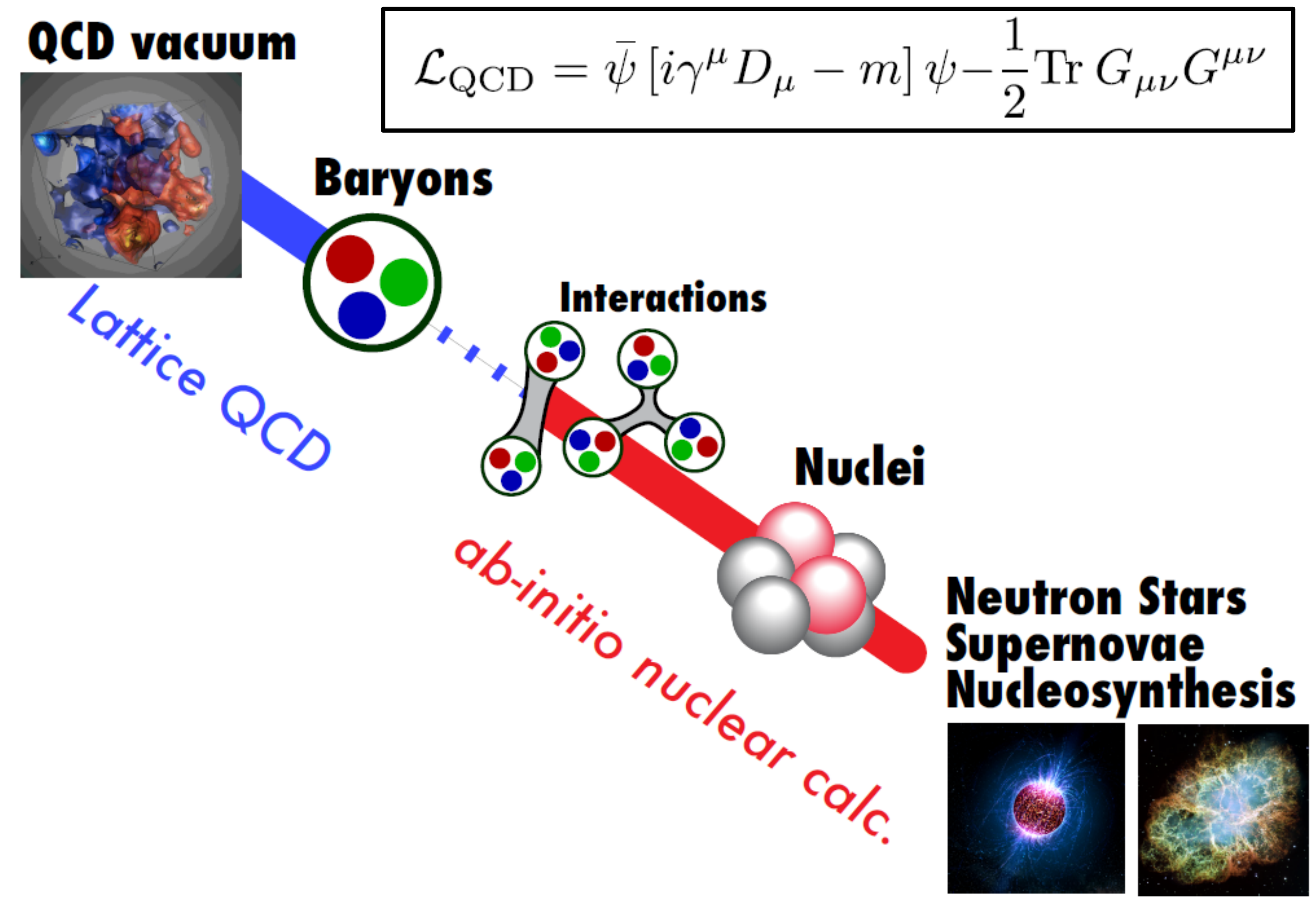
Baryon Interactions from Lattice QCD

$$\mathcal{L}_{\rm QCD} = \bar{\psi}\left[i\gamma^\mu D_\mu - m\right]\psi - \frac{1}{2}{\rm Tr}\, G_{\mu\nu}G^{\mu\nu}$$



Tetsuo Hatsuda (iTHEMS, RIKEN)

"56 International Winter Meeting on Nuclear Physics" (Jan. 26, 2018)

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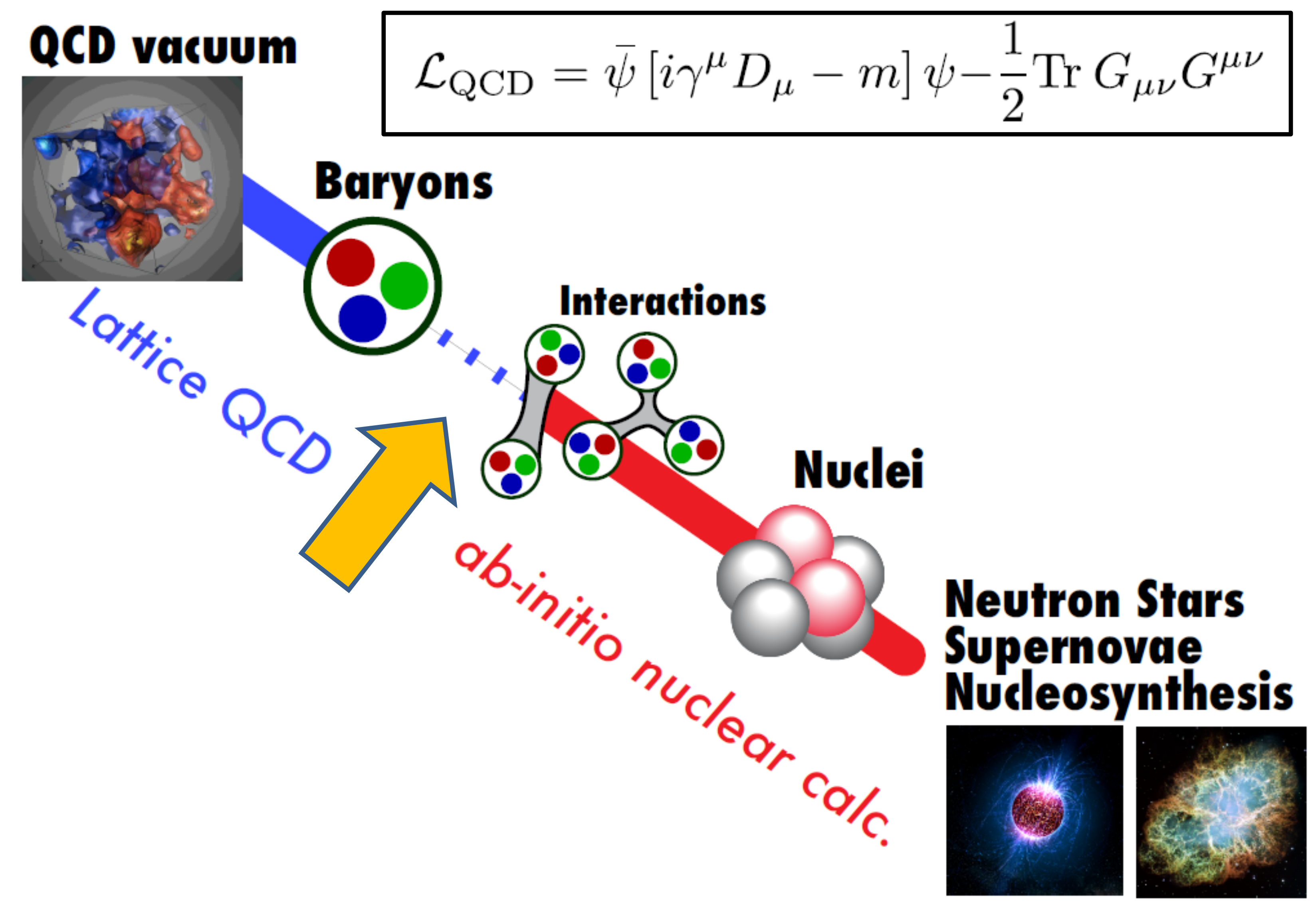

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Contents

I. Precision QCD (5min)

II. Lattice QCD simulations (10min)

III. Baryon interactions from LQCD (10min)

IV. Some examples (10min)

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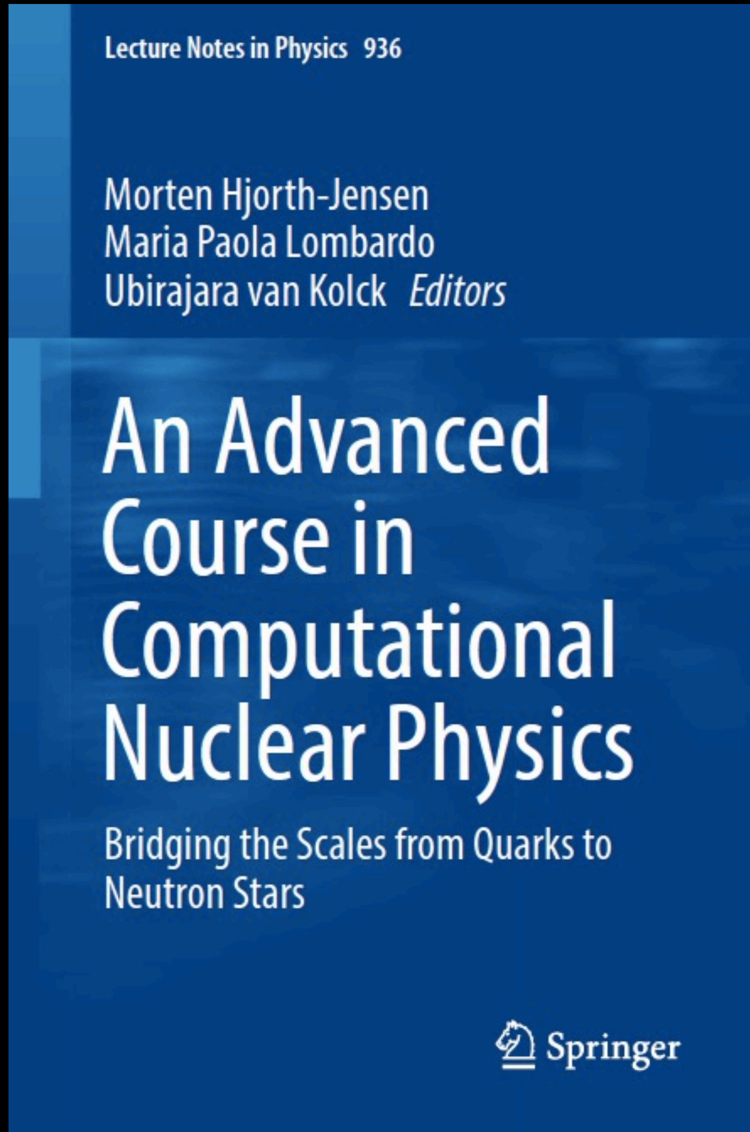

Chapt.3, p.55-91
Lattice QCD (T. Hatsuda)

I. Precision QCD

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

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Running masses: $m_q(Q)$

quark masses (from lattice QCD)	**[MeV]** (MS-bar @ 2GeV)
m_u	2.16 (9)(7)
m_d	4.68 (14)(7)
m_s	93.8 (1.5)(1.9)

FLAG Coll.(2015) http://itpwiki.unibe.ch/flag/

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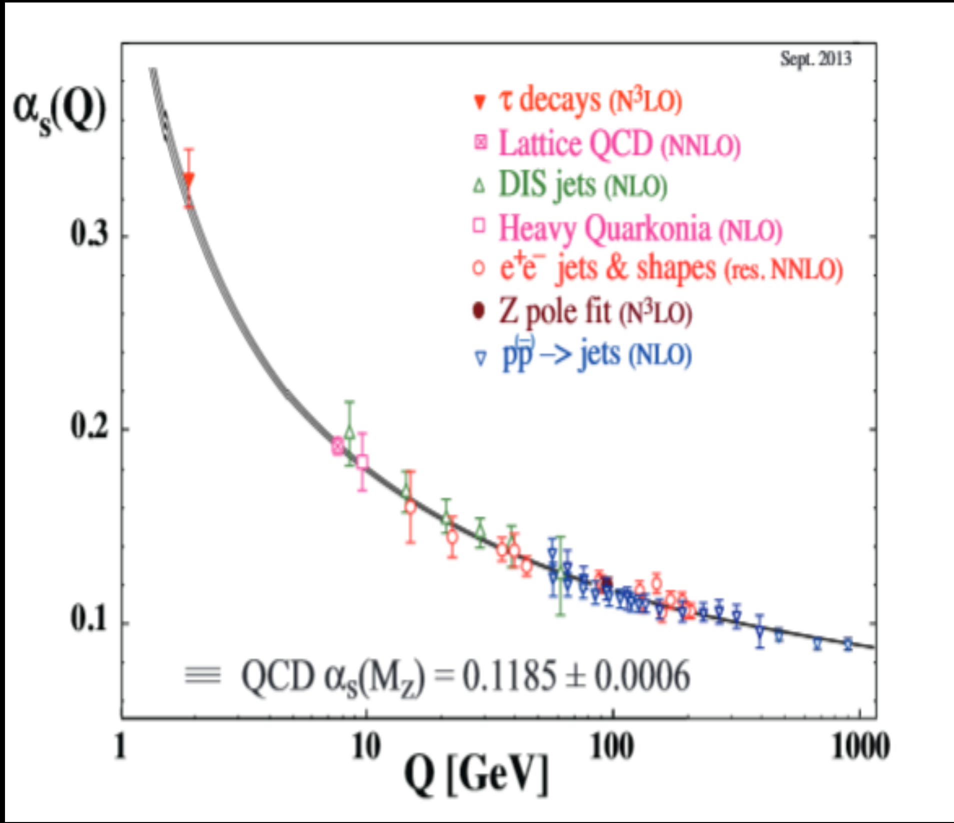
Running coupling: $\alpha_s(Q)=g^2/4\pi$



PDG (2014) http://pdg.lbl.gov/

II. Lattice QCD Simulations

$$Z = \int [dU][dqd\bar{q}] \ \exp\left[-\int d\tau d^3x \mathcal{L}_{\mathrm{E}}\right]$$

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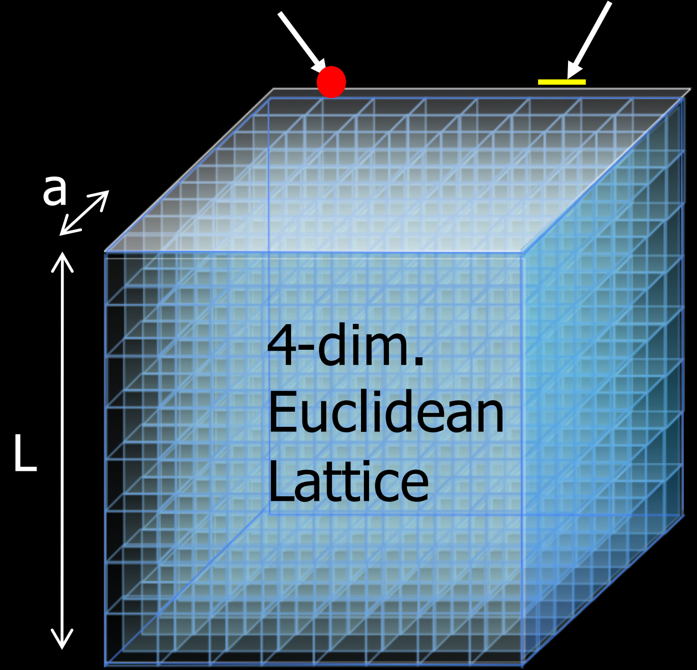
quarks *q* on the sites

gluons $U_\mu = exp(iaA_\mu)$ on the links



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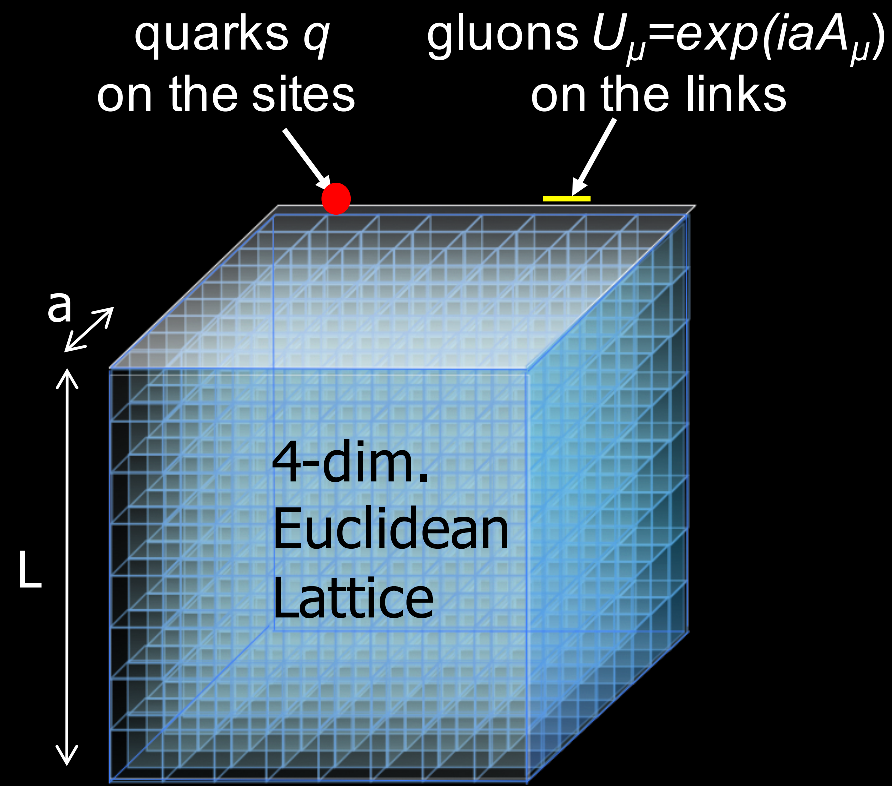

