Comparison of Hydrodynamics and Kinetic Transport Theory for p+A and A+A Collisions

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with Kai Gallmeister, Harri Niemi , Dirk Rischke

Bormio 56th winter meeting, january 2018

Hydrodynamics & BAMPS

Initial state

specific transversal distribution

longitudinal boost invariance

Results and Outlook

What can we learn?

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Bundesministerium für Bildung und Forschung

HIC for FAIR Helmholtz International Center

CRC-TR 211 Strong-interaction matter under extreme conditions

BAMPS

Boltzmann **A**pproach to **M**ulti-**P**arton **S**cattering

- (3+1)D Boltzmann equation

Z.Xu, C.Greiner, PRC 71 (2005) 064901
Z.Xu, C.Greiner, PRC 76 (2007) 024911

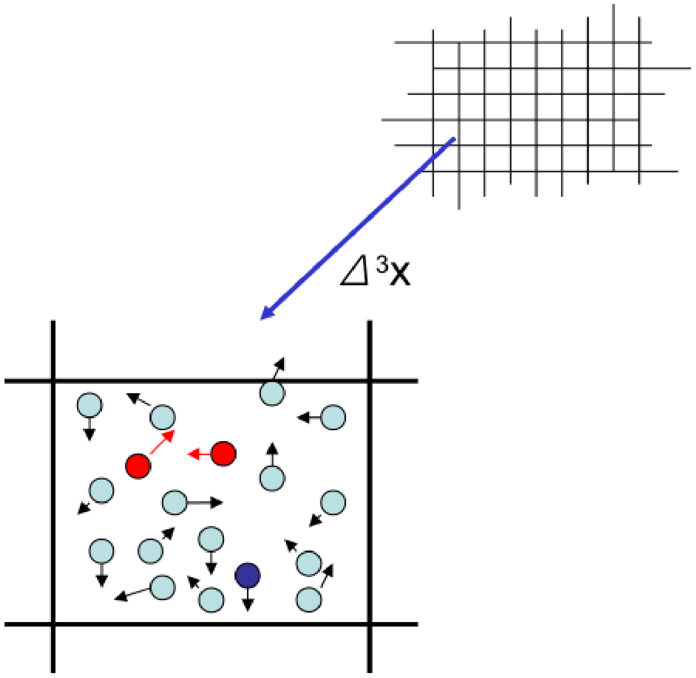
$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{E}\frac{\partial f}{\partial \vec{r}} = C_{2\leftrightarrow 2} + C_{2\leftrightarrow 3}$$

- Massless particles: partons / quarks & gluons
- Discretized space and time

$$\mathrm{P}_{2\to 2} = v_{\rm rel}\sigma_{2\to 2}\frac{\Delta t}{\Delta V}$$

$$\mathrm{P}_{2\to 3} = v_{\rm rel}\sigma_{2\to 3}\frac{\Delta t}{\Delta V}$$

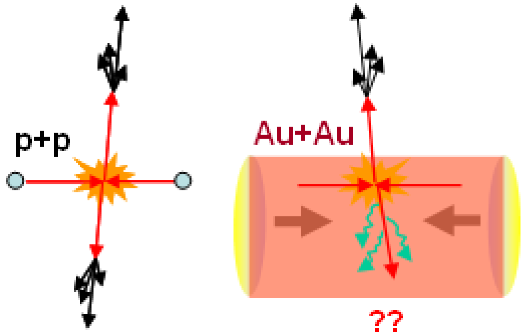
$$\mathrm{P}_{3\to 2} = \frac{I_{3\to 2}}{8E_1E_2E_3}\frac{\Delta t}{(\Delta V)^2}$$

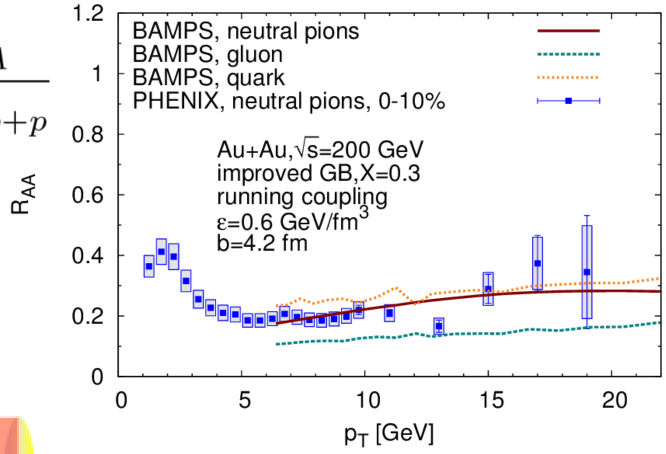


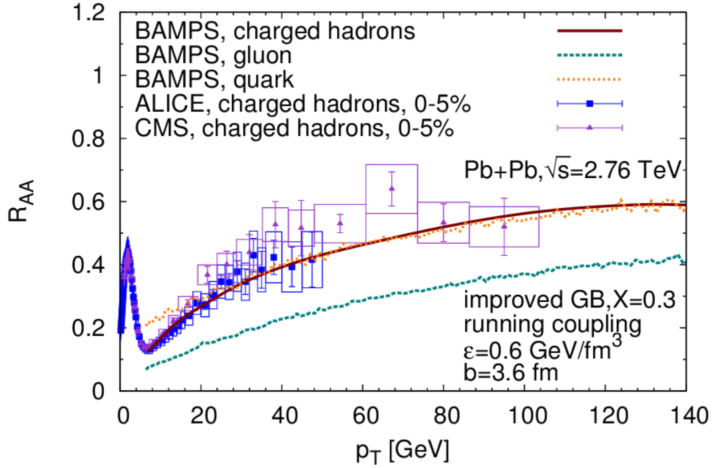
- Testparticle ansatz: $N_{\rm test}$

Nuclear modification factor R_{AA}

$$R_{AA} = \frac{dN/dp_T dy|_{A+A}}{N_{bin}\, dN/dp_T dy|_{p+p}}$$

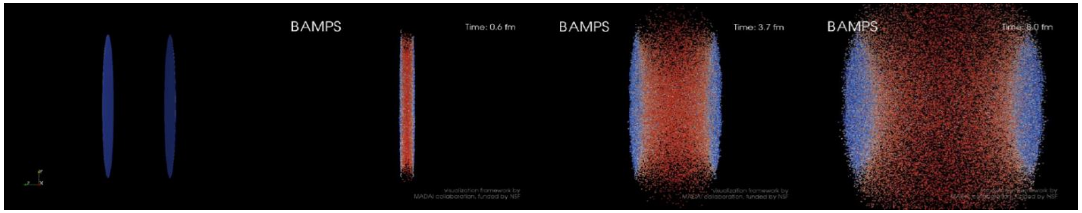






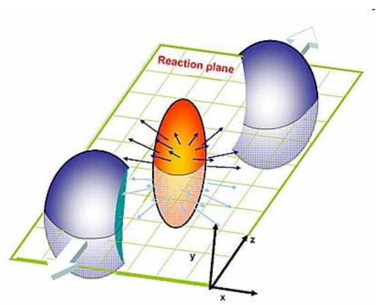
Phys. Rev. Lett. 114 (2015) 112301

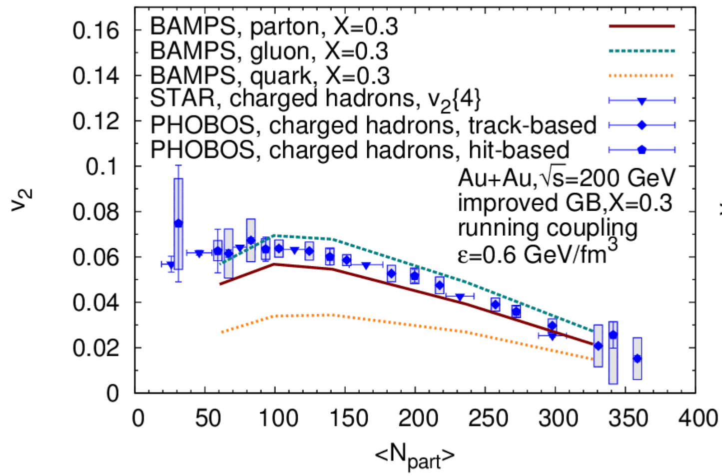
- Hadronization of high p_t partons with AKK fragmentation functions
- LPM parameter fixed by comparison to RHIC data
- Realistic suppression both for RHIC and LHC

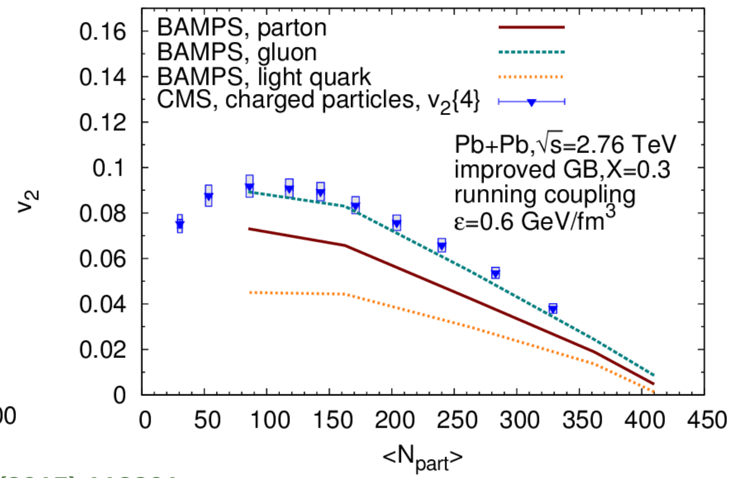


Elliptic flow v_2

$$\frac{d^3N}{p_T dp_T dy d\phi}(p_T, y, \phi) = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left[1 + 2v_2(p_T, y)\cos(2\phi) + \ldots\right]$$

