Recent progress in the NNLO evaluation of A_{LR} in Moller scattering

in collaboration with:
Ayres Freitas,
Michael Ramsey-Musolf,
Yong Du

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MITP Workshop

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(would be) outline

1. **Context —**
   Standard Model, E158 / MOLLER experiments

2. **Theory history —**
   (tree-level) Derman & Marciano
   (one loop) Marciano & Czarnecki

3. **Status of the NNLO calculation:**
   <u>Organization:</u>
   (closed fermion loops) + (rest)
   <u>Methods/techniques:</u>
   (Expansion by regions, integration by parts,
   dispersion relations)

4. Result

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   None that are meaningful yet…

(would be) outline

1. **Context —** Standard Model, E158 / MOLLER experiments

   Already covered by: (K.K. and others)

2. **Theory history —**
   (tree-level) Derman & Marciano
   (one loop) Marciano & Czarnecki

   (A. Freitas, A. Aleksejevs)

3. **Status of the NNLO calculation:**
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   Methods/techniques:
   (Expansion by regions, integration by parts, **(A. Freitas.)**
   dispersion relations) **(A. Aleksejevs)**

4. Result

   None that are meaningful yet…

Thank You

Any questions?

Behavior of a perturbation series

Perturbation series

$$\mathcal{O} = a^2 + a^4 \Big(\qquad \qquad \Big)$$
$$+ a^6 \Big(\qquad \qquad \qquad \qquad \Big)$$
$$+ \dots$$

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Behavior of a perturbation series

Perturbation series two scales

$$\mathcal{O} = a^2 + a^4\left(\frac{1}{6} + \ln(\frac{M_Z^2}{s}) \qquad\qquad \right) \\ + a^6\left(\frac{1}{10} + \ln^2(\frac{M_Z^2}{s}) \qquad\qquad\qquad\qquad + \ln(\frac{M_Z^2}{s})\right) \\ + \dots$$

Behavior of a perturbation series

Perturbation series <u>two scales</u>

≈ 15.0
(secular term)

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$$+ \ldots$$

Behavior of a perturbation series

Perturbation series two scales

≈15.0
(secular term)

$$
\begin{aligned}
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&+ a^6\Big(\frac{1}{10} + \ln^2(\frac{M_Z^2}{s}) \qquad\qquad\qquad\qquad + \ln(\frac{M_Z^2}{s})\Big) \\
&+ \ldots
\end{aligned}
$$

Secular terms threaten uniformity of perturbative series w.r.t. to kinematic variables.

Two (three) options —

- If log is small, do nothing.
- If a is large, resum logs. Classical RG analysis $a \to a(s)$
- If a is small, use large logs to guess dominant terms, and est. theory errors.

Behavior of a perturbation series

Perturbation series <u>two or more scales</u>

$$\mathcal{O} = a^2 + a^4\Big(\frac{1}{6} + \ln(\frac{M_Z^2}{s}) + \ln(\frac{m_b^2}{s})\Big)$$
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$$+ \dots$$

Can get complicated.

EFT is the organizing principle for handling the logs.

A. FALKOWSKI

Energy Scale

1 TeV

100 GeV

10 GeV

1 GeV

100 MeV

10 MeV

1 MeV

Theory

SMEFT

wEFT

(aka Fermi theory, LEFT, WET)