Integration variables

N_S^3 x N_T x $(4_L x 8_C + 4_S x 3_C x N_F)$

~10^8 for $N_S = N_T = 32$ and $N_F = 3$

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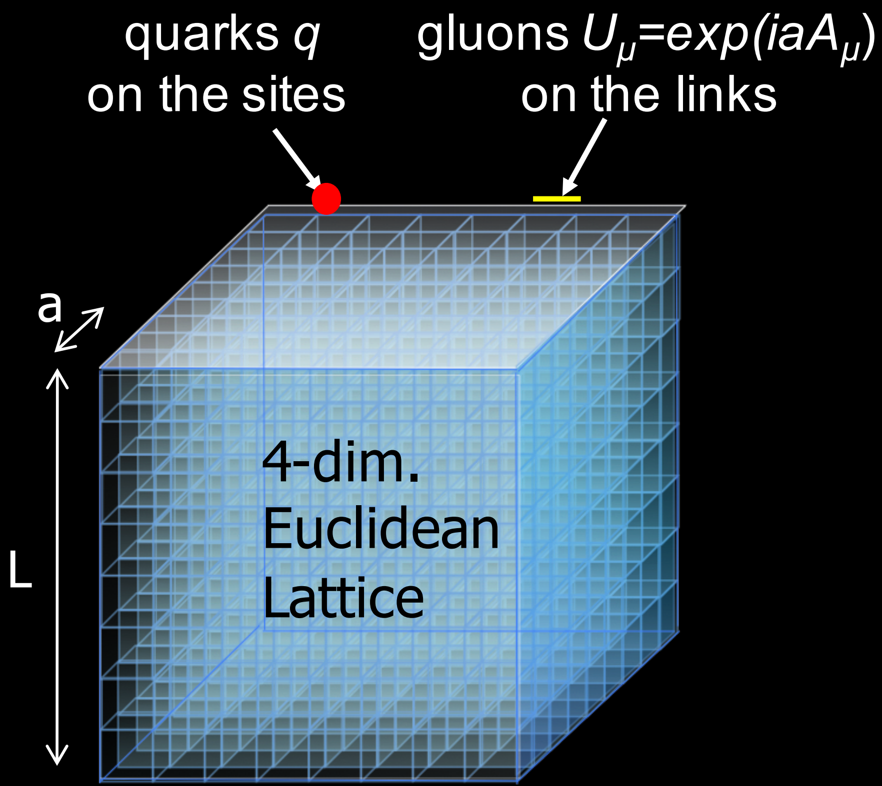

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Importance Sampling

(In practice; Hybrid MC = MD + Metropolis)

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int [d\phi] \, \mathcal{O}(\phi) e^{-S(\phi)}$$

$$= \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}^{(n)} \pm \sqrt{\frac{\sigma^2}{N}}$$

Signal　　Noise

$$\sigma^2 = \frac{1}{N-1} \sum_{n=1}^{N} \langle \mathcal{O}^{(n)} - \langle \mathcal{O} \rangle \rangle^2$$

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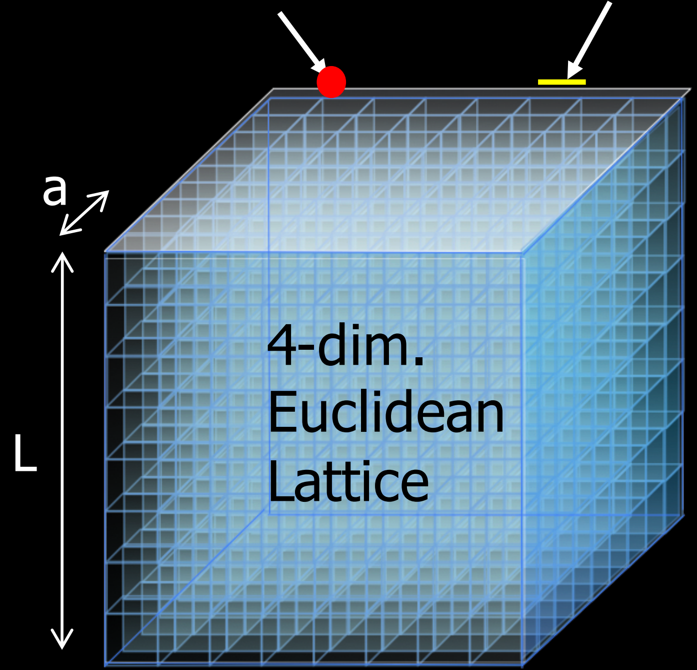

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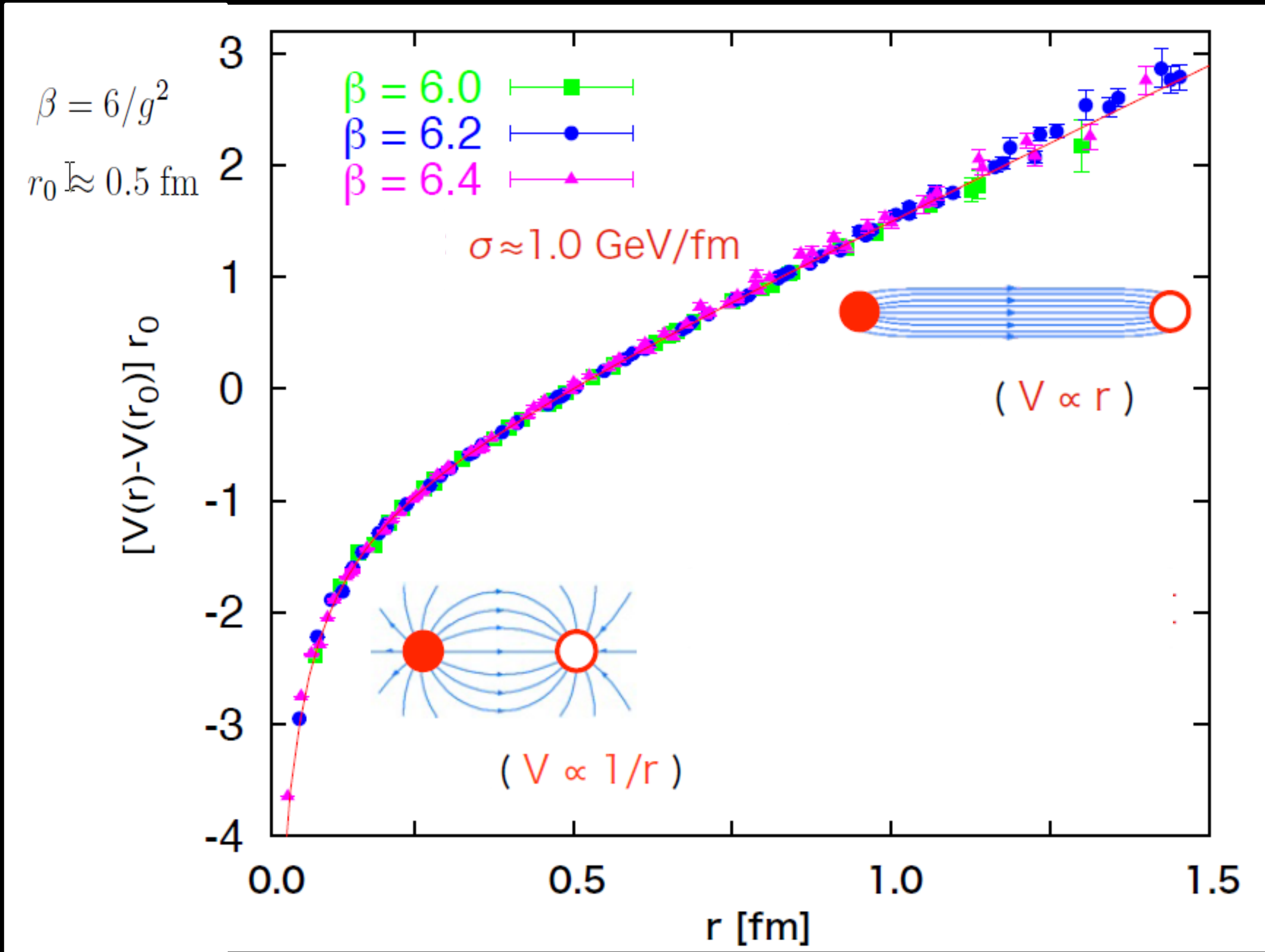
Continuum & Thermodynamic Limits

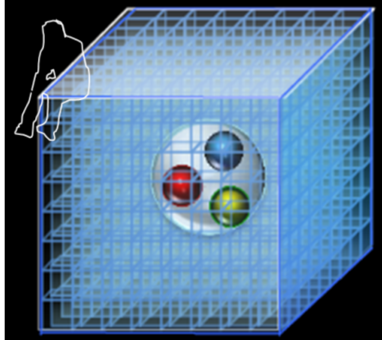
$(a \rightarrow 0 \ \& \ L \rightarrow \infty)$

Linear Confinement in LQCD with $N_F=0$



adapted from G. Bali, Phys. Rep. 343 (2001) 1

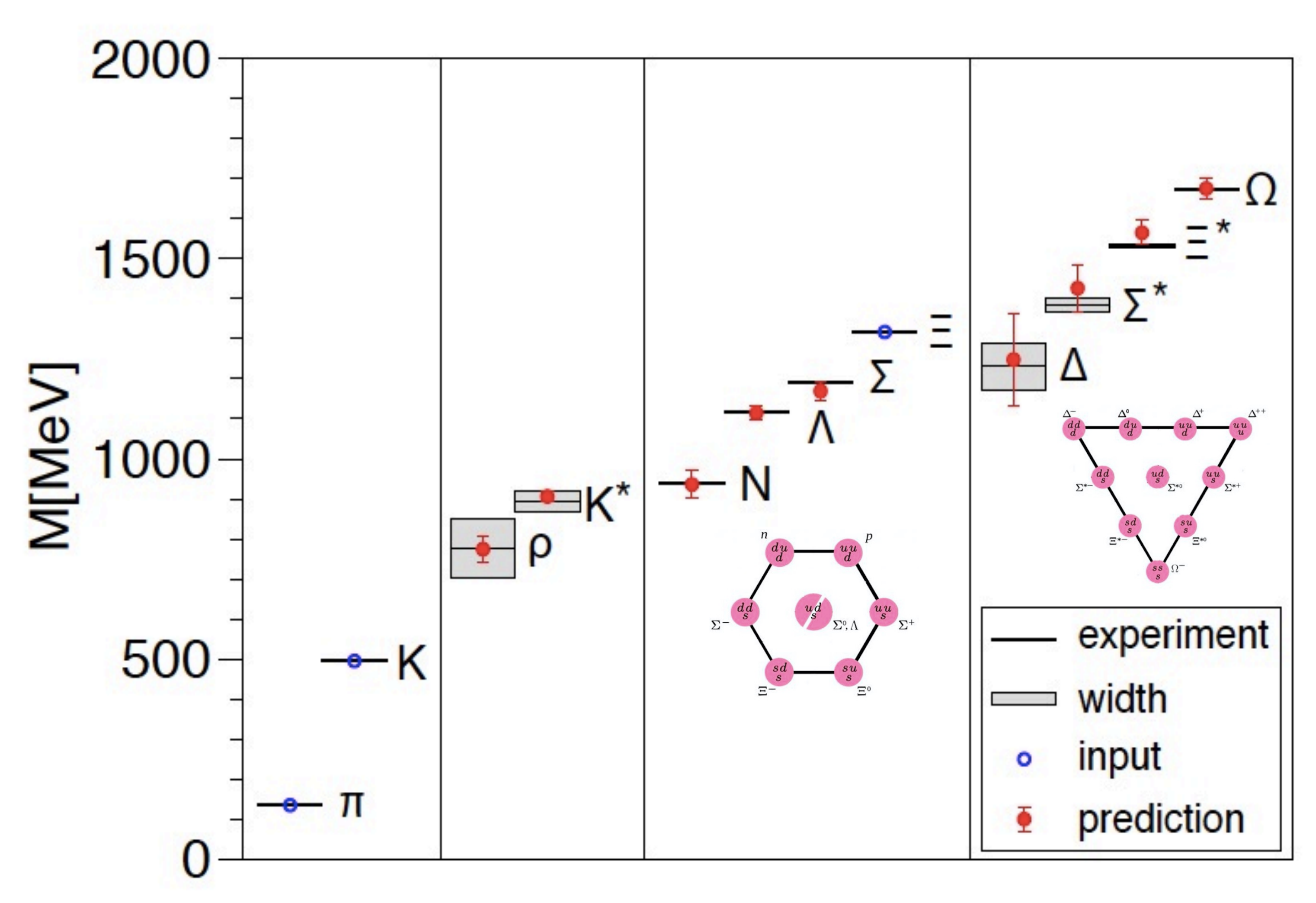
Light Hadron Masses in LQCD with N_F=2+1



