

Overview of γZ box corrections in PVES

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Bridging the Standard Model to New Physics with the
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Outline

- γZ box contributions to PV electron scattering
 - amenable to dispersion analysis in forward limit ($Q^2 \rightarrow 0$)
 - distinction between axial and vector hadron coupling
 - use of inelastic PV data in resonance and DIS regions
- From Thomson to scattering limits: a few issues
 - Vertex diagrams
 - Elastic γZ contribution (Coulomb distortions)

Parity-violating e - p scattering

- Left-right polarization asymmetry in $\vec{e} p \rightarrow e p$ scattering

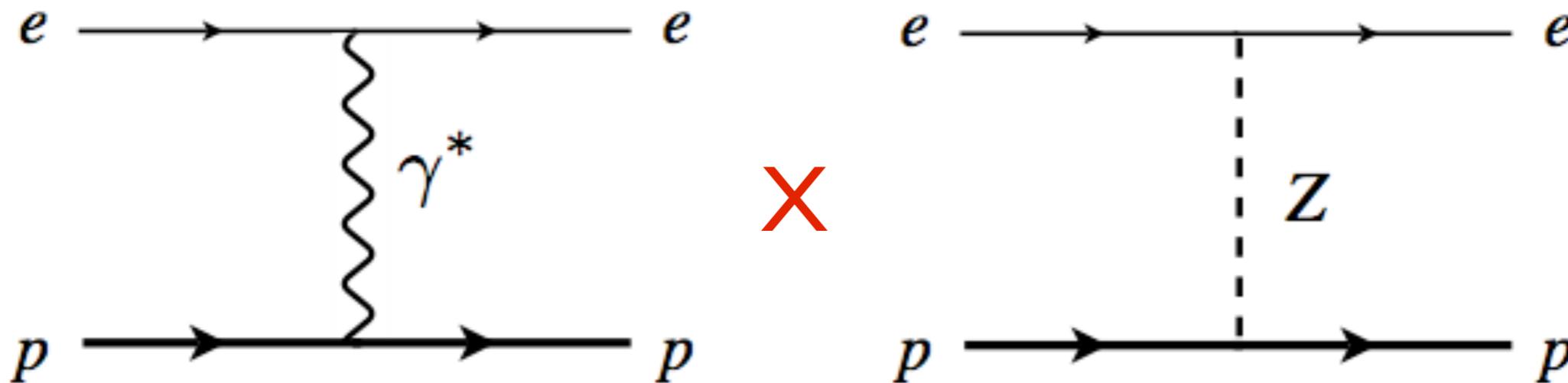
$$A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \rightarrow -\frac{G_F Q_W^p}{4\sqrt{2}\pi\alpha} Q^2$$

Forward limit:

$$Q^2 = -(k - k')^2 \rightarrow 0$$

$$s = (k + p)^2$$

$$= M^2 + 2ME$$



→ in forward limit, gives proton “weak charge”

$$A(e) \times V(h)$$

$$Q_W^p = 1 - 4 \sin^2 \theta_W$$

(tree level only)

Correction to proton weak charge

- including one-loop radiative corrections

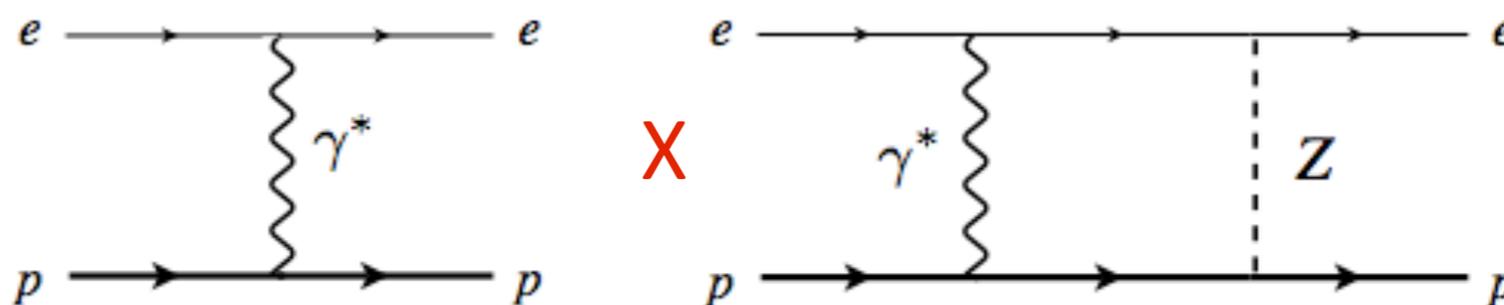
$$Q_W^p = \rho \left(1 - 4\kappa_{PT}(0)\hat{s}^2 + \Delta'_e + \Delta_W \right)$$

$$+ \square_{WW} + \square_{ZZ} + \square_{\gamma Z}$$

e.w. vertex corrections

← box diagrams

Box corrections



→ WW and ZZ box diagrams large but dominated by short distances; can be evaluated perturbatively

→ γZ box diagram sensitive to long distance physics, has two contributions:

$$\square_{\gamma Z} = \square_{\gamma Z}^A + \square_{\gamma Z}^V$$

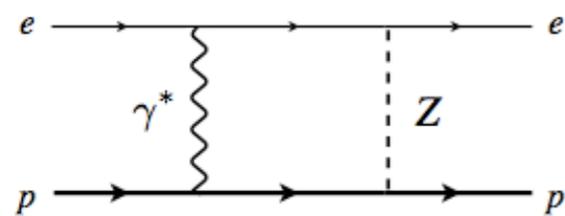
$V(e) \times A(h)$
(finite at $E=0$)

$A(e) \times V(h)$
(inelastic vanishes at $E=0$)

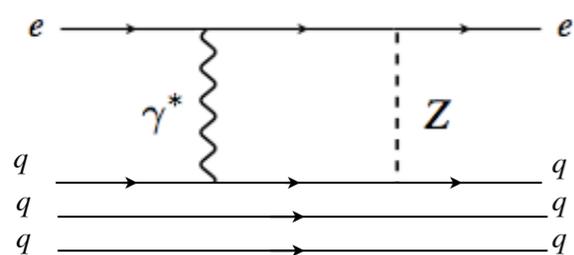
Axial h correction

- axial h correction $\square_{\gamma Z}^A$ dominant γZ correction in atomic parity violation at very low (zero) energy

→ computed by Marciano & Sirlin in 1983 as sum of two parts:



- ★ low-energy part approximated by *Born* contribution (elastic intermediate state)



- ★ high-energy part (above scale $\Lambda \sim 1$ GeV) computed perturbatively in terms of scattering from *free quarks*

$$\square_{\gamma Z}^A = (1 - 4\hat{s}^2) \underbrace{\frac{5\alpha}{2\pi} \int_{\Lambda^2}^{\infty} \frac{dQ^2}{Q^2(1 + Q^2/M_Z^2)} \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)}_{\sim \log \frac{M_Z^2}{\Lambda^2} + c}$$

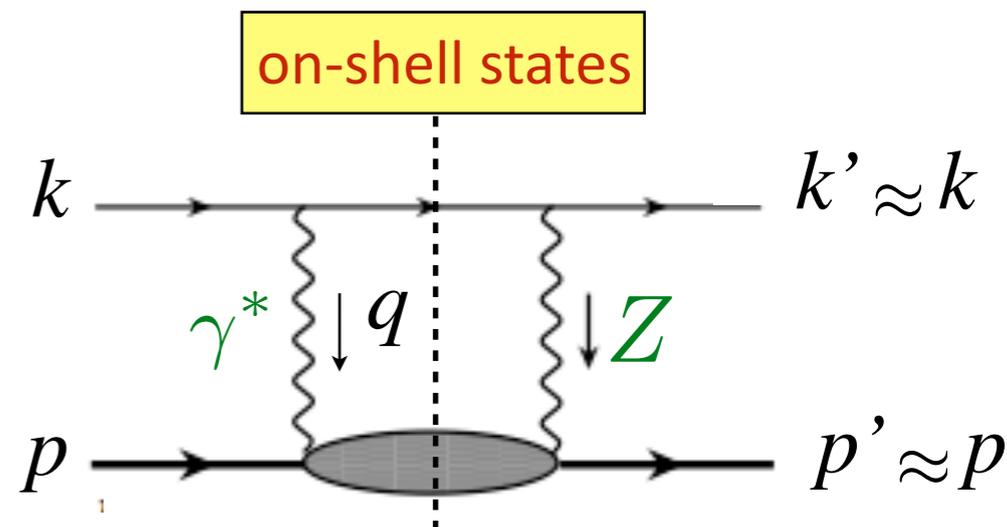
Forward angle dispersion method

Gorchtein, Horowitz, PRL 102 (2009) 091806

$$S = 1 + i\mathcal{M}$$

$$S^\dagger = 1 - i\mathcal{M}^\dagger$$

$$SS^\dagger = 1$$



Unitarity $\rightarrow -i(\mathcal{M} - \mathcal{M}^\dagger) = 2\Im m \mathcal{M} = \mathcal{M}^\dagger \mathcal{M}$

$$\Im m \langle f | \mathcal{M} | i \rangle = \frac{1}{2} \int d\rho \sum_n \langle f | \mathcal{M}^* | n \rangle \langle n | \mathcal{M} | i \rangle$$

Forward scattering amplitude: $|f\rangle \approx |i\rangle$

$$\Im m \langle i | \mathcal{M} | i \rangle = \frac{1}{2} \int d\rho \sum_n |\langle n | \mathcal{M} | i \rangle|^2 \sim \int d^3 k_1 \frac{L_{\mu\nu} W^{\mu\nu}}{q^2 (q^2 - M_Z^2)}$$

hadronic tensor:

$$MW_{\gamma Z}^{\mu\nu} = -g^{\mu\nu} \underbrace{\left(F_1^{\gamma Z} \right)}_{\text{vector } h} + \frac{p^\mu p^\nu}{p \cdot q} \underbrace{\left(F_2^{\gamma Z} \right)}_{\text{vector } h} - i \epsilon^{\mu\nu\lambda\rho} \frac{p_\lambda q_\rho}{2p \cdot q} \underbrace{\left(F_3^{\gamma Z} \right)}_{\text{axial } h}$$

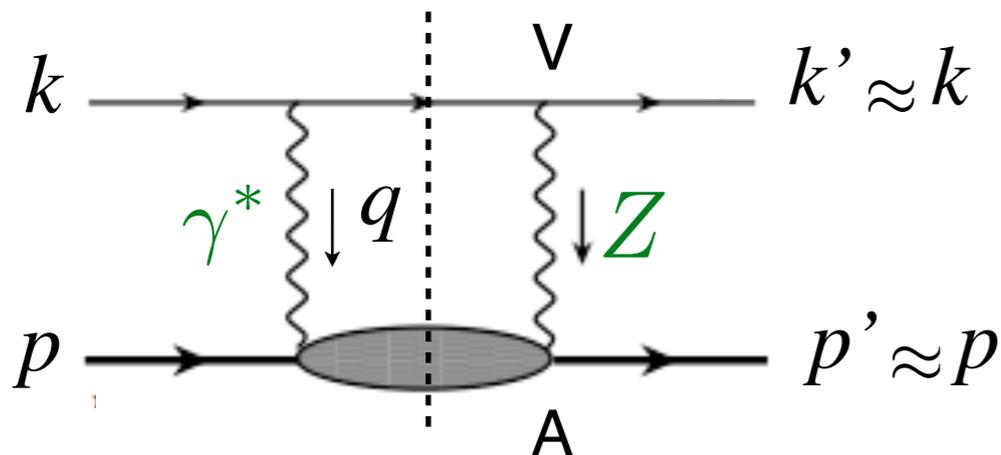
Axial h correction

- At low energy, dominant $V_e \times A_h$ correction evaluated using forward dispersion relations

$$\Re \square_{\gamma Z}^A(E) = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2 - E^2} \Im m \square_{\gamma Z}^A(E')$$

→ imaginary part given by $F_3^{\gamma Z}$ structure function

$$\Im m \square_{\gamma Z}^A(E) = \frac{1}{(2ME)^2} \int_{M^2}^s dW^2 \int_0^{Q_{\max}^2} dQ^2 \frac{v_e(Q^2) \alpha(Q^2)}{1 + Q^2/M_Z^2} \times \left(\frac{2ME}{W^2 - M^2 + Q^2} - \frac{1}{2} \right) \circledast F_3^{\gamma Z}$$



with $v_e(Q^2) = 1 - 4\kappa(Q^2)\hat{s}^2$

Axial h correction DIS part (dominant contribution)

- DIS part dominated by leading twist PDFs at small x

(MSTW, CTEQ, Alekhin parametrizations)

$$F_3^{\gamma Z(\text{DIS})}(x, Q^2) = \sum_q 2 e_q g_A^q (q(x, Q^2) - \bar{q}(x, Q^2))$$

→ in DIS region ($Q^2 \gtrsim 1 \text{ GeV}^2$), expand integrand in powers of x

$$\Re \square_{\gamma Z}^{\text{A(DIS)}}(E) = \frac{3}{2\pi} \int_{Q_0^2}^{\infty} dQ^2 \frac{v_e(Q^2) \alpha(Q^2)}{Q^2 (1 + Q^2/M_Z^2)} \\ \times \left[M_3^{(1)}(Q^2) + \frac{2M^2}{9Q^4} (5E^2 - 3Q^2) M_3^{(3)}(Q^2) + \dots \right]$$

with moments $M_3^{\gamma Z(n)} = \int_0^1 dx x^{n-1} F_3^{\gamma Z}(x, Q^2)$

Axial h correction

■ structure function moments

$$\underline{n=1} \quad M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3} \left(1 - \frac{\alpha_s(Q^2)}{\pi} \right)$$

→ γZ analog of Gross-Llewellyn Smith sum rule

$$\mathcal{R}e \square_{\gamma Z}^{A(\text{DIS})} \approx (1 - 4\hat{s}^2) \frac{5\alpha}{2\pi} \int_{Q_0^2}^{\infty} \frac{dQ^2}{Q^2(1+Q^2/M_Z^2)} \left(1 - \frac{\alpha_s(Q^2)}{\pi} \right)$$

→ precisely result from Marciano & Sirlin! $\sim \log \frac{M_Z^2}{Q_0^2}$

$$\underline{n=3} \quad M_3^{\gamma Z(3)}(Q^2) = \frac{1}{3} (2\langle x^2 \rangle_u + \langle x^2 \rangle_d) \left(1 + \frac{5\alpha_s(Q^2)}{12\pi} \right)$$

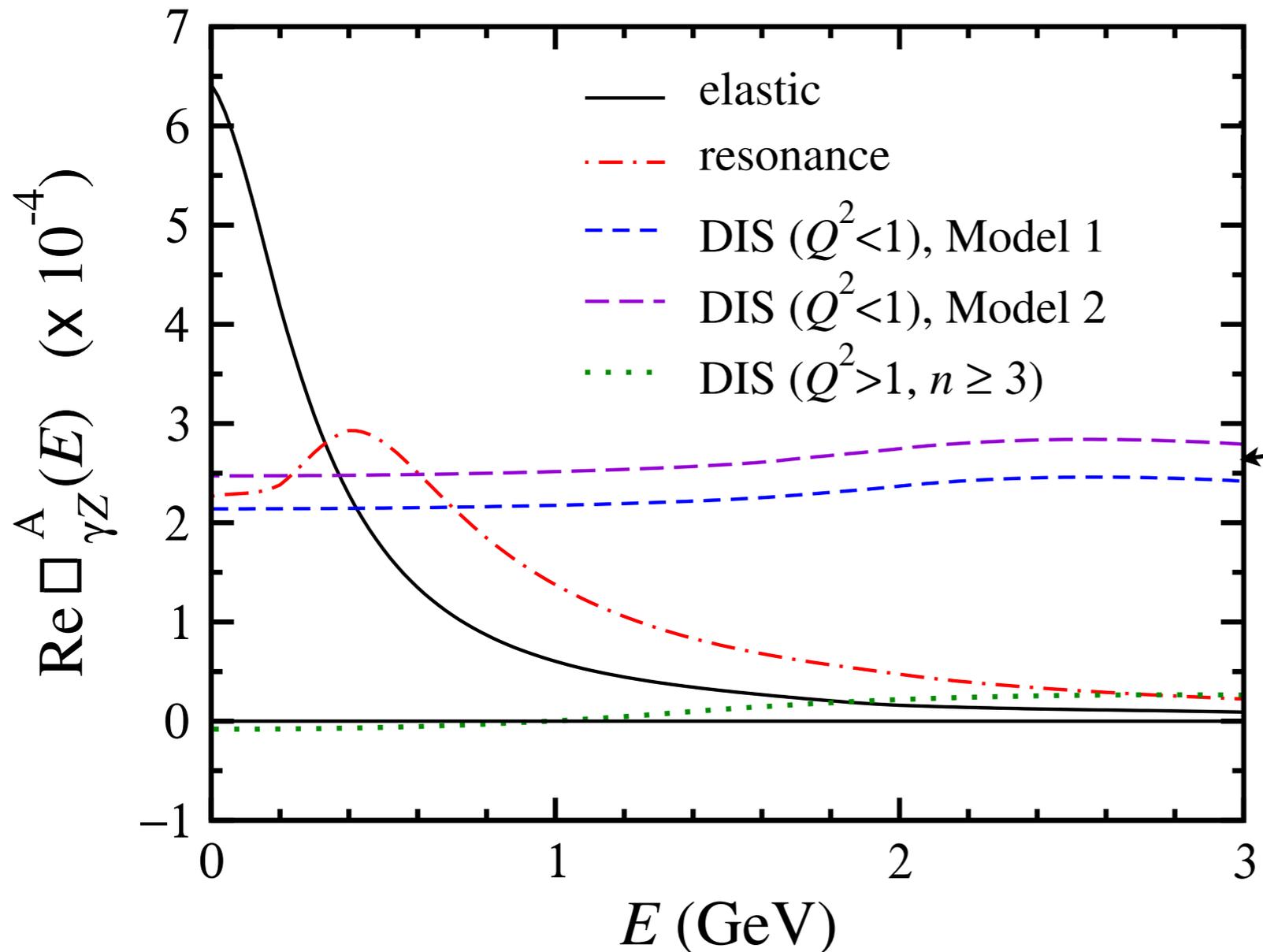
→ related to x^2 -weighted moment of valence quarks

Axial h elastic + resonance correction

★ elastic part: $F_3^{\gamma Z(\text{el})}(Q^2) = -Q^2 G_M^p(Q^2) G_A^Z(Q^2) \delta(W^2 - M^2)$

★ resonance part from parametrization of ν scattering data; includes lowest four spin 1/2 and 3/2 resonances

[P₃₃(1232), P₁₁(1440), D₁₃(1520), S₁₁(1535)] *Lalakulich, Paschos PRD 74, 014009 (2007)*



$n = 1$ moment gives
 32.8×10^{-4}
 (no E dependence)

Take this as
 estimate of
 uncertainty

*Blunden et al.,
 PRL 107, 081801 (2011)*

Axial h correction

■ correction at $E = 0$

$$\Re \square_{\gamma Z}^A = 0.00063 + 0.00023 + 0.00350 \rightarrow 0.0044(2)$$

elastic resonance DIS

■ correction at $E = 1.165 \text{ GeV}$ (Qweak)

$$\Re \square_{\gamma Z}^A = 0.00005 + 0.00011 + 0.00352 \rightarrow 0.0037(2)$$

cf. MS value: 0.0052(5) ($\sim 1\%$ shift in Q_W^p)

Marciano, Sirlin, PRD 29, 75 (1984)

■ shifts Q_W^p by -0.0008

Vector h correction

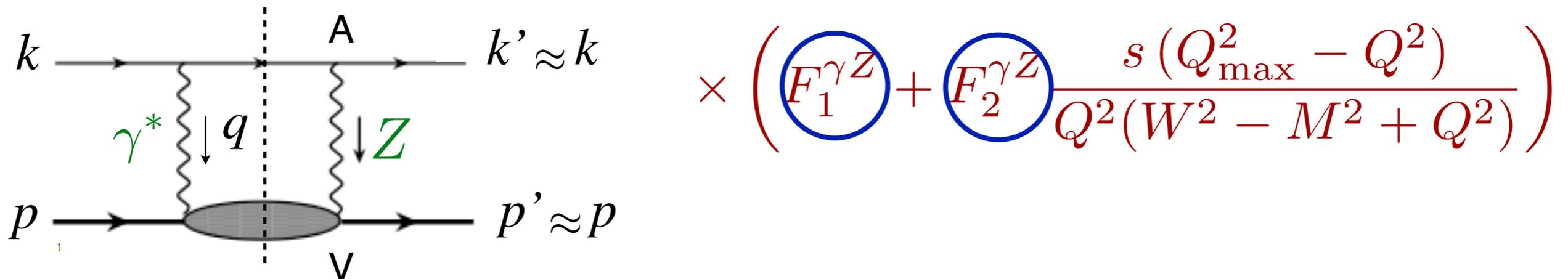
- vector h correction $\square_{\gamma Z}^V$ vanishes at $E = 0$ (inelastic only), but experiment has $E \sim 1$ GeV. What is energy dependence?

