Dispersive Approach To The Gamma-W Box Diagram In Beta Decay

Work in progress with M. Gorshtyen, H. Patel and M.J.Ramsey-Musolf

Chien-Yeah Seng

Institute of Nuclear, Particle, Astronomy and Cosmology (INPAC),

Department of Physics and Astronomy,

Shanghai Jiao Tong University

"Bridging the Standard Model to New Physics with the Parity Violation Program at MESA", MITP

03 May 2018

Outline

1. Neutron Beta Decay: Background
2. Radiative Corrections: Pre-SM
3. Radiative Corrections: Modern Treatment
4. Dispersive Approach: Formalism
5. Dispersive Approach: Separation of Regions
6. Relation to PV Scattering Processes
7. Brief Summary

Neutron Beta Decay: Background

- Beta decay is a perfect venue for precision test of SM as well as search of BSM physics.
- Super-allowed beta decays are powerful in the determination of the CKM matrix element V_{ud}; neutron beta decay is currently less competitive for V_{ud} but still powerful in probing new physics due to the existence of extra structures:

Neutron Alphabet

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{(G_F V_{ud})^2}{(2\pi)^5}(g_V^2 + 3g_A^2)|\vec{p}_e| E_e E_\nu^2 (1 + a\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + b\frac{m_e}{E_e} + \hat{s}_n \cdot (A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu} + D\frac{\vec{p}_e \times \vec{p}_\nu}{E_e E_\nu}))$$

- Future experiments in neutron beta decay are aiming for precision level of 10^{-3}-10^{-4}, so all higher-order corrections up to order $\alpha/4\pi$ (on top of the LO contribution) should be precisely determined before one can fully make use of such results.

Neutron Beta Decay: Background

- Tree-level differential decay rate of polarized neutron:

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{G_F^2 V_{ud}^2}{(2\pi)^5}(g_V^2 + 3g_A^2)|\vec{p}_e|E_e(E_m - E_e)^2 \left[1 + a\vec{\beta}\cdot\hat{p}_\nu + \hat{s}\cdot\left(A\vec{\beta} + B\hat{p}_\nu\right)\right]$$

$$a = \frac{g_V^2 - g_A^2}{g_V^2 + 3g_A^2}, \quad A = \frac{2(g_V g_A - g_A^2)}{g_V^2 + 3g_A^2}, \quad B = \frac{2(g_V g_A + g_A^2)}{g_V^2 + 3g_A^2}$$

$$\text{CVC}: g_V = 1$$

- Typical higher-order corrections:
  - Isospin breaking: tiny for neutron (Ademollo-Gatto Theorem)
  - Recoil correction: can be implemented systematically
  - Radiative correction (RC)

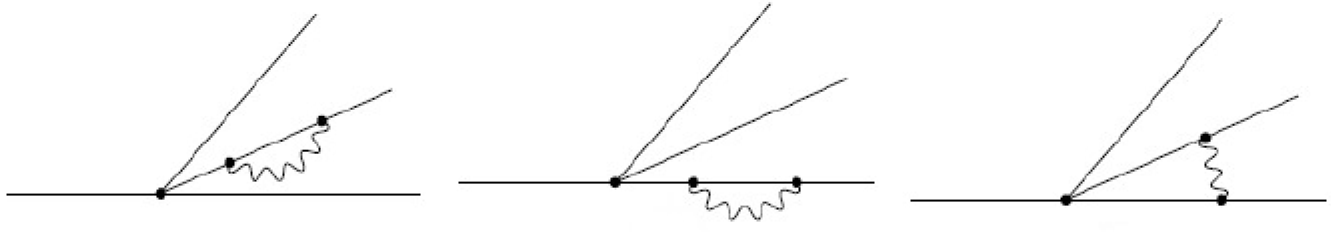
Holstein, Rev.Mod.Phys 46,789 (1974)
Ando et al, PLB 595 (2004) 250

We shall focus on the last one!

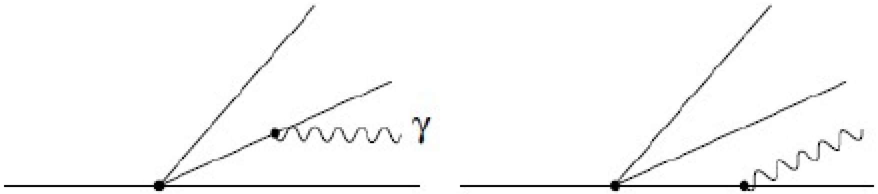
- This study is also important for super-allowed decays because the RC of the latter are inferred from free nucleon result.

Radiative Corrections:Pre-SM

- Pioneering work by Sirlin (*Phys.Rev. 164, 1767 (1967)*, before the establishment of SM) was to separate RC into two pieces:
  1. **"Outer" correction**: depends critically on the electron spectrum but not on the details of strong and weak interaction
  2. **"Inner" correction**: depends on the details of strong and weak interaction but not so much on the electron spectrum
- The "outer" contributions are obtained by retaining only the **IR-singular pieces** in the loop diagrams



- Bremsstrahlung diagrams are also needed to cancel IR divergence



Diagrams taken from Ando et al, PLB 595 (2004) 250

Radiative Corrections:Pre-SM

- Modified differential cross-section after including "outer" corrections:

Garcia and Maya, PRD 17 (1978) 1376

Ando et al, PLB 595 (2004) 250

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{G_F^2 V_{ud}^2}{(2\pi)^5}(g_V^2 + 3g_A^2)|\vec{p}_e|E_e(E_m - E_e)^2 F(\beta)(1 + \frac{\alpha}{2\pi}\delta^{(1)}) \times$$
$$\left[1 + \left(1 + \frac{\alpha}{2\pi}\delta^{(2)}\right) a\vec{\beta}\cdot\hat{p}_\nu + \hat{s}\cdot\left(\left(1 + \frac{\alpha}{2\pi}\delta^{(2)}\right) A\vec{\beta} + B\hat{p}_\nu\right)\right]$$

$F(\beta)$: Fermi's function (Coulomb interaction)

$\delta^{(1)}(m_e, \beta)$: Outer correction for total decay rate The Sirlin's universal function g(E)

$\delta^{(2)}(m_e, \beta)$: Outer correction for angular correlation

Radiative Corrections:Pre-SM

- All RC not included in the previous slide are called **"inner" corrections**. They are **independent of electron spectrum** upon neglecting $(m_e/m_W)^2$-terms.
- Inner correction can be parameterized as an effective amplitude:

$$i\delta M_{\text{inner}} = -i\sqrt{2}G_F V_{ud}\frac{\alpha}{2\pi}\bar{u}_{eL}\gamma^\mu v_{\nu L}\bar{u}_p\gamma_\mu(g_V c - g_A d\gamma_5)u_n.$$

where c, d are unknown constants. The only effect of inner correction is to replace:

$$g_V \quad \to \quad g_V\left(1+\frac{\alpha}{2\pi}\mathrm{Re}c\right)$$ Correction to Fermi amplitude

$$g_A \quad \to \quad g_A\left(1+\frac{\alpha}{2\pi}\mathrm{Re}d\right).$$ Correction to Gamow-Teller amplitude

I will stick to these old, 1967 notations…

They involve details of strong and weak interaction, thus are highly non-trivial.

