Delta r'(m_t, m_H))$$

$$\Delta r'(m_t, m_H) = 0.0598(2) \rightarrow \sin^2\theta_W(m_Z)_{MS} = \underline{0.23124(6)} \\ \pm 0.03\%$$

Error Expected to be reduced (improved m_t) to $\sim \pm 0.01\%$

Corresponds to $m_W = 80.362(6)$

Any significant difference with other precise $\sin^2\theta_W$ measurement implies “New Physics”

Currently $\sin^2\theta_W(m_Z)_{ave} = 0.23125(16)$ Excellent Agreement

Evidence for Grand Unification?

In simplest GUTS (SU(5), SO(10)...) $\sin^2\theta_W^0=3/8$ at unification

$$\sin^2\theta_W(m_Z)_{MS} = 3/8 [1 - 109\alpha/18\pi \ln(m_X/m_Z) + \dots]$$

$\approx 0.21!$ (Great Desert?)

$$\mu = m_X \sim 2 \times 10^{14} \text{ GeV in minimal SU(5)}$$

But later, minimal SU(5) ruled out by proton decay

$$\text{exps } \tau(p \rightarrow e^+ \pi^0) > 10^{33} \text{ yr} \rightarrow m_X > 3 \times 10^{15} \text{ GeV}$$

SUSY GUT Unification ($m_{\text{susy}} \sim 1 \text{ TeV}$) $\rightarrow m_X \sim 10^{16} \text{ GeV}$
 $\tau_p \sim 10^{35} - 10^{36} \text{ yr}$

SUSY **$\sin^2\theta_W(m_Z)_{MS} = 0.233$ (Current Agreement!)**
(Where is SUSY at the LHC?)

S & T Constraints

Gauge Boson Self-Energy Loops

Experimental Averages

$$\sin^2\theta_W(m_Z)_{MS} = \underline{0.23125(15)} \quad \& \quad m_W = \underline{80.385(15)\text{GeV}}$$

Standard Model + S & T

$$\sin^2\theta_W(m_Z)_{MS} = \underline{0.23124(6)} [1 + 0.016S - 0.011T]$$

$$m_W = \underline{80.362(6)\text{GeV}} [1 - 0.0036S + 0.0056T]$$

$$S \approx 0.7T \quad (\text{from Z pole } \sin^2\theta_W(m_Z)_{MS} = \underline{0.23125(15)})$$

$$S = 0.07(8) \quad T = 0.10(11) \quad \Delta N_{\text{doublets}} = 2(2)$$

Little (but some) room available for “New Physics”

Constrains: New Dynamics (Technicolor),

4th Generation, SUSY, Z', Z'' (mixing)...

Best Off Z Resonance Measurements of $\sin^2\theta_W$

(Not Competitive with Z Pole)

Reaction	$\sin^2\theta_W(m_Z)_{MS}$	$\langle Q \rangle$
Cs APV	0.2283(20)	2.5MeV
E158 ee	0.2329(13)	160MeV
Q_{weak} ep	0.2329(30)	160MeV
6GeV Dis eD	0.2299(43)	1.5GeV
NuTeV $\nu_\mu N$	0.2356(16)	3-4GeV

NuTeV $\sin^2\theta_W(m_Z)_{MS} = \underline{0.2356(16)}$ (2+ sigma High)

$A_{RL}(ee) = -131(14)(10) \times 10^{-9} \propto (1 - 4\sin^2\theta_W)$

Best Low Q^2 Determination $\sin^2\theta_W(m_Z)_{MS} = \underline{0.2329(13)}$

Waiting for Qweak

E158 at SLAC Pol $ee \rightarrow ee$ Moller)

$E_e \approx 50 \text{ GeV}$ on fixed target, $Q^2 = 0.02 \text{ GeV}^2$

$$A_{LR}(ee) = -131(14)(10) \times 10^{-9} \propto (1 - 4 \sin^2 \theta_W)$$

EW Radiative Corrections $\sim -40\%$! (Czarnecki & WJM 1996)

More $\sin^2 \theta_W$ Sensitivity!

Measured to $\pm 12\%$ $\rightarrow \sin^2 \theta_W$ to $\pm 0.6\%$ (20 to 1)

$\rightarrow \sin^2 \theta_W(m_Z)_{MS} = \underline{0.2329(13)}$ slightly high

Best Low Q^2 Determination of $\sin^2 \theta_W$

$$A_{LR}(ee)^{\text{exp}} = A_{LR}(ee)^{\text{SM}} (1 + 0.13T - 0.20S + 7(m_Z/m_{Z\chi})^2 \dots)$$

Constrains “New Physics” eq $m_{Z\chi} > 0.6 \text{ TeV}$, H^- , S , Anapole Moment, ...

Together APV(Cs) & E158 $\rightarrow \sin^2 \theta_W(Q^2)$ running

$\sin^2 \theta_W(m_Z)_{MS} = \underline{0.232(1)}$ Good agreement with Z Pole

1 loop contributions to $\sin^2\theta_W(Q^2)$ running

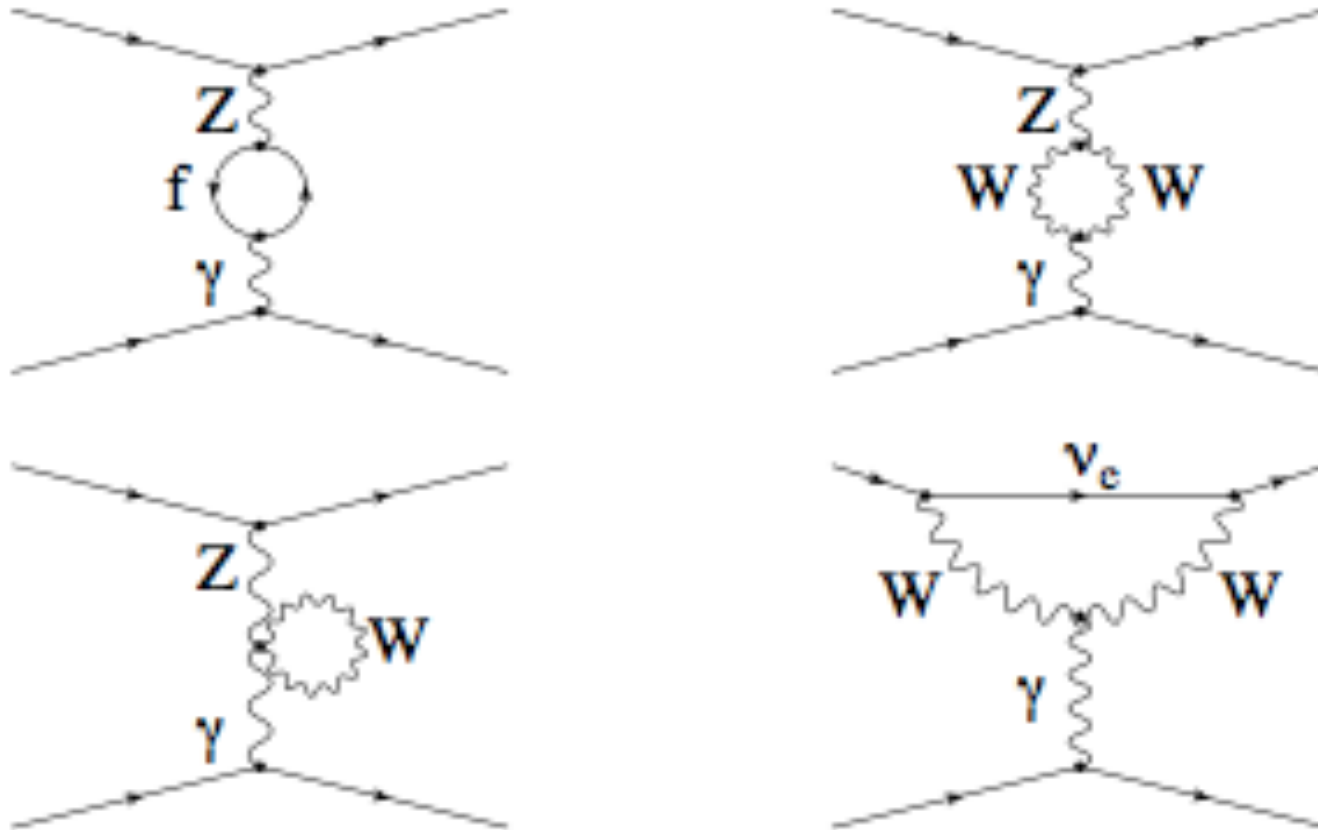


Fig. 2. $\gamma - Z$ mixing diagrams and W -loop contribution to the anapole moment.

