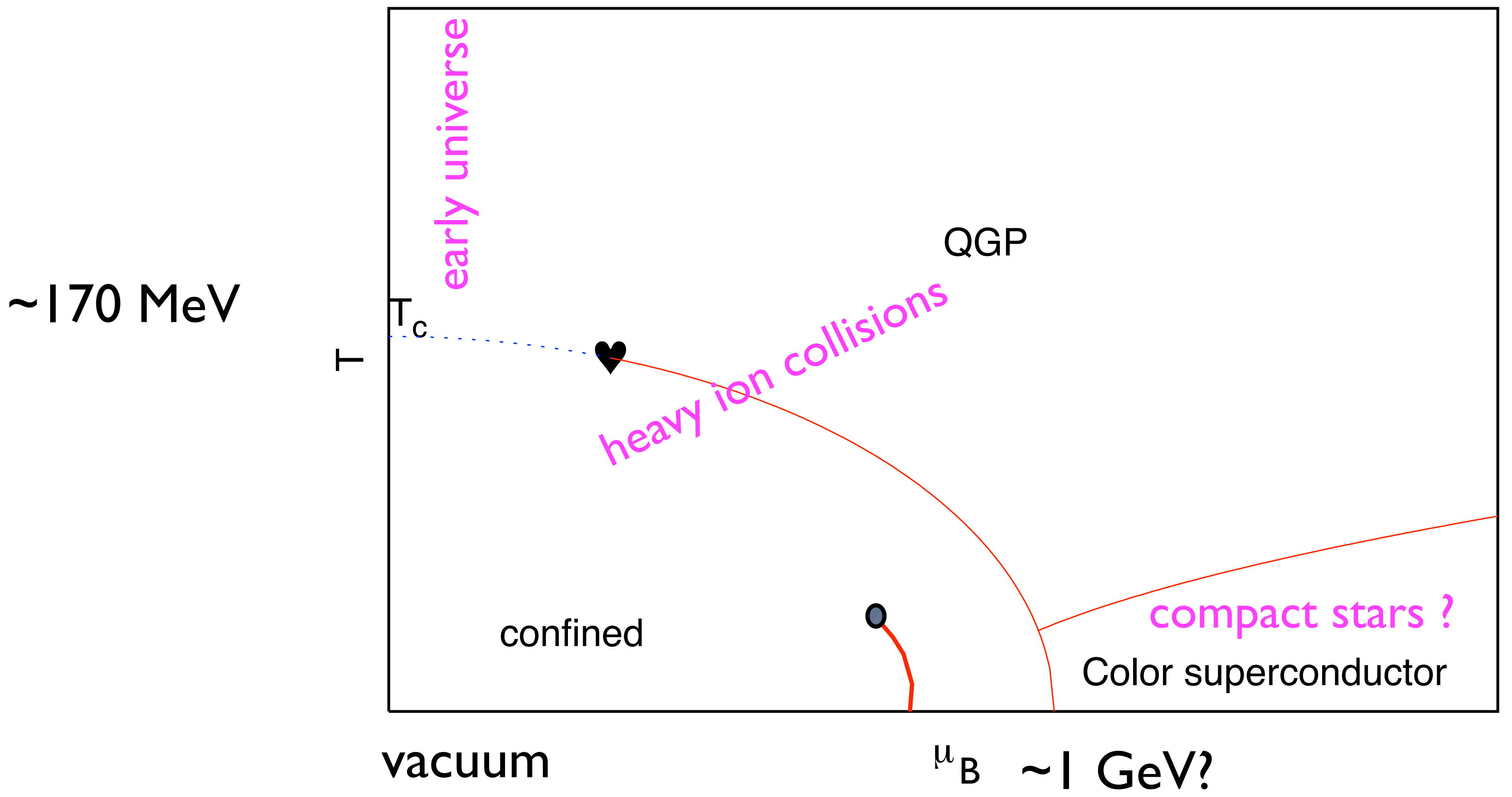
Towards a description of nuclear matter by finite density lattice QCD

Owe Philipsen

- Effective lattice theory for heavy quarks at finite density
- Effective lattice theory for chiral quarks at finite density

QCD phase diagram: theorist's view (science fiction)



Until 2001: no finite density lattice calculations, sign problem!

Expectation based on simplifying models (NJL, linear sigma model, random matrix models, ...)

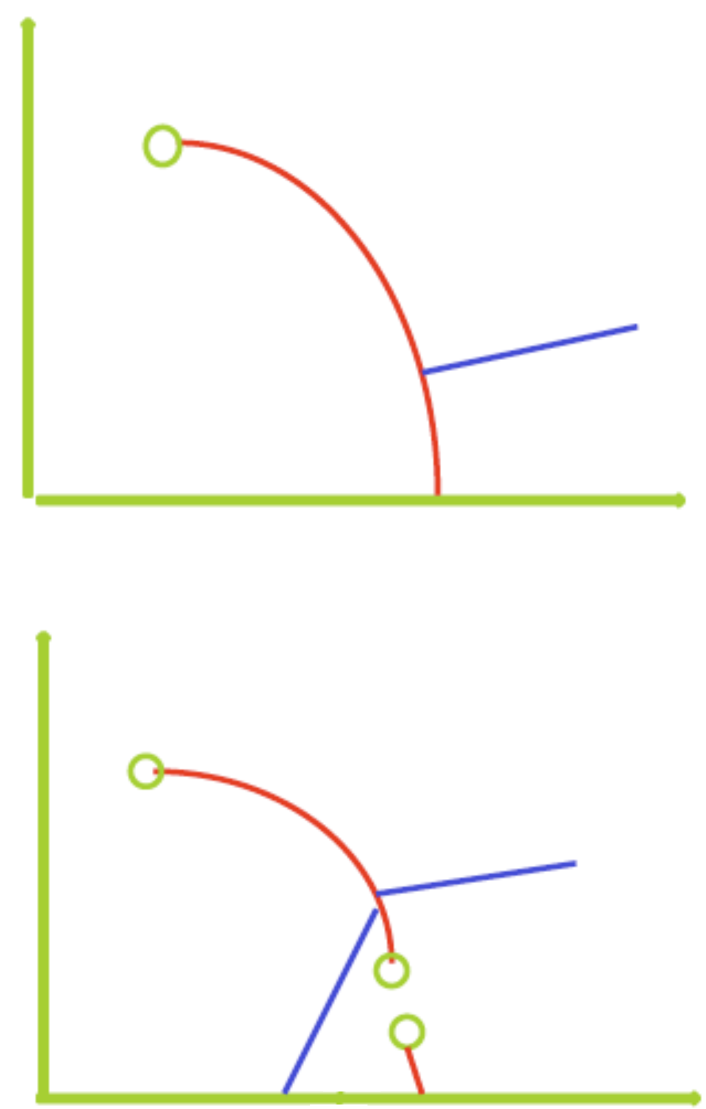
Check this from first principles QCD!