Radiative Corrections: Modern Treatment

Sirlin, Rev. Mod. Phys 50, 573 (1978)

Current Algebra (CA) approach is an efficient way to obtain all the RCs to the **Fermi amplitude** that scale as $G_F\alpha$

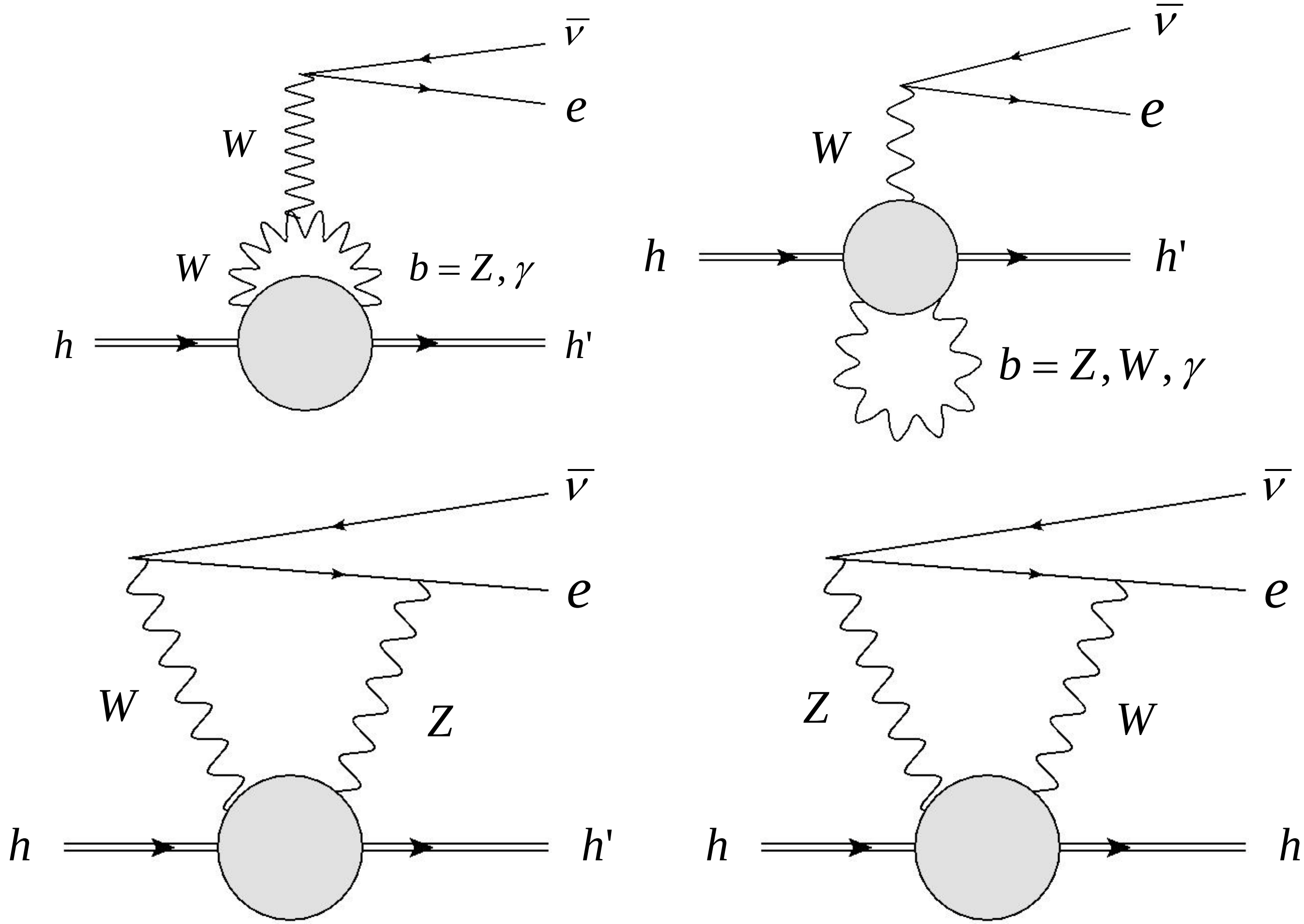
	Fermi amplitude	Gamow-Teller amplitude
Outer correction	Old, CA	Old
Inner correction	$\boxed{\text{CA}}$	

Therefore the improvement of CA on top of old calculation is its ability to obtain $\mathrm{Re}\, c$.

$$\left[J_W^0(x), J_Z^\mu(x')\right]_{x^0=x'^0} = \cos^2\theta_W J_W^\mu(x)\delta^3(\vec{x}-\vec{x}')$$
$$\left[J_W^0(x), J_\gamma^\mu(x')\right]_{x^0=x'^0} = J_W^\mu(x)\delta^3(\vec{x}-\vec{x}')$$
$$\left[J_W^0(x), J_W^{+\mu}(x')\right]_{x^0=x'^0} = -2\left(\sin^2\theta_W J_\gamma^\mu(x) + J_Z^\mu(x)\right)\delta^3(\vec{x}-\vec{x}') + S.T$$

Use of **equal-time commutation relation** effectively reduces pieces in loop diagrams into terms proportional to zeroth order amplitude with known coefficients.

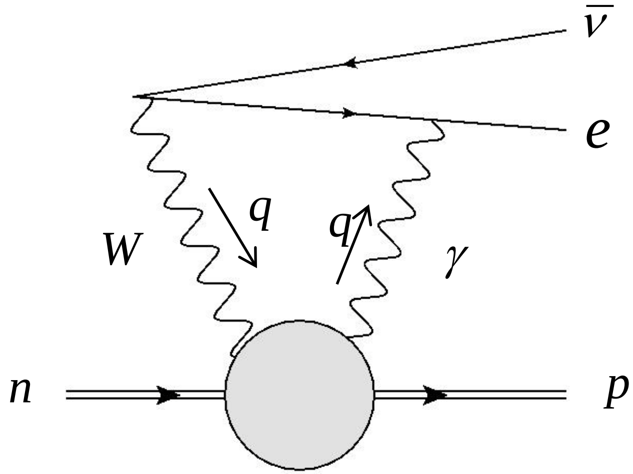
Radiative Corrections: Modern Treatment



Contributions of these diagrams are either **exactly known** (by CA) **or depend only on UV physics** which can be computed perturbatively

Radiative Corrections: Modern Treatment

The only piece that depends on **physics at hadronic scale** is **the V*A term** in the **Wγ–box diagram:**



$$\nu = \frac{p \cdot q}{m_N}$$

Its contribution to Rec (“m.d”: model-dependent) is:

$$\left(\mathrm{Re}\, c\right)_{\mathrm{m.d}} = 8\pi^2\, \mathrm{Re} \int \frac{d^4 q}{(2\pi)^4} \frac{m_W^2}{m_W^2 - q^2} \frac{\nu^2 - q^2}{\left(q^2\right)^2} \frac{T_3(\nu, -q^2)}{m_N \nu}$$

where the **forward Compton amplitude** is defined as:

$$\int \frac{d^4 q}{(2\pi)^4} e^{iq\cdot x} \left\langle p \left| T\{ J_{em}^{\mu}(x) \left(J_W^{\nu}(0) \right)_A \} \right| n \right\rangle = \frac{i\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2 m_N \nu} T_3(\nu, Q^2)$$

Radiative Corrections: Modern Treatment

- State-of-the-art study of box contribution (Marciano and Sirlin, **M&S**):

$$\left(\mathrm{Re}\,c\right)_{\mathrm{m.d}} = \frac{1}{4}\int_0^\infty dQ^2 \frac{m_W^2}{Q^2+m_W^2} F(Q^2)$$

Marciano and Sirlin, Phys.Rev.Lett. 96 (2006) 032002

1. Short distance: **leading OPE + perturbative QCD**

$$F(Q^2) = \frac{1}{Q^2}\left[1 - \frac{\alpha_S(Q^2)_{\overline{MS}}}{\pi} - C_2\left(\frac{\alpha_S(Q^2)_{\overline{MS}}}{\pi}\right)^2 - C_3\left(\frac{\alpha_S(Q^2)_{\overline{MS}}}{\pi}\right)^3\right] \qquad (1.5\mathrm{GeV})^2 < Q^2 < \infty$$

$$\boxed{\left(\mathrm{Re}\,c\right)^{(1)}_{\mathrm{m.d}} = 1.84}$$

(negligible uncertainty)

2. Intermediate distance: **interpolating function**

$$F(Q^2) = \frac{-1.490}{Q^2+m_\rho^2} + \frac{6.855}{Q^2+m_A^2} - \frac{4.414}{Q^2+m_{\rho'}^2} \qquad (0.823\mathrm{GeV})^2 < Q^2 < (1.5\mathrm{GeV})^2$$

$$\boxed{\left(\mathrm{Re}\,c\right)^{(2)}_{\mathrm{m.d}} = 0.14(14)}$$

3. Long distance: **Born contribution** with nucleon EM and axial current dipole FFs:

$$\boxed{\left(\mathrm{Re}\,c\right)^{(3)}_{\mathrm{m.d}} = 0.83(8)} \qquad 0 < Q^2 < (0.823\mathrm{GeV})^2$$

Dispersive Approach: Formalism

- The "**intermediate distances**" contribution gives rise to most of the theoretical uncertainty:

$$\sqrt{\underline{0.14}^2 + 0.08^2} \approx 0.16$$

- Some limitations in M&S approach:
  - **Conceptually**: separation of physical regions based on just Q^2 is questionable. E.g. how about many-particle virtual state effect at low Q^2? A more consistent separation of regions should also involve the variable $W^2=(p+q)^2$.
  - **Practically**: there is **almost no data input** during the estimation of the "intermediate distance" contribution, which means that its **error bar cannot be reduced** even with future experiments.
- Alternative approach: **dispersion relation**. The main spirit is to express T_3 in terms of **single-current on-shell matrix elements**, such that its uncertainty could be reduced following the **improvement of experimental precision**/**model** !

