Weak Mixing Angle in the Thomson Limit

Bridging the SM to New Physics with the
Parity Violation Program at MESA 2018

Outline

- Introduction.
- Renormalization group evolution.
- Singlet contribution.
- Flavor separation.
- Theoretical uncertainties.
- Results and conclusions.

Introduction

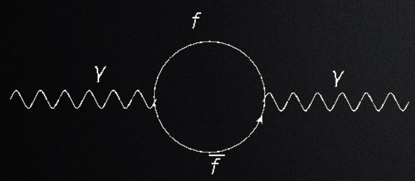
- The on shell scheme, preserves the mass relation to all orders.
- The $\overline{\text{MS}}$ scheme preserves the dependence on the couplings. Counter terms only absorb divergent terms. We will use this scheme.

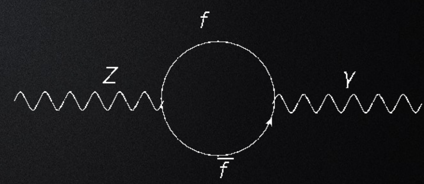
$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

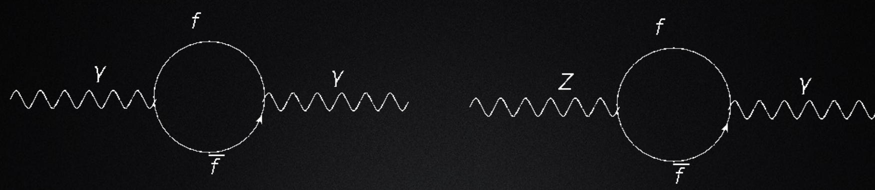
$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2}$$

Renormalization group evolution

Main idea: Relate the running of α to the running of the weak mixing angle. Based on Phys.Rev. D72 (2005) 073003.
Due to the same Lorentz structure, we can obtain the running of the vector coupling from the running of α. Then from here the running of the WMA.







$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi}\left[\frac{1}{24}\sum K_i \gamma_i Q_i^2 + \sigma\left(\sum_q Q_q\right)^2\right]$$

$$\mu^2 \frac{d\hat{v}_f}{d\mu^2} = \frac{\hat{\alpha}}{24\pi} Q_f \left[\sum K_i \gamma_i \hat{v}_i Q_i + 12\sigma\left(\sum_q \hat{v}_q\right)\left(\sum_q Q_q\right)\right]$$

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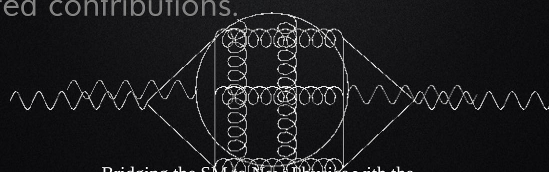
Renormalization group evolution

- Electromagnetic Running coupling constant.
- Ki contains information about higher order loops.
- Numerical factor depends on the type of particle.
- Charge of the particle.
- Disconnected contributions.

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi}\left[\frac{1}{24}\sum K_i \gamma_i Q_i^2 + \sigma\left(\sum_q Q_q\right)^2\right]$$



Renormalization group evolution

Key idea: We mix both equations in order to absorb the explicit dependence on α_s in the RGE of the vector coupling.

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi} \left[\frac{1}{24} \sum K_i \gamma_i Q_i^2 + \sigma \left(\sum_q Q_q \right)^2 \right]$$

$$\mu^2 \frac{d\hat{v}_f}{d\mu^2} = \frac{\hat{\alpha}}{24\pi} Q_f \left[\sum K_i \gamma_i \hat{v}_i Q_i + 12\sigma \left(\sum_q \hat{v}_q \right) \left(\sum_q Q_q \right) \right]$$

$$\hat{s}^2(\mu) = \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} \hat{s}^2(\mu_0) + \lambda_1 \left[1 - \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} \right] + \frac{\hat{\alpha}(\mu)}{\pi} \left[\frac{\lambda_2}{3} \ln \frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4} \ln \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \tilde{\sigma}(\mu_0) - \tilde{\sigma}(\mu) \right]$$

This is the solution to the running of the weak angle in terms of alpha.