Box Diagrams (tend to cancel) →
 very small 2 loop uncertainty ($\Delta A_{RL}/A_{RL} \sim \pm 0.3\%$)

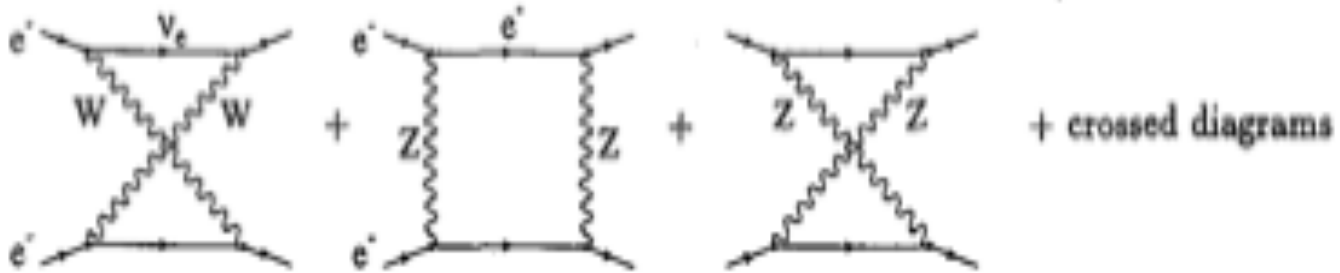


FIG. 3. Box diagrams with two heavy bosons.

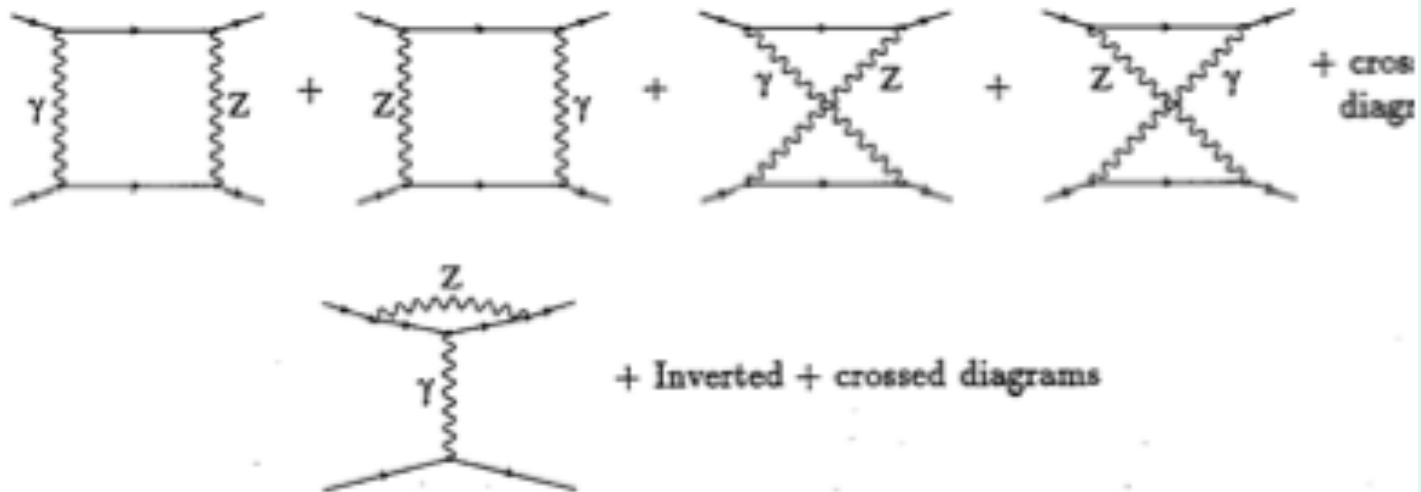


FIG. 4. Boxes containing one photon and Z-loop contribution to the anapole moment.

EW Radiative Corrections to Moller A_{RL} (-40%)

A. Czarnecki & WJM PRD(1996)

- $A_{RL}(ee) = \alpha (1 - 4\sin^2\theta_W)$ $\sin^2\theta_W(m_Z)_{MS} = \underline{0.23124(6)}$ or
running + $3.01(25)_{\text{hadronic}}\%$
 $\sin^2\theta_W(Q=0) = \underline{0.23820(60)}$
+ WWbox (+3.6%) γZ box...(-5.5%) partial cancellation
+ other small 1 loop corrections \rightarrow -40(3)% reduction!
E158 $\Delta A_{RL}/A_{RL} = \pm 12.5\%$ vs Running unc. $\pm 6\%$?

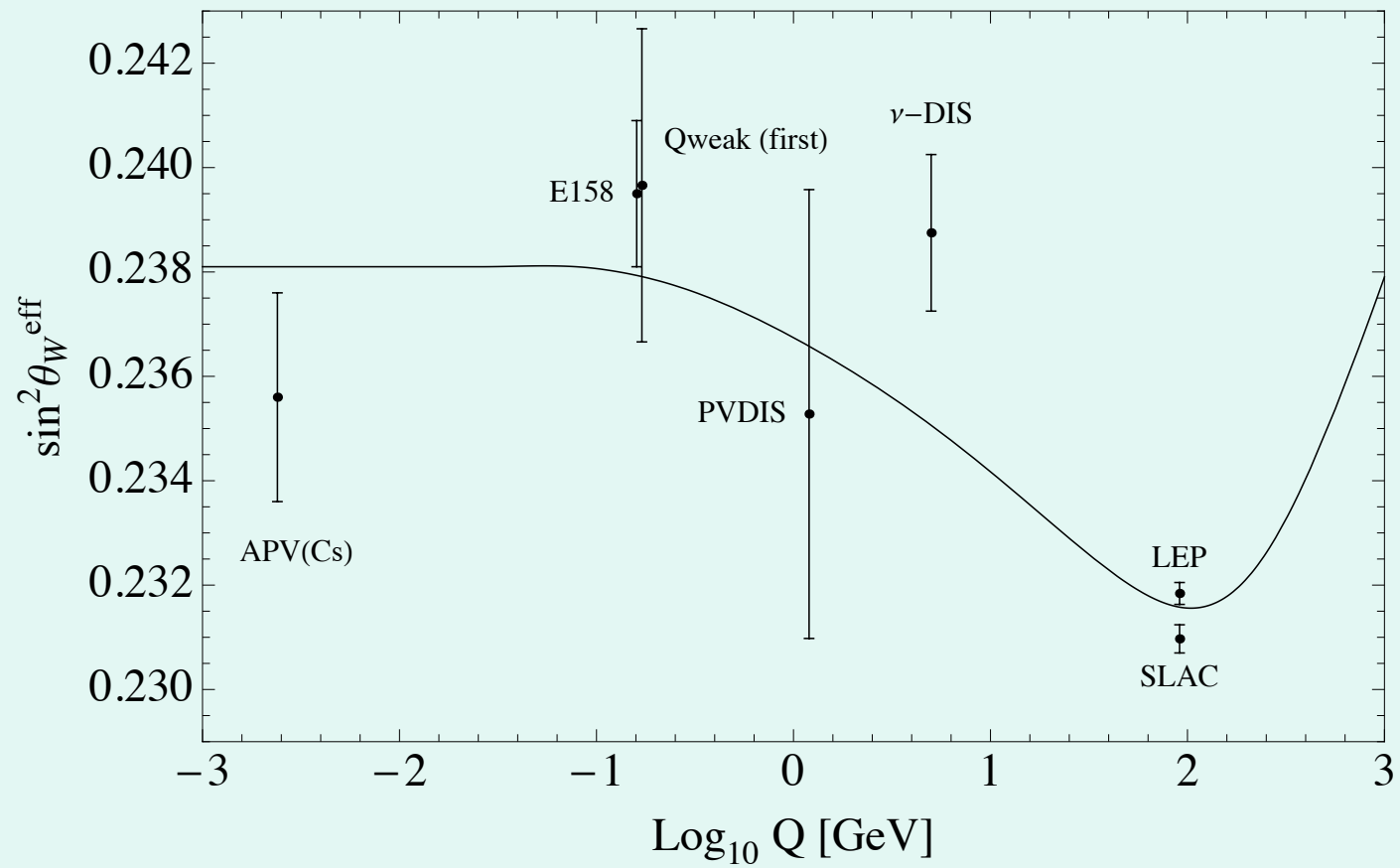
Erlar & Ramsey-Musolf \rightarrow factor of 8.6 error reduction!

+3.01(25)% \rightarrow +2.99(3)% Theory $\pm 0.6\%$ vs Moller exp $\pm 2.4\%$

$\Delta\sin^2\theta_W^{RC} \sim \pm 0.00007!$ Pristine

Potentially another factor of 2 reduction via lattice

Measurements of running $\sin^2\theta_W(Q^2)$



Goals of Future Low Q^2 Experiments

- High Precision: $\Delta \sin^2 \theta_W \sim \pm 0.0002 - 0.0004$ or better
Low Q^2 Sensitivity to “New Physics” (eg Dark PV)

SUSY Loops Sensitivity in Moller vs SM & Z pole
(Ramsey-Musolf)

$S \approx 0.7T$?, Confirm Z Pole A_{ve}/m_W determination?

