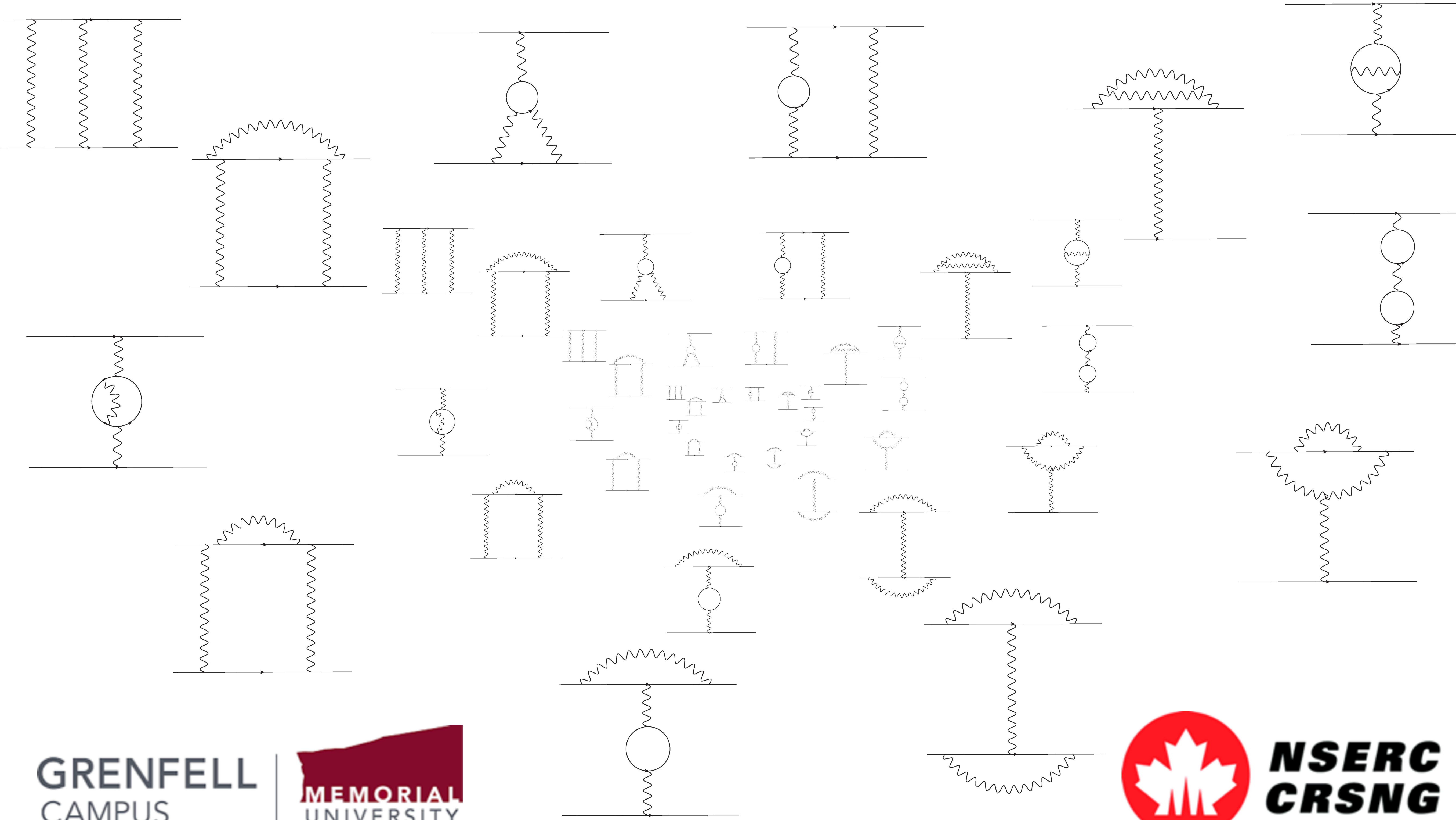
Role of Two-Loop EWC in Moller Scattering and Dispersive Approach

A. Aleksejevs and S. Barkanova, Grenfell Campus of Memorial University



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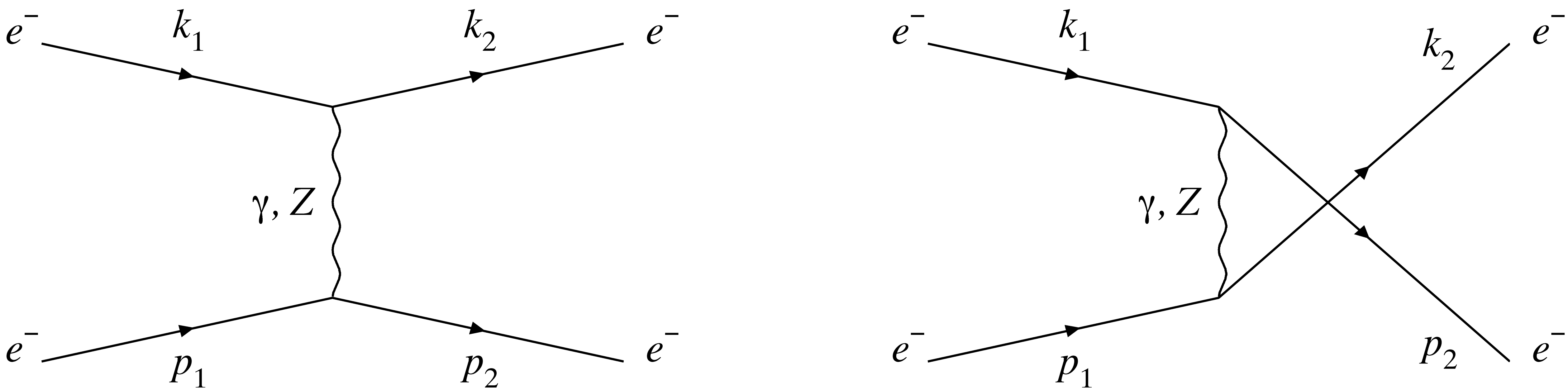
Møller scattering at the tree level

The process of electron–electron scattering (Møller process)
C. Møller, Annalen der Physik 406, 531 (1932)



$$A_{LR} = \frac{\sigma_{LL} + \sigma_{LR} - \sigma_{RL} - \sigma_{RR}}{\sigma_{LL} + \sigma_{LR} + \sigma_{RL} + \sigma_{RR}} = \frac{\sigma_{LL} - \sigma_{RR}}{\sigma_{LL} + 2\sigma_{LR} + \sigma_{RR}}$$

$$A^0_{LR} = \frac{s}{2m_W^2} \frac{y(1-y)}{1+y^4+(1-y)^4} \frac{1-4s_W^2}{s_W^2}, \quad y = -t/s$$

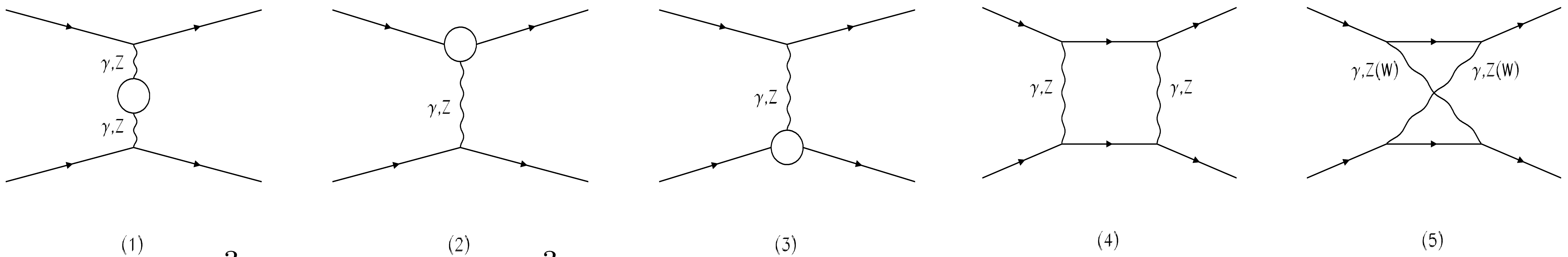
Møller scattering at one-loop

Although PV asymmetry ($A_{LR} \sim 10^{-7}$) is very small, the accuracy of modern experiments exceeds the accuracy of the theoretical result in Born approximation.

One–loop contribution was found to be rather big in the previous works:

A. Czarnecki, W. J. Marciano, Phys. Rev. D53, 1066 (1996)
A. Denner, S. Pozzorini, Eur. Phys. J. C7, 185 (1999)
A. A, S. Barkanova, A. Ilyichev, V. Zykunov, Phys. Rev. D82, 093013 (2010)

First Stage:One-Loop Corrections for MOLLER



$$\sigma = \frac{\pi^3}{2s}|M_0 + M_1|^2 = \frac{\pi^3}{2s}(\underbrace{M_0M_0^+}_{\propto\,\alpha^2} + \underbrace{2\mathrm{Re}M_1M_0^+}_{\propto\,\alpha^3} + \underbrace{M_1M_1^+}_{\propto\,\alpha^4}) = \sigma_0 + \sigma_1 + \sigma_Q$$

$$\sigma_1 = \sigma_1^{BSE} + \sigma_1^{Ver} + \sigma_1^{Box}$$

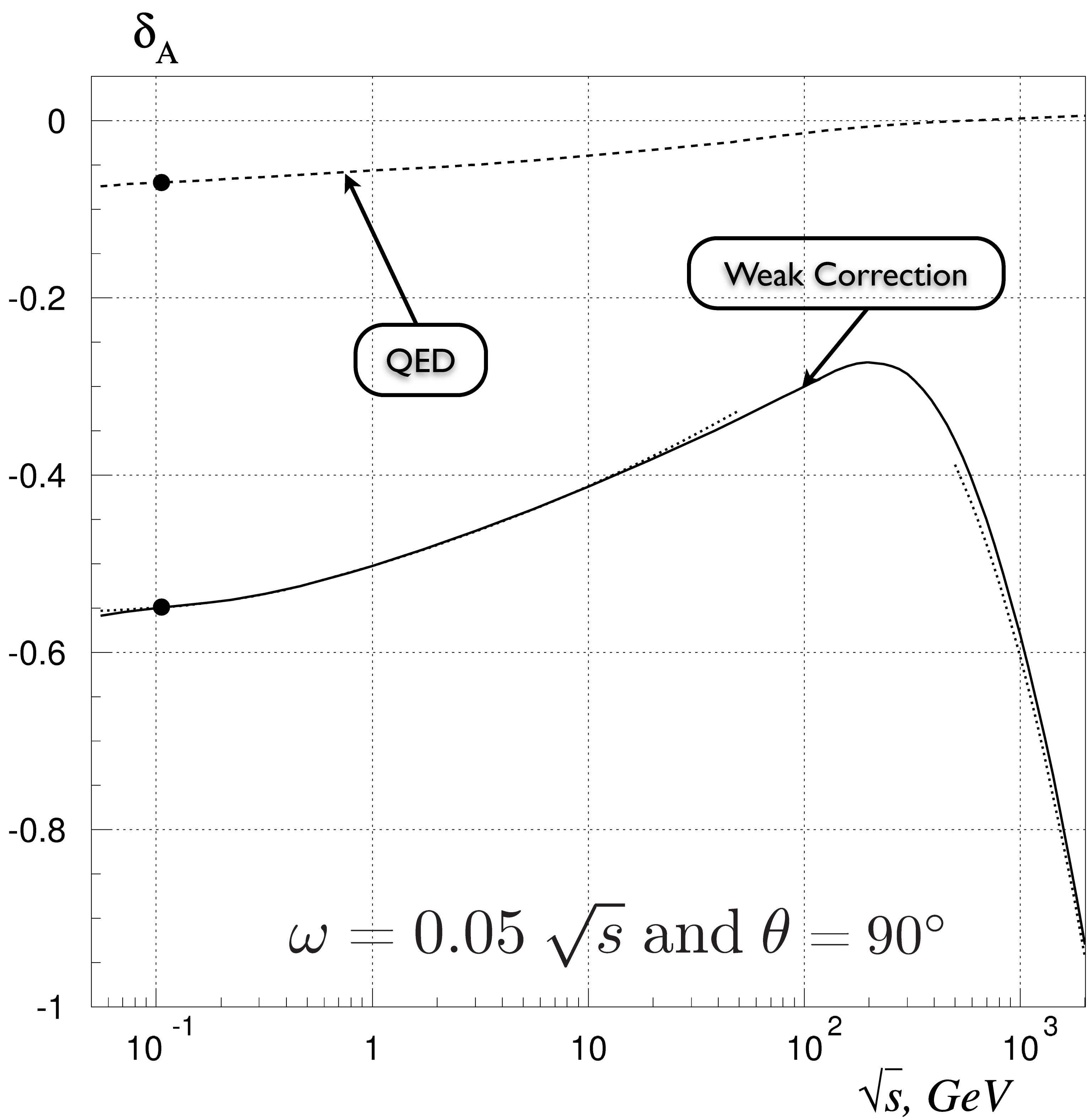
- Calculated in the on-shell renormalization, using both:
  - Computer-based approach, with Feynarts, FormCalc, LoopTools and Form

  T. Hahn, Comput. Phys. Commun. 140 418 (2001);
  T. Hahn, M. Perez-Victoria, Comput. Phys. Commun. 118, 153 (1999);
  J. Vermaseren, (2000) [arXiv:math-ph/0010025]

  - "On paper", with approximations in small energy region $\frac{\{t,u\}}{m_{Z,W}^2} \ll 1$, for $\sqrt{s} \ll 30\ GeV$ and high energy approximation for $\sqrt{s} \gg 500\ GeV$

A. Aleksejevs, S. Barkanova, A. Ilyichev, V. Zykunov, Phys. Rev. D82 (2010) 093013

One-Loop Corrections for MOLLER



$$\delta_A = \frac{A^C_{LR} - A^0_{LR}}{A^0_{LR}}$$

The relative weak (solid line in DRC (semi-automated) and dotted line in HRC ("on paper")) and QED (dashed line) corrections to the Born asymmetry A^0_{LR} versus $\sqrt{s}$ at $\theta = 90^\circ$.

The filled circle corresponds to our predictions for the MOLLER experiment.

A. Aleksejevs, S. Barkanova, A. Ilyichev, Y. Kolomensky, V. Zykunov, Phys. Part. Nucl. 44(2013) 161-174

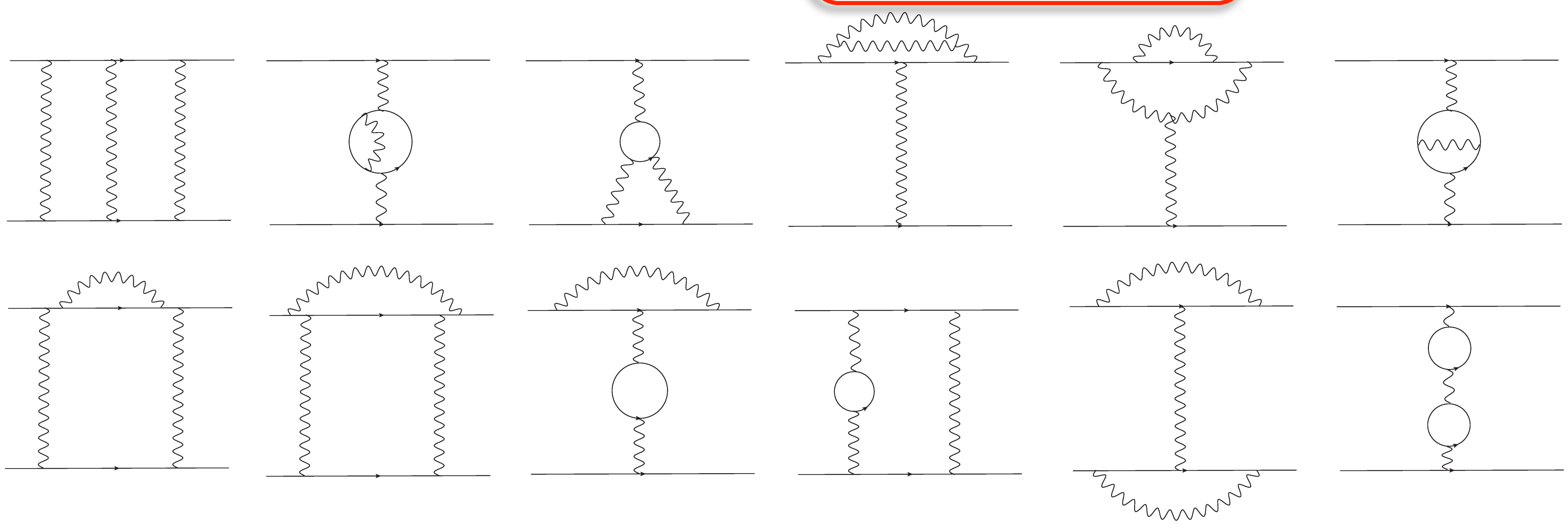
Second Stage: NNLO Corrections for MOLLER

The Next-to-Next-to-Leading Order (NNLO) EWC to the Born ($\sim M_0 M_0^+$) cross section can be divided into two classes:

- Q-part induced by quadratic one-loop amplitudes $\sim M_1 M_1^+$, and
- T-part – the interference of Born and two-loop diagrams $\sim 2\mathrm{Re} M_0 M_{2\text{-loop}}^+$.

$$\sigma = \frac{\pi^3}{2s}|M_0 + M_1|^2 = \frac{\pi^3}{2s}(\underbrace{M_0 M_0^+}_{\propto\,\alpha^2} + \underbrace{2\mathrm{Re} M_1 M_0^+}_{\propto\,\alpha^3} + \underbrace{M_1 M_1^+}_{\propto\,\alpha^4}) = \sigma_0 + \sigma_1 + \sigma_Q$$

$$\sigma_T = \frac{\pi^3}{s}\ \mathrm{Re} M_2 M_0^+ \propto \alpha^4$$



+ ...