Renormalization group evolution

$$\hat{s}^2(\mu) = \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\hat{s}^2(\mu_0) + \lambda_1\left[1 - \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\right] + \frac{\hat{\alpha}(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4}\ln\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \tilde{\sigma}(\mu_0) - \tilde{\sigma}(\mu)\right]$$

- Weak mixing angle at a scale μ.
- Fine structure constant at some scale μ.
- Numerical constants that depend on the number of particles in the EFT.
- Disconnected contributions.

This is the only term that contains explicit dependence on α_s!!
We can not use it at low energies! We need to estimate its size.

Where does the uncertainty comes from?

$$\hat{s}^2(\mu) = \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\hat{s}^2(\mu_0) + \lambda_1\left[1-\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\right] + \frac{\hat{\alpha}(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4}\ln\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \tilde{\sigma}(\mu_0) - \tilde{\sigma}(\mu)\right]$$

- We need to calculate the fine structure constant from hadronic data.
- How much does the strange contributes? different λ in this case!!
- What is the size of the explicit α_s dependent OZI contribution?
- What is the perturbative error?
- Are there any other sources?

Δα from hadronic data

The running of the fine structure constant can be written as

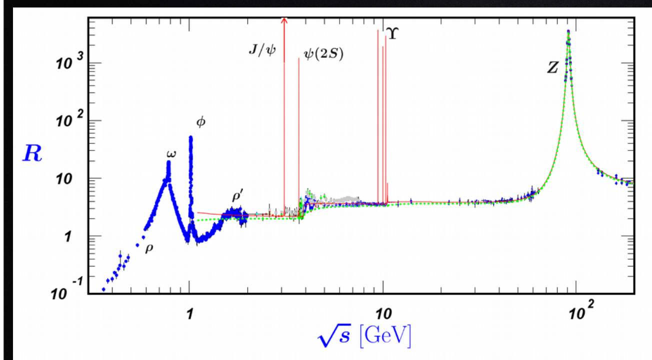
$$\hat{\alpha}\left(\mu\right) = \frac{\alpha}{1 - \Delta\alpha\left(\mu\right)}$$

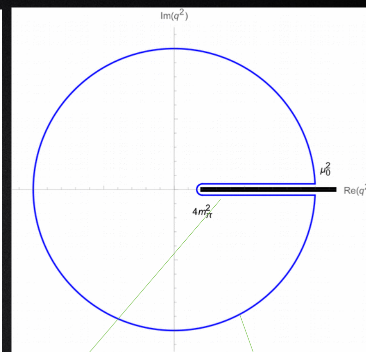
where

$$\Delta\alpha(\mu) = 4\pi\alpha\hat{\Pi}(0, \mu)$$

Where ∏ is the vacuum polarization function, to compute this for the three light quarks we use a contour in the complex q^2 plane.

Δα from hadronic data





$$\Delta\hat{\alpha}^{(3)}(\mu_0) = \frac{\alpha}{3\pi}\int_{4m_\pi^2}^{\mu_0^2} ds\,\frac{R(s)}{s - i\epsilon} + 4\pi I^{(3)},$$

Δα from hadronic data

We computed the integral, and did a cross check the RGE and the perturbative expantion for R.