→ forward dispersion relation

$$\Re \square_{\gamma Z}^V(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2} \Im m \square_{\gamma Z}^V(E')$$

→ inelastic imaginary part given by

$$\Im m \square_{\gamma Z}^V(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^s dW^2 \int_0^{Q_{\max}^2} \frac{dQ^2}{1 + Q^2/M_Z^2}$$



γZ box: Vector h correction

- Several groups doing independent analyses
- At Q_{weak} energy $E = 1.165$ GeV (relative to weak charge of 0.0713)

Gorchtein/Horowitz (2009)

Sibirtsev *et al.* (2010) $(4.7^{+1.1}_{-0.4}) \times 10^{-3}$

Gorchtein *et al.* (2011) $(5.4 \pm 2.0) \times 10^{-3}$

Rislow/Carlson (2011) $(5.7 \pm 0.9) \times 10^{-3}$

AJM-I: Hall *et al.* (2013) $(5.6 \pm 0.4) \times 10^{-3}$

Gorchtein *et al.* (2015) $(5.6 \pm 1.4) \times 10^{-3}$

AJM-II: Hall *et al.* (2016) $(5.4 \pm 0.4) \times 10^{-3}$

}

new results

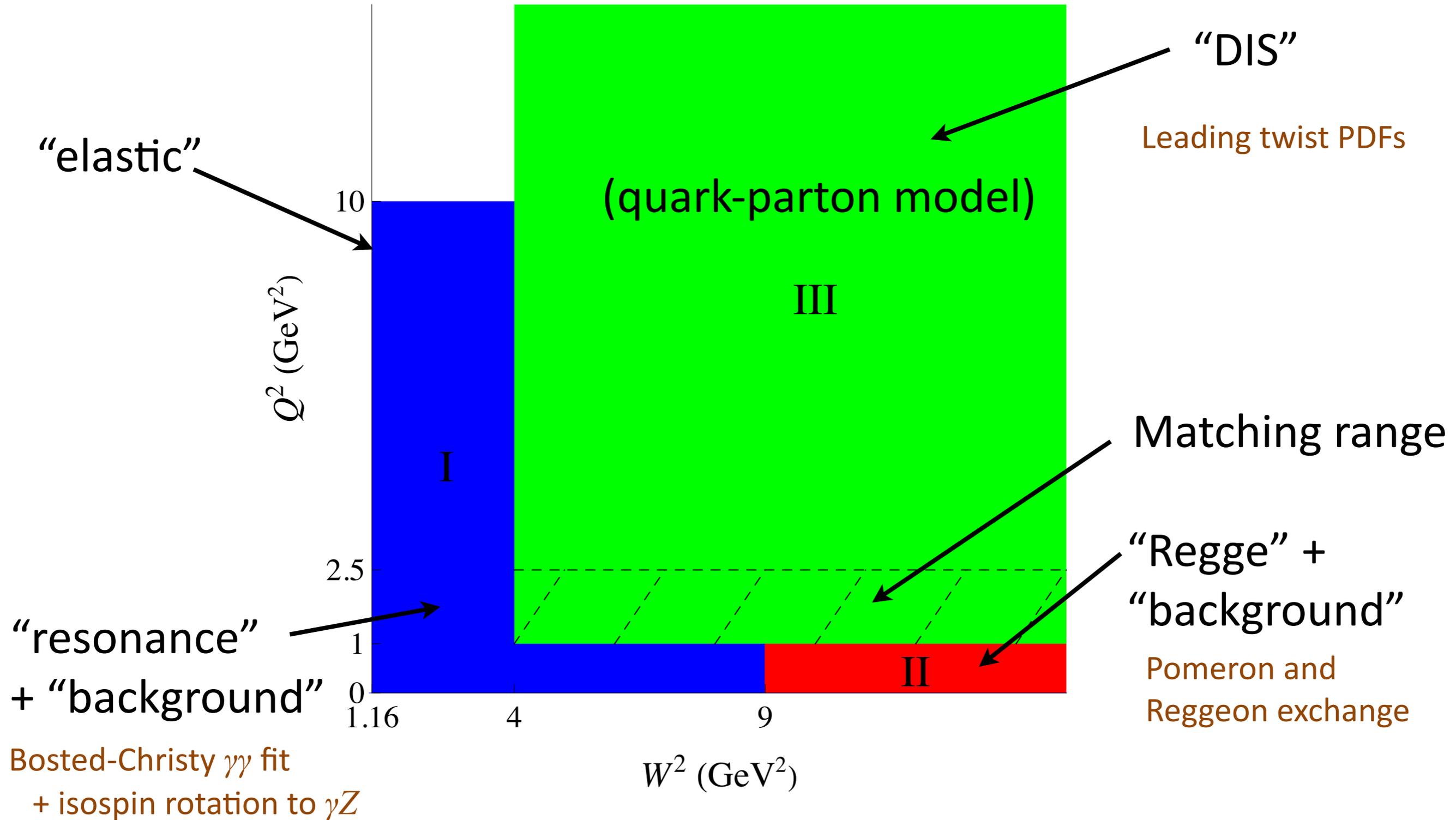
- Mainly different treatments of low Q^2 , low W region background contributions
- Agree on overall magnitude, but disagree on errors and details

AJM structure function model

- Accurate knowledge of $\gamma\gamma$ and γZ structure functions (at all kinematics) vital for determination of radiative corrections
 - Wealth of data on $F_i^{\gamma\gamma}$ structure functions over large range of kinematics in Q^2 and W (or x) – with some gaps
 - Relatively little known about $F_i^{\gamma Z}$ interference structure functions below HERA measurements, with $Q^2 \geq 1500 \text{ GeV}^2$
 - Fit $F_i^{\gamma\gamma}$ over all kinematics in Q^2 and W , then “rotate” to $F_i^{\gamma Z}$ using available theoretical/phenomenological constraints
- *e.g.* isospin symmetry

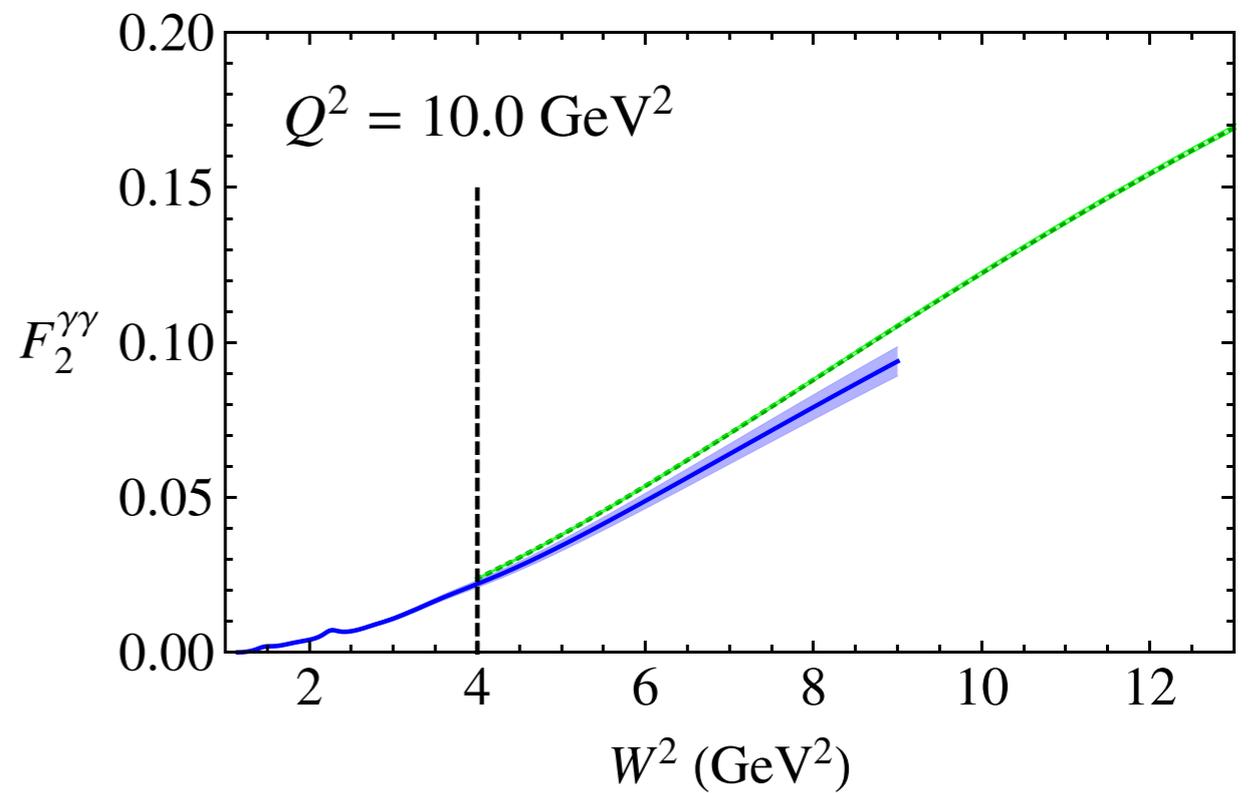
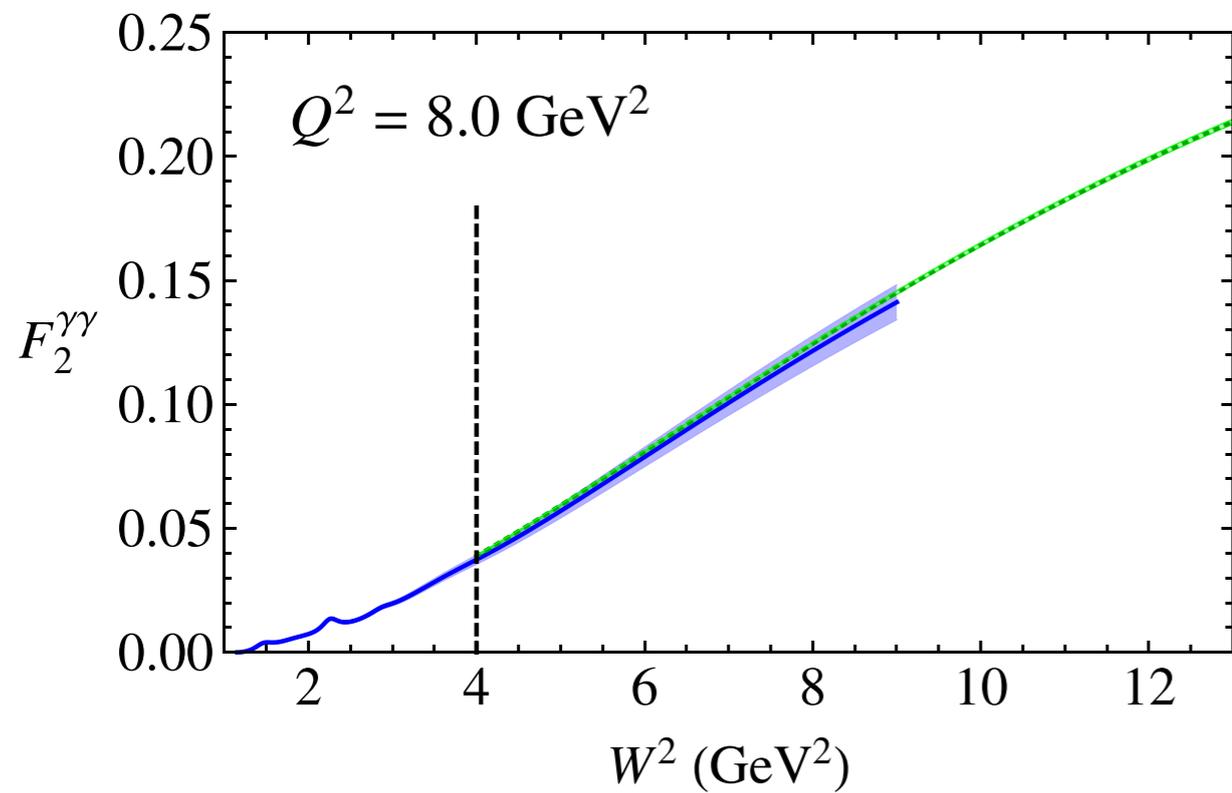
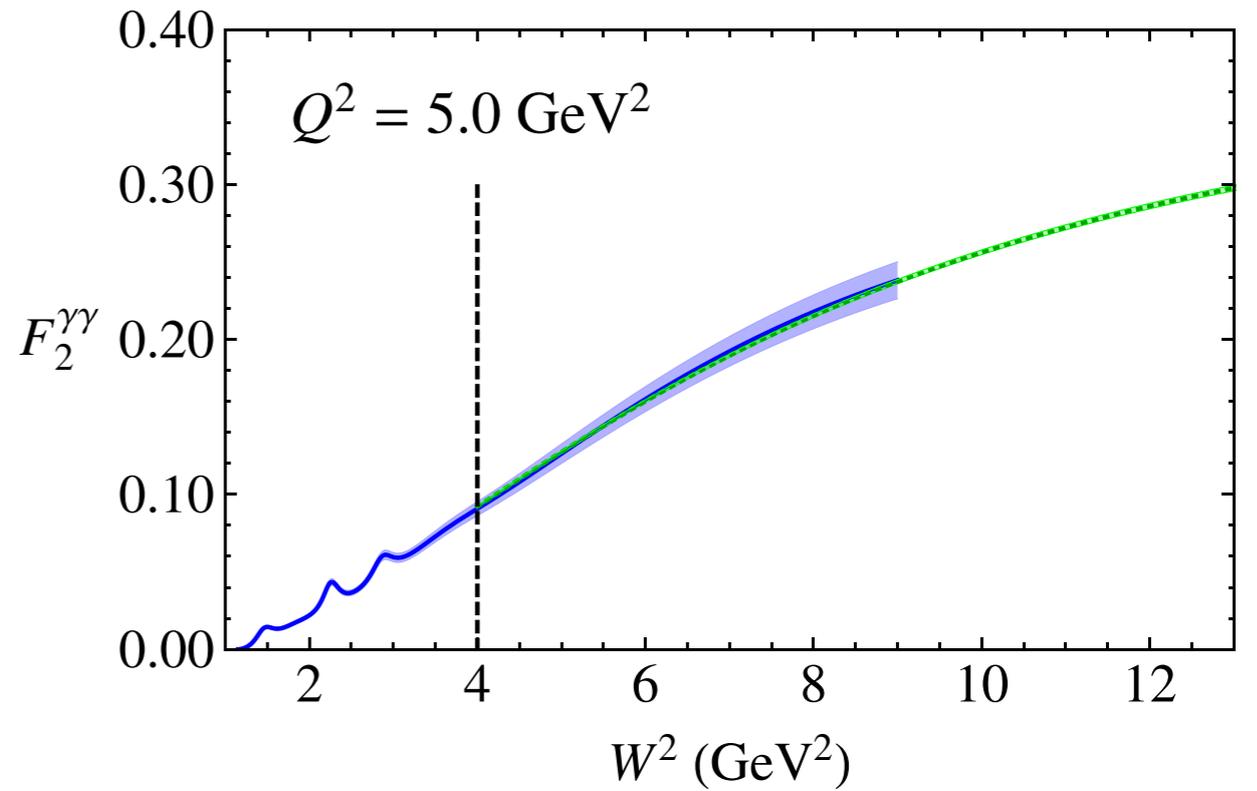
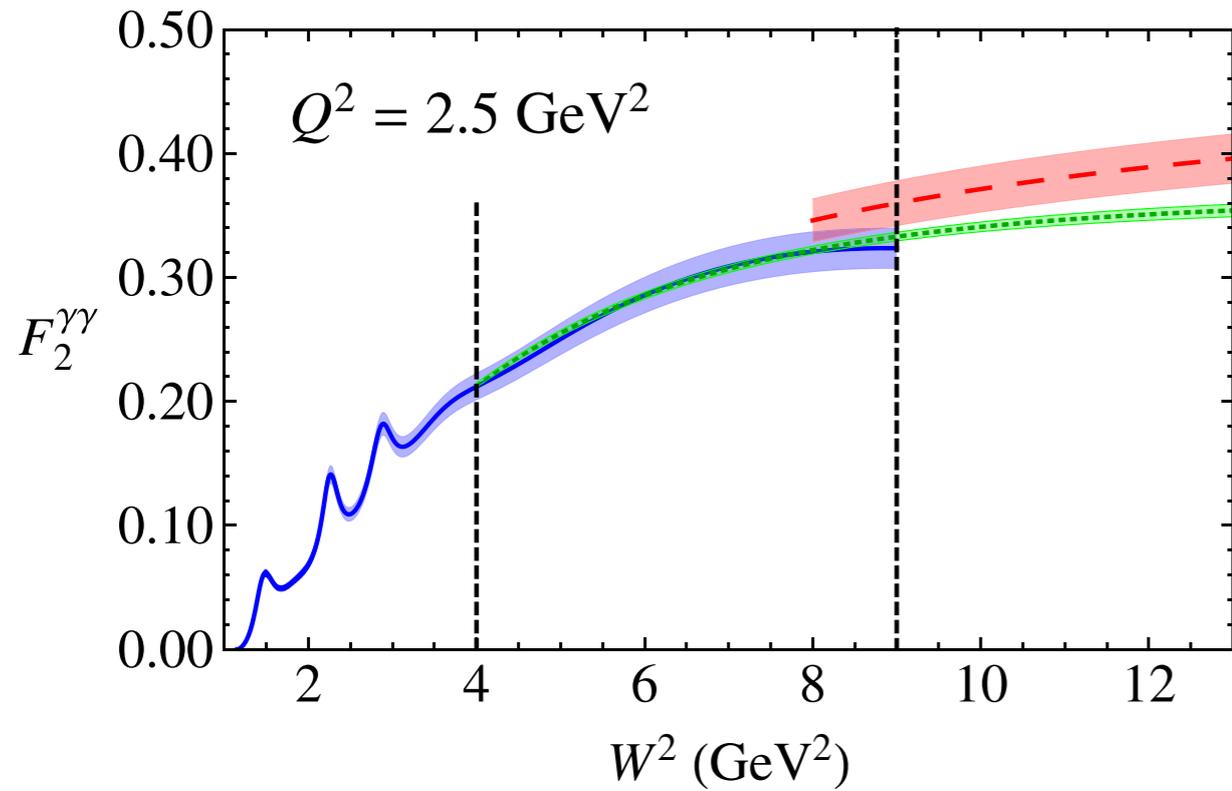
$$\langle N^* | J_Z^\mu | p \rangle = (1 - 4 \sin^2 \theta_W) \langle N^* | J_\gamma^\mu | p \rangle - \langle N^* | J_\gamma^\mu | n \rangle$$

Integration region (structure function map)



VMD model for “background”

$\gamma\gamma$ structure function matching



Basic issue: how to relate $F_{1,2}^{\gamma Z}$ to $F_{1,2}^{\gamma}$?

Scaling region III

$$F_2^{\gamma} = \sum_q e_q^2 x(q + \bar{q})$$
$$F_2^{\gamma Z} = \sum_q 2e_q g_V^q x(q + \bar{q})$$
$$x = \frac{Q^2}{W^2 - M^2 + Q^2}$$

Resonance region I largest contribution (unlike $F_3^{\gamma Z}$)

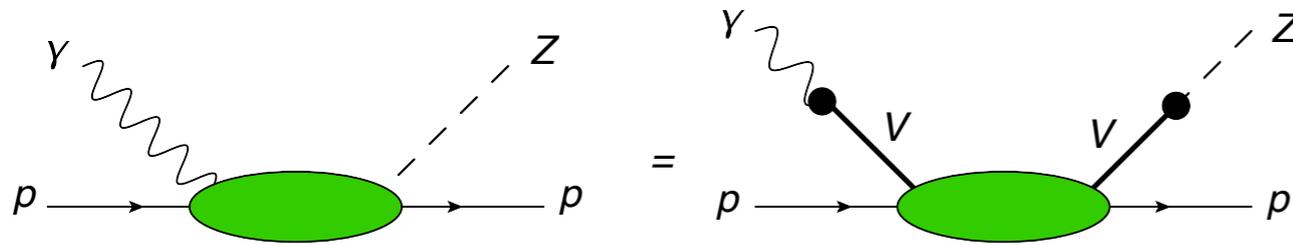
For $\gamma\gamma$ use Christy-Bosted (CB) fit to $e-p$ cross sections

$$\sigma_{T,L} = \sigma_{T,L}(\text{res}) + \sigma_{T,L}(\text{bg})$$

- $\sigma_{T,L}(\text{res})$
- Includes 7 most prominent N^* resonances below 2 GeV.
 - Generally agrees with data to $\sim 5\%$
 - For γZ modify fit by ratio of weak to e.m. transition amplitudes.

Background $\sigma_{T,L}(\text{bg})$

- Use Vector Meson Dominance (VMD) models fit to high energy data, plus isospin rotations



$$V = \rho, \omega, \phi + \text{continuum}$$

$$\sigma_V^{\gamma Z} = \kappa_V \sigma_V^{\gamma\gamma} \quad \frac{\sigma^{\gamma Z}}{\sigma^{\gamma\gamma}} = \frac{\kappa_\rho + \kappa_\omega R_\omega + \kappa_\phi R_\phi + \kappa_C R_C}{1 + R_\omega + R_\phi + R_C}$$

Isospin rotation: $\kappa_\rho = 2 - 4 \sin^2 \theta_W$, $\kappa_\omega = -4 \sin^2 \theta_W$, $\kappa_\phi = 3 - 4 \sin^2 \theta_W$

→ continuum parameter κ_C not constrained in VMD

- **GHRM**: assign 100% uncertainty on continuum contribution (dominates errors)

- **AJM model**: constrain continuum (higher Q^2) contribution by matching with PDF ratios (γZ to $\gamma\gamma$) across boundaries of Regions I, II and III.

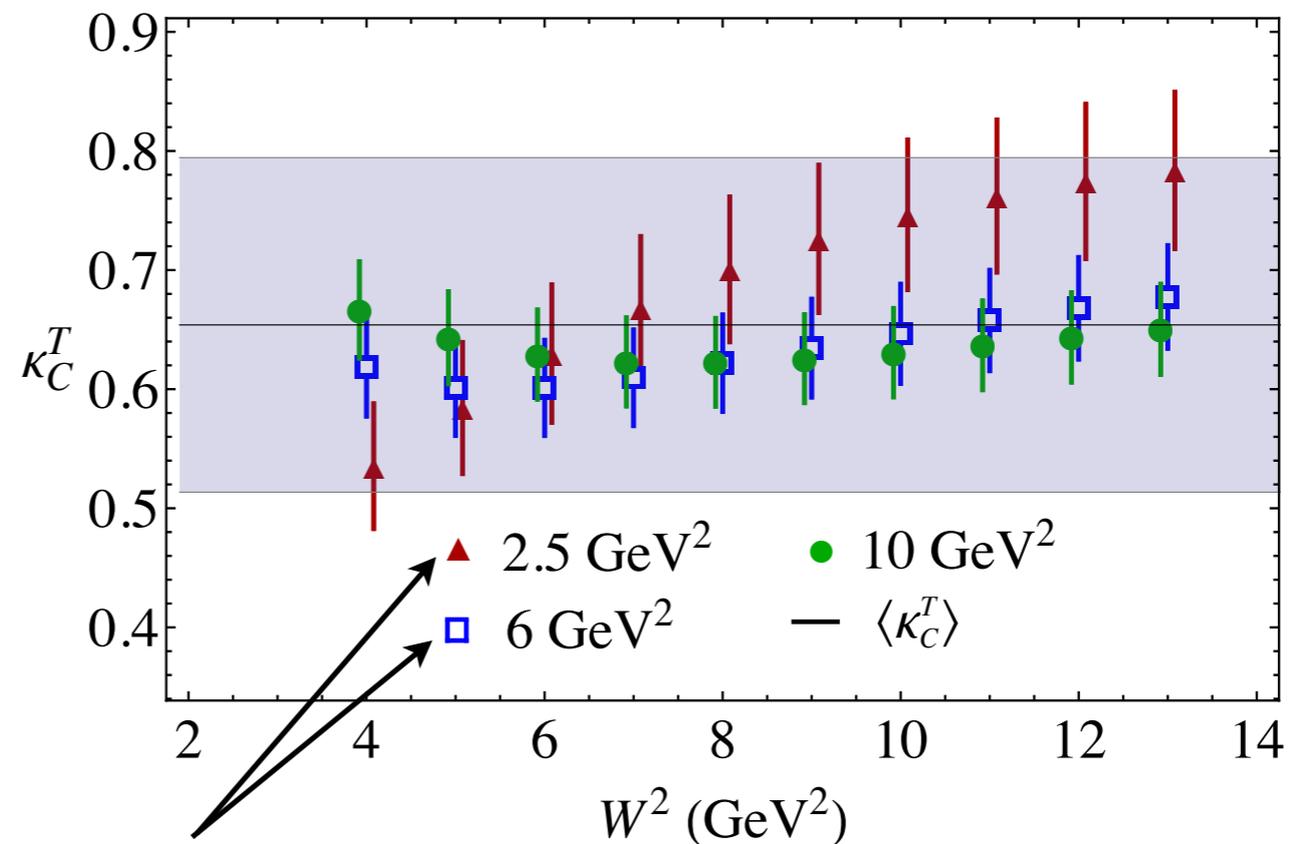
AJM γZ model

- Region where continuum contributions are relevant overlaps with typical reach of global PDF fits

→ constrain κ_C using PDF parametrizations by requiring matching of $F_{1,2}^{\gamma Z}$ to DIS structure functions:

$$\frac{\sigma_T^{\gamma Z}(\kappa_C^T)}{\sigma_T^{\gamma\gamma}} = \frac{F_1^{\gamma Z}}{F_1^{\gamma\gamma}} \Big|_{\text{LT}}$$

$$\frac{\sigma_L^{\gamma Z}(\kappa_C^T)}{\sigma_L^{\gamma\gamma}} = \frac{F_L^{\gamma Z}}{F_L^{\gamma\gamma}} \Big|_{\text{LT}}$$