Quadratic correction: IR part

Differential quadratic cross section σ_Q written as sums of λ-dependent (IRD-terms) and λ-independent (infrared-finite) parts:

$$\sigma_Q = \frac{\pi^3}{2s} M_1 M_1^+ = \underline{\sigma_Q^\lambda} + \underline{\sigma_Q^f}$$

$$\frac{\pi^3}{2s} M_1^{\lambda +} \left(M_1^\lambda + 2 M_1^f \right) = \frac{1}{4} \left(\frac{\alpha}{\pi} \right)^2 \mathrm{Re} \left[\delta_1^{\lambda *} (\delta_1^\lambda + 2 \delta_1^f) \right] \sigma_0 .$$

$$\left(\frac{\alpha}{\pi} \right)^2 \delta_Q^f \, \sigma_0$$

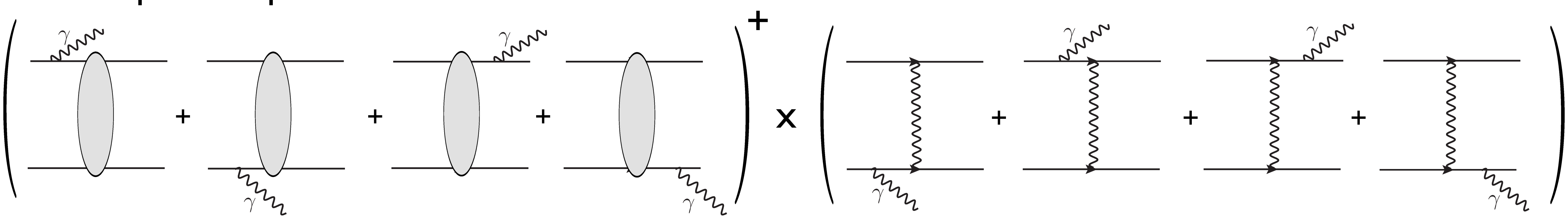
$$\delta_1^\lambda = 4B \log \frac{\lambda}{\sqrt{s}}$$

$$B = \log \frac{tu}{m^2 s} - 1 + i\pi$$

Quadratic correction: photon emission

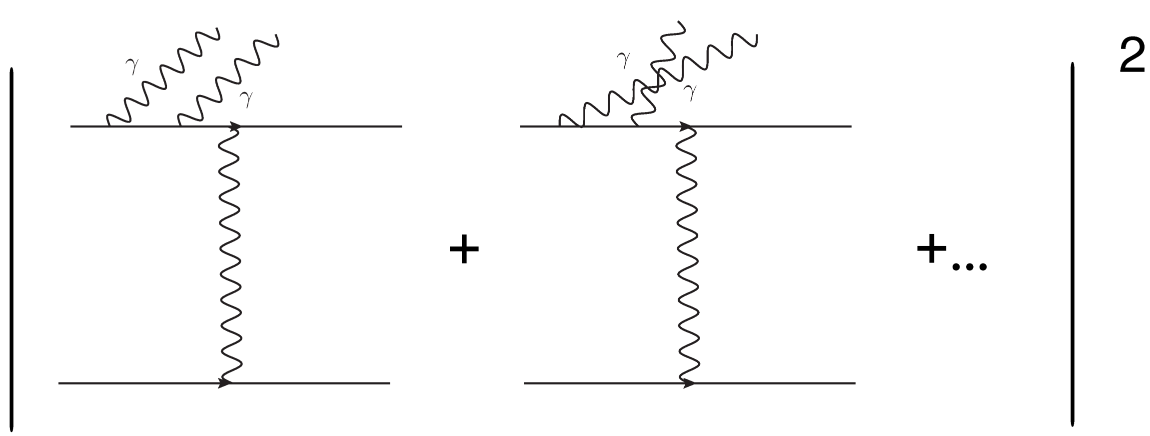
In order to remove the IR-divergent terms in quadratic cross section, wee need to consider:

1. Photon emission from one-loop diagrams
2. Two photon photon emission



$$\sigma_Q^\gamma = \tfrac{1}{2}\sigma^\gamma = \frac{\pi^2}{s}\ \mathrm{Re}\ [(-\delta_1^\lambda + R_1)^* M_1^+ M_0]$$

$$R_1 = -4B\log\frac{\sqrt{s}}{2\omega} - \log^2\frac{s}{em^2} + 1 - \frac{\pi^2}{3} + \log^2\frac{u}{t}$$



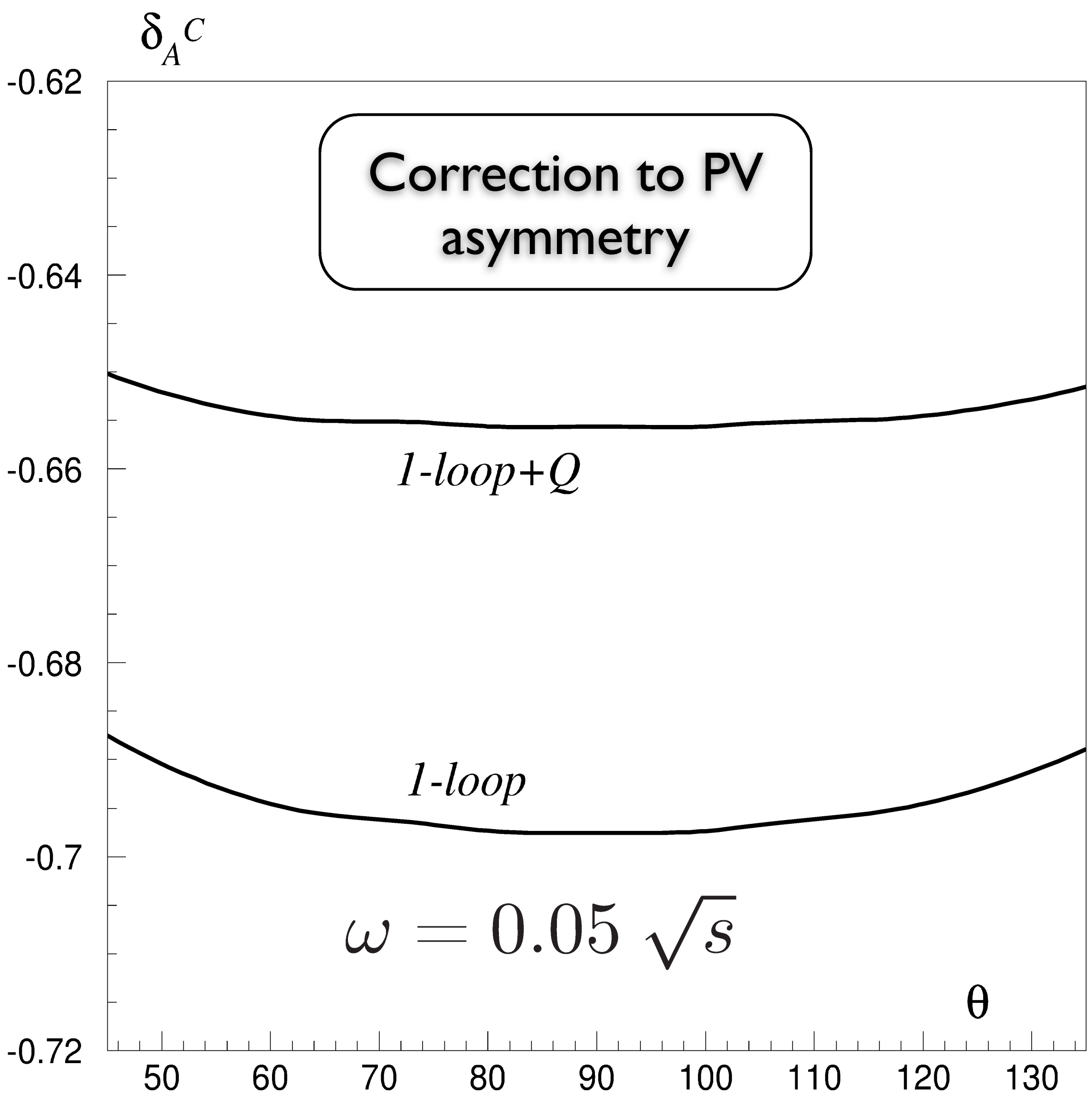
$$\sigma_Q^{\gamma\gamma} = \frac{1}{2}\sigma^{\gamma\gamma} = \frac{1}{4}\left(\frac{\alpha}{\pi}\right)^2\left(\left|-\delta_1^\lambda + R_1\right|^2 - R_2\right)\sigma_0$$

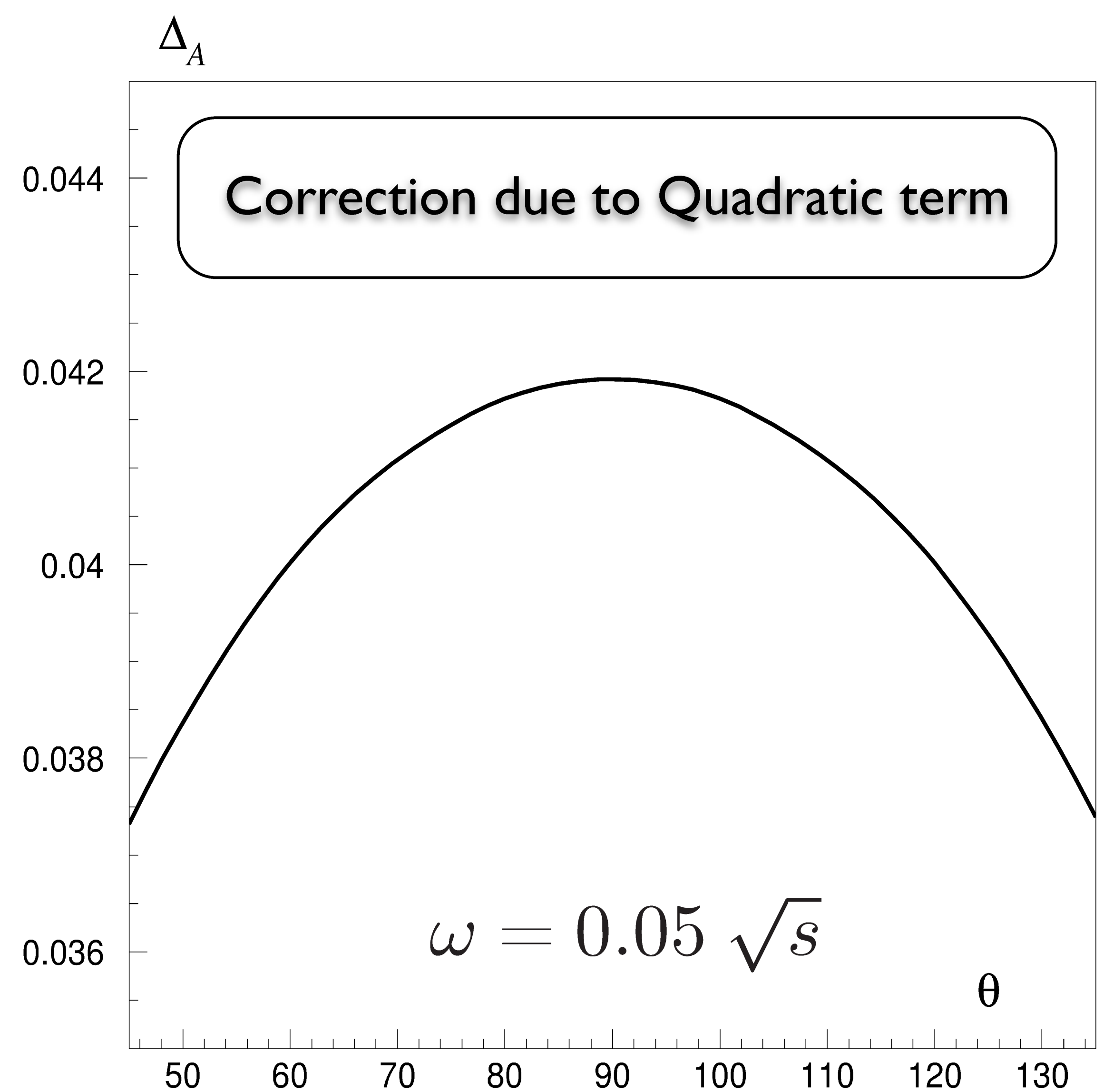
$$R_2 = \frac{8}{3}\pi^2(\log\frac{tu}{m^2 s} - 1)^2$$

Quadratic correction: results

$$\delta_A^{\mathrm{C}} = (A_{LR}^{\mathrm{C}} - A_{LR}^0)/A_{LR}^0$$

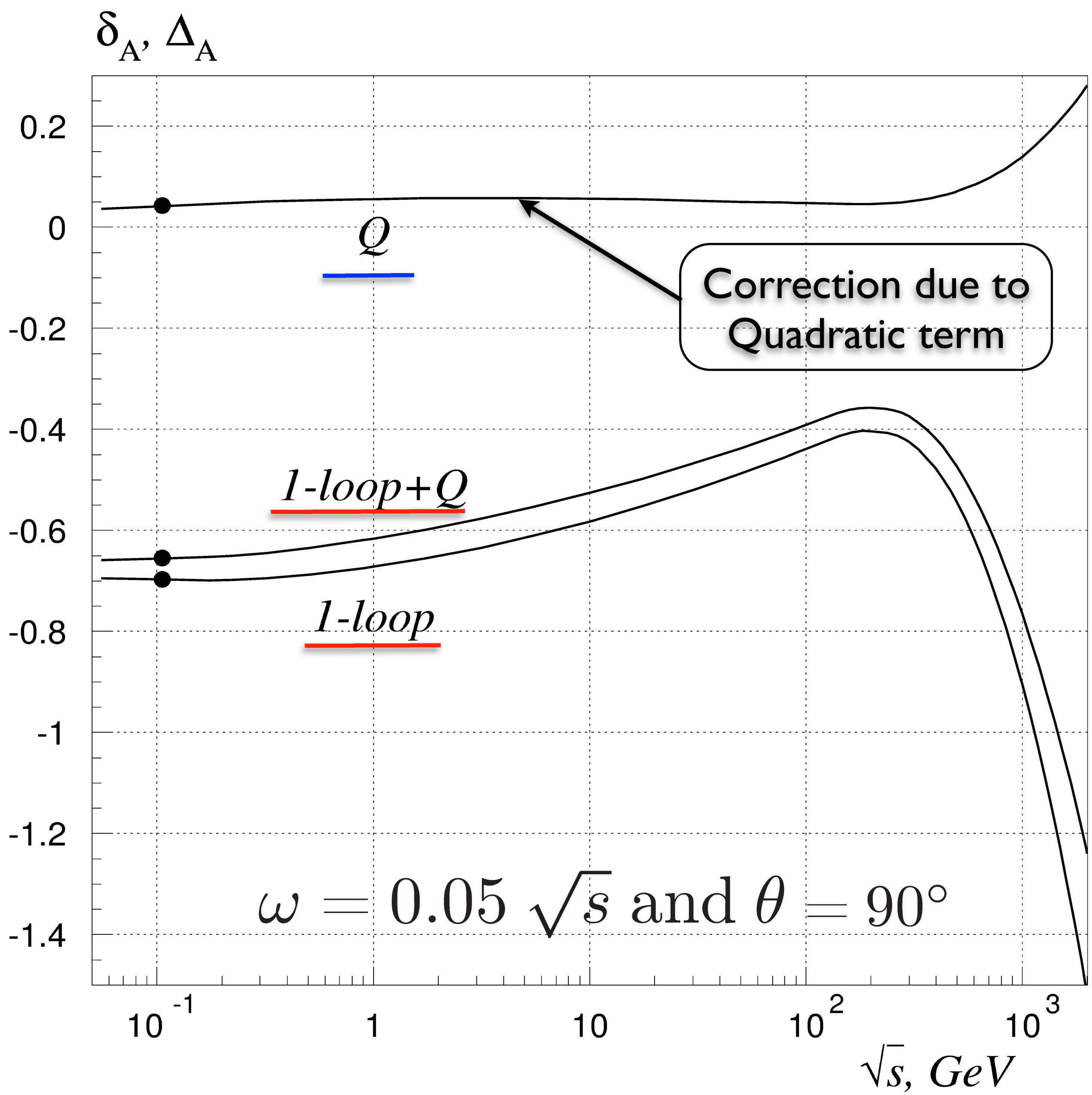
$$\Delta_A = (A_{LR}^{1-\mathrm{loop}+\mathrm{Q}} - A_{LR}^{1-\mathrm{loop}})/A_{LR}^0$$





E_{lab} = 11 GeV

Quadratic correction: results



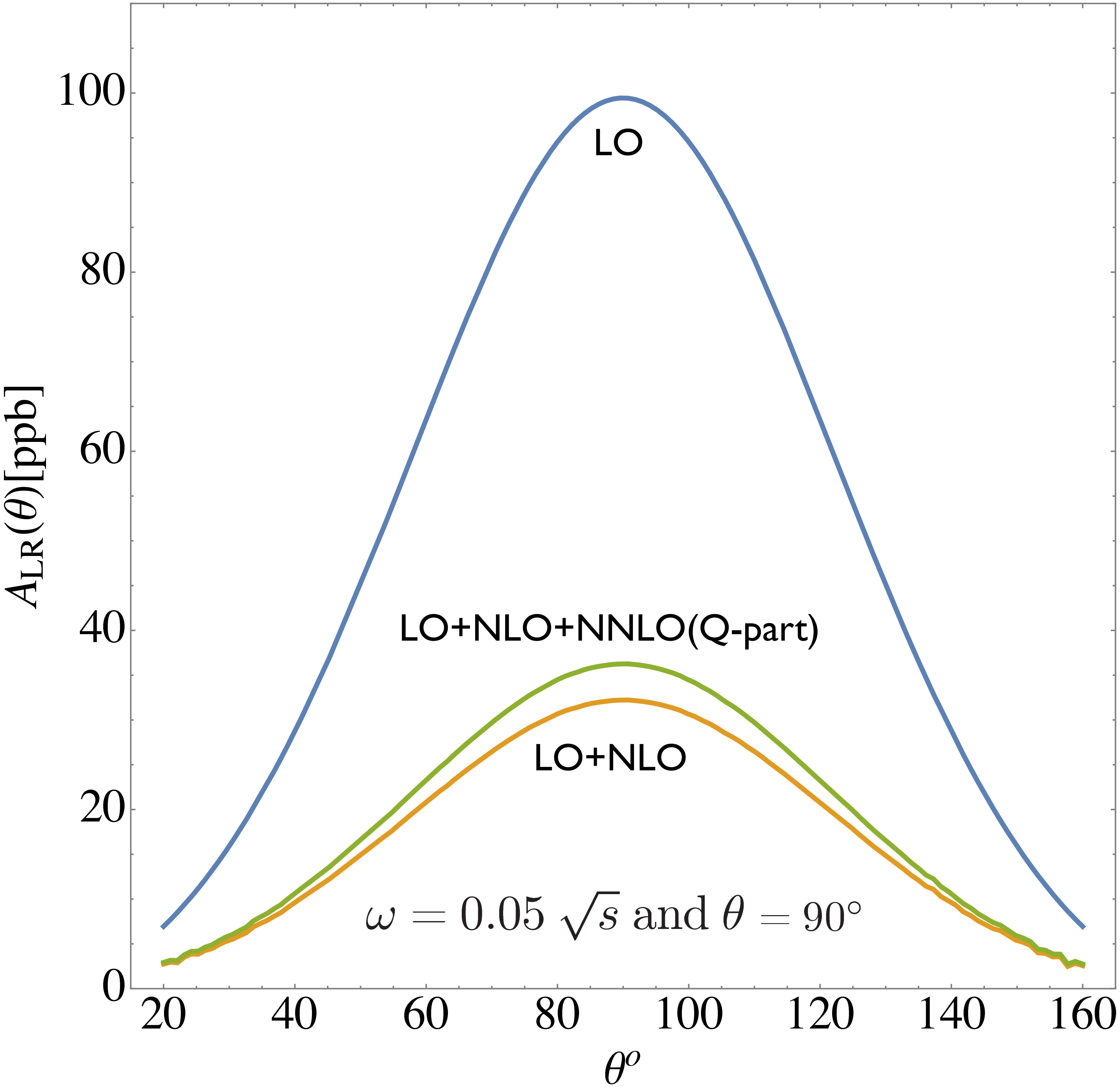
$$\delta_A^C = (A_{LR}^C - A_{LR}^0)/A_{LR}^0$$

$$\Delta_A = (A_{LR}^{1-\mathrm{loop+Q}} - A_{LR}^{1-\mathrm{loop}})/A_{LR}^0$$

The scale of the Q-part contribution in the low-energy region is approximately constant, but starting from $\sqrt{s} \geq m_Z$, where the weak contribution becomes comparable with electromagnetic, the effect of Q-part grows sharply.