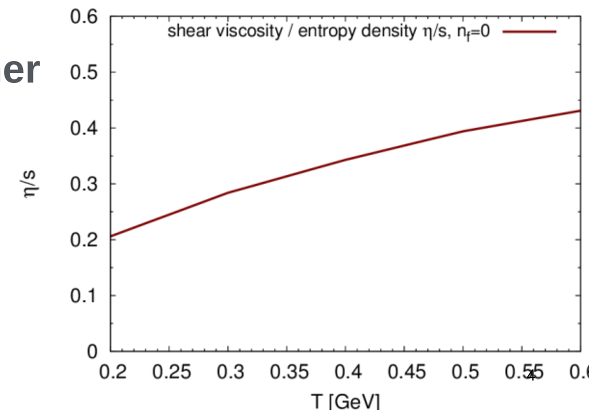






Phys. Rev. Lett. 114 (2015) 112301

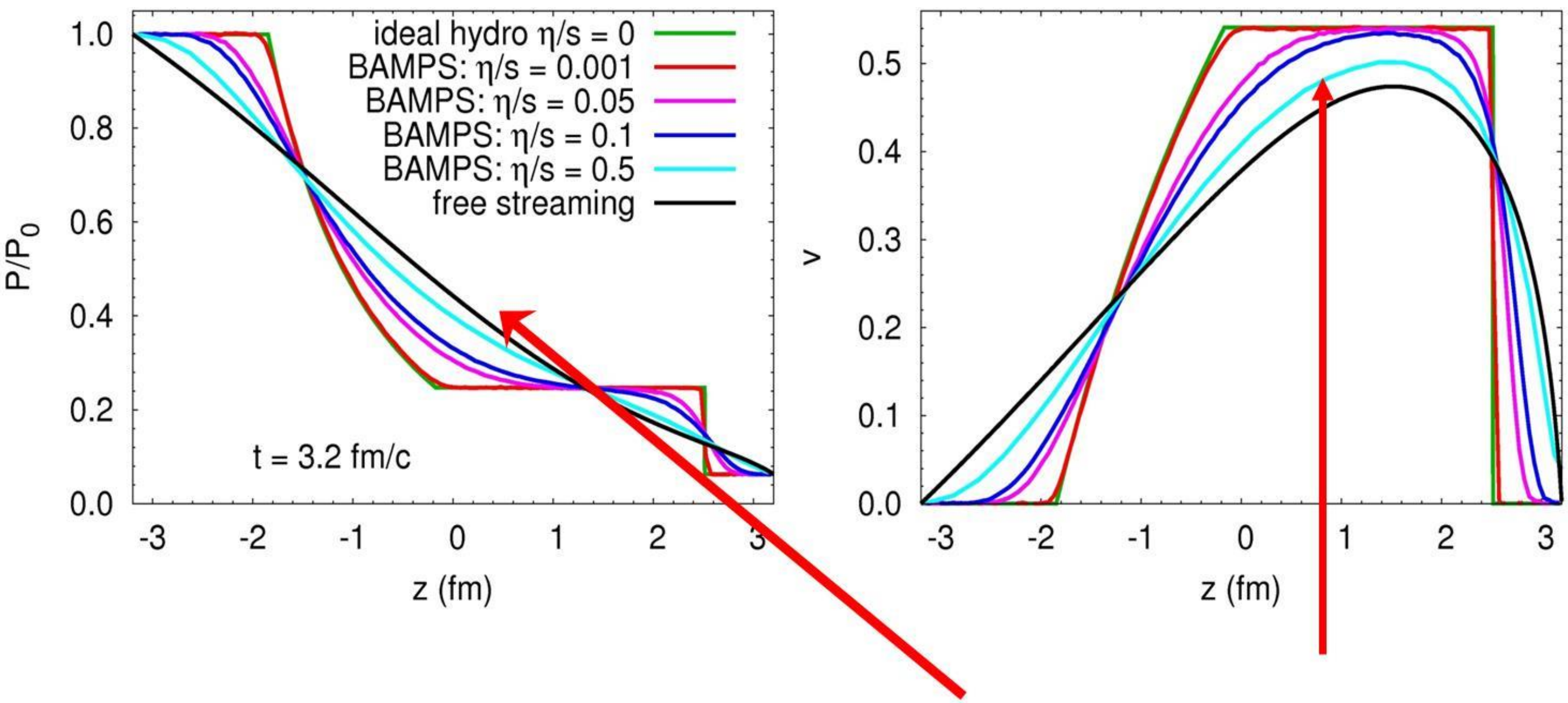
- **Same pQCD interactions lead to a sizeable elliptic flow for bulk medium**
- **No hadronization for bulk medium $\longrightarrow$ no hadronic after-burner**



Riemann problem at finite viscosity

$$p^{\mu}\partial_{\mu}f = C$$

I. Bouras et al, PRL 103:032301 (2009)



Tleft = 400 MeV
Tright = 200 MeV
t = 1.0 fm/c

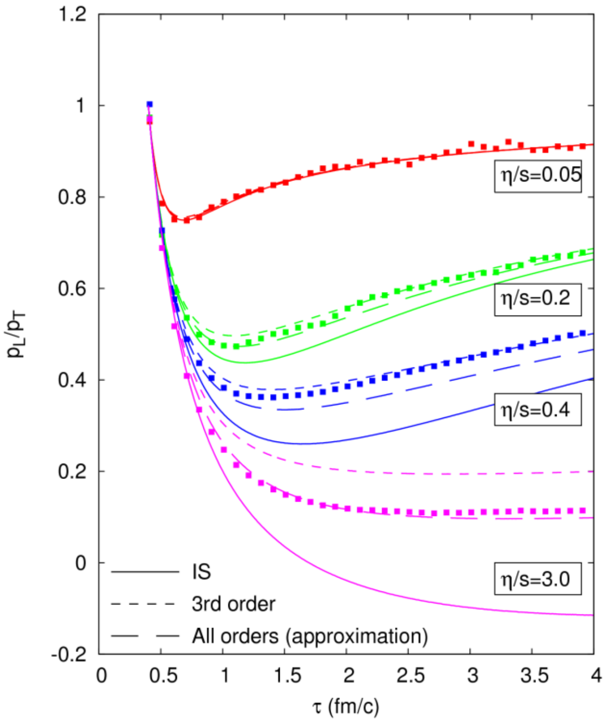
Development of a shock plateau

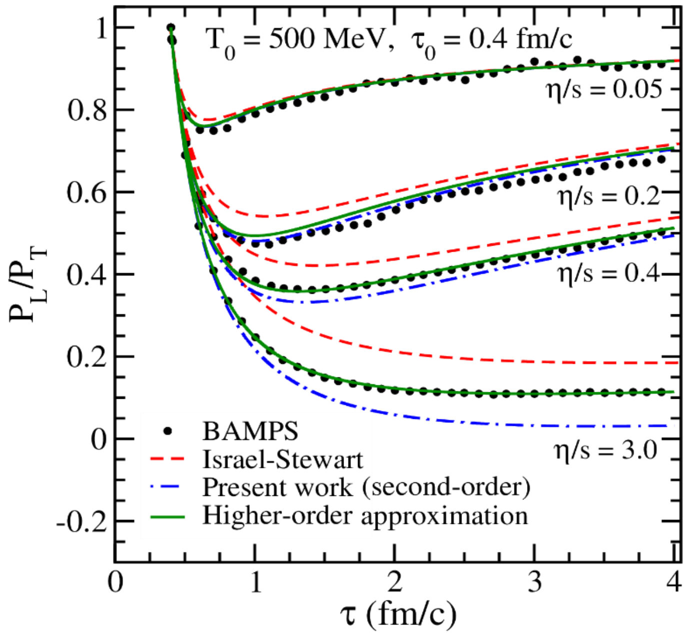
η/s less than 0.1-0.2

Hydro vs BAMPS in 1D

A. El, Z. Xu, C. Greiner, *PRC 81 (2010) 041901*

$$\frac{p_L}{p_T} = \frac{p - \pi}{p + \pi/2}$$





A. Jaiswal, Phys.Rev.C87:051901,2013

x=0: Israel-Stewart

x=3: third-order rel. diss. hydro

x=5/3: approximative 'all-orders'

> Resummation works at strong dissipation (large Knudsen number!).

Relativistic Fluid Dynamics

Conservation laws & tensor decompositions

$$\partial_\mu N^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$

$$N^\mu = n\, u^\mu + V^\mu$$
$$T^{\mu\nu} = e\, u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$n = u_\mu N^\mu$	LRF particle density
$e = u_\mu T^{\mu\nu} u_\nu$	LRF energy density
$V^\mu = \Delta^\mu_\alpha N^\alpha$	particle diffusion current
$p(e,n) + \Pi = -\frac{1}{3}\Delta_{\mu\nu}T^{\mu\nu}$	isotropic pressure $(p_{eq} + bulk)$
$\pi^{\mu\nu} = T^{\langle\mu\nu\rangle}$	shear stress tensor

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

$$T^{\langle\mu\nu\rangle} = \left[\frac{1}{2}\left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\nu_\alpha \Delta^\mu_\beta\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}\right] T^{\alpha\beta}$$

Relativistic Fluid Dynamics

- Transient / second order fluid dynamics (e.g. Israel & Stewart)

$$\tau_\pi \frac{d}{d\tau}\pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \tau_\pi C \pi^{\mu\nu}\left(\nabla_\alpha u^\alpha\right) + \cdots$$

$$\tau_V \frac{d}{d\tau}V^{\langle\mu\rangle} + V^{\mu} = \kappa\nabla^{\mu}\alpha + \tau_V C' V^{\mu}\left(\nabla_\alpha u^\alpha\right) + \cdots$$

($\pi^{\mu\nu}$ and V^{μ} independent variables)

  - Second order coefficients from
    G.S.Denicol, H.Niemi, E.Molnar, D.H.Rischke, PRD 85, 114047 (2012)

  - Expansion in Knudsen and (inverse) Reynolds number

$$\mathrm{Kn} = \frac{\ell_{\mathrm{micr}}}{L_{\mathrm{macr}}} \qquad \mathrm{R}_V^{-1} \sim \frac{|V^\mu|}{n_0}, \; \mathrm{R}_\pi^{-1} \sim \frac{|\pi^{\mu\nu}|}{P_0}$$

  - Hydrodynamical limit: $\mathrm{Kn} \ll \mathcal{O}(1)$ and $\mathrm{R}^{-1} \ll \mathcal{O}(1)$

Comparison Hydro / BAMPS in 3D

Collectivity in Heavy Ion Collision?
Fast Thermalization?
Flow?

How small can system be,
how large can gradients be,
until disrepancies occur?

- Longitudinal: Boost invariant
- Transversal:
  - Radial symmetric, large/small system
  - Glauber; overlapping Woods-Saxon

A+A　　p+A, p+p

$$\langle r^2 \rangle = (3\mathrm{fm})^2,\ (1\mathrm{fm})^2$$

Comparison 1: Radial symmetric

- Longitudinal: boost invariant
- Transversal:
  - Rotational symmetric
  - Gaussian density profile, $\langle r^2 \rangle = (3\mathrm{fm})^2$ or $\langle r^2 \rangle = (1\mathrm{fm})^2$

A+A → $\langle r^2 \rangle = (3\mathrm{fm})^2$

p+A, p+p → $\langle r^2 \rangle = (1\mathrm{fm})^2$

- Temperature $\mathrm{T}_0(r)\,,\ T_0(0) = 500\,\mathrm{MeV}$

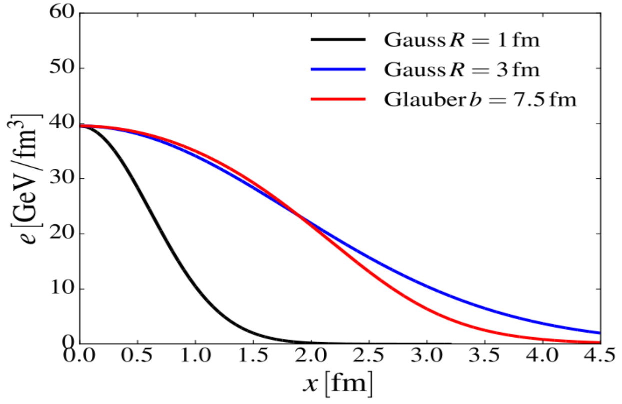
Fugacity $\lambda_0(r) = 1$

$$\tau_0 = 0.2\,\mathrm{fm}$$

start in full equilibrium

- only gluons
- Cross section:
  - Elastic
  - Isotropic
  - Constant

$$\sigma = 1,\ 5,\ 20,\ 50,\ 100\,\mathrm{mb}$$



Comparison 2: Glauber

- Longitudinal: boost invariant
- Transversal:
  - Overlapping Woods-Saxon ← A+A
  - Impact parameter dependence   selected value: 7.5 fm   = (“nBC“)

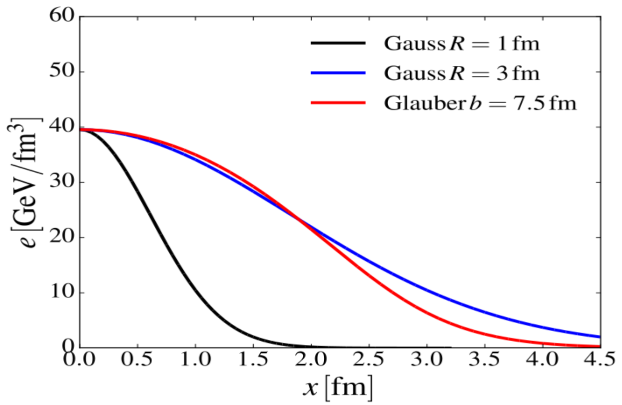
- Temperature $T_0(r)\,,\ T_0(0) = 500\,\mathrm{MeV}$ $\tau_0 = 0.2\,\mathrm{fm}$