Z pole implies $|0.25T - 0.34S| \leq 10^{-3}$; **TRUE?**

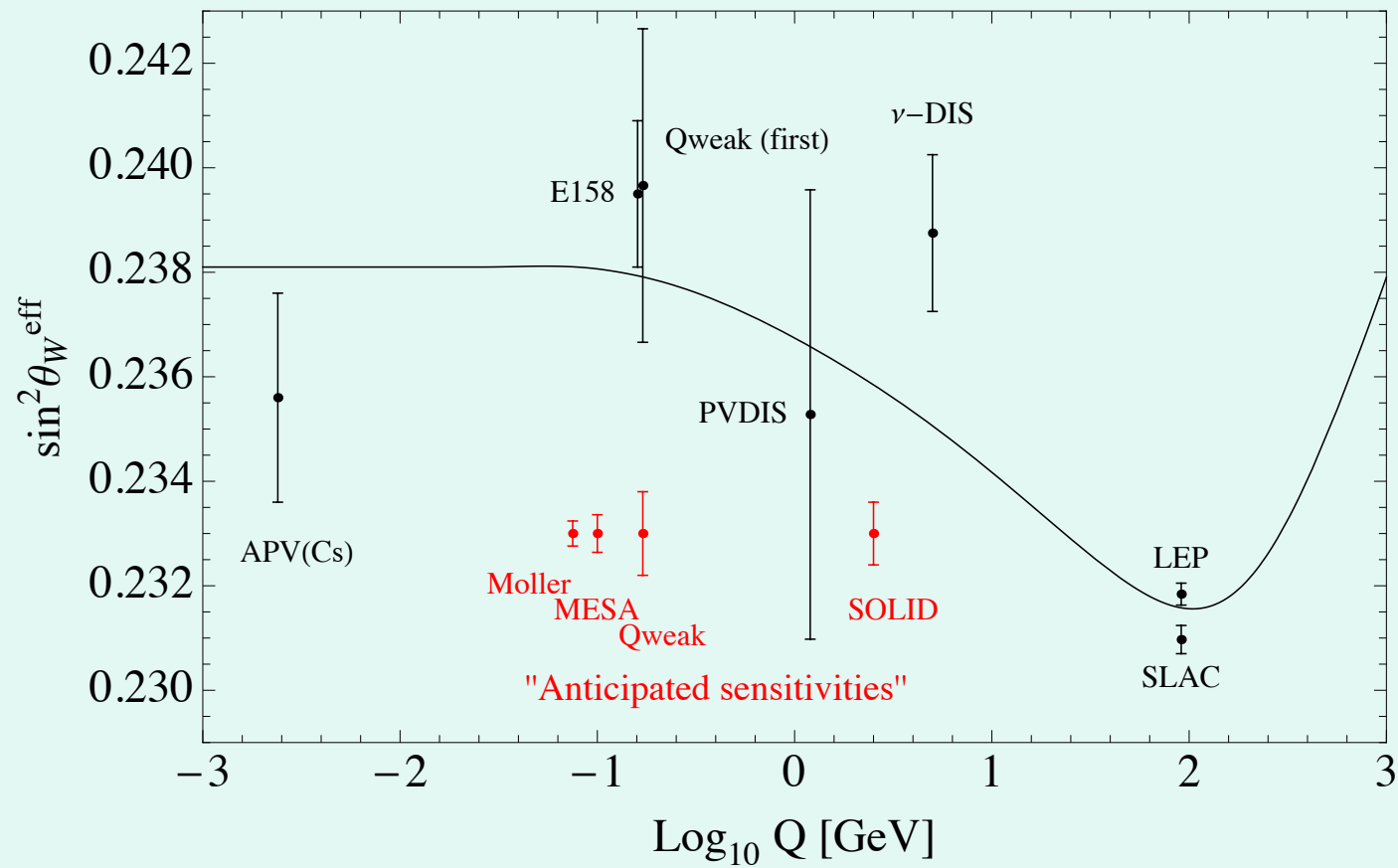
If so, Charge radius $X(Q^2)$, Z' ... of prime importance

Heavy $m_{Z'}$ $> 1-8\text{TeV}$, Model Dependent Sensitivities

(Do not interfere at Z Pole)

***Light $m_{Z'}$** $< 10\text{GeV}$ (Dark Parity Violation) *Sensitivity*

$\sin^2\theta_W(Q^2)$ Measurements & expected Future Sensitivities



Z' Boson Sensitivity (extra U(1)' gauge symmetry)

Consequences of $A_{RL}(ee \rightarrow ee)$ to $\pm 2.4\%$

$$\Delta \sin^2 \theta_W(m_Z)_{MS} = \pm 0.00027!$$

$$A_{RL}(ee)^{\text{exp}} = A_{RL}(ee)^{\text{SM}} (1 + 0.25T - 0.34S + 7(m_Z/m_{Z\chi})^2 \dots \\ + 0.7R(0) + \text{SUSY} + H^- \dots)$$

Z pole $\sin^2 \theta_W(m_Z)_{MS} \rightarrow 0.25T - 0.34S$ very small. *Is it?*

SM agreement significantly constrains BSM

Deviation Implies New Physics (Many Z' Examples)

Complements LHC Z' Direct Discovery 1-8TeV

(LHC Requires relatively large Z' production & I^+I^- BR)

(Potentially) Watered down by BSM decay modes

Examples

$U(1)_\chi, U(1)_\psi$ of E_6 GUT (Mix) $\leq 2\text{TeV}$ sensitivity

Much Stronger coupling \rightarrow Better Sensitivity $\rightarrow O(8\text{TeV})$

$U(1)_{B-L}, U(1)_{L_e-L_\mu}, U(1)_{SUSY}\dots$ Many Heavy Z' Examples

LHC Sensitivity may be diluted by reduced I^+I^- BRs

Light Z' bosons with very weak SM couplings $\sim 10^{-3}-10^{-6}g$

I will illustrate **dark Z** model of **(DAVOUDIASL, LEE, MARCIANO)**

GENERALIZATION OF DARK PHOTON MODEL

The Dark Boson – A Portal to Dark Matter

- What if some $U(1)_d$ gauge symmetry from the Dark or some Other Sector contains a “Light” *Dark Photon, U Boson, Hidden Boson... Dark Z (Z_d)*

- Introduced for:
- 1) Sommerfeld Enhancement $D+D \rightarrow Z_d+Z_d$
 - 2) $Z_d \rightarrow e^+e^-$ (source of positrons, *γ -rays*)
 - 3) Cosmic Dark Matter Stability via global $U(1)_d$
 - *4) Light Dark Matter Abundance
 - *5) *Muon Anomalous Magnetic Moment*

Can we find direct evidence for such a light boson in the laboratory?

Dark Photon & Dark Z

Interacts with our particle world via

1) Kinetic Mixing $U(1)_Y \times U(1)_d$ $\epsilon e Z_d^\mu J_\mu^{\text{em}}$ $\epsilon \approx \alpha/\pi \approx 2 \times 10^{-3}$

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 288(80) \times 10^{-11} \text{ (3.6}\sigma \text{ discrepancy!)}$$

$$\approx \frac{1}{4}(\alpha/\pi)\epsilon^2 \quad m_{Z_d} \approx 20\text{-}300\text{MeV (see figure)}$$

2) Z-Z_d Mass Mixing: $\epsilon_Z g/2 \cos\theta_W Z_d^\mu J_\mu^{\text{NC}}$ $\epsilon_Z = m_{Z_d}/m_Z \delta$ $\delta \approx 10^{-3}$ ($\sim v_1^2/v_2 v_s$)

Induced by extended Higgs (2 doublets + sing.) Portal

Rare Higgs $\rightarrow ZZ_d$, $K \rightarrow \pi Z_d$ & $B \rightarrow K Z_d$ decays $\sim \delta^2$,

Dark Parity Violation (probes $\delta \approx 2 \times 10^{-3}$ $\epsilon \approx 2 \times 10^{-3}$)

(DAVOUDIASL, LEE, MARCIANO) Enhanced Phenomenology

3) Small direct coupling eg. $U(1)_{L_e-L_\mu}$ $\sim \text{few } \times 10^{-3} e$

Example

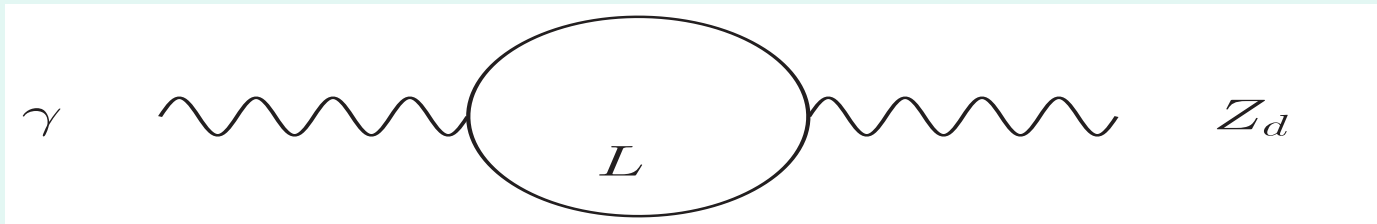
One Loop gamma- γ_d Kinetic Mixing

$$\varepsilon F_{\mu\nu} D^{\mu\nu}$$

(eg Through Heavy Charged Leptons)