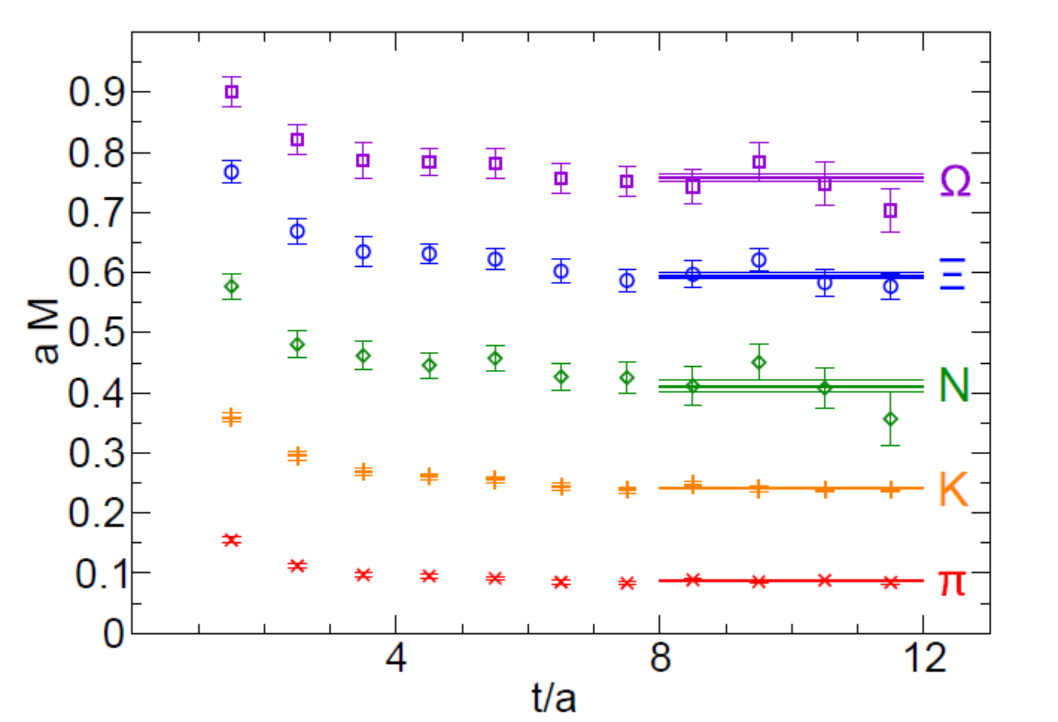
a_{min} = 0.065 fm

L_{max} = 4.1 fm

$m_{\pi,min}$ =190 MeV



taken from Fodor and Hoelbling, Rev. Mod. Phys. 84 (2012) 449

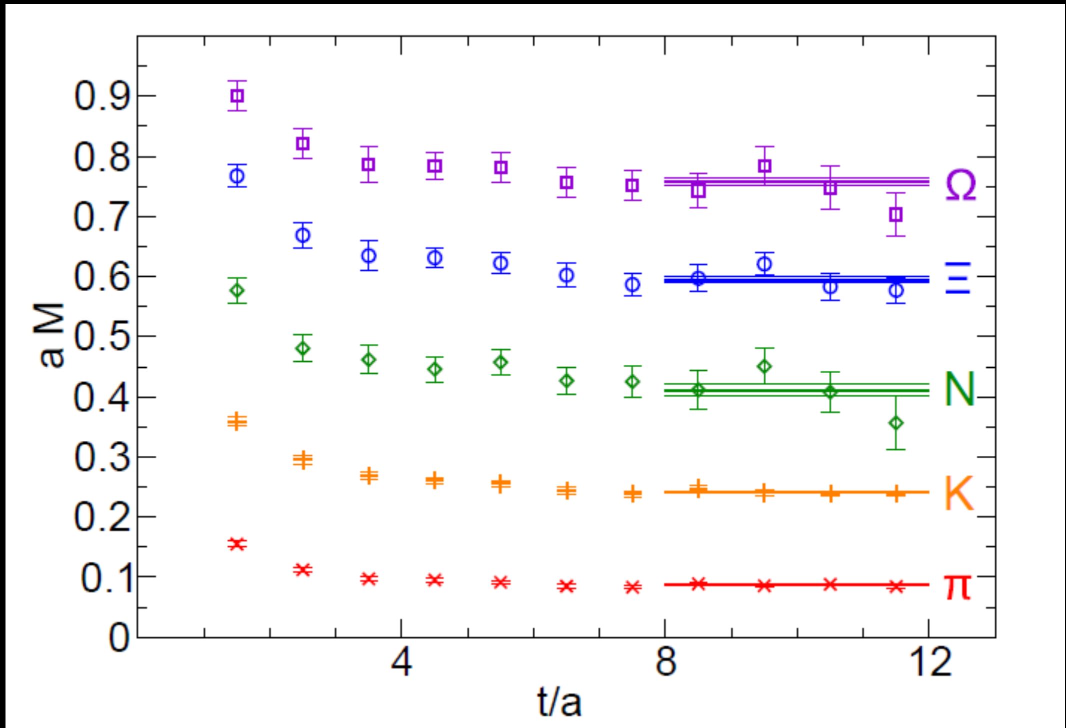


Effective Mass plot

$$C_H(\tau) \xrightarrow[\tau\to\infty]{} |Z_H|^2 e^{-M_H \tau}$$

$$aM_H^{\text{eff}}(\tau) = \ln\left(\frac{C_H(\tau)}{C_H(\tau+a)}\right)$$

BMW Collaboration
Science 322 (2008) 1224



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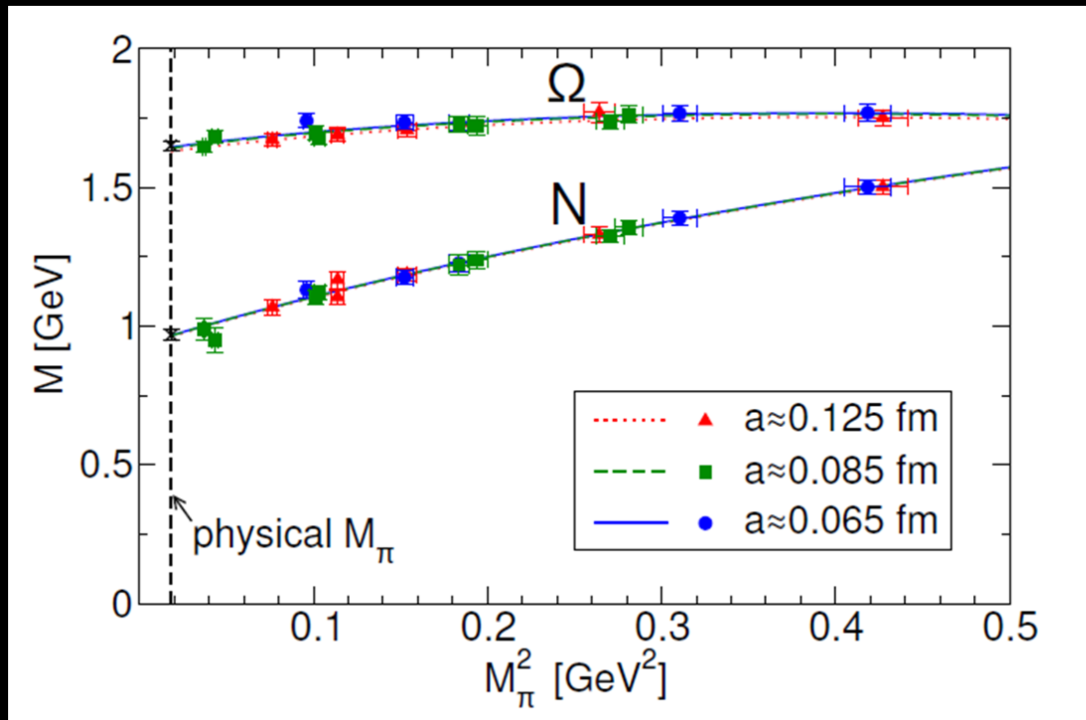
Chiral Extrapolation



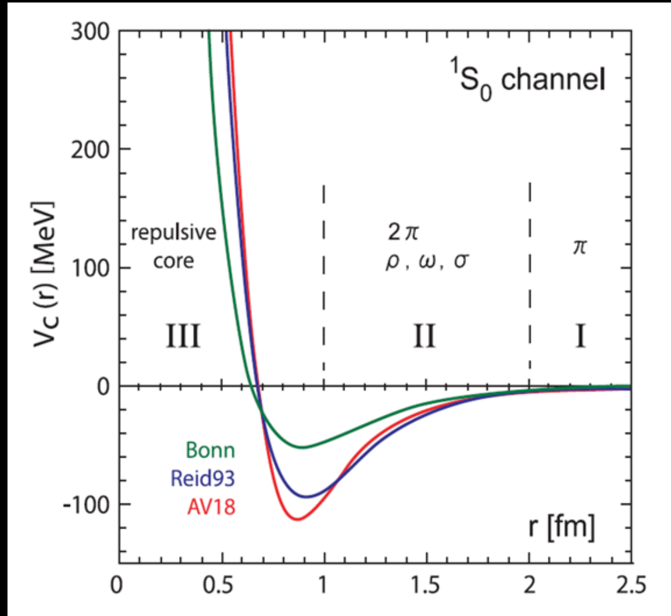
BMW Collaboration
Science 322 (2008) 1224

III. Baryon interactions from LQCD ?

- NN int.: about 4500 np and pp scatt. data

"high precision" NN interactions		# of parameters
CD Bonn	(p space)	38
AV18	(r space)	40
EFT in N^3LO	($n\pi$+contact)	24

R. Machleidt, arXiv:0704.0807 [nucl-th]



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- YYN, YNN, YYY : data none

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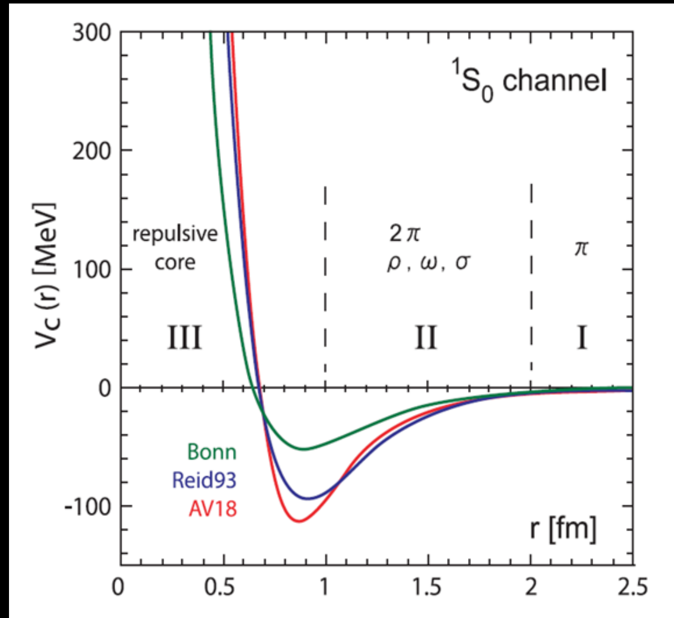

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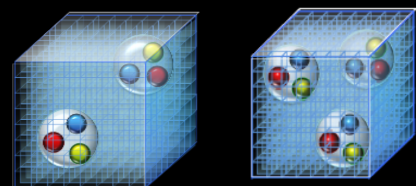
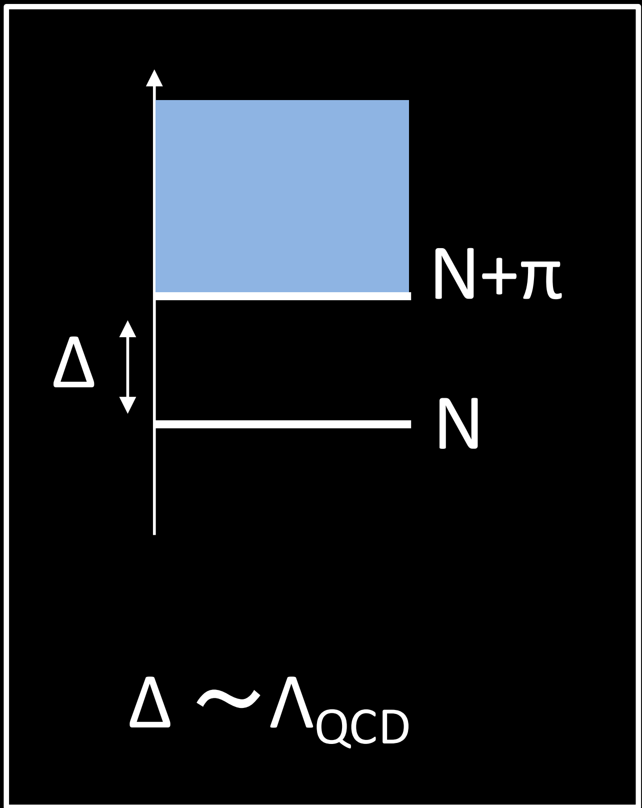
QCD has only 4 parameters: g , $m_{u,d,s}$

→ Derivation of the BB and BBB interactions from LQCD ?

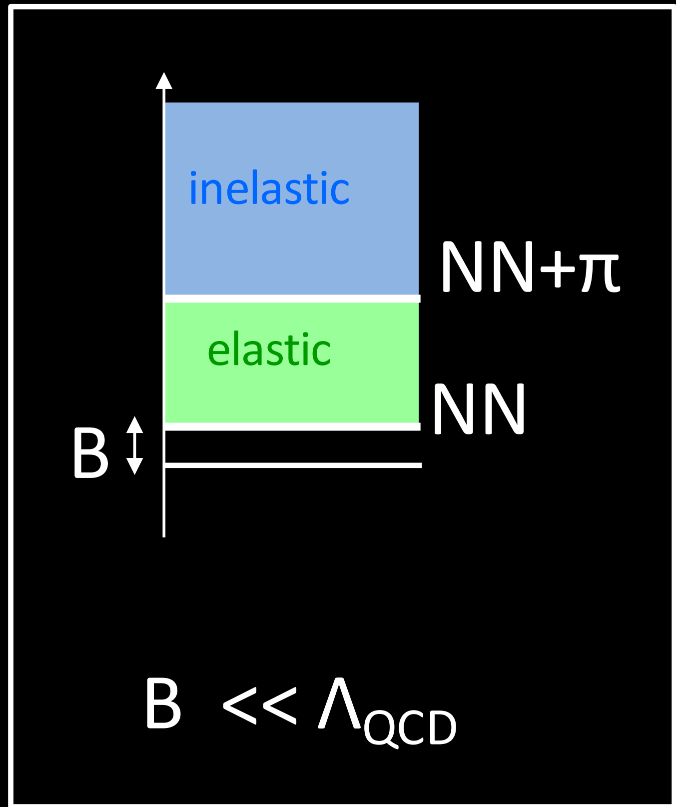


Difference between single-baryon and multi-baryon in QCD

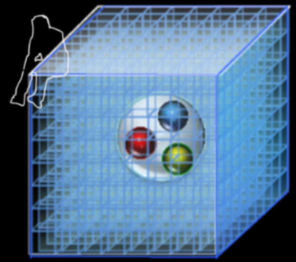
Single nucleon

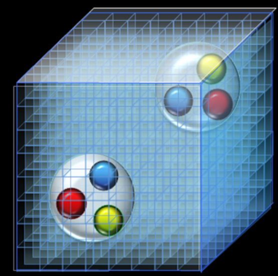


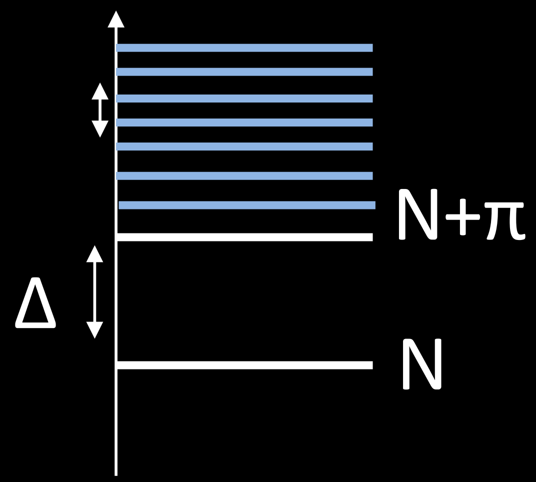
Two nucleons



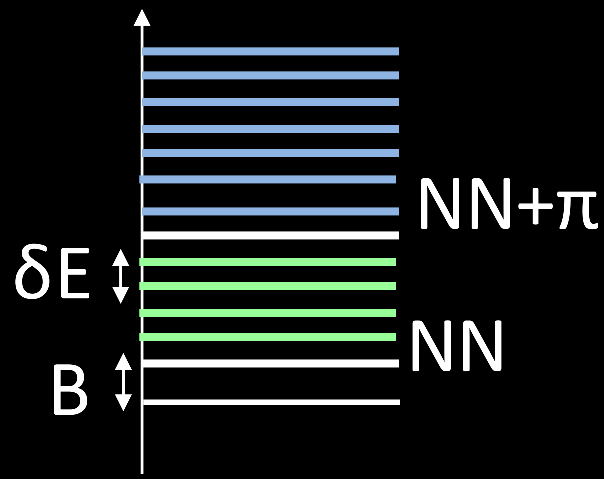
Difference between single-baryon and multi-baryon in LQCD







$\tau^{-1} = \Delta \sim 200$ MeV



$\tau^{-1} = \delta E \sim 20\,(9/L)^2$ MeV

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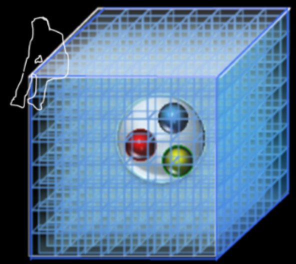