Fugacity $\lambda_0(r) = 1$

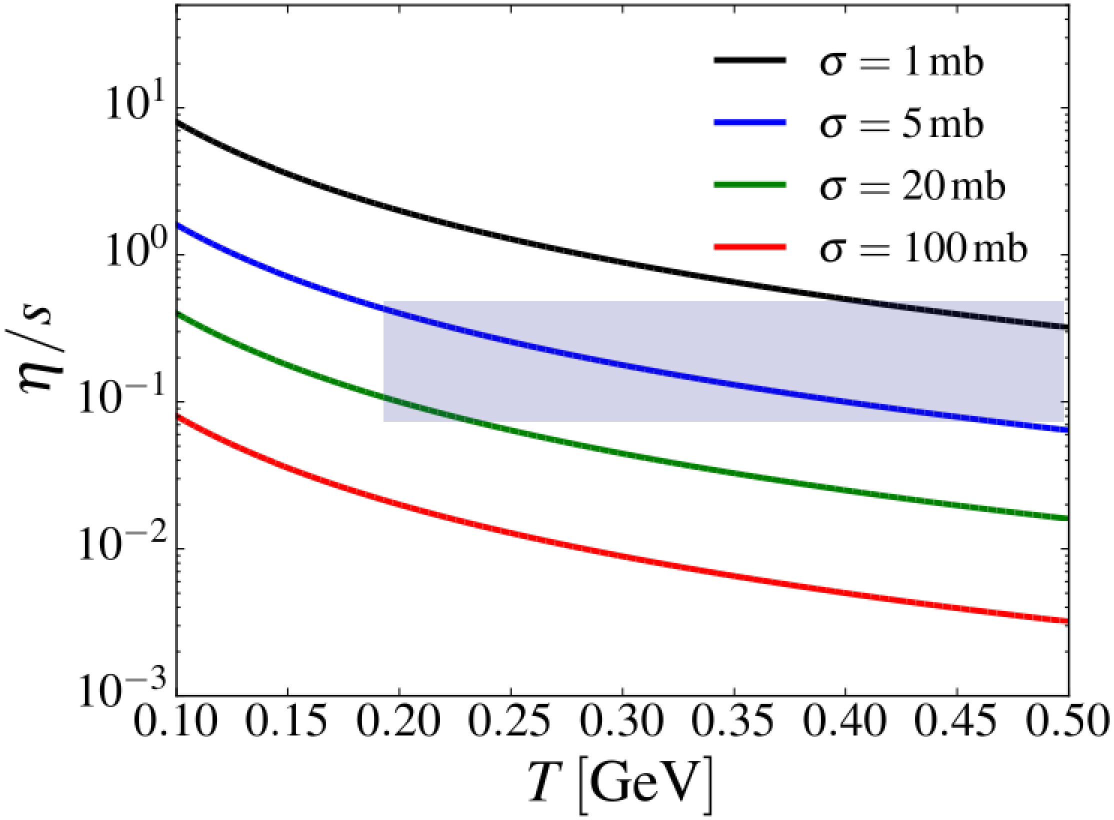
start in full equilibrium

- only gluons
- Cross section:
  - Elastic
  - Isotropic
  - Constant

$$\sigma = 1,\ 5,\ 20,\ 50,\ 100\,\mathrm{mb}$$



Available eta/s



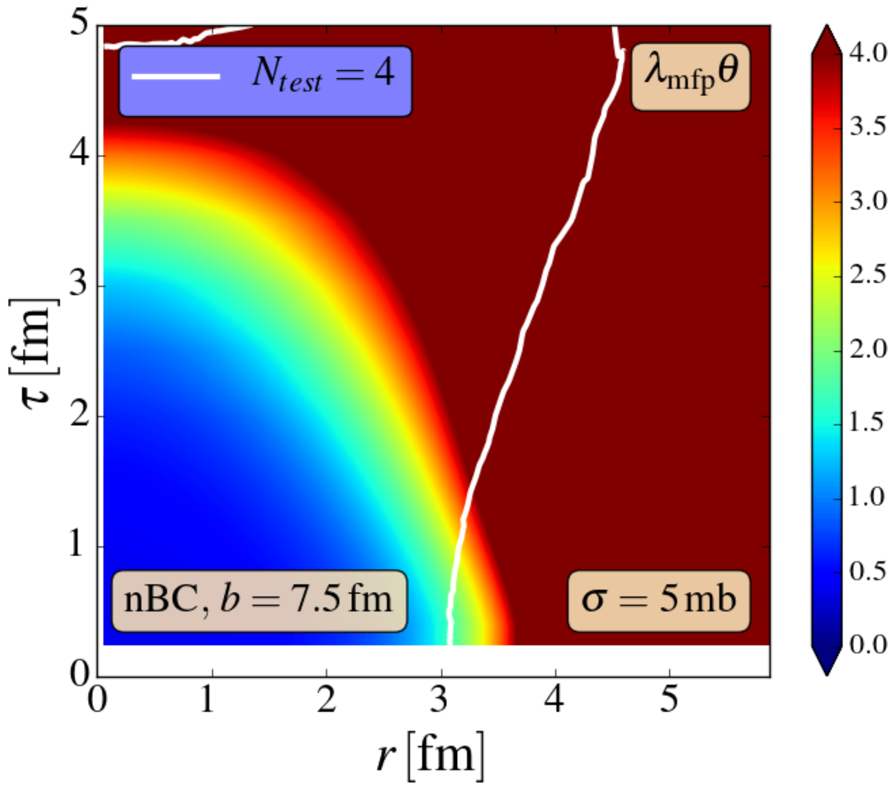
$$\sigma = 1,\ 5,\ 20,\ 50,\ 100\,\mathrm{mb}$$

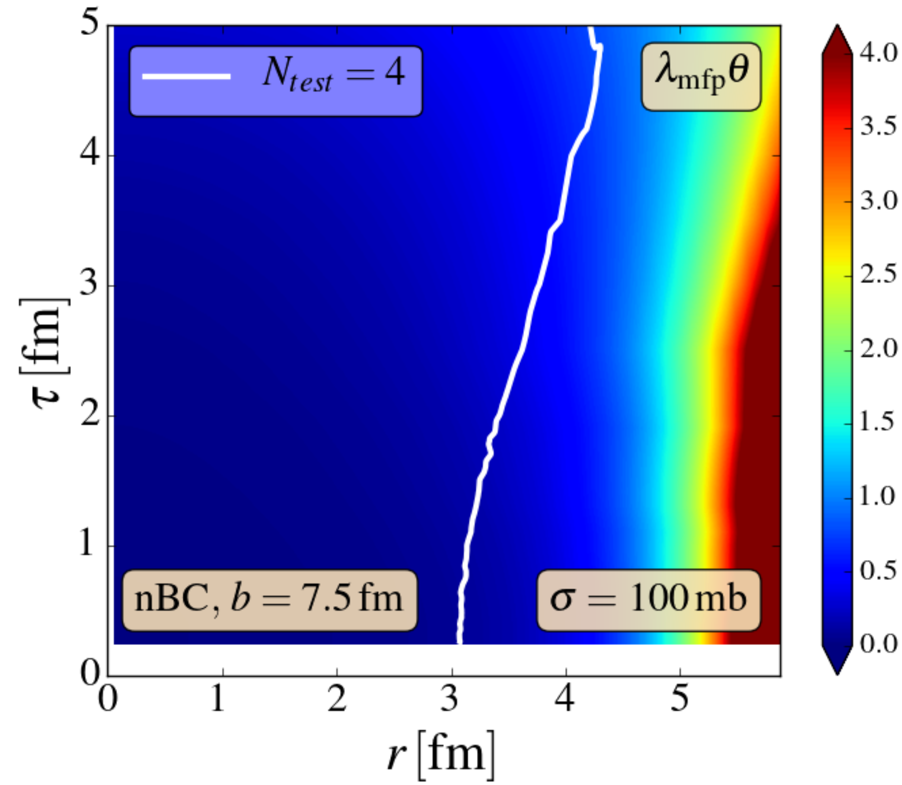
$$\Longrightarrow \sigma = 1 \ldots 20\,\mathrm{mb}$$

$$\eta = \frac{4}{3}\frac{T}{\sigma} \qquad s = \frac{4g}{\pi^2}T^3 \qquad (g = 16)$$

Comparison: Glauber A+A

- Knudsen number





Hydrodynamical limit:

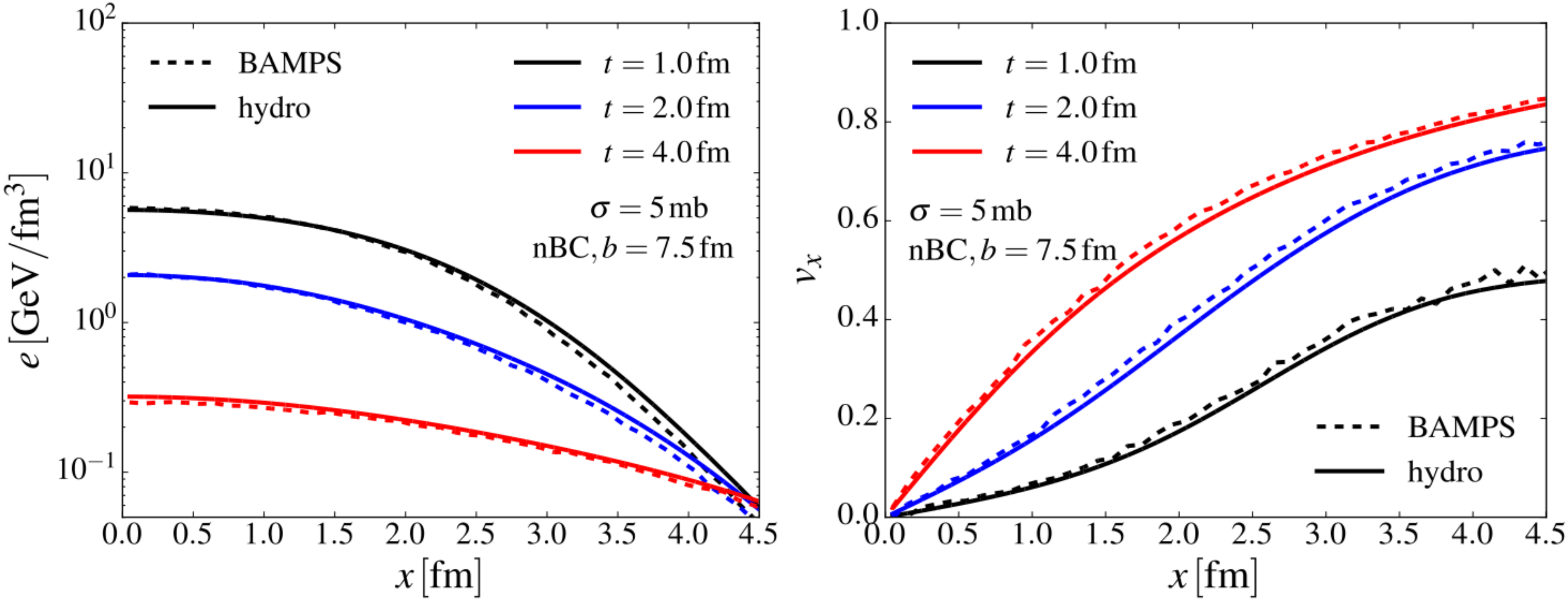
$$\mathrm{Kn} = \frac{\ell_{\rm micr}}{L_{\rm macr}} \ll \mathcal{O}(1)$$

$$\mathrm{Kn} \equiv \lambda_{\rm mfp}\,\theta \quad , \qquad \theta = \partial_\mu u^\mu$$