Less conservative views….

NJL with vector interactions,
Ginzburg-Landau approach
for quark condensates,
beyond mean field methods...

Zhang, Kunihiro, Fukushima 09
Baym et al. 06

Ferroni, Koch, Pinto 10

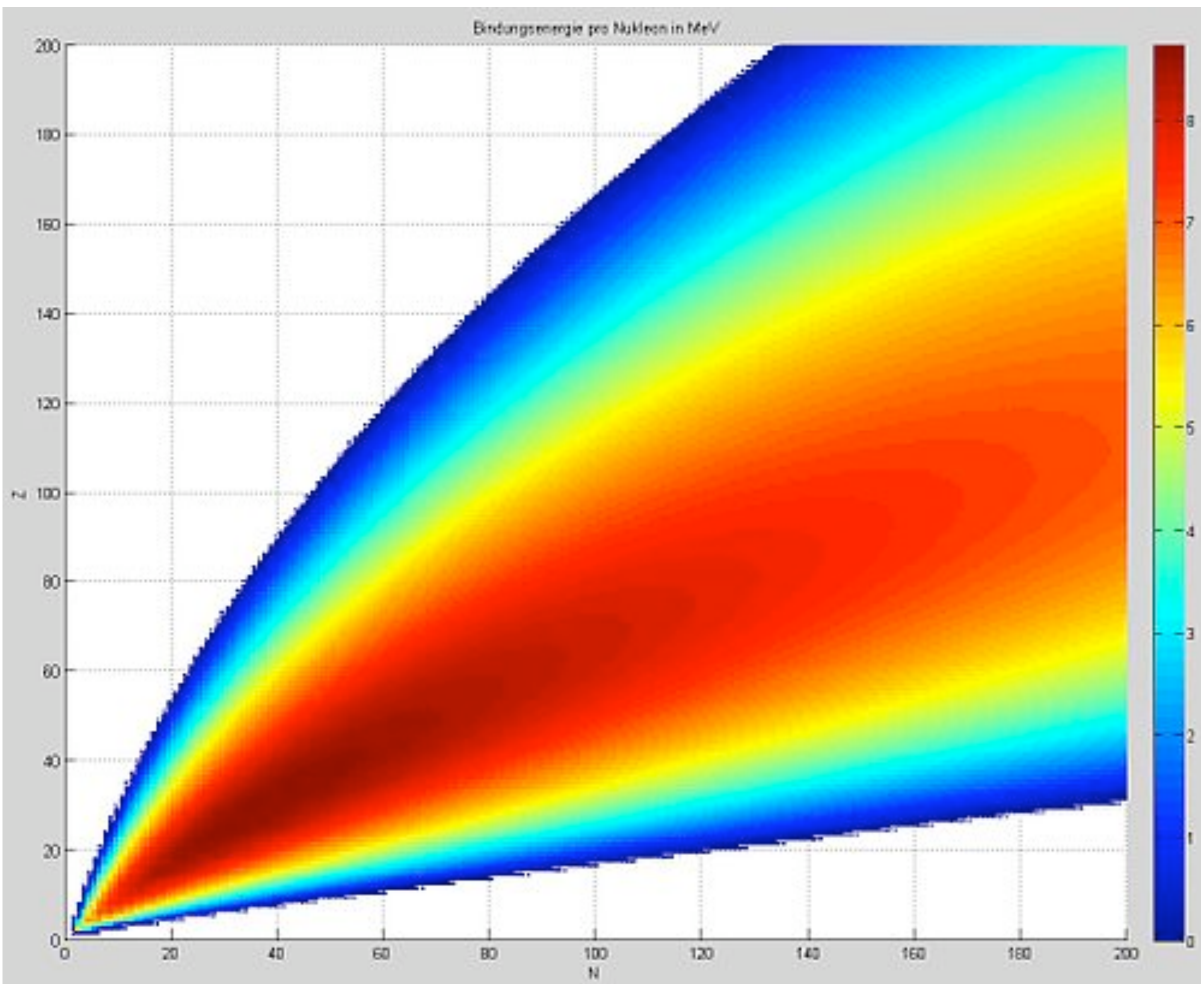


+ inhomogeneous phases, quarkyonic phases,…. you name it!

Completely unsolved: bulk nuclear matter

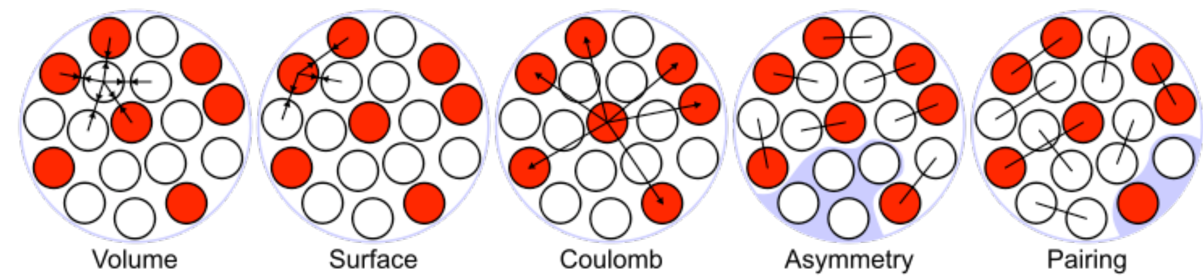
~100 years old, still no fundamental description, Bethe-Weizsäcker droplet model:

Z



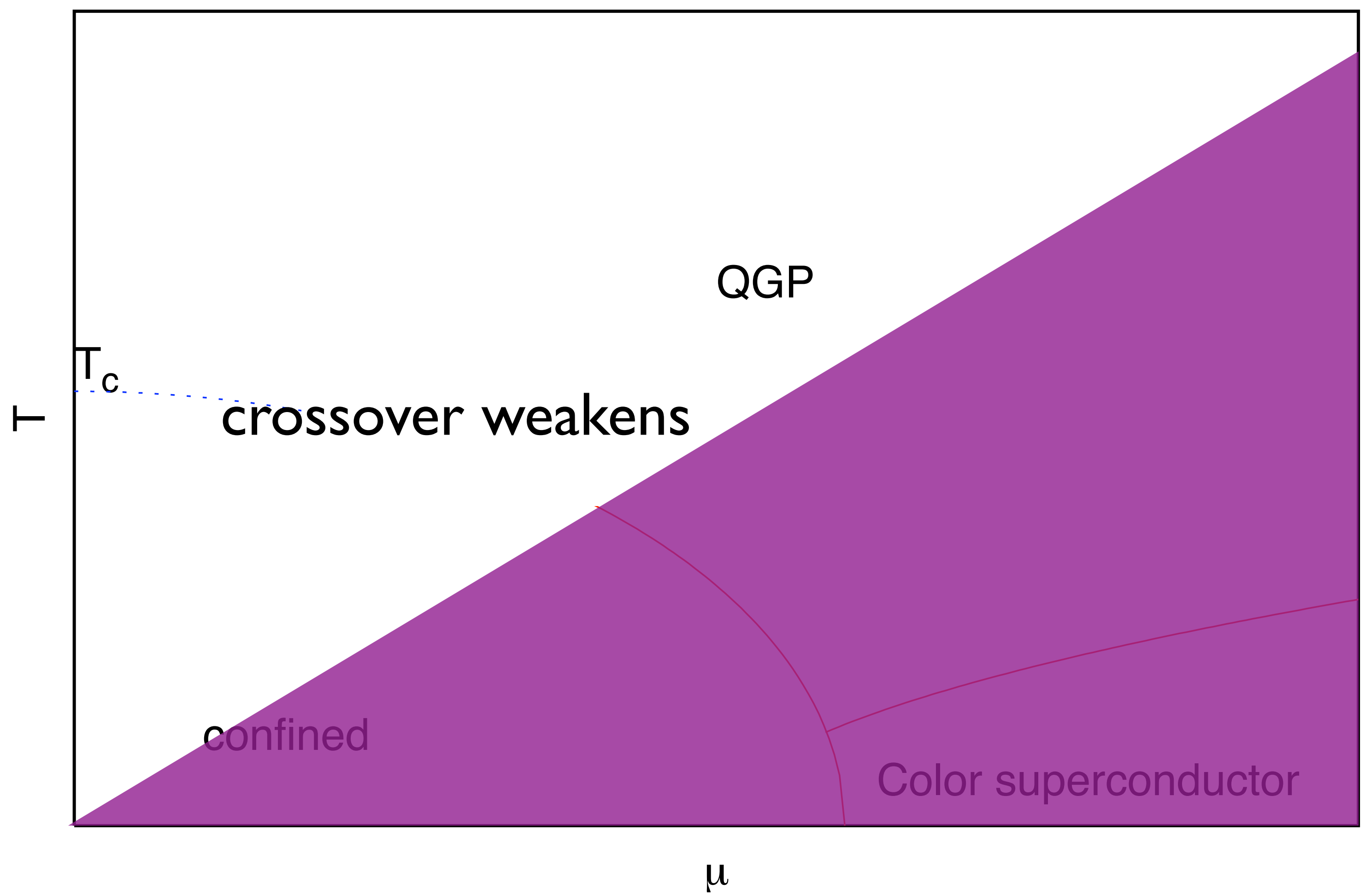
N

Binding energy per nucleon



QFT descriptions: Fetter-Walecka model, Skyrme model, ...

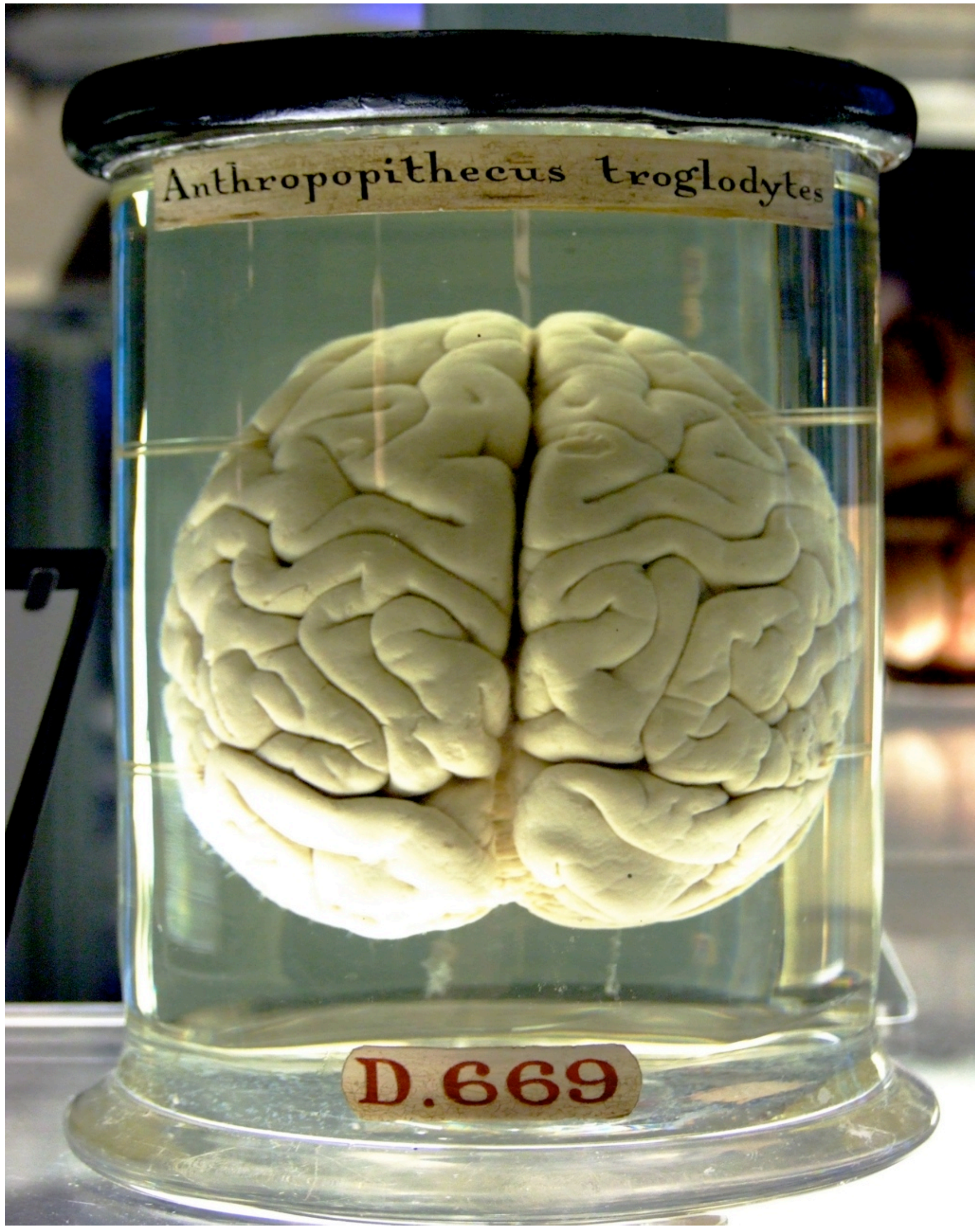
The lattice-calculable region of the phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- No critical point in the controllable region