That also carry $U(1)_d$ charge

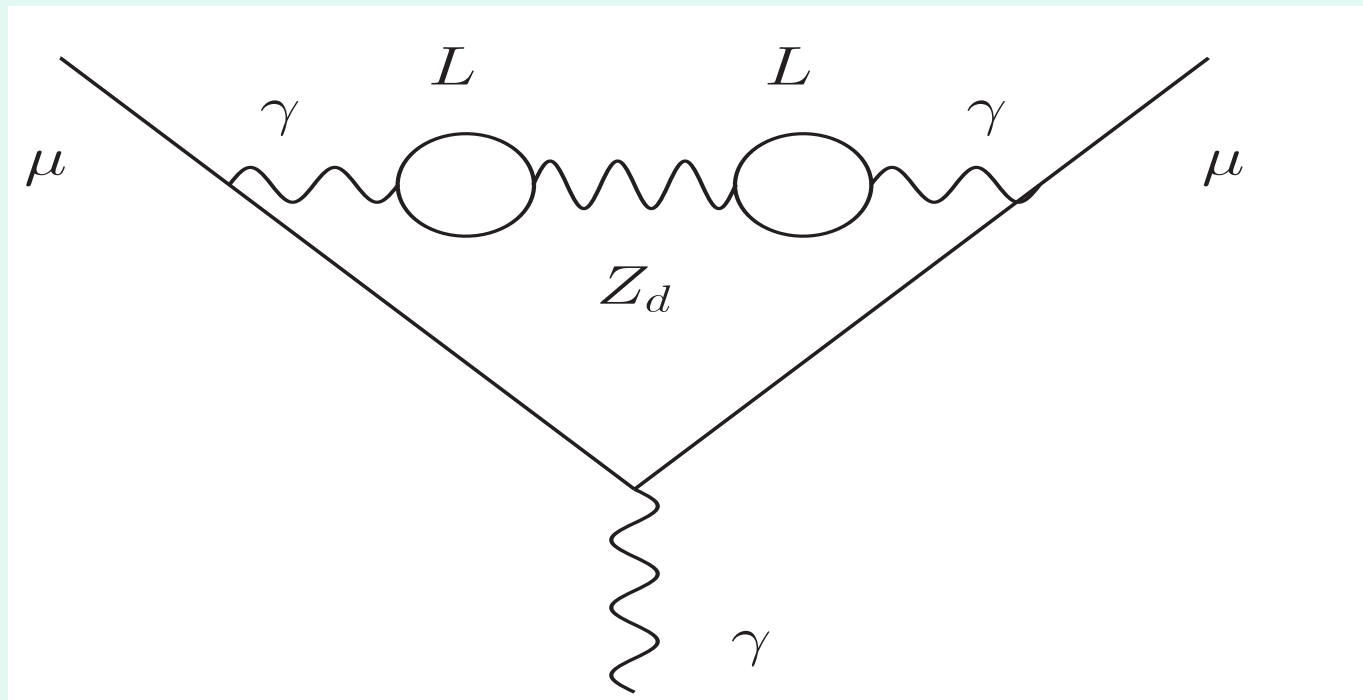
Expect $\varepsilon \sim e g_d Q Q_d / 8\pi^2 \leq O(10^{-3})$



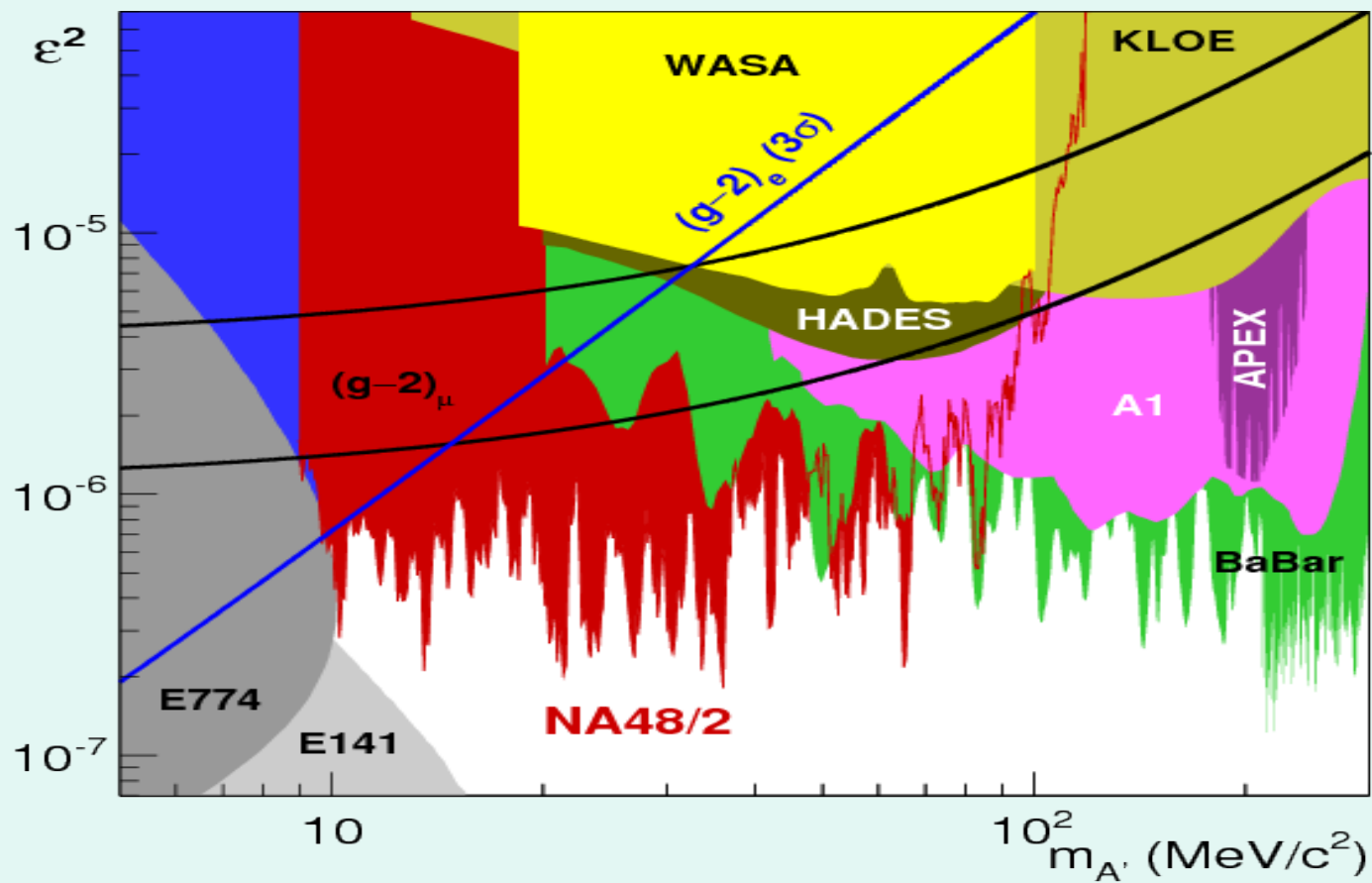
Muon Anomalous Magnetic Moment

$$a_{\mu}^{Z_d} = \alpha/2\pi\epsilon^2 F(m_{Z_d}/m_{\mu}), F(0)=1$$

solves $(g_{\mu}-2)/2$ discrepancy $\approx 288(80)\times 10^{-11}$
for $\epsilon^2 \approx 10^{-6}-10^{-4}$ & $m_{Z_d} \approx 10-300\text{MeV}$ (see figure)



NA48/2 Updated Bounds on Dark Photon
Simple $g_\mu-2$ discrepancy solution ruled out
Assumes $BR(\gamma_d \rightarrow e+e-) \sim 1$



Dark Parity Violation

H. DAVOUDI ASL, H-S LEE, W. MARCIANO

Effect of ε & ε_z together: (at low $Q^2 \ll m_Z^2$)

$$\Delta \sin^2 \theta_W(Q^2) = -0.42 \varepsilon \delta m_Z m_{Z_d} / (Q^2 + m_{Z_d}^2)$$

For $\delta \approx m_{Z_d} / m_Z$, $\Delta \sin^2 \theta_W(Q^2) = \pm 0.42 \varepsilon m_{Z_d}^2 / (Q^2 + m_{Z_d}^2)$

Shift largest at small $Q^2 < m_{Z_d}^2$ ($\approx O(1\%)$! Eg APV)

(1.5 sigma APV deviation) fit $\rightarrow \varepsilon \delta = 4 \times 10^{-6}$