Dispersive Approach: Formalism

First step: symmetry argument

$$\left(\mathrm{Re}\, c\right)_{\mathrm{m.d}} = 8\pi^2\, \mathrm{Re} \int \frac{d^4 q}{(2\pi)^4} \frac{m_W^2}{m_W^2 - q^2} \frac{\nu^2 - q^2}{\left(q^2\right)^2 m_N \nu} T_3(\nu, -q^2)$$

$$\int d^4 q = \int d^3 q \int_{-\infty}^{\infty} d\nu$$

- By **crossing-symmetry**:

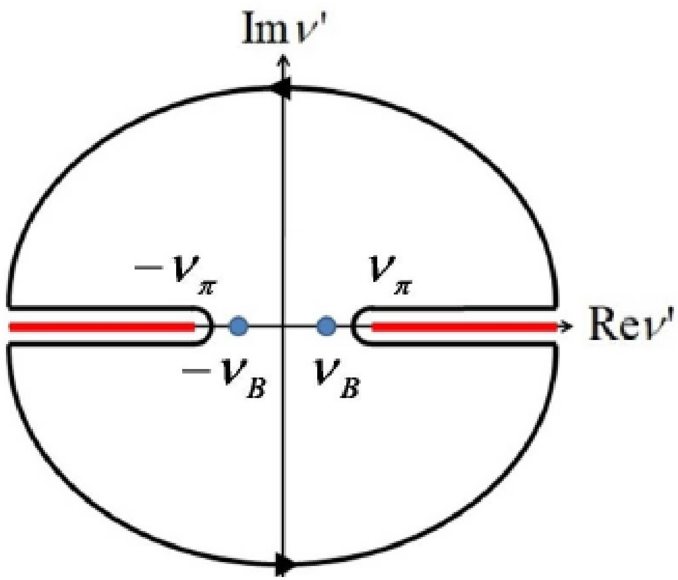
$$T_3(\nu, Q^2) = T_3^{(0)}(\nu, Q^2) + T_3^{(3)}(\nu, Q^2)$$

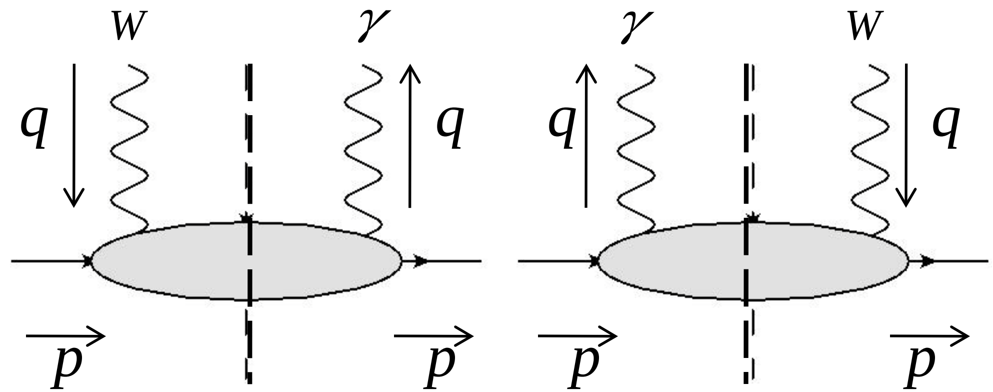
Isosinglet EM current: Odd wrt $\nu \rightarrow -\nu$

Isotriplet EM current: Even wrt $\nu \rightarrow -\nu$

- Only the (0) piece is needed. It involves the product between the axial charged weak current and the **isoscalar component** of the EM current.

Dispersive Approach: Formalism





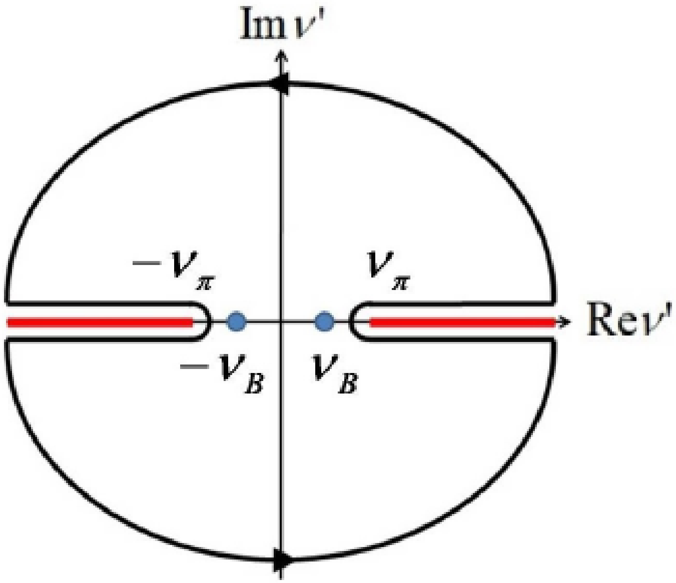
$$T_3^{(0)}(\nu,Q^2) = -\frac{2\nu \mathrm{Res} T_3^{(0)}(\nu_B,Q^2)}{\nu_B^2-\nu^2} - \frac{i\nu}{\pi}\int_{\nu_\pi}^{\infty} d\nu' \frac{\mathrm{Dis} T_3^{(0)}(\nu',Q^2)}{\nu'^2-\nu^2}$$

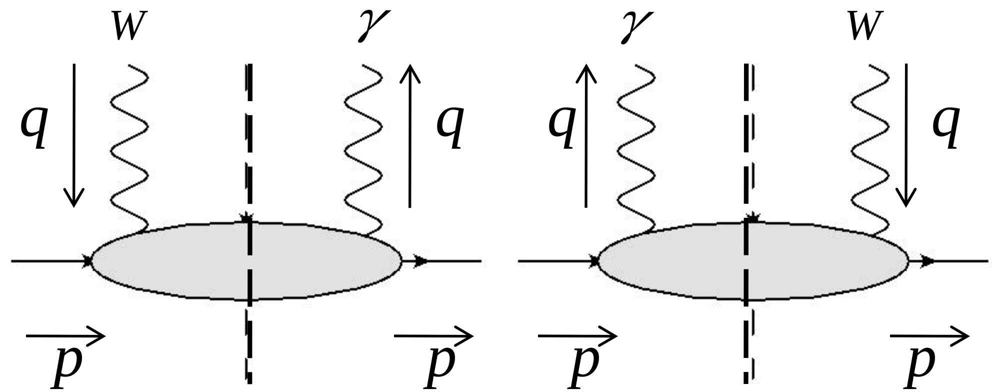
No constant subtraction term since $T_3^{(0)}$ is odd wrt ν

Optical theorem: $\mathrm{Dis}T_3^{(0)}(\nu,Q^2) = 4\pi F_3^{(0)}(\nu,Q^2)$

$$\frac{1}{4\pi}\sum_X (2\pi)^4 \delta^4(p+q-p_X)\langle p|J^{\mu}_{EM,0}|X\rangle\langle X|\left(J_W^{\nu}\right)_A|n\rangle = \frac{i\varepsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2m_N\nu}F_3^{(0)}(\nu,Q^2)$$