χPT

eQED

nrQED

Euler-Heisenberg

Experiment

LHC

LEP

TRISTAN

ν beams

PVDIS

PVES

Meson decays

Moller scattering

APV

$$\frac{E}{\Lambda}$$

$$\frac{E}{m_W}$$

$$\frac{E}{4\pi f_\pi}$$

$$\frac{E}{m_e}$$

Assumption: below ~1 TeV scale, no new degrees of freedom beyond those of the SM

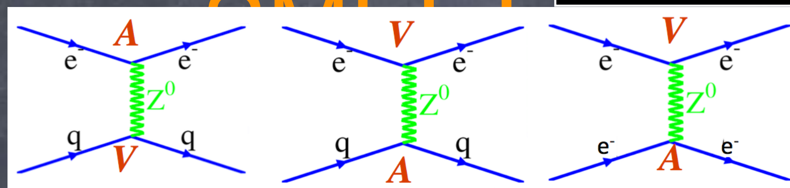
A. FALKOWSKI

Energy Scale

Theory

Experiment

P. SOUDER



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A. FALKOWSKI

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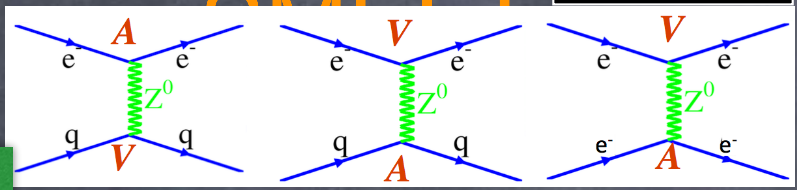

Integrate out W/Z, ...

$g_{AV}^{ef}\,[\bar{e}\gamma^\mu\gamma_5 e]\,[\bar{f}\gamma_\mu f]$

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Energy Scale | Theory | Experiment

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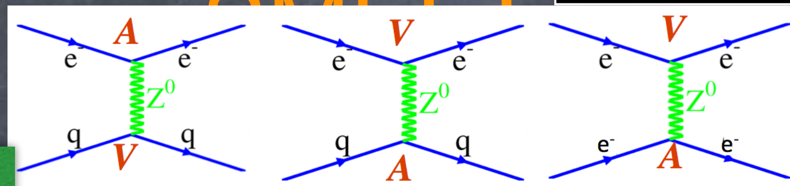

LHC

LEP

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10 GeV

, LEFT, WET)

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$g^{ef}_{\mathrm{AV}} = \frac{1}{2} - 2|Q_f|\sin^2\bar{\theta}(\mu)$

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APV

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Energy Scale | Theory | Experiment

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1 TeV

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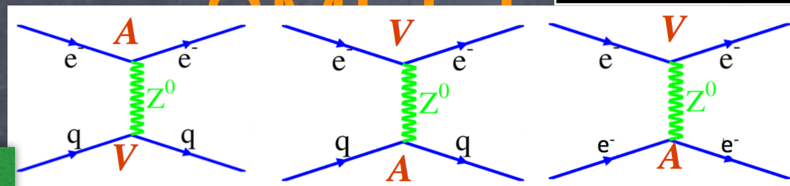

LHC

LEP

TRISTAN

$\frac{E}{\Lambda}$

E

Integrate out W/Z, ...

10 GeV

$$g_{AV}^{ef}\,[\bar{e}\gamma^{\mu}\gamma_5 e]\,[\bar{f}\gamma_{\mu} f]$$
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LEFT, WET

