Higher-order electroweak corrections for PV asymmetries

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MITP Workshop PVES 2018

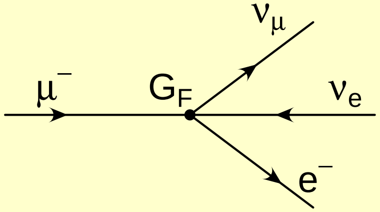
1. Z and W boson physics

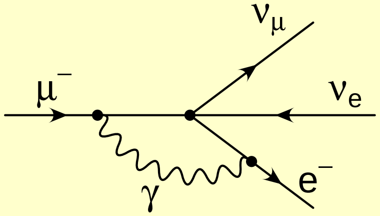
2. Calculational challenges and techniques

3. Low-energy parity violation

W mass

μ decay in Fermi Model



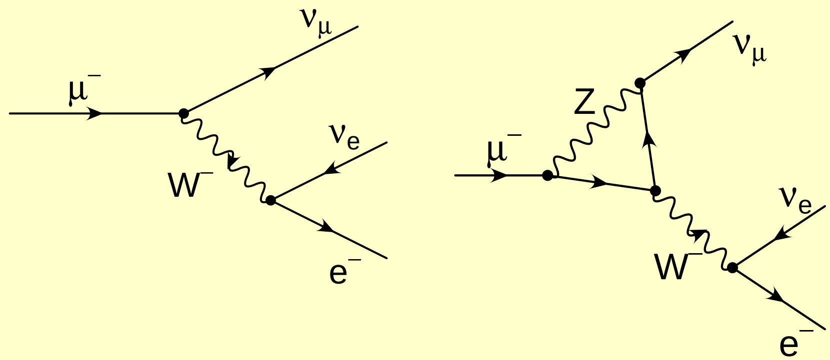


← QED corr.
(2-loop)

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)(1 + \Delta q)$$

Ritbergen, Stuart ’98
Pak, Czarnecki ’08

μ decay in Standard Model



$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}(1 + \Delta r)$$

electroweak corrections

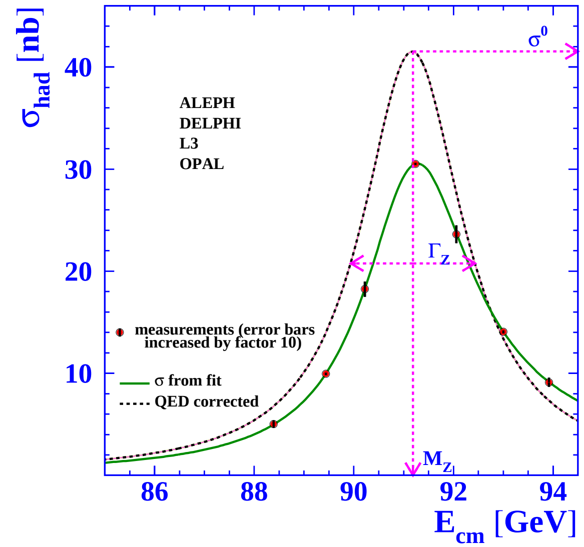
Z-pole observables

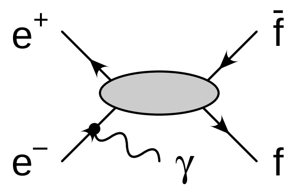
- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Skrzypek '92
Montagna, Nicrosini, Piccinini '97

LEP EWWG '05





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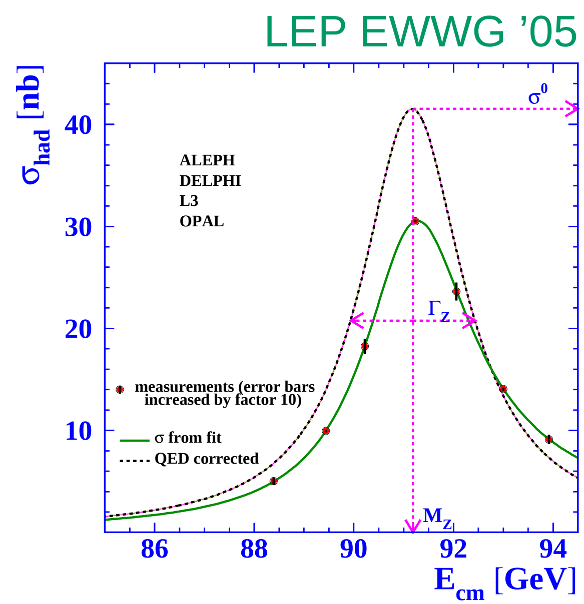
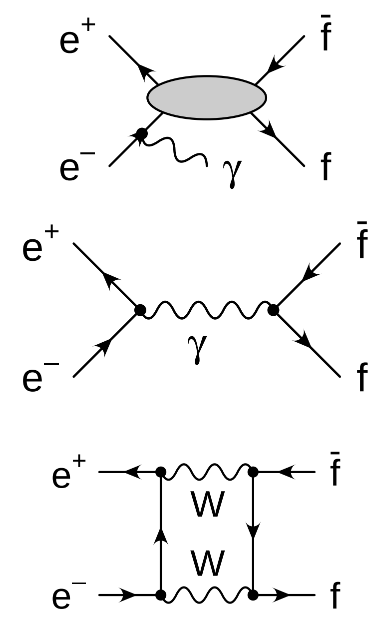
$$\sigma_{\text{hard}} = \sigma_Z + \sigma_\gamma + \sigma_{\gamma Z} + \sigma_{\text{box}}$$





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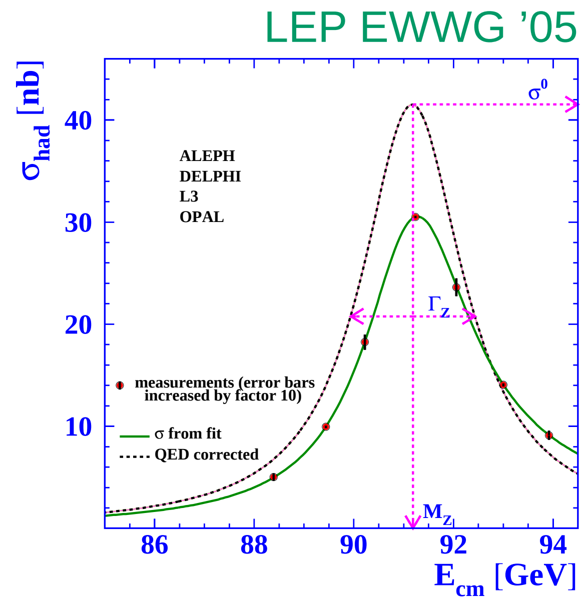
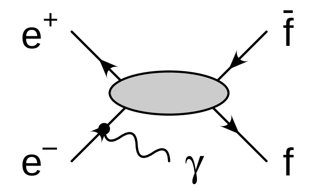
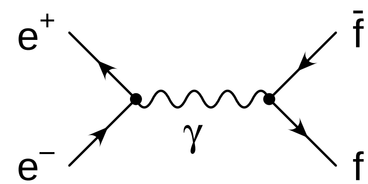
$$\sigma_{\text{hard}} = \sigma_{\text{Z}} + \sigma_{\gamma} + \sigma_{\gamma\text{Z}} + \sigma_{\text{box}}$$

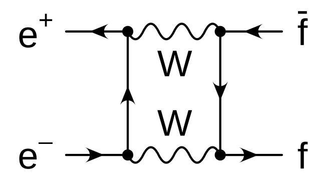
- Z-pole contribution:

$$\sigma_{\text{Z}} = \frac{R}{(s - \overline{M}_{\text{Z}}^2)^2 + \overline{M}_{\text{Z}}^2\overline{\Gamma}_{\text{Z}}^2} + \sigma_{\text{non-res}}$$