Dispersive Approach: Formalism





Final expression:

$$\left(\mathrm{Re}\,c\right)_{\mathrm{m.d}} = \left(\mathrm{Re}\,c\right)_{\mathrm{m.d}}^{\mathrm{Born}} + 2\int_0^\infty dQ^2 \frac{m_W^2}{m_W^2+Q^2}\int_{\nu_\pi}^\infty d\nu \frac{1}{m_N \nu} \frac{2\sqrt{\nu^2+Q^2}+\nu}{\left(\sqrt{\nu^2+Q^2}+\nu\right)^2} F_3^{(0)}(\nu, Q^2)$$

$$\nu_\pi = \frac{m_\pi^2 + 2m_N m_\pi + Q^2}{2m_N}$$

Dispersive Approach: Separation of Regions

M&S treatment

Q^2

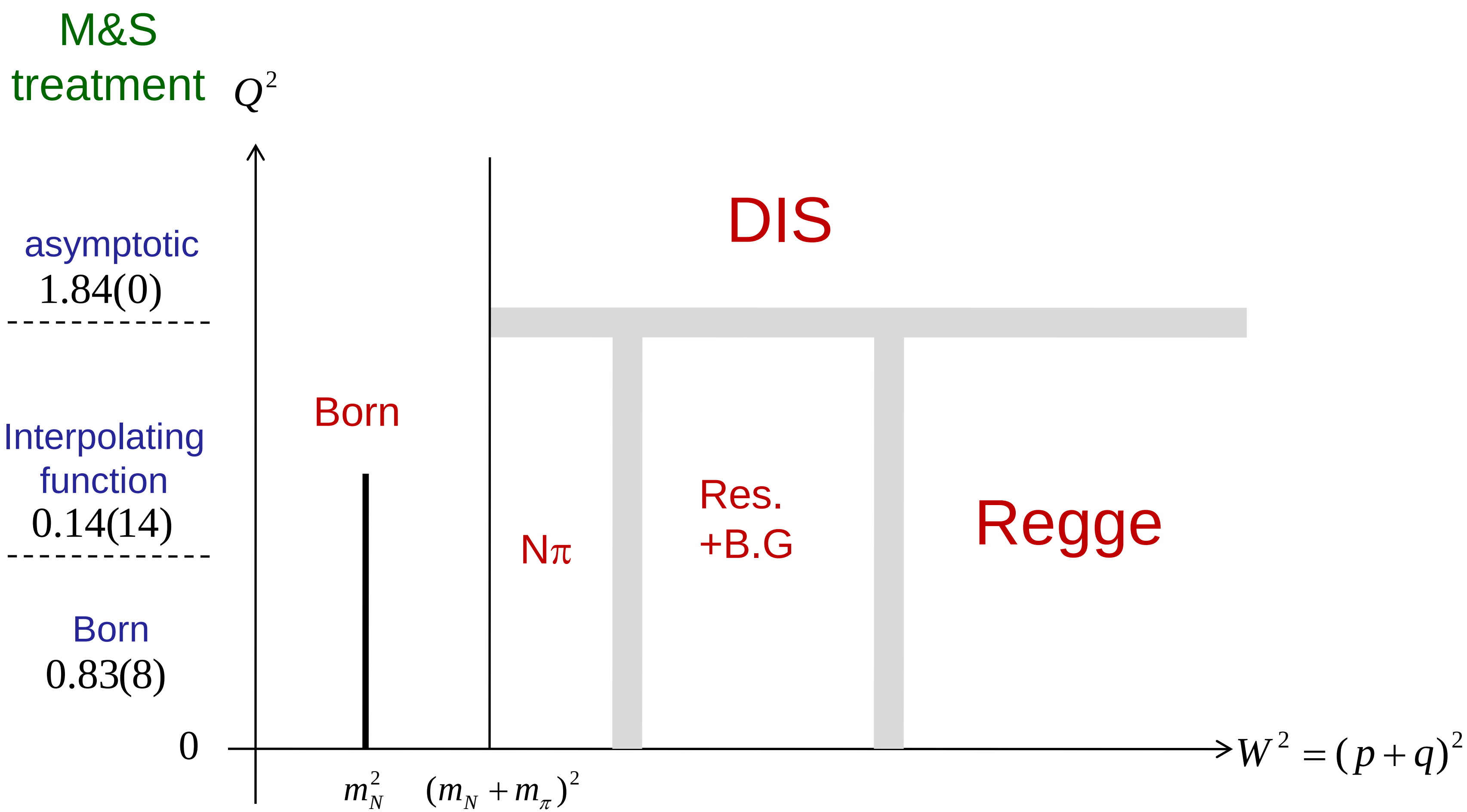
asymptotic
1.84(0)

Interpolating function
0.14(14)

Born
0.83(8)

0

Dispersive Approach: Separation of Regions



Dispersive Approach: Separation of Regions

(1) Born contribution:

$$\left(\mathrm{Re}\,c\right)^{\mathrm{Born}}_{\mathrm{m.d}} = -2\int_0^\infty dQ \frac{m_W^2}{m_W^2+Q^2} \frac{2\sqrt{4m_N^2+Q^2}+Q}{\left(\sqrt{4m_N^2+Q^2}+Q\right)^2} G_A(Q^2) G_M^S(Q^2)$$

Axial F.F

Isosinglet magnetic Sachs F.F

To illustrate, consider simple parameterizations of (magnetic) Sachs and axial FF:

$$G_M^N = \mu_N \frac{1+a_1^N\tau}{1+b_1^N\tau+b_2^N\tau^2+b_3^N\tau^3} \quad \left(\tau = \frac{\mathrm{Q}^2}{4m_N^2}\right)$$

Kelly, PRC 70 (2004) 068202

$$G_A = -g_A\left(1+\frac{Q^2}{m_A^2}\right)^{-2}$$

Disputable

$$m_A = 1.026(21)\,\mathrm{GeV}$$

(ν–scattering data)

Bernard, Elouadrhiri and Meissner, J.Phys.G28 (2002) R1

One obtains: $\boxed{\left(\mathrm{Re}\,c\right)^{\mathrm{Born}}_{\mathrm{m.d}} = 0.88(9)}$

Central value larger than M&S as we take $Q_{max}=\infty$. Updated studies of FFs may further improve it.

Dispersive Approach: Separation of Regions

(2) Large Q^2-limit (asymptotic region):

$$Q^2 \to \infty,\ \nu > \nu_\pi = \frac{m_\pi^2 + 2m_N m_\pi + Q^2}{2m_N} \to \infty$$

approaches DIS regime; PDF description is appropriate

The ν–integral is simplified:

$$\left(\mathrm{Re}\, c\right)^{\mathrm{asym}}_{\mathrm{m.d}} = 2\int_{\Lambda^2}^{\infty} dQ^2 \frac{m_W^2}{m_W^2 + Q^2} \int_{\nu_\pi}^{\infty} d\nu \frac{1}{m_N \nu} \frac{2\sqrt{\nu^2 + Q^2} + \nu}{\left(\sqrt{\nu^2 + Q^2} + \nu\right)^2} F_3^{(0)}(\nu, Q^2)$$

$$\to 2\int_{\Lambda^2}^{\infty} dQ^2 \frac{m_W^2}{m_W^2 + Q^2} \frac{1}{16Q^2} \int_0^1 dx d_V^n(x) = \frac{1}{4}\int_{\Lambda^2}^{\infty} dQ^2 \frac{m_W^2}{m_W^2 + Q^2} \frac{1}{Q^2}$$

Agrees with the M&S result.