$$
\begin{aligned}
4\pi I^{(3)} &= 2\alpha \int_0^{2\pi} d\theta\, \hat{\Pi}^{(3)}(\mu^2 e^{i\theta}) \\
&= \frac{2\alpha}{3\pi}\left[\frac{5}{3} + \left(\frac{55}{12} - 4\zeta(3) + 2\frac{\hat{m}_s^2}{\mu^2}\right)\left(\frac{\hat{\alpha}_s}{\pi} + \frac{\hat{\alpha}}{4\pi}\right)\right. \\
&+ \left(\frac{34525}{864} - \frac{9}{4}\zeta(2) - \frac{715}{18}\zeta(3) + \frac{25}{3}\zeta(5) + \frac{125}{12}\frac{\hat{m}_s^2}{\mu^2} + F(\hat{m}_c, \hat{m}_b)\right)\frac{\hat{\alpha}_s^2}{\pi^2} \\
&+ \left(\frac{7012579}{13824} - \frac{961}{16}\zeta(2) - \frac{76681}{144}\zeta(3) + \frac{12515}{288}\zeta(5)\right. \\
&\left.\left. - \frac{665}{36}\zeta(7) + \frac{81}{2}\zeta(2)\zeta(3) + \frac{155}{2}\zeta(3)^2\right)\frac{\hat{\alpha}_s^3}{\pi^3}\right] \\
&= (24.86 \pm 0.18 - 43\,\Delta\hat{\alpha}_s) \times 10^{-4},
\end{aligned}
$$

Perturbative uncertainty

Δα from hadronic data

Including the integration of the cross section ratio R we get

$$\Delta^{(3)}\hat{\alpha}(2.0\mathrm{GeV}) = (83.56 \pm 0.45 \pm 0.18) \times 10^{-4}$$

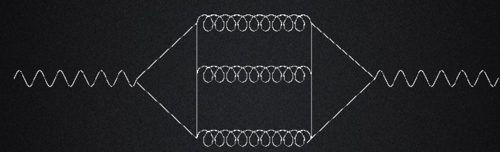
Using the master equation, we can easily propagate this uncertainty to the low energy weak mixing angle.

$$\delta \sin^2\hat{\theta}_W(0) = \left(\frac{1}{2} - \sin^2\hat{\theta}_W\right) \delta\Delta^{(3)}\alpha(2.0\,\mathrm{GeV})$$

$$\delta \sin^2\hat{\theta}_W(0) = \pm 1.2 \times 10^{-5}$$

Disconnected contributions

These comes from diagrams like



Known to be explicitly small in the perturbative limit. In the hadronic region, these are responsible of effects like the suppresion of ϕ going to three pions.

Disconnected contributions

We can roughly estimate the contribution of these diagrams assuming α_s to be of order one. This implies,

$$\Delta_{\mathrm{disc}} \sin\hat{\theta}(0) \approx \; 10^{-6}$$

By construction the contribution of the disconnected diagrams to the weak angle is related to the one in α. For μ less than the charm mass we get the useful relation

$$\tilde{\sigma}(\mu) - \tilde{\sigma}(\mu_0) = -\lambda_1 \frac{\pi}{\alpha} [\Delta_{\mathrm{disc}} \alpha(\mu) - \Delta_{\mathrm{disc}} \alpha(\mu_0)]$$

Disconnected contributions

$$\tilde{\sigma}(\mu) - \tilde{\sigma}(\mu_0) = -\lambda_1 \frac{\pi}{\alpha}[\Delta_{\rm disc}\alpha(\mu) - \Delta_{\rm disc}\alpha(\mu_0)]$$