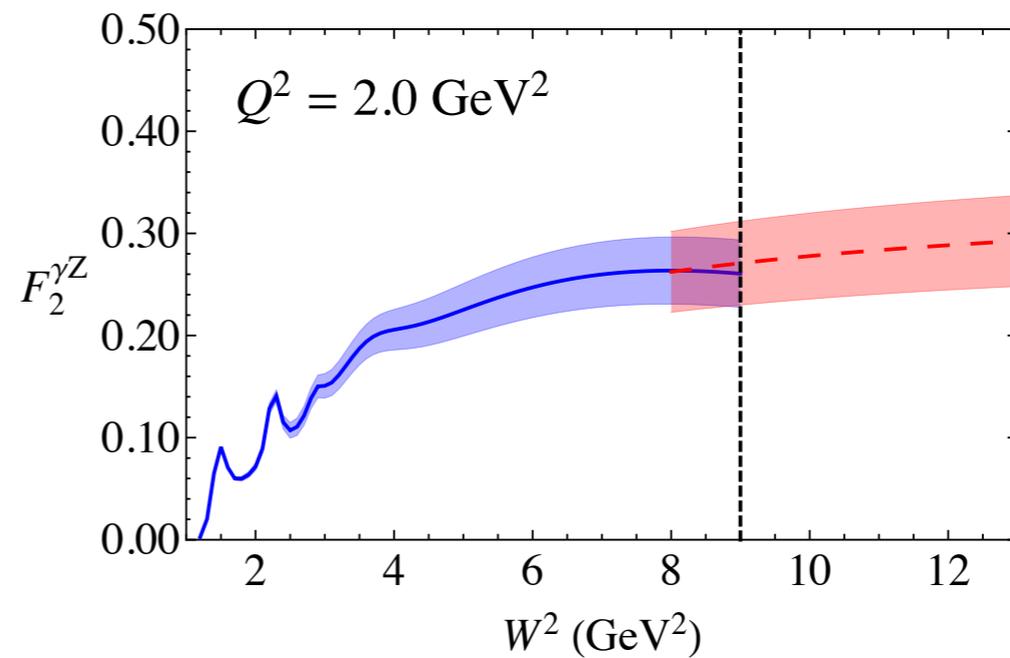
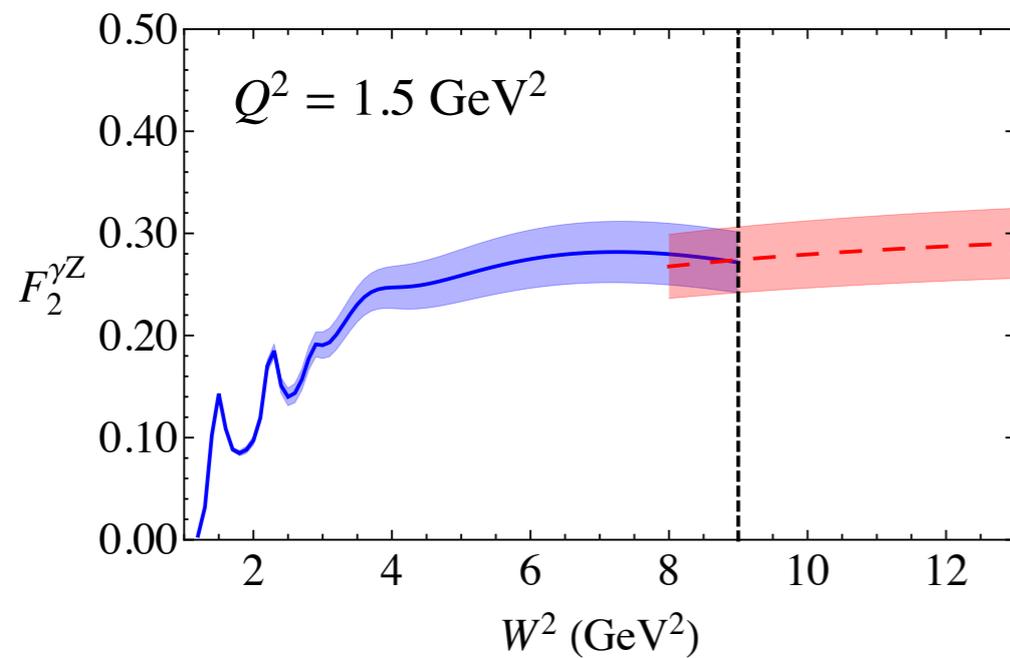
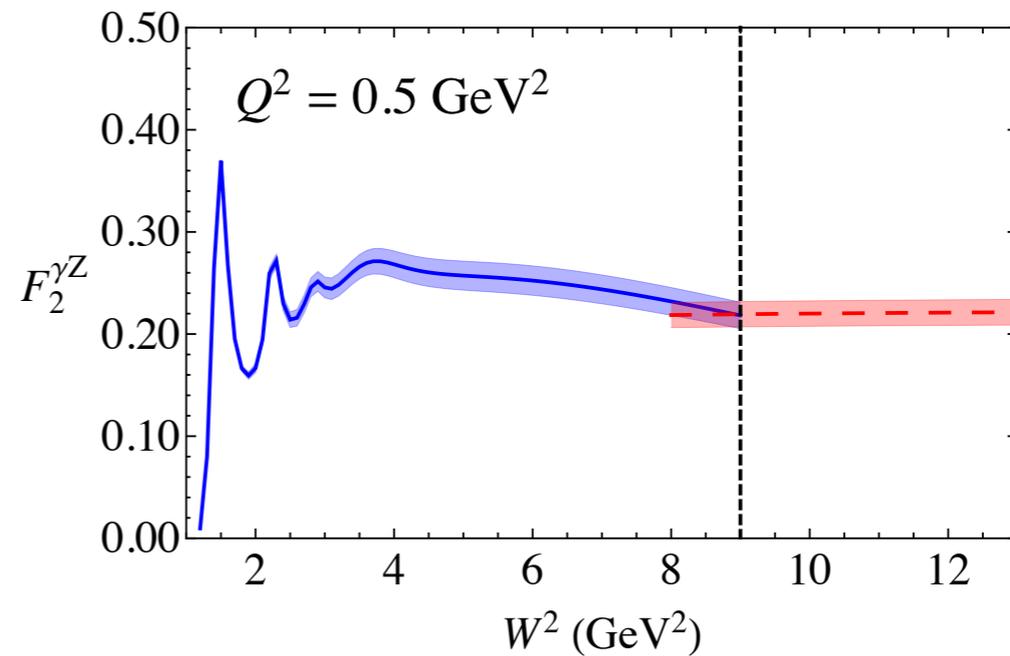
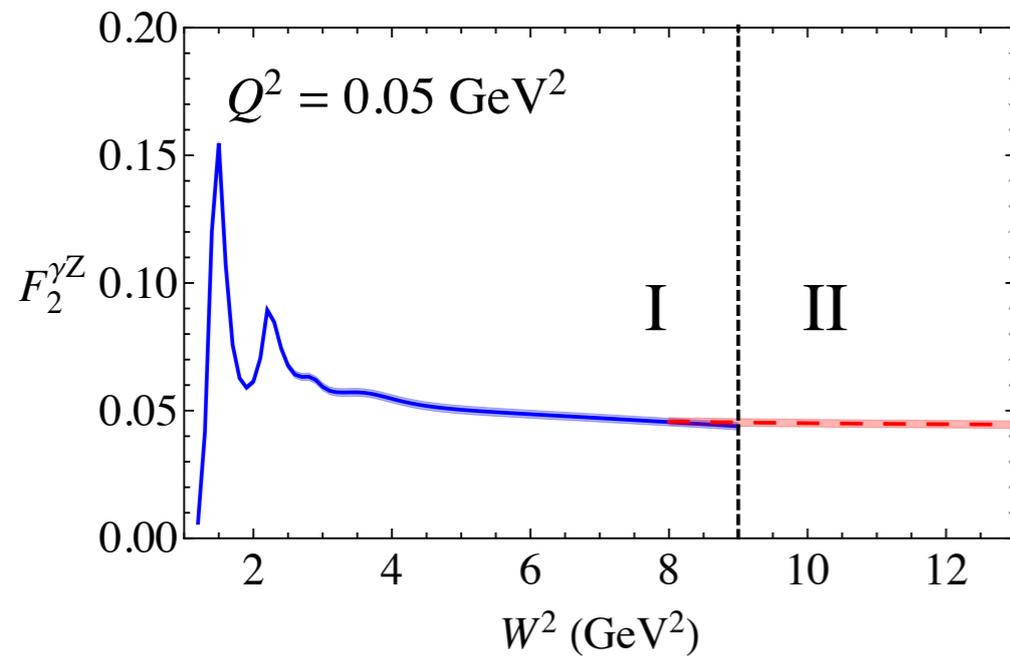


matching at
different Q^2

→ $\kappa_C^T = 0.65 \pm 0.14, \quad \kappa_C^L = -1.3 \pm 1.7$

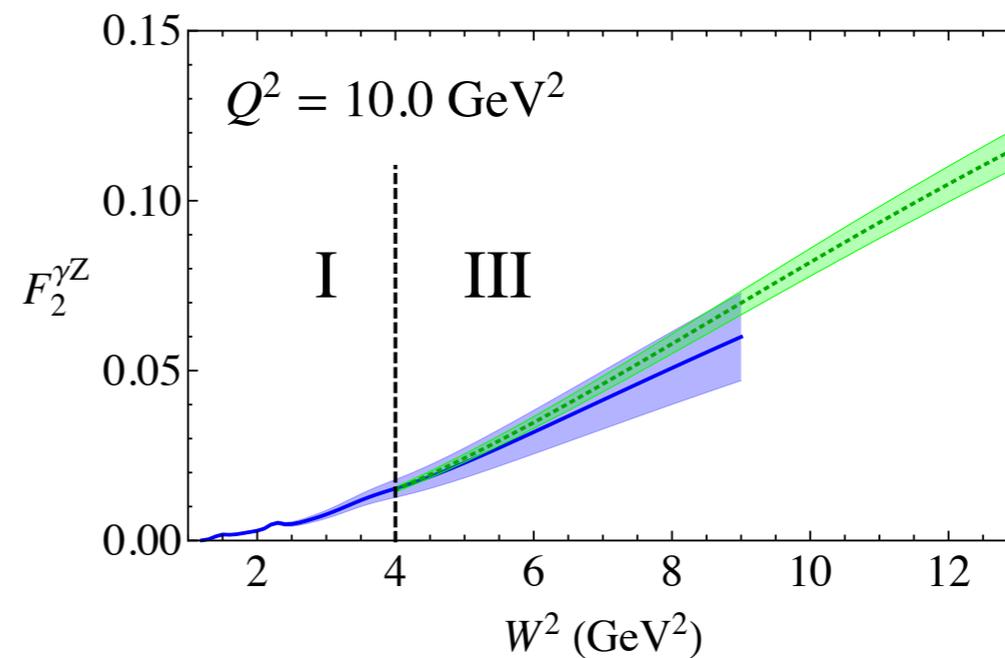
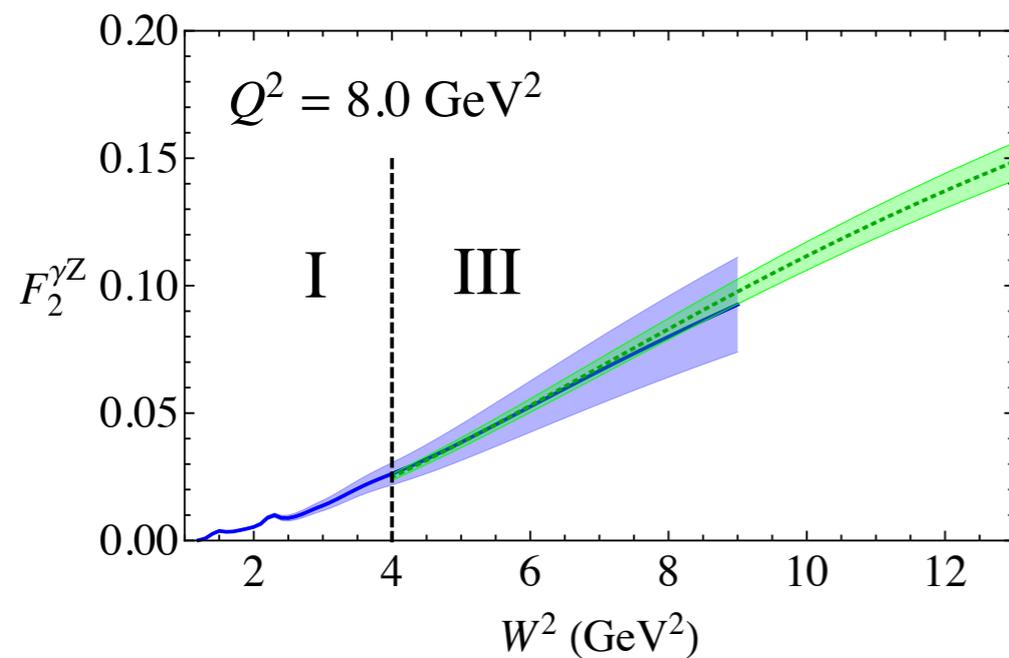
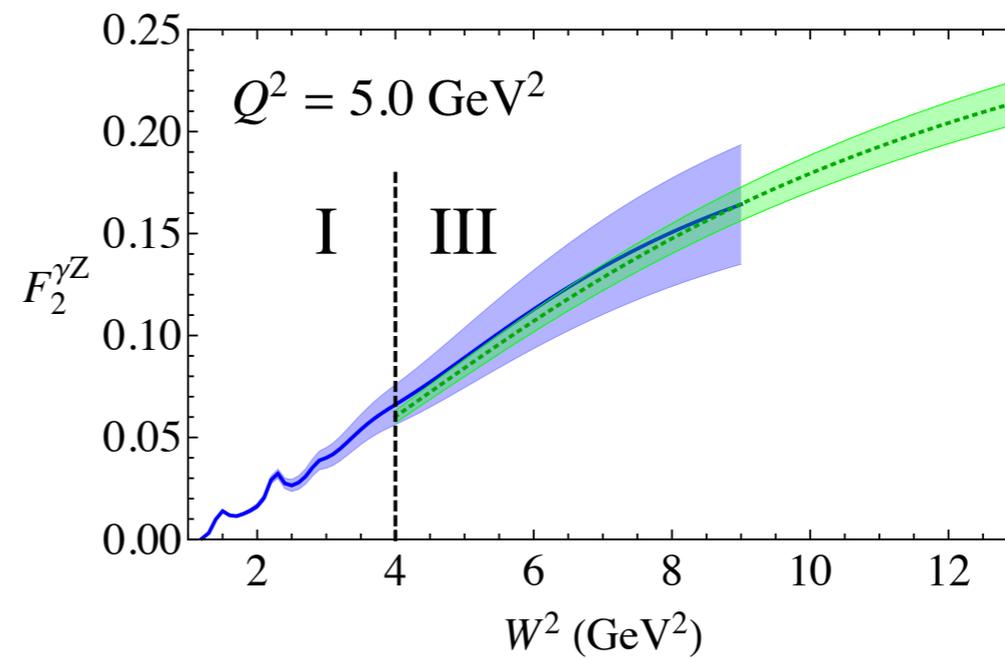
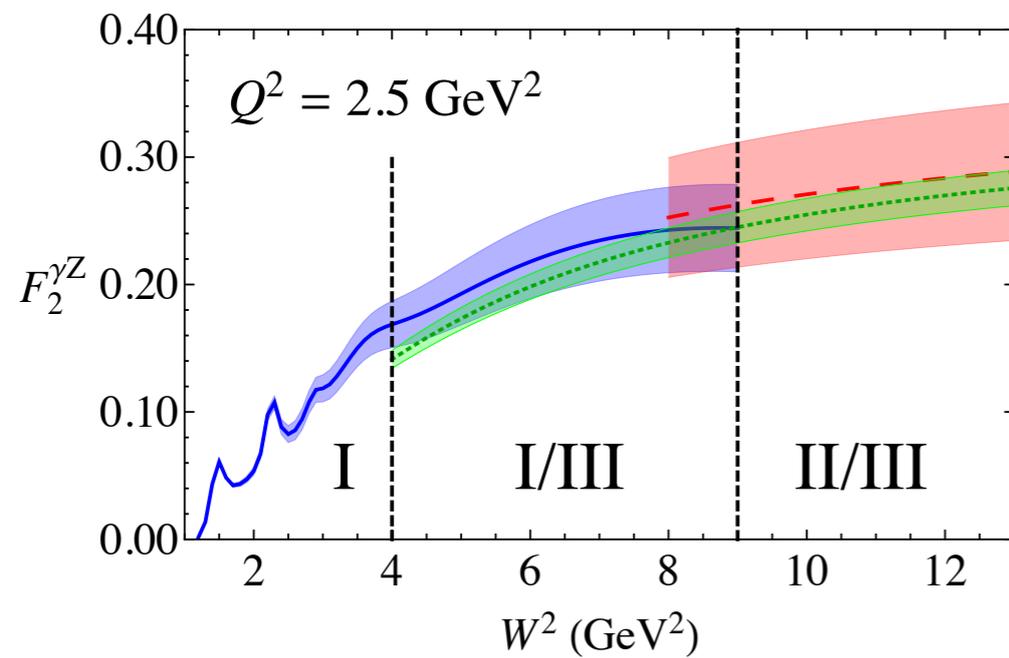
(small contribution to asymmetry)

γZ structure function matching



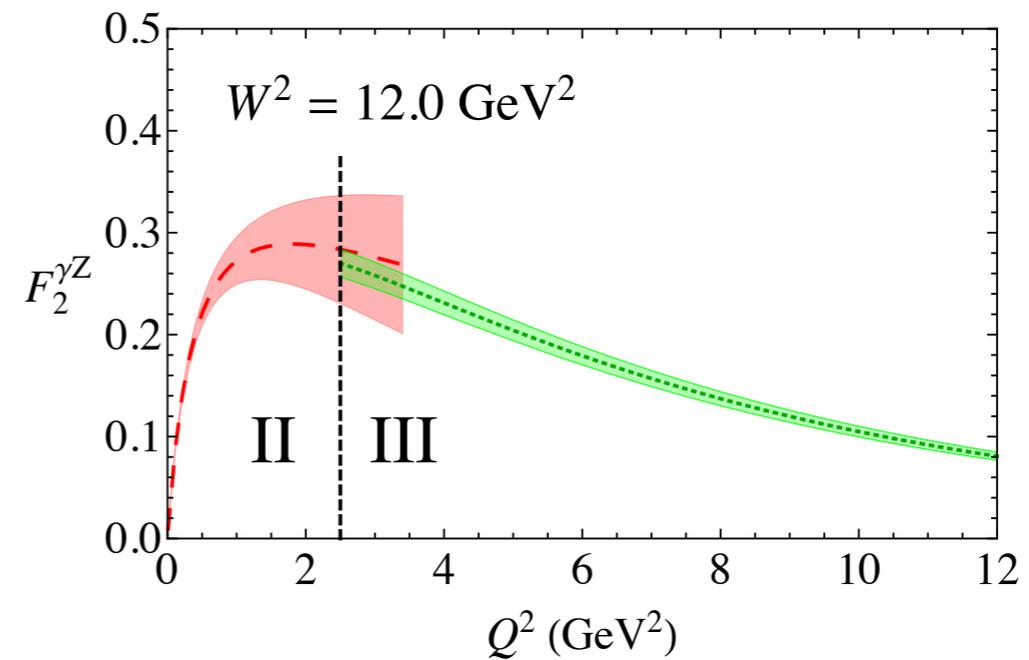
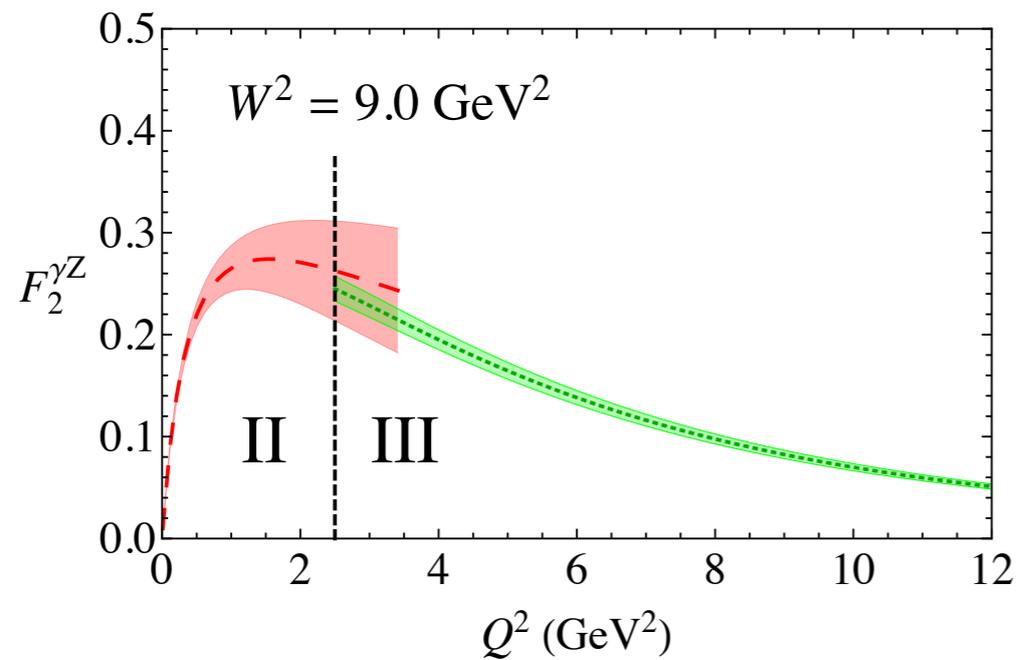
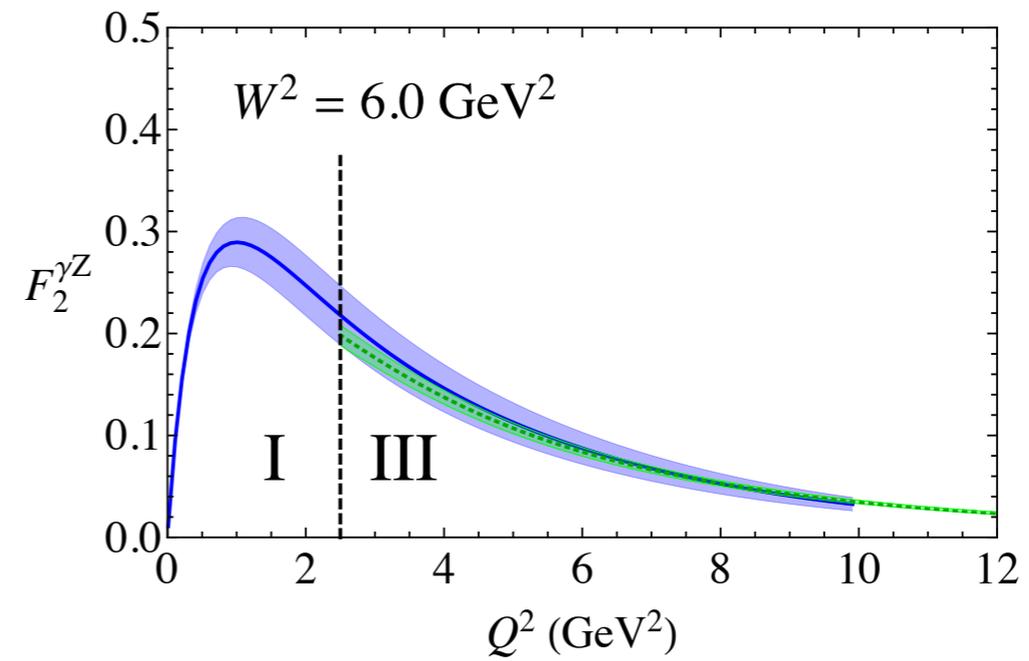
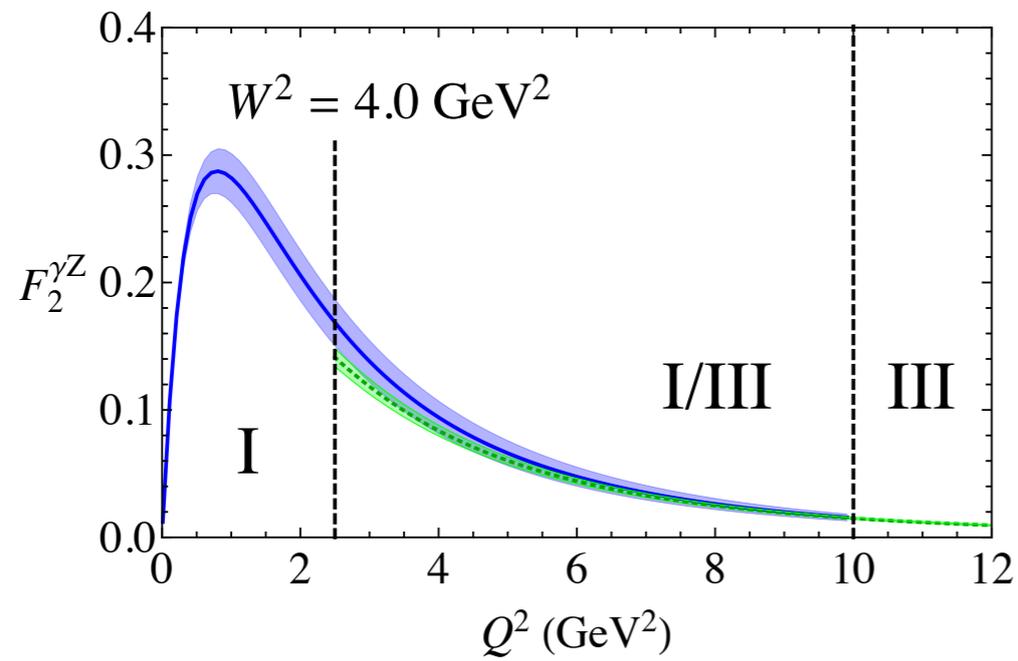
→ structure functions well matched at boundary of regions I and II, within larger errors *cf.* $\gamma\gamma$ functions