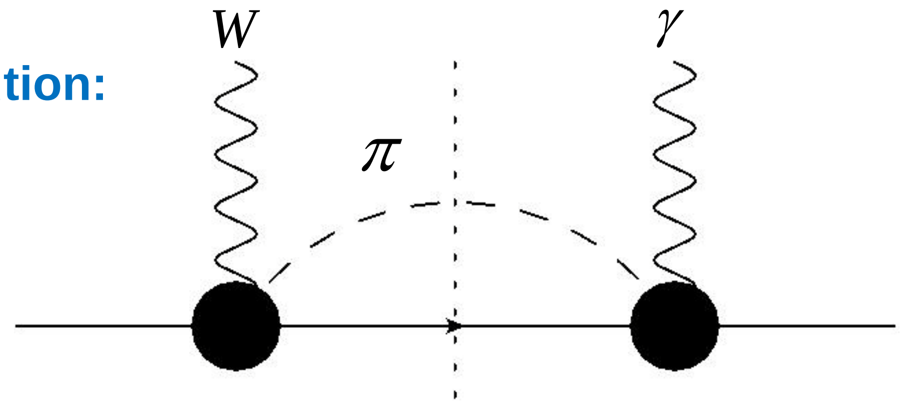
Scale at which PDF description becomes valid

Valence d-quark PDF in a neutron

Dispersive Approach: Separation of Regions

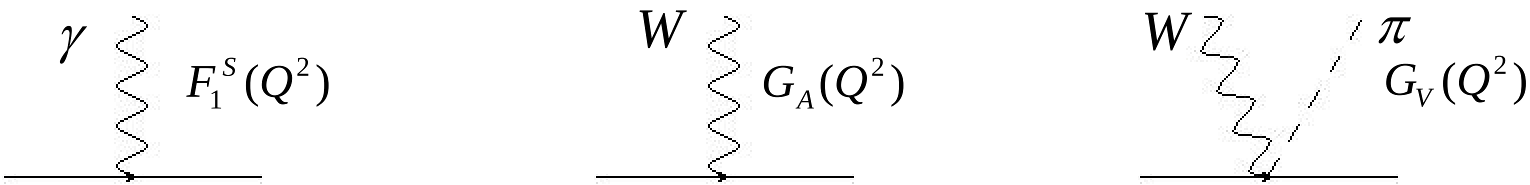
(3) Nπ contribution:

$$W_\pi^2 < W^2 < \infty$$
$$0 < Q^2 < \Lambda^2$$



Calculable using **baryon chiral perturbation theory at tree level**.

Q-dependence modeled by **electroweak form factor** insertion:

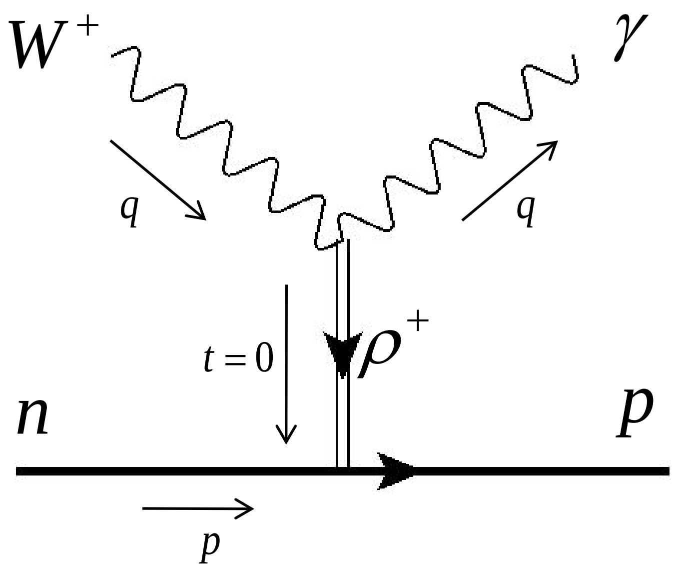


Numerically: with Λ=1.5GeV it gives:

$$\boxed{\left(\mathrm{Re}\, c\right)_{\mathrm{m.d}}^{N\pi} \approx 0.044}$$

Dispersive Approach: Separation of Regions

(4) Regge contribution:



Choosing the appropriate exchanged particle:

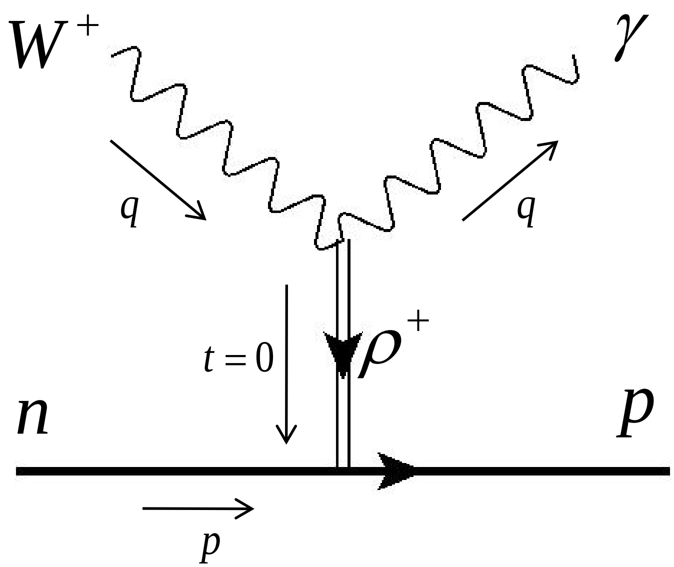
1. It should carry **I=1**
2. Its coupling to nucleon should take the form $\bar{N}\gamma_\alpha N$, i.e. **vector coupling** in order to give rise of the factor p_α in

$$\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta T_3^{(0)}$$

That leads to the **ρ-exchange picture**

Dispersive Approach: Separation of Regions

Version 1:



P-odd W$\gamma\rho$ coupling: Harvey, Hill and Hill, PRD 77 (2008) 085017

$$S = \frac{N_c}{24\pi^2} g e g_{\rho NN} \varepsilon^{\mu\nu\alpha\beta} \int d^4x (W_\mu^+ \rho_\nu^- \partial_\alpha A_\beta + h.c.)$$

Regge propagator for the **ρ-trajectory**:

Kashevarov, Ostrick and Tiator, PRC 96 (2017) 035207

$$\frac{1}{t-m_\rho^2} \to \frac{\pi\alpha'\left(e^{-i\pi\alpha(t)}-1\right)}{2\sin[\pi\alpha(t)]\Gamma[\alpha(t)]}\left(\frac{\nu}{1\mathrm{GeV}}\right)^{\alpha(t)-1}$$

Regge description is valid only at **large W^2**. Thus we introduce a cutoff function:

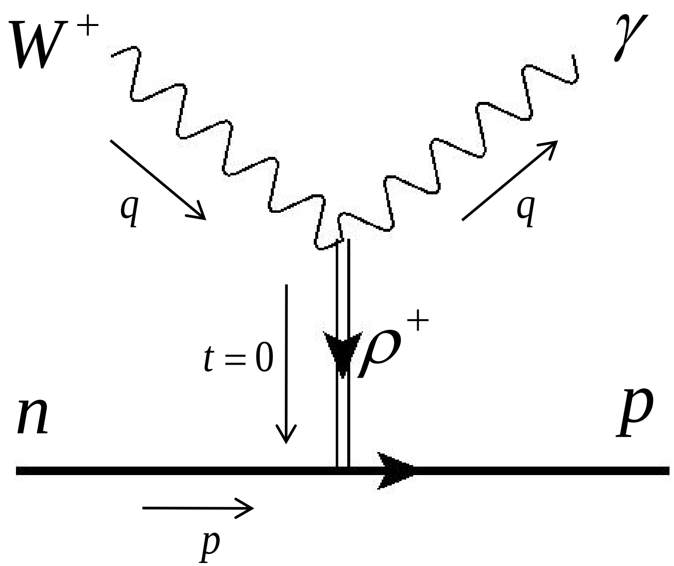
$$\Theta\left(W^2 - W_{\mathrm{cut}}^2\right)\left(1-\exp\left[\frac{W_{\mathrm{cut}}^2 - W^2}{\Lambda_{\mathrm{cut}}^2}\right]\right)$$

Bianchi et al, PRC 54 (1996) 1688

A natural choice of cutoff in W: $W_{\mathrm{cut}}^2 = (m_N + 2m_\pi)^2$

Dispersive Approach: Separation of Regions

Version 2:



Same coupling, but with an extra multiplicative factor:

$$S = \boxed{C}\,\frac{N_c}{24\pi^2} g e g_{\rho NN} \varepsilon^{\mu\nu\alpha\beta} \int d^4x (W_\mu^+ \rho_\nu^- \partial_\alpha A_\beta + h.c.)$$

Regge propagator for the **ρ-trajectory**:

Kashevarov, Ostrick and Tiator, PRC 96 (2017) 035207

$$\frac{1}{t - m_\rho^2} \rightarrow \frac{\pi\alpha'\left(e^{-i\pi\alpha(t)} - 1\right)}{2\sin[\pi\alpha(t)]\Gamma[\alpha(t)]}\left(\frac{\nu}{1\mathrm{GeV}}\right)^{\alpha(t)-1}$$