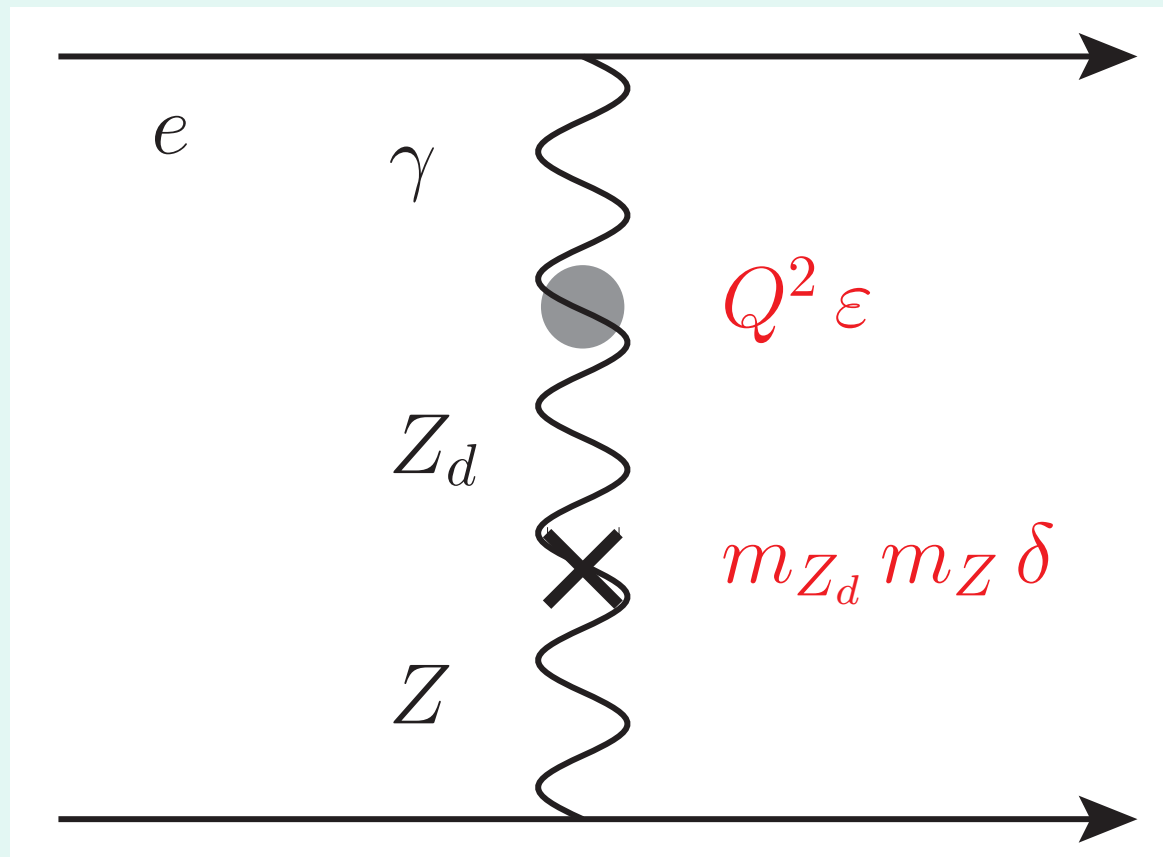
or $\varepsilon \approx \delta \approx 2 \times 10^{-3}$ for $(g_\mu - 2)$ & APV $\rightarrow m_{Z_d} \approx 50 \text{ MeV}$ region

$\sin^2 \theta_W(Q \approx 75 \text{ MeV})$ shift by $\pm O(0.5-1\%)$!!

$\varepsilon \delta$ down to $\approx 10^{-6}$ Potentially Observable

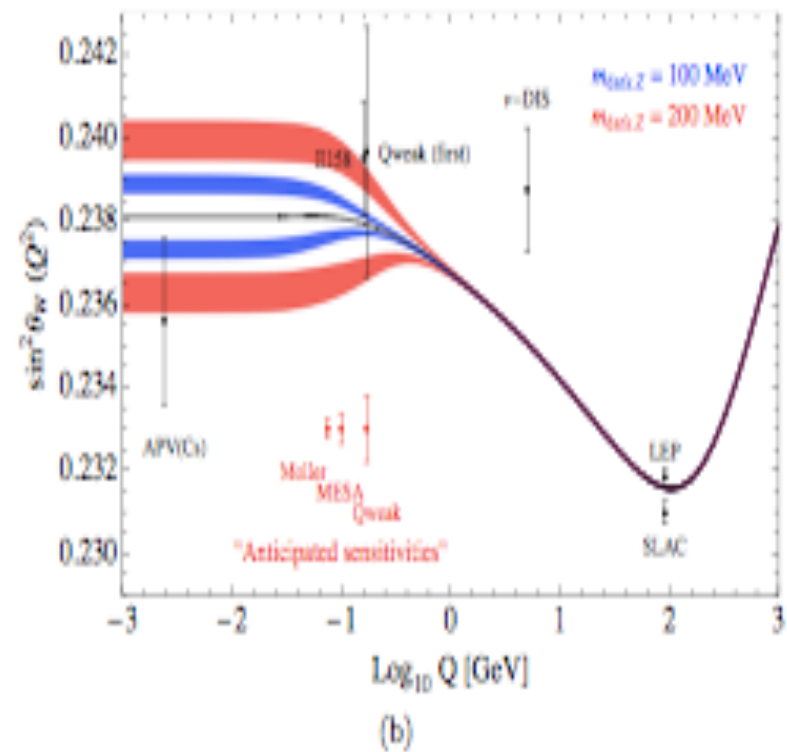
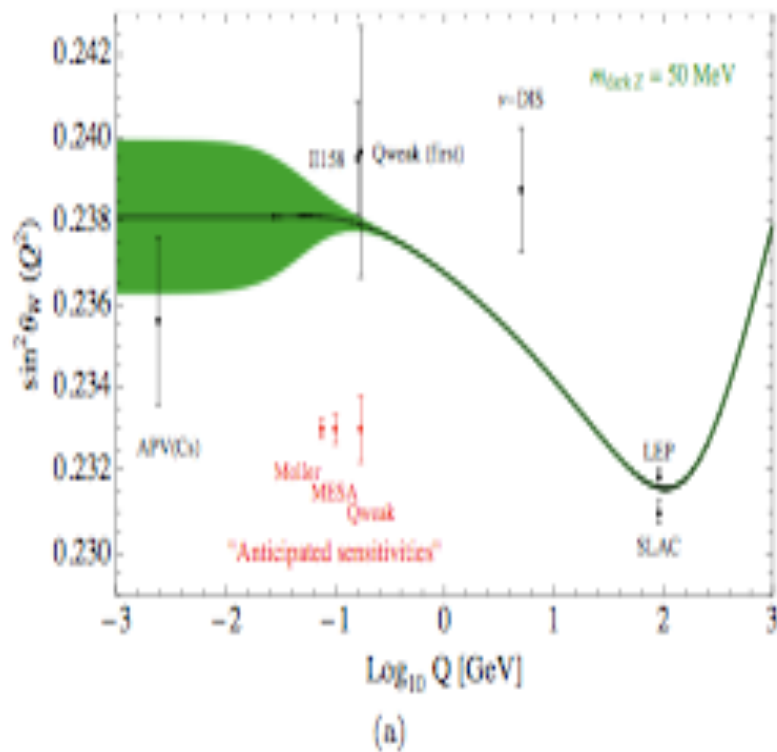
$A_{RL}(ee)$ at low Q^2 very Important