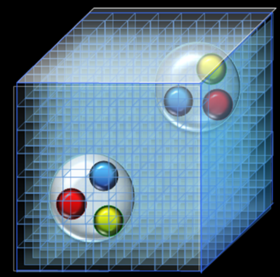

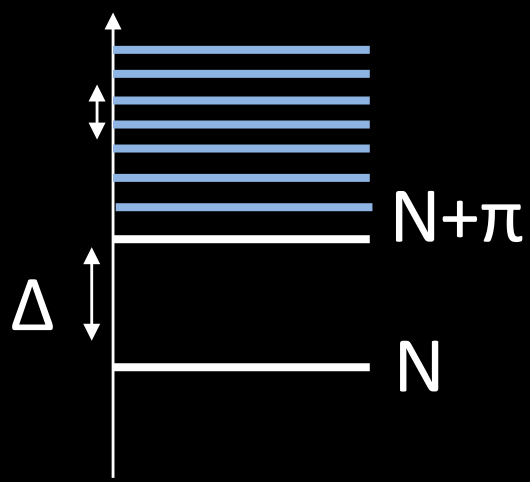

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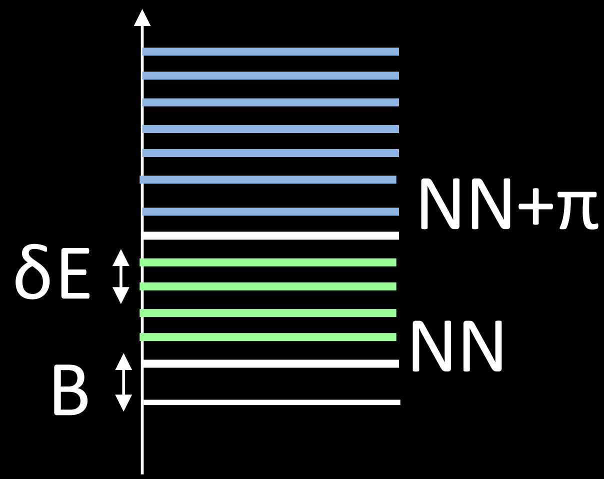


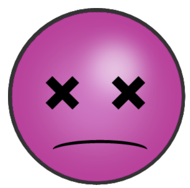
$\tau^{-1} = \delta E \sim 20\,(9/L)^2$ MeV

$S/N \sim \exp(-m_N\,\tau) \times \sqrt{N}$

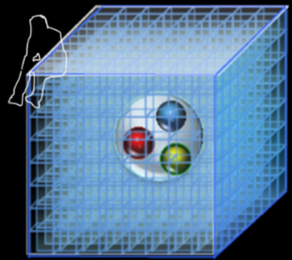
$\sim 10^{-2} \times \sqrt{N}$

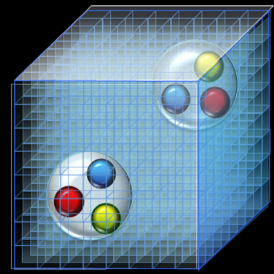
$S/N \sim \exp(-2m_N\,\tau) \times \sqrt{N}$

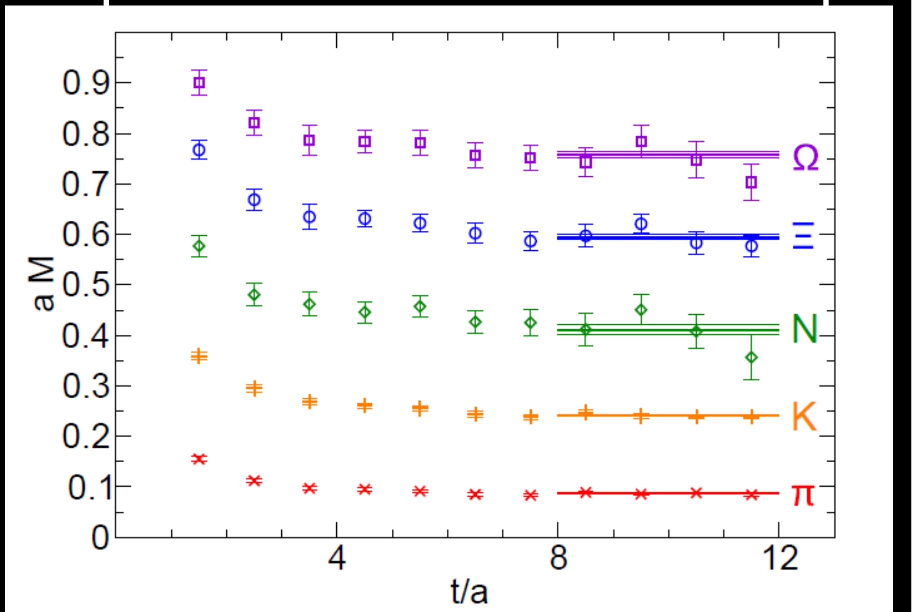
$\sim 10^{-41} \times \sqrt{N}$

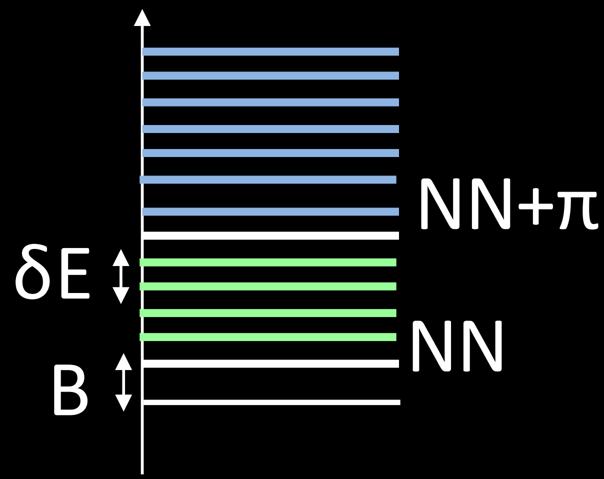


Difference between single-baryon and multi-baryon in LQCD





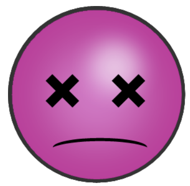




$\tau^{-1} = \delta E \sim 20\ (9/L)^2$ MeV

S/N ~ $\exp(-m_N \tau) \times \sqrt{N}$
~ $10^{-2} \times \sqrt{N}$

S/N ~ $\exp(-2 m_N \tau) \times \sqrt{N}$
~ $10^{-41} \times \sqrt{N}$



Space-Time Hadronic Correlation to overcome the difficulty

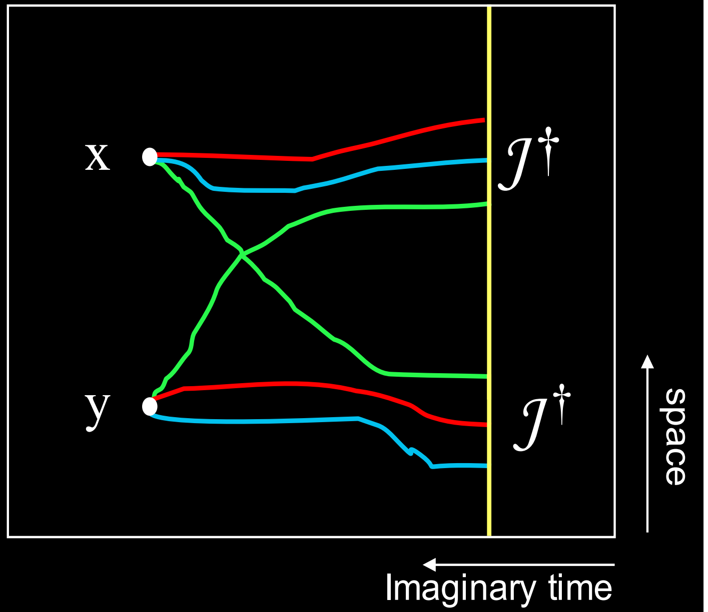
$$\left\{\frac{1}{4M_B}\frac{\partial^2}{\partial\tau^2}-\frac{\partial}{\partial\tau}-H_0\right\}\mathscr{R}(\mathbf{r},\tau)=\int d^3r' U(\mathbf{r},\mathbf{r}')\mathscr{R}(\mathbf{r}',\tau)$$

R(r, τ)→U(r,r')→ phase shift, binding energy

$$U(\mathbf{r},\mathbf{r}') = V(\mathbf{r},\mathbf{v})\delta(\mathbf{r}-\mathbf{r}'),$$

$$V(\mathbf{r},\mathbf{v}) = \underbrace{V_C(r)+V_T(r)S_{12}}_{\text{LO}}+\underbrace{V_{LS}(r)\mathbf{L}\cdot\mathbf{S}}_{\text{NLO}}+\underbrace{O(\mathbf{v}^2)}_{\text{N}^2\text{LO}}+\cdots$$

Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001
Ishii et al. [HAL QCD Coll.], PLB 712 (2012) 437



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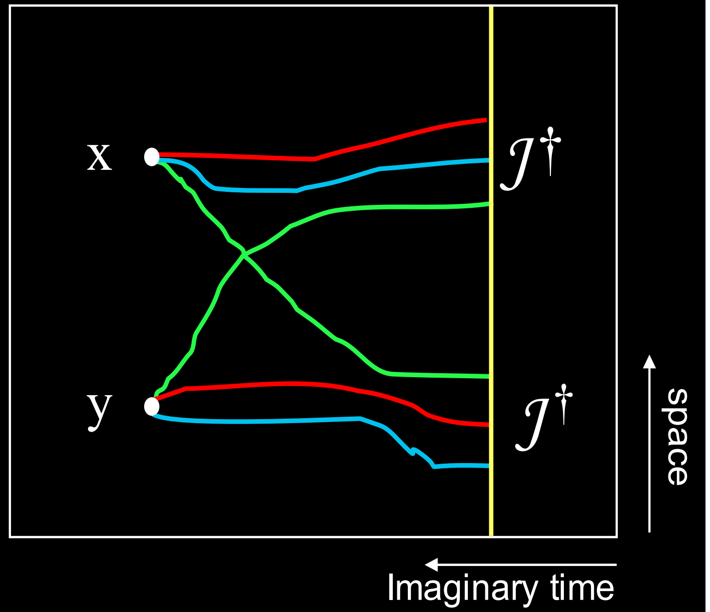
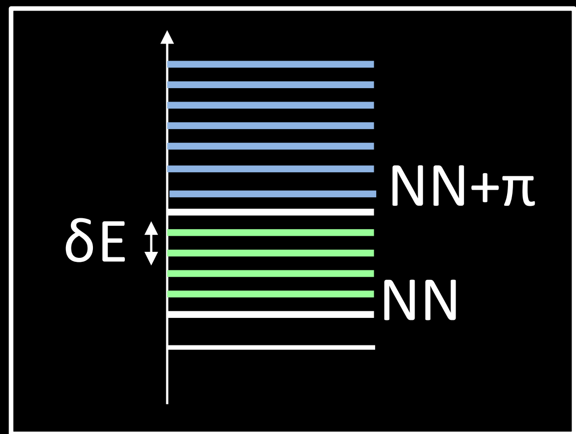
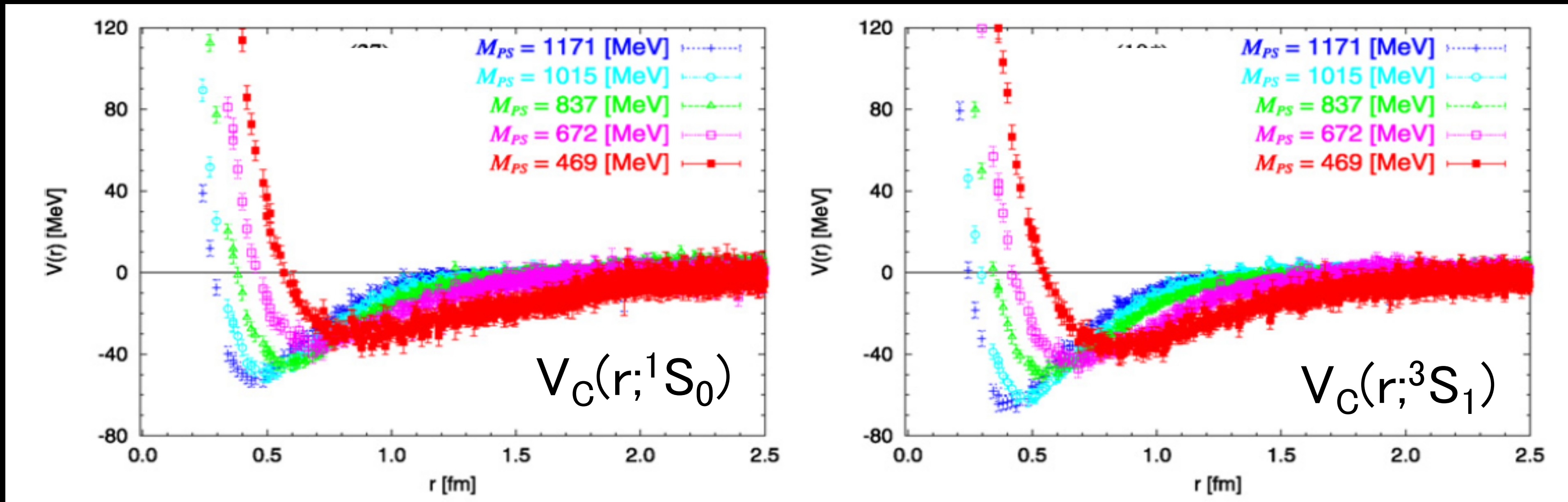


	time correlation	Space-time correlation
inelastic	Noise	Noise
elastic	Noise	Signal
ground	Signal	Signal
necessary τ	τ > 10 fm	τ ~ 1 fm



IV. Some examples
Pilot study (L=3.9 fm) in LQCD with N_F=3



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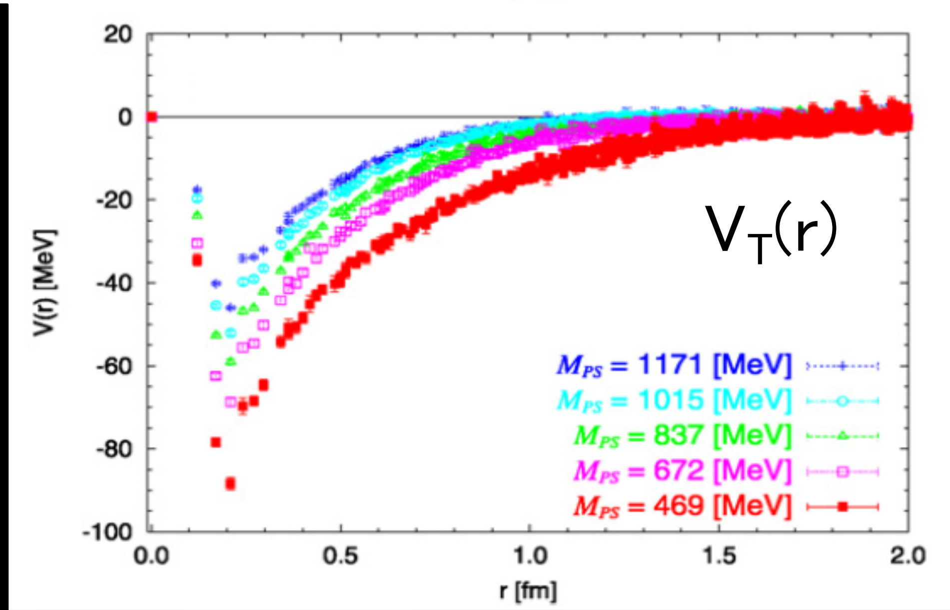
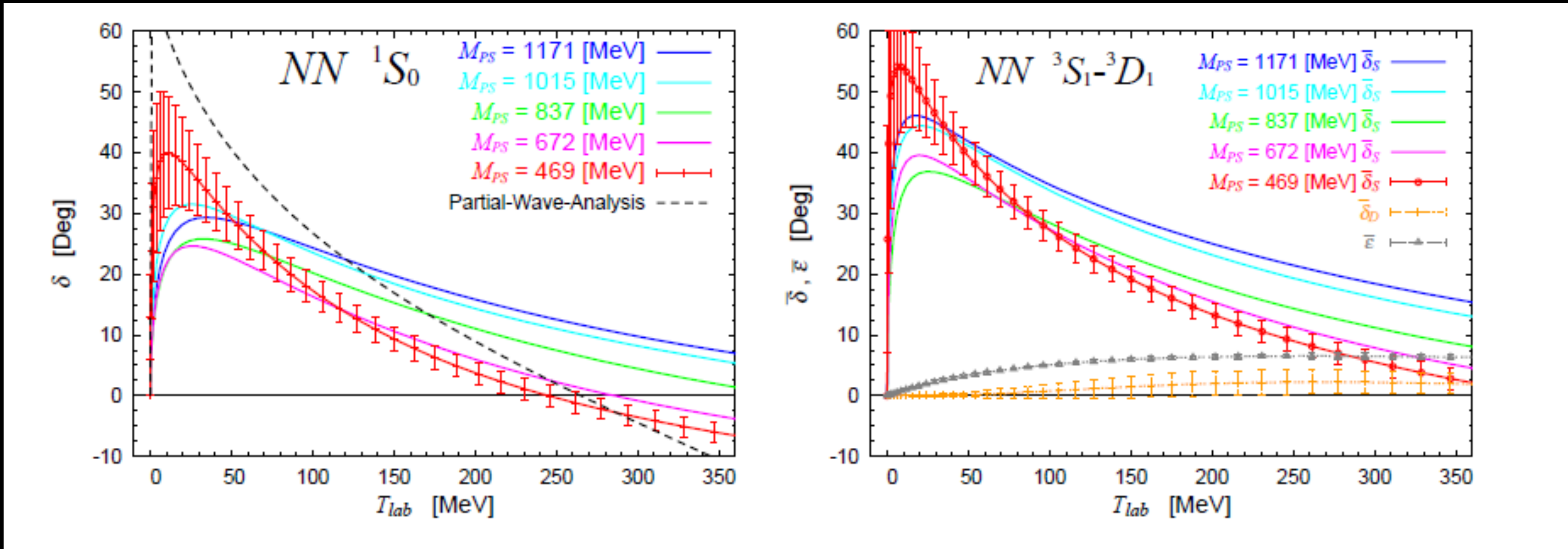