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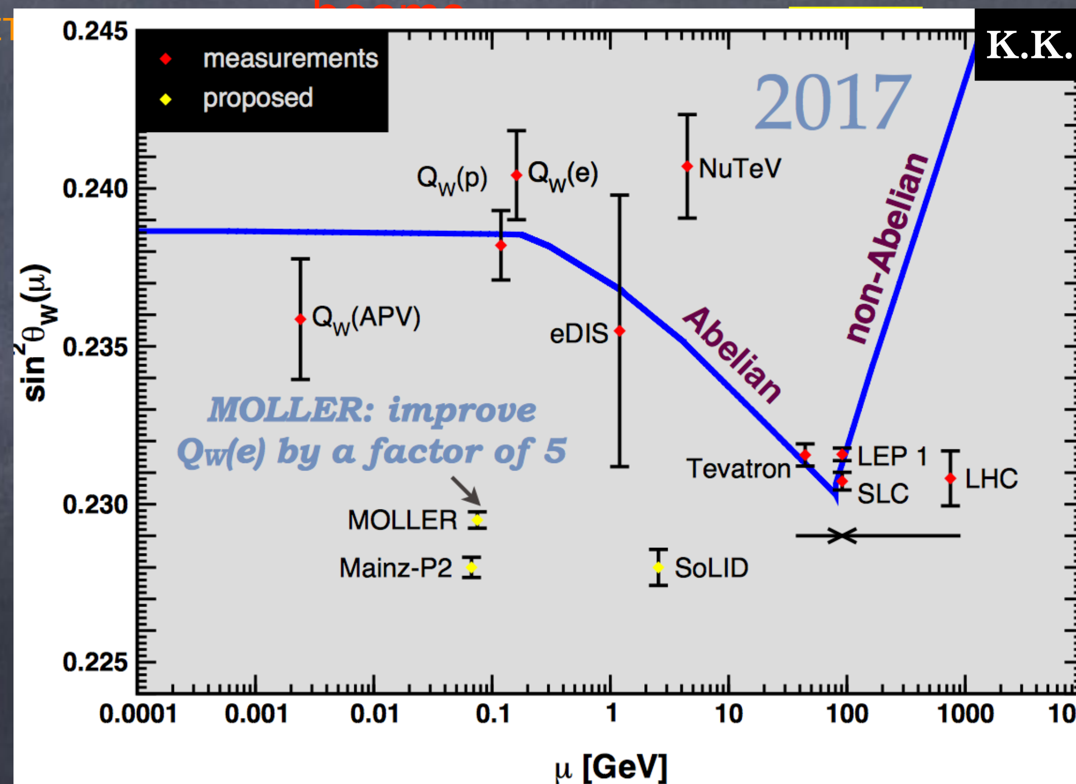
K.K.



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I question current practice of connecting EW and low energy scales.

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   After all, the Fermi theory is not a gauge theory.

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1. Strictly speaking, **the concept of weak mixing angle does not exist** below the Z/W pole (EW scale).

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2. The correct RG analysis requires **whole Wilson coefficients** to run rather than the parameters on which they depend.

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1. Capture all logs in a systematic manner

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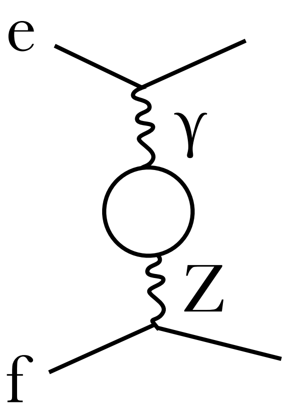
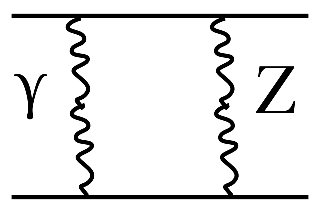
1. Capture <u>all</u> logs in a systematic manner

   "Running $\sin^2(\theta)$"





$\ln(m_f^2/s)$ all accounted

$\ln(m_Z^2/s)$ missed

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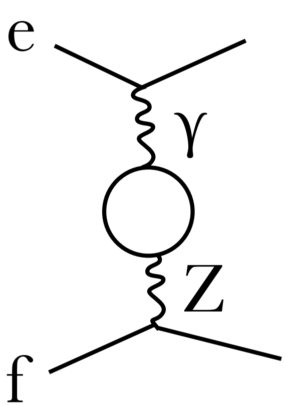

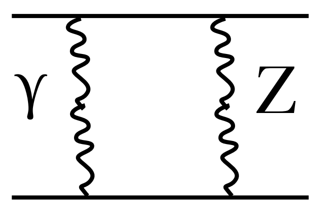

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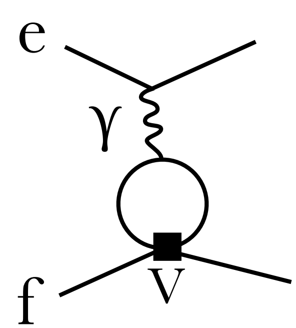
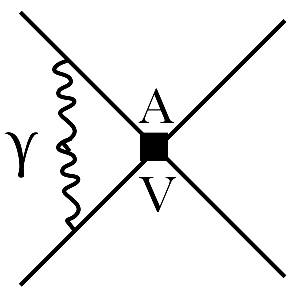
"Strict EFT"