Comparison: Glauber

A+A

- Glauber, 5mb: energy density & velocity



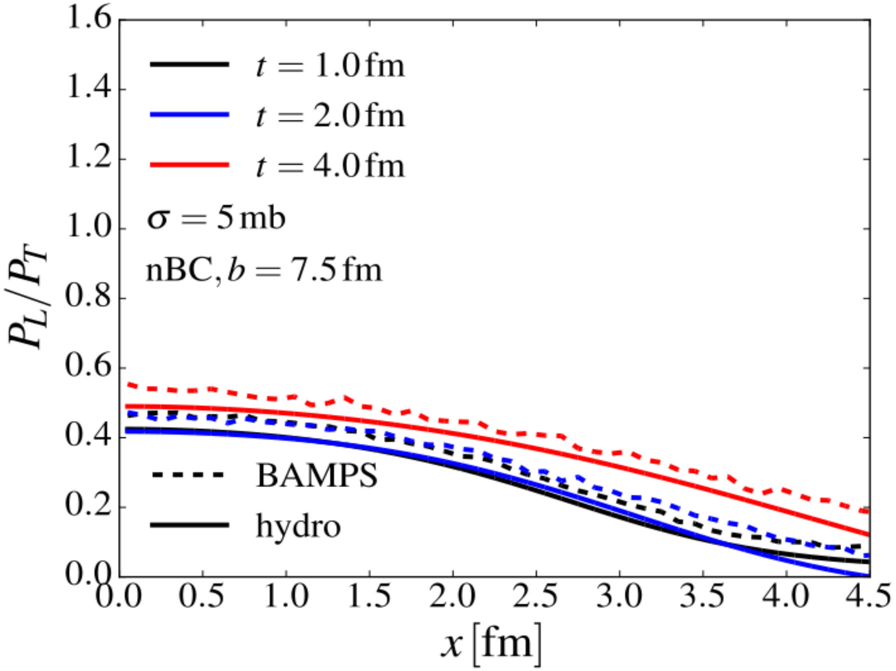
5mb: still very nice agreement

Comparison: Glauber

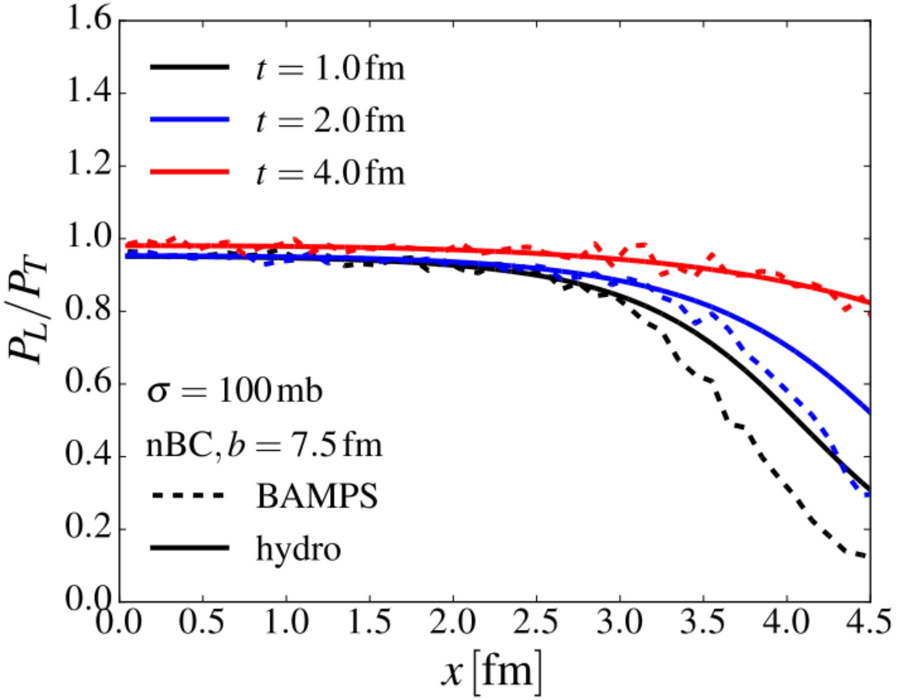
A+A

- Pressure ratio: P_L/P_T (in the LRF)

5 mb



100 mb

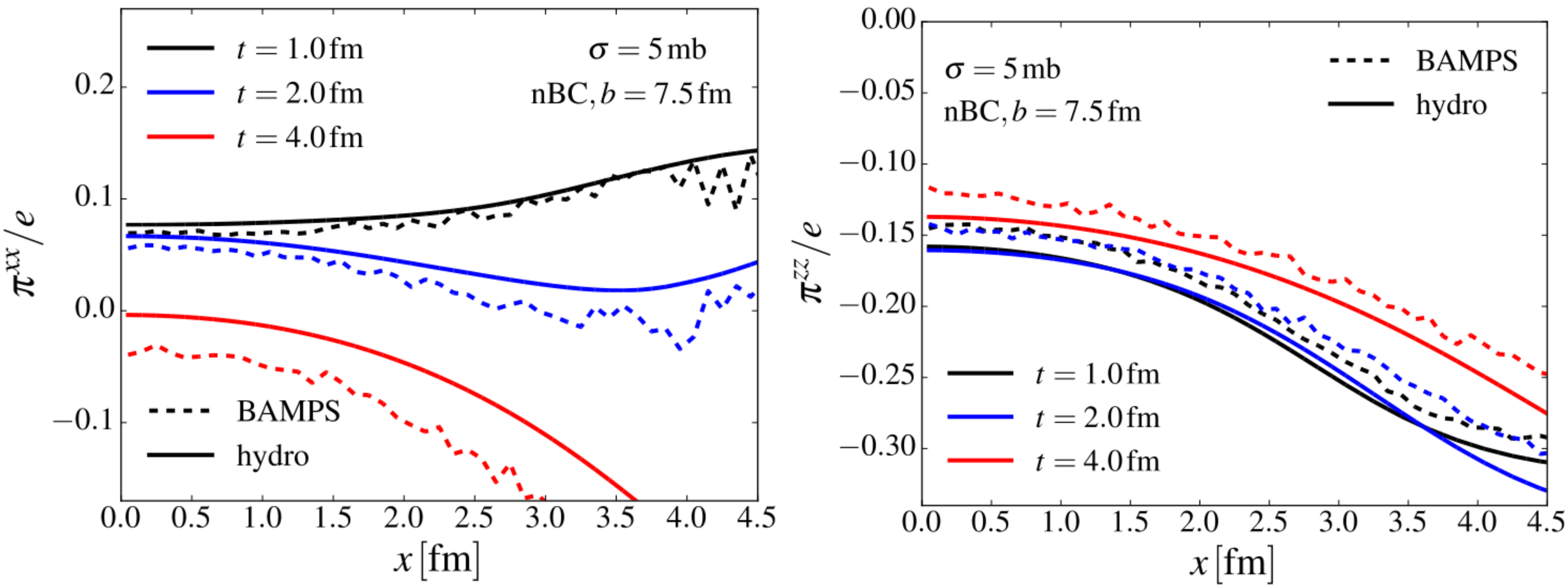


5mb: still very nice agreement

Comparison: Glauber

A+A

- Glauber, 5mb: shear stress tensor

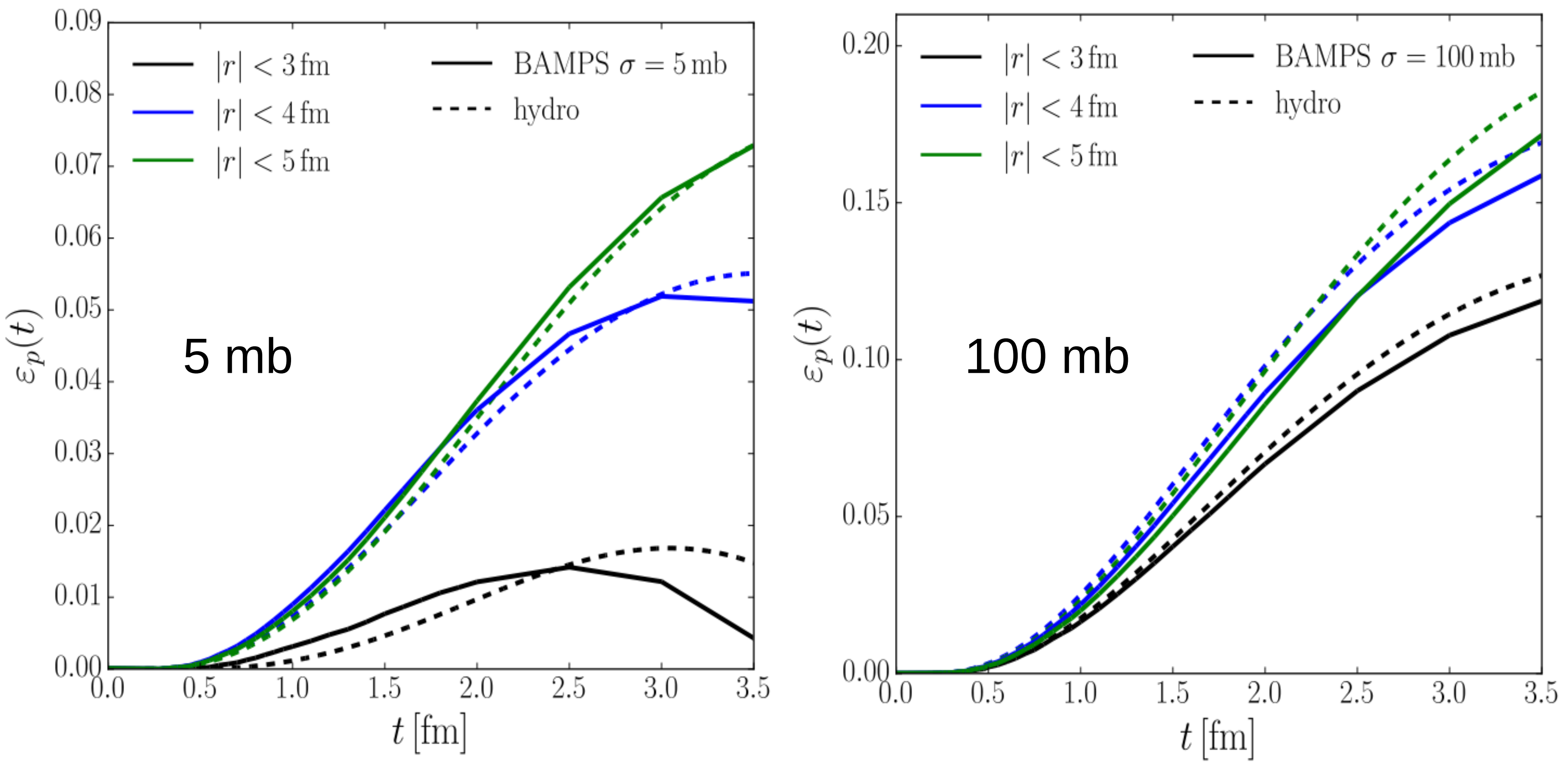


5mb: still very nice agreement

Comparison: Glauber

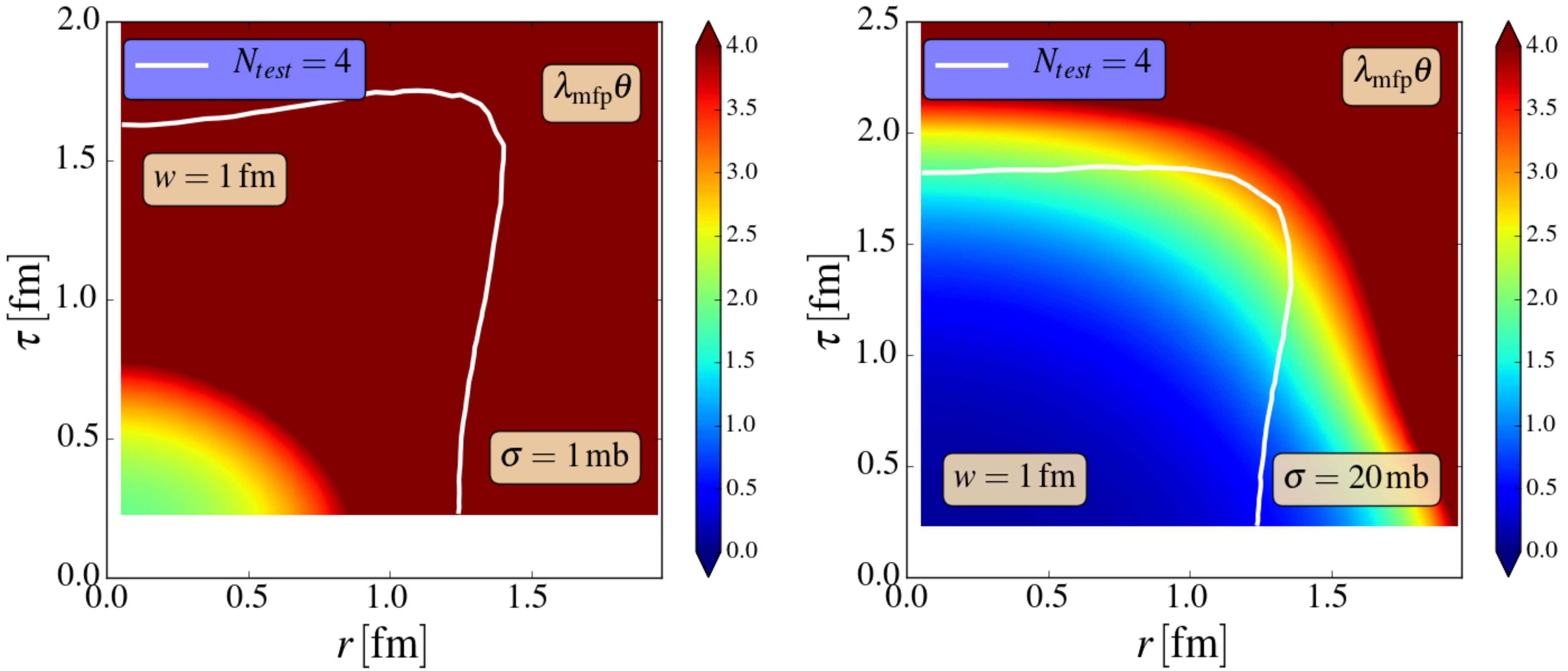
A+A

- Asymmetry: $$\varepsilon_P = \frac{\langle T^{xx} - T^{yy} \rangle_{xy}}{\langle T^{xx} + T^{yy} \rangle_{xy}}$$