Dark Z Effect on electron scattering
Photon-Z Mixing through Z_d
from H-S Lee

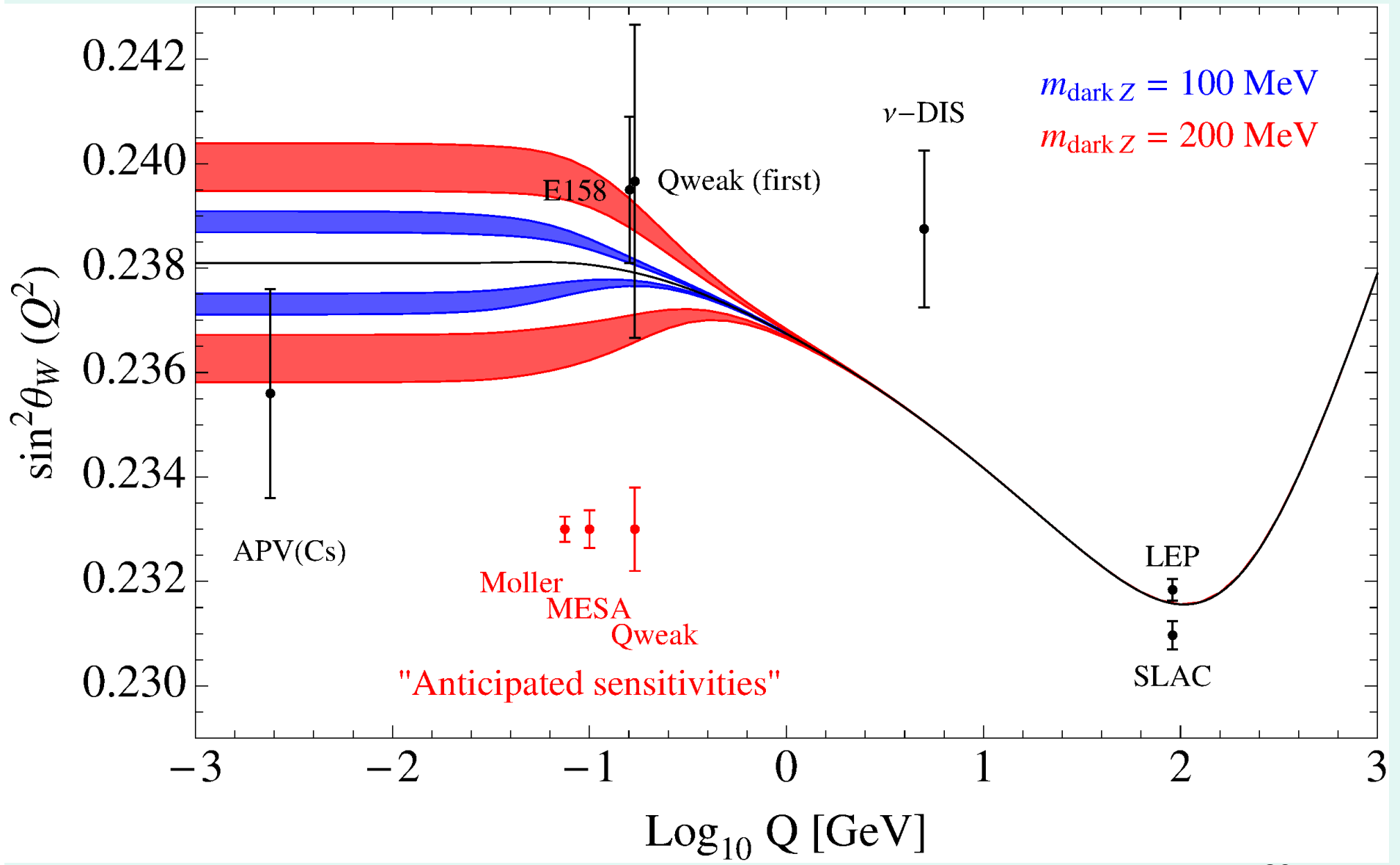


Effect of “Light” Z_d on Running

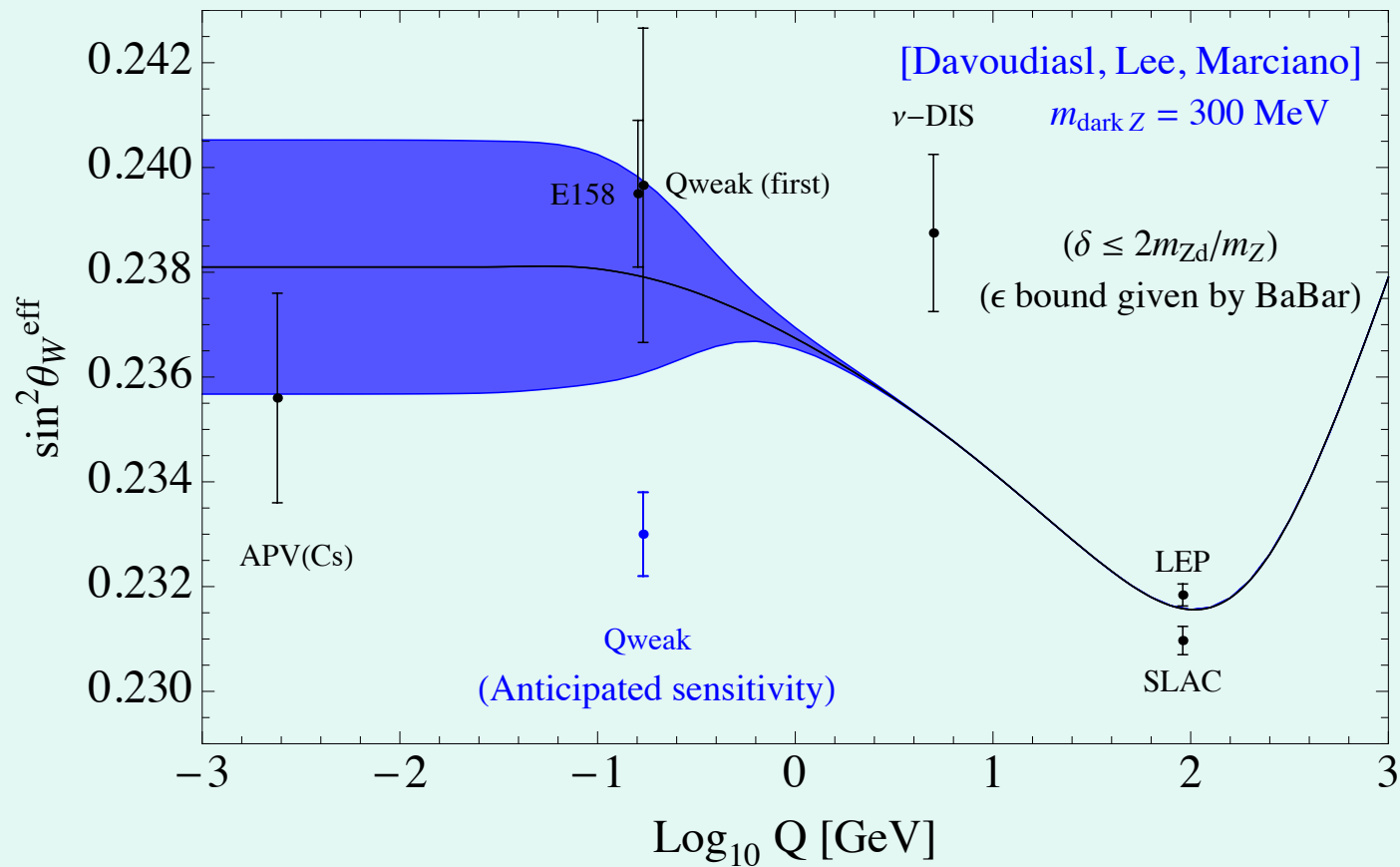
H. DAVOUDIASL, H-S LEE, W. MARCIANO



Blowup



Potential 300MeV Dark Z Effects on Running
 $0 < |\sin^2\theta_W(0)| < 0.002$ Start to show up in Qweak



The Dark Z_d Model

DAVOUDI ASL, LEE, MARCIANO

$\gamma_d - Z$ Mass Mixing $\rightarrow Z_d$ (dark Z) & Z

Add second $SU(2)_L$ Dark Higgs Doublet H_2

Three Higgs Multiplets H_1 , H_2 & ϕ_d

Vacuum expectation values v_1 , v_2 & v_d

Mixing $\epsilon_Z = m_{Z_d}/m_Z \delta$

$\delta = v_2^2/v_1 v_d$ small $\sim O(m_{Z_d}/m_Z) \sim O(10^{-3})$

Find $\Delta \sin^2 \theta_W / \sin^2 \theta_W \approx -2\epsilon(m^2 \gamma_d / Q^2 + m^2 \gamma_d)$

Potentially of order 10^{-3} for low Q^2

Υ_d -Z Mass Mixing $\rightarrow \epsilon_Z = \delta m_{Z_d} / m_Z$

- Potentially Observable Effects, for $\delta \sim O(10^{-3})$, over a range of $10\text{MeV} < m_{Z_d} < 15\text{GeV}$ in

***Weak mixing angle running at low $\langle Q \rangle$**

$$\text{BR}(K \rightarrow \pi Z_d) \approx 4 \times 10^{-4} \delta^2$$

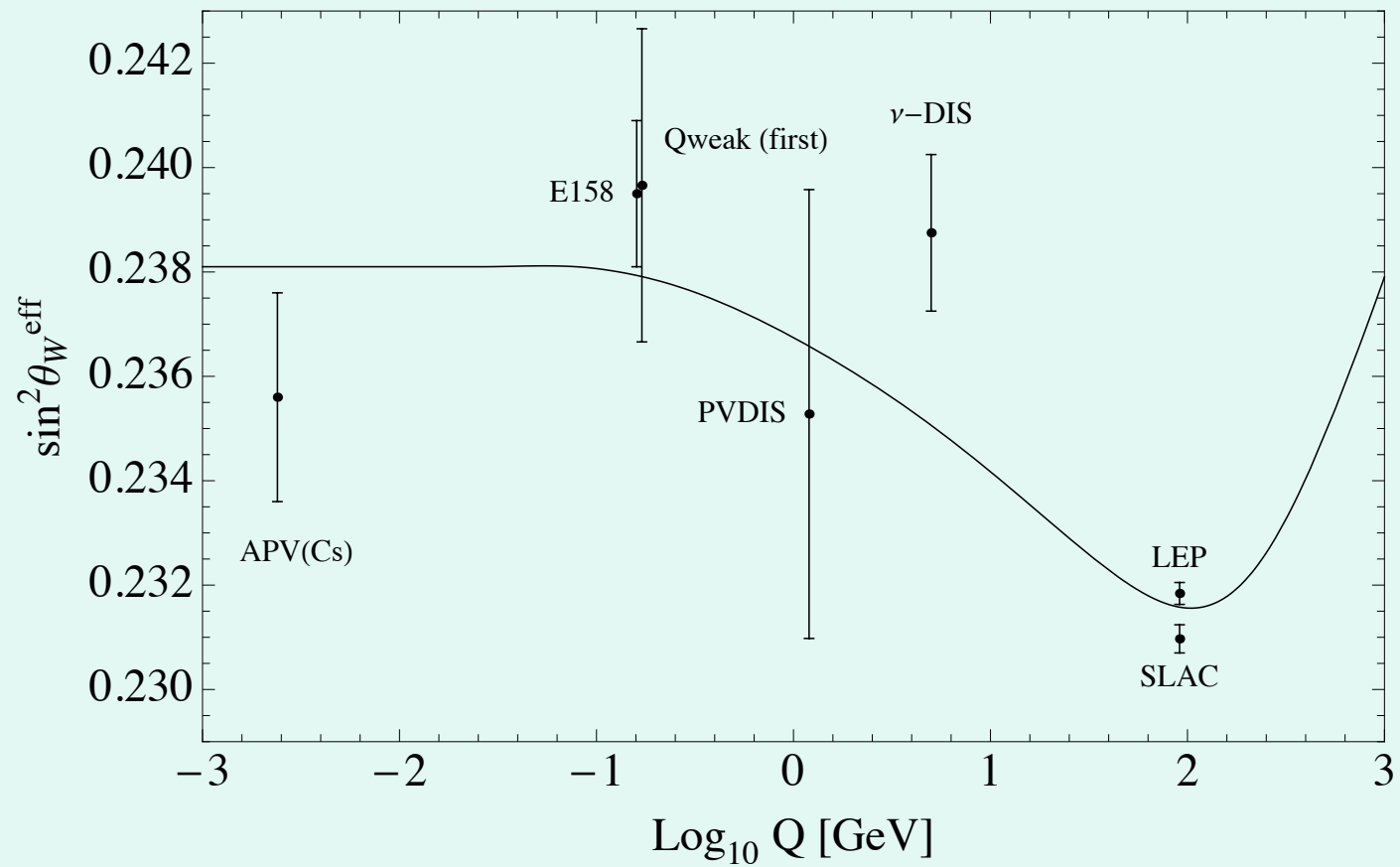
$$\text{BR}(B \rightarrow K Z_d) \approx 0.1 \delta^2$$