$\ln(m_f^2/s) + \ln(m_Z^2/s)$

all accounted

Advantages

$$\cancel{\begin{aligned} g_{\mathrm{AV}}^{ef} &= \tfrac{1}{2} - 2|Q_f|\sin^2\bar{\theta}(\mu) \\ g_{\mathrm{VA}}^{ef} &= \tfrac{1}{2} - 2\sin^2\bar{\theta}(\mu) \end{aligned}}$$

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2. Less prone to making silly mistakes —
   Anapole moment is gauge dependent.

One loop matching generates
(apparently gauge-dependent Wilson coefficients)

$$\frac{a(\xi)}{m_Z^2}(\bar{\psi}\gamma_\mu\gamma_5\psi)\partial_\nu F^{\mu\nu} + \frac{g_{AV}(\xi)}{m_Z^2}(\bar{\psi}\gamma_\mu\gamma_5\psi)(\bar{\psi}\gamma_\mu\psi)$$

Advantages

$$\cancel{g^{ef}_{\mathrm{AV}} = \tfrac{1}{2} - 2|Q_f|\sin^2\bar{\theta}(\mu)}$$
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tree-level EOM: $\partial_\nu F^{\mu\nu} \to \bar{\psi}\gamma_\mu\psi$

$$\frac{g^{\mathrm{tot}}_{AV}}{m_Z^2}(\bar{\psi}\gamma_\mu\gamma_5\psi)(\bar{\psi}\gamma_\mu\psi)$$

Ramsey-Musolf—Holstein mechanism in EFT

Advantages

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$$\cancel{\begin{aligned} g_{\mathrm{AV}}^{ef} &= \tfrac{1}{2} - 2|Q_f|\sin^2\bar{\theta}(\mu) \\ g_{\mathrm{VA}}^{ef} &= \tfrac{1}{2} - 2\sin^2\bar{\theta}(\mu) \end{aligned}}$$

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3. Easier to calculate. After the EFT is set up.
   All radiative corrections are pure QED.

Can focus on the relevant degrees of freedom without
getting caught up in the complications of the heavy W/Z bosons.

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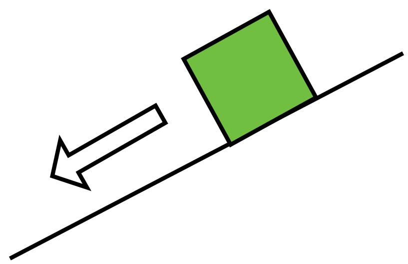

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vs

atomic motions (incl. W/Z)

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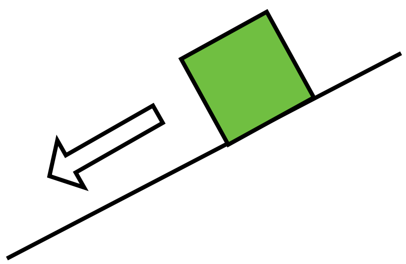