Comparison: Radial symmetric (small)

- Knudsen number

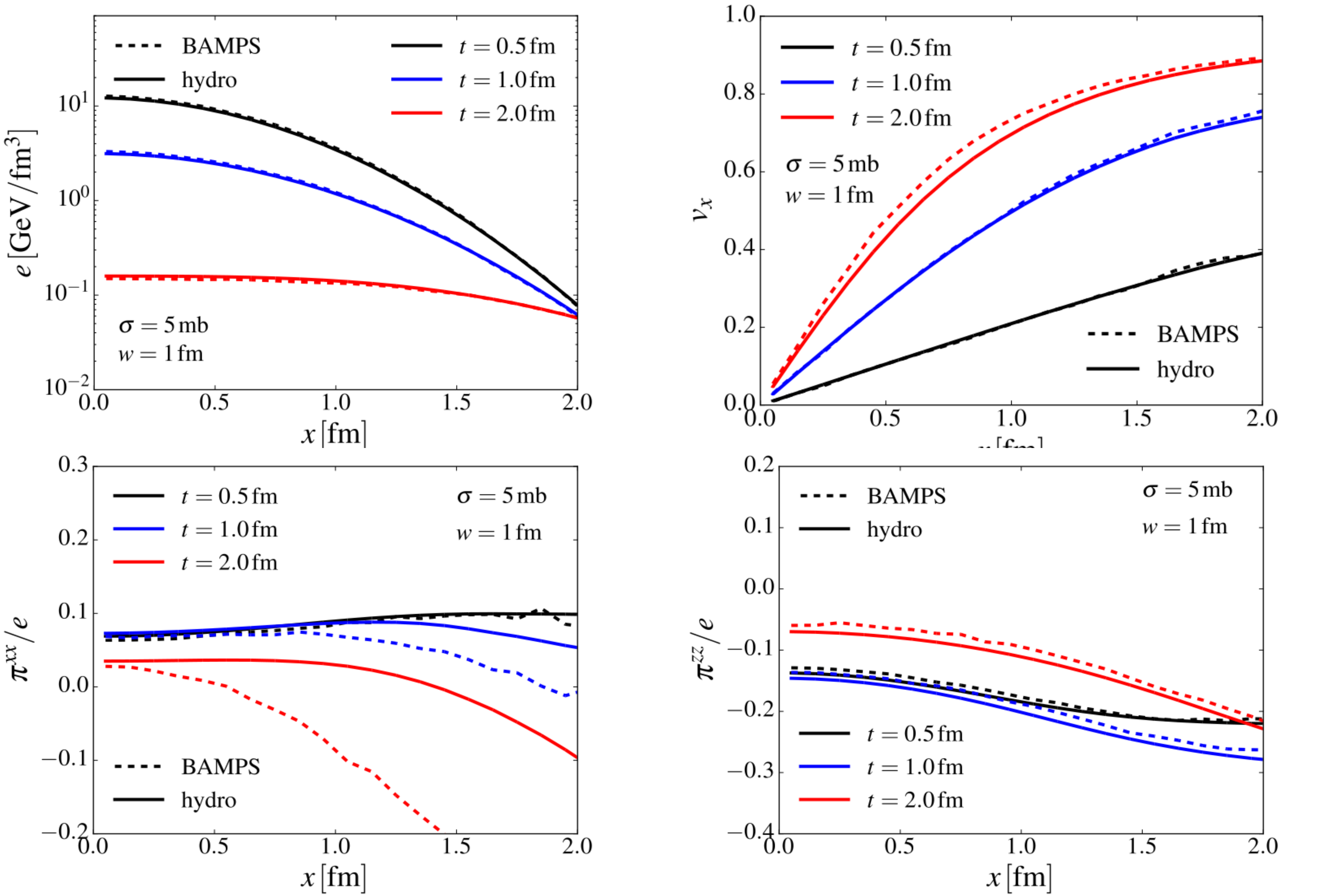


Hydrodynamical limit:

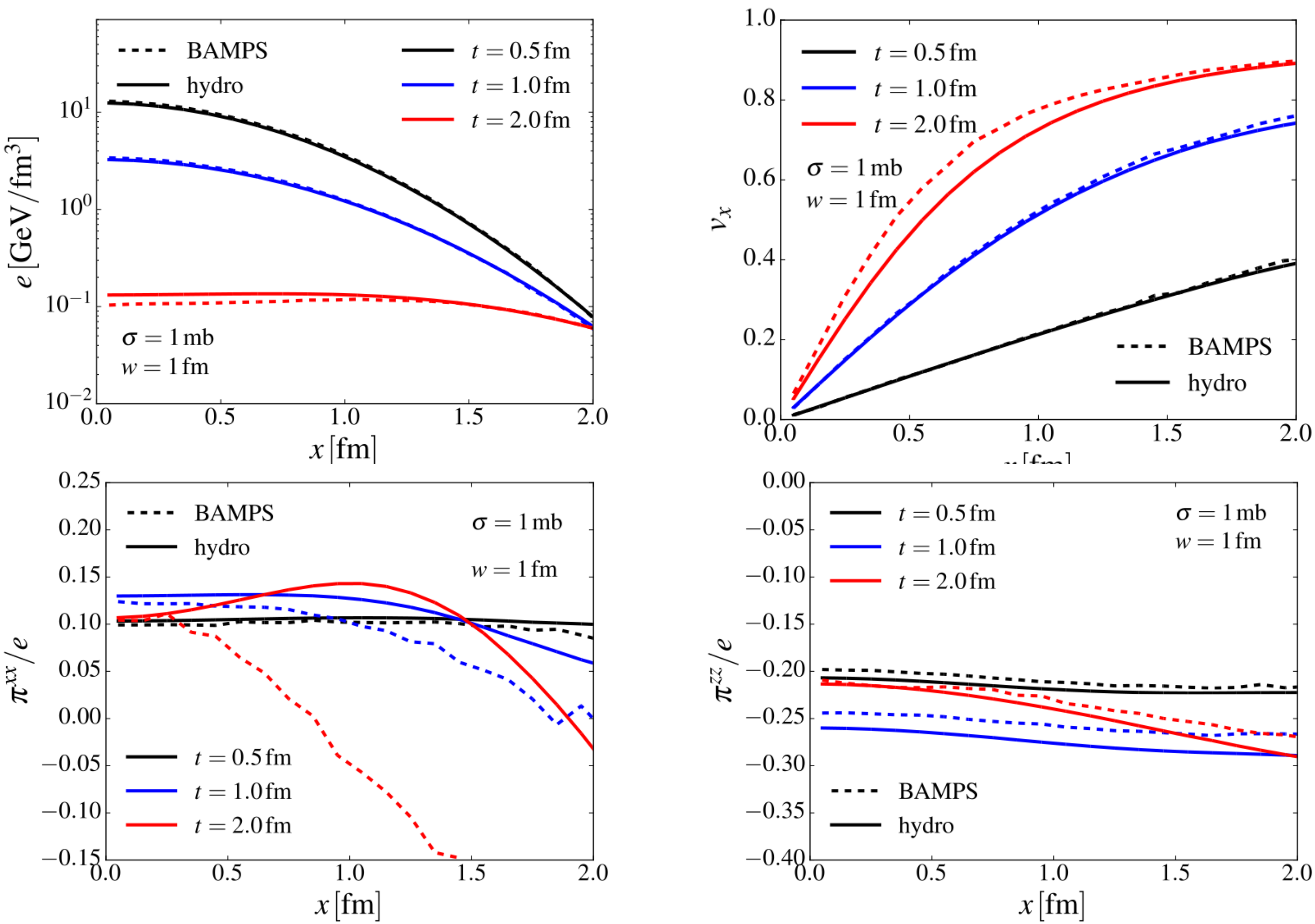
$$\mathrm{Kn} = \frac{\ell_{\rm micr}}{L_{\rm macr}} \ll \mathcal{O}(1)$$

$$\mathrm{Kn} \equiv \lambda_{\rm mfp}\,\theta \quad , \qquad \theta = \partial_\mu u^\mu$$

Comparison: Radial symmetric (small, 5mb)



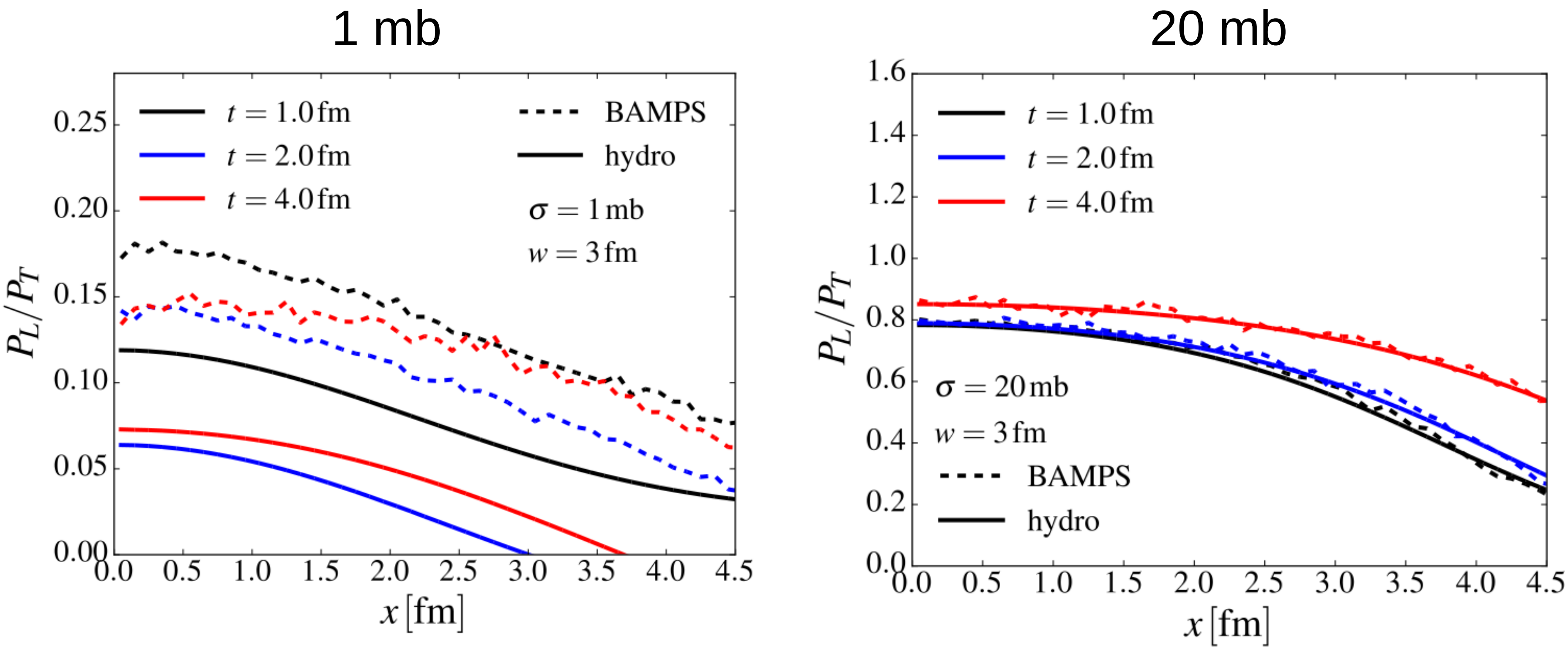
Comparison: Radial symmetric (small, 1mb)



Comparison: Radial symmetric (large)