use BPU's

Biological Processing Unit!



Large densities? Effective theories!

Effective lattice theory for heavy dense QCD

O.P. with Fromm, Langelage, Lottini, Neuman, Glesaaen

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: split temporal and spatial link integrations:

$$Z = \int DU_0 DU_i \, \det Q \, e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL \, e^{-S_{eff}[L]}$$

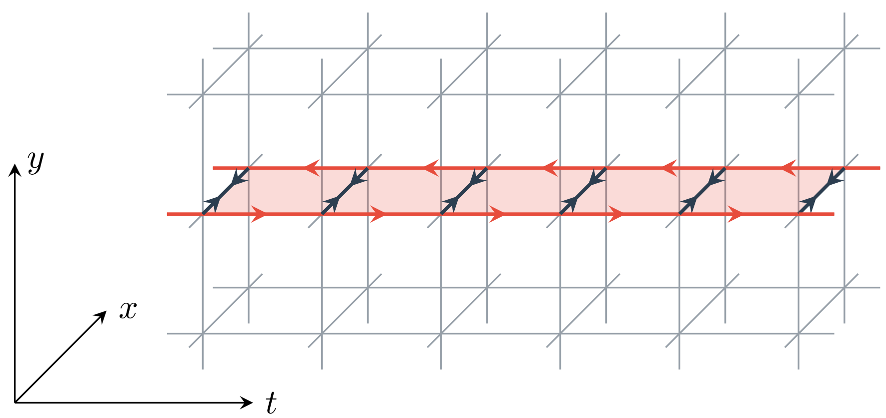
  Spatial integration after analytic strong coupling and hopping expansion $\sim \frac{1}{g^2}, \frac{1}{m_q}$

- Truncation valid for heavy quarks on reasonably fine lattices, a~0.1 fm

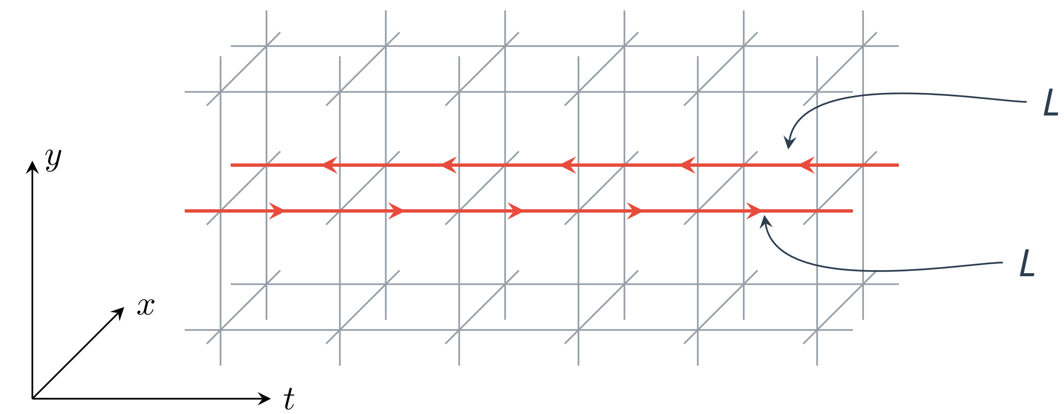
- Mild sign problem of effective theory: complex Langevin, Monte Carlo

- Analytic solution by cluster expansion

LO 3d effective theory for lattice YM



Integrate over all spatial gauge links



What remains is an interaction between Polyakov Loops

$$-S_1 = u^{N_\tau} \sum_{\langle ij \rangle} \mathrm{tr}\, W_i \, \mathrm{tr}\, W_j$$

Polonyi, Szachlanyi 82

Character expansion: $u = \frac{\beta}{18} + O(\beta^2) < 1$ $\qquad \beta = \frac{2N}{g^2}$

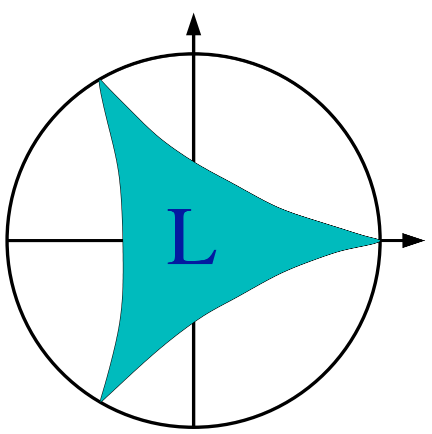
- larger distances between loops, higher power of loops
- higher representations of loops
- decorations of LO graphs by additional plaquettes

Effective one-coupling theory for SU(3) YM

Langelage, Lottini, O.P. 10

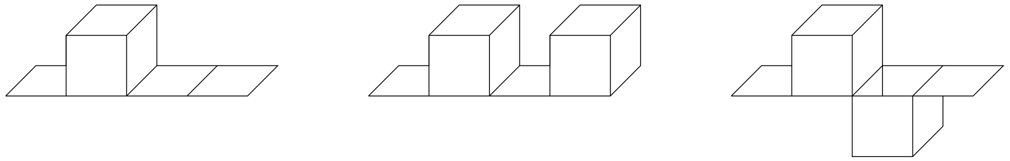
(L= Tr W)

$$Z = \int [dL] \exp\left[-S_1 + V_{SU(3)}\right]$$
$$= \int [dL] \prod_{\langle ij \rangle} \left[1 + 2\lambda_1 \mathrm{Re}\left(L_i L_j^*\right)\right] *$$
$$* \prod_i \sqrt{27 - 18|L_i|^2 + 8\mathrm{Re}L_i^3 - |L_i|^4}$$