Include a Q^2-dependence modeled by VDM:

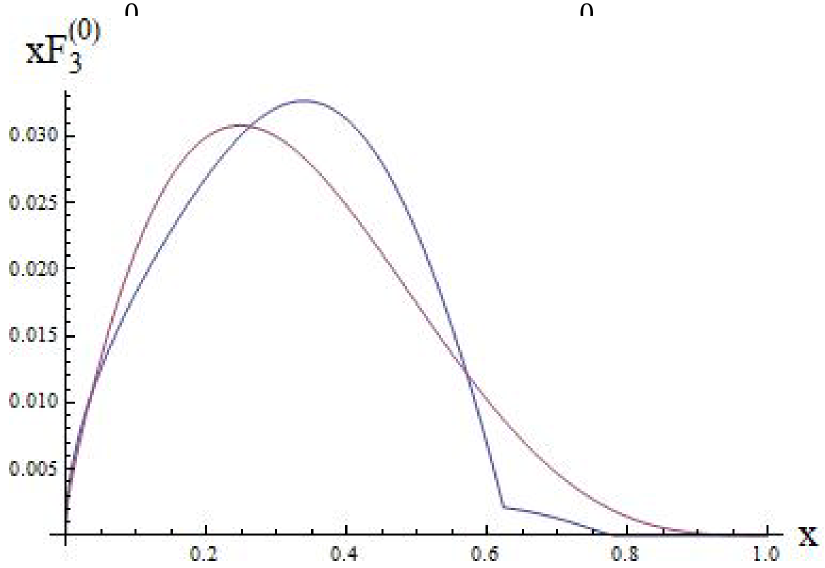
$$\boxed{\frac{m_{a_1}^2}{m_{a_1}^2 + Q^2}\,\frac{m_\omega^2}{m_\omega^2 + Q^2}}$$

Dispersive Approach: Separation of Regions

Matching between Regge+Nπ and PDF: (Choosing the values of Λ_{cut})

$$\text{Criteria}: \int_0^1 dx x\left(F_{3,\text{Regge}}^{(0)} + F_{3,N\pi}^{(0)}\right) = \int_0^1 dx x F_{3,\text{PDF}}^{(0)}$$

Version 1



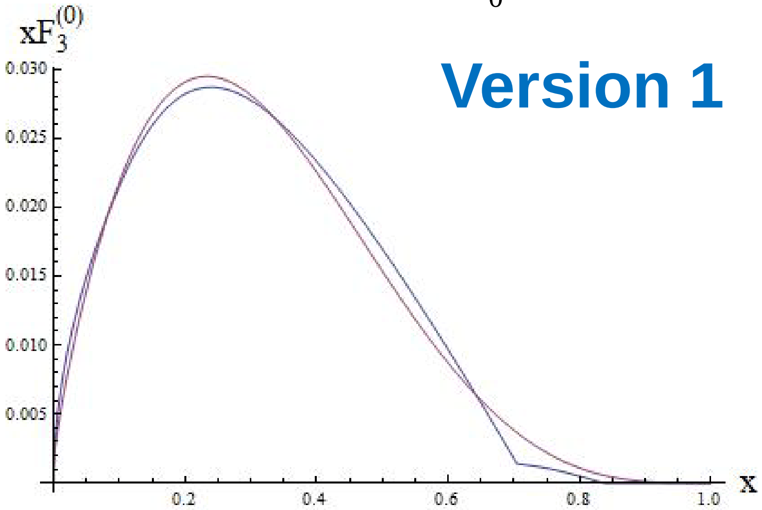
Match at Q=1.0GeV: Λ_{cut}=0.78GeV

Dispersive Approach: Separation of Regions

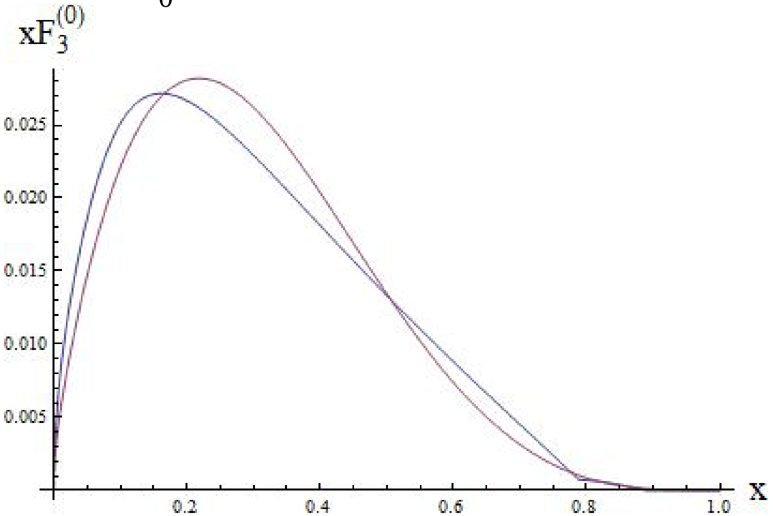
Matching between Regge+Nπ and PDF: (Choosing the values of Λ_{cut})

$$\text{Criteria}: \int_0^1 dx x\left(F_{3,\text{Regge}}^{(0)} + F_{3,N\pi}^{(0)}\right) = \int_0^1 dx x F_{3,\text{PDF}}^{(0)}$$

Version 1



(a) Match at Q=1.2GeV: Λ_{cut}=1.51GeV



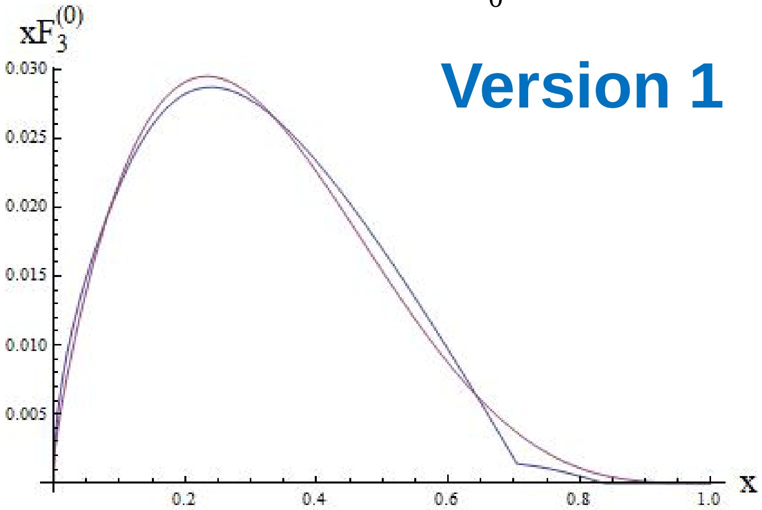
(b) Match at Q=1.5GeV: Λ_{cut}=2.68GeV

Dispersive Approach: Separation of Regions

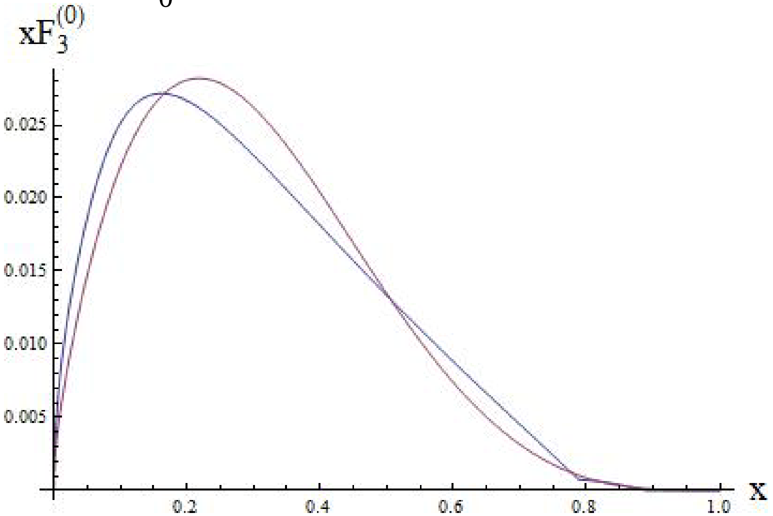
Matching between Regge+Nπ and PDF: (Choosing the values of $\Lambda_{\rm cut}$)

$$\text{Criteria}: \int_0^1 dx x\left(F_{3,\rm Regge}^{(0)} + F_{3,N\pi}^{(0)}\right) = \int_0^1 dx x F_{3,\rm PDF}^{(0)}$$