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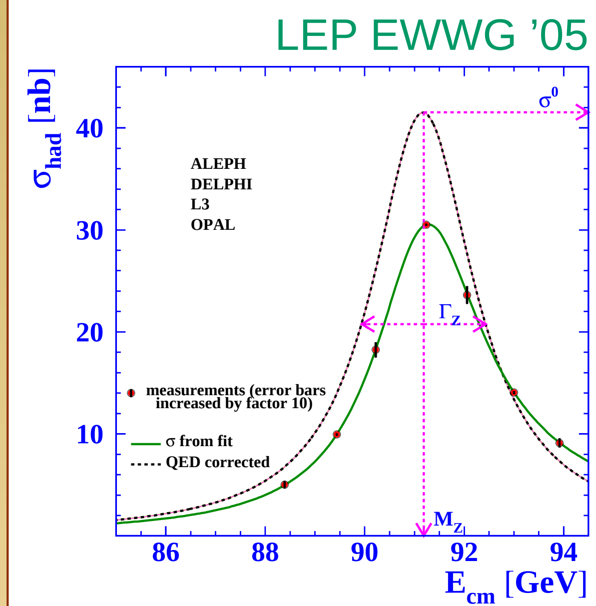
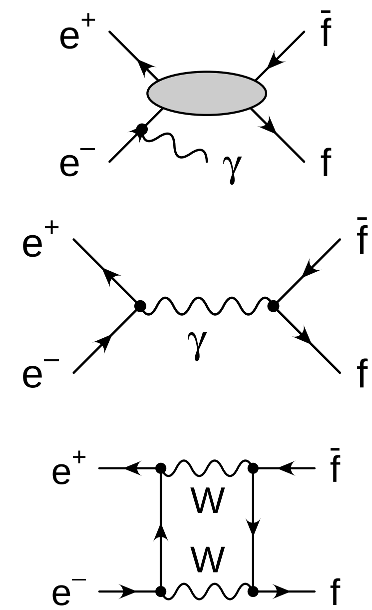
- In experimental analyses:

$$\sigma \sim \frac{1}{(s-M_{\text{Z}}^2)^2 + s^2\Gamma_{\text{Z}}^2/M_{\text{Z}}^2}$$

$$\overline{M}_{\text{Z}} = M_{\text{Z}}\Big/\sqrt{1+\Gamma_{\text{Z}}^2/M_{\text{Z}}^2} \approx M_{\text{Z}} - 34\ \text{MeV}$$

$$\overline{\Gamma}_{\text{Z}} = \Gamma_{\text{Z}}\Big/\sqrt{1+\Gamma_{\text{Z}}^2/M_{\text{Z}}^2} \approx \Gamma_{\text{Z}} - 0.9\ \text{MeV}$$





Z-pole observables

Relevant pseudo-observables:

- Total width $\overline{\Gamma}_Z$
- Partial widths $\overline{\Gamma}_f = \Gamma[Z \to f\bar{f}]_{s=\overline{M}_Z^2}$
- Peak cross-section $\sigma^0_{\text{had}} = \sigma_Z(s = \overline{M}_Z^2)$
- Branching ratios:

  $R_q = \Gamma_q/\Gamma_{\text{had}}$ ($q = b, c$, probes heavy quark generations)

  $R_\ell = \Gamma_{\text{had}}/\Gamma_\ell$ ($\ell = e, \mu, \tau$)

Z-pole observables

Effective weak mixing angle:

Z-pole asymmetries:

$$A_{\mathrm{FB}}^{f} \equiv \frac{\sigma(\theta < \frac{\pi}{2}) - \sigma(\theta > \frac{\pi}{2})}{\sigma(\theta < \frac{\pi}{2}) + \sigma(\theta > \frac{\pi}{2})} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$

$$A_{\mathrm{LR}} \equiv \frac{\sigma(\mathcal{P}_e > 0) - \sigma(\mathcal{P}_e < 0)}{\sigma(\mathcal{P}_e > 0) + \sigma(\mathcal{P}_e < 0)} = \mathcal{A}_e$$

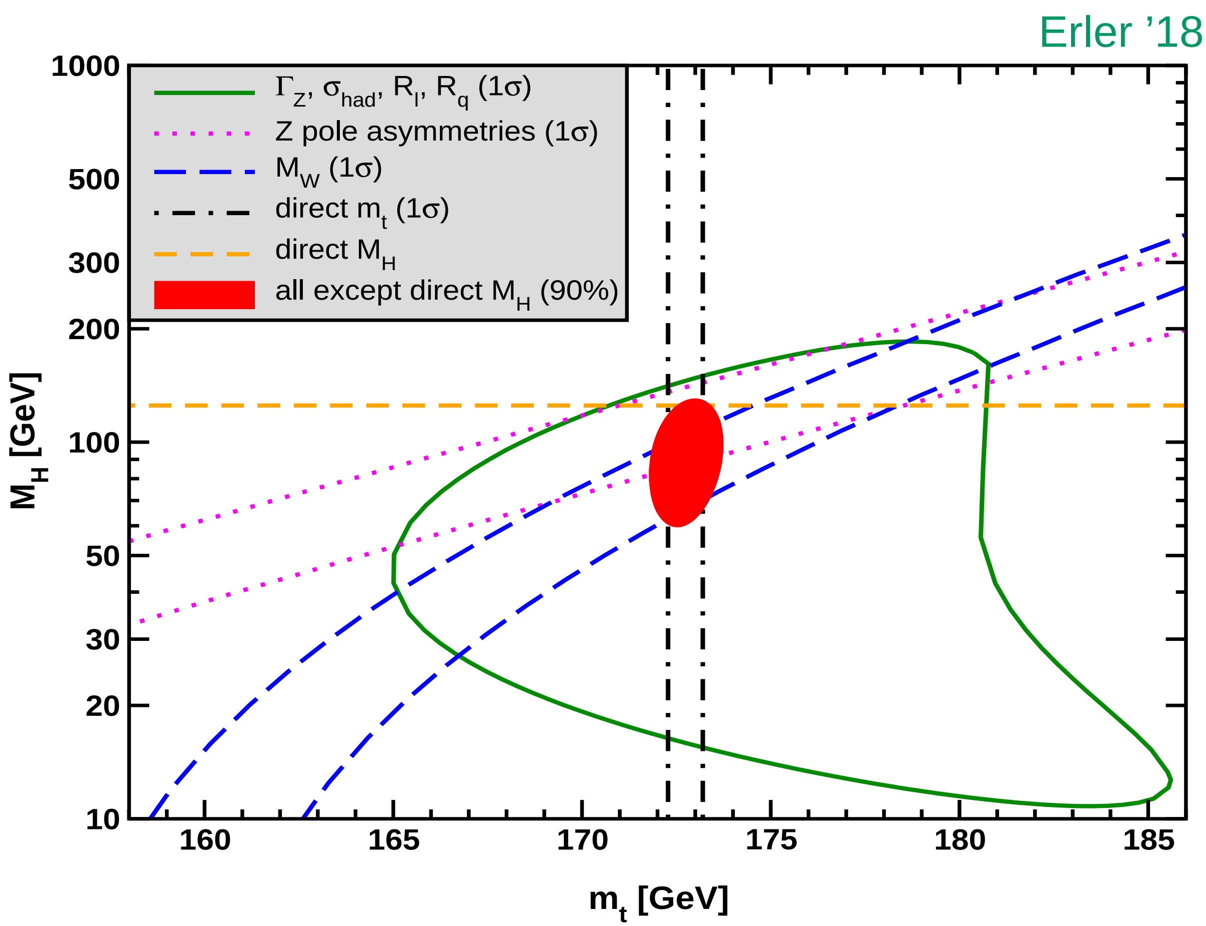
$$\mathcal{A}_f = 2\frac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2} = \frac{1 - 4|Q_f|\sin^2\theta_{\mathrm{eff}}^{f}}{1 - 4|Q_f|\sin^2\theta_{\mathrm{eff}}^{f} + 8(|Q_f|\sin^2\theta_{\mathrm{eff}}^{f})^2}$$

Most precisely measured for $f = \ell$ (also $f = b, c$)

Current status of electroweak precision tests

Standard Model after Higgs discovery:

- Good agreement between measured mass and indirect prediction
- Very good agreement over large number of observables



Direct measurements:

$M_H = 125.14 \pm 0.15$ GeV

$m_t = 172.74 \pm 0.46$ GeV

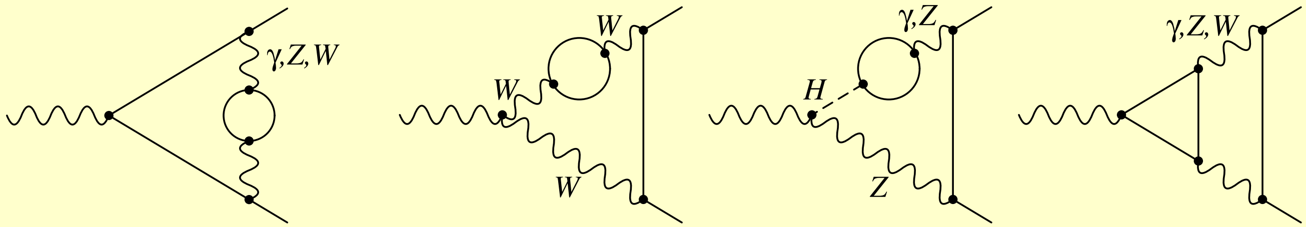
Indirect prediction:

$M_H = 90^{+17}_{-16}$ GeV

$m_t = 176.4 \pm 1.8$ GeV

Current status of SM loop results

Known corrections to Δr, $\sin^2\theta^f_{\rm eff}$, g_{Vf}, g_{Af}:



- Complete NNLO corrections

Freitas, Hollik, Walter, Weiglein '00
Awramik, Czakon '02; Onishchenko, Veretin '02
Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14
Freitas '13,14; Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16,18

- Partial 3/4-loop corrections to ρ/T-parameter
  $\mathcal{O}(\alpha_t\alpha_s^2)$, $\mathcal{O}(\alpha_t^2\alpha_s)$, $\mathcal{O}(\alpha_t\alpha_s^3)$

Chetyrkin, Kühn, Steinhauser '95
Faisst, Kühn, Seidensticker, Veretin '03
Boughezal, Tausk, v. d. Bij '05
Schröder, Steinhauser '05; Chetyrkin et al. '06
Boughezal, Czakon '06

$(\alpha_t \equiv \frac{y_t^2}{4\pi})$

Current uncertainties

	Experiment	Theory error	Main source
M_W	80.385 ± 0.015 MeV	4 MeV	α^3, $\alpha^2\alpha_\text{s}$
Γ_Z	2495.2 ± 2.3 MeV	0.4 MeV	α^3, $\alpha^2\alpha_\text{s}$, $\alpha\alpha_\text{s}^2$
σ^0_had	41540 ± 37 pb	6 pb	α^3, $\alpha^2\alpha_\text{s}$
$R_b \equiv \Gamma^b_\text{Z}/\Gamma^\text{had}_\text{Z}$	0.21629 ± 0.00066	0.0001	α^3, $\alpha^2\alpha_\text{s}$
$\sin^2\theta^\ell_\text{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_\text{s}$

- Theory error estimate is not well defined, ideally $\Delta_\text{th} \ll \Delta_\text{exp}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence
- Also parametric error from external inputs (m_t, m_b, α_s, $\Delta\alpha_\text{had}$, ...)

Calculational challenges and techniques

Full SM corrections at $\geq$2-loop:

- Large number of diagrams and tensor integrals, $\mathcal{O}(100) - \mathcal{O}(10000)$
- Many different scales (masses and ext. momenta)

Computer algebra methods:

- Generation of diagrams with *FeynArts*, *QGraf*, ...
  Küblbeck, Eck, Mertig '92, Hahn '01
  Nogueira '93
- Dirac/Lorentz algebra with *Form*, *FeynCalc*, ...
  Vermaseren '89,00
  Mertig '93

Evaluation of loop integrals:

- In general not possible analytically
- Numerical methods must be automizable, stable, fastly converging
- Need procedure for isolating divergent pieces

Asymptotic expansions

- Exploit large mass ratios,
  $e.\,g.\ M_{\rm Z}^2/m_{\rm t}^2 \approx 1/4$
- Evaluate coeff. integrals analytically
- Fast numerical evaluation

Current status:

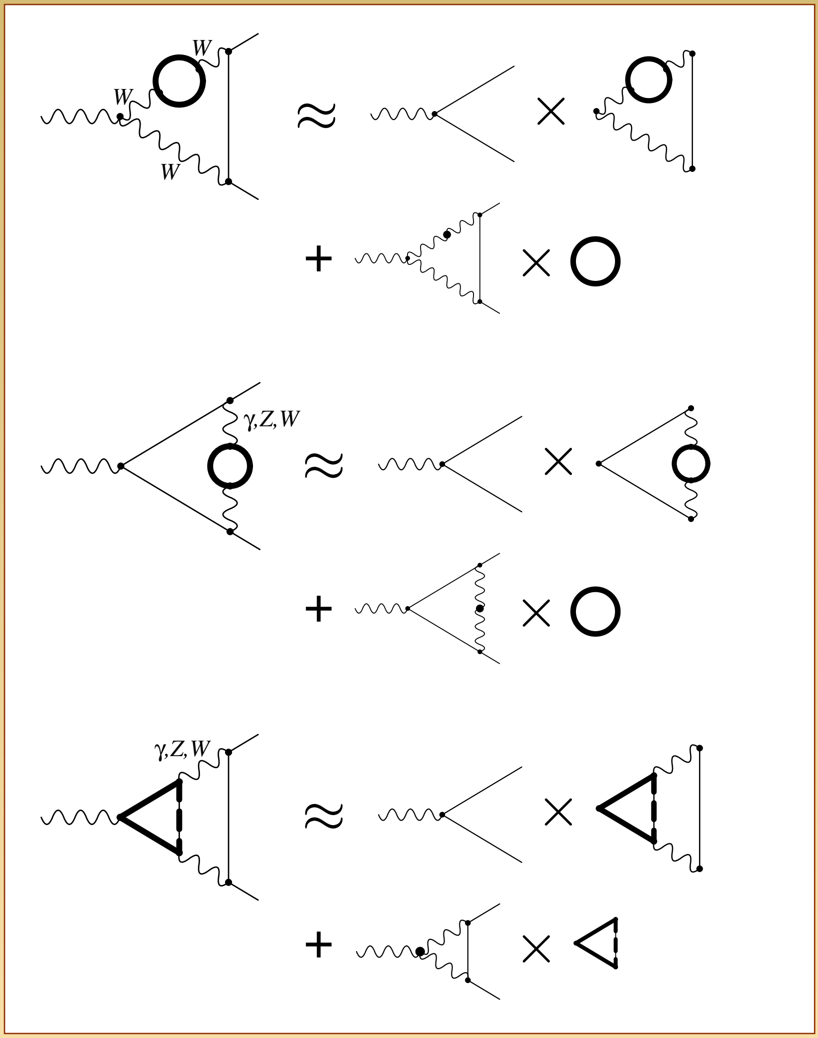
Two-scale problems: $\mathcal{O}(\alpha\alpha_{\rm s}^n)$ for $\Delta\rho$, Δr

$\longrightarrow$ Several expansion terms up to 3-loop, leading term up to 4-loop

Djouadi, Verzegnassi ’87; Bardin, Chizhov ’88
Chetyrkin, Kühn, Steinhauser ’95
Faisst, Kühn, Seidensticker, Veretin ’03; ...