This effect of increasing importance of two-loop contribution at higher energies may have a significant effect on the asymmetry measured at the future $e^- e^-$-colliders.

PV Asymmetry



Predicted PV asymmetry (E_{lab} = 11GeV):

$A_{PV}^{(LO+NLO)}$ (90°) = 32.2 (ppb)

$A_{PV}^{(LO+NLO+Q\text{-part})}$ (90°) = 36.2 (ppb)

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu \sin^2\theta_W(1-\Delta r)}$$

Two-Loops Contribution

We split the two-loops contribution into subsets of the gauge invariant classes:

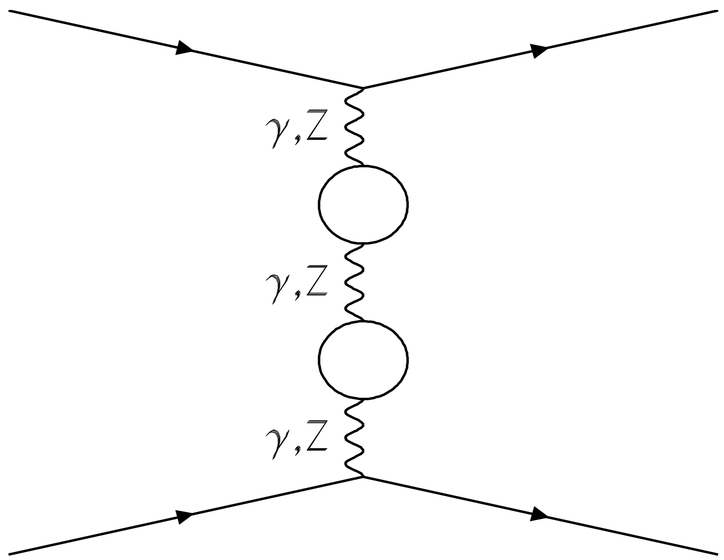
- Reducible contribution $(\mathrm{BSE} + \mathrm{Ver})^2$.

- Irreducible ladder, decorated boxes and boxes with electron self-energies.

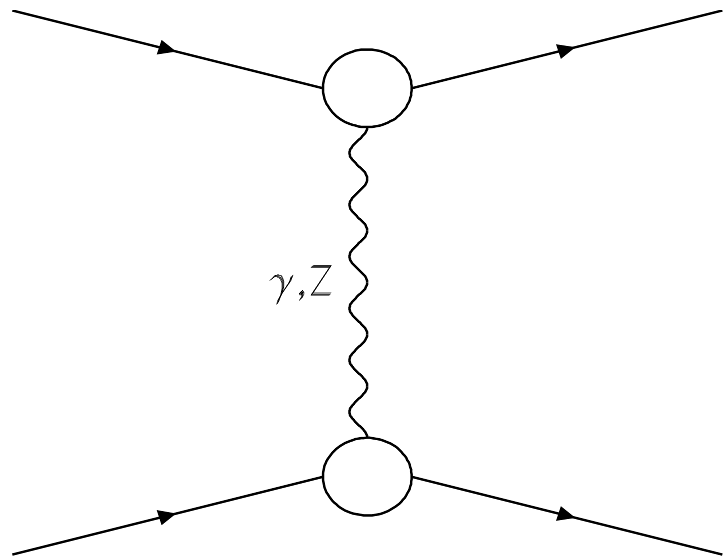
- Irreducible two-loops vertex correction (double vertices) and self energy diagrams.

$$\sigma_T = \frac{\pi^3}{s}\ \mathrm{Re} M_2 M_0^+ \propto \alpha^4$$

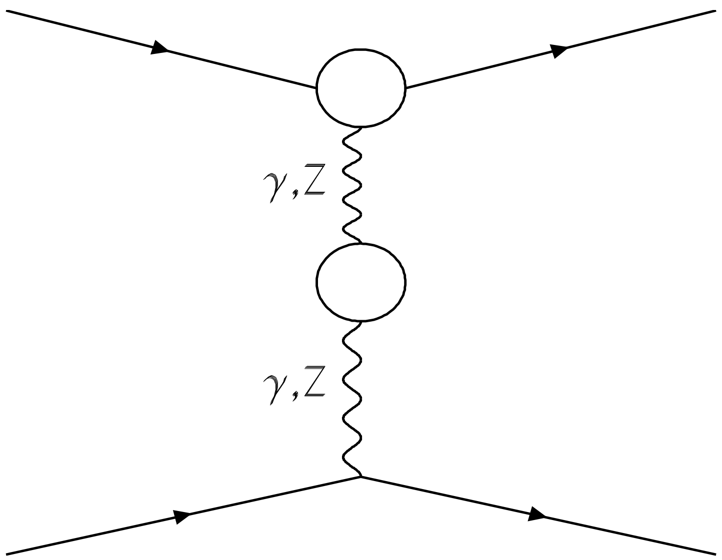
(BSE+Ver)^2 Two-Loops Contribution



(BBSE)



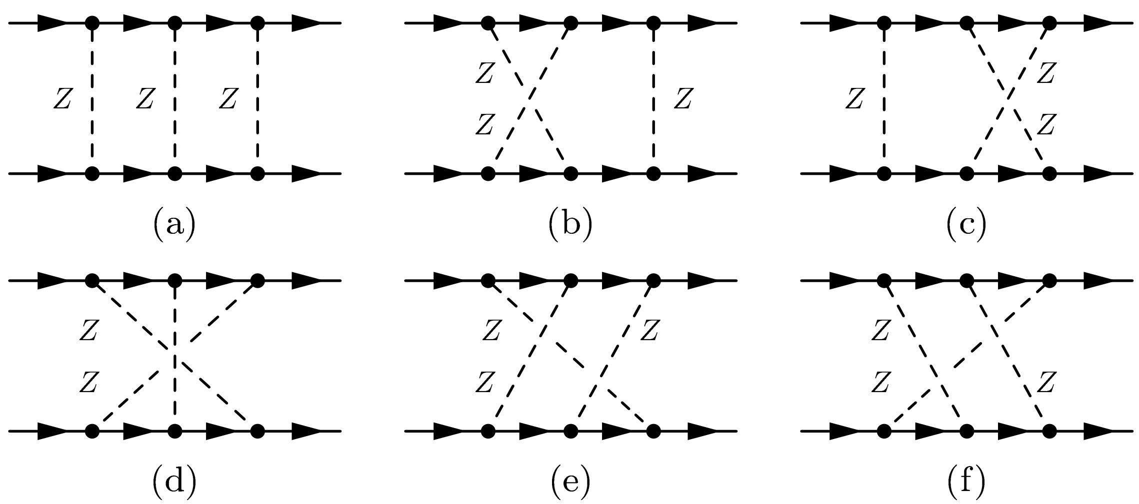
(VerVer)



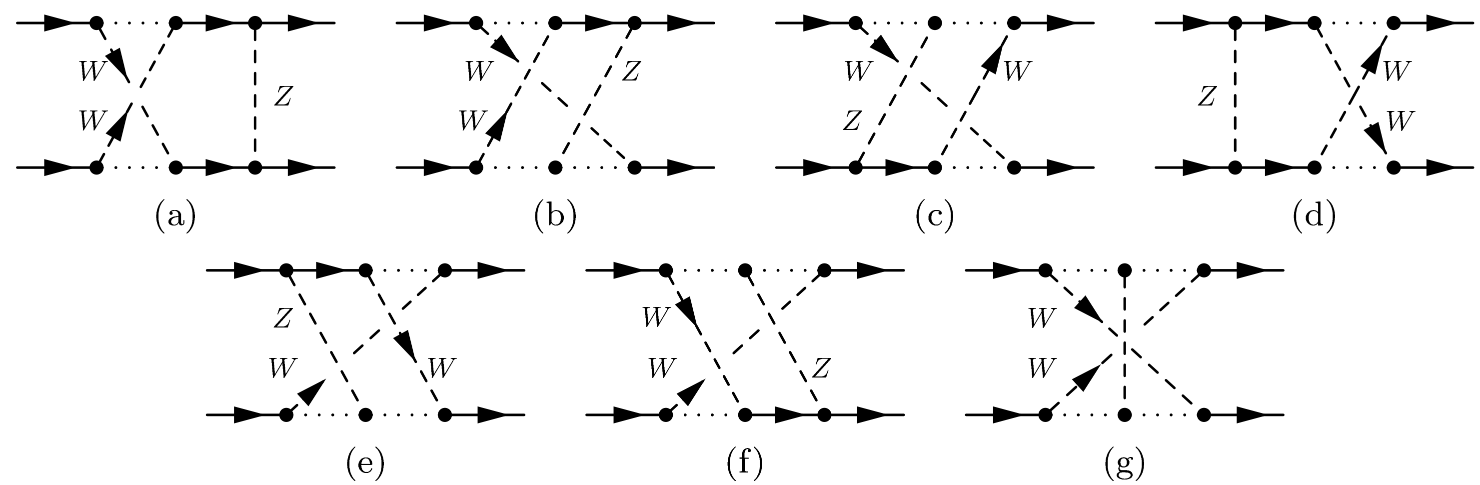
(VerBSE)

Two-loops t-channel diagrams from the gauge-invariant set of vertices and boson self-energies. Here, the circles represent the contributions of self-energies and vertex functions.

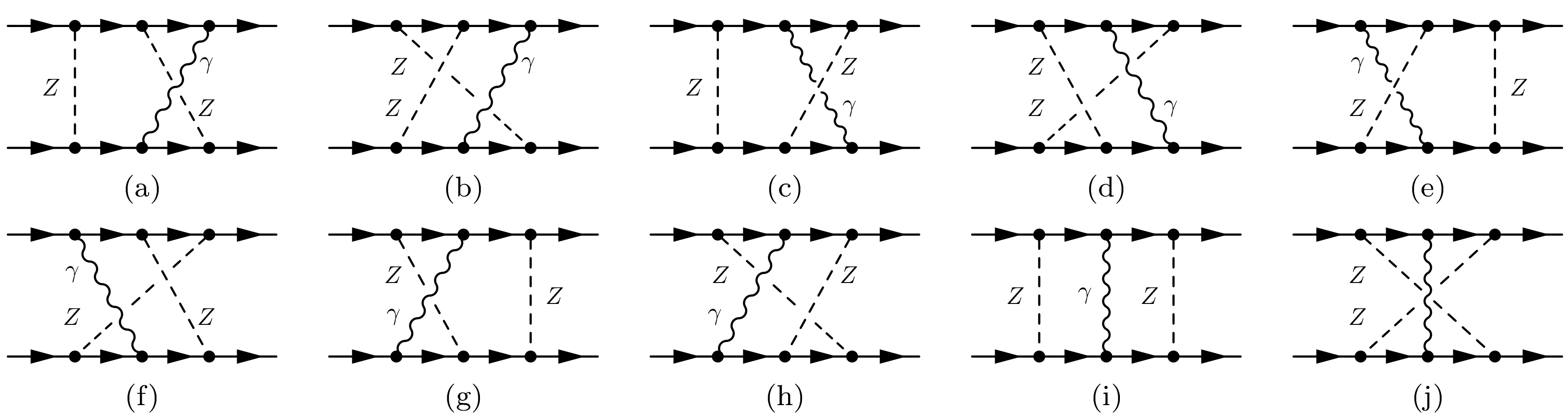
Ladder-Box Diagrams



Diagrams with ZZZ exchange.



Diagrams with WWZ exchange.



Diagrams with ZZγ exchange.

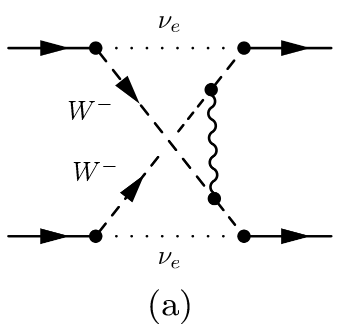
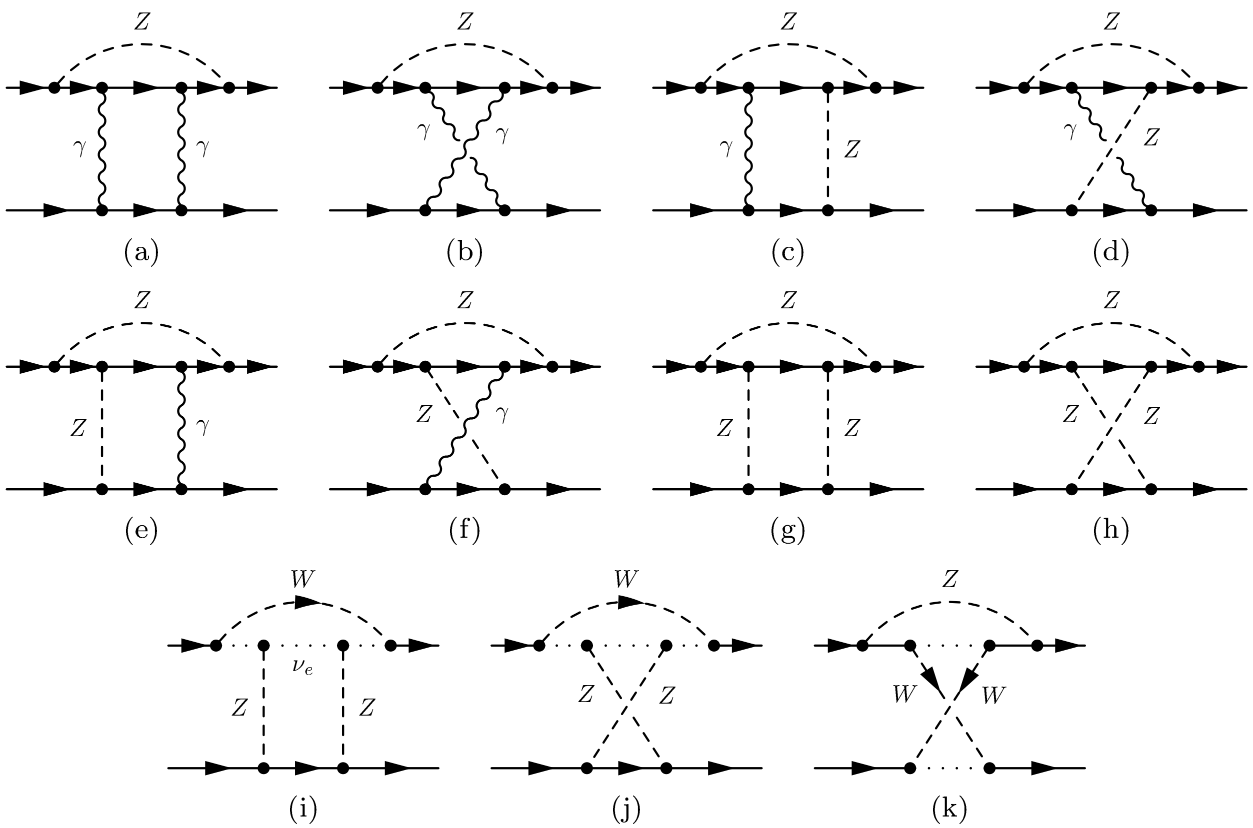
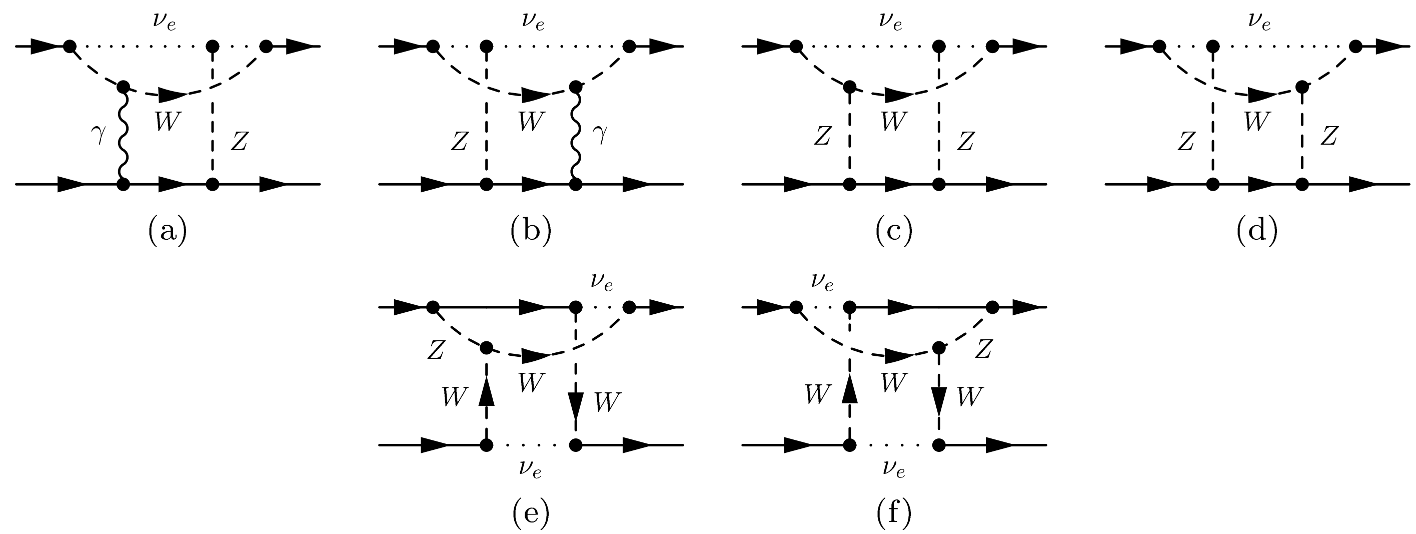


Diagram with W W γ exchange.