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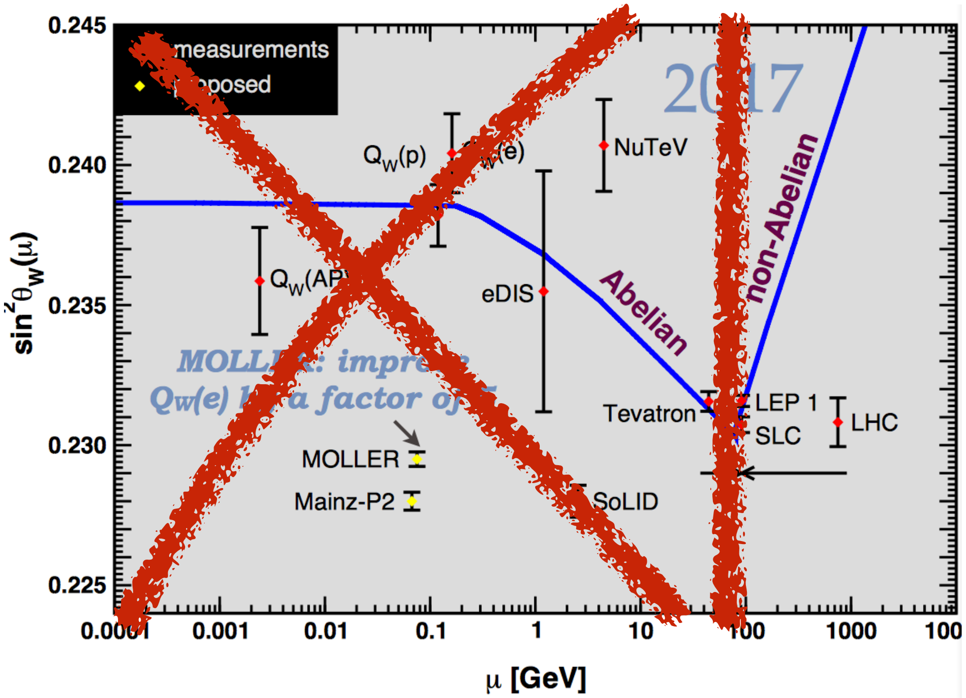
4. …and many more (see A. Falkowski's talk)

To summarize

Strictly speaking, shouldn't talk about $\sin^2(\theta)$ below the W/Z scale.



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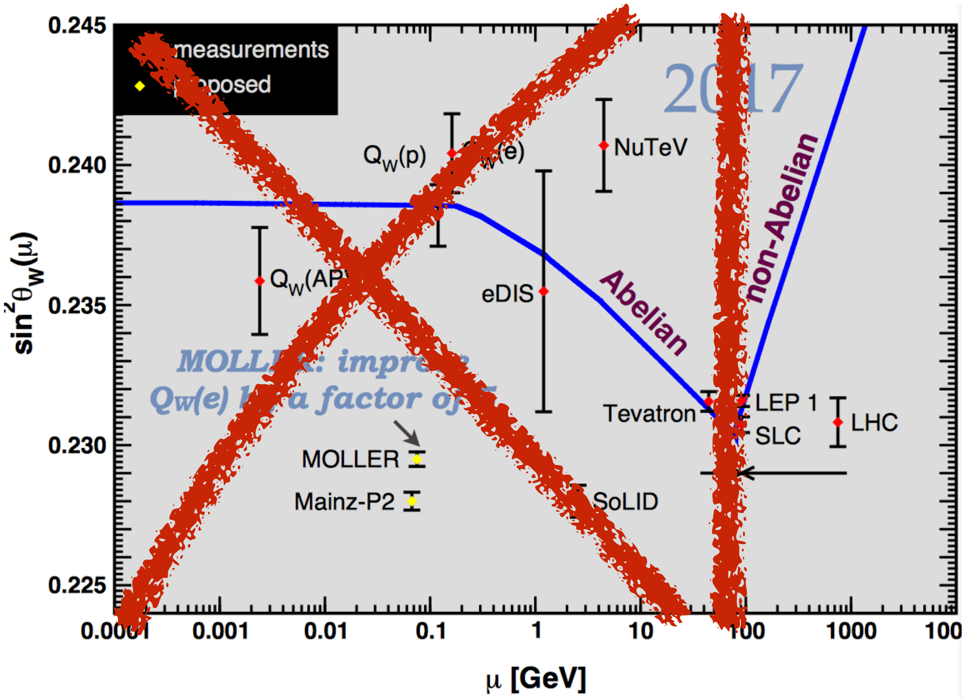

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Thank you

Special thanks to
Jens Erler,
Vincenzo Cirigliano,
Ayres Freitas,
Jordy De Vries

with whom I discussed by objections, and who confirm that I have not gone completely insane!

Any questions?
Discussions—