Three-scale problems: $Zf\bar{f}$ vertex at 2-loop

Barbieri et al. ’92,93
Fleischer, Tarasov, Jegerlehner ’93,95
Degrassi, Gambino, Sirlin ’97
Awramik, Czakon, Freitas, Weiglein ’04



Extendability: Promising, limited by computing/algorithms

Direct numerical integration

General form of Feynman integral:

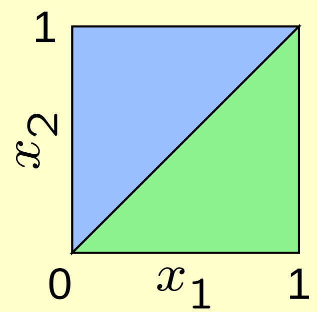
$$I = \int_0^1 dx_1 \dots dx_n \, \delta(1 - \sum_i x_i) \, \frac{N(x_i)}{D(x_i)^{r+\varepsilon}}$$

$\longrightarrow$ Can be integrated numerically (if finite)

Alternatives: Integration in momentum space, Mellin-Barnes space

Treatment of divergencies:

- **Sector decomposition:** Sub-divide integration space such that divergent terms factorize
  Binoth, Heinrich '00,03



- **Subtraction terms:** Remove divergencies with simple terms that can be integrated analytically

Nagy, Soper '03
Becker, Reuschle, Weinzierl '10; Freitas '12

Numerical integration over Feynman parameters

- After removal of singularities through sector decomposition:

$$I^{(1)}_{\text{reg}} = \int_0^1 dx_1 \dots dx_{n-1} \, (A - i\epsilon)^{-k}$$

- Physical thresholds: A changes sign in ingration region
  - $\longrightarrow$ Problematic for numerical integrators
  - $\longrightarrow$ Deform integration into complex plane: Nagy, Soper '06

$$x_i = z_i - i\lambda z_i(1 - z_i)\frac{\partial A}{\partial z_i}, \qquad 0 \leq z_i \leq 1.$$

$$A(\vec{x}) = A(\vec{z}) - i\lambda \sum_i z_i(1 - z_i)\left(\frac{\partial A}{\partial z_i}\right)^2 + \mathcal{O}(\lambda^2).$$

Typical choice: $\lambda \sim 0.5{-}1$

- Potential issues:
  - $\longrightarrow$ $\partial A/\partial z_i$ may vanish in certain sub-spaces
  - $\longrightarrow$ Thresholds may be at edge of integration region

Mellin-Barnes representations

Transform Feynman integral with Mellin-Barnes representation

$$\frac{1}{(A_0 + \ldots + A_m)^Z} = \frac{1}{(2\pi i)^m} \int_{\mathcal{C}_1} dz_1 \cdots \int_{\mathcal{C}_m} dz_m$$
$$\times A_1^{z_1} \cdots A_m^{z_m} A_0^{-Z-z_1-\ldots-z_m}$$
$$\times \frac{\Gamma(-z_1) \cdots \Gamma(-z_m) \Gamma(Z + z_1 + \ldots + z_m)}{\Gamma(Z)},$$

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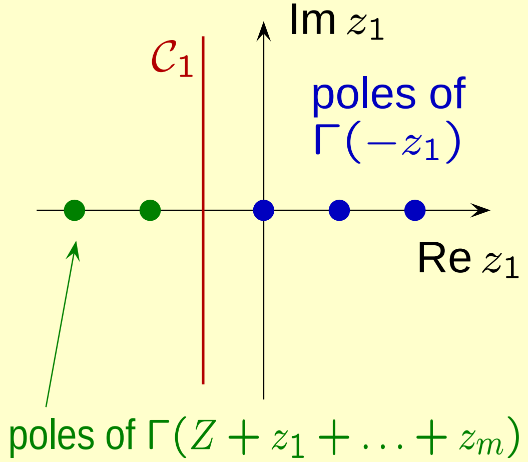

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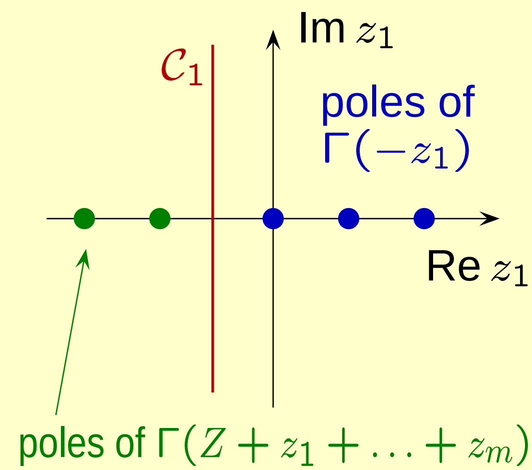
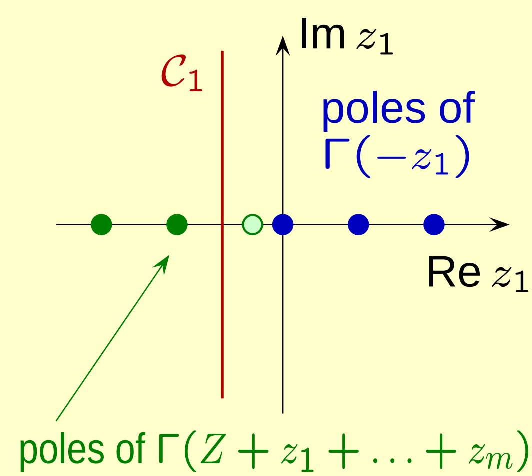


- For $\varepsilon \longrightarrow 0$: residues from pole crossings
  $\longrightarrow 1/\varepsilon^k$ terms

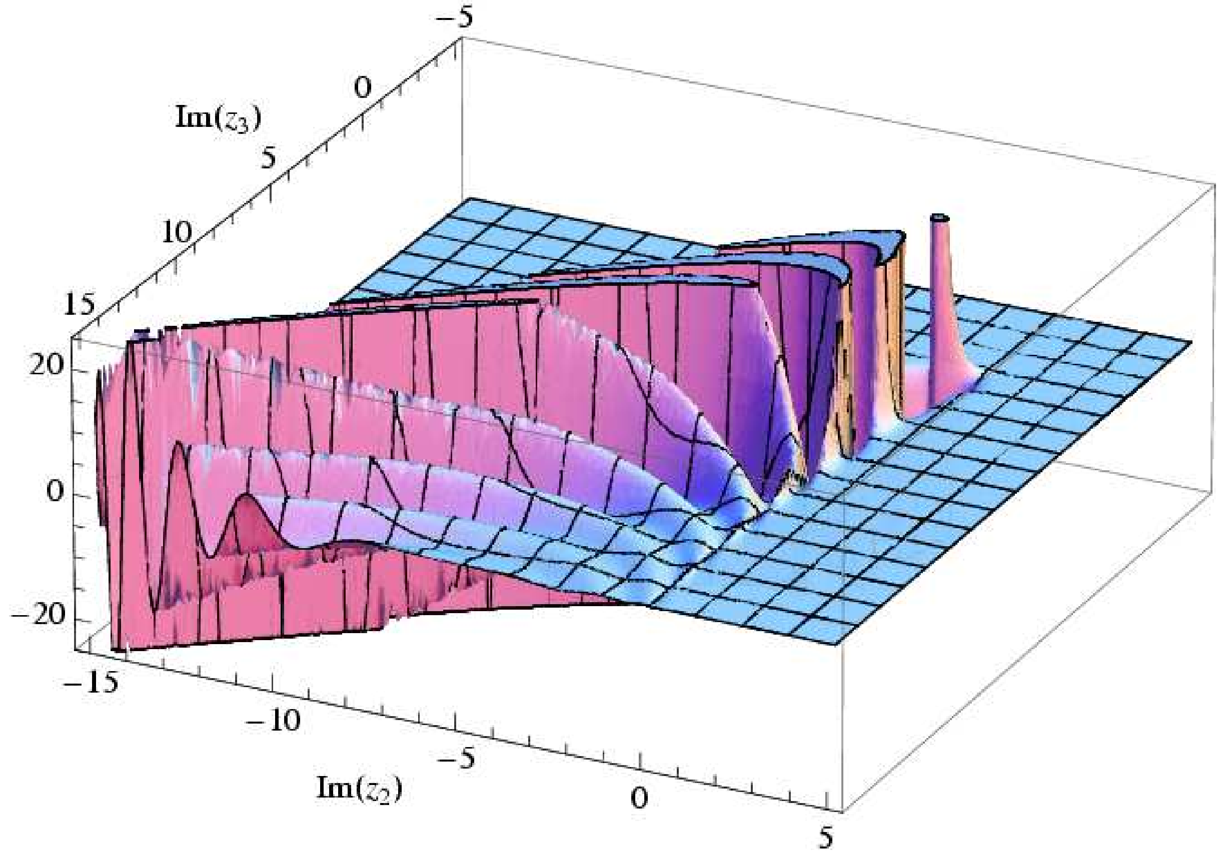
  Czakon '06
  Anastasiou, Daleo '06

- Do remaining $\mathcal{C}_i$ integrations numerically



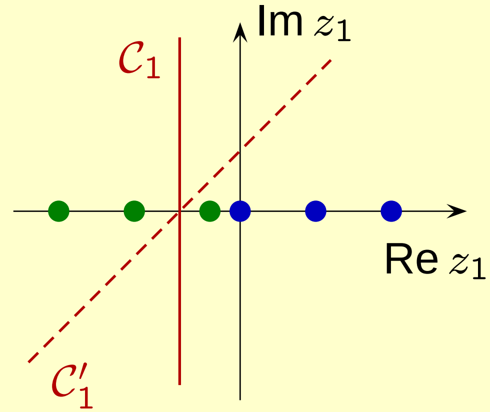
Mellin-Barnes representations

Numerical integration may not converge easily:



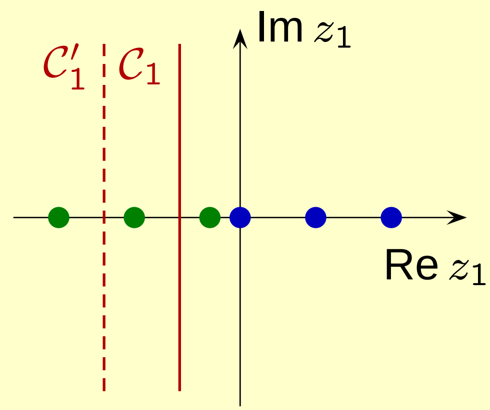
Improvements:

a) contour rotation:



Huang, Freitas '10

b) contour shifts:



Dubovyk et al. '16
Usovitsch '16-18

SM input parameters

- α_s:

  Most precise determination using Lattice QCD from υ spectroscopy:

  $\alpha_\mathsf{s} = 0.1184 \pm 0.0006$ HPQCD '10

  But e^+e^- event shapes and DIS prefer $\alpha_\mathsf{s} \sim 0.114$

  Alekhin, Blümlein, Moch '12; Abbate et al. '11; Gehrmann et al. '13

- Impact on EWPOs:

$$\delta\alpha_\mathsf{s} = 0.005 \quad \Rightarrow \quad \delta M_\mathsf{W} \approx 3.5\ \mathsf{MeV}$$

$$\delta \sin^2\theta^\ell_{\mathsf{eff}} \approx 2 \times 10^{-5}$$

  Currently not dominant, but similar order of magnitude as intrinsic theory error

Renormalization scheme dependence

Use of $\overline{\text{MS}}$ renormalization for m_t reduces h.o. QCD corrections of $\mathcal{O}(\alpha_\text{t}\alpha_\text{s}^n)$:

loops $(n+1)$	$\Delta\rho^{\overline{\text{MS}}}_{(n)} \Big/ \left(\frac{3G_F\overline{m}_\text{t}^2}{8\sqrt{2}\pi^2}\right)$	$\Delta\rho^{\text{OS}}_{(n)} \Big/ \left(\frac{3G_F m_\text{t}^2}{8\sqrt{2}\pi^2}\right)$	
2	$-0.193\left(\frac{\alpha_\text{s}}{\pi}\right)$	$-3.970\left(\frac{\alpha_\text{s}}{\pi}\right)$	Djouadi, Verzegnassi '87 Kniehl '90
3	$-2.860\left(\frac{\alpha_\text{s}}{\pi}\right)^2$	$-14.59\left(\frac{\alpha_\text{s}}{\pi}\right)^2$	Avdeev, Fleischer, et al. '94 Chetyrkin, Kühn, Steinhauser '95
4	$-1.680\left(\frac{\alpha_\text{s}}{\pi}\right)^3$	$-93.15\left(\frac{\alpha_\text{s}}{\pi}\right)^3$	Schröder, Steinhauser '05 Chetyrkin, Faisst, Kühn, et al. '06 Boughezal, Czakon '06

No clear pattern of this kind known for $\mathcal{O}(\alpha^n)$

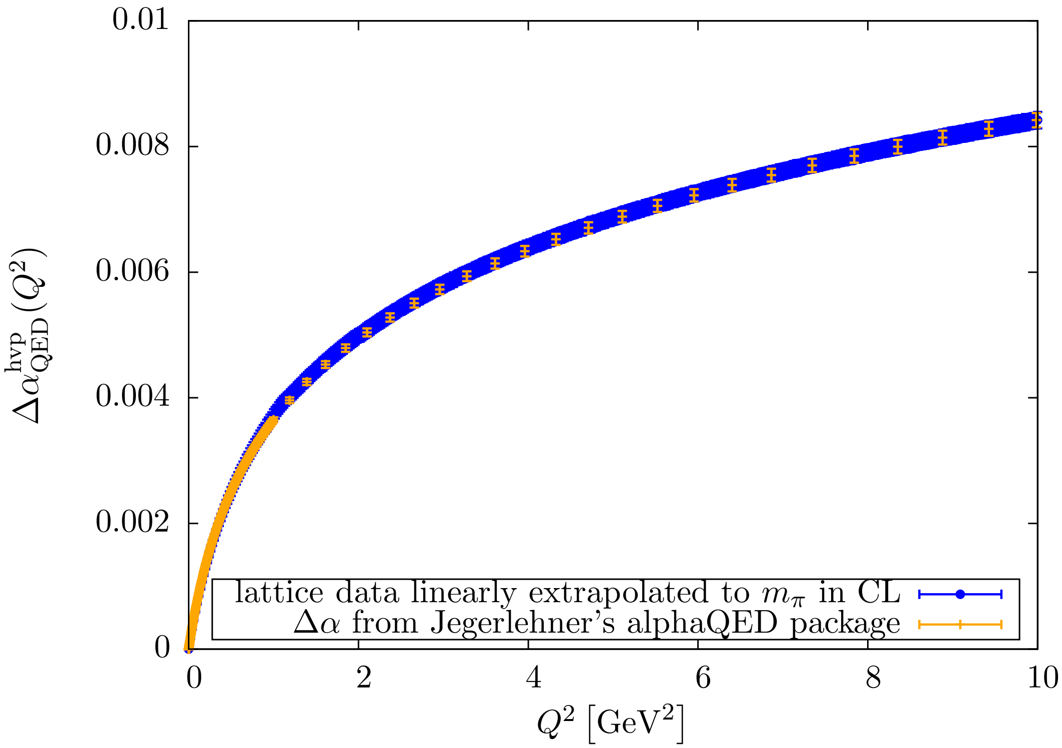
$\longrightarrow$ Only few results available that allow direct comparison

e.g. Faisst, Kühn, Seidensticker, Veretin '03

Shift of finestructure constant

- $\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 = 0.0315_{\text{lept}} + 0.0276_{\text{had}}$
- Hadronic effects from $e^+e^- \to$ had. data
- Last 5 years: new data from BaBar, VEPP, BES
- No significant improvement from including τ data
- Robust precision $\sim 10^{-4}$