$$*\Gamma(H \rightarrow ZZ_d) / \Gamma_H(125\text{GeV})_{\text{SM}} = 16 \delta^2$$

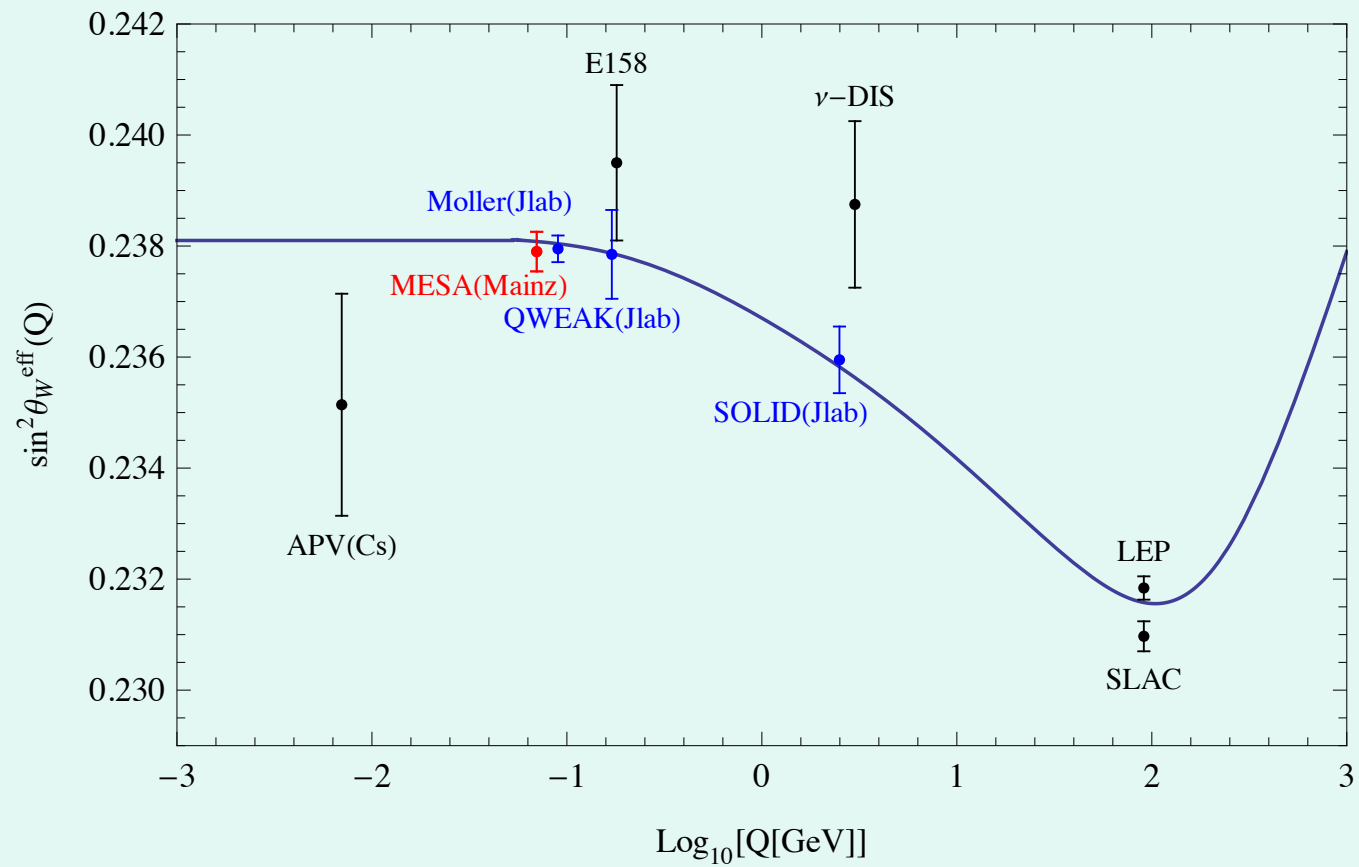
δ roughly probed to $< 10^{-3}$

Z_d Discovery would revolutionize particle physics

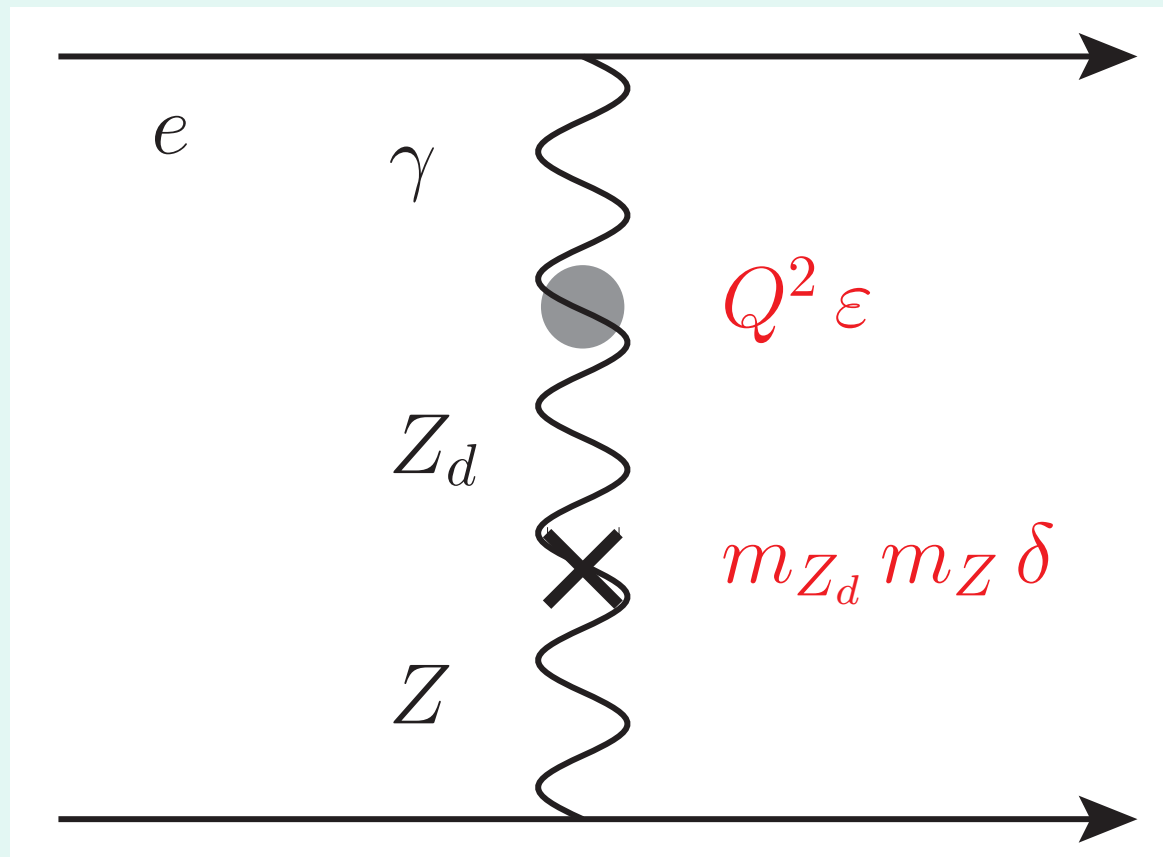
Measurements of running $\sin^2\theta_W(Q^2)$ Pre New Qweak