γZ structure function matching



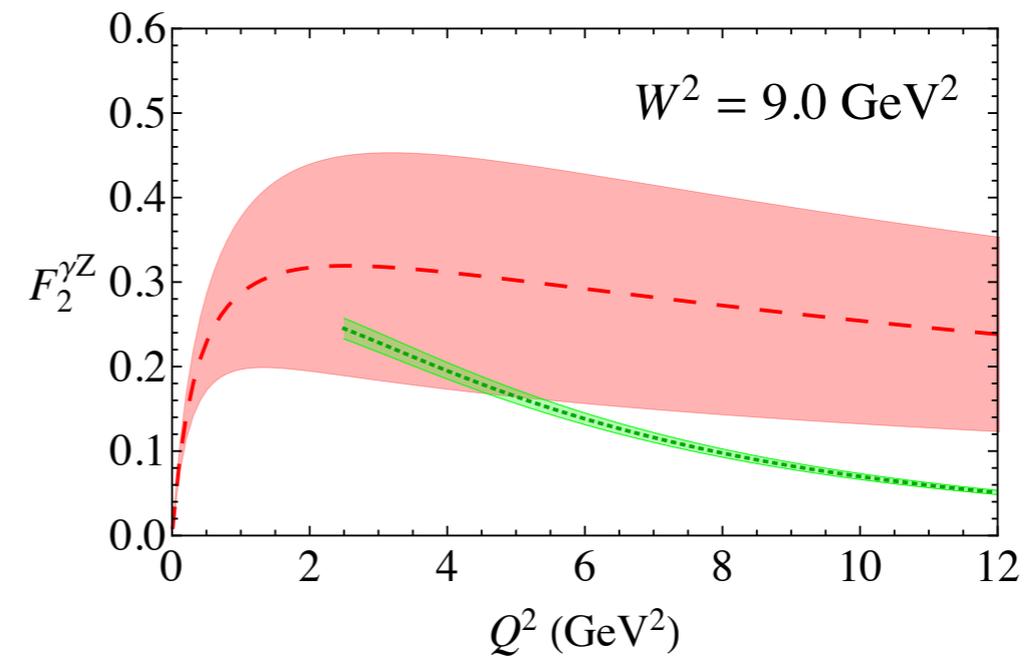
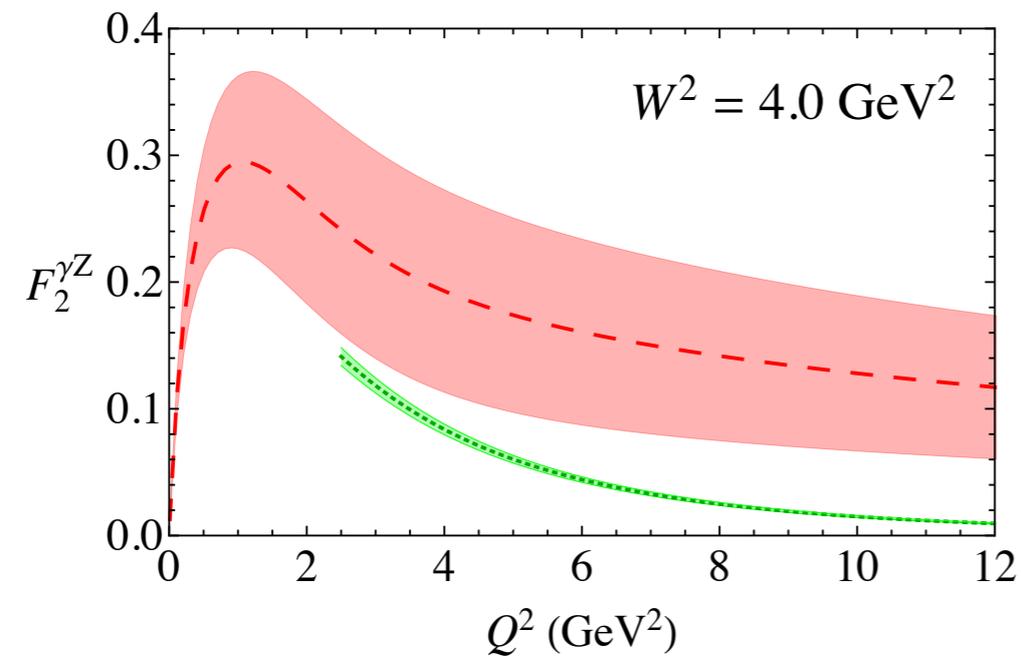
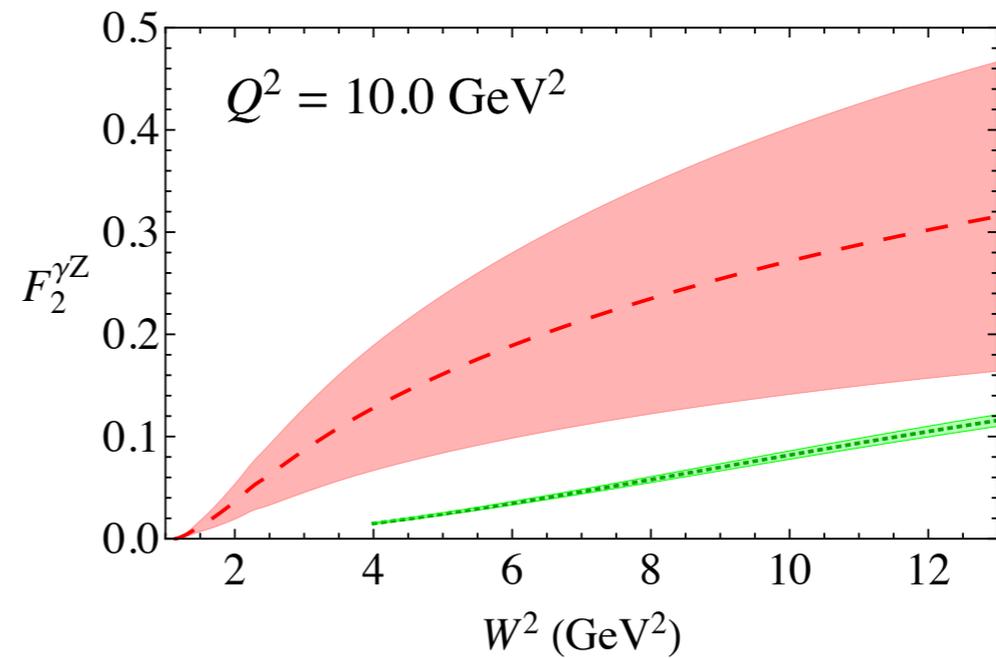
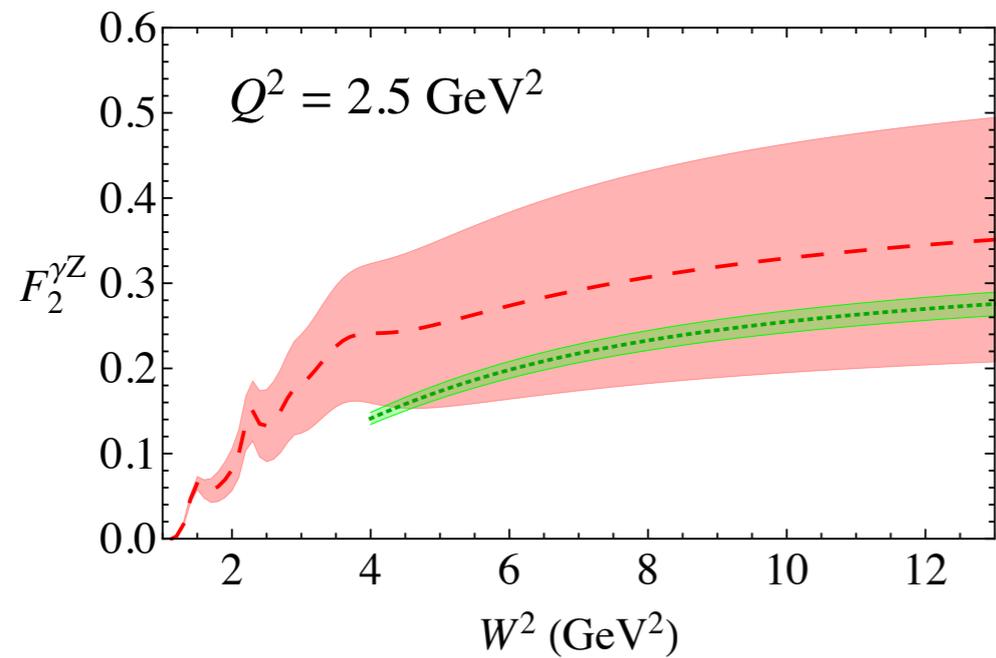
→ structure functions well matched at boundaries of regions I, II and III

γZ structure function matching

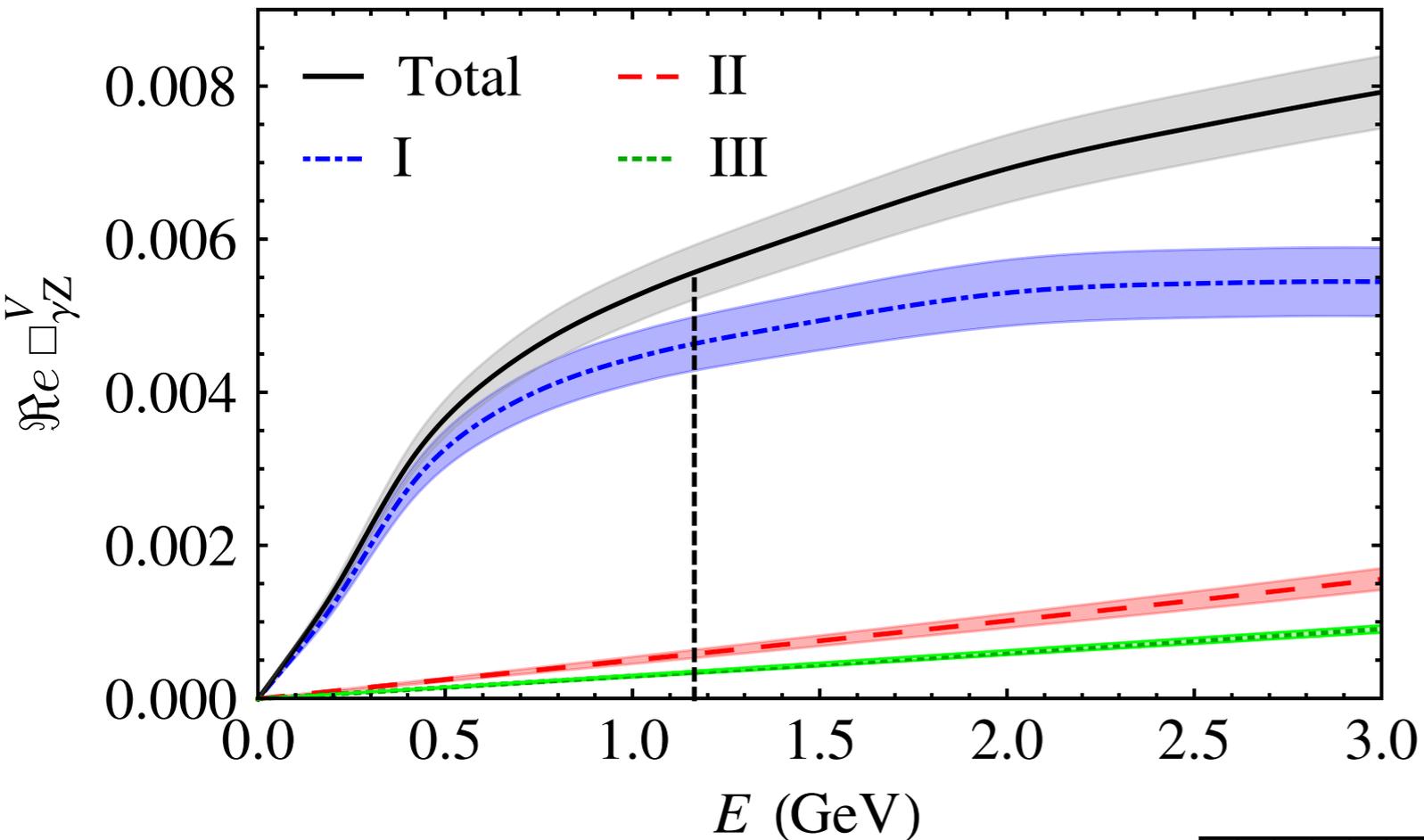


→ structure functions well matched at boundaries of regions I, II and III

γZ structure function matching



→ if do *not* impose any constraint on κ_C (e.g. GHRM model), have far greater mismatch between Regge and DIS regions

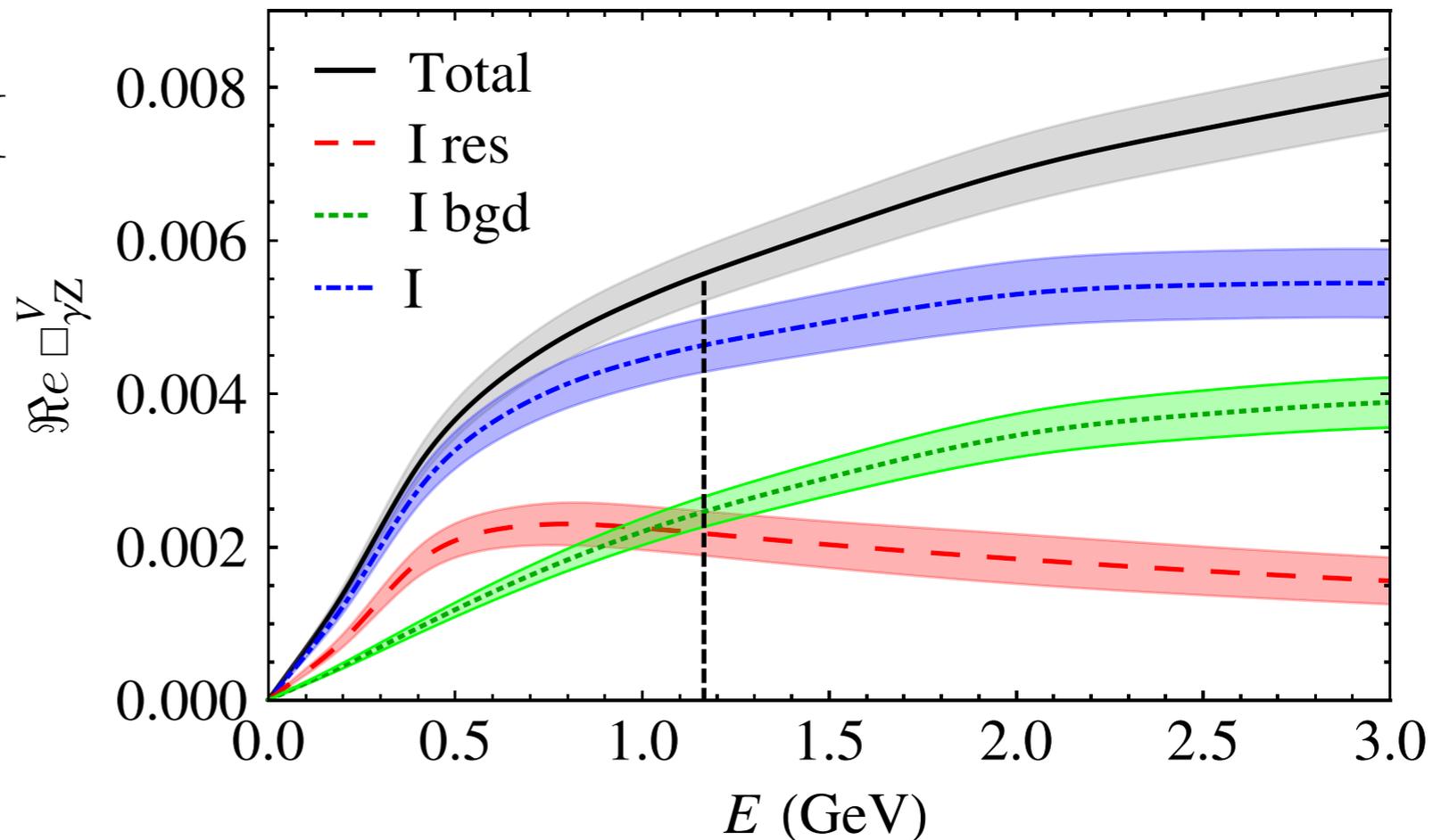


Contribution from
different regions to $\Pi_{\gamma Z}^V$

(relative to weak
charge of 0.0713)

Breakdown of region I res + bgd

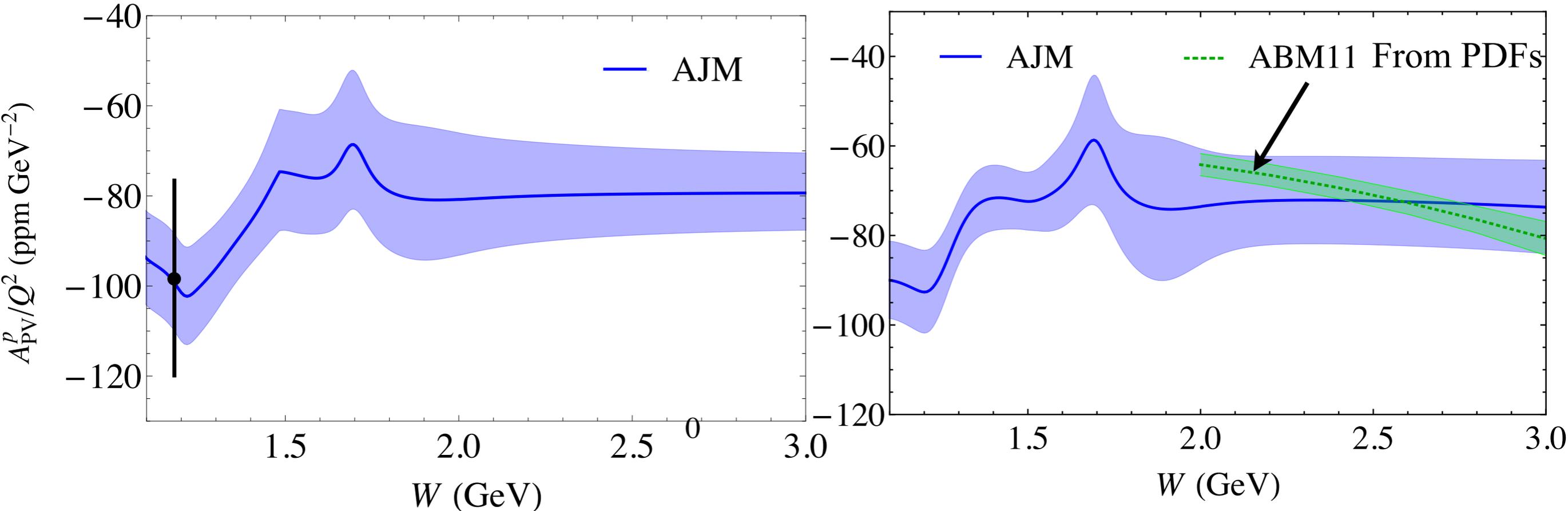
Region	$\Pi_{\gamma Z} (\times 10^{-3})$
I (res)	2.18 ± 0.29
I (bgd)	2.46 ± 0.20
I (total)	4.64 ± 0.35
II	0.59 ± 0.05
III	0.35 ± 0.02
Total	5.57 ± 0.36