Davier et al. ’17; Jegerlehner ’17
Keshavarzi, Nomura, Teubner ’18



Burger, Jansen, Petschlies, Pientka ’15

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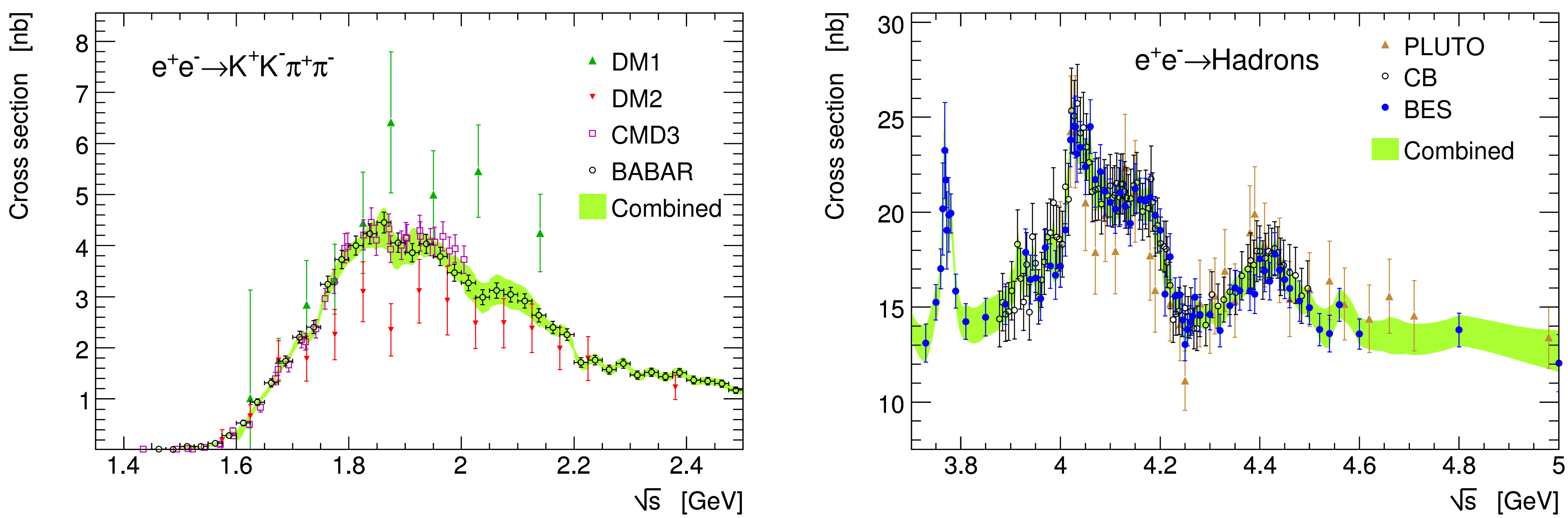
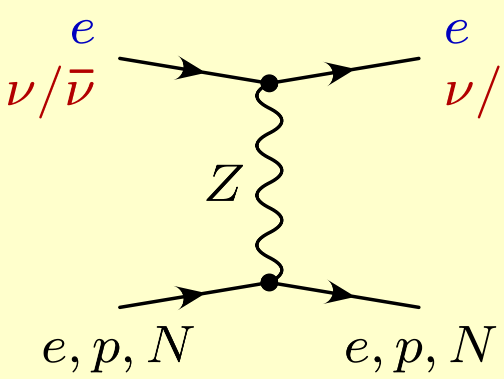


Davier et al. ’17

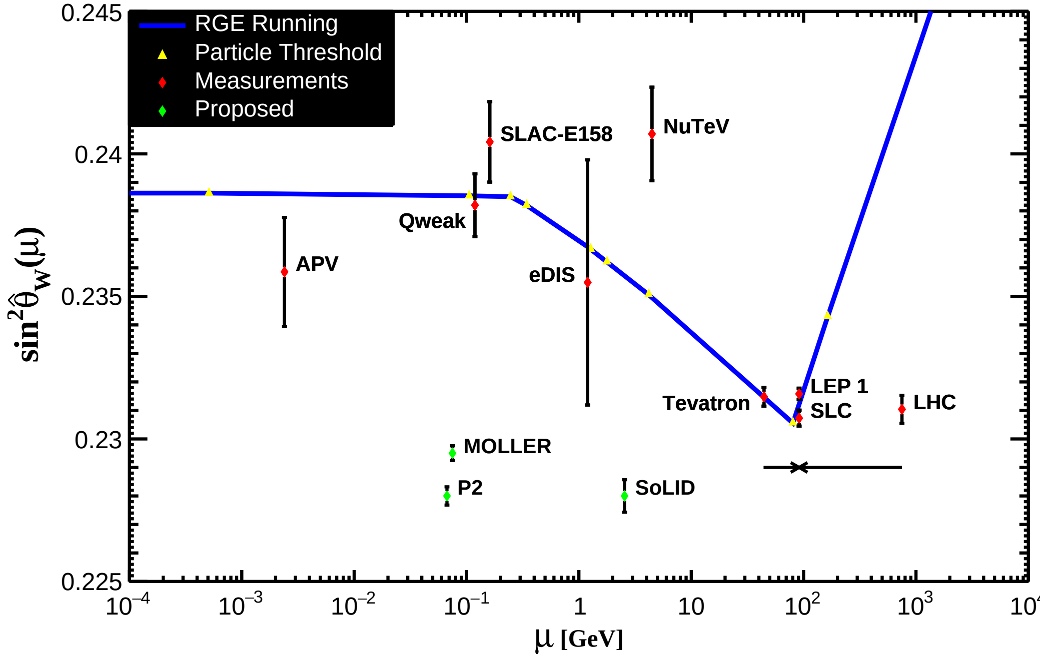
Low-energy parity violation



$$g_{AV}^{ef}\,[\bar{e}\gamma^{\mu}\gamma_5 e]\;[\bar{f}\gamma_{\mu} f]$$
$$g_{VA}^{ef}\,[\bar{e}\gamma^{\mu} e]\;[\bar{f}\gamma_{\mu}\gamma_5 f]$$

$$g_{AV}^{ef} = \frac{1}{2} - 2|Q_f|\sin^2\bar{\theta}(\mu)$$
$$g_{VA}^{ef} = \frac{1}{2} - 2\sin^2\bar{\theta}(\mu)$$

Erler, Ferro-Hernández ’17

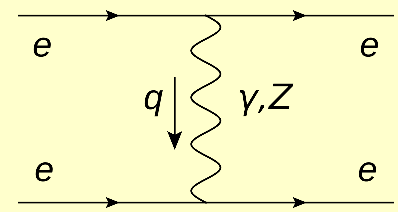


Electroweak corrections to Møller scattering

- $e^- e^-$ PV scattering is very clean channel with minimal hardonic effects
- EW corrections to ee and ep scattering are similar (except box graphs)
- LR asymmetry:

$$A_{\mathrm{LR}} = \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}} = \frac{G_\mu(-q^2)}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4}(1 - 4\sin^2\theta_{\mathrm{W}})$$

$$y \approx \tfrac{1}{2}\cos\theta$$

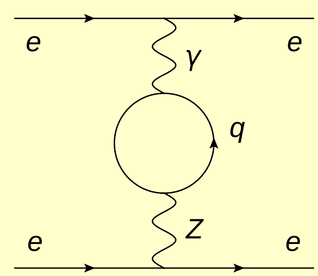


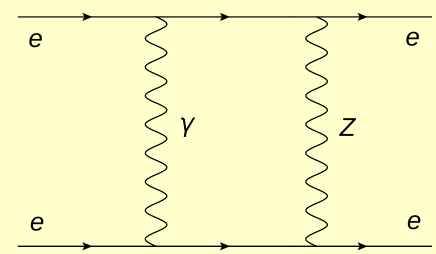
- One-loop correction $\delta_{1l} A_{\mathrm{LR}} \sim 40\%$
  $\delta_{1l} \sin^2\theta_{\mathrm{W}} \sim 3\%$

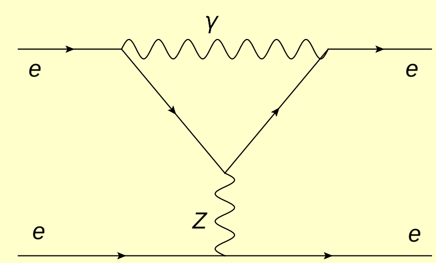
  Czarnecki, Marciano '96

- IR radiation cancels in A_{LR}
  $\longrightarrow$ No real emission corrections

- MOLLER exp. target:
  $\delta_{\mathrm{exp}} \sin^2\theta_{\mathrm{W}} \sim 0.1\%$
  $\longrightarrow$ 2-loop corrections needed