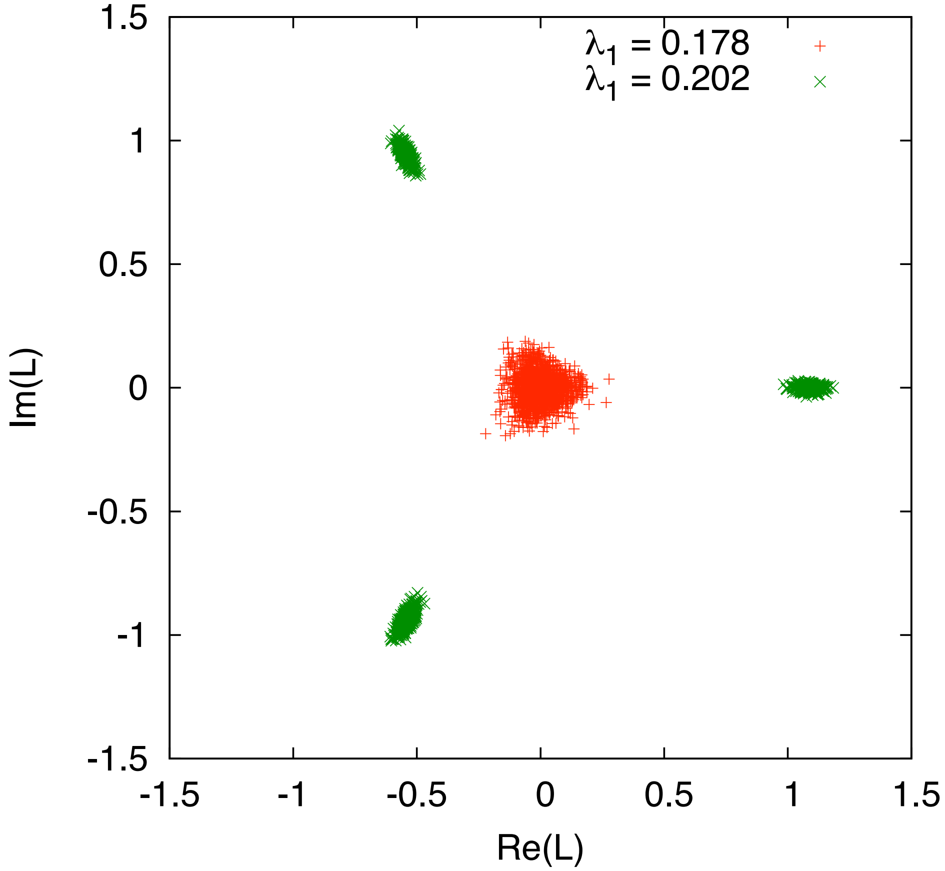
Resummations: $\sum_{<ij>} \left(\lambda_1 L_i L_j - \frac{\lambda_1^2}{2} L_i^2 L_j^2 + \frac{\lambda_1^3}{3} L_i^3 L_j^3 - \ldots\right) = \sum_{<ij>} \ln\left(1 + \lambda_1 L_i L_j\right)$

$$\lambda(u, N_\tau \geq 5) = u^{N_\tau} \exp\left[N_\tau \left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1055797}{5120}u^{10}\right)\right]$$

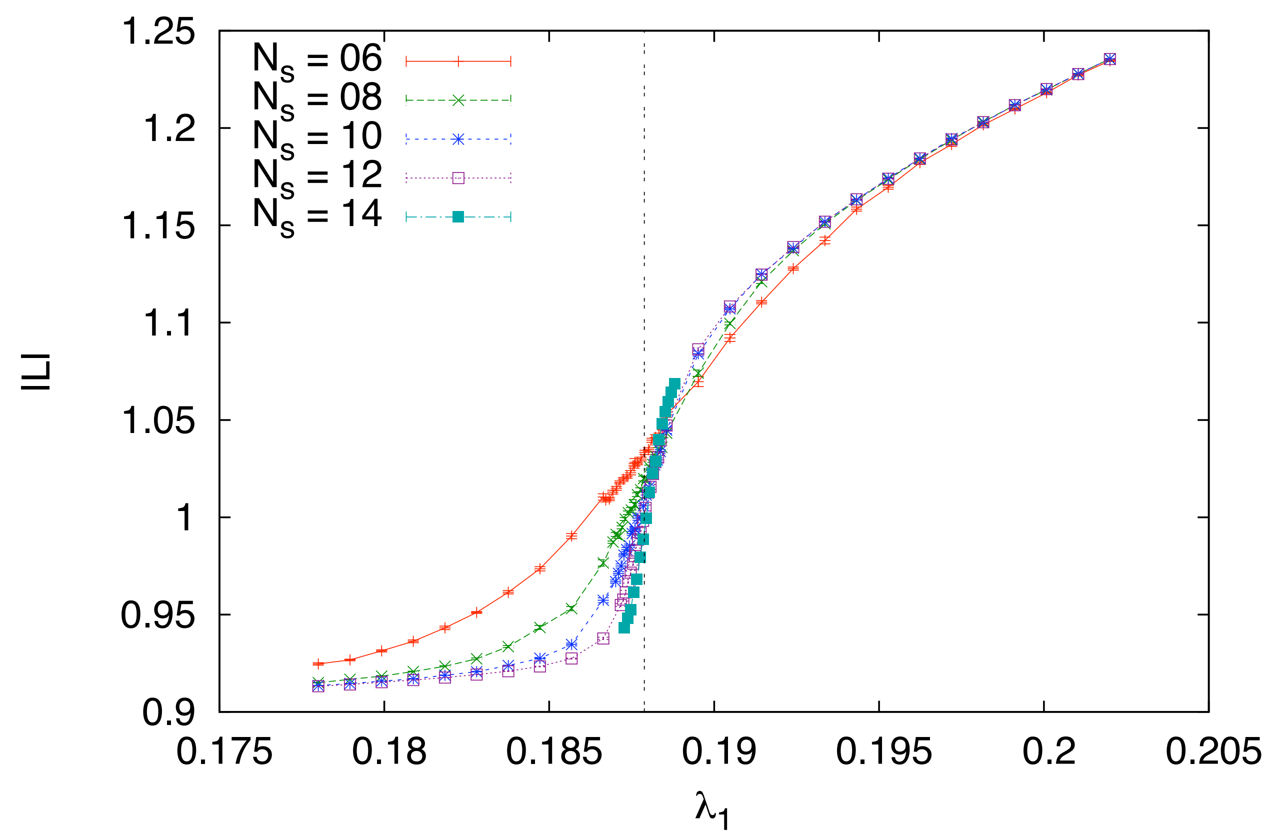


...