AJM γZ model direct test

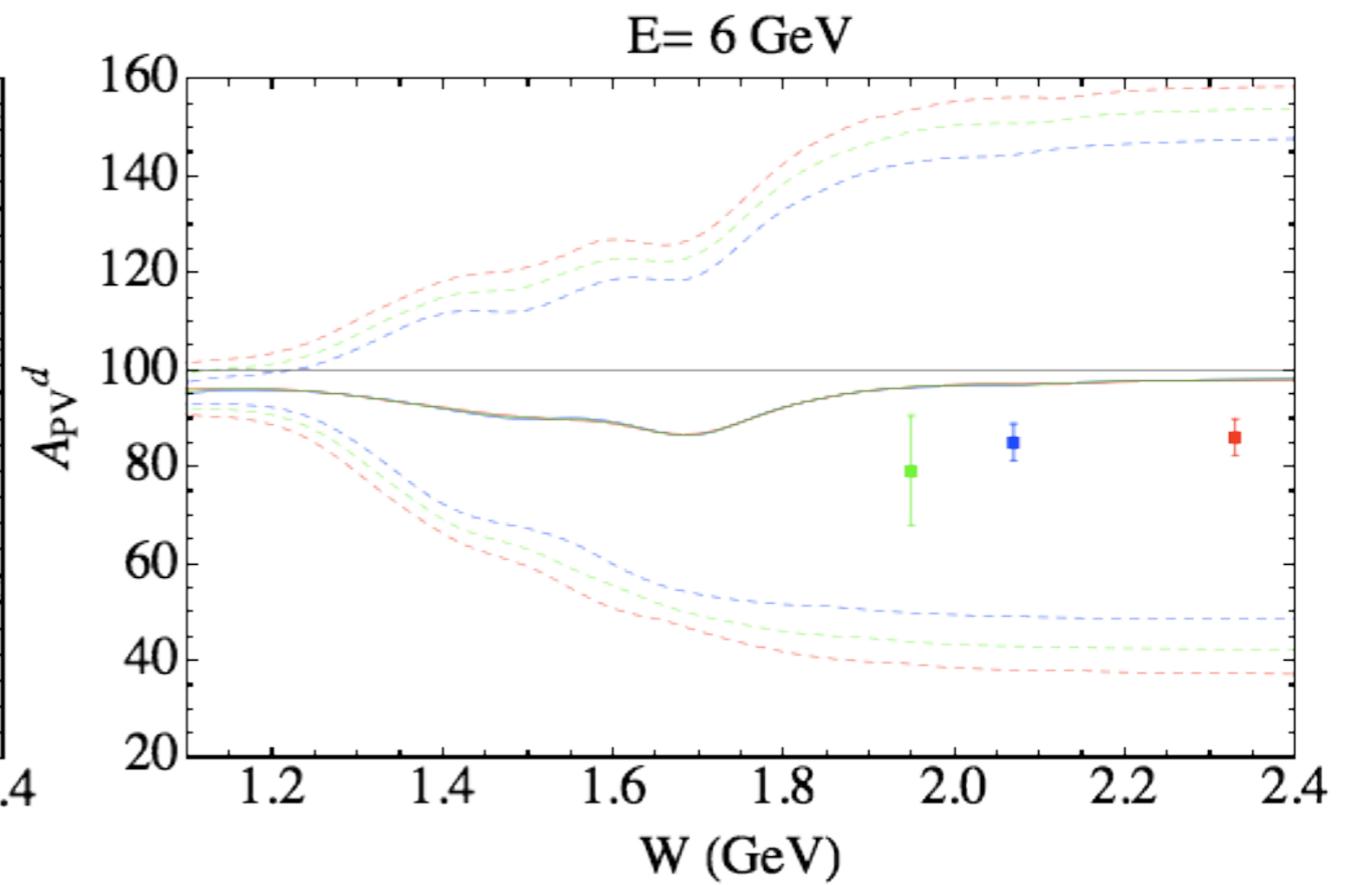
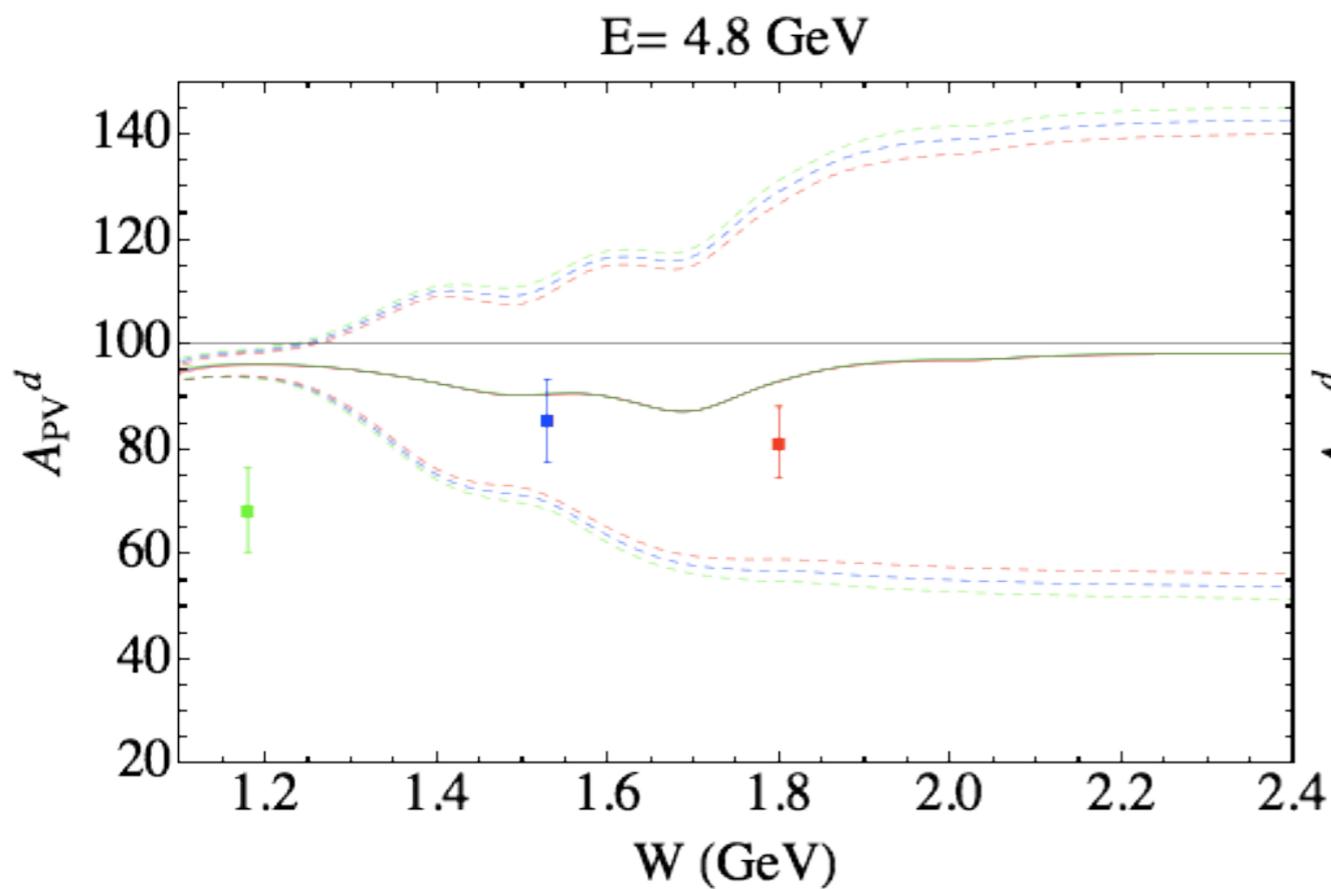
- Parity-violating Deep Inelastic Scattering (PVDIS) asymmetry allows a direct measurement of the γZ structure functions

$$A_{PV} = g_A^e \left(\frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \right) \frac{xy^2 F_1^{\gamma Z} + (1-y)F_2^{\gamma Z} + \frac{g_V^e}{g_A^e} (y - y^2/2)x F_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + (1-y)F_2^{\gamma\gamma}}$$



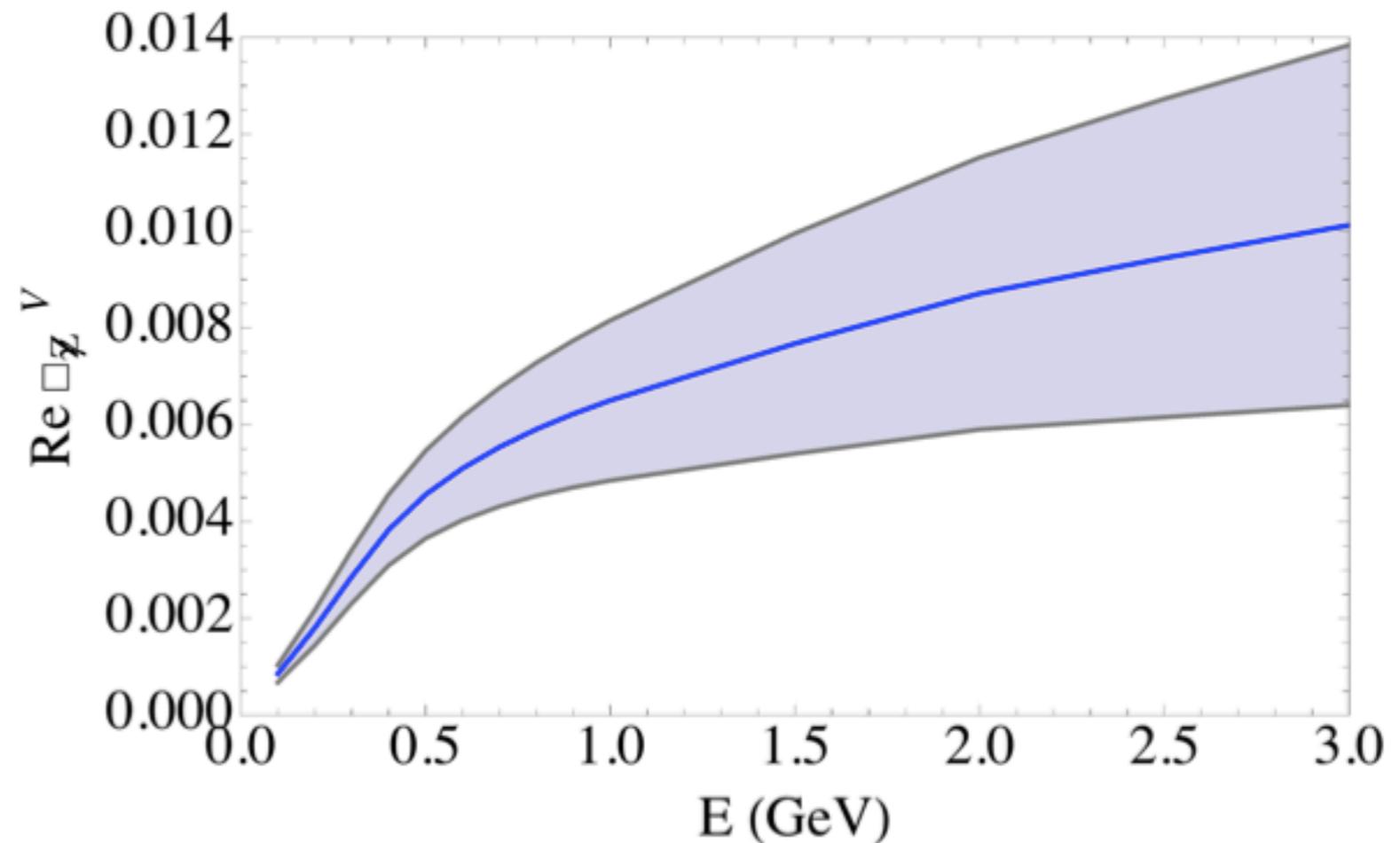
$Q^2 = 0.34 \text{ GeV}^2, E = 0.69 \text{ GeV}$

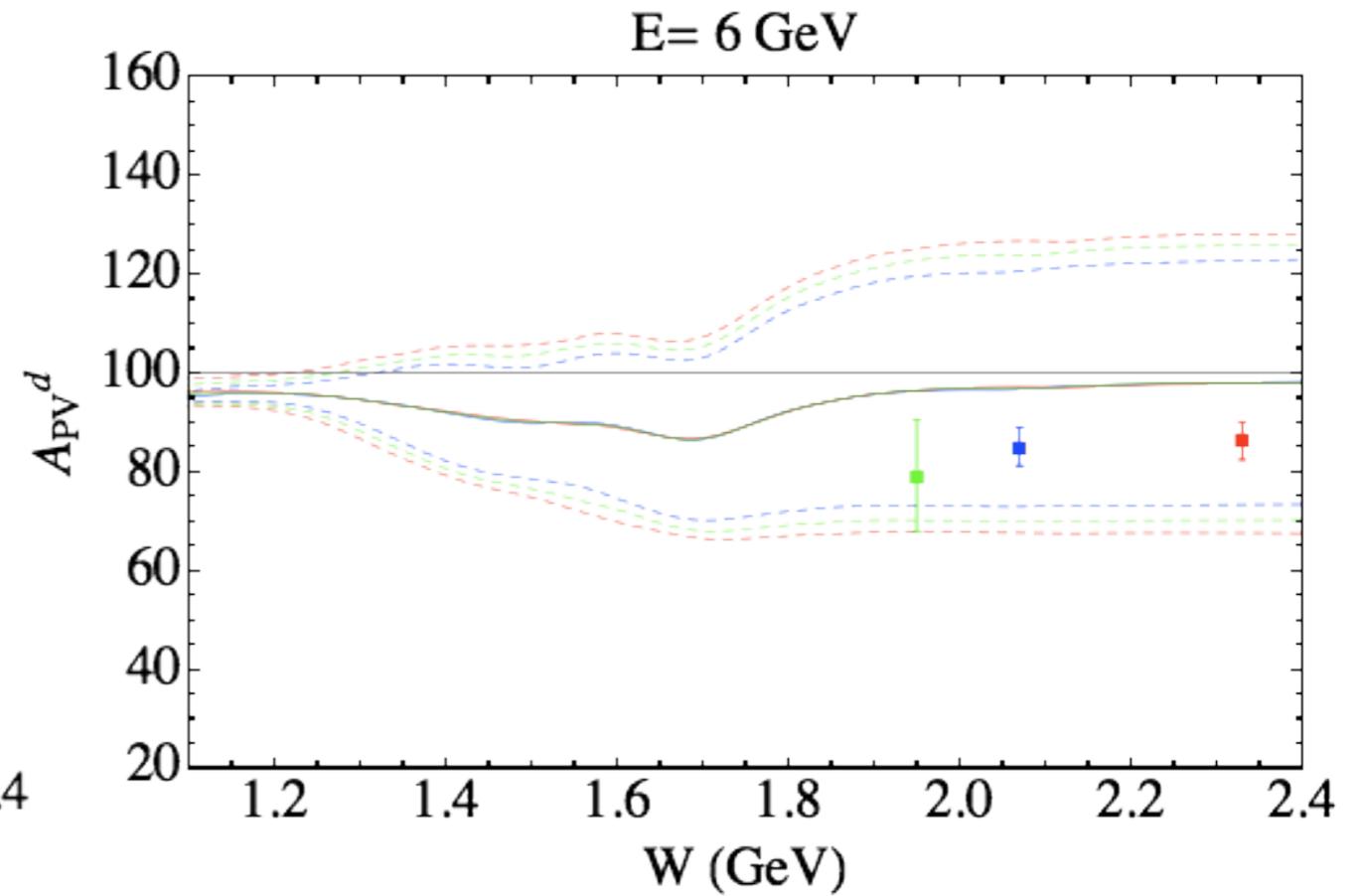
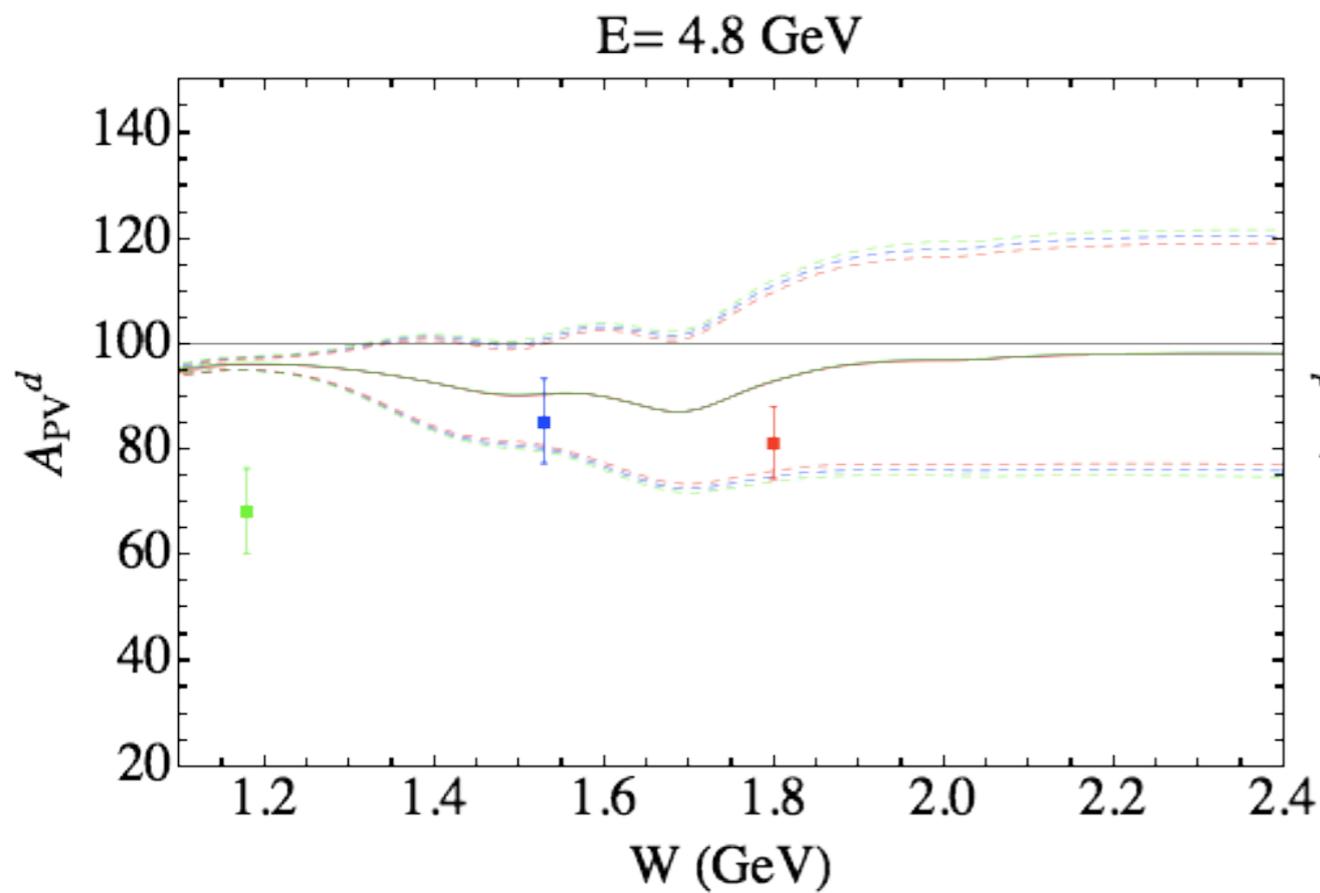
$Q^2 = 2.5 \text{ GeV}^2, E = 6 \text{ GeV}$



Potential impact of
constraints from
deuteron PV inelastic
asymmetries

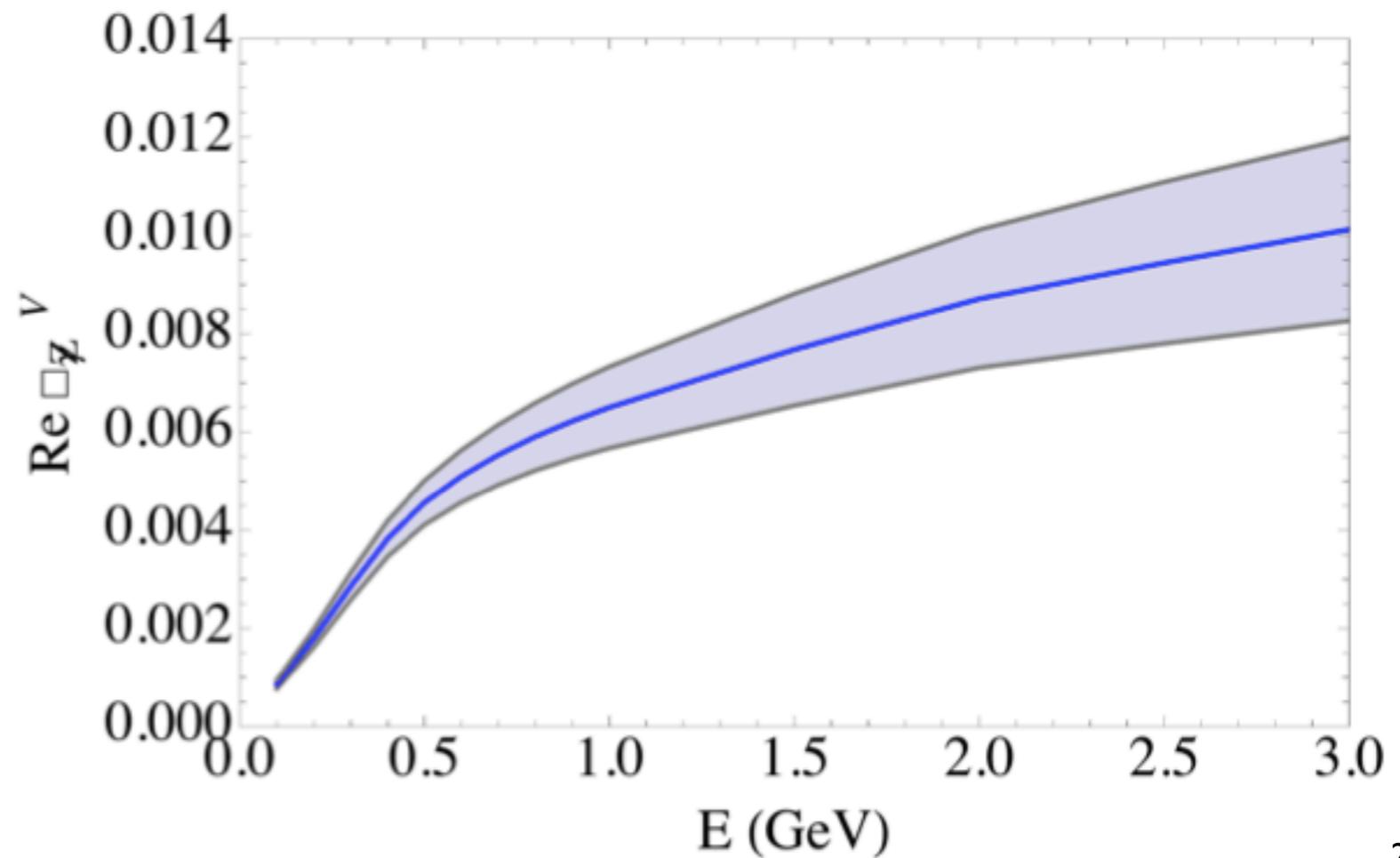
100% uncertainty on
continuum background



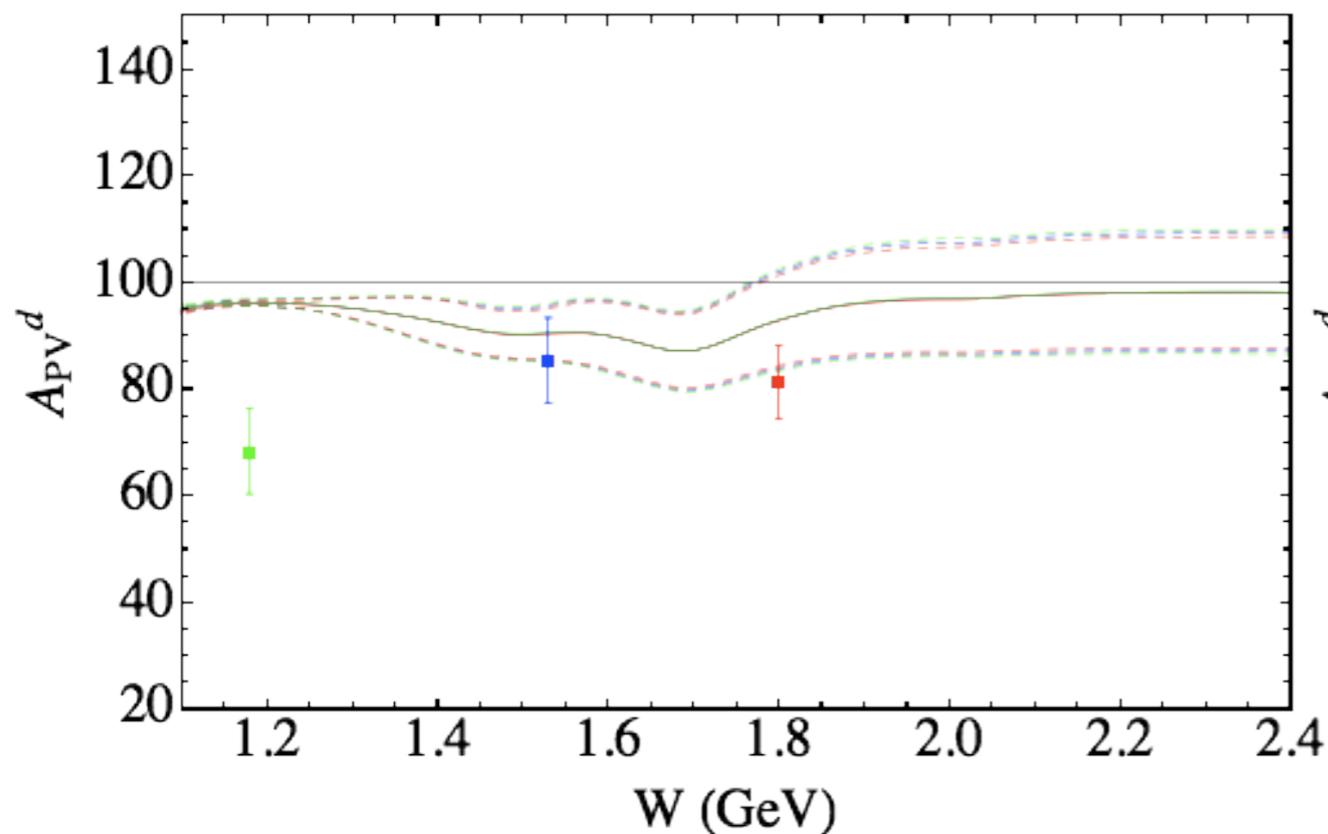


Potential impact of
constraints from
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asymmetries