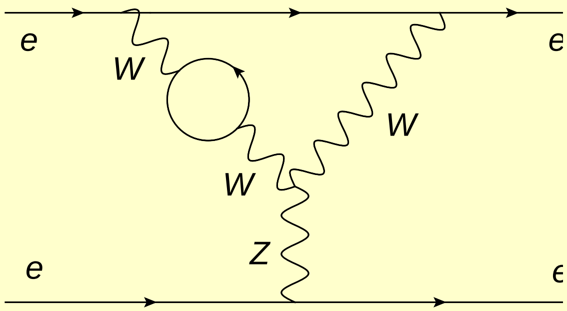




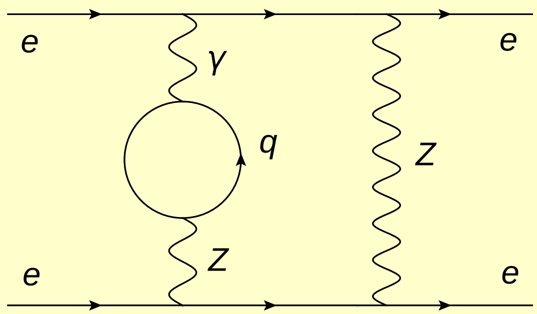


Electroweak corrections to Møller scattering

- As first step: EW 2-loop corrections with closed fermion loops
  $\longrightarrow$ Enhanced by N_f
  $\longrightarrow$ Experience from Z-pole EWPO:
  $\mathcal{O}(\alpha^2_{\text{ferm}})\,/\,\mathcal{O}(\alpha^2_{\text{bos}}) \gtrsim 5$



- Computer tools to handle many diagrams and large expressions
  - Diagram generation
  - Lorentz and Dirac algebra
  - Integral simplication and expansion
  - Numerical integration of final set of loop functions



EW corrections to Møller scattering: Scales

- At Z pole: $s = M_Z^2 \sim M_W^2 \sim M_H^2 \sim m_t^2 \gg m_f^2 \quad (f \neq t)$
  $\longrightarrow$ Neglect all light fermion masses (except in $\Delta\alpha$)

- Low-energy ee scattering: $|q^2| \sim m_f^2 \ll M_{\text{weak}}^2$
  $\longrightarrow$ Expansion in large M_{weak}^2

- Techical realization: expansion by regions — Beneke, Smirnov '97
  $\longrightarrow$ Expansion in integrand, different categories for loop momenta $k_{1,2}$
  $\longrightarrow$ Here only soft+hard regions needed:

  hard–hard: $|k_1| \sim |k_2| \sim M_{\text{weak}} \gg Q, m_f \qquad Q = \sqrt{|q^2|}$

  soft–soft: $|k_1| \sim |k_2| \sim Q, m_f \ll M_{\text{weak}}$

  soft–hard: $|k_1| \sim Q, m_f \ll |k_2| \sim M_{\text{weak}}$ (and permutations)

- Coefficients are simpler integrals:
  hard–hard: 2-loop vacuum
  soft–hard: (1-loop) $\times$ (1-loop)
  soft–soft: 2-loop with fewer masses

EW corrections to Møller scattering: Renormalization

- Form of one-loop result: Czarnecki, Marciano '96

$$A_{\text{LR}} = \frac{\rho G_\mu q^2}{\sqrt{\pi}\alpha} \frac{-1+y}{1+y^4+(1-y)^4}\left[1 - 4\kappa(q^2)\,\hat{s}^2(M_Z^2) + \text{boxes, QED}\right]$$

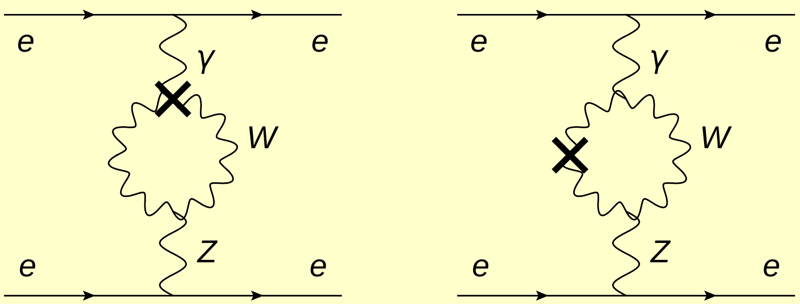
- G_μ absorbs dependence on $\Delta\alpha$

- Corrections to G_μ known at 2-loop Freitas, Hollik, Walter, Weiglein '00; Awramik, Czakon '02; Onishchenko, Veretin '02

- $\hat{s}^2(M_Z^2) \equiv \sin^2\theta_{\text{W}}^{\overline{\text{MS}}}\Big|_{\mu^2=M_Z^2}$ numerically close to $\sin^2\theta_{\text{eff}}^f$
  $\longrightarrow$ Connection to Z-pole EWPOs

- ρ contains remaining renormalization (Z mass)

- At 2-loop level: need sub-loop renormalization
- Dependence $\Delta\alpha$ re-enters
- Also need renormalization of M_{W}, but since $\hat{s}^2$ is used as input, M_{W} is computed from $\hat{s}^2$ and M_{Z}

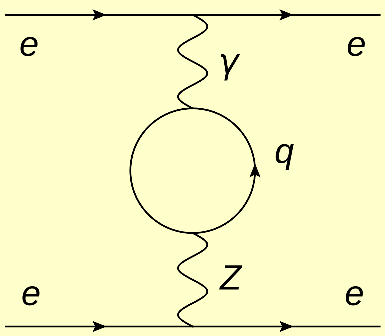


EW corrections to Møller scattering: Hadronic effects

$$A_{\mathrm{LR}} = \frac{\rho G_\mu q^2}{\sqrt{\pi}\alpha} \frac{-1+y}{1+y^4+(1-y)^4}\left[1 - 4\kappa(q^2)\,\hat{s}^2(M_Z^2) + \text{boxes, QED} + \ldots\right]$$

- $\kappa(q^2)$ contains effect of γ–Z self-energy
- $\kappa(q^2) - \kappa(0)$ is small
- At 1-loop: $\kappa(0) = 1 - \frac{\alpha}{6\pi s^2}\Delta_{\gamma Z} + \text{bosonic}$,

$$\Delta_{\gamma Z} = \sum_f (I_{3f}Q_f - 2s^2Q_f^2)\ln\frac{m_f^2}{M_Z^2}$$

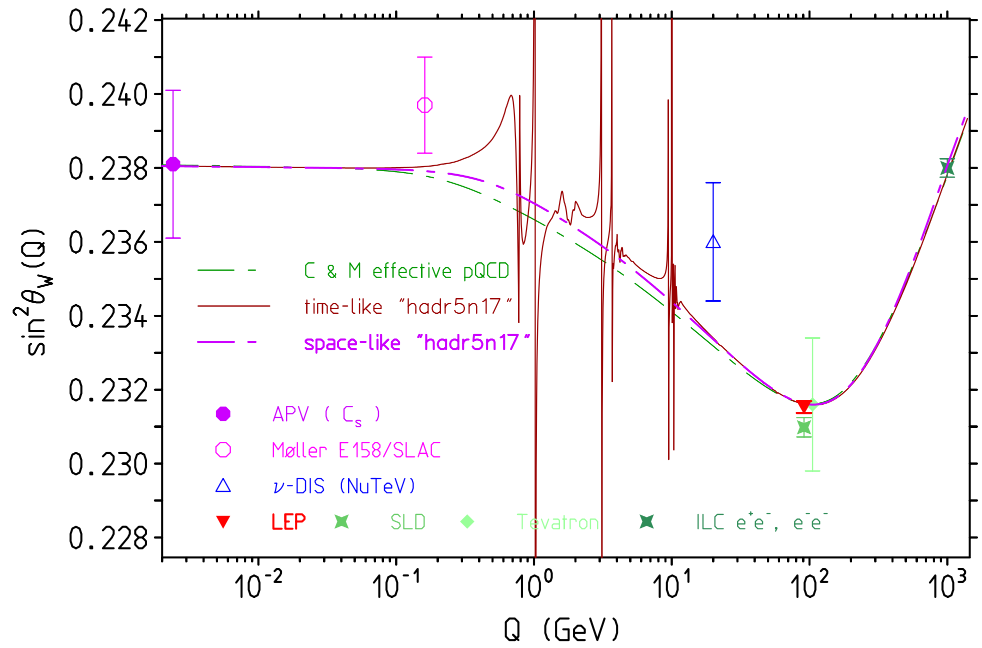


- Sensitivity to m_q: non-perturbative hadron physics
- $\Delta_{\gamma Z}$ described running of $\hat{s}^2(\mu^2)$ from $\mu = M_Z$ to 0

Determination of $\Delta_{\gamma Z} = \sum_f (I_{3f} Q_f - 2s^2 Q_f^2) \ln m_f^2 / M_Z^2$:

a) Directly from e^+e^- data using reweighting of different flavors [SU(3)$_{u,d,s}$ symmetry, pQCD for u, d, s at c, b thrsh.]