Numerical results for SU(3), one coupling

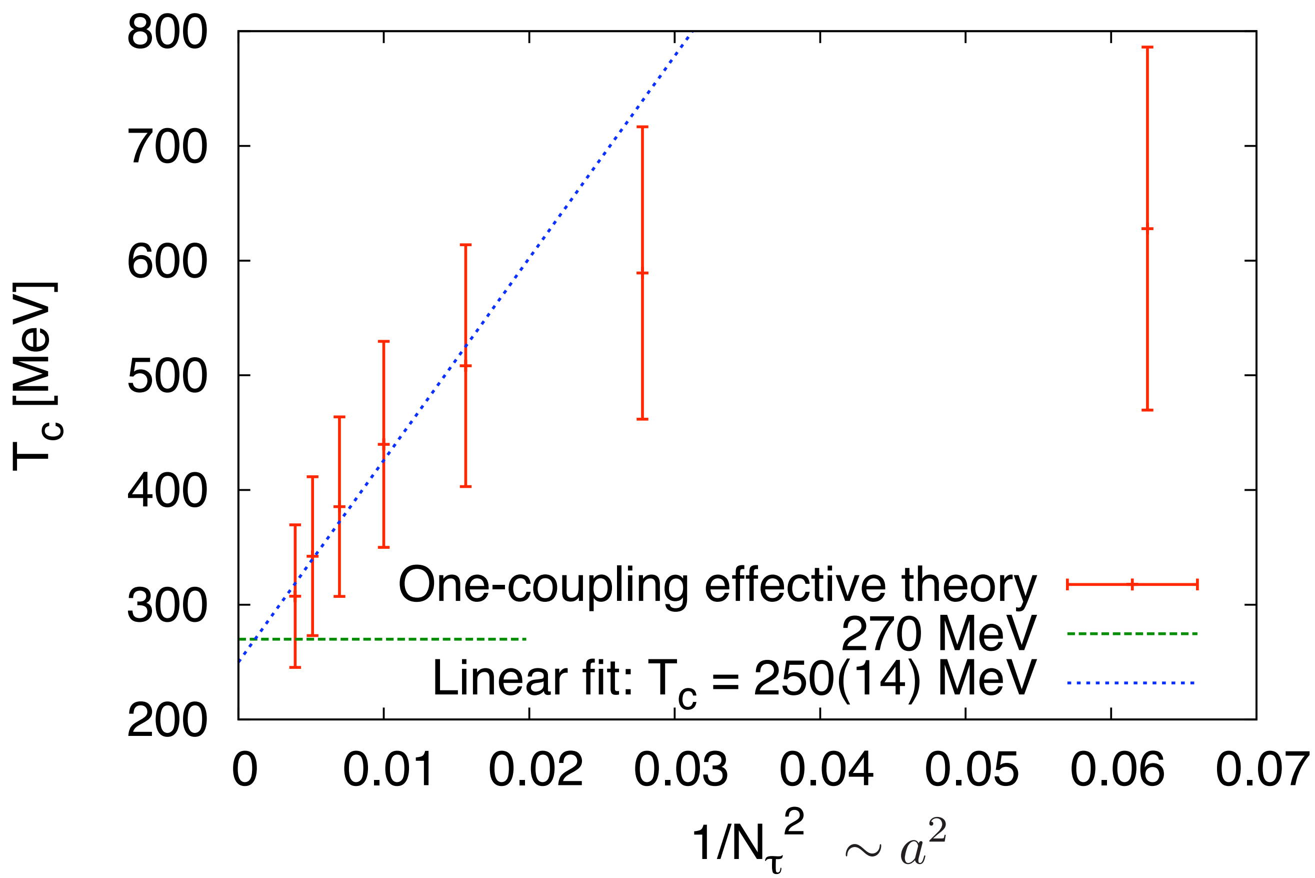


Order-disorder transition
=Z(3) breaking



Mapping back to 4d Yang-Mills

$$\lambda_1(N_\tau, \beta) \rightarrow \beta_c(\lambda_{1,c}, N_\tau)$$



-error bars: difference between last two orders in strong coupling exp.

-using non-perturbative beta-function (4d T=0 lattice)

-all data points from one single 3d MC simulation!

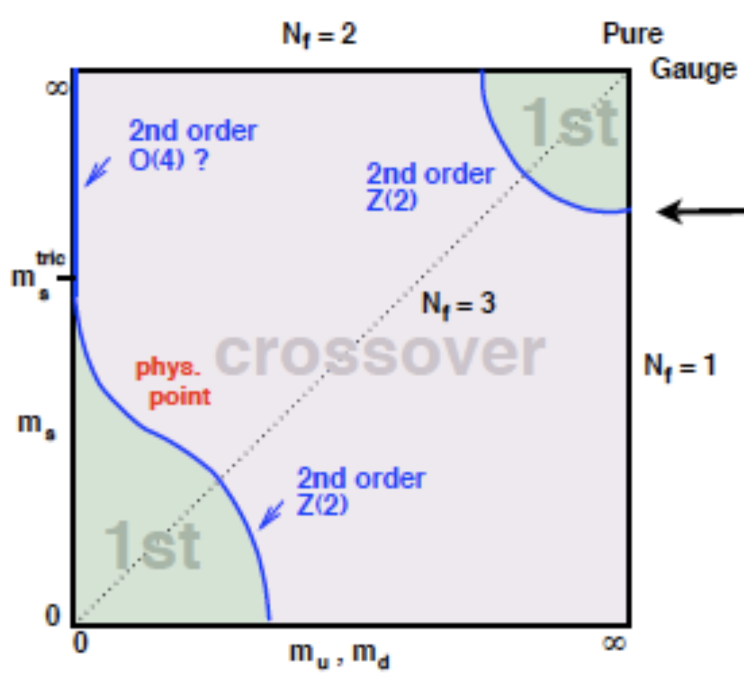
Including heavy, dynamical Wilson fermions