HAL QCD Coll.
Phys. Rev. Lett. 106 (2011) 162002
Nucl. Phys. A881 (2012) 28
Phys. Rev. Lett. 111 (2013) 112503

Pilot study (L=3.9 fm) in LQCD with N_F=3: NN phase shifts



- Stronger attraction in the deuteron channel
- Physical point is close to the unitary region

HAL QCD Coll.,
Phys. Rev. Lett. 106 (2011) 162002
Nucl. Phys. A881 (2012) 28
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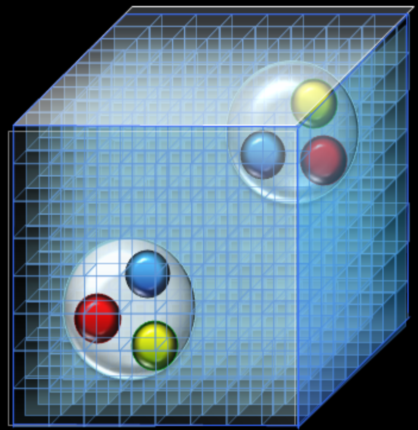
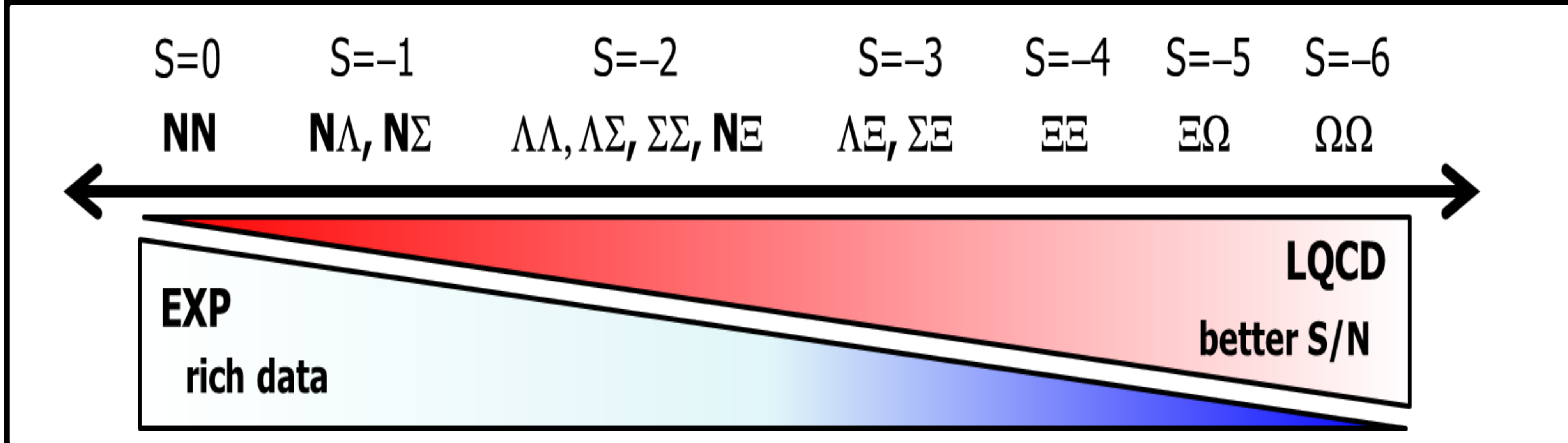
Large scale LQCD simulations with $N_F=2+1$



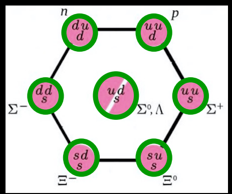
$a = 0.085$ fm

$L = 8.1$ fm

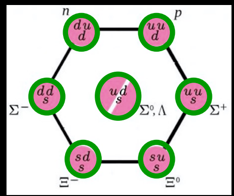
$m_\pi = 146$ MeV
$M_K = 525$ MeV



Flavor SU(3) Classification : Two-baryon



x



$$8 \times 8 = 27 + 8_s + 1 + 10^* + 10 + 8_a$$

$H_{\Lambda\Lambda-N\Xi-\Lambda\Sigma}$ (J=0) → 1

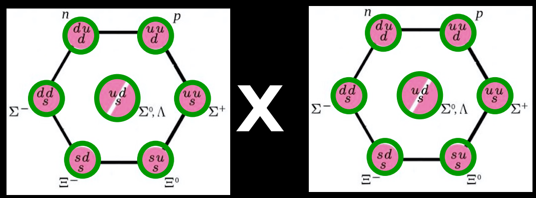
Jaffe (1977)

D (J=1) → 10^*

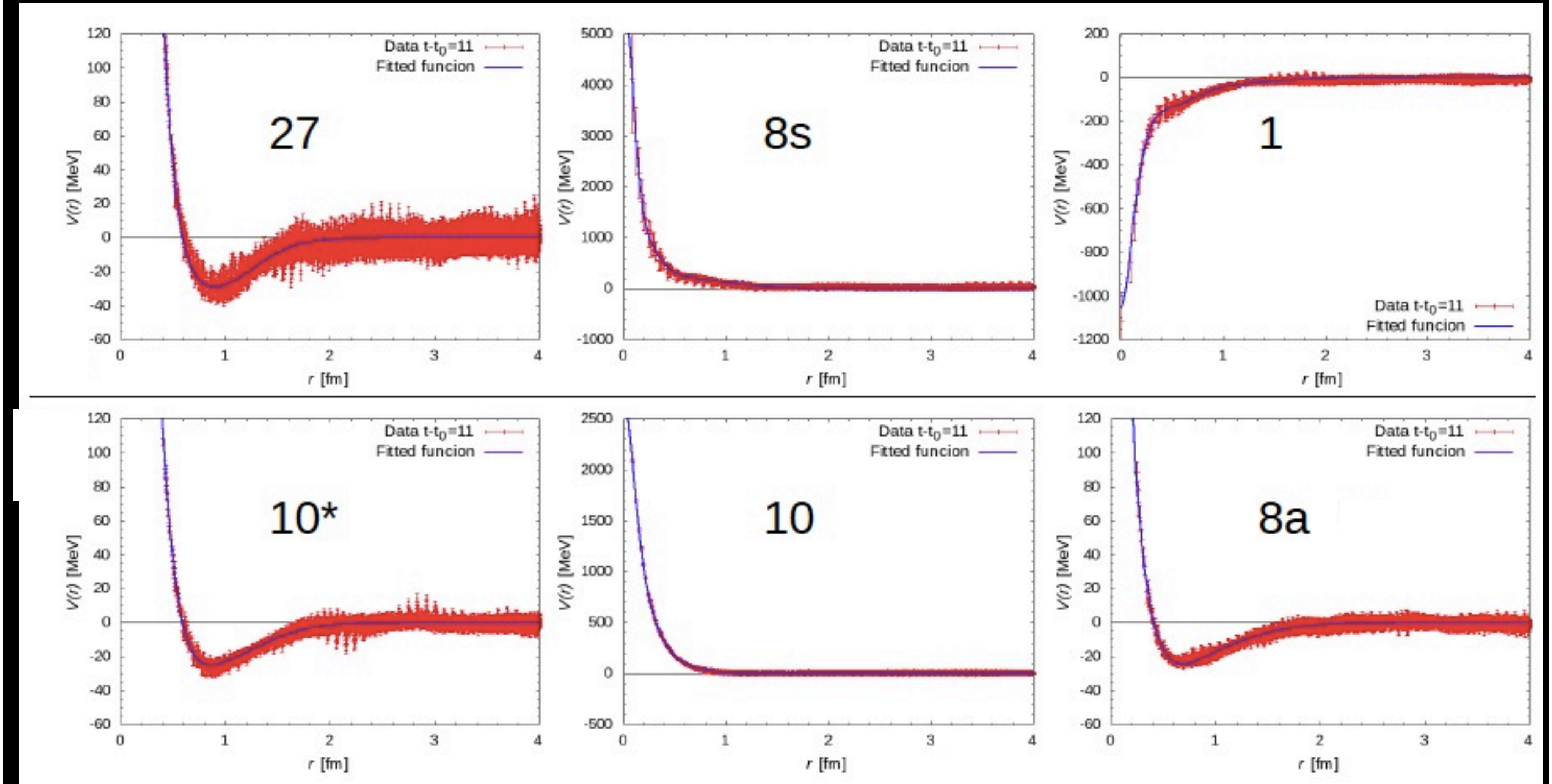
Rarita-Schwinger (1941)

BB interactions in the flavor basis: $V_C(r)$

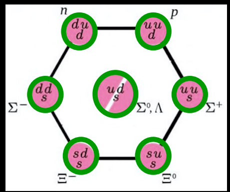
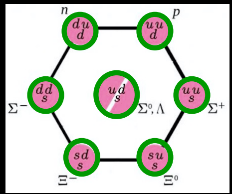
preliminary



$$8 \times 8 = \underbrace{27 + 8s + 1}_{^1S_0} + \underbrace{10^* + 10 + 8a}_{^3S_1,\ ^3D_1}$$



Flavor SU(3) Classification : Two-baryon

 x 

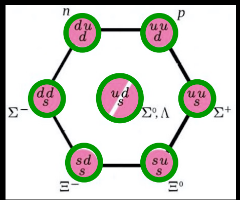
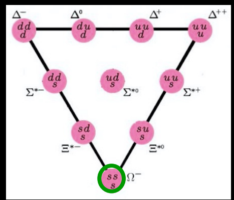
$$8 \times 8 = 27 + 8_s + 1 + 10^* + 10 + 8_a$$

$H_{\Lambda\Lambda-N\Xi-\Lambda\Sigma}$ (J=0) → 1

Jaffe (1977)

D (J=1) → 10^*

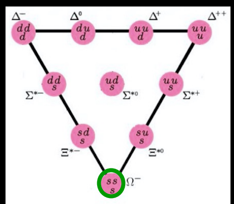
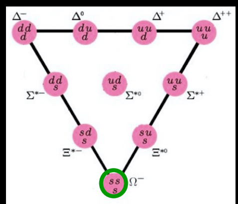
Rarita-Schwinger (1941)

 x 

$$8 \times 10 = 35 + 8 + 10 + 27$$

NΩ (J=2) → 8

Goldman et al (1987)

 x 

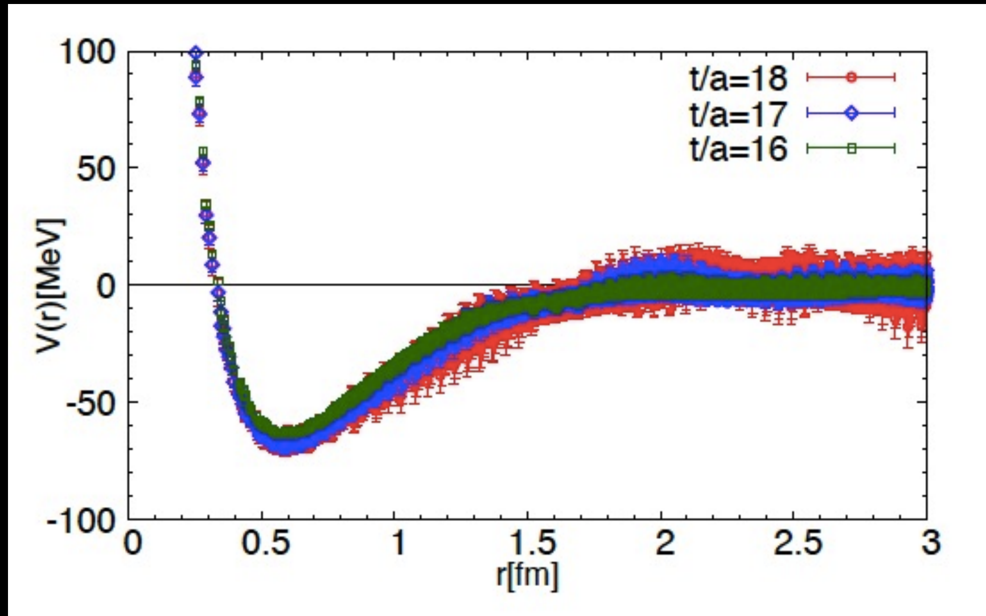
$$10 \times 10 = 28 + 27 + 35 + 10^*$$

ΩΩ(J=0) → 28

Zhang et al (1997)

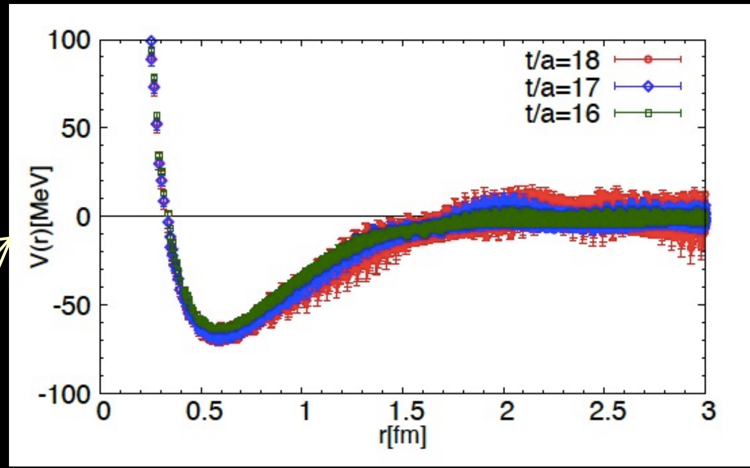
Most strange dibaryon ΩΩ

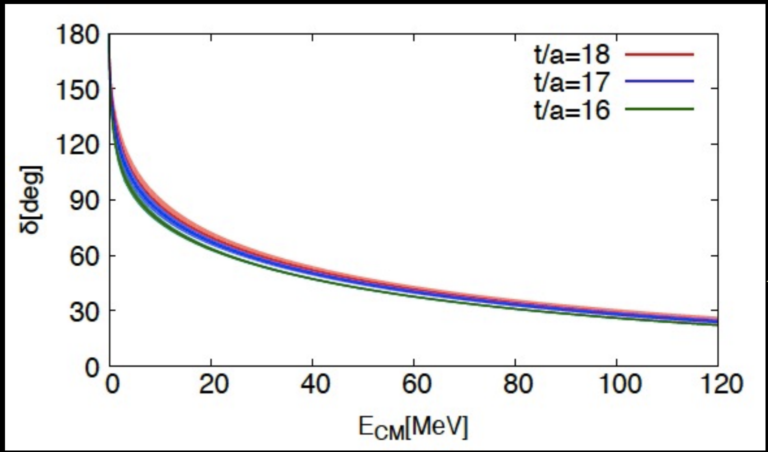
HAL QCD Coll.,
arXiv:1709.00654 [hep-lat]