(Double Boxes)

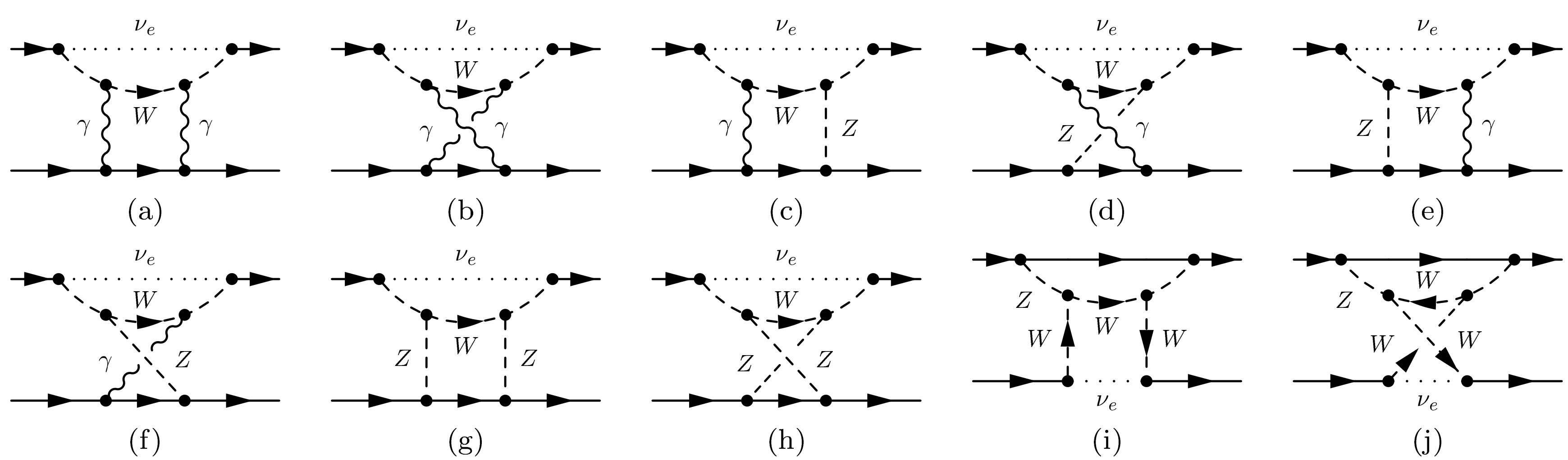
Decorated-Box Diagrams





Decorated boxes of type III.

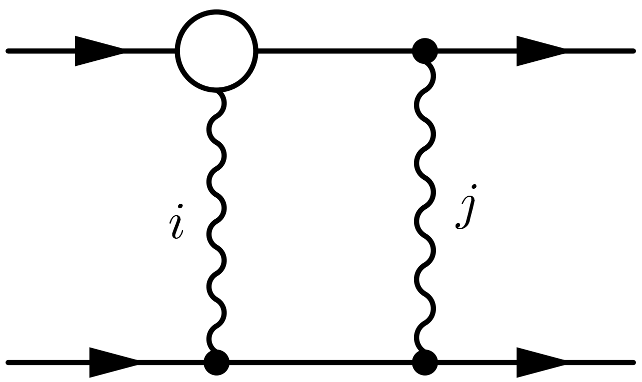
Decorated boxes of type I.

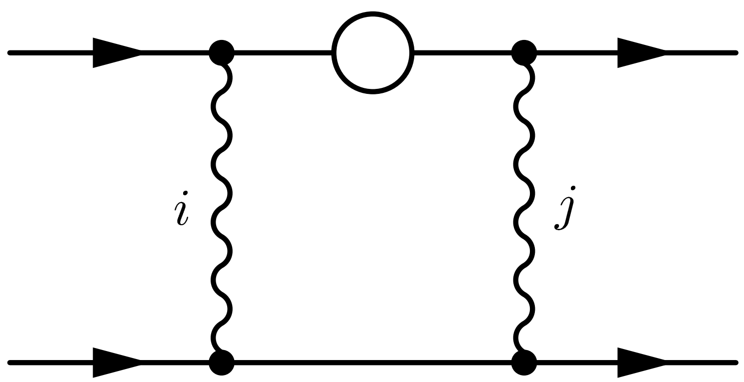


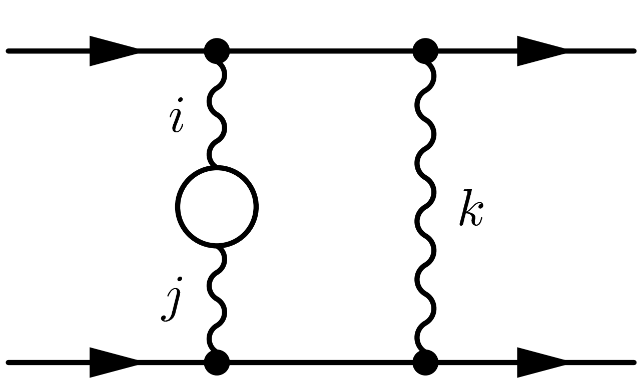
Decorated boxes of type II.

(Double Boxes)

Boxes with Lepton Self-Energy and Vertex Insertions



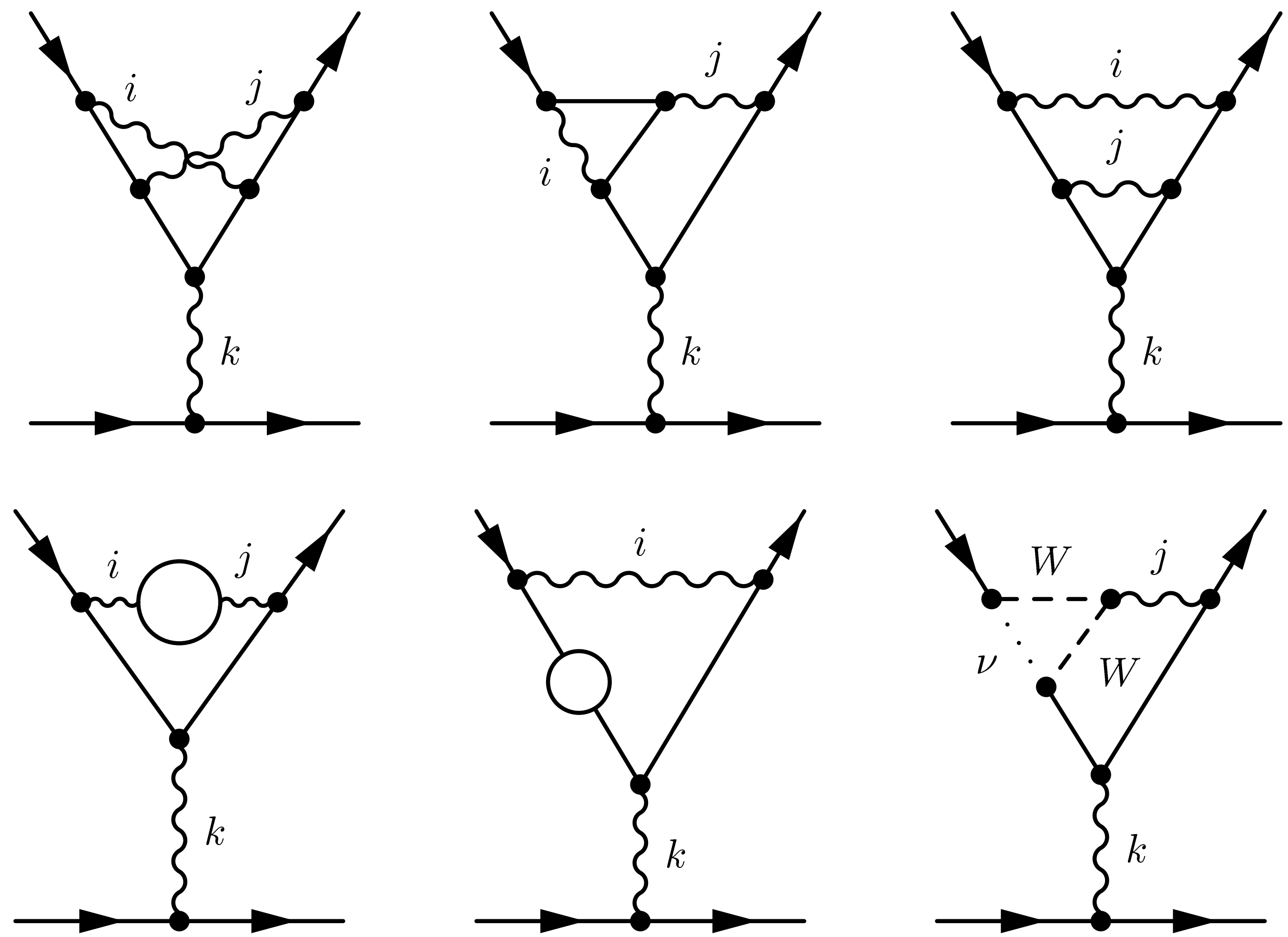




Boxes with vertices (VB), fermion self-energy boxes FSEB and boson self-energy boxes BSEB.

(SE and Ver in boxes)

Double Vertices



Two loops electron vertices
(NNLO EW Vert)

Final Expressions

Combining all the terms together, we get the infrared-finite result at one-loop level

$$\sigma_{NLO} = \frac{\alpha}{\pi}\Re[R_1 + \delta_1^f]\sigma_0$$

and NNLO level

$$\sigma_{NNLO} = \sigma_Q + \sigma_T + \sigma^\gamma + \sigma^{\gamma\gamma} = \left(\frac{\alpha}{\pi}\right)^2 \Re\left[R_1^*\delta_1^f + \frac{1}{2}|R_1|^2 - \frac{1}{2}R_2 + \delta_Q^f + \delta_T^f\right]\sigma_0$$

$$= \quad \sigma_O^f + \sigma_B^f + \sigma_Q^f + \sigma_T^f,$$

where:

$$\sigma_O^f = \frac{\alpha}{\pi}\Re[R_1^*\sigma_{NLO}] \qquad \sigma_B^f = -\frac{1}{2}\left(\frac{\alpha}{\pi}\right)^2\Re(|R_1|^2 + R_2)\sigma_0$$

$$\sigma_Q^f = \left(\frac{\alpha}{\pi}\right)^2\delta_Q^f\sigma_0 \qquad \sigma_T^f = \left(\frac{\alpha}{\pi}\right)^2\delta_T^f\sigma_0$$

$$R_1 = -4B\log\frac{\sqrt{s}}{2\omega} - \left(\log\frac{s}{m^2} - 1\right)^2 + 1 - \frac{\pi^2}{3} + \log^2\frac{u}{t}, \;\; R_2 = \frac{8}{3}\pi^2 B^2, \;\; B = \log\frac{tu}{m^2 s} - 1 + i\pi$$

Combination of Corrections

For the orthogonal kinematics: $\theta = 90^\circ$

Type of contribution	$\delta_A{}^C$
NLO	-0.6953
...+Q+ BBSE+VVer+	-0.6420
...+ double boxes	-0.6534
...+NNLO QED	-0.6500
...+SE and Ver in boxes	-0.6539
...+NNLO EW Ver	-0.6574

Correction to PV asymmetry:

$$\delta_A^C = \frac{A_{LR}^C - A_{LR}^0}{A_{LR}^0}$$

Soft-photon bremsstrahlung cut:

$$\omega = 0.05\sqrt{s}$$

"..." means all contributions from the lines above

A. Aleksejevs, S. Barkanova, Y. Kolomensky, E. Kuraev, V. Zykunov, Phys. Rev. D 85 (2012) 013007
A. Aleksejevs, S. Barkanova, Y. Kolomensky, E. Kuraev, V. Zykunov, Nuovo Cim. C035N04 (2012) 192-197
A. Aleksejevs, S. Barkanova, V. Zykunov, Phys. Atom. Nucl., 75(2012) 209-226
A. Aleksejevs, S. Barkanova, Y. Bystritskiy, A. Ilyichev, E. Kuraev, V. Zykunov, Phys. Rev. D 85 (2012) 013007
A. Aleksejevs, S. Barkanova, Y. Bystritskiy, E. Kuraev, V. Zykunov, Phys. Part. Nucl. Lett. 12(2015) 5 645-656
A. Aleksejevs, S. Barkanova, Y. Bystritskiy, E. Kuraev, V. Zykunov, Phys. of Part. and Nucl. Letters, (2016), 13-3, 310–317

Third Stage: Computer Algebra

- The most of the leading two-loop EWC corrections to Moller process has been completed.

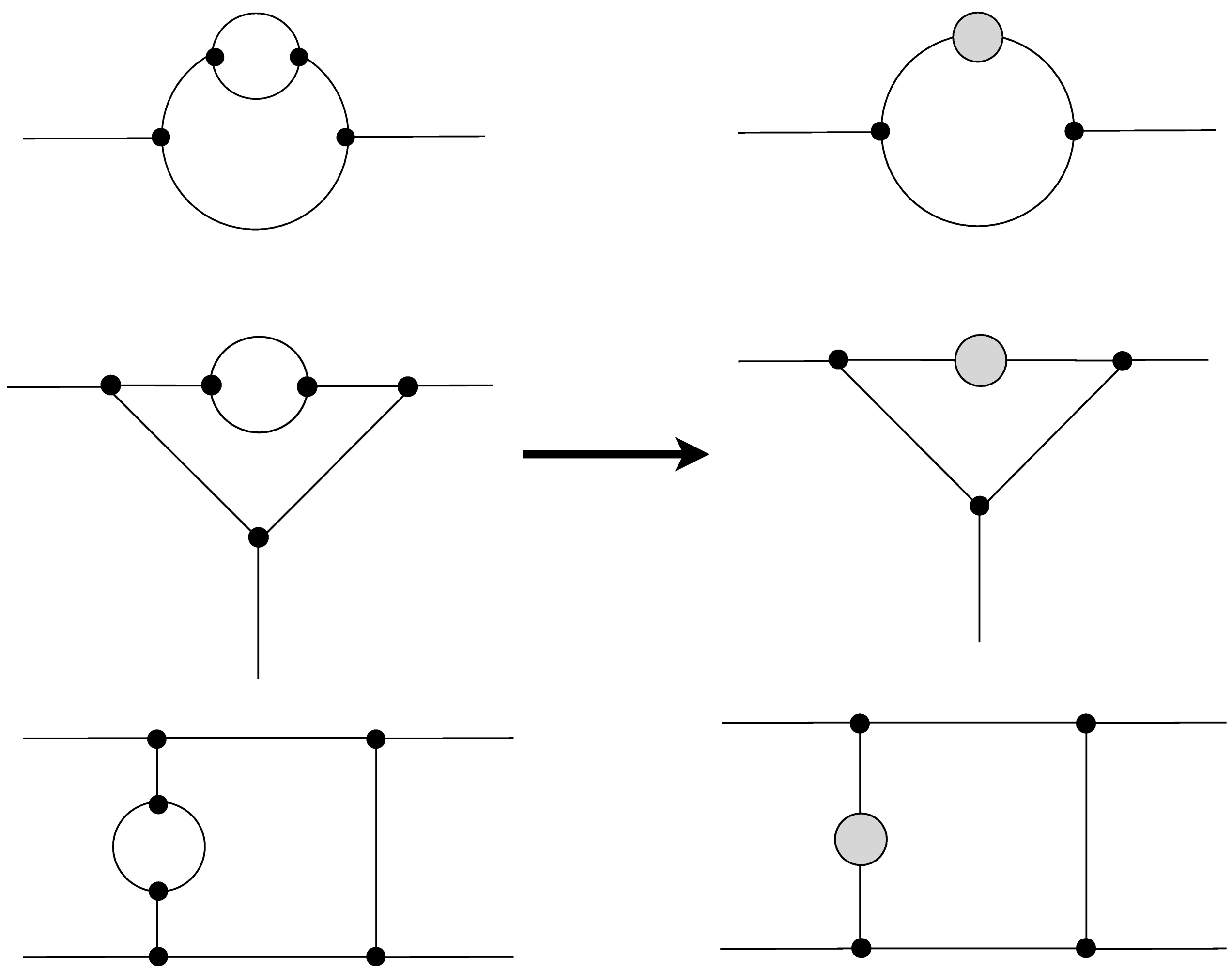
- It is essential to apply alternative approaches in two-loop EWC calculations for the cross-check purposes.