- Pressure ratio: P_L/P_T (in the LRF)

$$\langle r^2 \rangle = (3\text{fm})^2$$

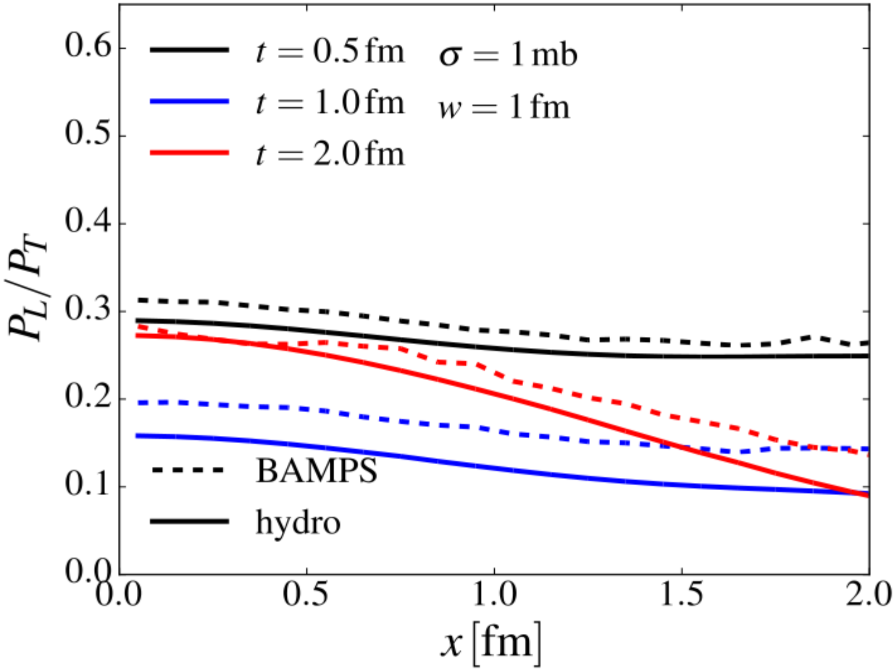


Comparison: Radial symmetric (small)

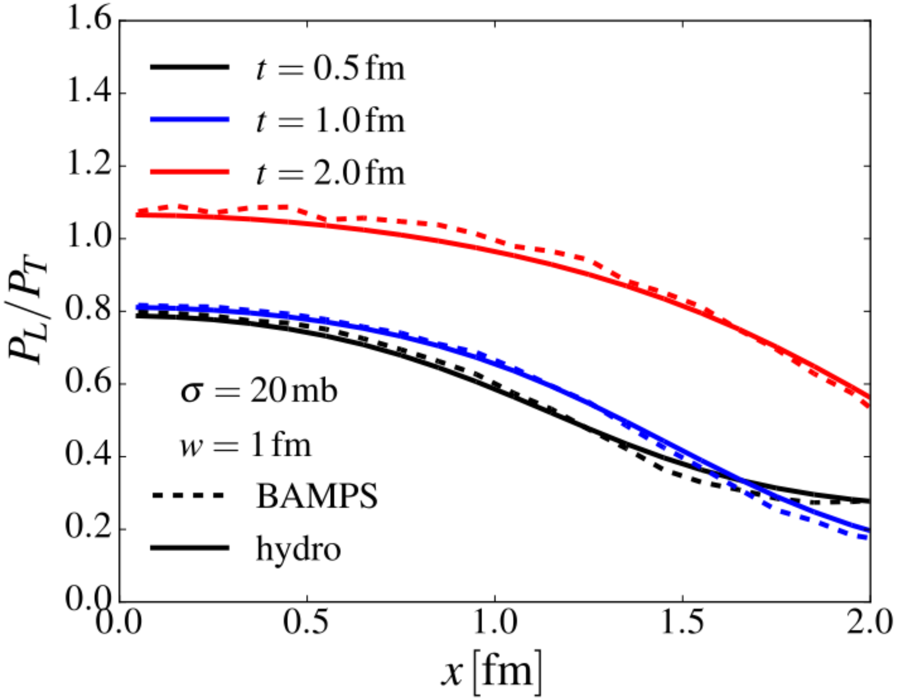
- Pressure ratio: P_L/P_T (in the LRF)

$$\langle r^2 \rangle = (1\mathrm{fm})^2$$

1 mb



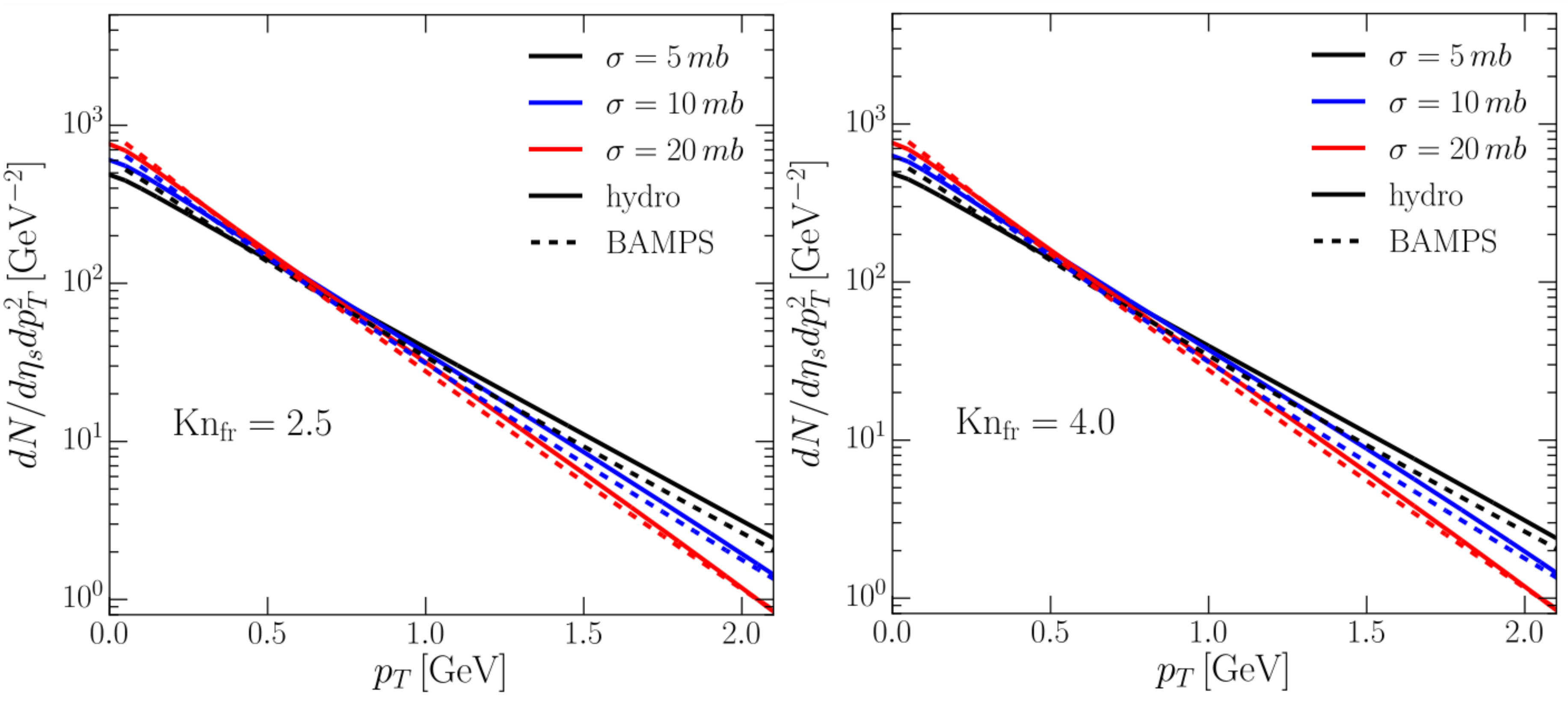
20 mb



Comparison 2: Glauber

A+A

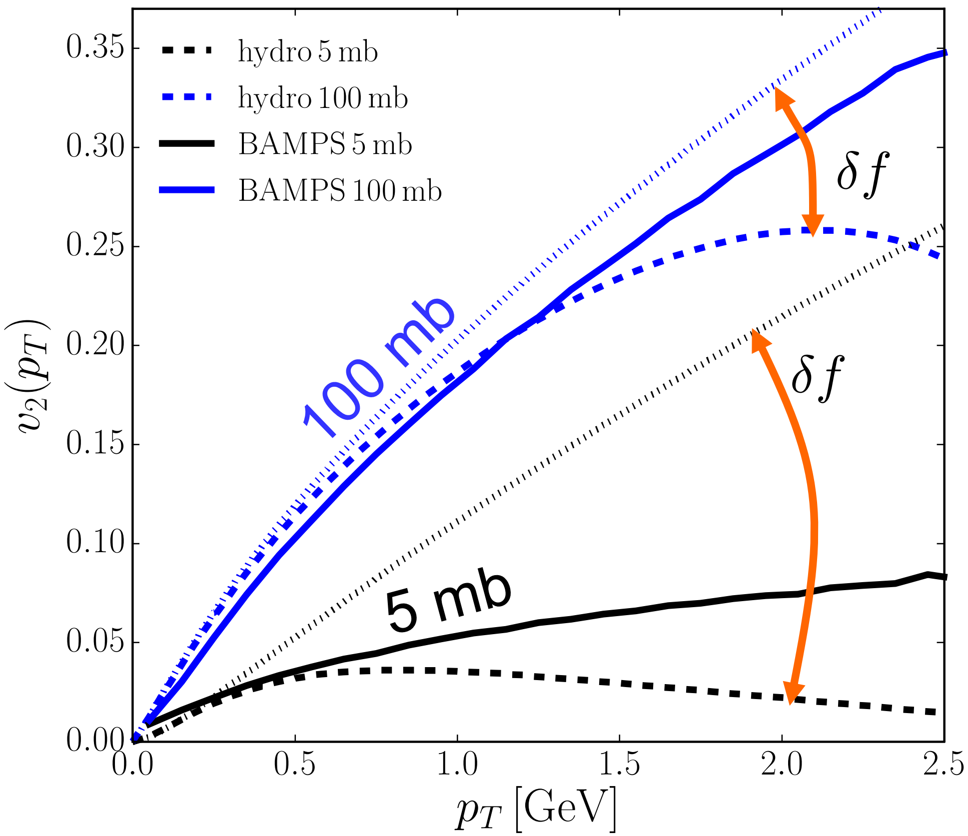
- Spectra:



Comparison 2: Glauber

A+A

- Flow:

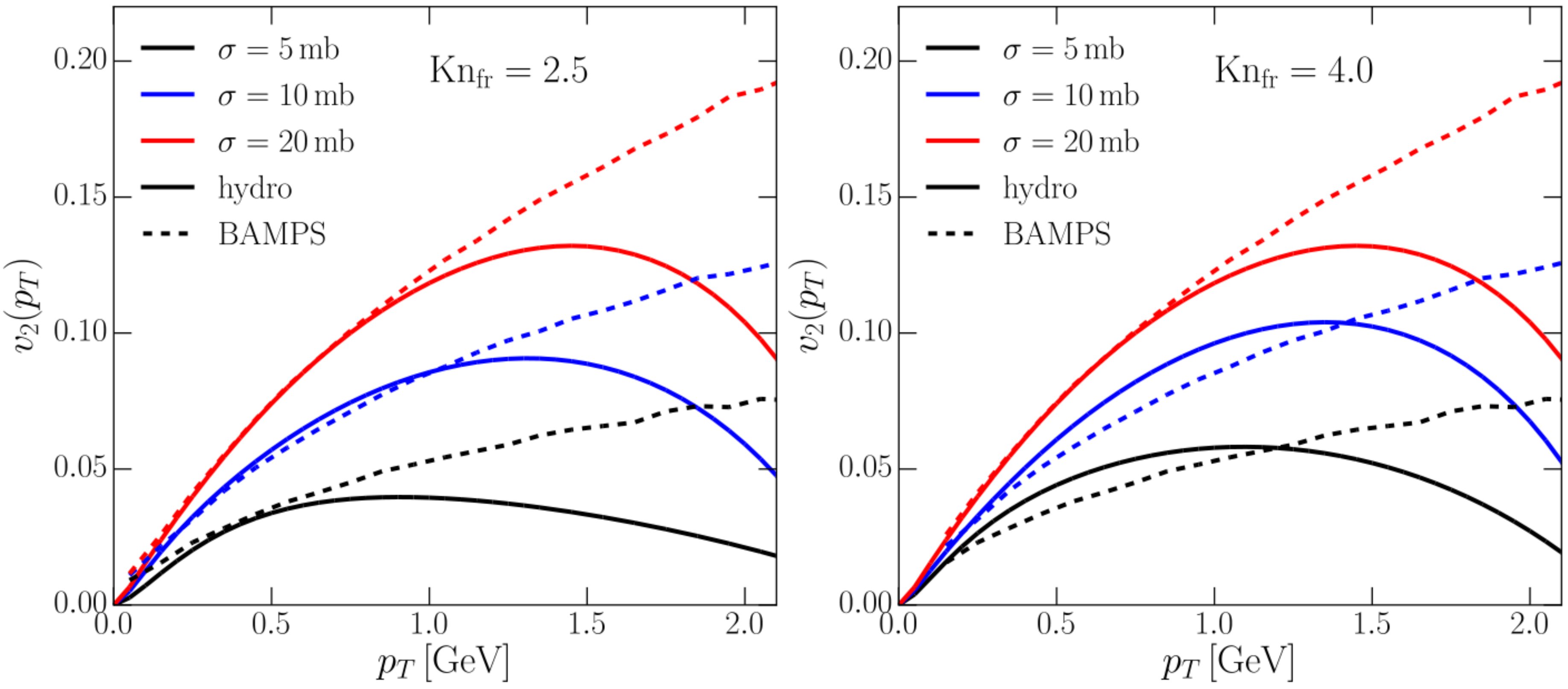


- Large uncertainty due to viscous correction terms
- Strong dependence on freeze out conditions

$$\delta f_{\mathbf{k}} = f_{0\mathbf{k}} \left(\frac{1}{8 p_0 T^2} k_{\langle\mu} k_{\nu\rangle} \pi^{\mu\nu} - \frac{5}{p_0} k_\mu n^\mu + \frac{1}{p_0 T} E_{\mathbf{k}} k_\mu n^\mu \right)$$