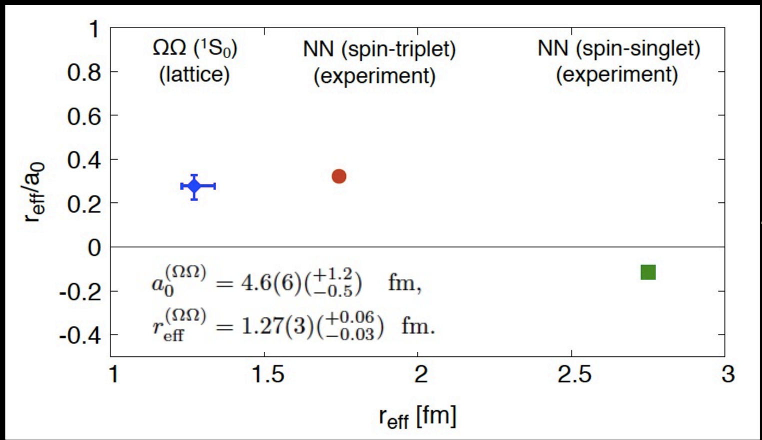
Most strange dibaryon ΩΩ

HAL QCD Coll.,
arXiv:1709.00654 [hep-lat]

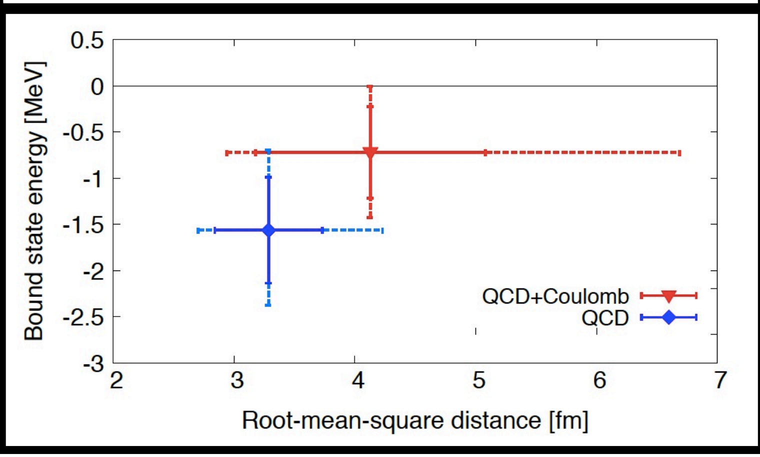




Phase shift



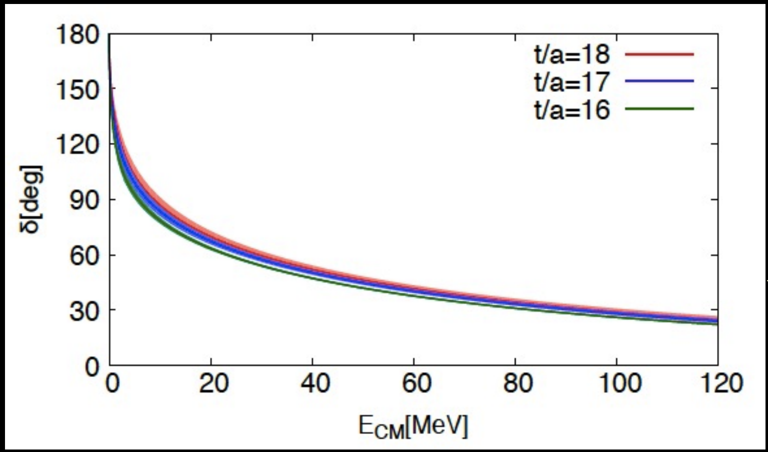
Scatt.length
Effective range

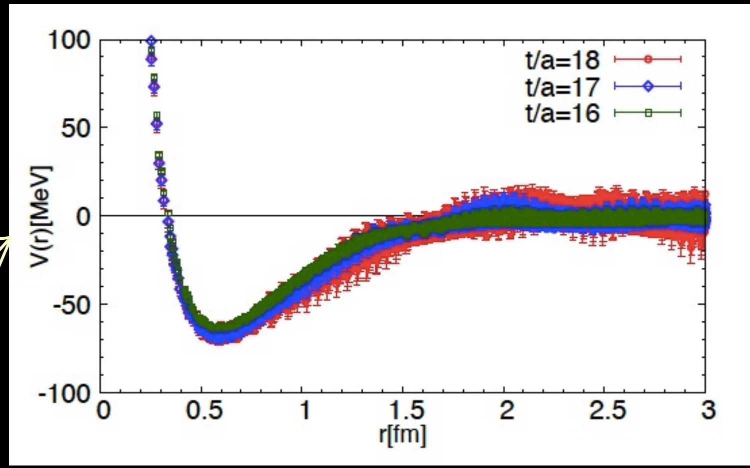


Binding energy
Matter size

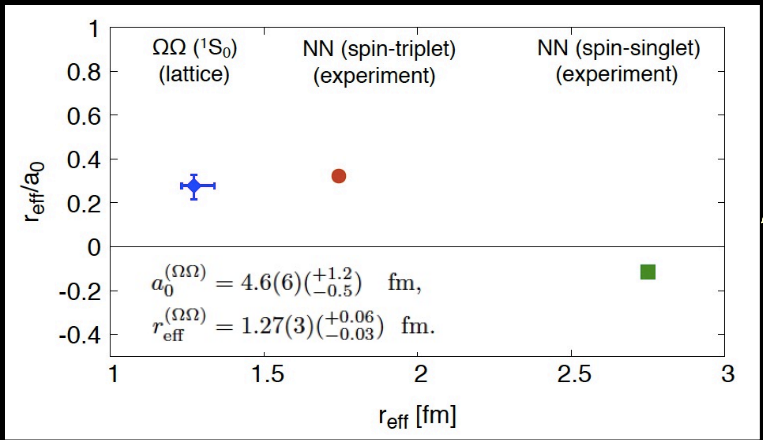
Most strange dibaryon ΩΩ

HAL QCD Coll.,
arXiv:1709.00654 [hep-lat]

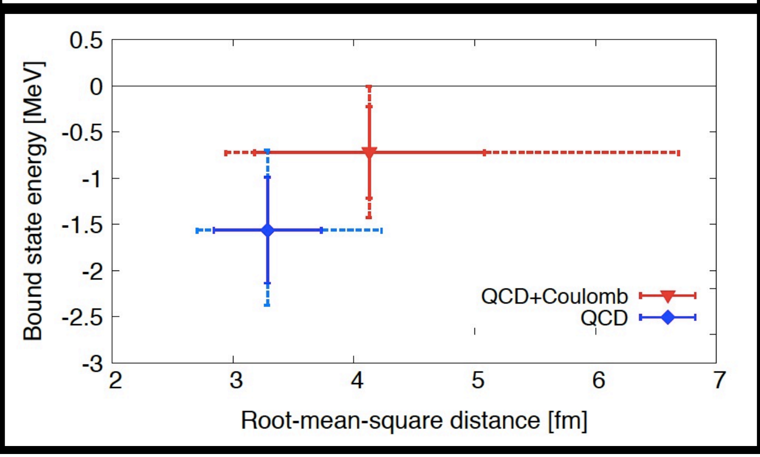




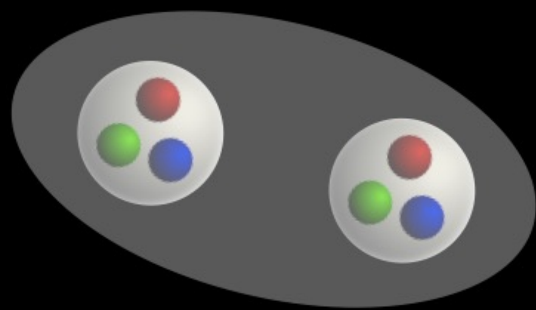
Phase shift



Scatt.length
Effective range

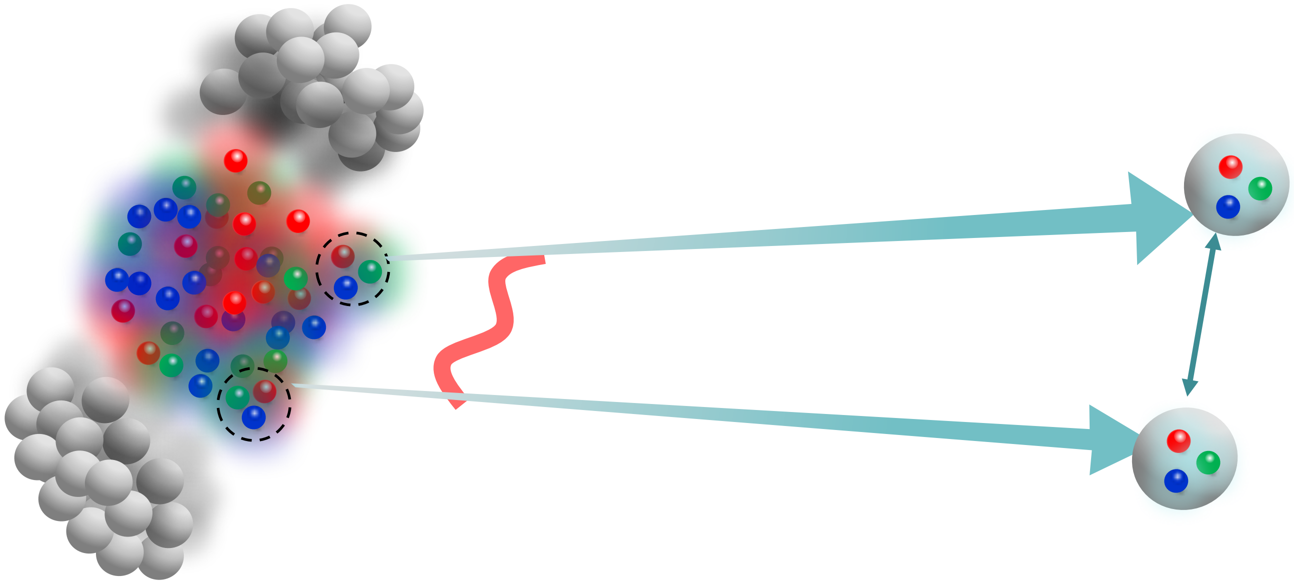


Binding energy
Matter size



Deuteron (Urey, 1931) and Ω Ω
are only two stable dibaryons?

How HIC can tell us about interaction?



$$Q = \sqrt{-\left(\frac{p_1 - p_2}{2} - \frac{(p_1 - p_2)\cdot P}{P^2} P\right)^2}$$

Measuring Pair Correlation → Constraint on Pairwise Interaction

$$C_{AB}(Q) = \frac{N_{AB}^{\mathrm{pair}}(Q)}{N_A N_B(Q)} = \begin{cases} 1 & \textbf{No Correlation} \\ \text{others} & \textbf{Interaction, Quantum Interference etc} \end{cases}$$

Summary: From QCD to Nuclear Physics

Lattice QCD

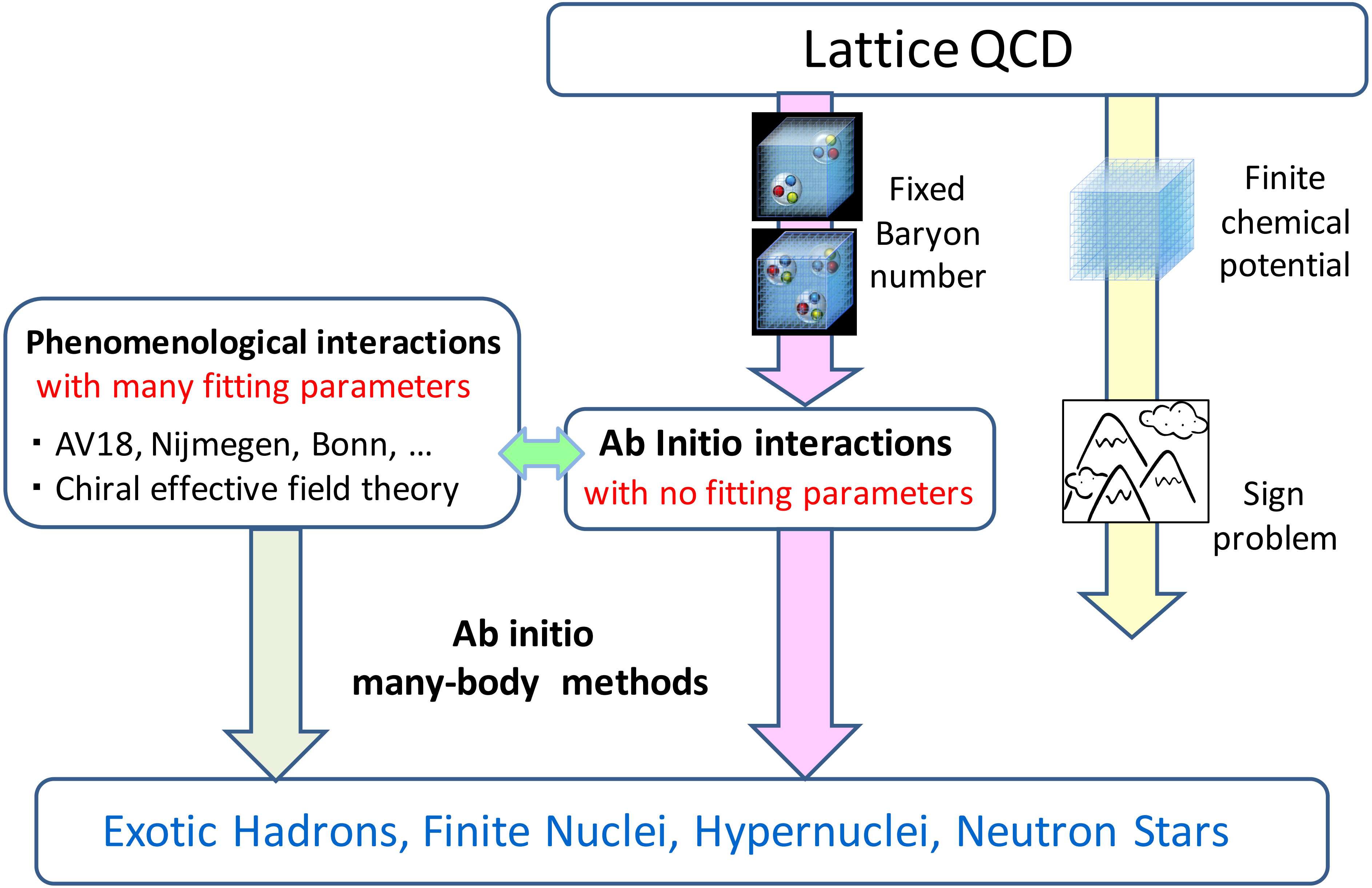
Phenomenological interactions
with many fitting parameters