- We develop the third stage method which is based on the dispersive representation of many-point Passarino-Veltman functions.

- Advantages include not only cross checking previous results, but also our ability to retain kinematical dependence of two-loop EWC and inclusion of broader sets of two-loops graphs.

Sub-Loop Insertions: Self-Energy

W. Hollik, U. Meier, S. Uccirati, Nucl.Phys. B731 213-224 (2005)

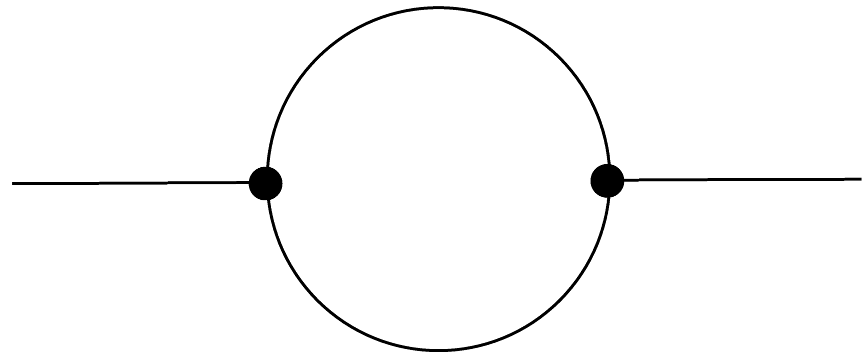


$$L(q^2) = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\Im L(s)}{s - q^2 - i\epsilon}$$

•Replace self-energy insertion by effective propagator

• Dispersive representation of self-energy sub-loop has propagator like structure with mass s

Self-Energy Sub-Loop



Vector boson: $\Sigma_{\mu\nu}^{V-V}(q) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Sigma_T^{V-V}\left(q^2\right) + \frac{q_\mu q_\nu}{q^2}\Sigma_L^{V-V}\left(q^2\right)$

Fermion: $\Sigma^f(q) = \not{q}\,\omega_-\Sigma_L^f\left(q^2\right) + \not{q}\,\omega_+\Sigma_R^f\left(q^2\right) + m_f\Sigma_S^f\left(q^2\right)$

Each of the Σ terms are functions of:

$$B_{i,ij,ijk}\left(q^2, m_\alpha^2, m_\beta^2\right) = \frac{1}{\pi}\int_{\left(m_\alpha+m_\beta\right)^2}^{\infty} ds \frac{\Im B_{i,ij,ijk}\left(s, m_\alpha^2, m_\beta^2\right)}{s - q^2 - i\epsilon}$$

Sub-Loop: Vector Boson SE

First loop insertion:

$$\Sigma_{\mu\nu}^{V-V}(q) = \frac{1}{\pi}\sum_{\alpha,\beta}\int_{(m_\alpha+m_\beta)^2}^{\infty} ds \frac{1}{s-q^2-i\epsilon}\left[\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\Im\Sigma_T^{V-V}\left(s,m_\alpha^2,m_\beta^2\right)+\frac{q_\mu q_\nu}{q^2}\Im\Sigma_L^{V-V}\left(s,m_\alpha^2,m_\beta^2\right)\right]$$

Second loop integration:

$$I_{\mu_1\mu_2,\nu_1\ldots\nu_R}^{1,M,1} = \frac{1}{\pi}\frac{\mu^{(4-D)}}{\left(i\pi^{D/2}\right)}\sum_{\alpha,\beta}\int_{(m_\alpha+m_\beta)^2}^{\infty} ds \int d^D q_2 \cdot \frac{q_{2,\nu_1}\ldots q_{2,\nu_R}}{(s-q_2^2-i\epsilon)\prod_{j=0}^{M}\left[(q_2+k_{j,M})^2-m_{j,M}^2\right]} F_{\mu_1\mu_2}\left(q_2,s,m_\alpha,m_\beta\right)$$

$$F_{\mu_1\mu_2}\left(q_2,s,m_\alpha,m_\beta\right) = \left(g_{\mu_1\mu_2}-\frac{q_{2\mu_1}q_{2\mu_2}}{q_2^2}\right)\Im\Sigma_T^{V-V}\left(s,m_\alpha^2,m_\beta^2\right)+\frac{q_{2\mu_1}q_{2\mu_2}}{q_2^2}\Im\Sigma_L^{V-V}\left(s,m_\alpha^2,m_\beta^2\right)$$

Sub-Loop: Fermion SE

First loop insertion:

$$\Sigma^f(q) = \frac{1}{\pi} \sum_{\alpha,\beta} \int\limits_{(m_\alpha+m_\beta)^2}^{\infty} ds \frac{1}{s - q^2 - i\epsilon} \left[\not{q}\omega_- \Im\Sigma_L^f\left(s, m_\alpha^2, m_\beta^2\right) + \not{q}\omega_+ \Im\Sigma_R^f\left(s, m_\alpha^2, m_\beta^2\right) + m_f \Im\Sigma_S^f\left(s, m_\alpha^2, m_\beta^2\right) \right]$$

Second loop integration:

$$I^{1,M,1}_{\nu_1 \ldots \nu_R} = \frac{1}{\pi} \frac{\mu^{(4-D)}}{\left(i\pi^{D/2}\right)} \sum_{\alpha,\beta} \int\limits_{(m_\alpha+m_\beta)^2}^{\infty} ds \int d^D q_2 \cdot \frac{q_{2,\nu_1} \ldots q_{2,\nu_R}}{\left(s - q_2^2 - i\epsilon\right) \prod\limits_{j=0}^{M} \left[(q_2 + k_{j,M})^2 - m_{j,M}^2\right]} G\left(q_2, s, m_\alpha, m_\beta\right)$$

$$G\left(q_2, s, m_\alpha, m_\beta\right) = \left[\not{q}_2\omega_- \Im\Sigma_L^f\left(s, m_\alpha^2, m_\beta^2\right) + \not{q}_2\omega_+ \Im\Sigma_R^f\left(s, m_\alpha^2, m_\beta^2\right) + m_f \Im\Sigma_S^f\left(s, m_\alpha^2, m_\beta^2\right) \right]$$

Self-Energy Sub-Loop: General Structure

Vector boson SE sub-loop insertion two-loop result:

$$I^{1,M,1}_{\mu_1\mu_2,\nu_1\ldots\nu_R} = \frac{1}{\pi} \sum_{\alpha,\beta} \int\limits_{(m_\alpha+m_\beta)^2}^{\infty} ds \cdot \Big[L^{1,M,1}_{a,\mu_1\mu_2,\nu_1\ldots\nu_R}(D,E,F)\, \Im\Sigma^{V-V}_T\left(s, m^2_\alpha, m^2_\beta\right) +$$

$$L^{1,M,1}_{b,\mu_1\mu_2,\nu_1\ldots\nu_R}(D,E,F)\, \Im\Sigma^{V-V}_L\left(s, m^2_\alpha, m^2_\beta\right) \Big]$$

Fermion SE sub-loop insertion two-loop result:

$$I^{1,M,1}_{\nu_1\ldots\nu_R} = \frac{1}{\pi} \sum_{\alpha,\beta} \int\limits_{(m_\alpha+m_\beta)^2}^{\infty} ds \cdot \Big[N^{1,M,1}_{a,\nu_1\ldots\nu_R}(D,E,F)\, \Im\Sigma^{f}_L\left(s, m^2_\alpha, m^2_\beta\right) \omega_- +$$

$$N^{1,M,1}_{b,\nu_1\ldots\nu_R}(D,E,F)\, \Im\Sigma^{f}_R\left(s, m^2_\alpha, m^2_\beta\right) \omega_+ + N^{1,M,1}_{c,\nu_1\ldots\nu_R}(D,E,F)\, \Im\Sigma^{f}_S\left(s, m^2_\alpha, m^2_\beta\right) \Big]$$

Vector Boson SE Subtractions

{Z-Z} or {W-W} mixings:

$$\hat{\Sigma}^{V-V}(q^2) = \Sigma^{V-V}(q^2) - \Sigma^{V-V}(m_V^2) - \frac{\partial}{\partial q^2}\Sigma^{V-V}(q^2)\Big|_{q^2=m_V^2}(q^2 - m_V^2) =$$

$$\frac{(q^2 - m_V^2)^2}{\pi}\sum_{\alpha,\beta}\int_{(m_\alpha+m_\beta)^2}^{\infty} ds \frac{\Im\Sigma^{V-V}(s, m_\alpha, m_\beta)}{(s - m_V^2)^2 (s - q^2 - i\epsilon)}$$

ϒ-Z mixing:

$$\hat{\Sigma}^{\gamma-Z}(q^2) = \Sigma^{\gamma-Z}(q^2) - \frac{1}{m_Z^2}\left[\Sigma^{\gamma-Z}(0)q^2 - \Sigma^{\gamma-Z}(m_Z^2)(q^2 - m_Z^2)\right] =$$

$$\frac{q^2 (q^2 - m_Z^2)}{\pi}\sum_{\alpha,\beta}\int_{(m_\alpha+m_\beta)^2}^{\infty} ds \frac{\Im\Sigma^{\gamma-Z}(s, m_\alpha, m_\beta)}{s(s - m_Z^2)(s - q^2 - i\epsilon)}$$

ϒ-ϒ mixing:

$$\hat{\Sigma}^{\gamma-\gamma}(q^2) = \frac{q^4}{\pi}\sum_{\alpha,\beta}\int_{(m_\alpha+m_\beta)^2}^{\infty} ds \frac{\Im\Sigma^{\gamma-\gamma}(s, m_\alpha, m_\beta)}{s^2 (s - q^2 - i\epsilon)}$$

Fermion SE Subtractions

$$\hat{\Sigma}^f(q) = \Sigma^f(\not{q}) - \Sigma^f(m_f) - \frac{\partial}{\partial \not{q}} \Sigma^f(\not{q}) \Big|_{\not{q}=m_f} (\not{q} - m_f) =$$

$$\not{q}\omega_- \left(I_L + a_L\right) + \not{q}\omega_+ \left(I_R + a_R\right) + m_f \left(I_S + a_S\right)$$

$$I_{L,R,S} = \frac{q^2 - m_f^2}{\pi} \sum_{\alpha,\beta} \int\limits_{\left(m_\alpha + m_\beta\right)^2}^{\infty} ds \frac{\Im \Sigma^f_{L,R,S}(s, m_\alpha, m_\beta)}{\left(s - m_f^2\right)\left(s - q^2 - i\epsilon\right)}$$

$$a_{L,R} = -2m_f^2 \left(\Sigma'_{L,R}\left(m_f^2\right) + \Sigma'_S\left(m_f^2\right)\right)$$

$$a_S = m_f^2 \left(\Sigma'_L\left(m_f^2\right) + \Sigma'_R\left(m_f^2\right) + 2\Sigma'_S\left(m_f^2\right)\right)$$

Effective SE Propagators

Vector boson effective propagator:

$$\Pi^{V-V}_{\mu\nu}(q) = \Pi^{V-V}_{T,\mu\nu} + \Pi^{V-V}_{L,\mu\nu}$$

$$\Pi^{V-V}_{T,\mu\nu} = \frac{-ig_{\rho\mu}}{q^2 - m_V^2}\left[\frac{g^{\rho\sigma} - \frac{q^\rho q^\sigma}{q^2}}{s - q^2 - i\epsilon}\Im\Sigma_T^{V-V}\left(s, m_\alpha^2, m_\beta^2\right)\right]\frac{-ig_{\sigma\nu}}{q^2 - m_V^2}$$

$$\Pi^{V-V}_{L,\mu\nu} = \frac{-ig_{\rho\mu}}{q^2 - m_V^2}\left[\frac{\frac{q^\rho q^\sigma}{q^2}}{s - q^2 - i\epsilon}\Im\Sigma_L^{V-V}\left(s, m_\alpha^2, m_\beta^2\right)\right]\frac{-ig_{\sigma\nu}}{q^2 - m_V^2}$$

Fermion effective propagator:

$$\Pi^f(q) = \frac{1}{\not{q} - m_f}\left[\frac{G\left(q, s, m_\alpha, m_\beta\right)}{s - q^2 - i\epsilon}\right]\frac{1}{\not{q} - m_f}$$

Effective Propagators: Subtracted VB

$$\hat{\Pi}^{V-V}_{\mu\nu}(q) = \hat{\Pi}^{V-V}_{T,\mu\nu} + \hat{\Pi}^{V-V}_{L,\mu\nu}$$

$$\hat{\Pi}^{V-V}_{T,\mu\nu} = -T^{V-V}\left(s, m_V^2\right)\left[\frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}}{s - q^2 - i\epsilon}\right]\Im\Sigma^{V-V}_{T}\left(s, m_\alpha^2, m_\beta^2\right)$$

$$\hat{\Pi}^{V-V}_{L,\mu\nu} = -T^{V-V}\left(s, m_V^2\right)\left[\frac{\frac{q_\mu q_\nu}{q^2}}{s - q^2 - i\epsilon}\right]\Im\Sigma^{V-V}_{L}\left(s, m_\alpha^2, m_\beta^2\right)$$

	$\gamma - \gamma$	$\{Z, W\} - \{Z, W\}$	$\gamma - Z$
T^{V-V}	$\frac{1}{s^2}$	$\frac{1}{\left(s - m^2_{\{Z,W\}}\right)^2}$	$\frac{1}{s\left(s - m_z^2\right)}$

Effective Propagators: Subtracted FF

$$\hat{\Pi}^f(q) = \hat{\Pi}_1^f(q) + \hat{\Pi}_2^f(q)$$

$$\hat{\Pi}_1^f(q) = (\not{q} + m_f)\left[\frac{y_L \not{q}\omega_- + y_R \not{q}\omega_+ + m_f y_S}{\left(q^2 - m_f^2\right)\left(s - q^2 - i\epsilon\right)}\right](\not{q} + m_f)$$

$$\hat{\Pi}_2^f(q) = \frac{1}{\not{q} - m_f}\left[d_L \not{q}\omega_- + d_R \not{q}\omega_+ + m_f d_S\right]\frac{1}{\not{q} - m_f},$$

$$y_{L,R,S} \equiv y_{L,R,S}\left(s, m_\alpha^2, m_\beta^2\right) = \frac{\Im\Sigma_{L,R,S}^f}{s - m_f^2}$$

$$d_{L,R} \equiv d_{L,R}\left(s, m_\alpha^2, m_\beta^2\right) = -2m_f^2\frac{\Im\Sigma_{L,R}^f + \Im\Sigma_S^f}{\left(s - m_f^2\right)^2}$$

$$d_S \equiv d_S\left(s, m_\alpha^2, m_\beta^2\right) = m_f^2\frac{\Im\Sigma_L^f + \Im\Sigma_R^f + 2\Im\Sigma_S^f}{\left(s - m_f^2\right)^2}$$

Self-Energy Sub-Loop: Subtracted

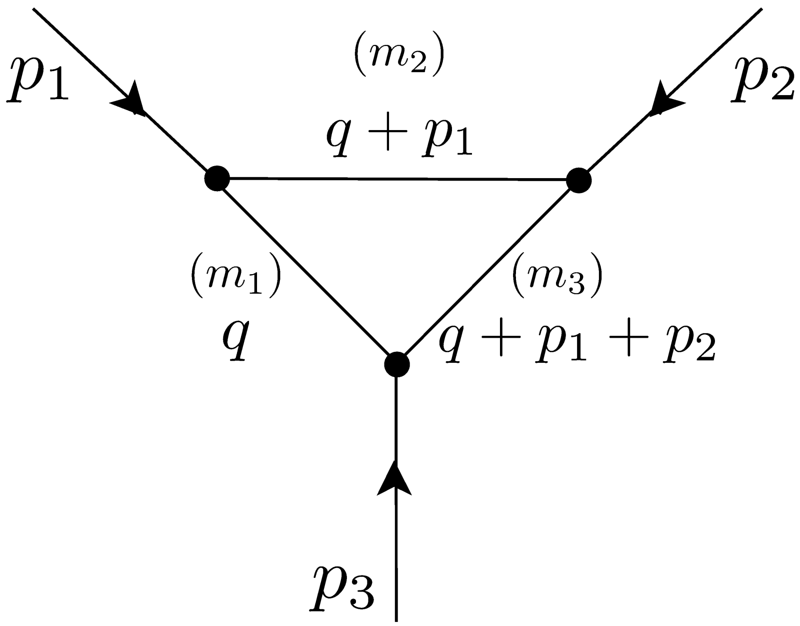
Vector boson SE sub-loop insertion two-loop result (subtracted):

$$\hat{I}^{1,M,1}_{\mu_1\mu_2,\nu_1\ldots\nu_R} = \frac{1}{\pi}\sum_{\alpha,\beta}\int\limits_{(m_\alpha+m_\beta)^2}^{\infty} ds \cdot \Big[L^{1,M,1}_{a,\mu_1\mu_2,\nu_1\ldots\nu_R}\left(\underline{B,C,D}\right)\Im\Sigma^{V-V}_T\left(s,m_\alpha^2,m_\beta^2\right) + L^{1,M,1}_{b,\mu_1\mu_2,\nu_1\ldots\nu_R}\left(\underline{B,C,D}\right)\Im\Sigma^{V-V}_L\left(s,m_\alpha^2,m_\beta^2\right)\Big]$$

Fermion SE sub-loop insertion two-loop result (subtracted):

$$\hat{I}^{1,M,1}_{\nu_1\ldots\nu_R} = \frac{1}{\pi}\sum_{\alpha,\beta}\int\limits_{(m_\alpha+m_\beta)^2}^{\infty} ds \cdot \Big[N^{1,M,1}_{a,\nu_1\ldots\nu_R}\left(\underline{C,D,E}\right)\Im\Sigma^{f}_L\left(s,m_\alpha^2,m_\beta^2\right)\omega_- + N^{1,M,1}_{b,\nu_1\ldots\nu_R}\left(\underline{C,D,E}\right)\Im\Sigma^{f}_R\left(s,m_\alpha^2,m_\beta^2\right)\omega_+ + N^{1,M,1}_{c,\nu_1\ldots\nu_R}\left(\underline{C,D,E}\right)\Im\Sigma^{f}_S\left(s,m_\alpha^2,m_\beta^2\right)\Big]$$