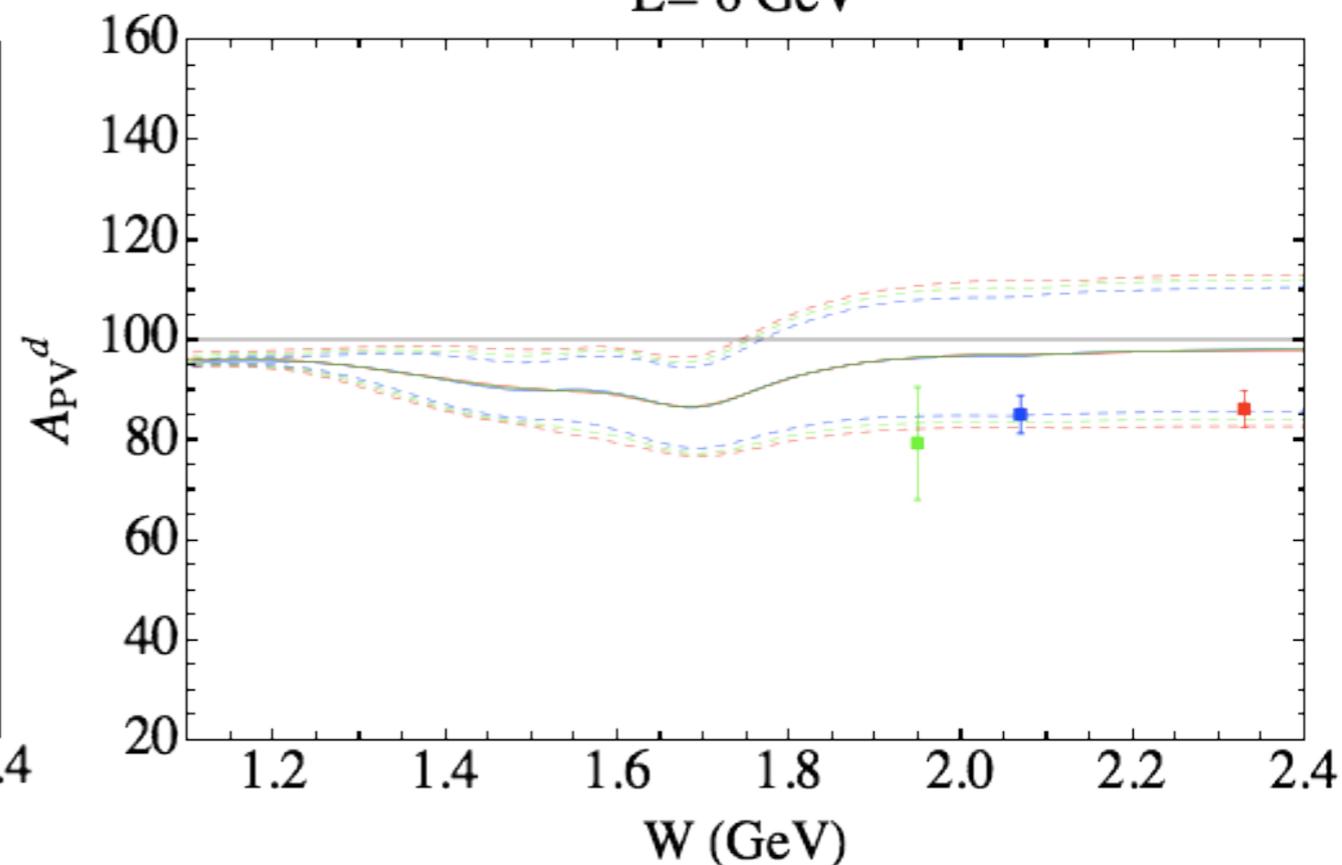
50% uncertainty on
continuum background



E= 4.8 GeV

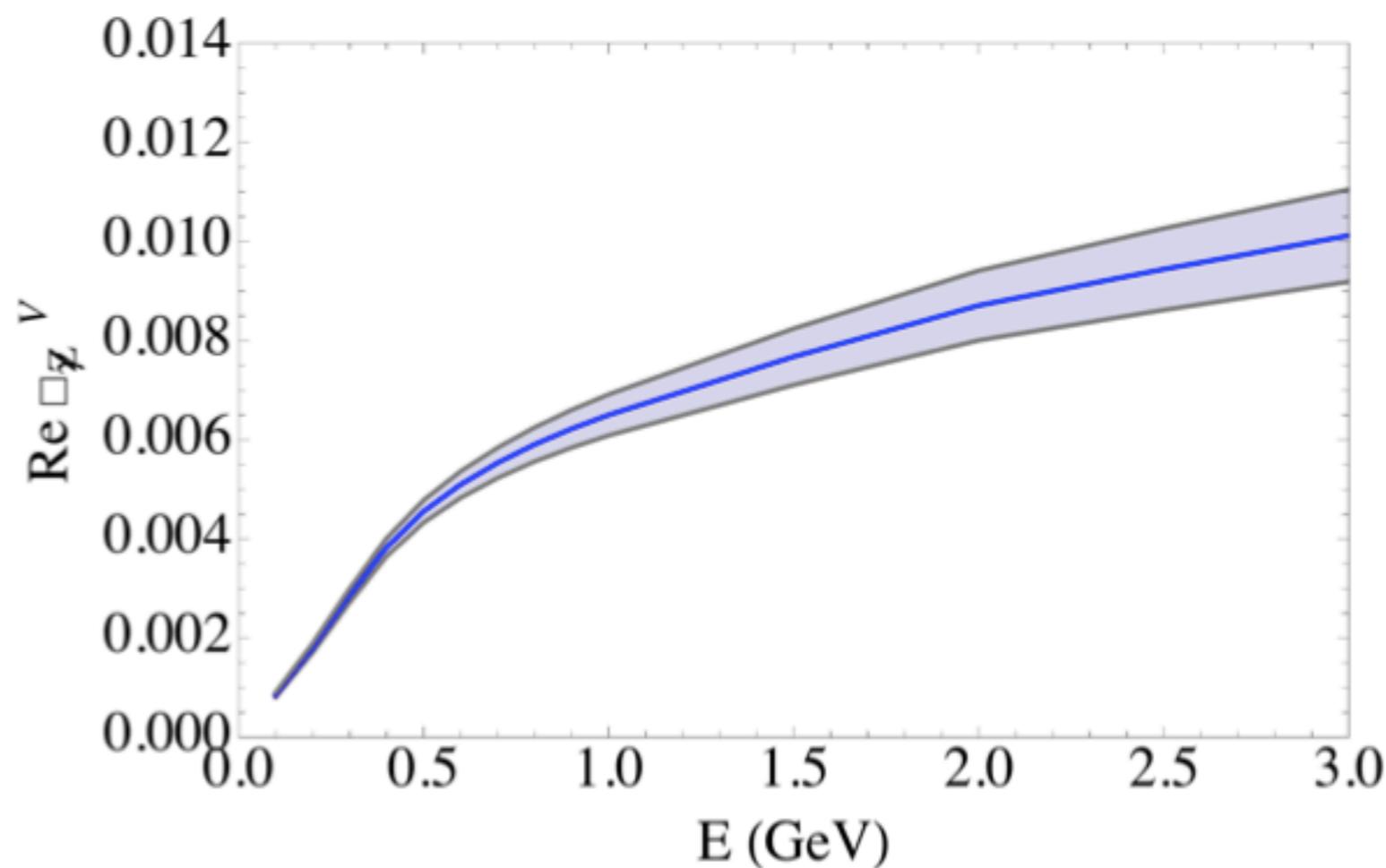


E= 6 GeV



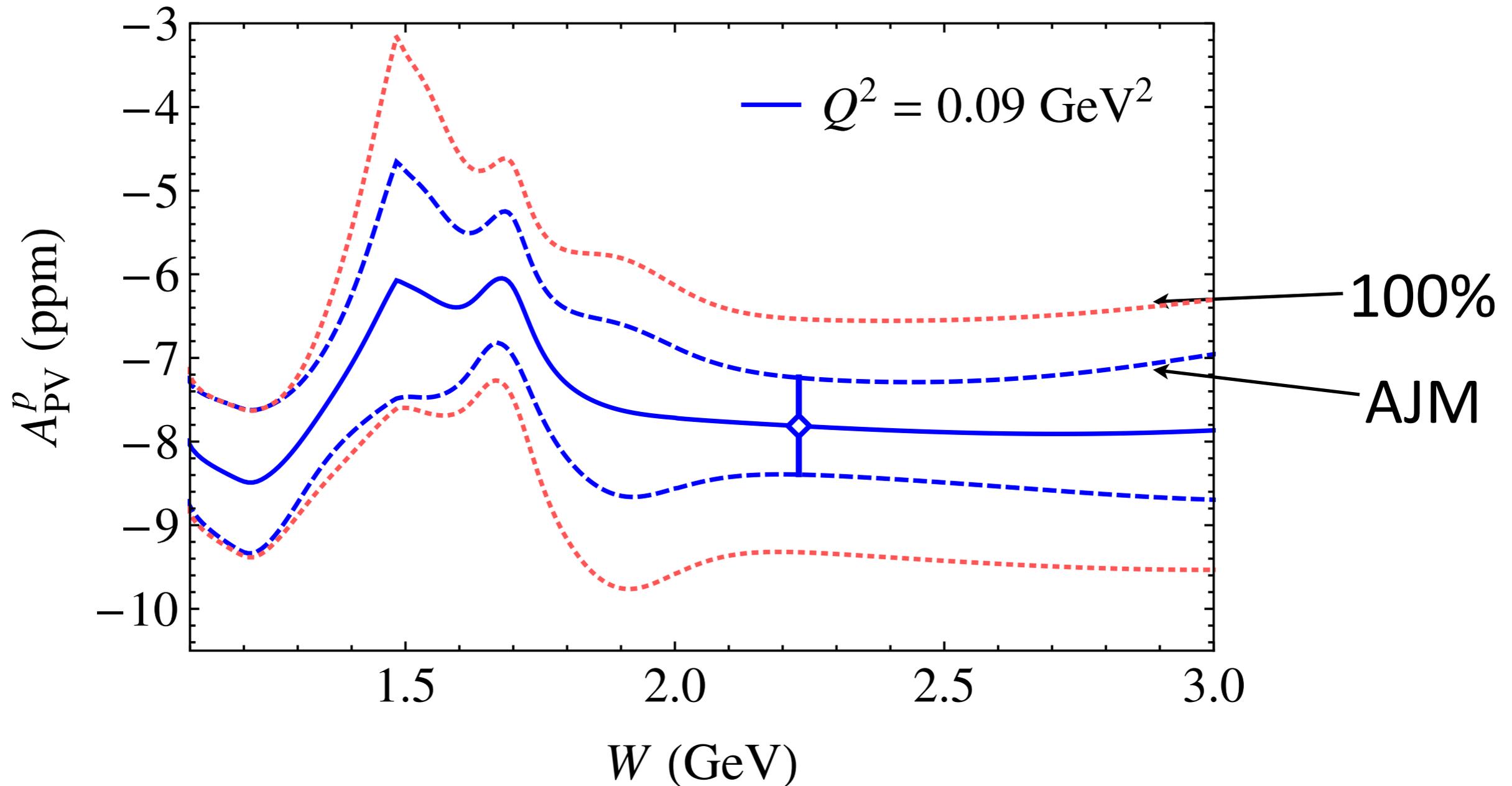
Potential impact of
constraints from
deuteron PV inelastic
asymmetries

25% uncertainty on
continuum background



Parity-violating inelastic asymmetries

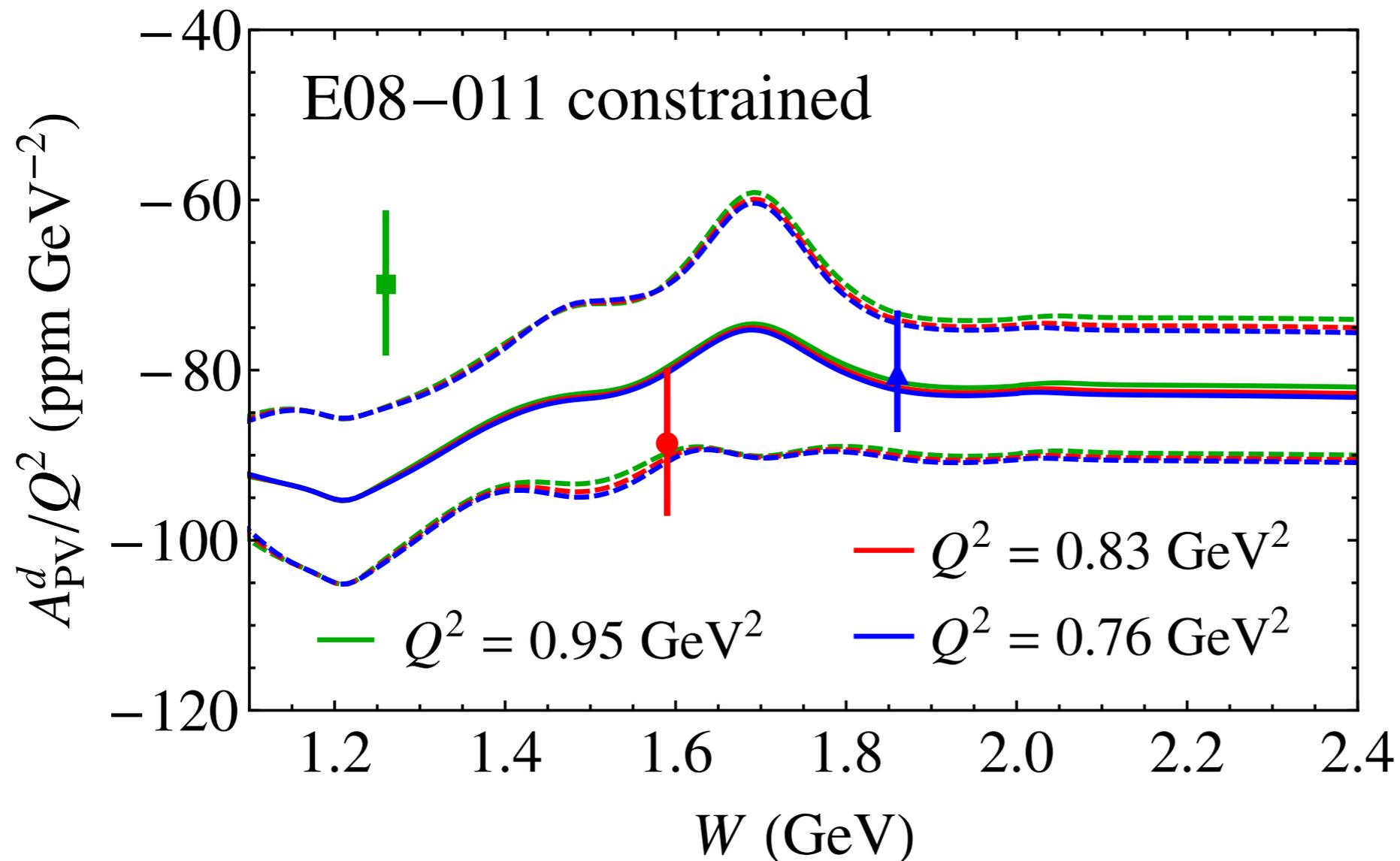
- Expected inelastic asymmetry data from Qweak



→ AJM model uncertainties compared with 100% on continuum contribution

Hall et al. (2013)

Constraints from PV inelastic asymmetries in the resonance region

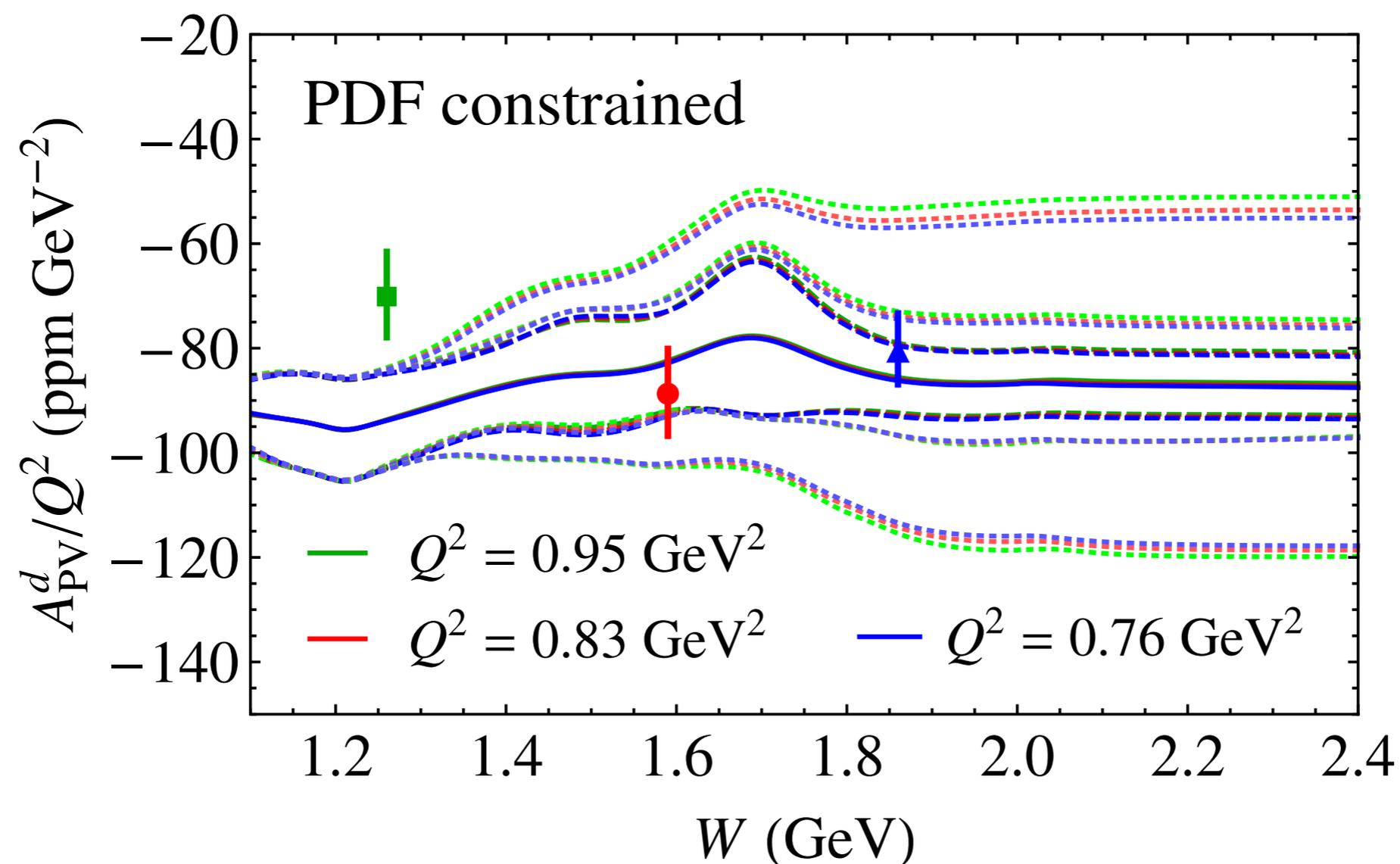


Wang et al. PRL 111, 082501 (2013)

AJM model asymmetries and uncertainties for PV
deuteron asymmetry constrained by fit to E08-011 data

Hall et al. (2013)

Predictions for PV deuteron asymmetry in DIS kinematics



Prediction: *Hall et al. (2013)*

$$A_{PV} = -92.4 \pm 6.8 \text{ ppm}$$

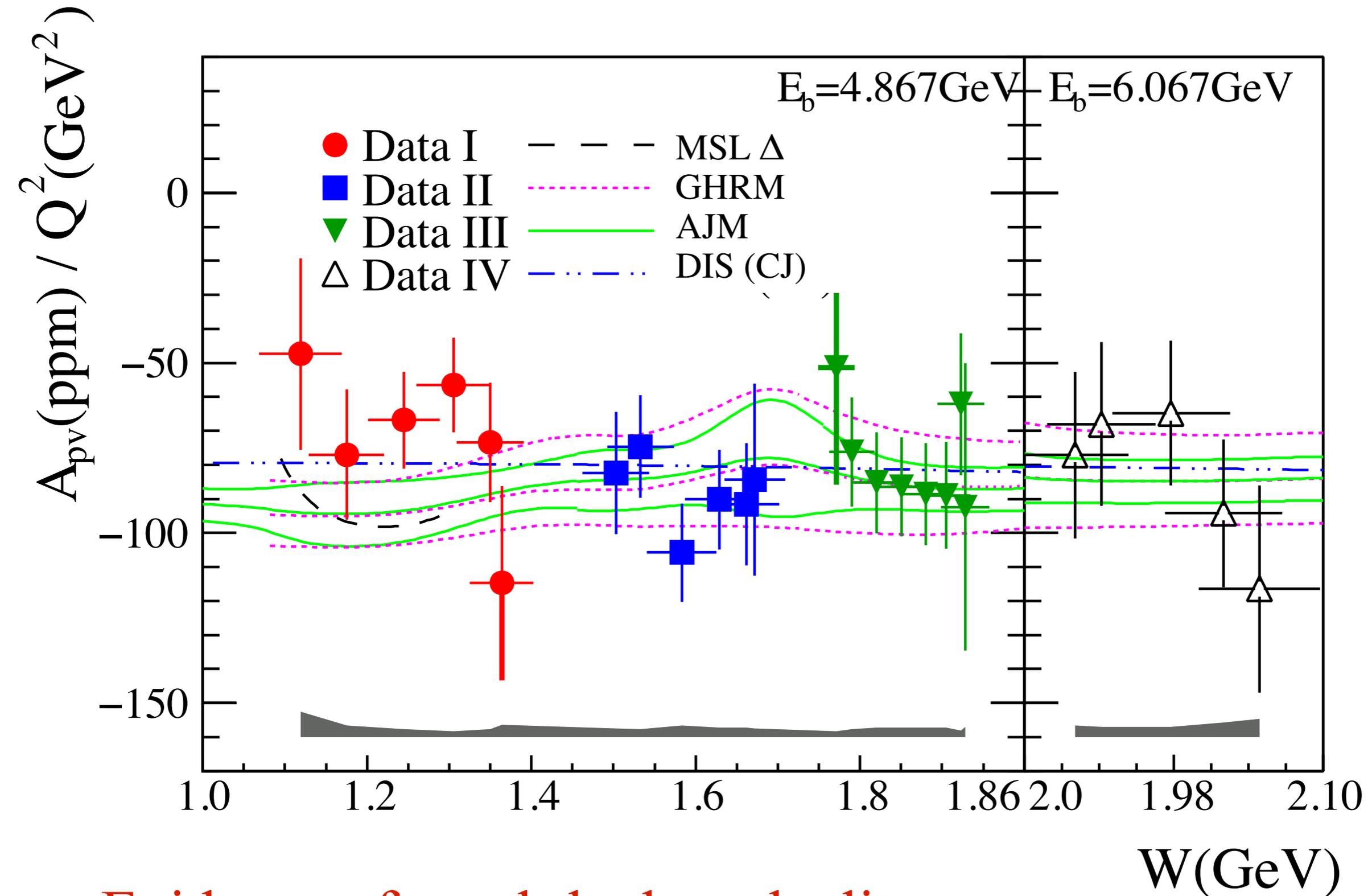
$$A_{PV} = -157.2 \pm 12.2 \text{ ppm}$$