Version 1



(a) Match at Q=1.2GeV: $\Lambda_{\rm cut}$=1.51GeV



(b) Match at Q=1.5GeV: $\Lambda_{\rm cut}$=2.68GeV

Contribution to $(\mathrm{Re}\,c)_{\rm m.d}$ at $0.823\,\mathrm{GeV} < Q < 1.5\,\mathrm{GeV}$:

(a) 0.254 (b) 0.211

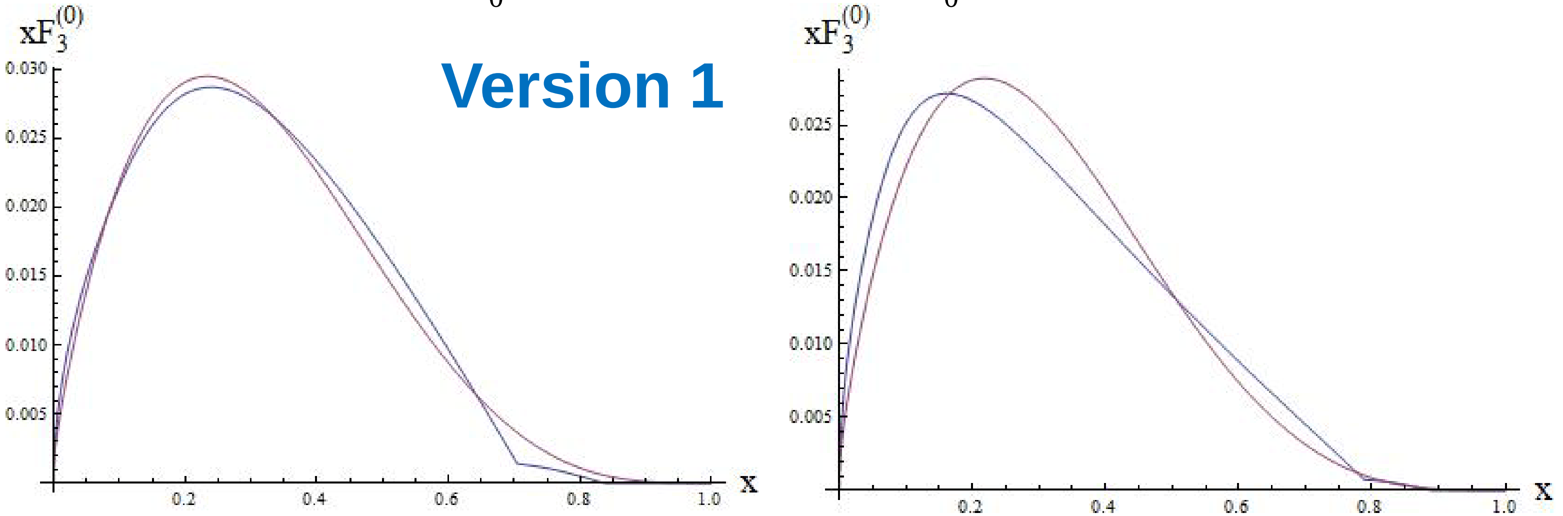
at $0 < Q < 0.823\,\mathrm{GeV}$:

(a) 0.189 (b) 0.145 This part could be missed in M&S!

Dispersive Approach: Separation of Regions

Matching between Regge+Nπ and PDF: (Choosing the values of $\Lambda_{\rm cut}$)

$$\text{Criteria}: \int_0^1 dx\, x\left(F^{(0)}_{3,\rm Regge} + F^{(0)}_{3,N\pi}\right) = \int_0^1 dx\, x F^{(0)}_{3,\rm PDF}$$



(a) Match at Q=1.2GeV: $\Lambda_{\rm cut}$=1.51GeV

(b) Match at Q=1.5GeV: $\Lambda_{\rm cut}$=2.68GeV

Contribution to $(\mathrm{Re}\,c)_{\rm m.d}$ at $0.823\,\mathrm{GeV} < Q < 1.5\,\mathrm{GeV}$:

(a) 0.254 (b) 0.211

at $0 < Q < 0.823\,\mathrm{GeV}$:

(a) 0.189 (b) 0.145

Integrating $F^{(0)}_{3,\rm PDF}$ (MSTW) at 0<Q<0.823GeV:

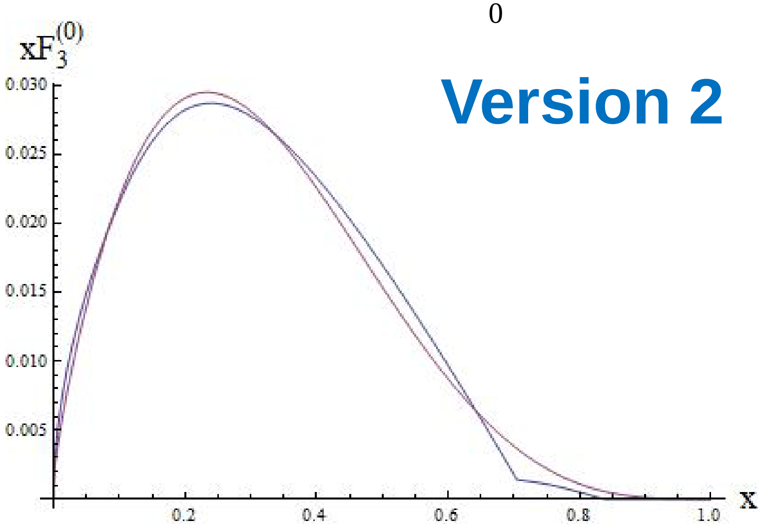
0.192

Dispersive Approach: Separation of Regions

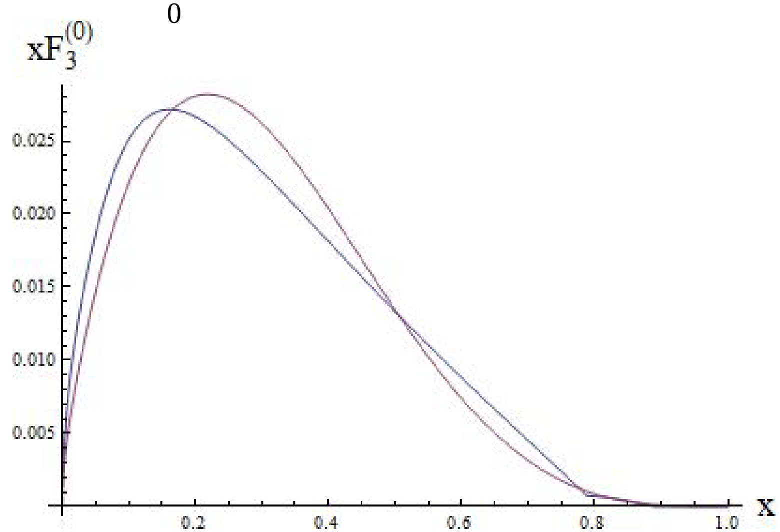
Matching between Regge+Nπ and PDF: (Choosing the values of Λ_{cut})

$$\text{Criteria}: \int_0^1 dx\,x\left(F^{(0)}_{3,\text{Regge}} + F^{(0)}_{3,N\pi}\right) = \int_0^1 dx\,x F^{(0)}_{3,\text{PDF}}$$

Version 2



(a) Match at Q=1.2GeV: C=6.4, Λ_{cut}=1.51GeV



(b) Match at Q=1.5GeV: C=11.3, Λ_{cut}=2.68GeV

Contribution to $(\text{Re}c)_{\text{m.d}}$ at $0.823\text{GeV} < Q < 1.5\text{GeV}$:

(a) 0.321 (b) 0.414

at $0 < Q < 0.823\text{GeV}$:

(a) 0.597 (b) 0.727

Dispersive Approach: Separation of Regions

(5) Resonance contribution: Assuming Breit-Wigner form

$$F_{3,R}^{(0)} = \frac{1}{2\pi}\frac{\nu}{\sqrt{\nu^2+Q^2}}\frac{m_R\Gamma_R}{(W^2-m_R^2)^2+m_R^2\Gamma_R^2}\times\{\left\langle p,\frac{1}{2}\middle|J_{EM,0}^{-}\middle|R,\frac{3}{2}\right\rangle\left\langle R,\frac{3}{2}\middle|\left(J_W^{+}\right)_A\middle|n,\frac{1}{2}\right\rangle$$
$$+\left\langle p,-\frac{1}{2}\middle|J_{EM,0}^{-}\middle|R,\frac{1}{2}\right\rangle\left\langle R,\frac{1}{2}\middle|\left(J_W^{+}\right)_A\middle|n,-\frac{1}{2}\right\rangle\}$$

Only $\boxed{\textbf{I=1/2 resonances}}$ contribute. EM currents can be related to well-known **helicity amplitudes**:

$$A_{1/2}^{R,N} = -\sqrt{\frac{\pi\alpha}{m_N(m_R^2-m_N^2)}}\left\langle R,\frac{1}{2}\middle|J_{EM}^{+}\middle|N,-\frac{1}{2}\right\rangle$$

$$A_{3/2}^{R,N} = -\sqrt{\frac{\pi\alpha}{m_N(m_R^2-m_N^2)}}\left\langle R,\frac{3}{2}\middle|J_{EM}^{+}\middle|N,\frac{1}{2}\right\rangle$$

which values are taken from the Unitary Isobar Model Drechsel, Kamalov and Tiator, EPJA 34 (2007) 69

Axial matrix elements can be estimated using PCAC, but only straightforward for low-spin resonances.

Lalakulich, Paschos and Piranishvili, PRD 74 (2006) 014009

Dispersive Approach: Separation of Regions

First estimate of contributions from lowest resonances:

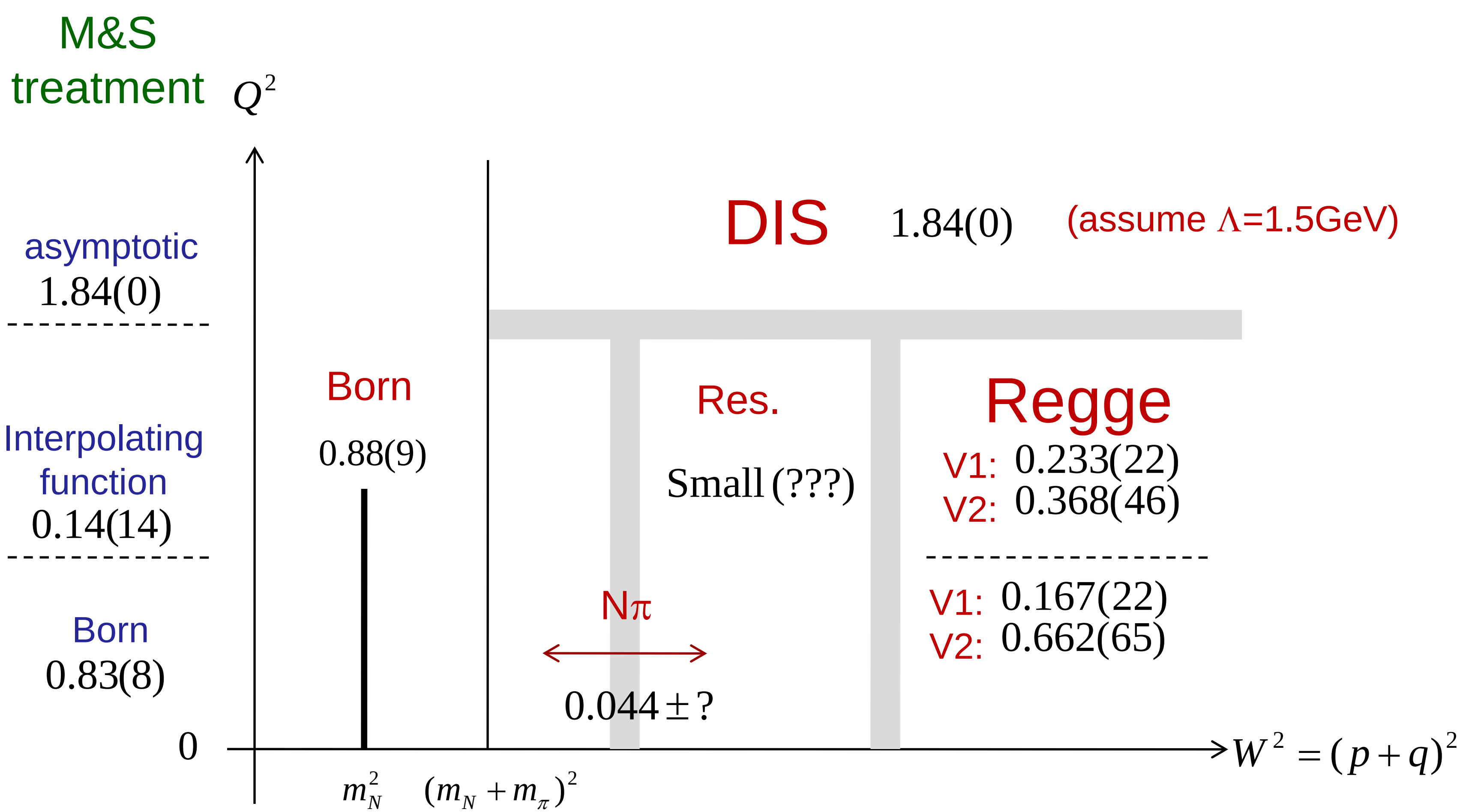
Resonance	Cont. to Rec
N(1440) $1/2^+$	-0.0011
N(1535) $1/2^-$	-0.00037
N(1520) $3/2^-$	0.0098
N(1650) $1/2^-$	-0.00094

Don't trust them yet!

Outcomes are unusually small. Although partially due to the isoscalar EM matrix elements are small for most resonances, but still need to double check the formalism.

Need also to figure out ways to include **spin- 5/2** resonances that seem to have large non-zero isosinglet EM helicity amplitudes.

Dispersive Approach: Separation of Regions



Relation to PV Scattering Processes

- One problem with the current formalism: the EM current involved in the matrix element is only the **isosinglet** component
- Using **isospin rotation**, it is possible in principle to build a connection between beta decay box diagram and PV scattering processes, involving the **FULL** EM current: Caveat: Negligence of strange neutral current

$$
\begin{aligned}
&\langle p | [J^{\mu}_{EM,0}(x), J^{\nu}_{W}(0)] | n \rangle_{V\times A} \\
&= \langle p | [J^{\mu}_{EM}(x), J^{\nu}_{Z}(0)] | p \rangle_{V\times A} - (p \to n) \qquad \boxed{F^{N}_{3,\gamma Z}} \qquad \text{inclusive eN-scattering} \\
&= -\frac{1}{2\sin^2\theta_W}\left[\langle p | [J^{\mu}_{Z}(x), J^{\nu}_{Z}(0)] | p \rangle - (p \to n)\right]_{V\times A} \qquad \boxed{F^{N}_{3,ZZ}}
\end{aligned}
$$

inclusive νN-scattering

- Example: in DIS regime,

γW-picture: $$\int_0^1 dx F_3^{(0)}(x) = \frac{1}{24}\int_0^1 dx d_V^n(x) = \frac{1}{12}$$

γZ-picture: $$\int_0^1 dx\left(F^{p}_{3,\gamma Z}(x) - F^{n}_{3,\gamma Z}(x)\right) = \frac{1}{4}\int_0^1 dx\left(\frac{2}{3}u_V^p(x) + \frac{1}{3}d_V^p(x) - (p \to n)\right) = \frac{1}{12}$$

Brief Summary

- $W\gamma$-box diagram gives the largest theoretical uncertainty to the radiative corrections in beta decay
- The separation of region by M&S based on just Q^2 is questionable. Also, they uses an interpolating function to describe the "intermediate distances" contribution, which does not allow for a systematically-improvable theoretical error
- We adopt a dispersive approach that matches the forward Compton-scattering amplitude to single-current on-shell matrix elements.
- Separation of regions is performed in the W^2-Q^2 plane. Main contributors in different regions are investigated; more works remains to be done.
- It is also interesting to investigate its connection to PV scattering experiments.