Second Loop Integration: Many-Point PV



$$C_0 \equiv C_0\left(p_1^2, p_2^2, (p_1+p_2)^2, m_1^2, m_2^2, m_3^2\right) =$$

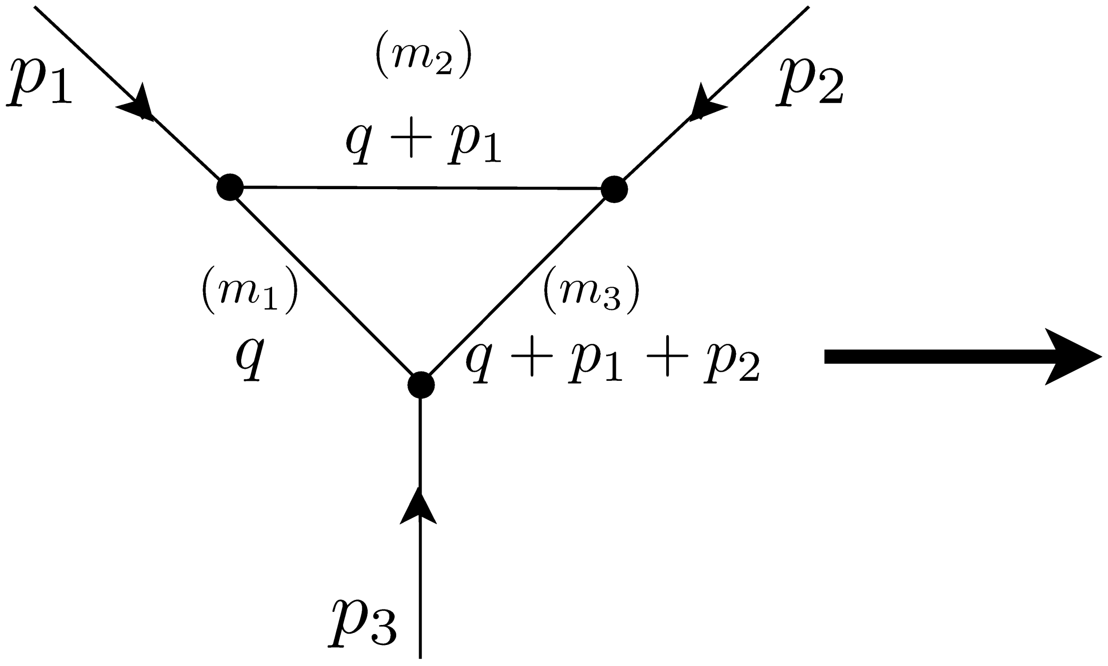
$$\frac{\mu^{4-D}}{i\pi^{D/2}} \int d^D q \frac{1}{\left[q^2 - m_1^2\right]\left[(q+p_1)^2 - m_2^2\right]\left[(q+p_1+p_2)^2 - m_3^2\right]}$$

$$C_0 = \frac{\mu^{4-D}}{i\pi^{D/2}} \int_0^1 dx \int d^D\tau \frac{1}{\left[(\tau - (p_1\bar{x} + p_2))^2 - m_{12}^2\right]^2 \left[\tau^2 - m_3^2\right]}, \text{ here } \quad m_{12}^2 = m_1^2\bar{x} + m_2^2 x - p_1^2 x\bar{x}$$

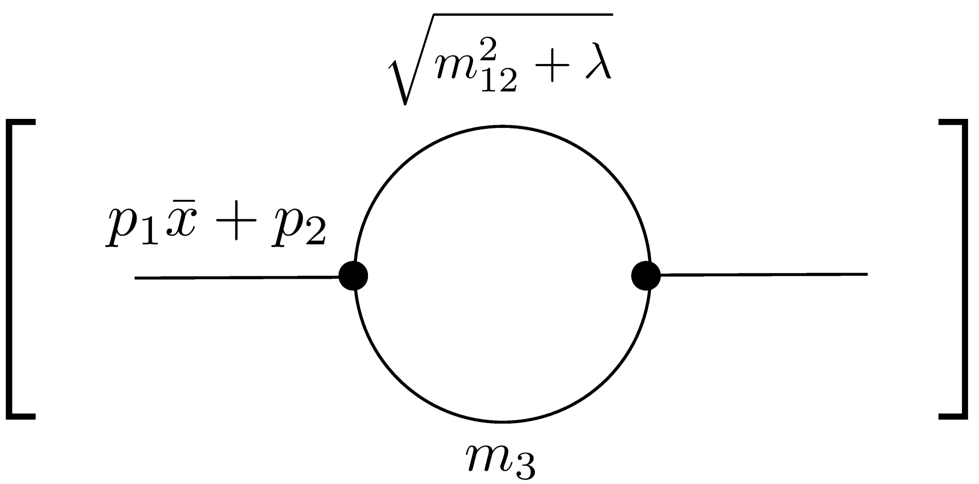
Using: $$\left((\tau - (p_1\bar{x}+p_2))^2 - m_{12}^2\right)^{-2} = \lim_{\lambda\to 0} \frac{\partial}{\partial\lambda} \left((\tau - (p_1\bar{x}+p_2))^2 - \left(m_{12}^2 + \lambda\right)\right)^{-1}$$

$$C_0 = \frac{\mu^{4-D}}{i\pi^{D/2}} \lim_{\lambda\to 0} \frac{\partial}{\partial\lambda} \int_0^1 dx \int d^D\tau \frac{1}{\left[(\tau - (p_1\bar{x}+p_2))^2 - (m_{12}^2+\lambda)\right]\left[\tau^2 - m_3^2\right]} = \lim_{\lambda\to 0} \frac{\partial}{\partial\lambda} \int_0^1 dx\ B_0\left((p_1\bar{x}+p_2)^2, m_3^2, m_{12}^2+\lambda\right)$$

Second Loop Integration: Many-Point PV



$$\lim_{\lambda \to 0} \frac{\partial}{\partial \lambda} \int_0^1 dx$$



Second Loop Integration: Many-Point PV

$$C_{\mu_1\ldots\mu_N} = \frac{\mu^{4-D}}{i\pi^{D/2}} \int d^D q \frac{q_{\mu_1}\ldots q_{\mu_N}}{\left[q^2 - m_1^2\right]\left[(q+p_1)^2 - m_2^2\right]\left[(q+p_1+p_2)^2 - m_3^2\right]}$$

<u>Vector case:</u>

Applying tensor decomposition to both sides of above equation:

$$p_{1\mu}C_1 + (p_{1\mu} + p_{2\mu})\, C_2 = \lim_{\lambda\to 0} \frac{\partial}{\partial\lambda} \int_0^1 dx \left[B_\mu - (p_{1\mu} + p_{2\mu})\, B_0\right]$$

$$\downarrow$$

$$B_\mu = -\left(p_{1\mu}\bar{x} + p_{2\mu}\right) B_1$$

Matching coefficients in front of $p_{1,2}$, we get:

$$C_1 = \lim_{\lambda\to 0} \frac{\partial}{\partial\lambda} \int_0^1 dx B_1 x \qquad C_2 = -\lim_{\lambda\to 0} \frac{\partial}{\partial\lambda} \int_0^1 dx \left[B_0 + B_1\right]$$

Second Loop Integration: Many-Point PV

$$\hat{\mathbf{I}}_C = \lim_{\lambda\to 0} \frac{\partial}{\partial\lambda} \int_0^1 dx ...$$

$C_{\mu\nu}$:

$$C_{00} = \hat{\mathbf{I}}_C \left[B_{00}\right] \qquad C_{12} = -\hat{\mathbf{I}}_C \left[\left(B_1 + B_{11}\right) x\right]$$

$$C_{11} = \hat{\mathbf{I}}_C \left[B_{11} x^2\right] \qquad C_{22} = \hat{\mathbf{I}}_C \left[B_0 + 2B_1 + B_{11}\right]$$

$C_{\mu\nu\alpha}$:

$$C_{001} = \hat{\mathbf{I}}_C \left[B_{001} x\right] \qquad C_{112} = -\hat{\mathbf{I}}_C \left[\left(B_{11} + B_{111}\right) x^2\right]$$

$$C_{002} = -\hat{\mathbf{I}}_C \left[B_{00} + B_{001}\right] \qquad C_{122} = \hat{\mathbf{I}}_C \left[\left(B_1 + 2B_{11} + B_{111}\right) x\right]$$

$$C_{111} = \hat{\mathbf{I}}_C \left[B_{111} x^3\right] \qquad C_{222} = -\hat{\mathbf{I}}_C \left[B_0 + 3\left(B_1 + B_{11}\right) + B_{111}\right]$$

$$B_{i,ij,ijk} \equiv B_{i,ij,ijk} \left[\left(p_1 \bar{x} + p_2\right)^2, m_3^2, m_{12}^2 + \lambda\right]$$

Second Loop Integration: Many-Point PV

$$D_0 \equiv D_0\left(p_1^2, p_2^2, p_3^2, p_4^2, (p_1+p_2)^2, (p_2+p_3)^2, m_1^2, m_2^2, m_3^2, m_4^2\right) =$$

$$\frac{\mu^{4-D}}{i\pi^{D/2}} \int d^D q \frac{1}{\left[q^2 - m_1^2\right]\left[(q+p_1)^2 - m_2^2\right]\left[(q+p_1+p_2)^2 - m_3^2\right]\left[(q+p_1+p_2+p_3)^2 - m_4^2\right]}$$

$$D_0 = 2\frac{\mu^{4-D}}{i\pi^{D/2}} \int_0^1 dx \int_0^{1-x} dy \int d^D\tau \frac{1}{\left[(\tau - (p_1(\bar{x}-y) + p_2\bar{y} + p_3))^2 - m_{123}^2\right]^3 \left[\tau^2 - m_4^2\right]}$$

$$m_{123}^2 = m_1^2(\bar{x}-y) + m_2^2 x + m_3^2 y - p_1^2 x\bar{x} - p_{12}^2 y\bar{y} + 2xy(p_1 p_{12})$$

↓

$$p_{12} = p_1 + p_2$$

$$D_0 = \lim_{\lambda\to 0} \frac{\partial^2}{\partial\lambda^2} \int_0^1 dx \int_0^{\bar{x}} dy B_0\left[(p_1(\bar{x}-y) + p_2\bar{y} + p_3)^2, m_4^2, m_{123}^2 + \lambda\right]$$

Second Loop Integration: Many-Point PV

$$\hat{\mathbf{I}}_D = \lim_{\lambda\to 0} \frac{\partial^2}{\partial\lambda^2} \int_0^1 dx \int_0^{\bar{x}} dy ...$$

$$B_{i,ij,ijk,ijkl} \equiv B_{i,ij,ijk,ijkl}\left[\left(p_1\left(\bar{x}-y\right)+p_2\bar{y}+p_3\right)^2, m_4^2, m_{123}^2+\lambda\right]$$

D_μ :

$$D_1 = \hat{\mathbf{I}}_D\left[B_1 x\right]$$

$$D_2 = \hat{\mathbf{I}}_D\left[B_1 y\right]$$

$$D_3 = -\hat{\mathbf{I}}_D\left[B_0 + B_1\right]$$

$D_{\mu\nu}$:

$$D_{00} = \hat{\mathbf{I}}_D\left[B_{00}\right]$$

$$D_{11} = \hat{\mathbf{I}}_D\left[B_{11}x^2\right]$$

$$D_{22} = \hat{\mathbf{I}}_D\left[B_{11}y^2\right]$$

$$D_{33} = \hat{\mathbf{I}}_D\left[B_0 + 2B_1 + B_{11}\right]$$

$$D_{12} = \hat{\mathbf{I}}_D\left[B_{11}xy\right]$$

$$D_{13} = -\hat{\mathbf{I}}_D\left[\left(B_1 + B_{11}\right)x\right]$$

$$D_{23} = -\hat{\mathbf{I}}_D\left[\left(B_1 + B_{11}\right)y\right].$$

$D_{\mu\nu\rho}$:

$$D_{001} = \hat{\mathbf{I}}_D\left[B_{001}x\right]$$

$$D_{002} = \hat{\mathbf{I}}_D\left[B_{001}y\right]$$

$$D_{003} = -\hat{\mathbf{I}}_D\left[B_{00} + B_{001}\right]$$

$$D_{111} = \hat{\mathbf{I}}_D\left[B_{111}x^3\right]$$

$$D_{112} = \hat{\mathbf{I}}_D\left[B_{111}x^2y\right]$$

$$D_{113} = -\hat{\mathbf{I}}_D\left[\left(B_{11} + B_{111}\right)x^2\right]$$

$$D_{122} = \hat{\mathbf{I}}_D\left[B_{111}xy^2\right]$$

$$D_{123} = -\hat{\mathbf{I}}_D\left[\left(B_{11} + B_{111}\right)xy\right]$$

$$D_{222} = \hat{\mathbf{I}}_D\left[B_{111}y^3\right]$$

$$D_{223} = -\hat{\mathbf{I}}_D\left[\left(B_{11} + B_{111}\right)y^2\right]$$

$$D_{233} = \hat{\mathbf{I}}_D\left[\left(B_1 + 2B_{11} + B_{111}\right)y\right]$$

$$D_{333} = -\hat{\mathbf{I}}_D\left[B_0 + 3\left(B_1 + B_{11}\right) + B_{111}\right]$$

$D_{\mu\nu\rho\sigma}$:

$$D_{0000} = \hat{\mathbf{I}}_D\left[B_{0000}\right]$$

$$D_{0011} = \hat{\mathbf{I}}_D\left[B_{0011}x^2\right]$$

$$D_{0012} = \hat{\mathbf{I}}_D\left[B_{0011}xy\right]$$

$$D_{0013} = -\hat{\mathbf{I}}_D\left[\left(B_{001} + B_{1111}\right)x\right]$$

$$D_{0022} = \hat{\mathbf{I}}_D\left[B_{0011}x^2\right]$$

$$D_{0023} = -\hat{\mathbf{I}}_D\left[\left(B_{001} + B_{1111}\right)y\right]$$

$$D_{0033} = \hat{\mathbf{I}}_D\left[B_{00} + 2B_{001} + B_{0011}\right]$$

$$D_{1111} = \hat{\mathbf{I}}_D\left[B_{1111}x^4\right]$$

$$D_{1112} = \hat{\mathbf{I}}_D\left[B_{1111}x^3y\right]$$

$$D_{1113} = -\hat{\mathbf{I}}_D\left[\left(B_{111} + B_{1111}\right)x^3\right]$$

$$D_{1122} = \hat{\mathbf{I}}_D\left[B_{1111}x^2y^2\right]$$

$$D_{1123} = -\hat{\mathbf{I}}_D\left[\left(B_{111} + B_{1111}\right)x^2y\right]$$

$$D_{1133} = \hat{\mathbf{I}}_D\left[\left(B_{11} + 2B_{111} + B_{1111}\right)x^2\right]$$

$$D_{1222} = \hat{\mathbf{I}}_D\left[B_{1111}xy^3\right]$$

$$D_{1223} = -\hat{\mathbf{I}}_D\left[\left(B_{111} + B_{1111}\right)xy^2\right]$$

$$D_{1233} = \hat{\mathbf{I}}_D\left[\left(B_{11} + 2B_{111} + B_{1111}\right)xy\right]$$

$$D_{1333} = -\hat{\mathbf{I}}_D\left[\left(B_1 + 3\left(B_{11} + B_{111}\right) + B_{1111}\right)x\right]$$

$$D_{2222} = \hat{\mathbf{I}}_D\left[B_{1111}y^4\right]$$

$$D_{2223} = -\hat{\mathbf{I}}_D\left[\left(B_{111} + B_{1111}\right)y^3\right]$$

$$D_{2233} = \hat{\mathbf{I}}_D\left[\left(B_{11} + 2B_{111} + B_{1111}\right)y^2\right]$$

$$D_{2333} = -\hat{\mathbf{I}}_D\left[\left(B_1 + 3\left(B_{11} + B_{111}\right) + B_{1111}\right)y\right]$$

$$D_{3333} = \hat{\mathbf{I}}_D\left[\left(B_0 + 4\left(B_1 + B_{111}\right) + 6B_{11} + B_{1111}\right)\right]$$

Second Loop Integration: Many-Point PV

$$E_0 \equiv E_0\left(p_1^2, p_2^2, p_3^2, p_4^2, p_5^2, p_{12}^2, p_{23}^2, p_{34}^2, p_{45}^2, p_{51}^2, m_1^2, m_2^2, m_3^2, m_4^2, m_5^2\right) =$$

$$\lim_{\lambda \to 0} \frac{\partial^3}{\partial \lambda^3} \int\limits_0^1 dx \int\limits_0^{\bar{x}} dy \int\limits_0^{\bar{x}-y} dz\, B_0\left[\left(p_1\left(\bar{x}-y-z\right)+p_2\left(\bar{y}-z\right)+p_3\bar{z}+p_4\right)^2, m_5^2, m_{1234}^2+\lambda\right]$$

$$p_{ij} = \left(p_i + p_j\right)^2, p_{ijk} = \left(p_i + p_j + p_k\right)^2$$

and $m_{1234}^2 = m_1^2\left(\bar{x}-y-z\right)+m_2^2 x+m_3^2 y+m_4^2 z-$

$$p_1^2 \bar{x} x - p_{12}^2 \bar{y} y - p_{123}^2 \bar{z} z + 2xy\left(p_1 p_{12}\right) + 2xz\left(p_1 p_{123}\right) + 2yz\left(p_{12} p_{123}\right)$$

Second Loop Integration: Many-Point PV

$$\hat{\mathbf{I}}_E = \lim_{\lambda\to 0}\frac{\partial^3}{\partial\lambda^3}\int_0^1 dx\int_0^{\bar{x}} dy\int_0^{\bar{x}-z} dz...$$

$$B_{i,ij,ijk,ijkl} \equiv B_{i,ij,ijk,ijkl}\left[\left(p_1\left(\bar{x}-y-z\right)+p_2\left(\bar{y}-z\right)+p_3\bar{z}+p_4\right)^2, m_5^2, m_{1234}^2+\lambda\right]$$