E08-011: *Wang et al. Nature 506, 67 (2014)*

$$A_{PV} = -91.1 \pm 4.3 \text{ ppm}$$

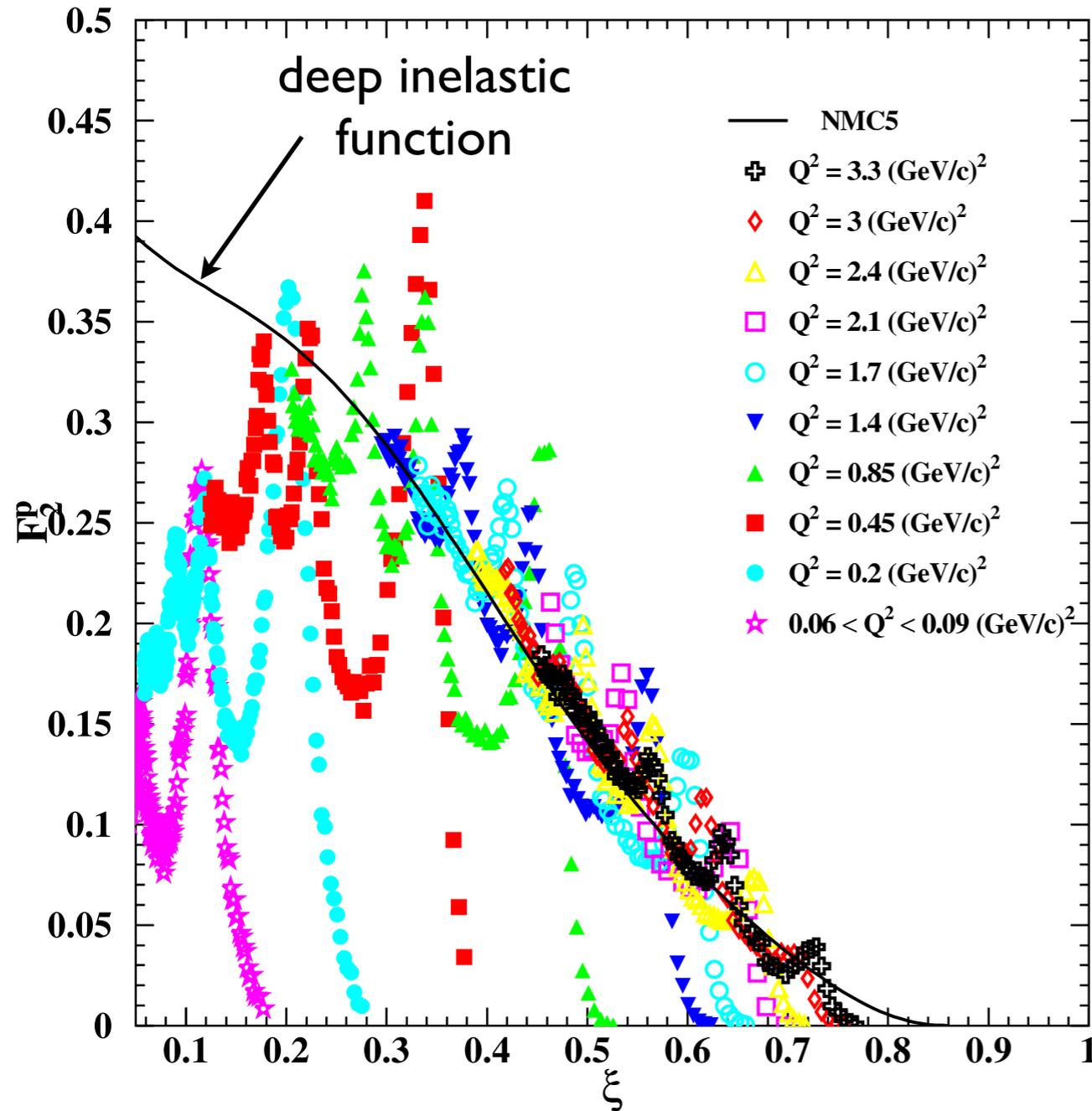
$$A_{PV} = -160.8 \pm 7.1 \text{ ppm}$$

PV deuteron asymmetry in DIS kinematics



Evidence of quark-hadron duality
at the 10-15% level

Duality in electron-nucleon scattering



Niculescu et al., PRL 85, 1182 (2000)
Melnitchouk, Ent, Keppel, PRep. 406, 127 (2005)

Empirical observation

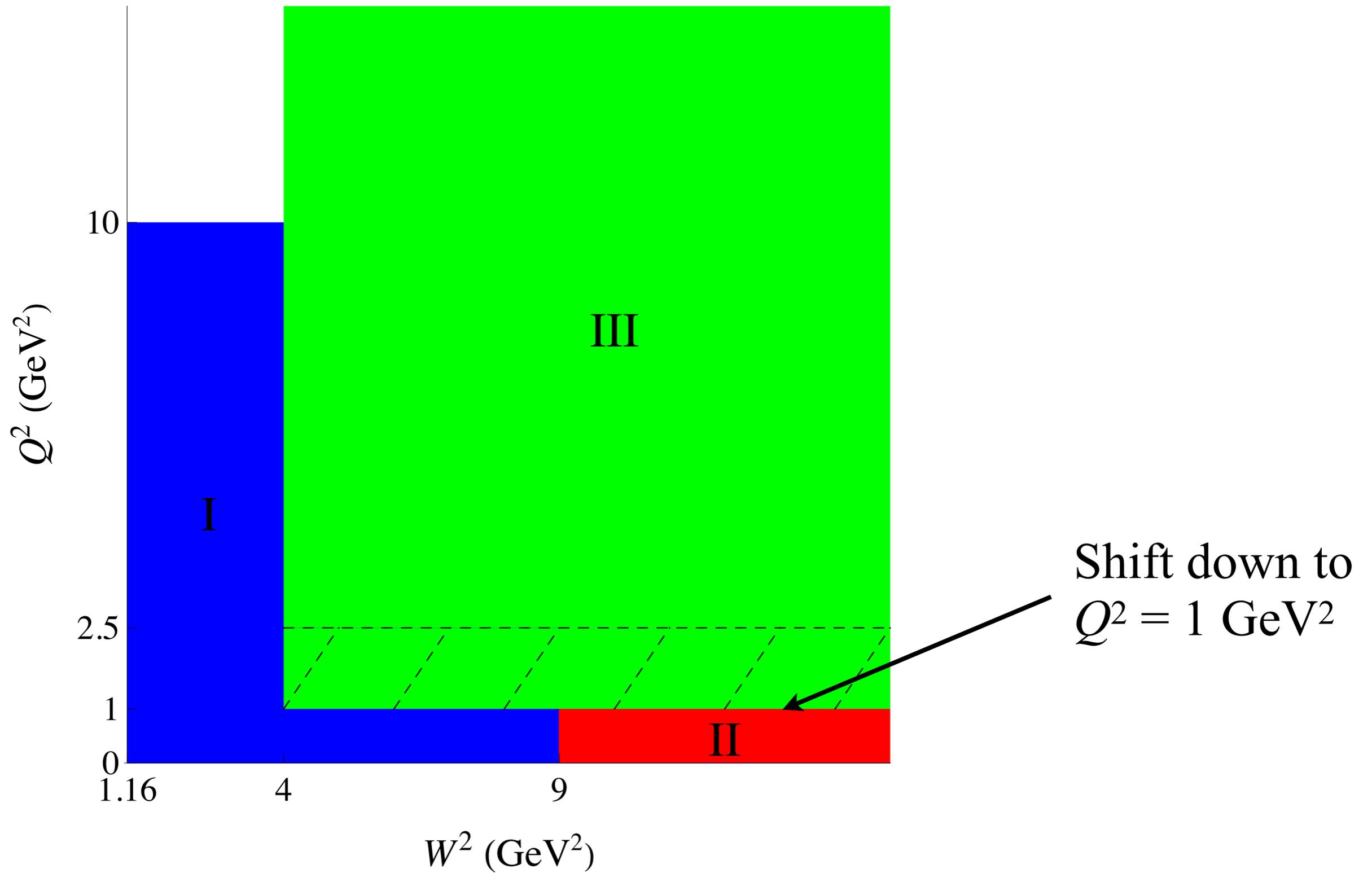
average over
 (strongly Q^2 -dependent)
 resonances
 $\approx Q^2$ independent
 scaling function

“Nachtmann” scaling variable

$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2 x^2 / Q^2}}$$

Separates higher twist (HT) effects
 from target mass corrections to
 leading twist (LT)

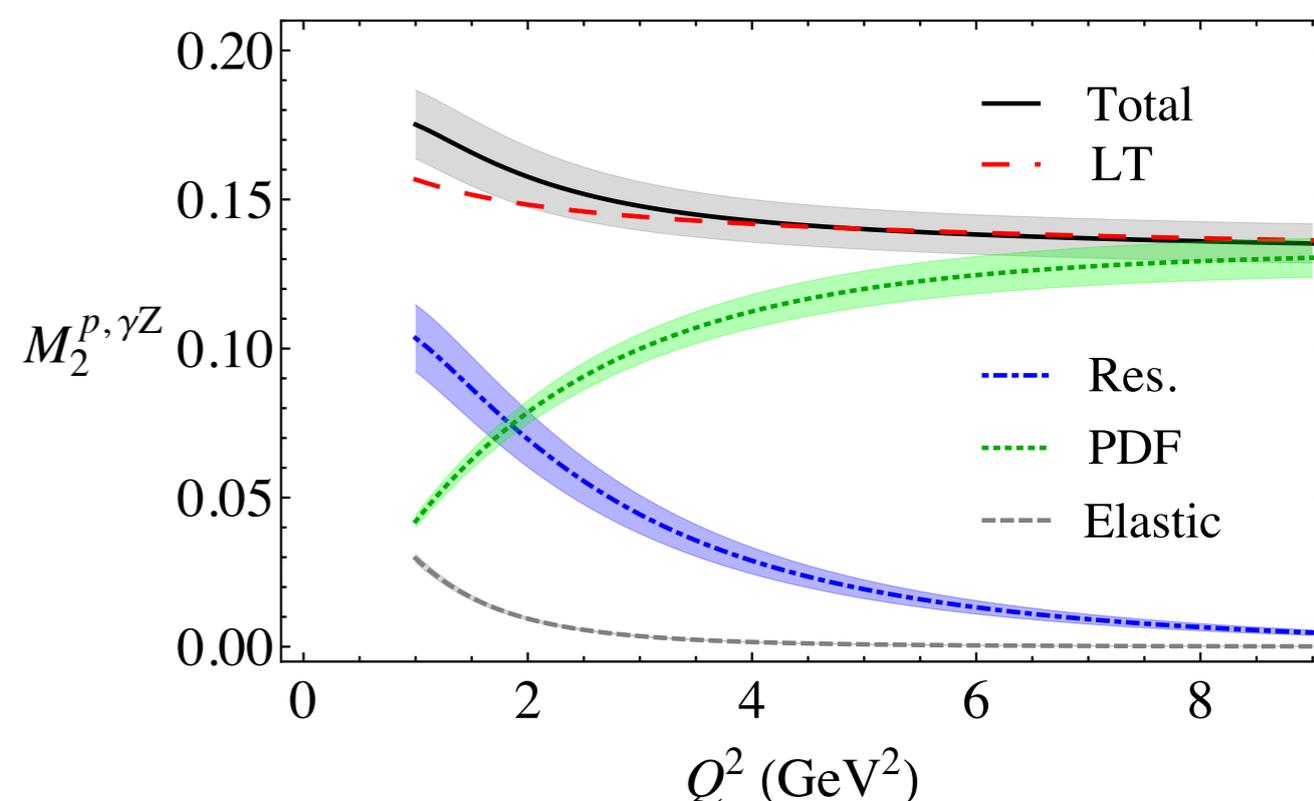
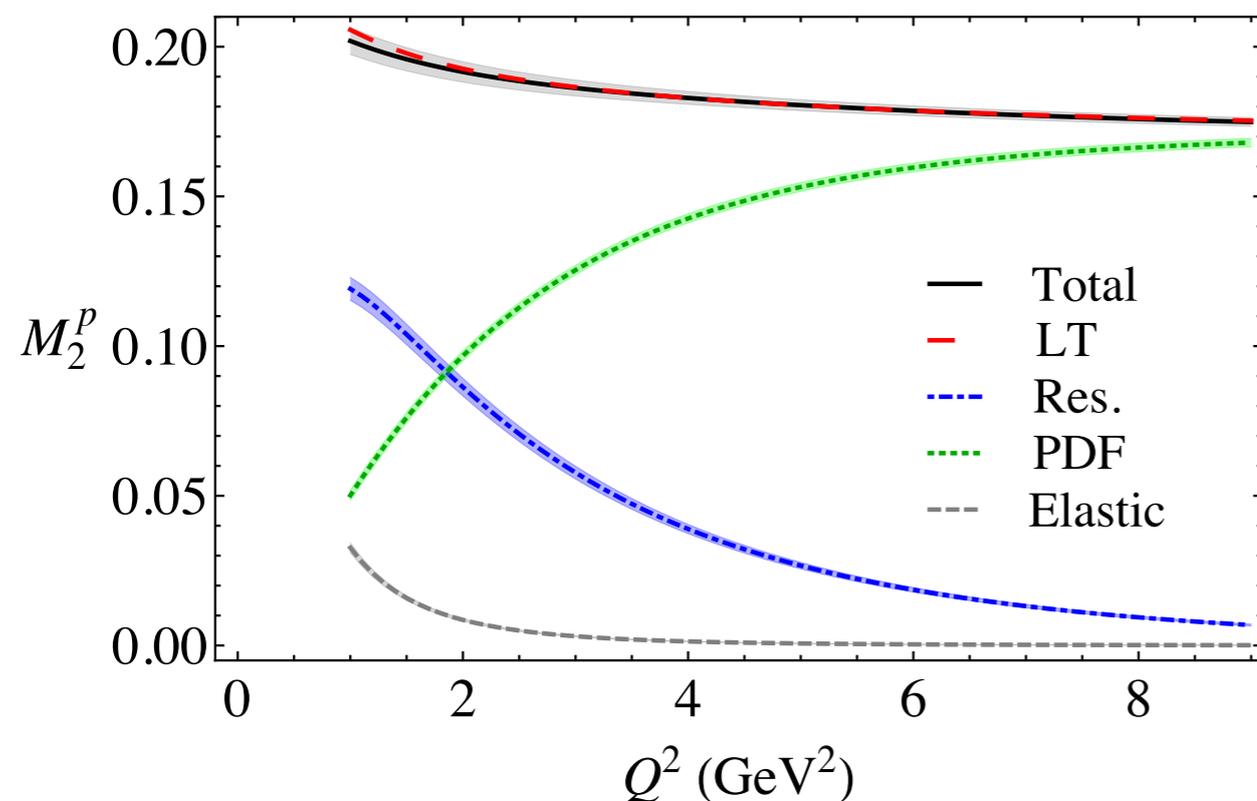
Using Quark-Hadron Duality in Resonance Region



■ Estimate higher twist corrections using AJM model

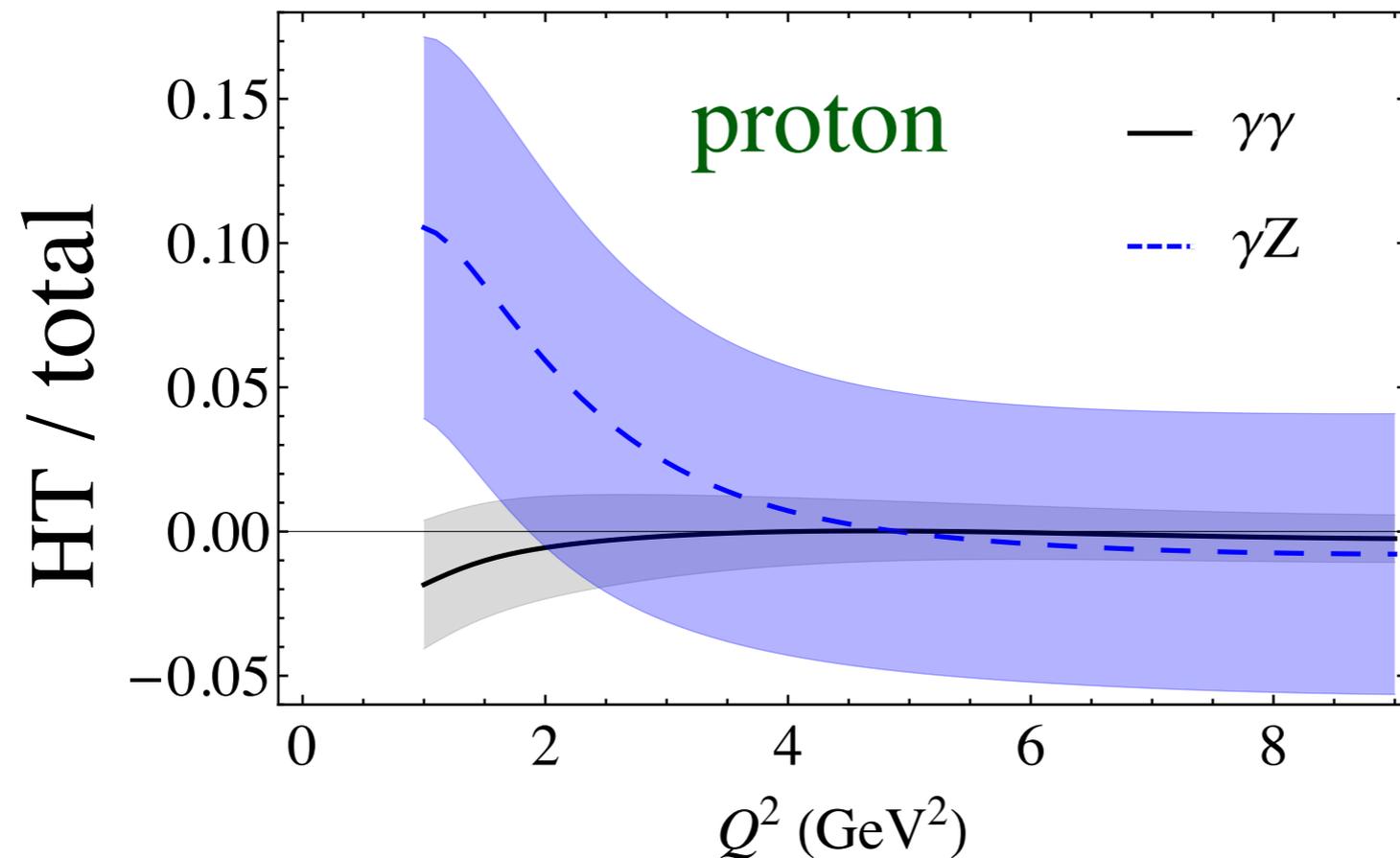
→ define HT as difference between *total* structure function and leading twist contribution

e.g. for F_2^p moment



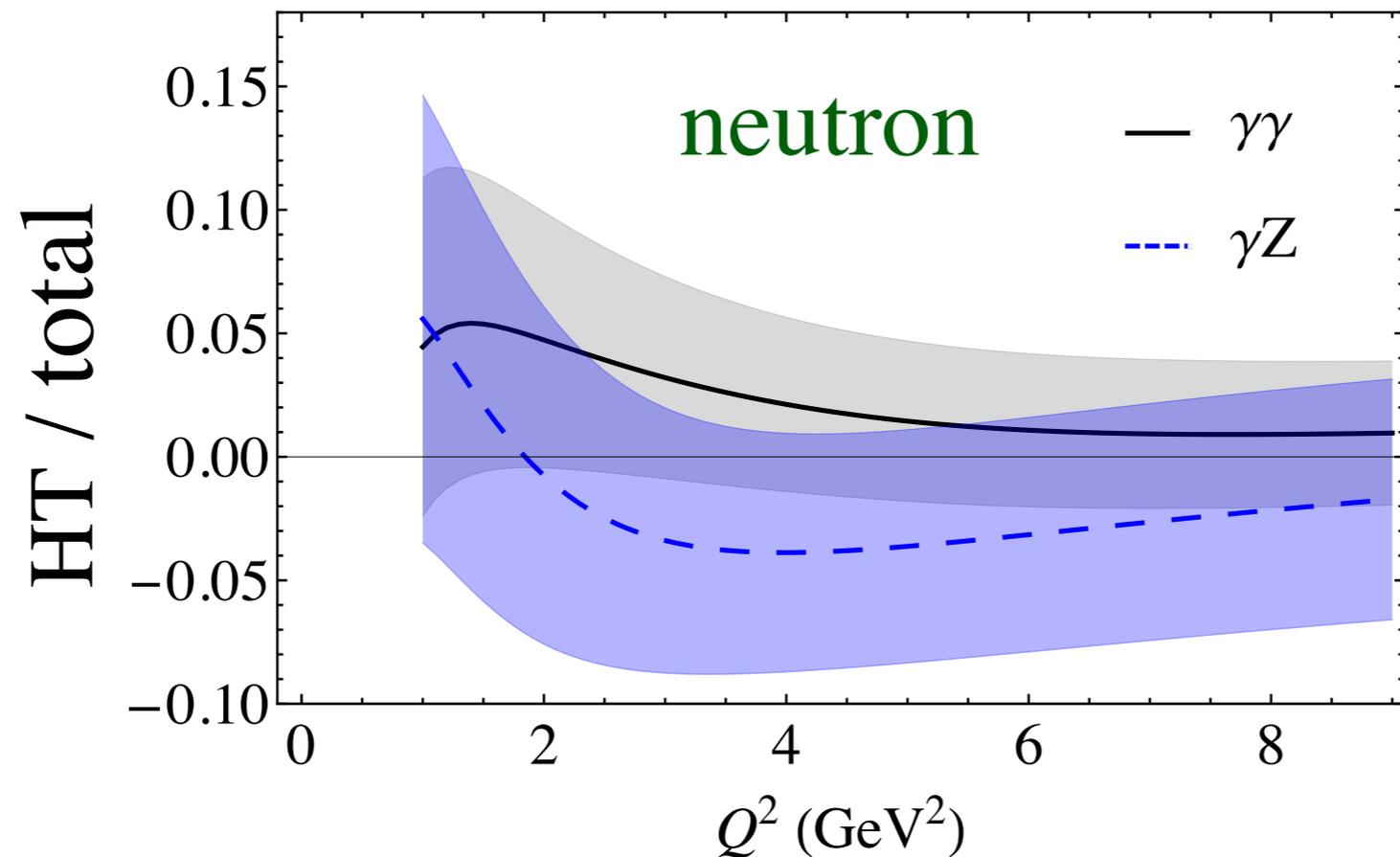
→ total moment is sum of resonance (low W), “PDF” (high W), and elastic contributions

- Estimate higher twist corrections using AJM model
 - define HT as difference between *total* structure function and leading twist contribution