- AV18, Nijmegen, Bonn, …
- Chiral effective field theory

Ab initio
many-body methods

Exotic Hadrons, Finite Nuclei, Hypernuclei, Neutron Stars

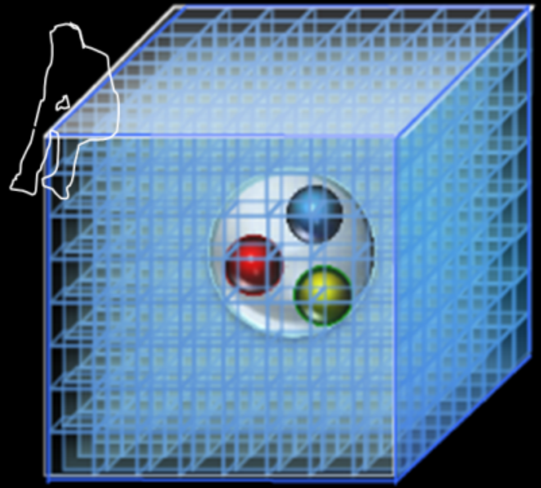
Summary: From QCD to Nuclear Physics



END

Precision LQCD
-- neutron-proton mass difference --

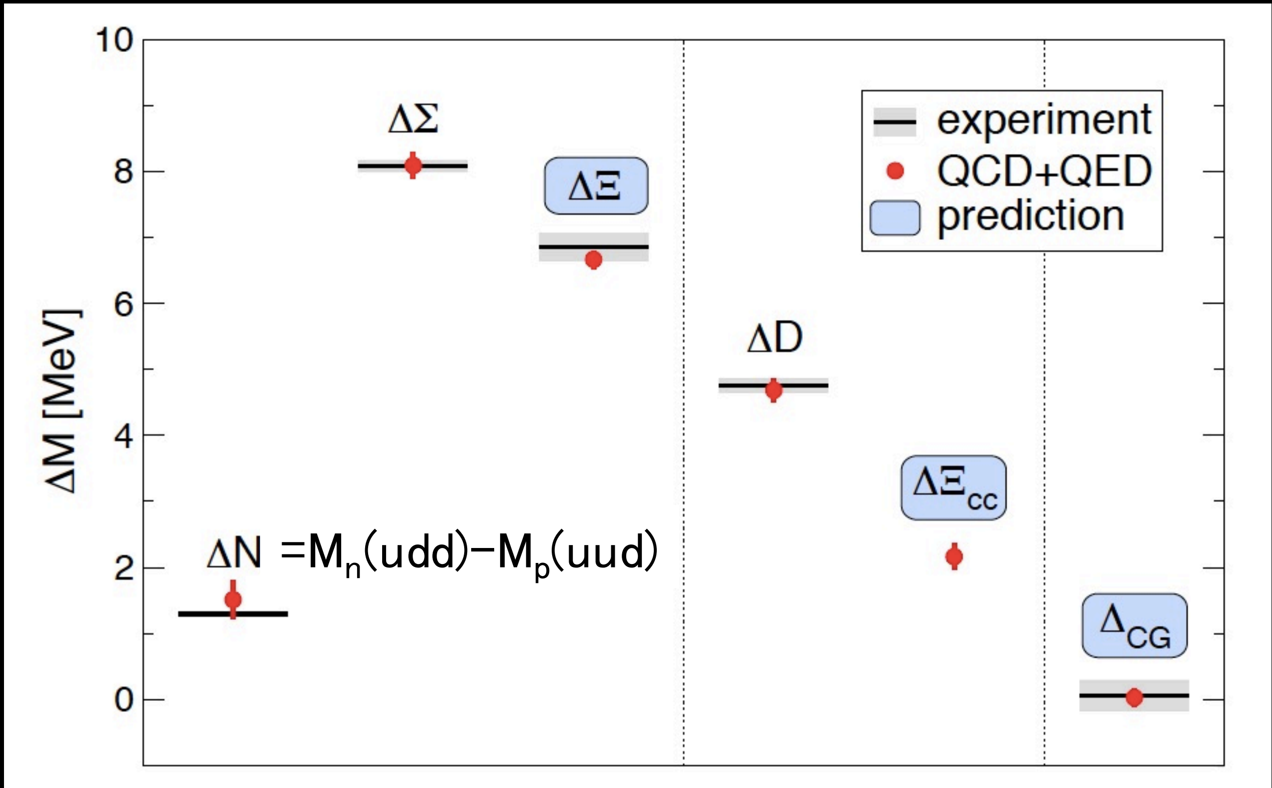
BMW Coll., Science 347 (2015) 1452



a_{min} = 0.054 fm

L_{max} = 8 fm

$m_{\pi,min}$ = 190 MeV



$(M_n - M_p)_{lat}$ = 1.51(16)(23) MeV

$(M_n - M_p)_{exp}$ = 1.29 MeV

Problem of Signal to Noise Ratio

Parisi, Lepage (1989)

$$G(r,t) = \langle 0|\mathcal{O}(r,t)\overline{\mathcal{O}}(0)|0|\rangle = \sum_n \alpha_n \psi_n(r) e^{-E_n t} \xrightarrow[t\to\infty]{} \alpha_0 \psi_0(r) e^{-E_0 t}$$

Single pion $\dfrac{\langle \pi(t)\pi(0)\rangle}{\sqrt{\langle \pi\pi(t)\pi\pi(0)\rangle}} \sim \dfrac{\exp(-m_\pi t)}{\sqrt{\exp(-2m_\pi t)}} \sim \text{const.}$ Signal/Noise ~ $\sqrt{N_{conf}}$

Multi pion Signal/Noise ~ $\sqrt{N_{conf}}$

Single nucleon $\dfrac{\langle N(t)\bar{N}(0)\rangle}{\sqrt{\langle |N(t)\bar{N}(0)|^2\rangle}} \sim \dfrac{\exp(-m_N t)}{\sqrt{\exp(-3m_\pi t)}} \sim \exp[-(m_N - 3/2 m_\pi)t]$

Signal/Noise ~ $\exp(- m_N t) \times \sqrt{N_{conf}}$

Multi nucleon $\dfrac{\langle N^{\mathbf{A}}(t)\bar{N}^{\mathbf{A}}(0)\rangle}{\sqrt{\langle |N^{\mathbf{A}}(t)\bar{N}^{\mathbf{A}}(0)|^2\rangle}} \sim \dfrac{\exp(-\mathbf{A} m_N t)}{\sqrt{\exp(-3\mathbf{A} m_\pi t)}} \sim \exp[-\mathbf{A}(m_N - 3/2 m_\pi)t]$

Signal/Noise ~ $\sqrt{\exp(- \mathbf{A}\, m_N t)} \times \sqrt{N_{conf}}$

Fake plateaus in temporal correlation for two baryons

"Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD",
JHEP 10 (2016) 101 by HAL QCD Coll.

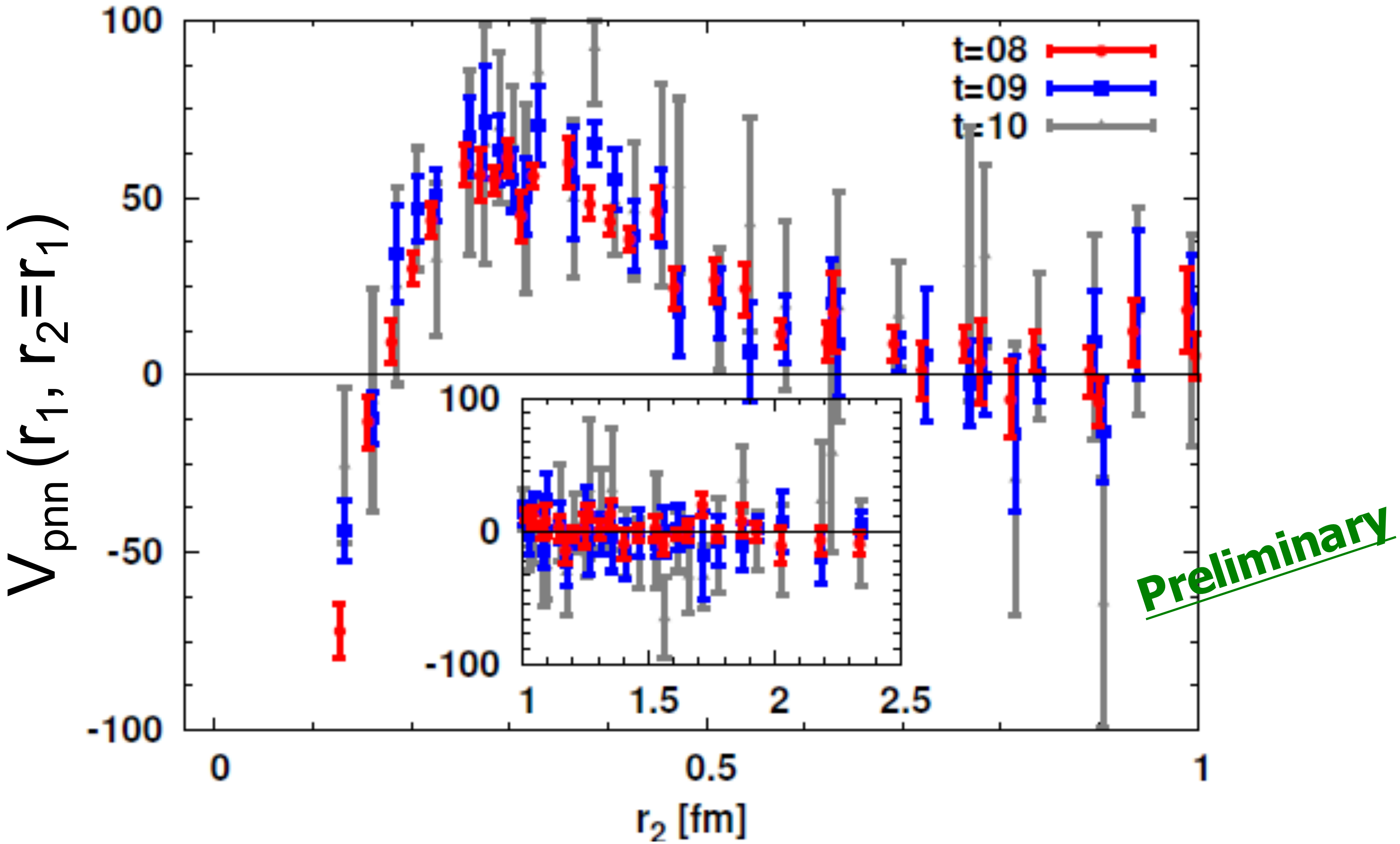
"Are two nucleons bound in lattice QCD for heavy quark masses ?
– Sanity check with Lucsher's finite volume formula –"
Phys. Rev. D96 (2017) 034521 by HAL QCD Coll.

"Sanity check for NN bound states in lattice QCD with Luscher's finite volume formula -- Exposing Symptoms of Fake Plateaux -- "
arXiv:1707.08800 [hep-lat] by Aoki, Doi, Iritani

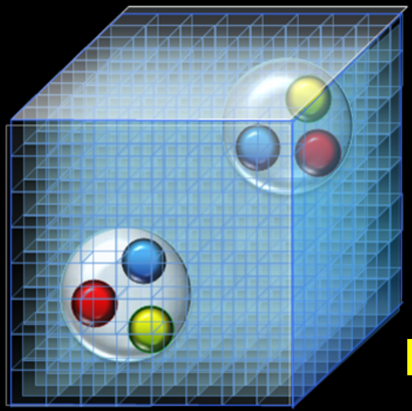
	$NN(^1S_0)$				$NN(^3S_1)$			
		Consistency check				Consistency check		
Data	Source independence	(i)	(ii)	(iii)	Source independence	(i)	(ii)	(iii)
YKU2011 [24]	†	No	No	*	†	No	No	*
YIKU2012 [25]	No	†	No	*	No	†	No	*
YIKU2015 [26]	†	†	No	*	†	†	No	No
NPL2012 [27]	†	†	No	*	†	†	*	*
NPL2013 [28,29]	No	*	*	No	No	*	*	?
NPL2015 [30]	†	No	*	No	†	No	*	No
CalLat2017 [31]	No	?	*	No	No	?	*	No

Pilot Study (L=2.9-5.8 fm) in (2+1)-flavor QCD :
3N force in Triton channel

<u>mπ=0.51 GeV</u>



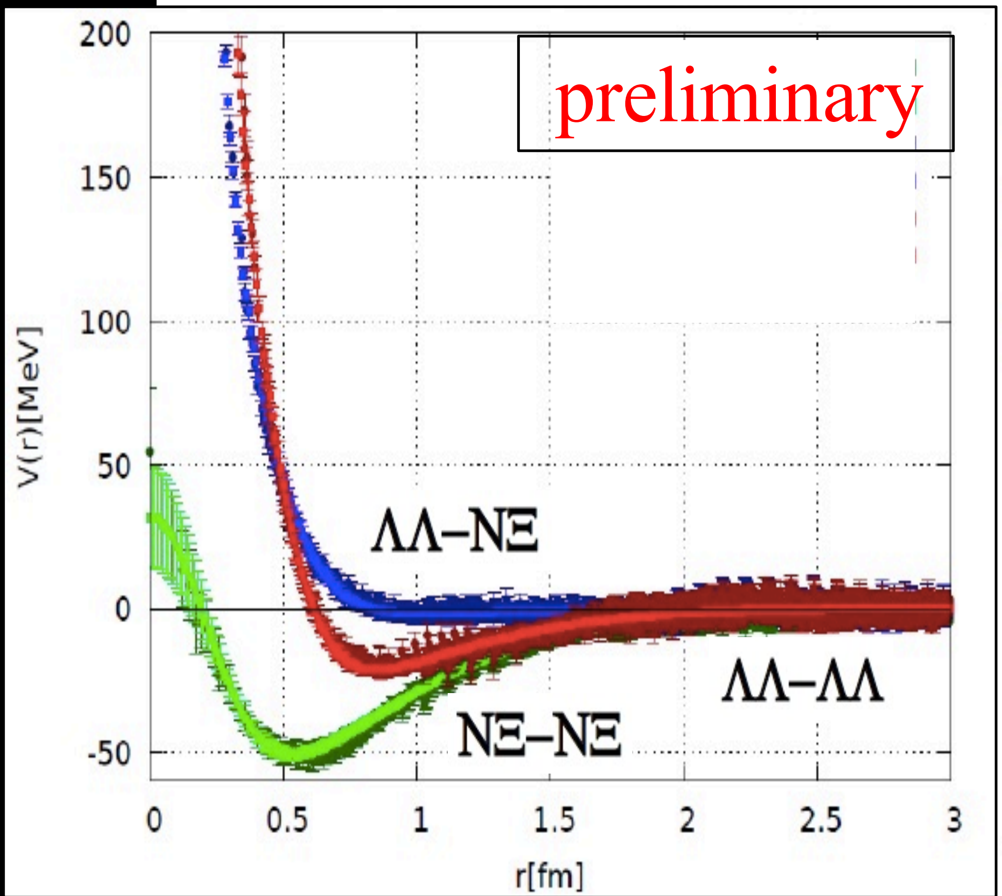
Example:
Hyperon Forces from LQCD
Hyperons in Nuclear Matter (LQCD+BHF)