$E_1 = \hat{\mathbf{I}}_E\left[B_1 x\right]$

$E_2 = \hat{\mathbf{I}}_E\left[B_1 y\right]$

$E_3 = \hat{\mathbf{I}}_E\left[B_1 z\right]$

$E_3 = -\hat{\mathbf{I}}_E\left[B_0 + B_1\right]$

$E_{00} = \hat{\mathbf{I}}_E\left[B_{00}\right]$

$E_{11} = \hat{\mathbf{I}}_E\left[B_{11} x^2\right]$

$E_{12} = \hat{\mathbf{I}}_E\left[B_{11} xy\right]$

$E_{13} = \hat{\mathbf{I}}_E\left[B_{11} xz\right]$

$E_{14} = -\hat{\mathbf{I}}_E\left[\left(B_1 + B_{11}\right) x\right]$

$E_{22} = \hat{\mathbf{I}}_E\left[B_{11} y^2\right]$

$E_{23} = \hat{\mathbf{I}}_E\left[B_{11} yz\right]$

$E_{24} = -\hat{\mathbf{I}}_E\left[\left(B_1 + B_{11}\right) y\right]$

$E_{33} = \hat{\mathbf{I}}_E\left[B_{11} z^2\right]$

$E_{34} = -\hat{\mathbf{I}}_E\left[\left(B_1 + B_{11}\right) z\right]$

$E_{44} = \hat{\mathbf{I}}_E\left[B_0 + 2B_1 + B_{11}\right]$

$E_{0000} = \hat{\mathbf{I}}_E\left[B_{0000}\right]$

$E_{0011} = \hat{\mathbf{I}}_E\left[B_{0011} x^2\right]$

$E_{0012} = \hat{\mathbf{I}}_E\left[B_{0011} xy\right]$

$E_{0013} = \hat{\mathbf{I}}_E\left[B_{0011} xz\right]$

$E_{0014} = -\hat{\mathbf{I}}_E\left[\left(B_{001} + B_{1111}\right) x\right]$

$E_{0022} = \hat{\mathbf{I}}_E\left[B_{0011} y^2\right]$

$E_{0023} = \hat{\mathbf{I}}_E\left[B_{0011} yz\right]$

$E_{0024} = -\hat{\mathbf{I}}_E\left[\left(B_{001} + B_{1111}\right) y\right]$

$E_{0033} = \hat{\mathbf{I}}_E\left[B_{0011} z^2\right]$

$E_{0034} = -\hat{\mathbf{I}}_E\left[\left(B_{001} + B_{1111}\right) z\right]$

$E_{0044} = \hat{\mathbf{I}}_E\left[B_{00} + 2B_{001} + B_{0011}\right]$

$E_{1111} = \hat{\mathbf{I}}_E\left[B_{1111} x^4\right]$

$E_{1224} = -\hat{\mathbf{I}}_E\left[\left(B_{111} + B_{1111}\right) xy^2\right]$

$E_{1233} = \hat{\mathbf{I}}_E\left[B_{1111} xyz^2\right]$

$E_{1234} = -\hat{\mathbf{I}}_E\left[\left(B_{111} + B_{1111}\right) xyz\right]$

$E_{1244} = \hat{\mathbf{I}}_E\left[\left(B_{11} + 2B_{111} + B_{1111}\right) xy\right]$

$E_{1333} = \hat{\mathbf{I}}_E\left[B_{1111} xz^3\right]$

$E_{1334} = -\hat{\mathbf{I}}_E\left[\left(B_{111} + B_{1111}\right) xz^3\right]$

$E_{1344} = \hat{\mathbf{I}}_E\left[\left(B_{11} + 2B_{111} + B_{1111}\right) xz\right]$

$E_{1444} = -\hat{\mathbf{I}}_E\left[\left(B_1 + 3\left(B_{11} + B_{111}\right) + B_{1111}\right) x\right]$

$E_{2222} = \hat{\mathbf{I}}_E\left[B_{1111} y^4\right]$

$E_{2223} = \hat{\mathbf{I}}_E\left[B_{1111} y^2 z\right]$

$E_{2224} = -\hat{\mathbf{I}}_E\left[\left(B_{111} + B_{1111}\right) y^3\right]$

$E_{2233} = \hat{\mathbf{I}}_E\left[B_{1111} y^2 z^2\right]$

$E_{001} = \hat{\mathbf{I}}_E\left[B_{001} x\right]$

$E_{002} = \hat{\mathbf{I}}_E\left[B_{001} y\right]$

$E_{003} = \hat{\mathbf{I}}_E\left[B_{001} z\right]$

$E_{004} = -\hat{\mathbf{I}}_E\left[B_{00} + B_{001}\right]$

$E_{111} = \hat{\mathbf{I}}_E\left[B_{111} x^3\right]$

$E_{112} = \hat{\mathbf{I}}_E\left[B_{111} x^2 y\right]$

$E_{113} = \hat{\mathbf{I}}_E\left[B_{111} x^2 z\right]$

$E_{114} = -\hat{\mathbf{I}}_E\left[\left(B_{11} + B_{111}\right) x^2\right]$

$E_{122} = \hat{\mathbf{I}}_E\left[B_{111} xy^2\right]$

$E_{123} = \hat{\mathbf{I}}_E\left[B_{111} xyz\right]$

$E_{124} = -\hat{\mathbf{I}}_E\left[\left(B_{11} + B_{111}\right) xy\right]$

$E_{133} = \hat{\mathbf{I}}_E\left[B_{111} xz^2\right]$

$E_{134} = -\hat{\mathbf{I}}_E\left[\left(B_{11} + B_{111}\right) xz\right]$

$E_{144} = \hat{\mathbf{I}}_E\left[\left(B_1 + 2B_{11} + B_{111}\right) x\right]$

$E_{222} = \hat{\mathbf{I}}_E\left[B_{111} y^3\right]$

$E_{223} = \hat{\mathbf{I}}_E\left[B_{111} y^2 z\right]$

$E_{224} = -\hat{\mathbf{I}}_E\left[\left(B_{11} + B_{111}\right) y^2\right]$

$E_{233} = \hat{\mathbf{I}}_E\left[B_{111} yz^2\right]$

$E_{234} = -\hat{\mathbf{I}}_E\left[\left(B_{11} + B_{111}\right) yz\right]$

$E_{244} = \hat{\mathbf{I}}_E\left[\left(B_1 + 2B_{11} + B_{111}\right) y\right]$

$E_{333} = \hat{\mathbf{I}}_E\left[B_{111} z^3\right]$

$E_{334} = -\hat{\mathbf{I}}_E\left[\left(B_{11} + B_{111}\right) z^2\right]$

$E_{344} = \hat{\mathbf{I}}_E\left[\left(B_1 + 2B_{11} + B_{111}\right) z\right]$

$E_{444} = -\hat{\mathbf{I}}_E\left[B_0 + 3\left(B_1 + B_{11}\right) + B_{111}\right]$

$E_{1112} = \hat{\mathbf{I}}_E\left[B_{1111} x^3 y\right]$

$E_{1113} = \hat{\mathbf{I}}_D\left[B_{1111} x^3 z\right]$

$E_{1114} = -\hat{\mathbf{I}}_E\left[\left(B_{111} + B_{1111}\right) x^3\right]$

$E_{1122} = \hat{\mathbf{I}}_E\left[B_{1111} x^2 y^2\right]$

$E_{1123} = \hat{\mathbf{I}}_E\left[B_{1111} x^2 yz\right]$

$E_{1124} = -\hat{\mathbf{I}}_E\left[\left(B_{111} + B_{1111}\right) x^2 y\right]$

$E_{1133} = \hat{\mathbf{I}}_E\left[B_{1111} x^2 z^2\right]$

$E_{1134} = -\hat{\mathbf{I}}_E\left[\left(B_{111} + B_{1111}\right) x^2 z\right]$

$E_{1144} = \hat{\mathbf{I}}_E\left[\left(B_{11} + 2B_{111} + B_{1111}\right) x^2\right]$

$E_{1222} = \hat{\mathbf{I}}_E\left[B_{1111} xy^3\right]$

$E_{1223} = \hat{\mathbf{I}}_E\left[B_{1111} xy^2 z\right]$

$E_{2234} = -\hat{\mathbf{I}}_E\left[\left(B_{111} + B_{1111}\right) y^2 z\right]$

$E_{2244} = \hat{\mathbf{I}}_E\left[\left(B_{11} + 2B_{111} + B_{1111}\right) y^2\right]$

$E_{2333} = \hat{\mathbf{I}}_E\left[B_{1111} yz^3\right]$

$E_{2334} = -\hat{\mathbf{I}}_E\left[\left(B_{111} + B_{1111}\right) yz^2\right]$

$E_{2344} = \hat{\mathbf{I}}_E\left[\left(B_{11} + 2B_{111} + B_{1111}\right) yz\right]$

$E_{2444} = -\hat{\mathbf{I}}_E\left[\left(B_1 + 3\left(B_{11} + B_{111}\right) + B_{1111}\right) y\right]$

$E_{3333} = \hat{\mathbf{I}}_E\left[B_{1111} z^4\right]$

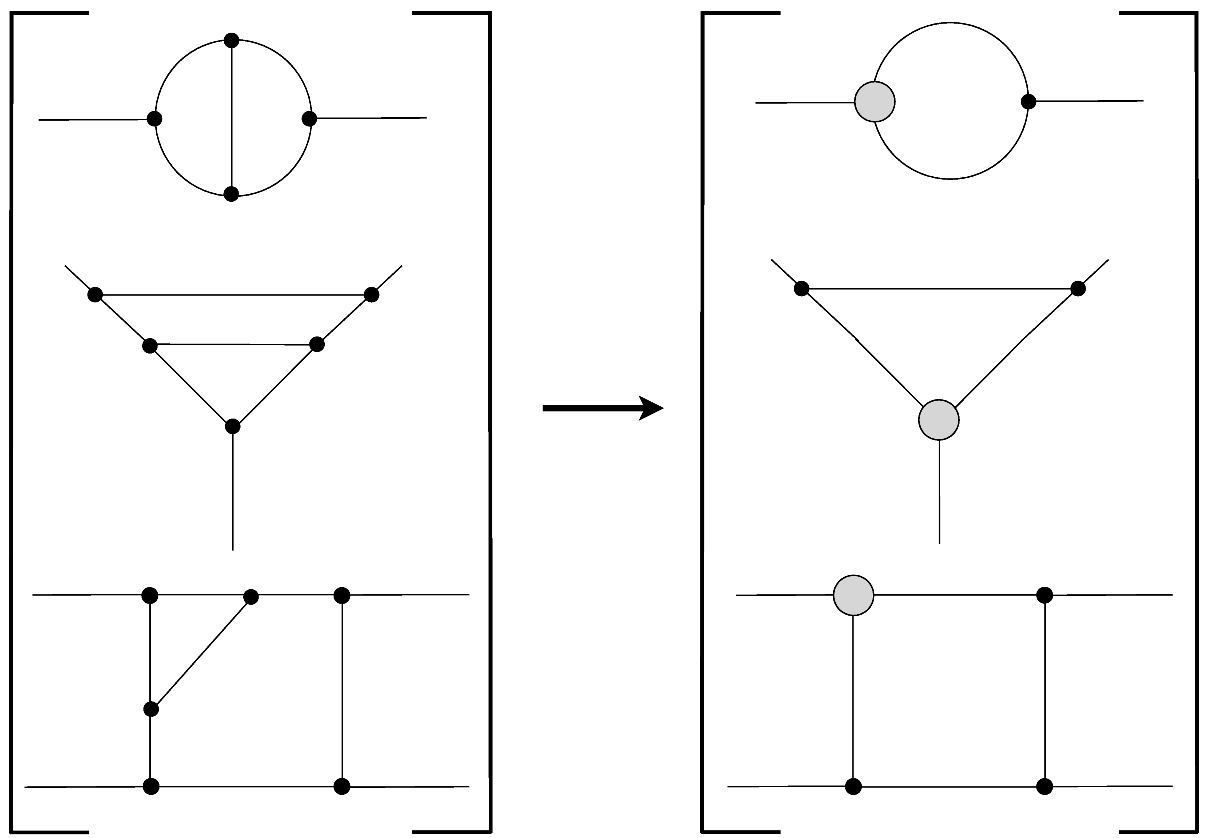
$E_{3334} = -\hat{\mathbf{I}}_E\left[\left(B_{111} + B_{1111}\right) z^3\right]$

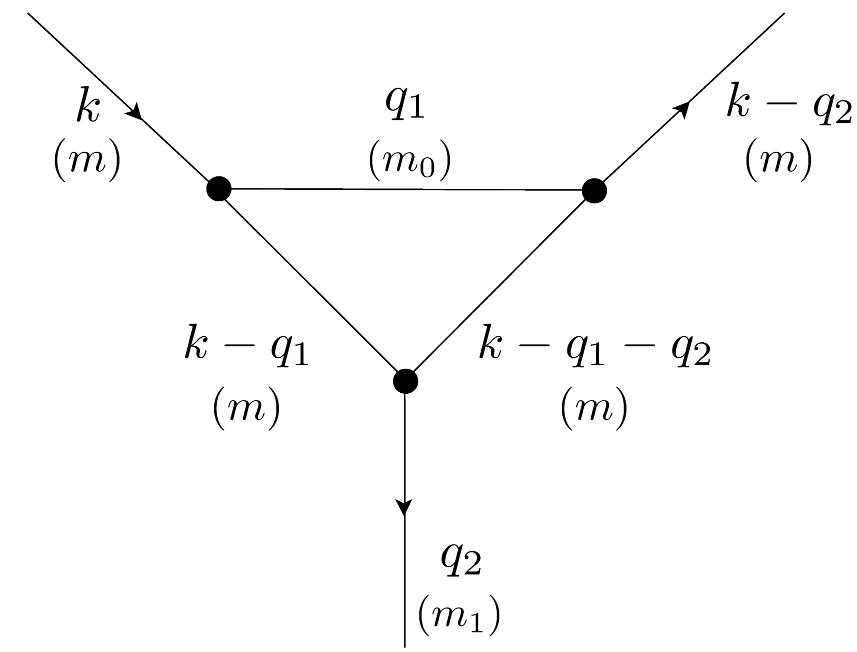
$E_{3344} = \hat{\mathbf{I}}_E\left[\left(B_{11} + 2B_{111} + B_{1111}\right) z^2\right]$

$E_{3444} = -\hat{\mathbf{I}}_E\left[\left(B_1 + 3\left(B_{11} + B_{111}\right) + B_{1111}\right) z\right]$

$E_{4444} = \hat{\mathbf{I}}_E\left[\left(B_0 + 4\left(B_1 + B_{111}\right) + 6B_{11} + B_{1111}\right)\right]$

Triangle Insertion: Dispersive Approach





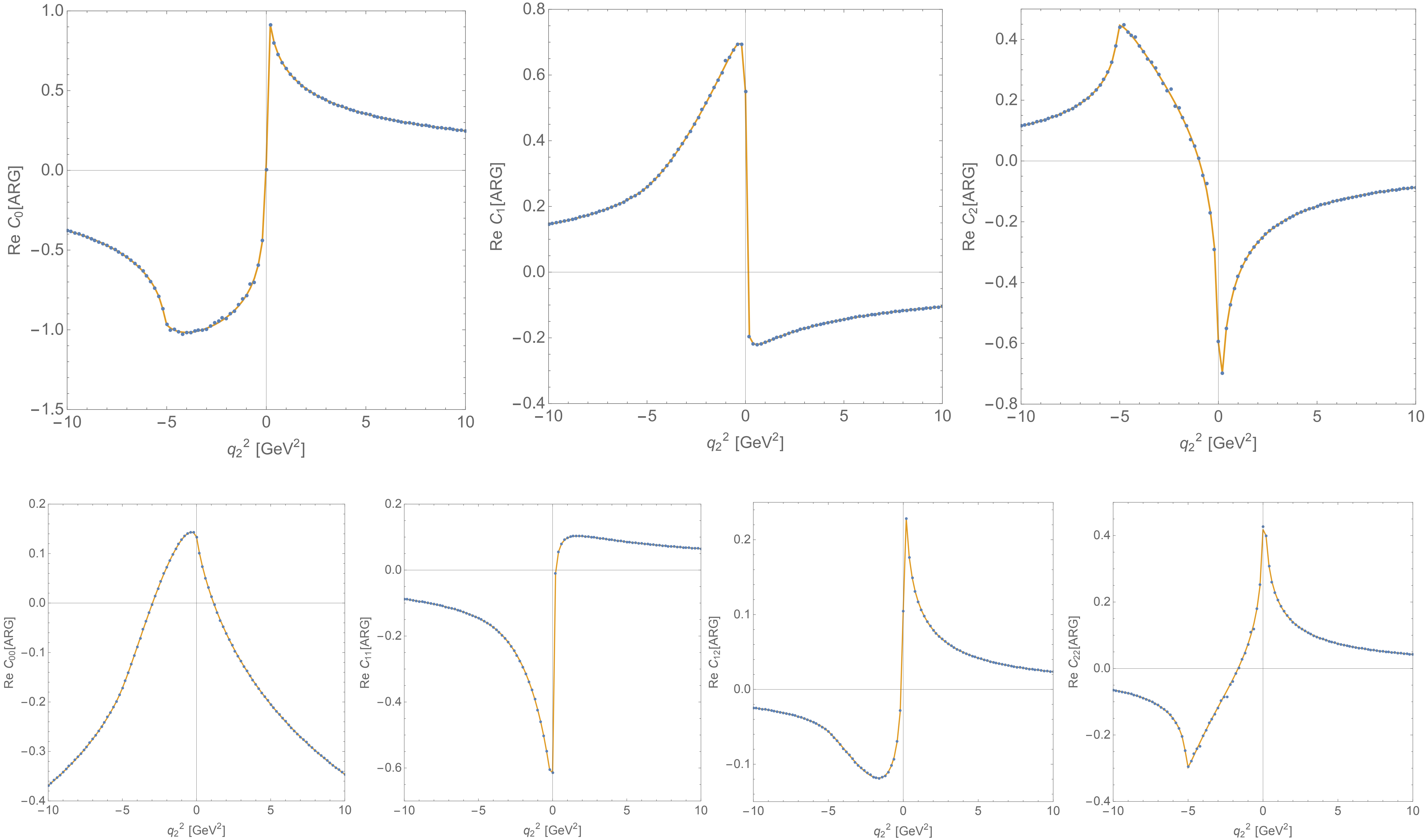
$$C_0\left(m^2, q_2^2, (q_2-k)^2, m_0^2, m^2, m^2\right) = \frac{\mu^{4-D}}{i\pi^{D/2}} \int \frac{d^D q_1}{\left[q_1^2 - m_0^2\right]\left[(q_1-k)^2 - m^2\right]\left[(q_1-k+q_2)^2 - m^2\right]}$$

join two propagators without q_2

$$C_0\left(m^2, q_2^2, (q_2-k)^2, m_0^2, m^2, m^2\right) = \lim_{\lambda\to 0} \frac{\partial}{\partial\lambda} \int_0^1 dx \int_{\left(m_3+\left(m_{12}^2+\lambda\right)^{1/2}\right)^2}^{\Lambda^2} ds \, \frac{\Im B_0\left(s, m_3^2, m_{12}^2+\lambda\right)}{s-(q_2-k\bar{x})^2 - i\epsilon}$$