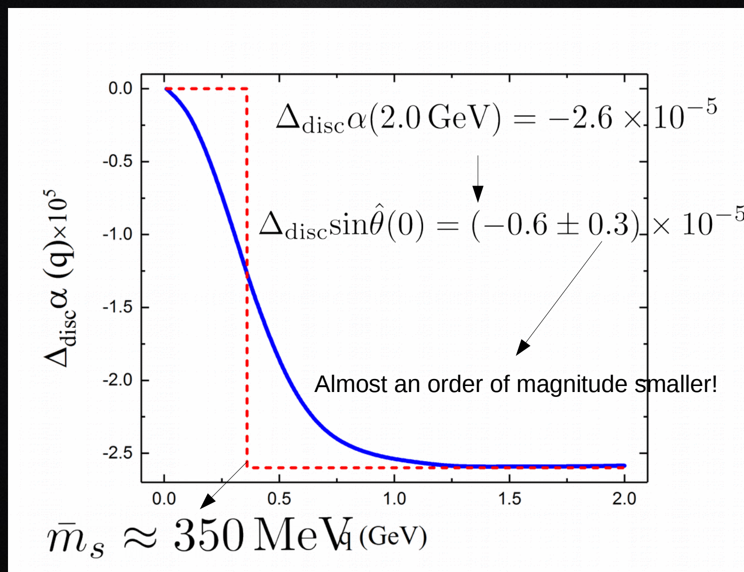
We can use lattice calculations to constraint these contribution! In Phys.Rev.Lett. 116 (2016) no.23, 232002, they calculate these contributions for a_μ.

As a first estimate we can use the fact that the kernel for a_μ is enhanced at low energies compared to the one for Δα. This gives us the an upper bound

$$|\Delta_{\rm disc}\alpha(1.8\mathrm{GeV})| < 8 \times 10^{-5}$$

Disconnected contributions

Using the lattice results, the dependence on q^2 for $\Delta\alpha$ can be obtained



Flavor separation

How much dœs the strange contributes relative to the up and down quarks? essentially we want to split the total contribution as:

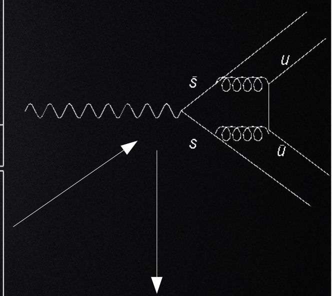
$$\Delta\hat{\alpha}^{(3)}\left(\bar{m}_c\right) = \Delta_s\hat{\alpha}\left(\bar{m}_c\right) + \Delta_{u,d}\hat{\alpha}\left(\bar{m}_c\right) = 6\Delta_s\hat{\alpha}\left(\bar{m}_c\right) + \Delta_{u,d}\hat{\alpha}\left(\bar{m}_s\right)$$

In the past this was constrained using the SU(3) limit, and the heavy quark limit. This is where the largest uncertainty came from.

Flavor separation

Now we use a data driven approach. First we look to specific channels that we can associate with a strange quark current. using the results from Eur.Phys.J. C77 (2017) no.12, 827

channel	$a_\mu \times 10^{10}$	$\Delta\alpha \times 10^4$
ϕ	38.43	5.13
$K\bar{K}\pi$	2.45	0.78
$\eta\phi$	0.36	0.13
PQCD [?] ($> 1.8\,\mathrm{GeV}$)	7.30	—
Total	48.54	6.04
$K\bar{K}$ (non $- \phi$)	3.62	0.76
$K\bar{K}2\pi$	0.85	0.30
$K\bar{K}3\pi$	-0.03	-0.01
$K\bar{K}\eta$	0.01	0.00
$K\bar{K}\omega$	0.01	0.00
Total	4.46	1.05



Hard to distinguish!!

Flavor separation

As a very conservative approach we can take 50%±50% of the contribution of these channels to come from the strange.

$$\Delta_s \alpha(1.8\ \mathrm{GeV}) = (6.56 \pm 0.11 \pm 0.19 \pm 0.53) \times 10^{-4} = (6.56 \pm 0.57) \times 10^{-4}.$$

Experimental uncertainties

Kaon channels.

Parametrization uncertainties.

Flavor separation

Nevertheless we expect this kaon contributions to come from strange currents.

We can use a lattice calculation (JHEP 1604 (2016) 063,), which computes the contribution of the strange to a_μ. To assign an error to this assumption.

$$\Delta_s \alpha(1.8 \text{ GeV}) = (7.09 \pm 0.11 \pm 0.19 \pm 0.23) \times 10^{-4} = (7.09 \pm 0.32) \times 10^{-4},$$

From another lattice calculation (Phys.Rev. D89 (2014) no.11, 114501) we estimate

$$\Delta_{\mathrm{s}}^{lattice} \alpha(2.0\,\mathrm{GeV}) \approx (6.9 \pm 0.5) \times 10^{-4}$$

Flavor separation

Now we can propagate this uncertainty to the weak mixing angle.