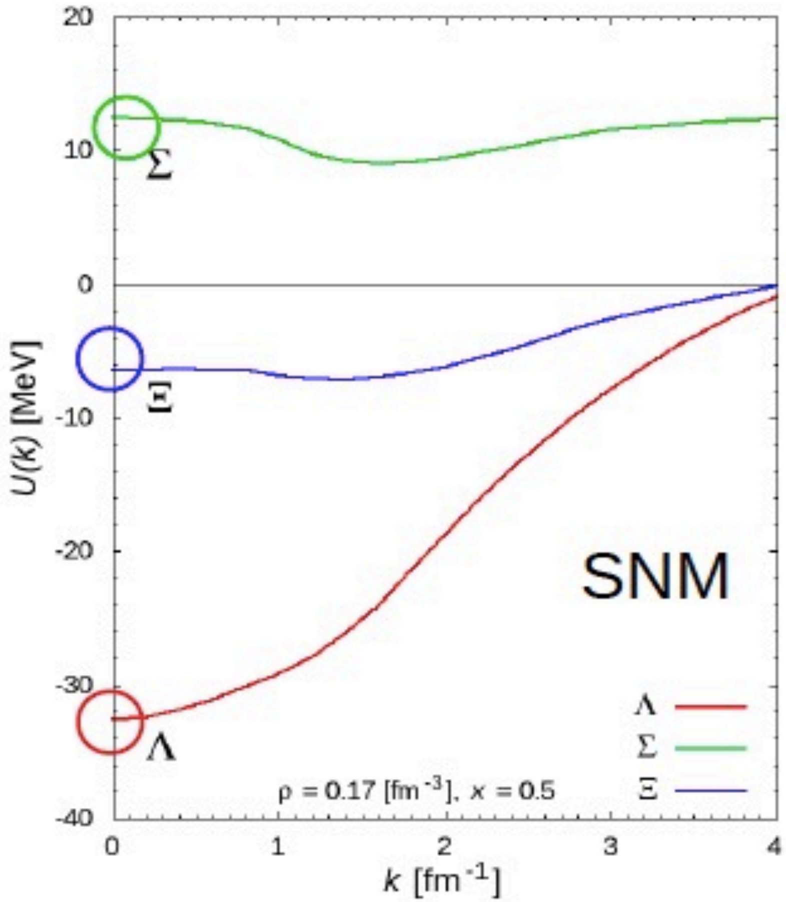


L = 8.1 fm

M_π ~146 MeV

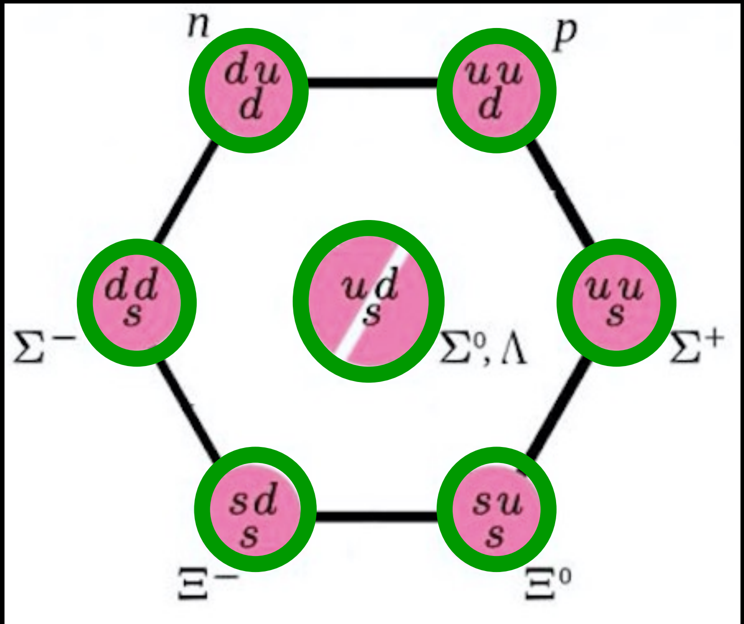
HAL QCD Coll. (2016)



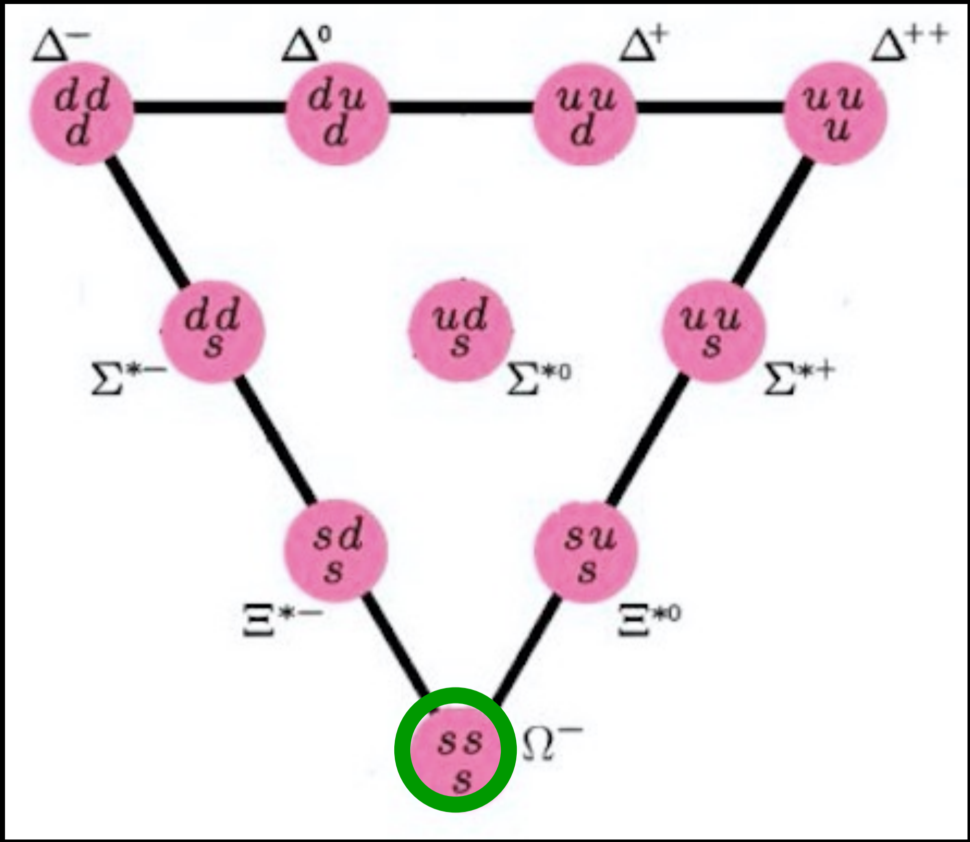


Flavor SU(3) Classification : single-baryon

8 (Octet)

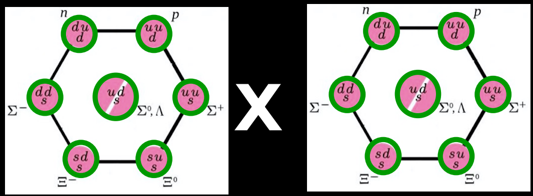


10 (Decuplet)

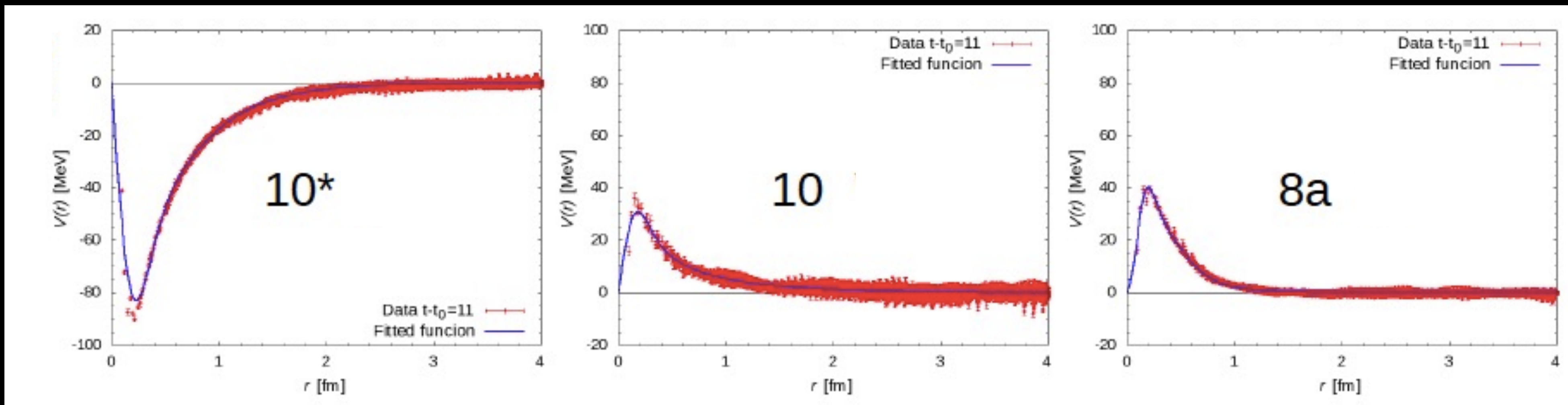


BB interactions in the flavor basis: $V_T(r)$

preliminary



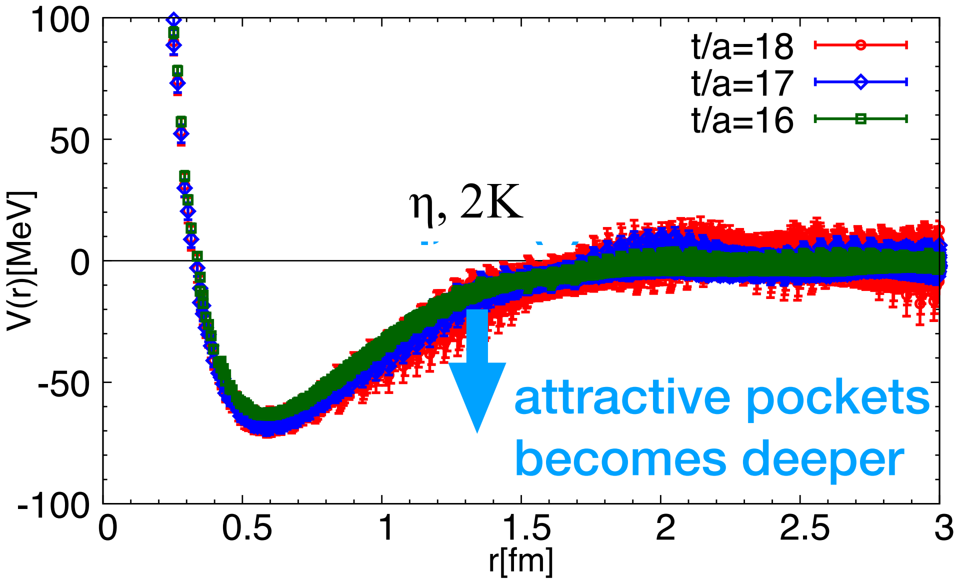
$$8 \times 8 = \underbrace{27 + 8s + 1}_{^1S_0} + \underbrace{10^* + 10 + 8a}_{^3S_1\ ,\ ^3D_1}$$



Conservative estimate at exact phys. pt.

m_{π}= 146 MeV -> 135 MeV, m_{Ω}= 1712MeV -> 1672 MeV

B.E. ↑



V.S.

B.E. ↓

$$\mathcal{H} = -\frac{\nabla^2}{m_\Omega} + V_{\Omega\Omega}^{\mathrm{LQCD}}(r)$$

kinetic term becomes more dominant
-> B.E. is reduced

conservative estimate:
only change the mass of schroedinger eq.

$$(B_{\Omega\Omega}^{(\mathrm{QCD})}, B_{\Omega\Omega}^{(\mathrm{QCD+Coulomb})}) = (1.6(6)\mathrm{MeV}, 0.7(5)\mathrm{MeV})$$
$$\to (1.3(5)\mathrm{MeV}, 0.5(5)\mathrm{MeV})$$

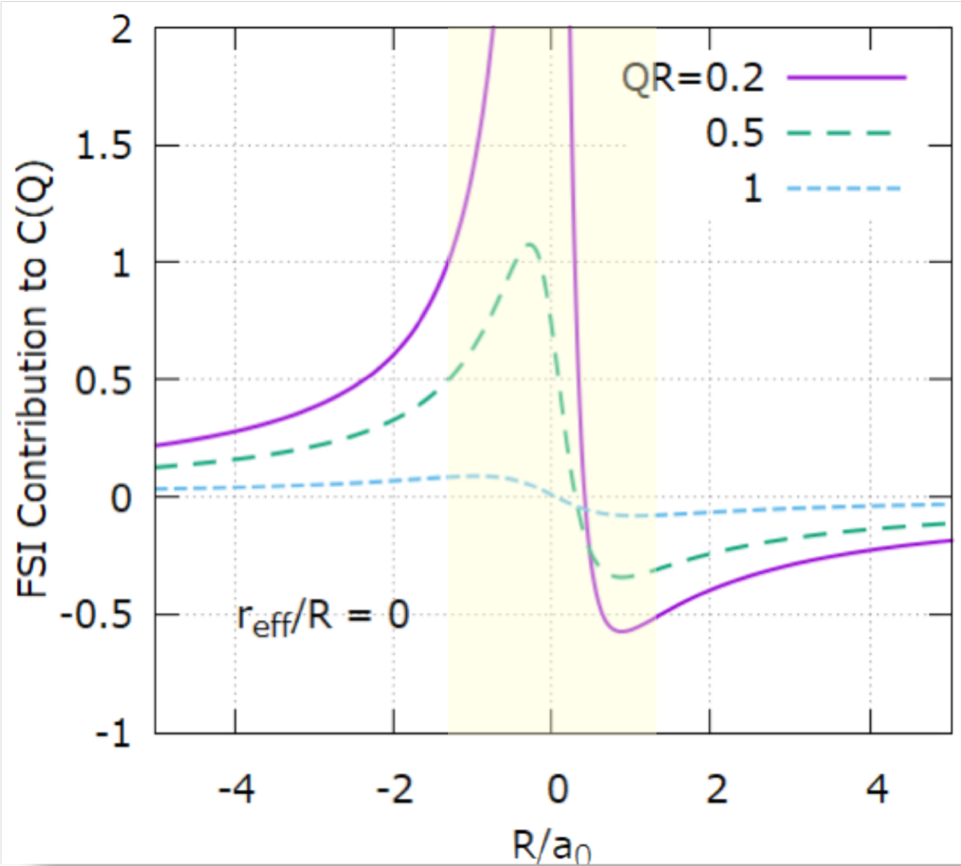
These changes are well within errors

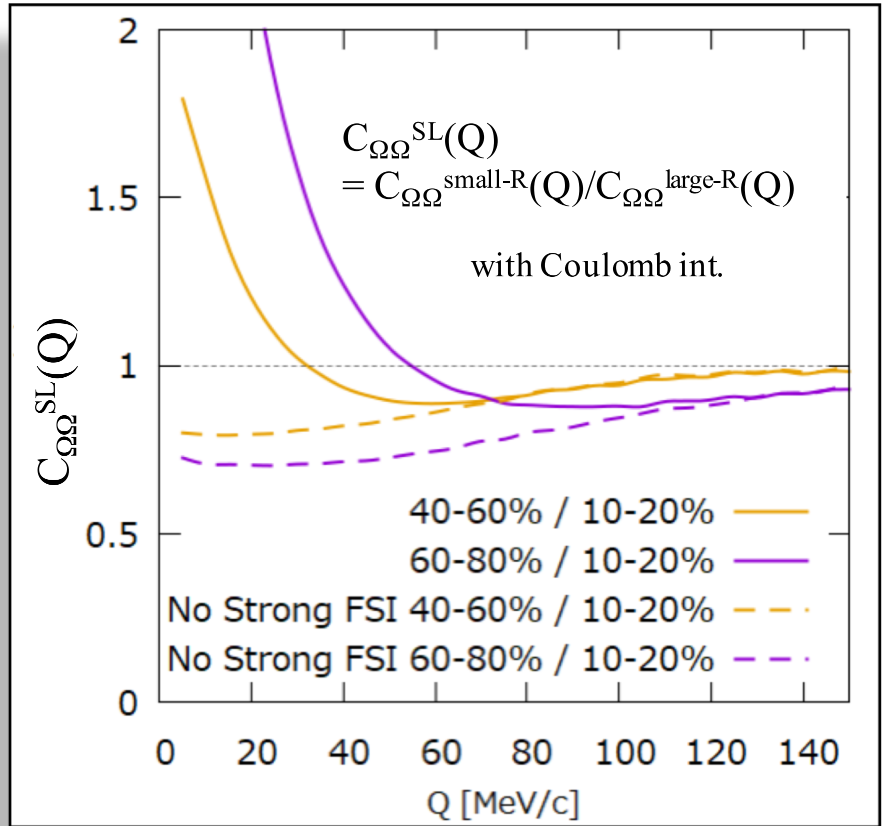
X

Correlation from FSI

$$C_{AB}(Q) - 1 = \frac{4\pi}{(2\pi R^2)^3} \int dr r^2 S^{\mathrm{rel}}(r) [|\chi_Q(r)|^2 - |j_0(Qr)|^2]$$

Lednicky+ 1982





Morita+ (in preparation)

Y=2 States in Su(6) Theory

Freeman J. Dyson and Nguyen-Huu Xuong

Phys. Rev. Lett. **13**, 815 – Published 28 December 1964; Erratum Phys. Rev. Lett. **14**, 339 (1965)

[From Dyson, Sep. 13, 2017]

Thank you very much for sending me your paper on the Omega-Omega calculation. This is a beautiful piece of work, and it will be a big step forward if the experimenters are able to confirm it.

Thank you also for referring to our 1964 paper. I am amazed that you remember that paper after 53 years. The predictions that we made in that paper turned out to be wrong, and the SU6 theory was soon abandoned. Luckily you did not assume SU6 symmetry when you made your prediction.

Now I wish you the joy of seeing it confirmed.

Yours sincerely,

Freeman Dyson.