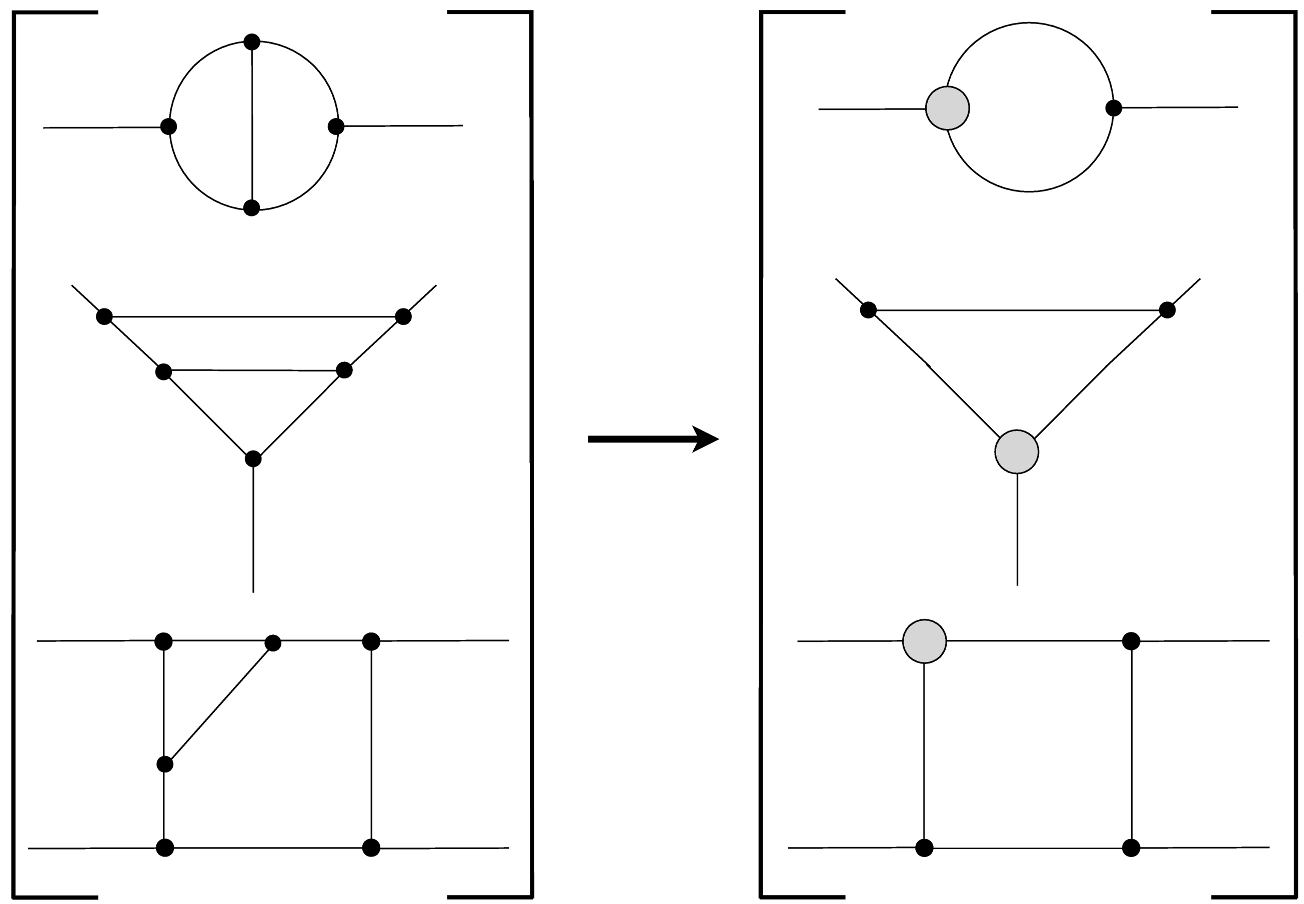
$$m_{12}^2 = m_0^2 \bar{x} + m^2 x^2$$

Triangle Insertion: Dispersive Approach



$$m_0 = 1.2 \text{ GeV}, \; m = 0.1 \text{ GeV and } (k \cdot q_2) = -3.4 \text{ GeV}^2$$

Triangle Insertion: Dispersive Approach



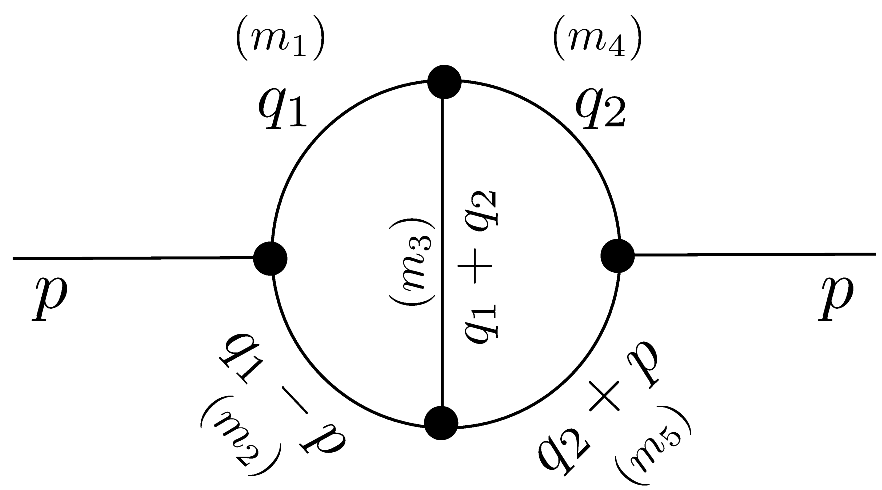
$$\hat{\mathbf{D}} = \lim_{\lambda\to 0} \frac{\partial}{\partial\lambda} \int_0^1 dx \int_{\left(m_3+\left(m_{12}^2+\lambda\right)^{1/2}\right)^2}^{\Lambda^2} ds \ldots$$

$$\Gamma = \hat{\mathbf{D}} \left[\frac{\Im F\left(s, m_3^2, m_{12}^2+\lambda\right)}{s-\left(p_2+p_1\bar{x}\right)^2 - i\epsilon} \right]$$

$$m_{12}^2 = m_1^2\bar{x} + m_2^2 x - p_1^2 x\bar{x}$$

Subtracted vertex at zero momentum: $\hat{\Gamma} = \hat{\mathbf{D}} \left[\frac{\Im F\left(s, m_3^2, m_{12}^2+\lambda\right)\left[\left(p_2+p_1\bar{x}\right)^2 - p_1^2\bar{x}^2\right]}{\left[s-\left(p_2+p_1\bar{x}\right)^2 - i\epsilon\right]\left[s-p_1^2\bar{x}^2\right]} \right]$

Numerical Examples



$$I_a = -\frac{1}{\pi^4} \int \frac{d^4q_1 d^4q_2}{\left[q_1^2 - m_1^2\right]\left[(q_1 - p)^2 - m_2^2\right]\left[(q_1 + q_2)^2 - m_3^2\right]\left[q_2^2 - m_4^2\right]\left[(q_2 + p)^2 - m_5^2\right]}$$

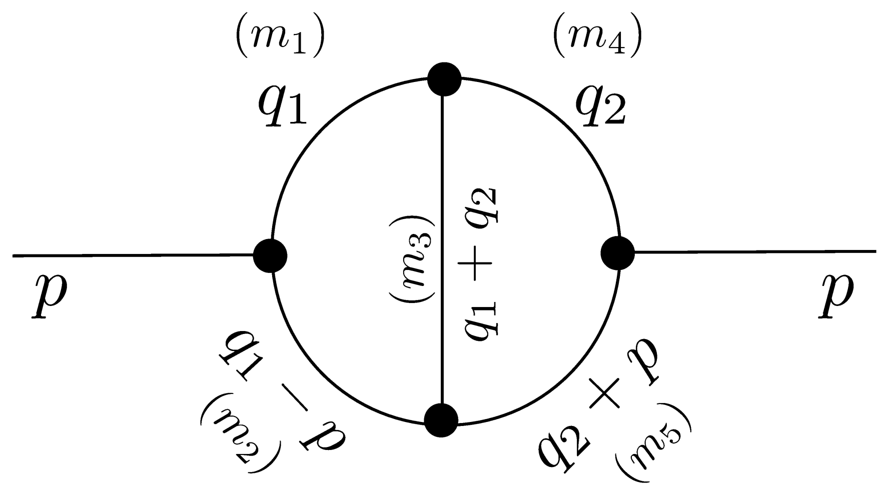
$$I_a = \frac{i}{\pi^3} \lim_{\lambda \to 0} \frac{\partial}{\partial \lambda} \int_0^1 dx \int_{\left(m_3 + \left(m_{12}^2 + \lambda\right)^{1/2}\right)^2}^{\Lambda^2} ds \, \Im B_0\left(s, m_3^2, m_{12}^2 + \lambda\right) \int d^4q_2 \frac{1}{\left[q_2^2 - m_4^2\right]\left[(q_2 + xp)^2 - s\right]\left[(q_2 + p)^2 - m_5^2\right]}$$

$$I_a = -\frac{1}{\pi} \lim_{\{\lambda, \xi\} \to 0} \frac{\partial^2}{\partial \lambda \partial \xi} \int_0^1 dx dy \int_{\left(m_3 + \left(m_{12}^2 + \lambda\right)^{1/2}\right)^2}^{\Lambda^2} ds \, \Im B_0\left(s, m_3^2, m_{12}^2 + \lambda\right) B_0\left(p^2 (x - y)^2, s, m_{45}^2 + \xi\right)$$

$$m_{12}^2 = m_1^2 \bar{x} + m_2^2 x - p^2 \bar{x} x$$

$$m_{45}^2 = m_4^2 \bar{y} + m_5^2 y - p^2 \bar{y} y$$

Numerical Examples



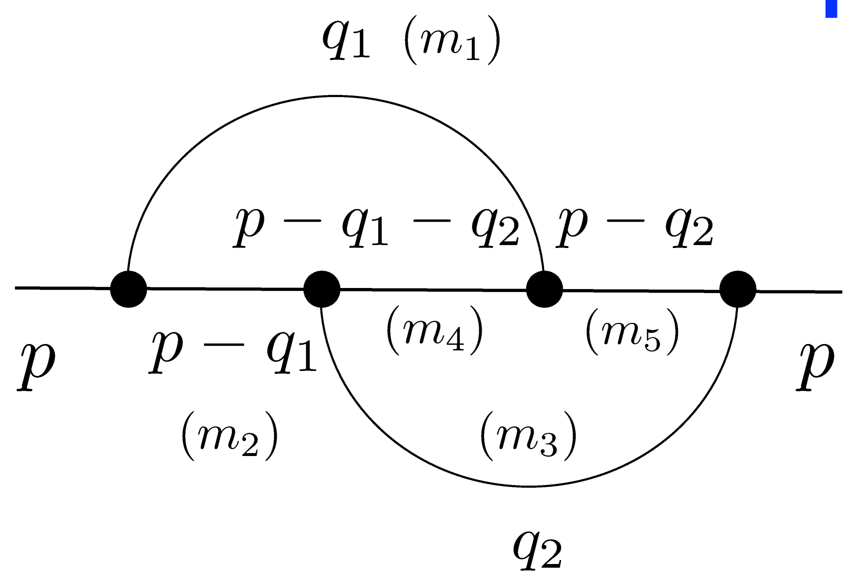
p^2 $(\mathrm{GeV})^2$	This work	[1]
-5.0	-0.22174	-
-1.0	-0.26925	-
-0.5	-0.27708	-
-0.1	-0.28371	-
0.1	-0.28706	-0.28701
0.5	-0.29450	-0.29479
1.0	-0.30439	-0.30493
5.0	-0.45231	-0.45241

[1] S. Bauberger, M. Bohm, Nucl. Phys. B 445, 25-46 (1995)

[This work] A. A, arXiv:1804.08914

$m_1^2 = 1$, $m_2^2 = 2$, $m_3^2 = 3$, $m_4^2 = 4$ and $m_5^2 = 5$ $(\mathrm{GeV})^2$

Numerical Examples



$$I_b = -\frac{1}{\pi} \lim_{\{\lambda,\xi\}\to 0} \frac{\partial^2}{\partial\lambda\partial\xi} \int_0^1 dxdy \int_{\left(m_4+\left(m_{12}^2+\lambda\right)^{1/2}\right)^2}^{\Lambda^2} ds\, \Im B_0\left(s, m_4^2, m_{12}^2+\lambda\right) B_0\left(p^2\left(\bar{x}-y\right)^2, s, m_{35}^2+\xi\right)$$

$$m_{12}^2 = m_1^2\bar{x} + m_2^2 x - p^2\bar{x}x$$

$$m_{35}^2 = m_3^2\bar{y} + m_5^2 y - p^2\bar{y}y$$

$m_1^2 = 1,\ m_2^2 = 2,\ m_3^2 = 3,\ m_4^2 = 4$ and $m_5^2 = 5\ (\mathrm{GeV})^2$

$p^2\ (\mathrm{GeV})^2$	Eq.(38)
-5.0	-0.22415
-1.0	-0.26911
-0.5	-0.28071
-0.1	-0.28760
0.1	-0.29346
0.5	-0.29908
1.0	-0.30945
5.0	-0.48510

Conclusion

- We are now in the last stage of the NNLO EWC calculations for the MOLLER experiment.

- Automatization of the NNLO EWC calculations for MOLLER is currently under way.

- Our next goal is a full gauge-invariant set of two-loop EW graphs with SE and triangles insertions.

- Results to be obtained will be cross checked with our previous calculations and other literature.

- We are looking for additional collaborative projects in two-loops calculations for various processes.

Additional Slides

Sensitivity to effective mixing angle

Sensitivity of Asymmetry to effective mixing angle

Representation of effective Born amplitude:

$$\mathfrak{M}_\gamma = \frac{\alpha(t) Q_e^2}{t} \left(\bar{u}_e \gamma_\mu u_e\right) \left(\bar{u}_e \gamma^\mu u_e\right),$$

D. Binosi, J. Papavassiliou, arXiv:0909.2536
D. Kennedy, B. Lynn, Nucl. Phys. B322 (1989)1
W. Hollik, DESY Report, DESY 88-188 (1988)

$$\mathfrak{M}_Z = \frac{G_\mu}{\sqrt{2}} \kappa \frac{m_Z^2}{t - m_Z^2 + i \frac{t}{m_Z} \Gamma_Z} \left(\bar{u}_e \gamma_\mu \left[I_3^e - 2\bar{s}_W^2(t) Q_e - I_3^e \gamma_5\right] u_e\right) \left(\bar{u}_e \gamma^\mu \left[I_3^e - 2\bar{s}_W^2(t) Q_e - I_3^e \gamma_5\right] u_e\right).$$

$$\kappa = \frac{1 - \Delta r}{1 + \Re\left[\frac{\partial}{\partial t} \hat{\Sigma}_{ZZ}(t)\right]} \qquad \alpha(t) = \frac{\alpha}{1 + \Re\left[\hat{\Sigma}_{\gamma\gamma}(t)\right] / t}$$

$$\Delta r = \frac{\Re[\hat{\Sigma}_{WW}(0)]}{m_W^2} + \frac{\alpha}{4\pi s_W^2} \left(6 + \frac{7 - 4s_W^2}{2s_W^2} \ln c_W^2\right) + \frac{c_W^2}{m_Z^2 s_W^2} \Re\left[\frac{\hat{\Sigma}_{\gamma Z}^2(m_Z^2)}{m_Z^2 + \hat{\Sigma}_{\gamma\gamma}(m_Z^2)}\right]$$

Sensitivity of Asymmetry to effective mixing angle

Effective Weinberg mixing angle up to NLO:

$$\bar{s}_W^2(t) = s_W^2 - s_W c_W \frac{\Re\left[\hat{\Sigma}_{\gamma Z}(t)\right]}{t + \Re\left[\hat{\Sigma}_{\gamma\gamma}(t)\right]}.$$

$$\sin^2\theta_W \equiv s_W^2 = 1 - \frac{m_W^2}{m_Z^2} \qquad m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu \sin^2\theta_W (1-\Delta r)}$$

$Q^2(GeV^2)$	$\bar{s}^2_{W(ON-SHELL)}$	$\bar{s}^2_{W(\bar{MS}-PT)}$[1]	$\bar{s}^2_{W(\bar{MS})}$ PDG(2015)
0	0.2383	0.2387	0.2386
m_z	0.2313	0.2320	0.2313

[1] A. Ferroglia, G. Ossola, A. Sirlin, EPJC., 10.1140, July, (2003).

Sensitivity of Polarization Asymmetry to effective mixing angle

$$\bar{s}_W^2\,(t) = s_W^2 - s_W c_W \frac{\Re\left[\hat{\Sigma}_{\gamma Z}(t)\right]}{t + \Re\left[\hat{\Sigma}_{\gamma\gamma}(t)\right]}.$$

Value of the effective s_W^2 for MOLLER kinematics:

$$\bar{s}_W^2(Q^2 = 0.0056\ GeV^2) = 0.2382$$

The 2% uncertainty on polarization asymmetry translates to 0.1% uncertainty for weak mixing angle