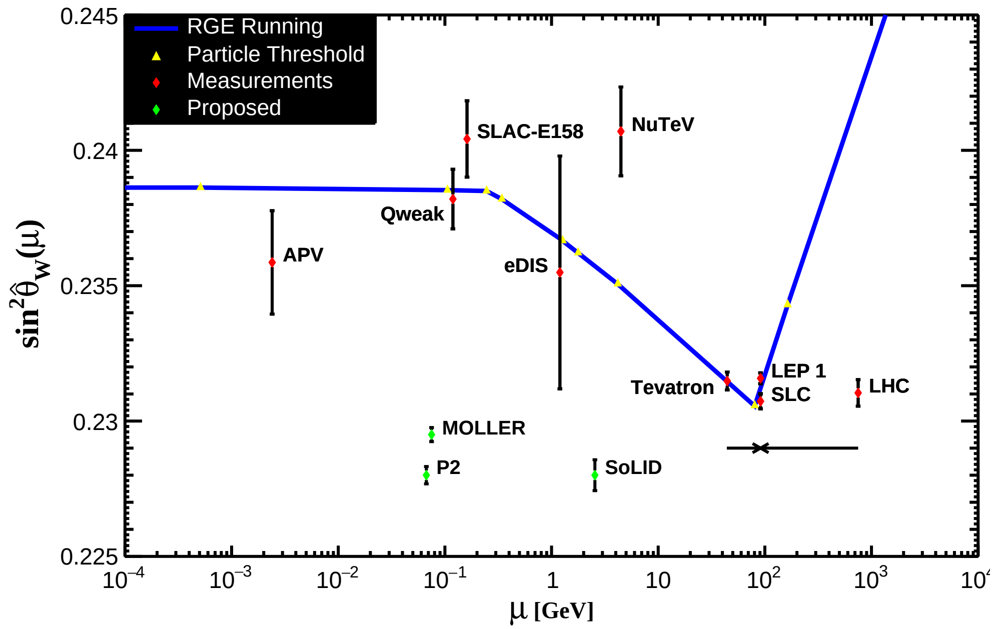
Wetzel '81; Marciano, Sirlin '84 Jegerlehner '86,17



Determination of $\Delta_{\gamma Z} = \sum_f (I_{3f} Q_f - 2s^2 Q_f^2) \ln m_f^2/M_Z^2$:

a) Directly from e^+e^- data using reweighting of different flavors
[SU(3)$_{u,d,s}$ symmetry,
pQCD for u, d, s at c, b thrsh.]
Wetzel '81; Marciano, Sirlin '84
Jegerlehner '86,17

b) Determine "threshold masses" $\bar{m}_{u,ds,c,b}$ from $\Delta\alpha(q^2)$;
pQCD RG running btw. thresholds
Erler, Ramsey-Musolf '04
Erler, Ferro-Hernández '17



EW corrections to Møller scattering: Hadronic effects

Determination of $\Delta_{\gamma Z} = \sum_f (I_{3f} Q_f - 2s^2 Q_f^2) \ln m_f^2 / M_Z^2$:

a) Directly from e^+e^- data using reweighting of different flavors

[SU(3)$_{u,d,s}$ symmetry, pQCD for u, d, s at c, b thrsh.]

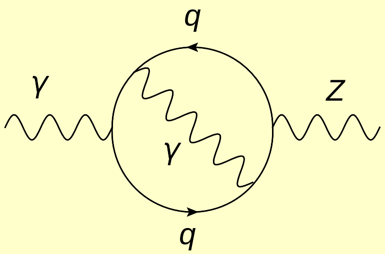
Wetzel '81; Marciano, Sirlin '84
Jegerlehner '86,17

b) Determine "threshold masses" $\bar{m}_{u,ds,c,b}$ from $\Delta\alpha(q^2)$;
pQCD RG running between thresholds

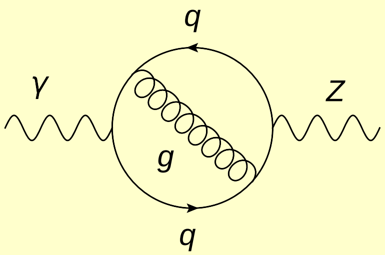
Erler, Ramsey-Musolf '04
Erler, Ferro-Hernández '17

c) Lattice QCD $\longrightarrow$ talk by K. Ottnad

Two-loop contributions to γ–Z self-energy:

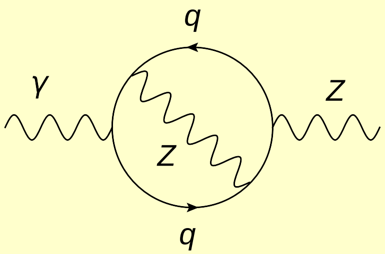


- Quark loops with photon or gluon
  $\longrightarrow$ Already contained in hadronic $\Delta_{\gamma Z}$



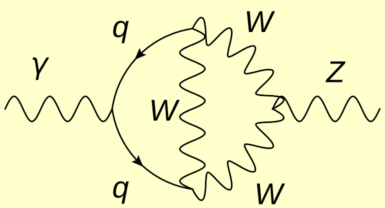
- Quark loops with W/Z boson

- In $\gamma\gamma$ SE:
  $\longrightarrow$ For $\Delta\alpha$ (γ–γ SE) limit $m_q \to 0$ is safe
  due to QED Ward id.



- In γZ SE:
  $\longrightarrow$ Sensitive to low-energy hadron physics

  Perform expansion by regions
  $\longrightarrow$ Factorization into $\Delta^{1-\text{loop}}_{\gamma Z} \times W/Z$-loop
  (except for terms $\propto m_t^2 \log m_b^2$)



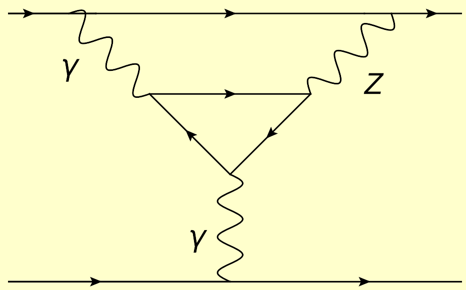
γ_5 and mass regulators

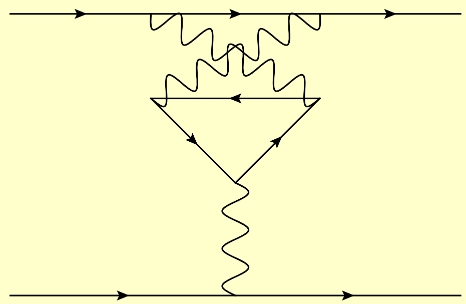
- Vertex diagrams with sub-loop triangles are sensitive to γ_5 problem

- DREG with naively anti-commuting γ_5:
  $\text{Tr}\{\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5\} = 0$

- Contributions $\propto \text{Tr}\{\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5\}$ are UV finite
  $\longrightarrow$ DREG not needed

- IR singularities from photon propagators
  (in individual diagrams, cancel in sum)
  $\longrightarrow$ Use mass regulator ($m_\gamma \neq 0$) in $D = 4$





Summary

- Sensitivity for Z-pole EWPOs and future low-energy EWPOs requires 2-loop SM corrections

- No fully automated procedure for 2-loop calculations
  $\longrightarrow$ Each process requires new work
  $\longrightarrow$ Combination of analytic and numerical methods
  $\longrightarrow$ Interesting subtleties around definition of input quantities & renormalization

- Evaluation of different methods for including hadronic effects?

$\longrightarrow$ Work is in progress and useful pheno results in $\lesssim$ 1 year