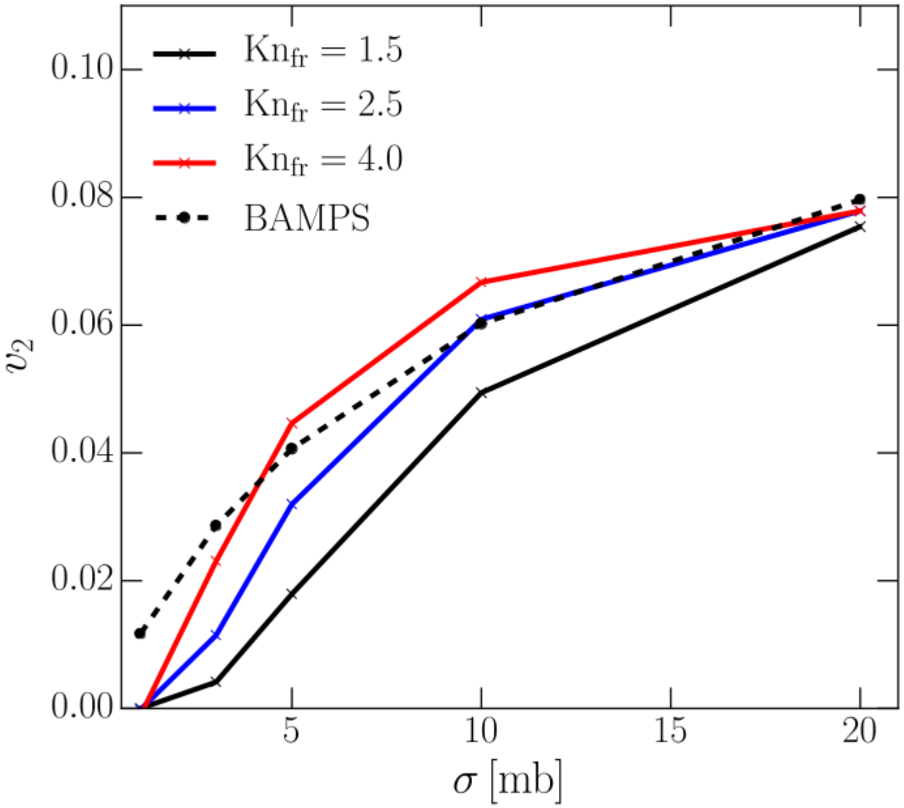
Comparison 2: Glauber

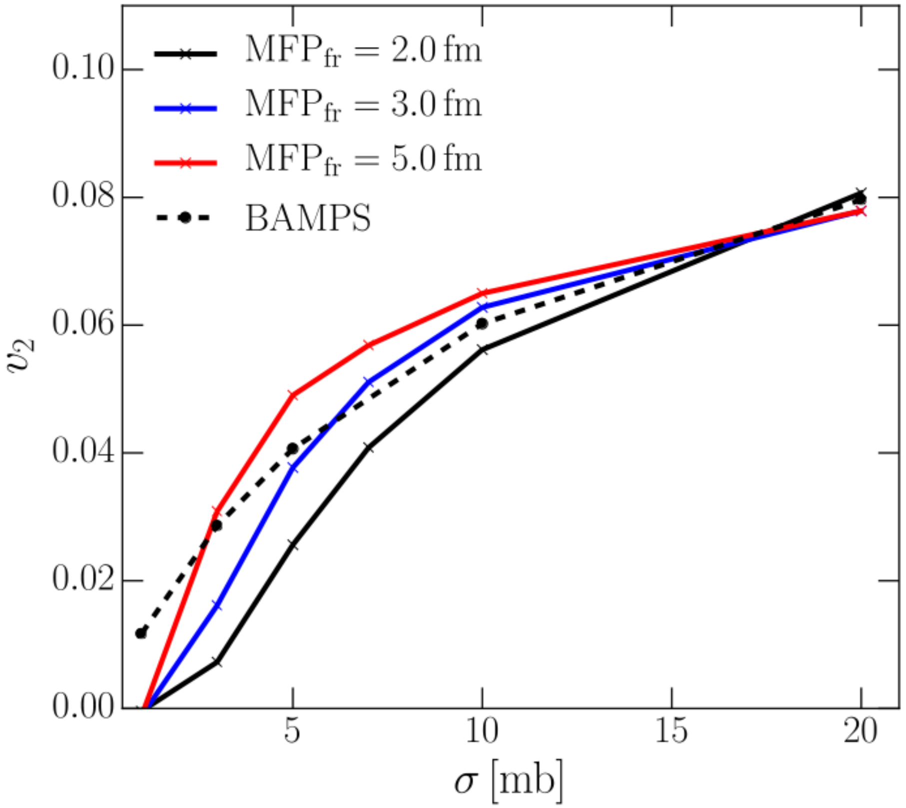
- Flow:



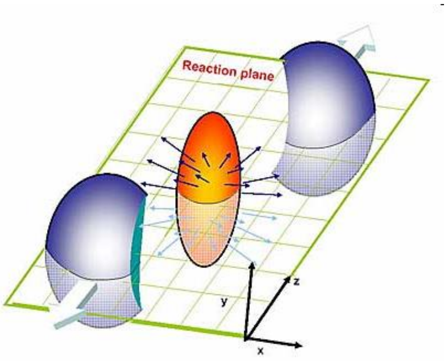
Comparison 2: Glauber

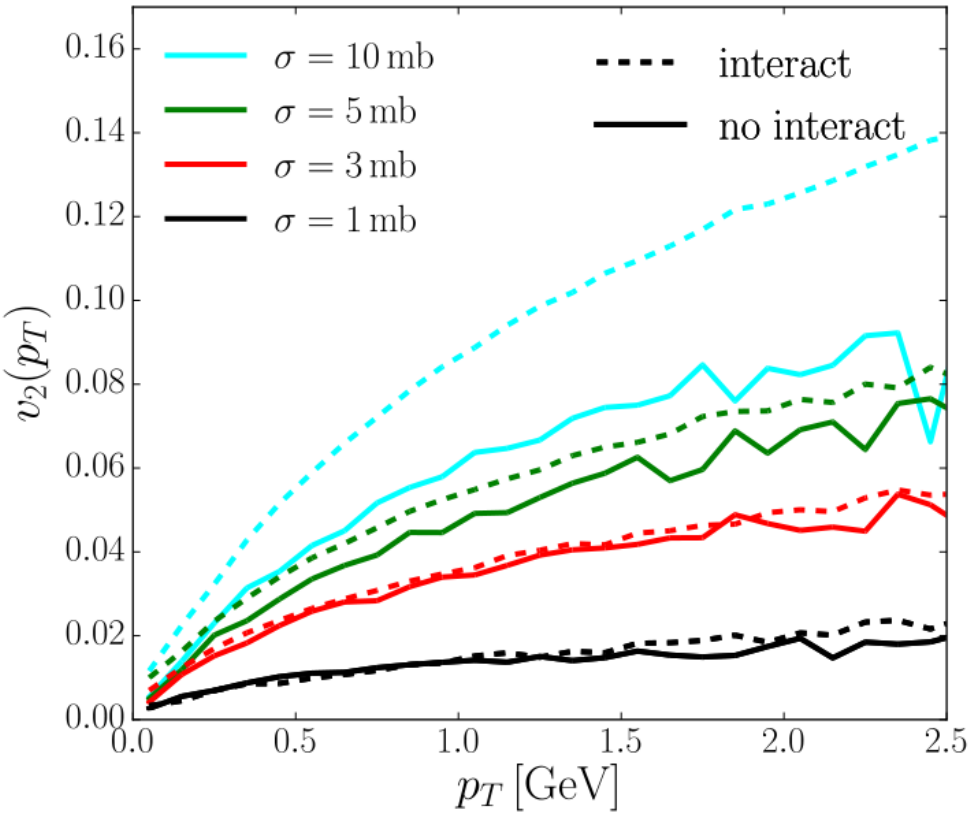
- Flow:





Comparison 2: Glauber, escaping probability





6 %

15 %

25 %

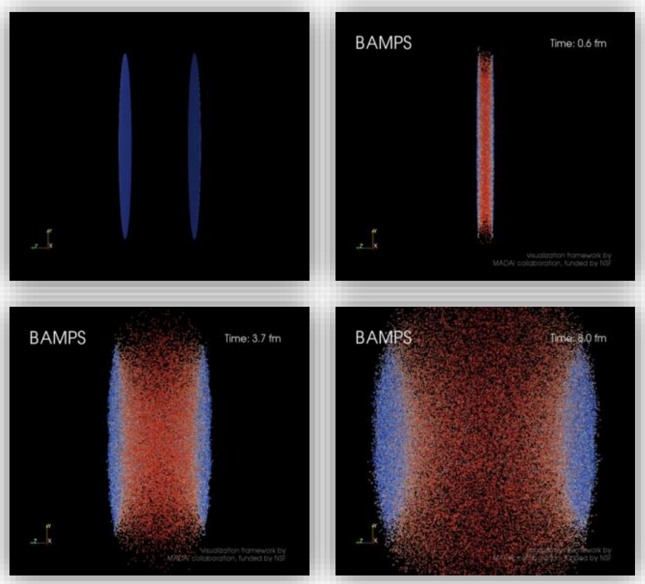
50 %

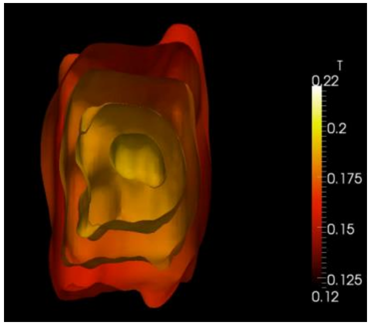
Conclusions

- Comparison of 3D Bjorken Scenario
- Radial symmetric configuration
  - Nice agreement (~10%) for densities, temperatures, velocities
  - Systematic deviation of fugacities
  - Deviations in components of shear-stress tensor
  - No difference between large and small system
- Asymmetric configuration
  - Same agreement as in radial symmetric case
  - ε_P and flow v_2: nice agreement, dependence on freeze-out
- Work in progress: quantify deviation as function of Knudsen number
- ToDo: hot spots, anisotropic hydro, …
- Work in progress: Greif, Schenke, …; IP-Glasma for p+A

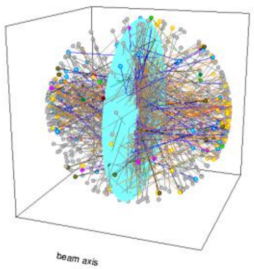
Heavy-ion collisions are complex !