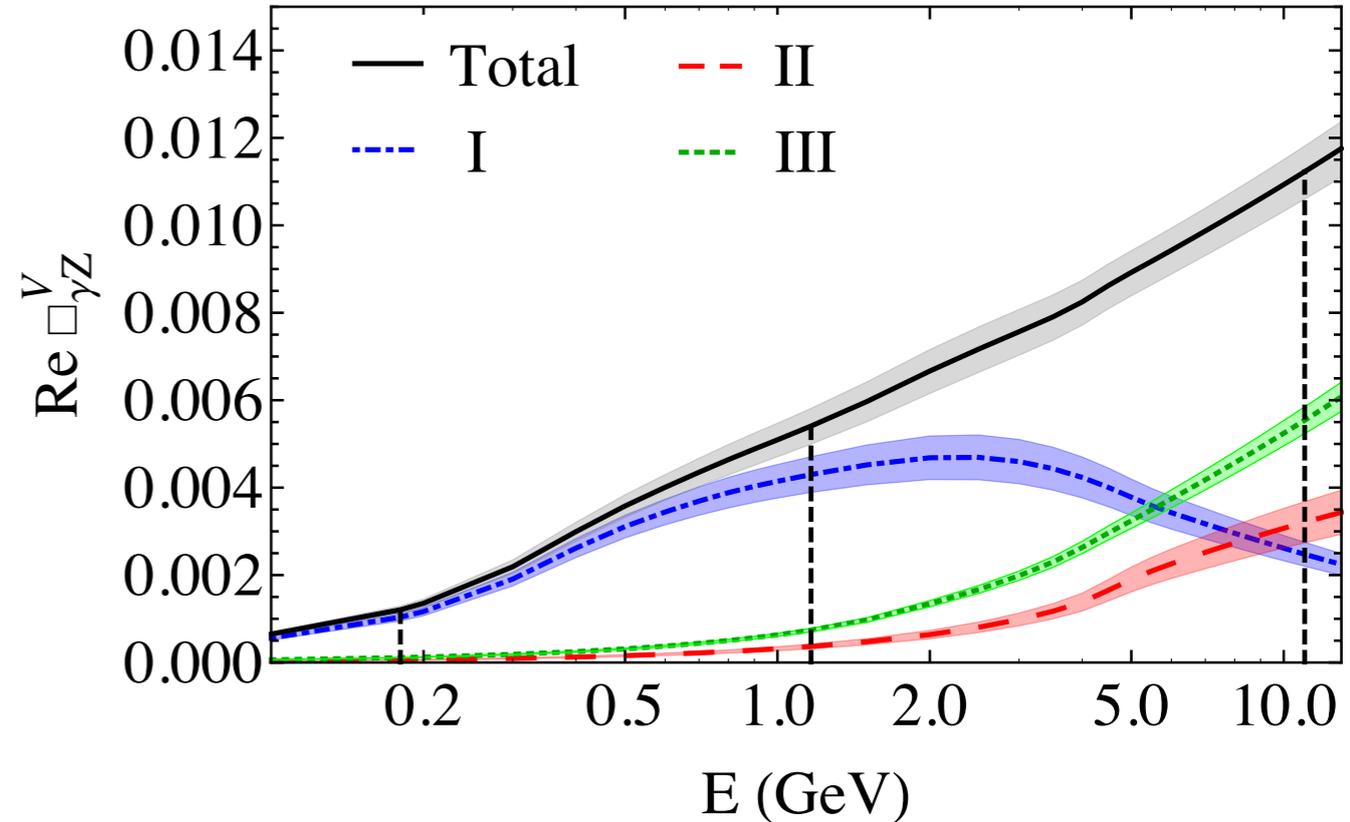
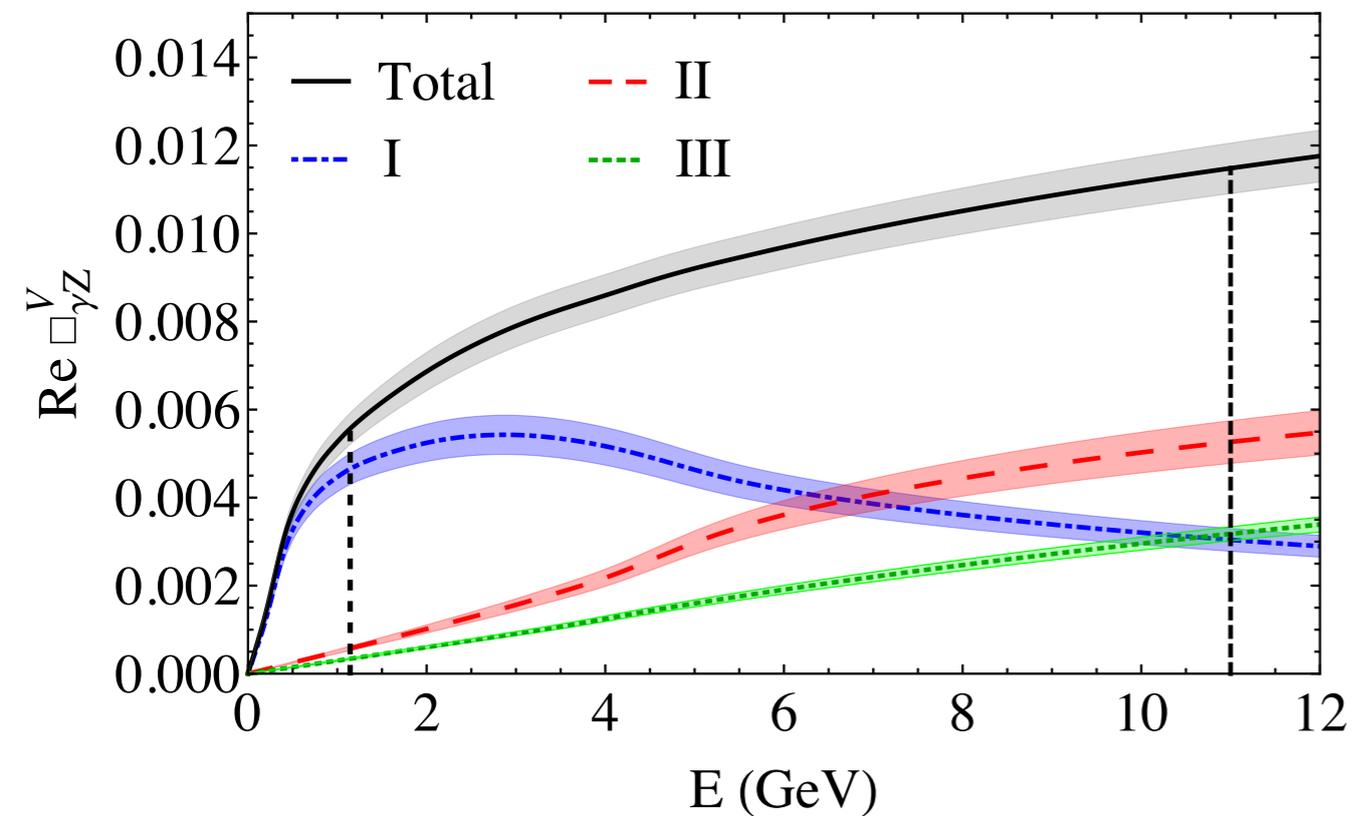


- HT in γZ for proton larger than in $\gamma\gamma$ but still $\sim 10\%$ at $Q^2 \sim 1 \text{ GeV}^2$

- Estimate higher twist corrections using AJM model
 - define HT as difference between *total* structure function and leading twist contribution



- for neutron HTs are comparable, within larger uncertainties



Match at $Q^2 = 2.5 \text{ GeV}^2$

(rel to $Q_W^p = 0.0713$)

Using duality down
to $Q^2 = 1 \text{ GeV}^2$

Region	$\Re \square_{\gamma Z}^V (\times 10^{-3})$	
	Q_{weak}	MOLLER
I	4.64 ± 0.35	3.04 ± 0.26
II	0.59 ± 0.05	5.26 ± 0.49
III	0.35 ± 0.02	3.18 ± 0.16
total	5.57 ± 0.36	11.5 ± 0.6

Region	$\Re \square_{\gamma Z}^V (\times 10^{-3})$		
	Q_{weak} ($E = 1.165 \text{ GeV}$)	MOLLER ($E = 11 \text{ GeV}$)	MESA ($E = 0.18 \text{ GeV}$)
I	4.3 ± 0.4	2.5 ± 0.3	1.0 ± 0.1
II	0.4 ± 0.05	3.2 ± 0.5	0.06 ± 0.01
III	0.7 ± 0.04	5.5 ± 0.3	0.1 ± 0.01
Total	5.4 ± 0.4	11.2 ± 0.7	1.2 ± 0.1

A few comments on Thomson (atomic) versus scattering limits

Thompson limit ($Q^2 \ll m_e^2$):

$$Q_W^p = (\rho + \Delta_e) (1 - 4\kappa^{\text{PT}}(0)\hat{s}^2 + \Delta'_e + \Delta_W) \\ + \square_{WW} + \square_{ZZ} + \square_{\gamma Z}(0)$$

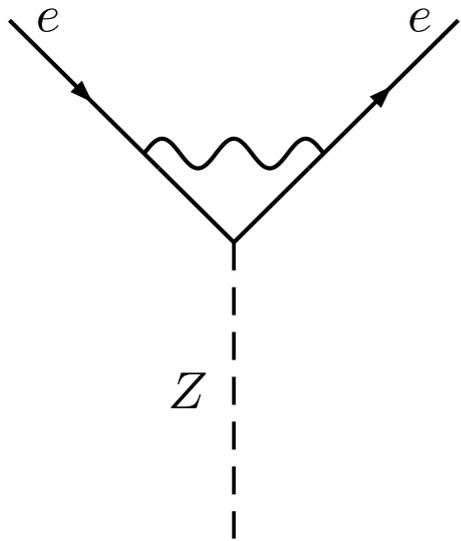
Scattering ($Q^2 \gg m_e^2$):

$$\mathcal{A}_{ep} = \mathcal{A}_0 [Q_W^p(ep) + Q^2 B(Q^2, E)], \quad \mathcal{A}_0 \equiv -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}}$$

$$Q_W^p(ep) = \lim_{E \rightarrow 0} \lim_{Q^2 \rightarrow 0} \frac{\mathcal{A}_{ep}}{\mathcal{A}_0}$$

Vertex correction: $Q_W^p = (\rho + \Delta_e) (1 - 4\kappa^{\text{PT}}(0)\hat{s}^2 + \Delta'_e + \Delta_W)$
 $+ \square_{WW} + \square_{ZZ} + \square_{\gamma Z}(0)$

Electromagnetic: $\Delta_e = -\frac{\alpha}{2\pi} \sim 0.00116$ (0.1% of 0.0708)



$$\gamma_\mu \rightarrow \gamma_\mu(1 + \delta f_1) + \frac{\delta f_2}{2m_e} \sigma_{\mu\nu}(-q^\nu)$$

$$\gamma_\mu \gamma_5 \rightarrow \gamma_\mu \gamma_5(1 + \delta g_1) + \frac{\delta g_2}{m_e} \gamma_5 \times (-q_\mu)$$

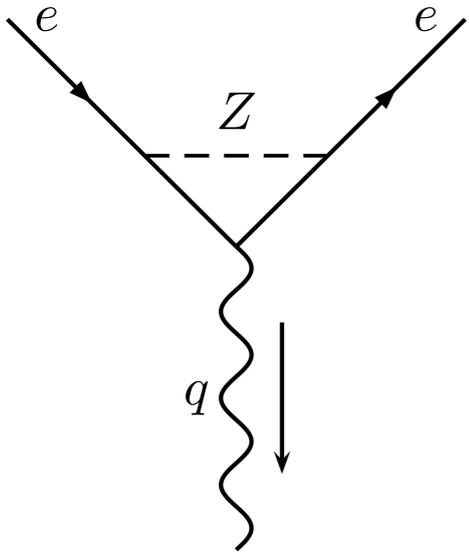
Thompson limit ($Q^2 \ll m_e^2$): $\delta f_1 = 0$; $\delta f_2 = \frac{\alpha}{2\pi}$; $\delta g_1 = -\frac{\alpha}{2\pi}$

Scattering ($Q^2 \gg m_e^2$): $\delta f_1 \sim \log \frac{Q^2}{m_e^2} \times \log \frac{\lambda^2}{m_e^2}$; $\delta f_2 \rightarrow 0$; $\delta g_1 \rightarrow \delta f_1$
 IR divergent
 (cancelled by soft photon emission)

Overall Zee vertex has same (divergent) factor as γee
 \rightarrow Cancels in A_{PV} (or include with standard rad. corr.).
 Therefore take $\Delta_e = 0$ for scattering.

Vertex correction: $Q_W^p = (\rho + \Delta_e) (1 - 4\kappa^{\text{PT}}(0)\hat{s}^2 + \Delta'_e + \Delta_W)$
 $+ \square_{WW} + \square_{ZZ} + \square_{\gamma Z}(0)$

Electroweak: $\Delta'_e = -0.0014$ (1.9% of 0.0708)



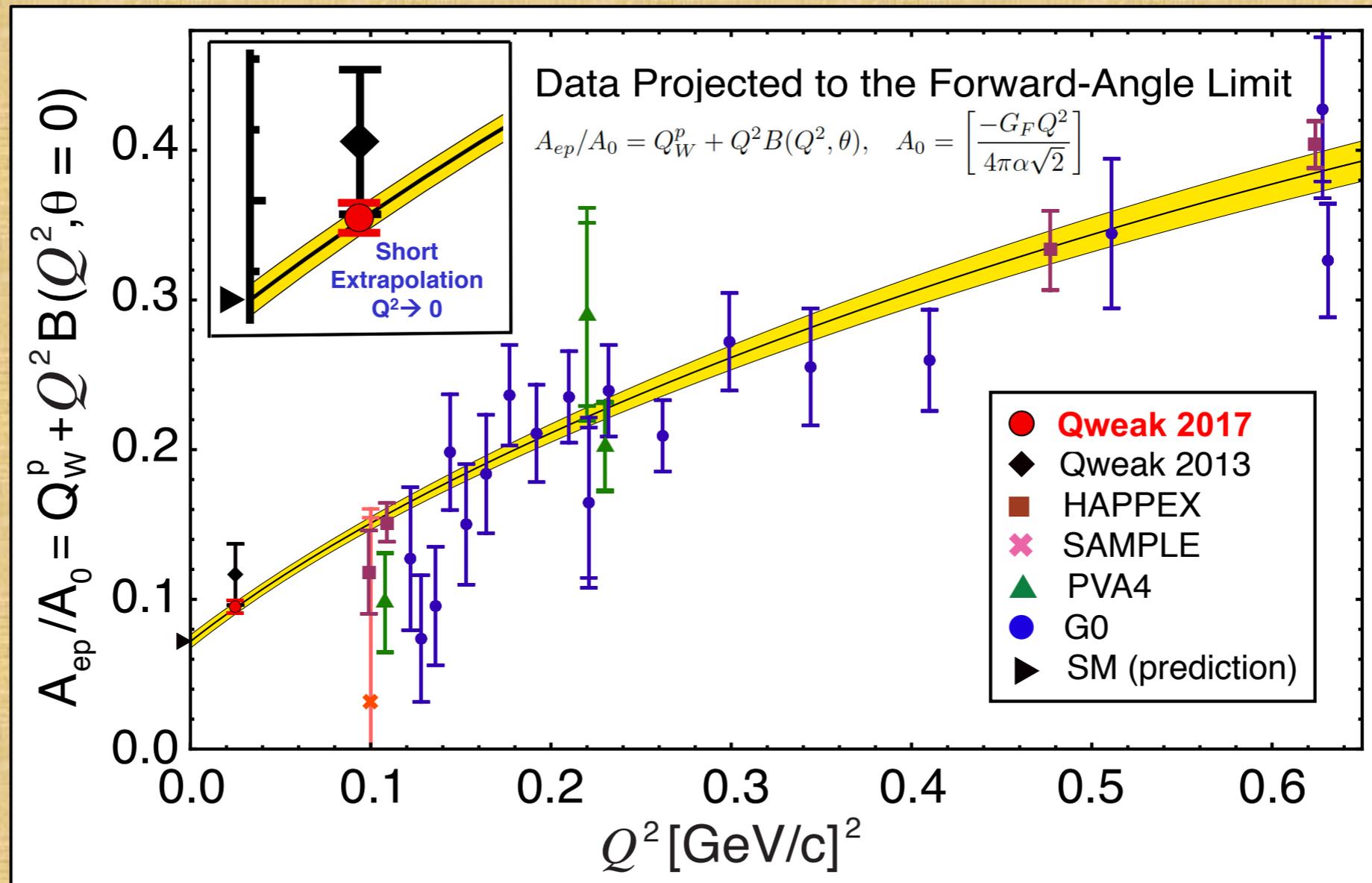
Thompson limit ($Q^2 \ll m_e^2$): $\Delta'_e(0) = -\frac{\alpha}{3\pi} (1 - 4\hat{s}^2) \left(\log \frac{M_z^2}{m_e^2} + \frac{1}{6} \right)$

Scattering ($Q^2 \gg m_e^2$): $\Delta'_e(Q^2) = -\frac{\alpha}{3\pi} (1 - 4\hat{s}^2) \left(\log \frac{M_z^2}{Q^2} + \frac{11}{6} \right)$

$$\Delta'_e(Q^2) = \Delta'_e(0) + \underbrace{\frac{\alpha}{3\pi} (1 - 4\hat{s}^2) \left(\log \frac{Q^2}{m_e^2} - \frac{5}{3} \right)}_{+0.0006 \text{ at } Q^2=0.025 \text{ GeV}^2}$$

Same size as electron contribution from running of $\sin^2 \theta_W$

Qweak Parity-Violating Asymmetry Extrapolated to $Q^2 = 0$

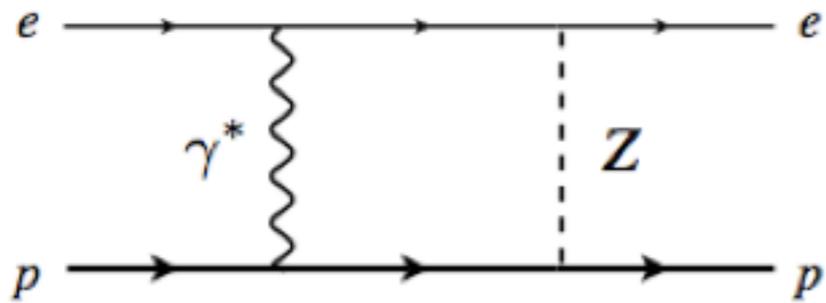


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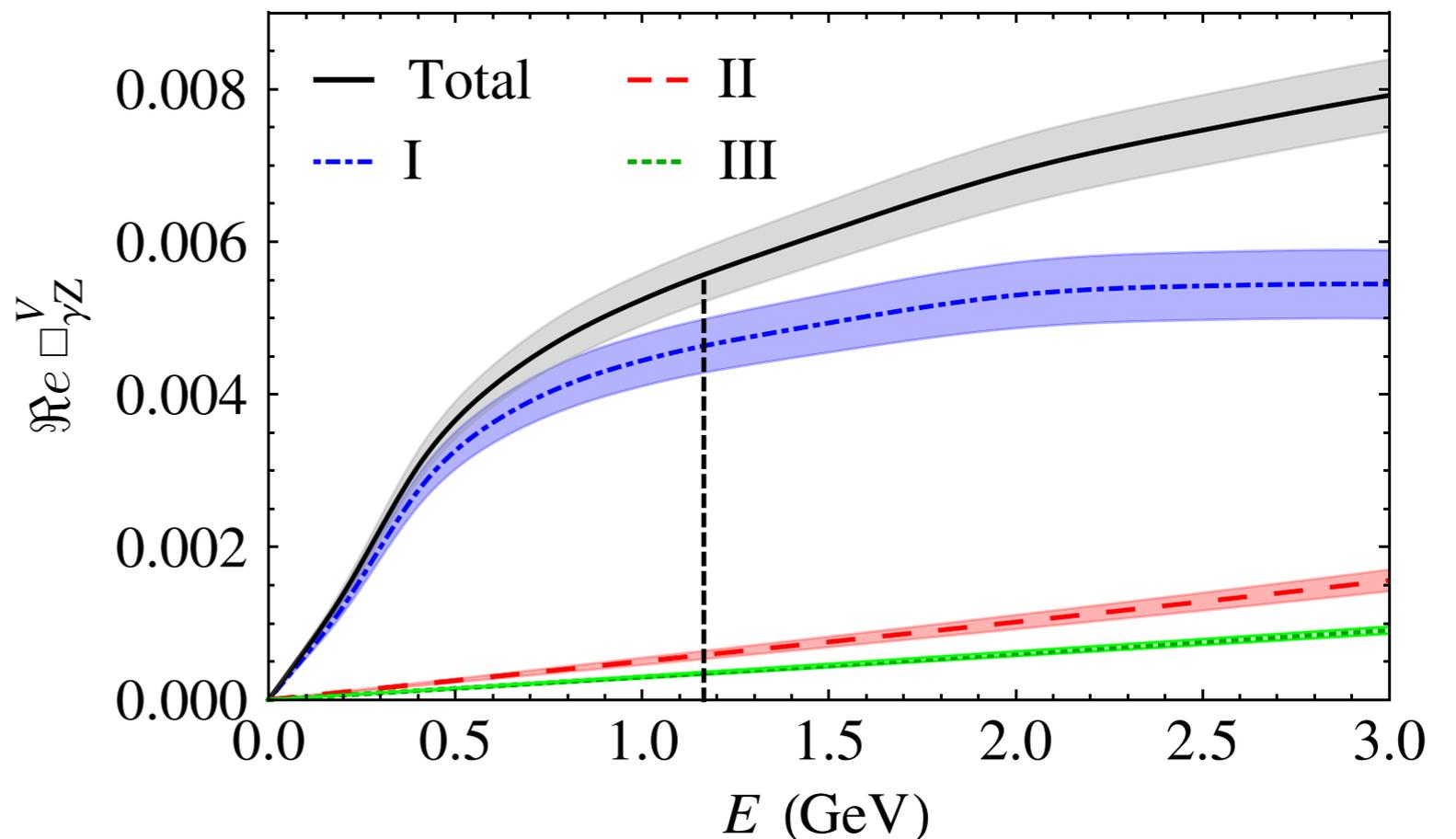
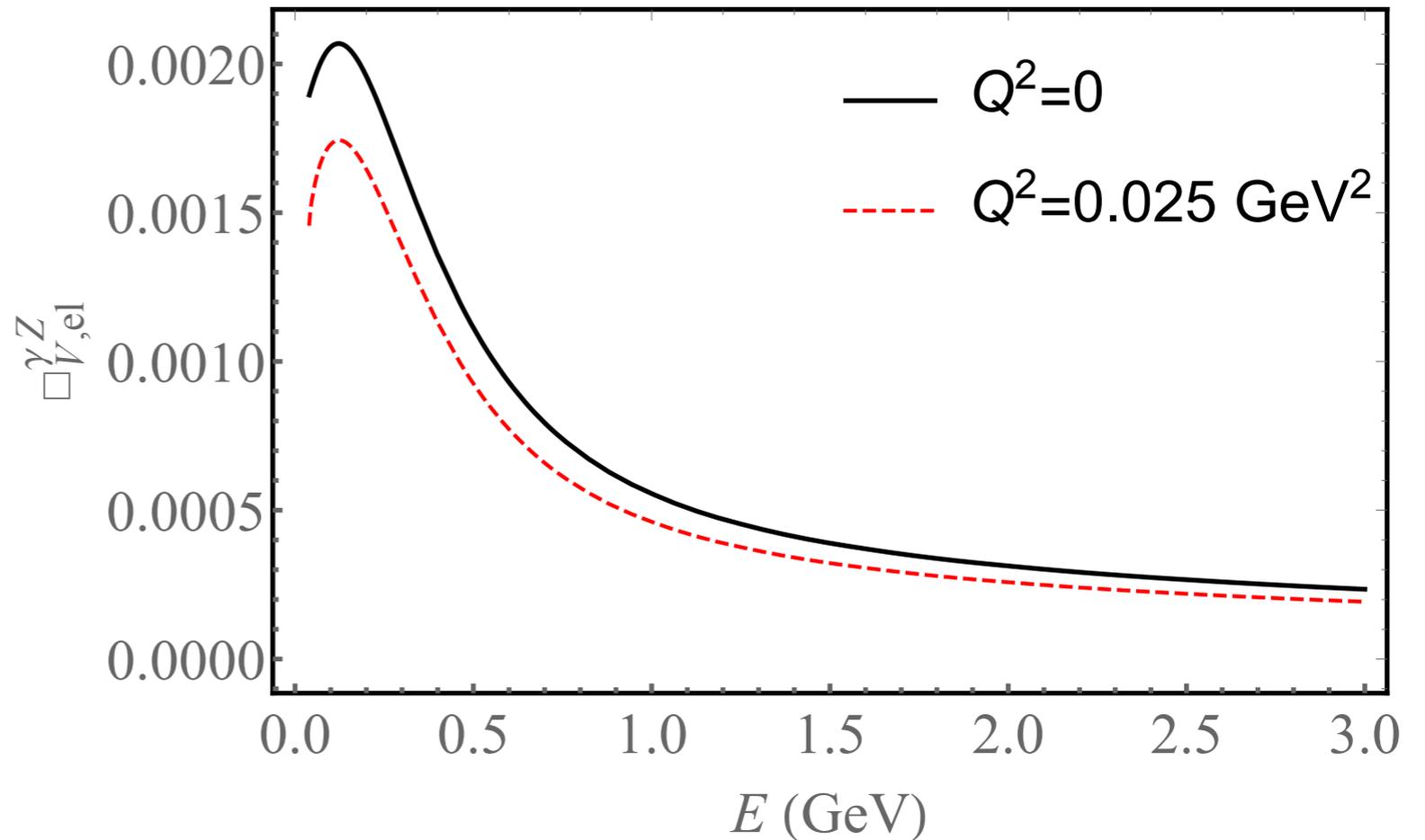
Assumption:
$$A_{PV} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} (Q_W^p + Q^2 B(Q^2, \theta_e))$$

Log terms most likely to affect extraction of additional parameters from fit, not extracted value of Q_W

Elastic part of Vector h correction



- Can be considered a “Coulomb distortion” effect
- Relatively insensitive to choice of nucleon form factors
- Equal to $\alpha\pi(1-4s^2)$ as $E \rightarrow 0$ independent of structure
- Does not cancel in A_{PV} ratio
- Excluded by Marciano-Sirlin in atomic systems (bound states)
- More important for P2 than Q_{weak}



Summary

- Dispersion approach significant improvement over old methods
- PDF region provides some constraints on model-dependence of isospin rotation
- Direct comparison with PV inelastic data in resonance and DIS regions
- $e-d$ PVDIS asymmetry strongly constrains the uncertainty
- checking Δ region for inelastic A_{PV} at Mainz or JLab would be useful
- quark-hadron duality approach allows further constraints on uncertainties