Expand in the *hopping parameter* $\kappa = 1/(2aM+8)$:

$$-S_{\mathrm{eff}} = \sum_i \lambda_i(u,\kappa,N_\tau) S_i^{\mathrm{S}} - 2N_f \sum_i \left[h_i(u,\kappa,\mu,N_\tau) S_i^{\mathrm{A}} + \bar{h}_i(u,\kappa,\mu,N_\tau) S_i^{\dagger\mathrm{A}} \right]$$

NLO: $\sim \kappa^2$

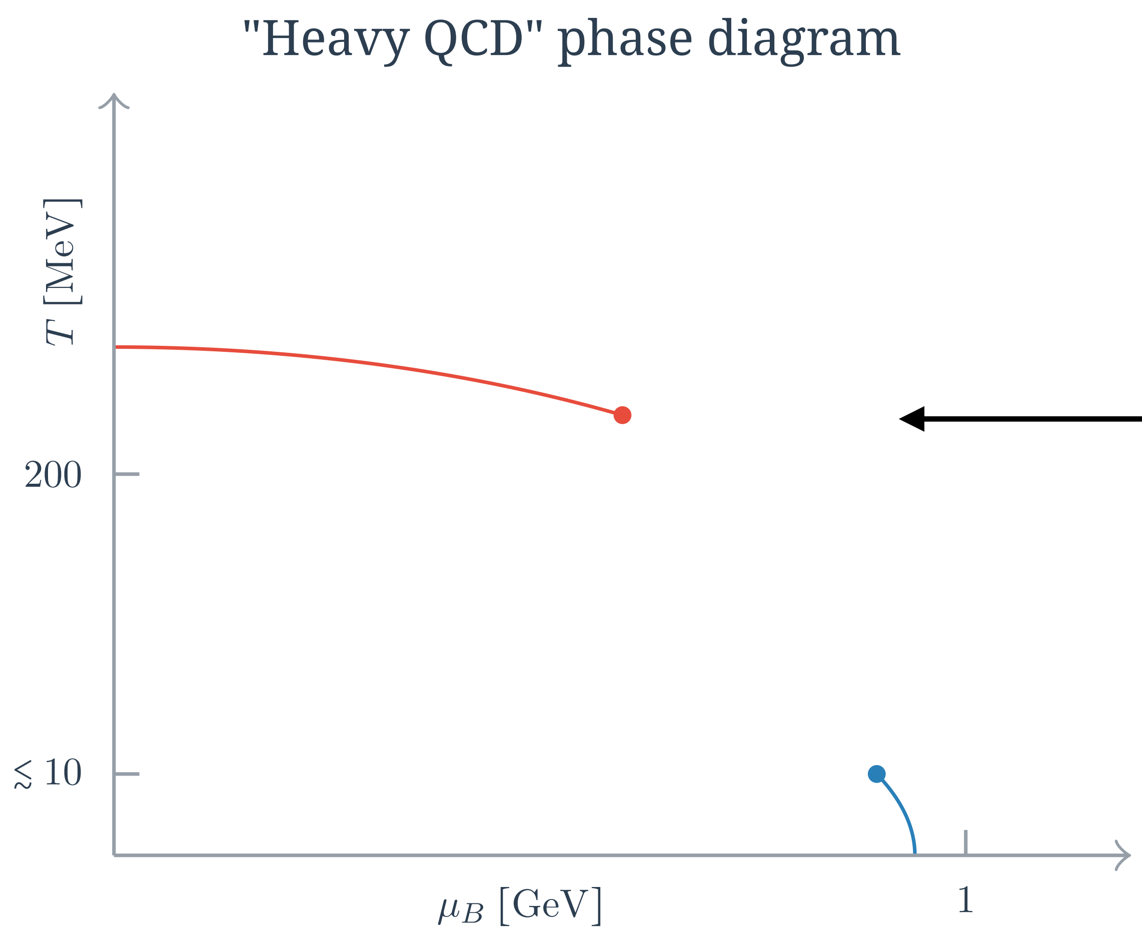
Deconfinement transition for heavy quarks