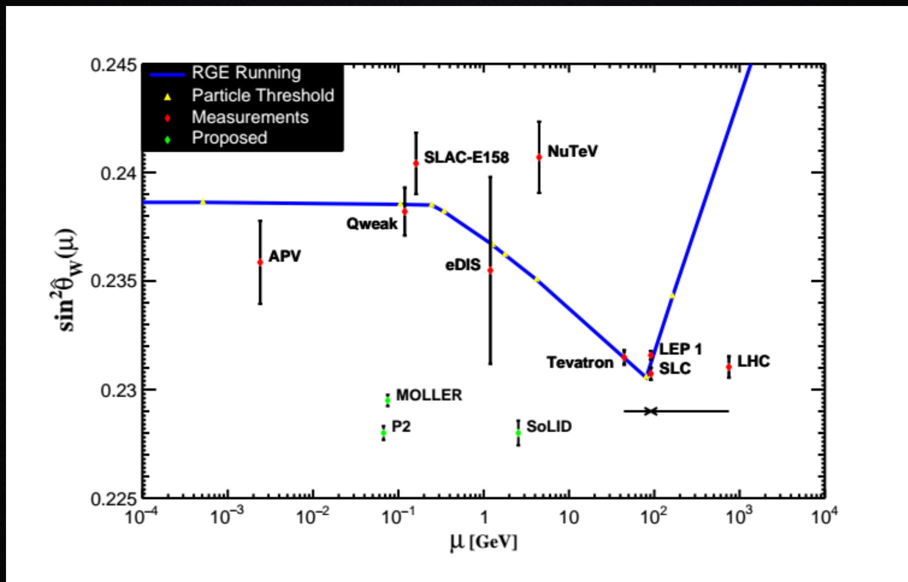
$$\delta\hat{s}^2(0) \simeq \frac{1}{20}\,\delta\Delta\hat{\alpha}^{(2)}(\bar{m}_c) = \pm 1.0 \times 10^{-5},$$

where

$$\Delta\hat{\alpha}^{(2)}(\bar{m}_s) = \Delta\hat{\alpha}^{(3)}(\bar{m}_c) - 6\Delta_s\hat{\alpha}(\bar{m}_s)$$

Uncertainties

source	$\delta \sin^2 \hat{\theta}_W(0) \times 10^5$
$\Delta\hat{\alpha}^{(3)}(2\ \text{GeV})$	1.2
flavor separation	1.0
isospin breaking	0.7
singlet contribution	0.3
PQCD	0.6
Total	1.8



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Final results

Using the result $\sin\hat{\theta}_w(M_Z) = 0.23129(5)$ from a global fit to electroweak data we get

$$\sin\hat{\theta}_w(0) = 0.23868(5)(2)$$

In terms of $\sin\hat{\theta}_W(0) \equiv \hat{\kappa}(0)\sin\hat{\theta}_W(M_Z)$ this result reads

$$\hat{\kappa}(0) = 1.03196 \pm 0.00006 + 1.14\,\tilde{\Delta}\alpha + 0.025\,\Delta\hat{\alpha}_s - 0.0016\,\Delta\hat{m}_c - 0.0012\,\Delta\hat{m}_b$$

Conclusions.

- We updated the inputs for different data.
- Included next order contributions for QCD in the RGE.
- Different method to handle OZI contribution.
- Different method to handle the flavour separation.
- The uncertainty is now four times smaller.

Extra stuff starts here….