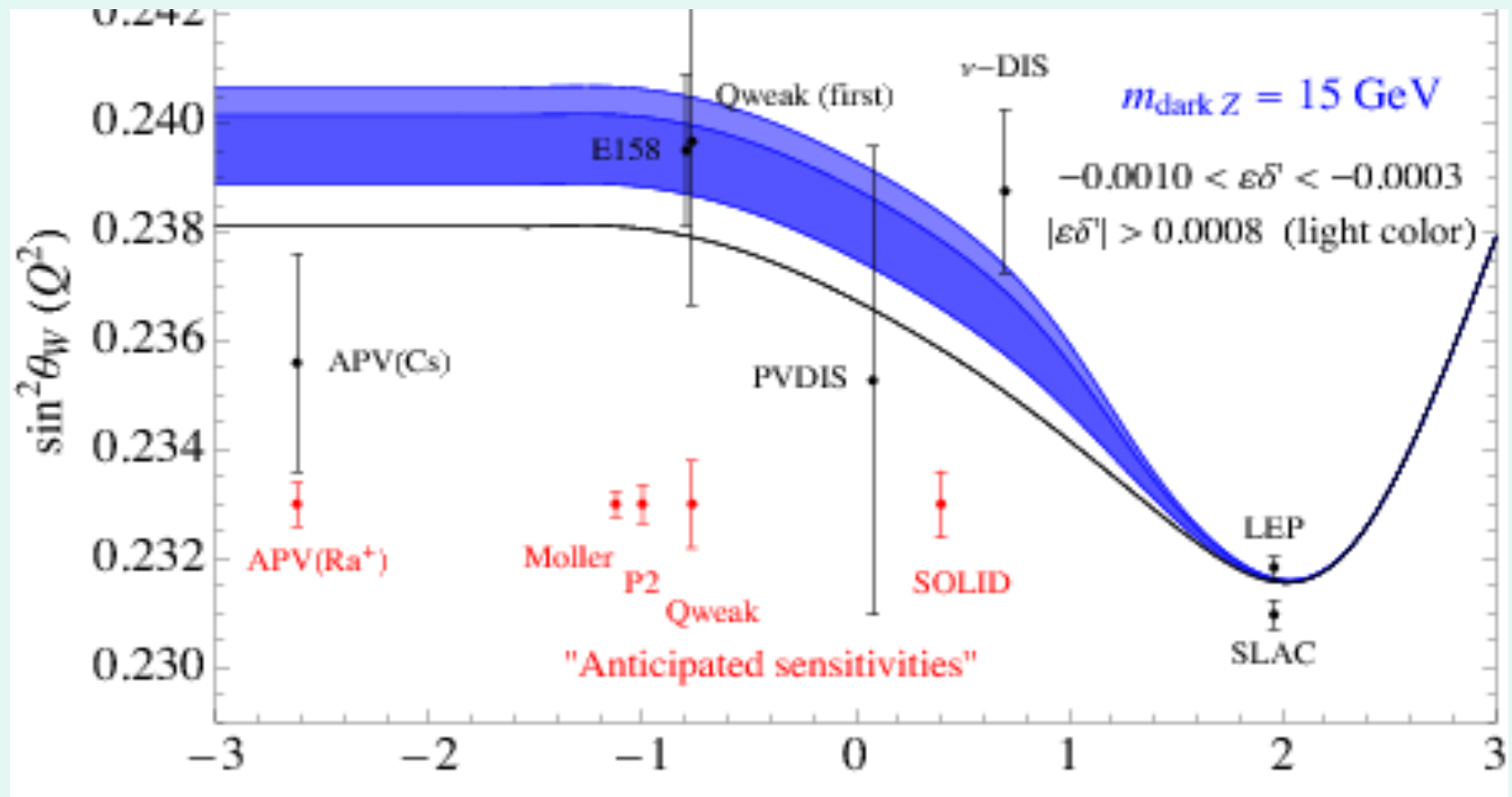
Possible A_{RL} Measurements



Dark Z Effect on electron scattering
Photon-Z Mixing through Z_d
Kinetic + Mass Mixing

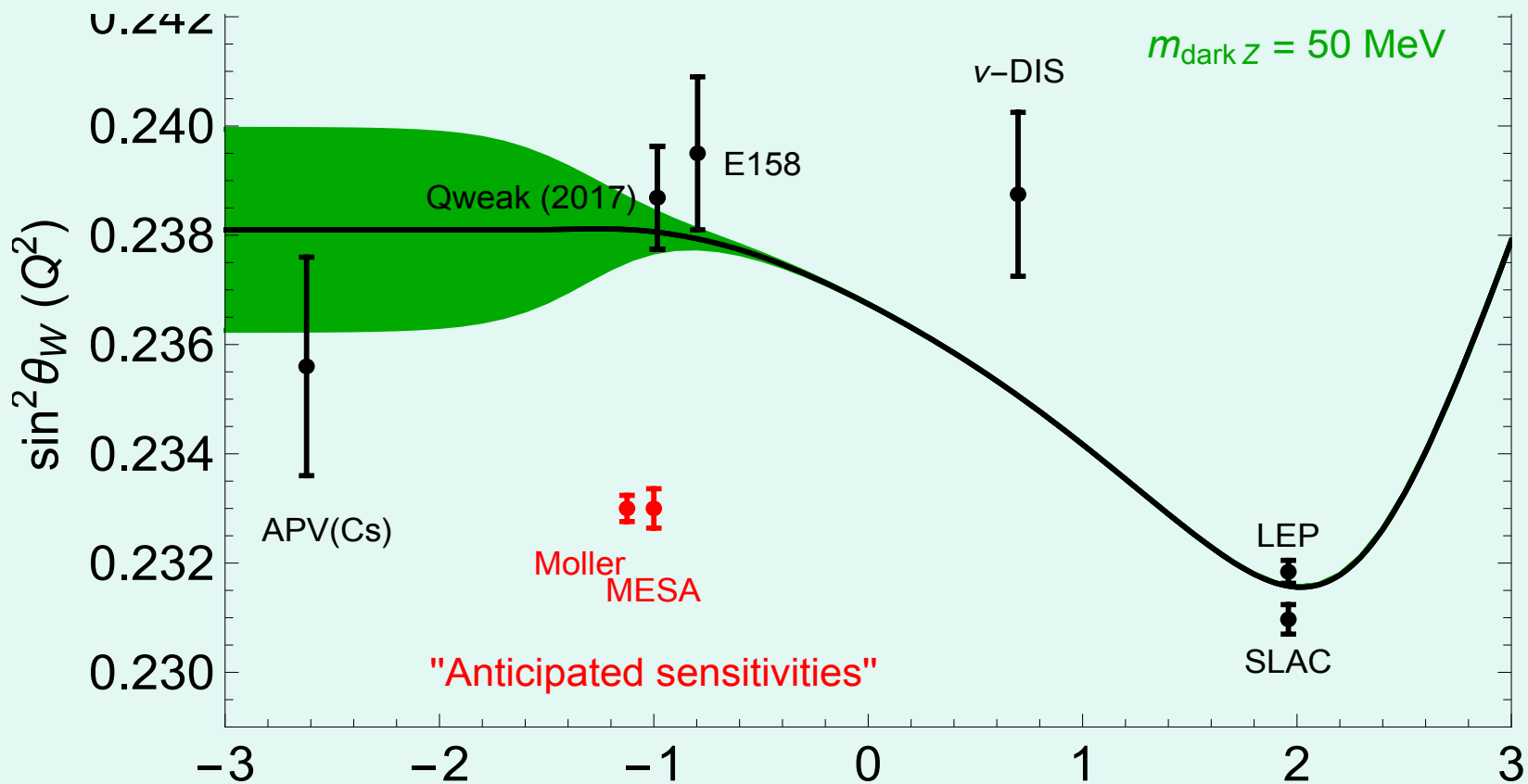


Pre- 2017 Qweak 15GeV Dark Z Fit to Low Energy Data



Examples of the effect of “Light” Z_d on Running

H. DAVOUDIASL, H-S LEE, W. MARCIANO



Present & Future

Precise, $\sin^2\theta_W(Q^2)$ PV Experiments at low Q^2

$Q_{\text{weak}} \sin^2\theta_W(m_Z)_{\text{MS}} = 0.2319(9)$ (with LQCD input)
Deviation of average low energy from pole: 0.0012(6)

Next PVES(^{12}C) MESA at Mainz elastic eC scattering
 $\Delta\sin^2\theta_W(m_Z)_{\text{MS}} = \pm 0.0007$ (Pol. Uncertainty!)

Future (Z pole competitive)
P2 in Mainz ($A_{\text{RL}}(ep)$) $\Delta\sin^2\theta_W(m_Z)_{\text{MS}} = \pm 0.00037$

Moller at JLAB Goal $\Delta\sin^2\theta_W(m_Z)_{\text{MS}} = \pm 0.00027!$

**“New Physics” in form of Light (150MeV -15GeV) Z_d
5+ sigma Discovery Potential**

Non PV $\sin^2\theta_W$ at very low Q^2

Eg. Vector – Like gauged B-L

No Parity Violating Effect

Reactor $\bar{\nu}_e$ -e scattering $Q \sim 2\text{MeV}$

Goal $\pm 0.5\%$ in $\sin^2\theta_W$ Currently $\sim \pm 10\%$

Explore low mass bosons & g_{B-L} of $O(10^{-7})$

Down to low masses $\sim 5\text{MeV}$

Long Term “Crazy” Challenge Based on Jet Lag

What prevents doing polarized A_{RL}

e^+p scattering at MESAll?

(Similar to Moller) **“I said it couldn’t be done”**

(Different than P2 at loop level)

Rate, polarization, polarimetry etc

Worthwhile R&D Project?