Dynamical bulk description

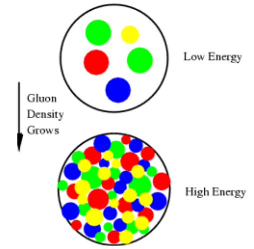




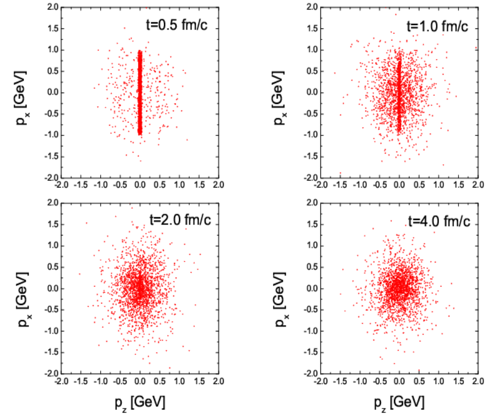
Glauber

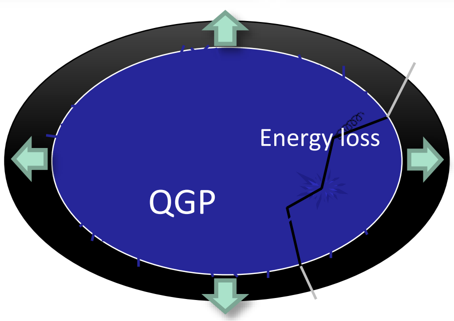


Gluon saturation



Early thermalisation

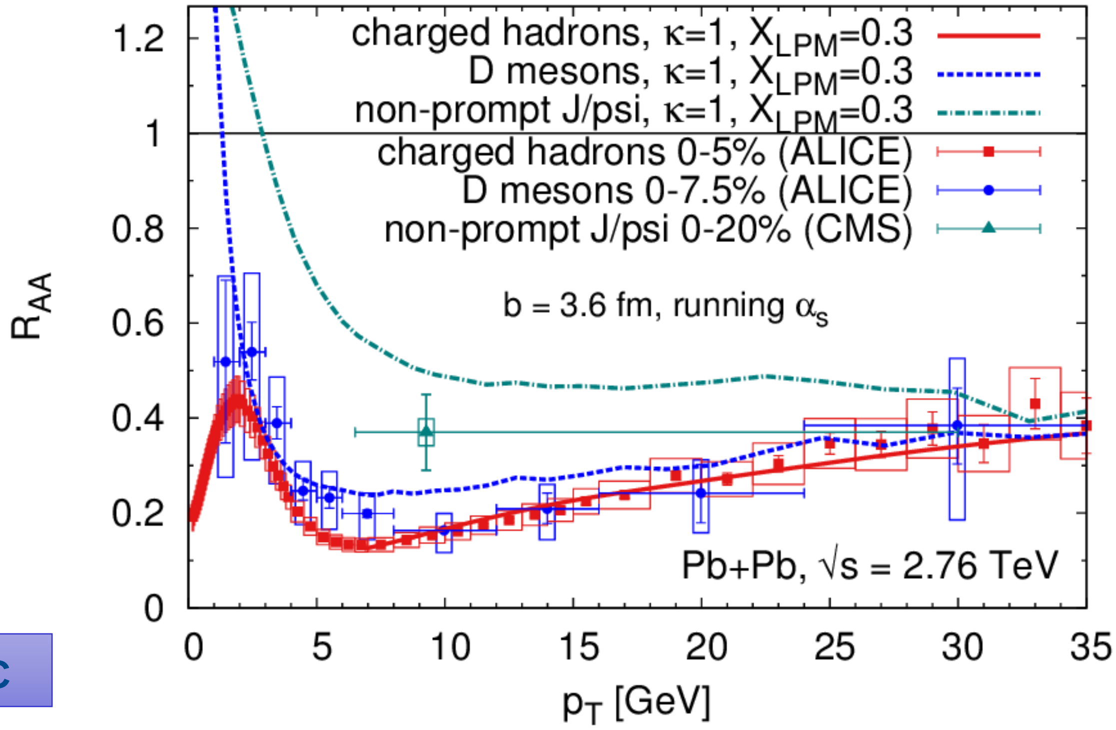




jet quenching and recovery

No model can describe all aspects of the QGP evolution

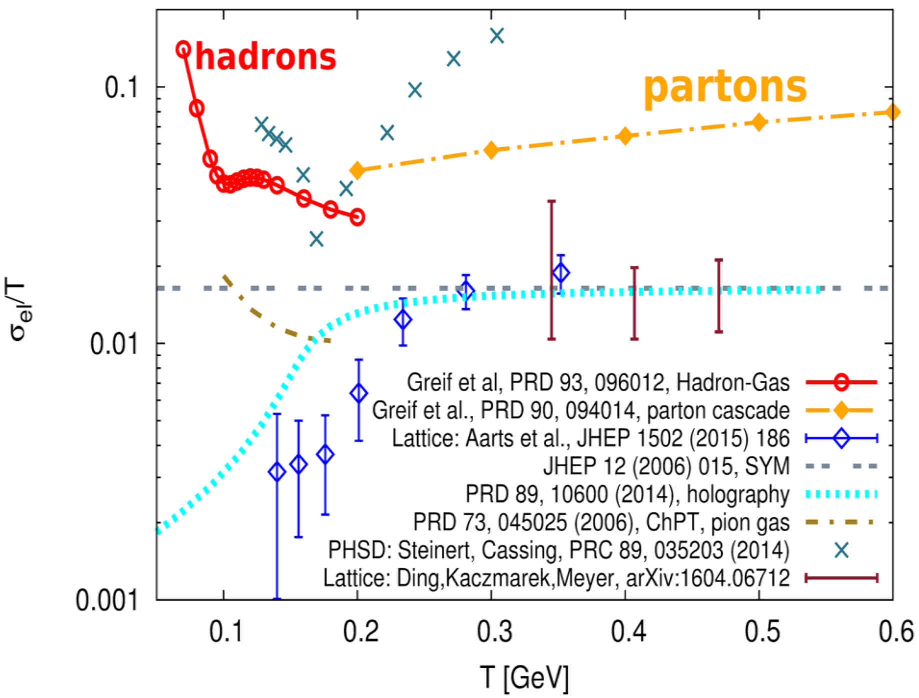
Heavy flavor and charged hadron R_{AA} at LHC



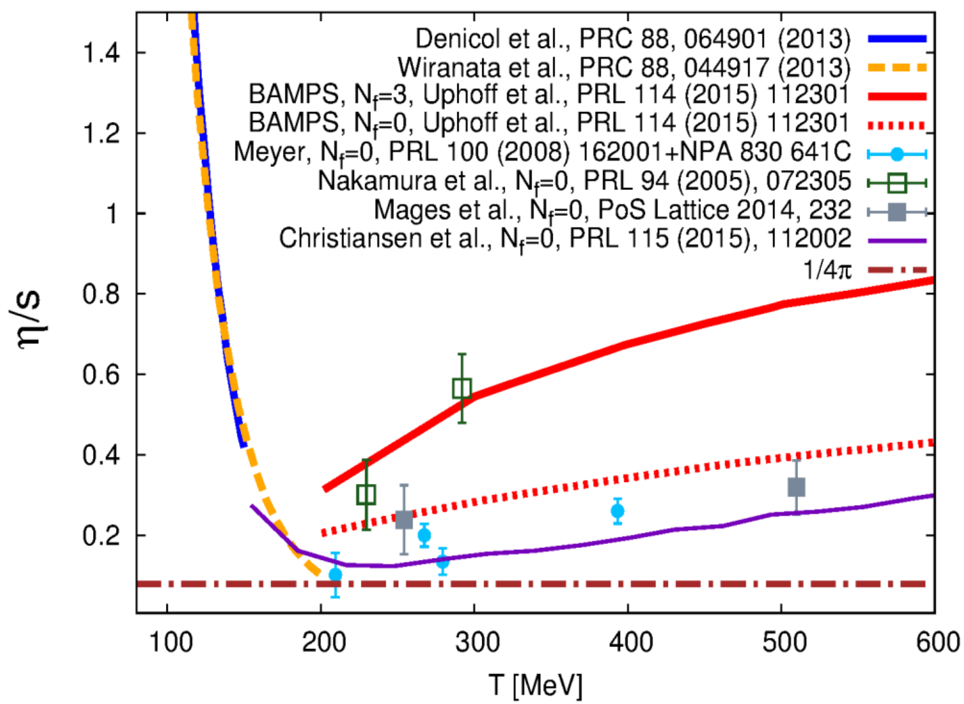
LHC

Transport coefficients

Electric conductivity of parton-hadron matter



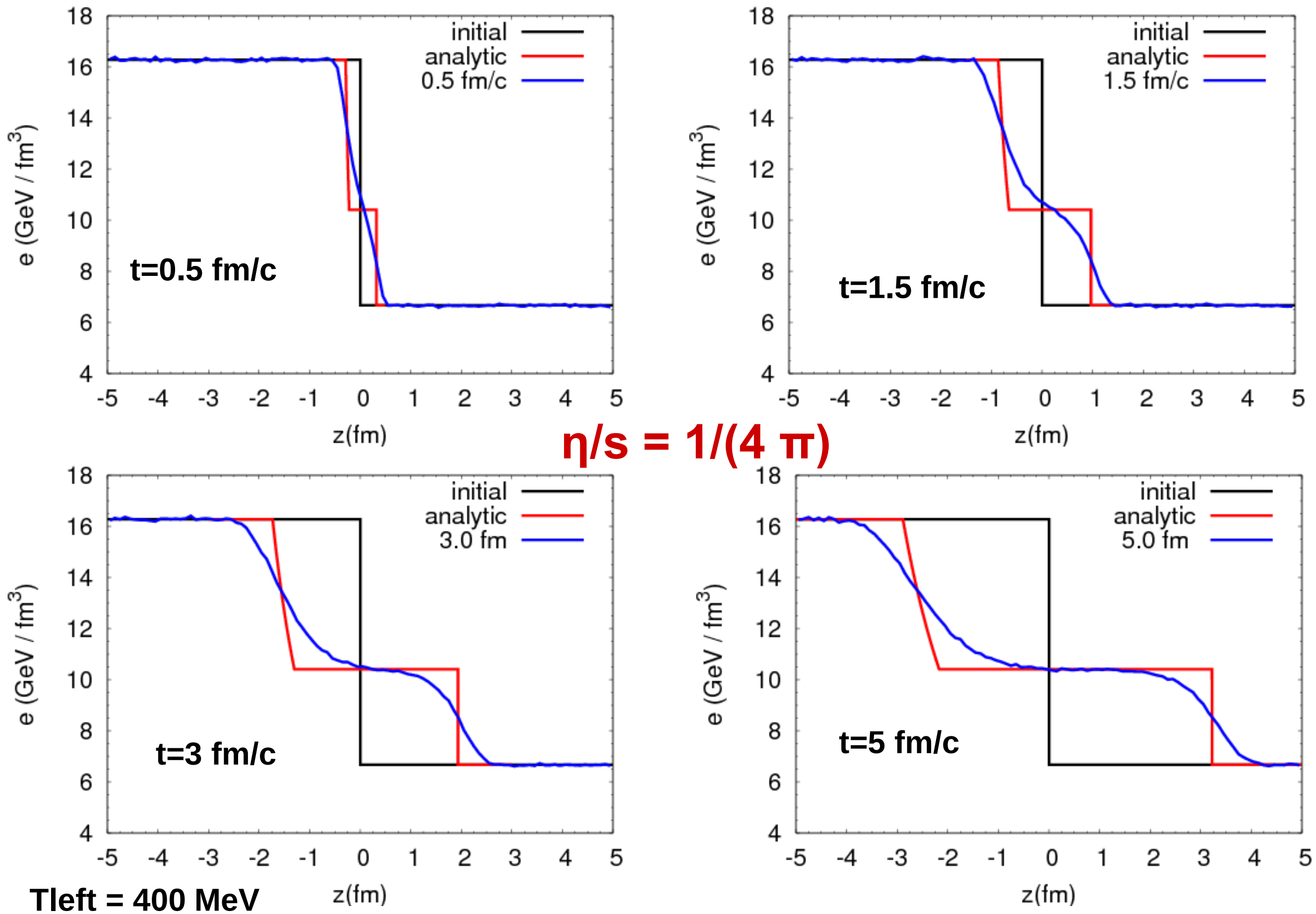
shear viscosity



ongoing projects:
- → baryon diffusion coefficient
- → charm diffusion coefficient
- → study effective couplings
- → momentum broadening: $\hat{q}$

we have studied:
- → shear viscosity
- → heat conductivity
- → electric conductivity

time evolution of viscous shocks



Tleft = 400 MeV
Tright = 320 MeV