To estimate the size of the SU(2) breaking we assume that the SU(2) breaking is as large as the SU(3) one. This gives us

$$\Delta\alpha^{(1)}(\bar{m}_d) < 14.8 \times 10^{-4}.$$

We propagate this to the weak mixing angle, giving us

$$\delta\hat{s}^2(0) = -\frac{3}{40}\Delta\alpha^{(1)}(\bar{m}_d) > -1.1 \times 10^{-4}.$$

Using as a measure if the SU(2) breaking relative to the SU(2) breaking the ratio

$$\left|\frac{M^2_{K^{*\pm}} - M^2_{K^{*0}}}{M^2_{K^{*\pm}} - M^2_{\rho^0}}\right| \approx 0.06,$$

$$\delta\hat{s}^2(0) = {}^{+0}_{-7} \times 10^{-6}.$$

Matching conditions

At each threeshold we have to match the electromagnetic coupling constant. This is very similar to the QCD mathcing. Using results from Chetyrkin 2006 we get

$$
\begin{aligned}
\frac{\pi}{\hat{\alpha}(m_f)^+} &= \frac{\pi}{\hat{\alpha}(m_f)^-} - \frac{15}{16} N_f^c \frac{\hat{\alpha}(m_f)}{\pi} Q_f^4 \\
&- \frac{N_f^c - 1}{2} \left[\frac{13}{12} \frac{\hat{\alpha}_s^+}{\pi} + \left(\frac{655}{144}\zeta_3 - \frac{3847}{864} + \frac{361}{1296} n_q \right) \frac{\hat{\alpha}_s^{+2}}{\pi^2} \right. \\
&+ \left. \left(-0.55739 - 0.92807\, n_q + 0.01928\, n_q^2 \right) \frac{\hat{\alpha}_s^{+3}}{\pi^3} \right] Q_f^2 \\
&- \frac{N_f^c - 1}{2} \left[\frac{295}{1296} \frac{\hat{\alpha}_s^{+2}}{\pi^2} + (\mathcal{K}_1 + \mathcal{K}_2 n_q) \frac{\hat{\alpha}_s^{+3}}{\pi^3} \right] \sum_\ell Q_\ell^2 \,.
\end{aligned}
$$

Threshold masses

We define a threshold mass as the t' hooft scale where the matching conditions become trivial. We obtain it up to next order in QCD. We computed this in the perturbative regime

$$\begin{aligned}\bar{m} &= \hat{m}\Big\{1 - \frac{13}{24}\frac{\hat{\alpha}_s}{\pi} + \left(\frac{10073}{3456} - \frac{655}{288}\zeta_3 - \frac{361}{2592}n_q\right)\frac{\hat{\alpha}_s^2}{\pi^2} \\ &+ \left(1.61024 + 0.59599\, n_q - 0.00964\, n_q^2\right)\frac{\hat{\alpha}_s^3}{\pi^3} \\ &+ \left[-\frac{295}{2592}\frac{\hat{\alpha}_s^2}{\pi^2} + \left(\frac{5767}{62208} - \frac{\mathcal{K}_1 + \mathcal{K}_2 n_q}{2}\right)\frac{\hat{\alpha}_s^3}{\pi^3}\right]\frac{\sum Q_\ell^2}{Q_h^2}\Big\}.\end{aligned}$$

Threshold masses light quarks

This defintion implies that for the light quarks we must have

$$\Delta_s \hat{\alpha}(\bar{m}_c) = Q_s^2 \frac{\alpha}{\pi} K_{\mathrm{QCD}}^s(\bar{m}_c) \ln \frac{\bar{m}_c^2}{\bar{m}_s^2}$$

$$K_{\mathrm{QCD}}^c(\bar{m}_c) < K_{\mathrm{QCD}}^s(\bar{m}_c).$$

$$\bar{m}_s < 390 \text{ MeV},$$

$$K_{\mathrm{QCD}}^s(\bar{m}_c) = 1.34 \pm 0.16,$$

$$\bar{m}_s = 342^{+48}_{-53} \text{ MeV}.$$