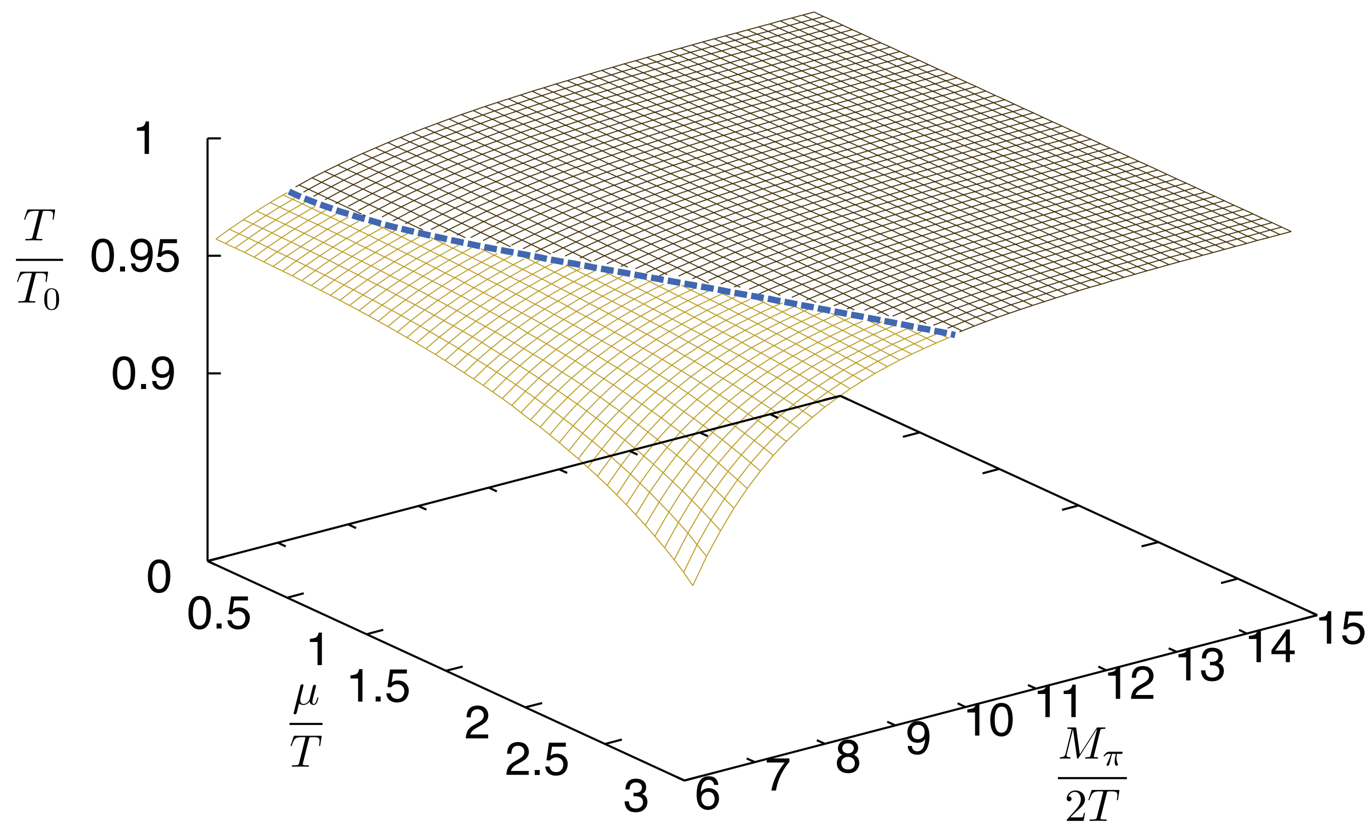


		eff. theory	4d MC, WHOT	4d MC, de Forcrand et al
N_f	M_c/T	$\kappa_c(N_\tau = 4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
1	7.22(5)	0.0822(11)	0.0783(4)	~ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

Accuracy ~5%, predictions for Nt=6,8,... available!

The fully calculated deconfinement transition





Continuum:

Friman, Lo, Redlich 14

Fischer, Lücker, Pawlowski 15

Fromm, Langelage, Lottini,O.P. 11

Cold and dense QCD: static strong coupling limit

Fromm, Langelage, Lottini, Neuman, O.P. , PRL 13

For T=0 (at finite density) anti-fermions decouple $N_f = 1, h_1 = C, h_2 = 0$

$$C \equiv (2\kappa_f e^{a\mu_f})^{N_\tau} = e^{(\mu_f - m_f)/T}, \ \bar{C}(\mu_f) = C(-\mu_f)$$

$$Z(\beta=0) \xrightarrow{T\to 0} \left[\prod_f \int dW \, (1 + C\, L + C^2 L^* + C^3)^2 \right]^{N_s^3}$$

$$= [1 + 4C^{N_c} + C^{2N_c}]^{N_s^3}$$

Free gas of baryons!

Quarkyonic?

$$n = \frac{T}{V}\frac{\partial}{\partial\mu}\ln Z = \frac{1}{a^3}\frac{4N_c C^{N_c} + 2N_c C^{2N_c}}{1 + 4C^{N_c} + C^{2N_c}}$$

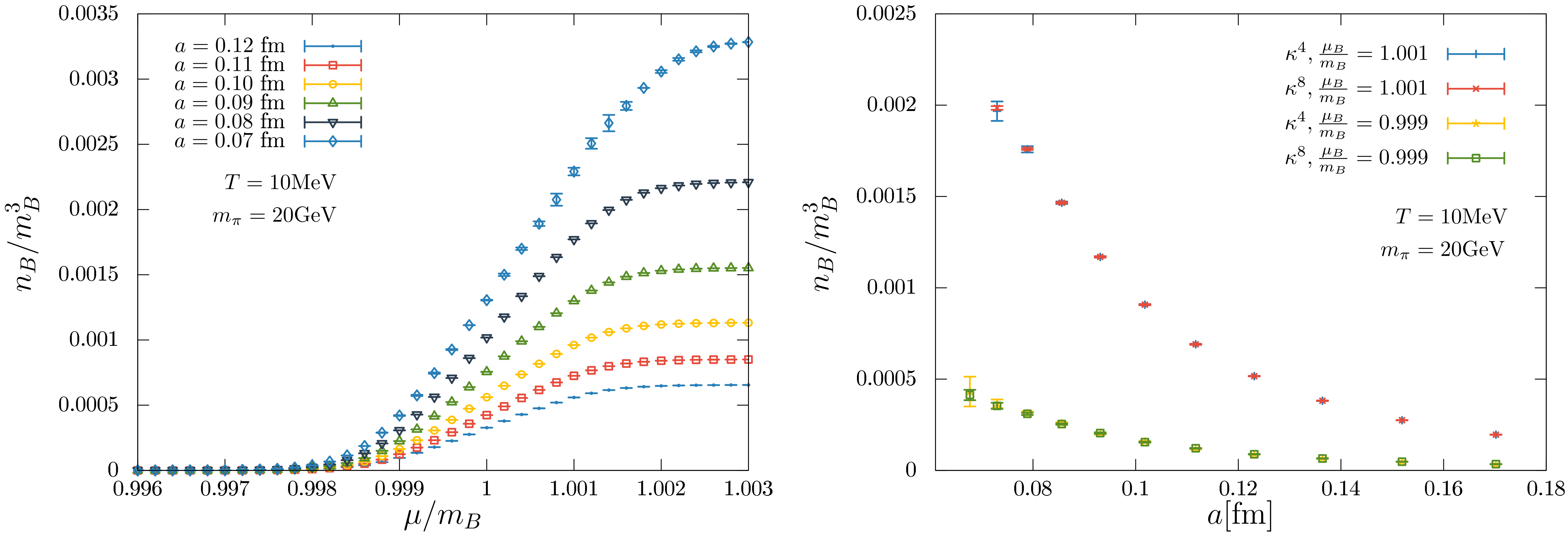
$$\lim_{\mu\to\infty}(a^3 n) = 2N_c$$

Sivler blaze property + saturation!

$$\lim_{T\to 0} a^3 n = \begin{cases} 0, & \mu < m \\ 2N_c, & \mu > m \end{cases}$$

Cold and dense regime

Fromm, Langelage, Lottini, Neuman, O.P. , PRL 13

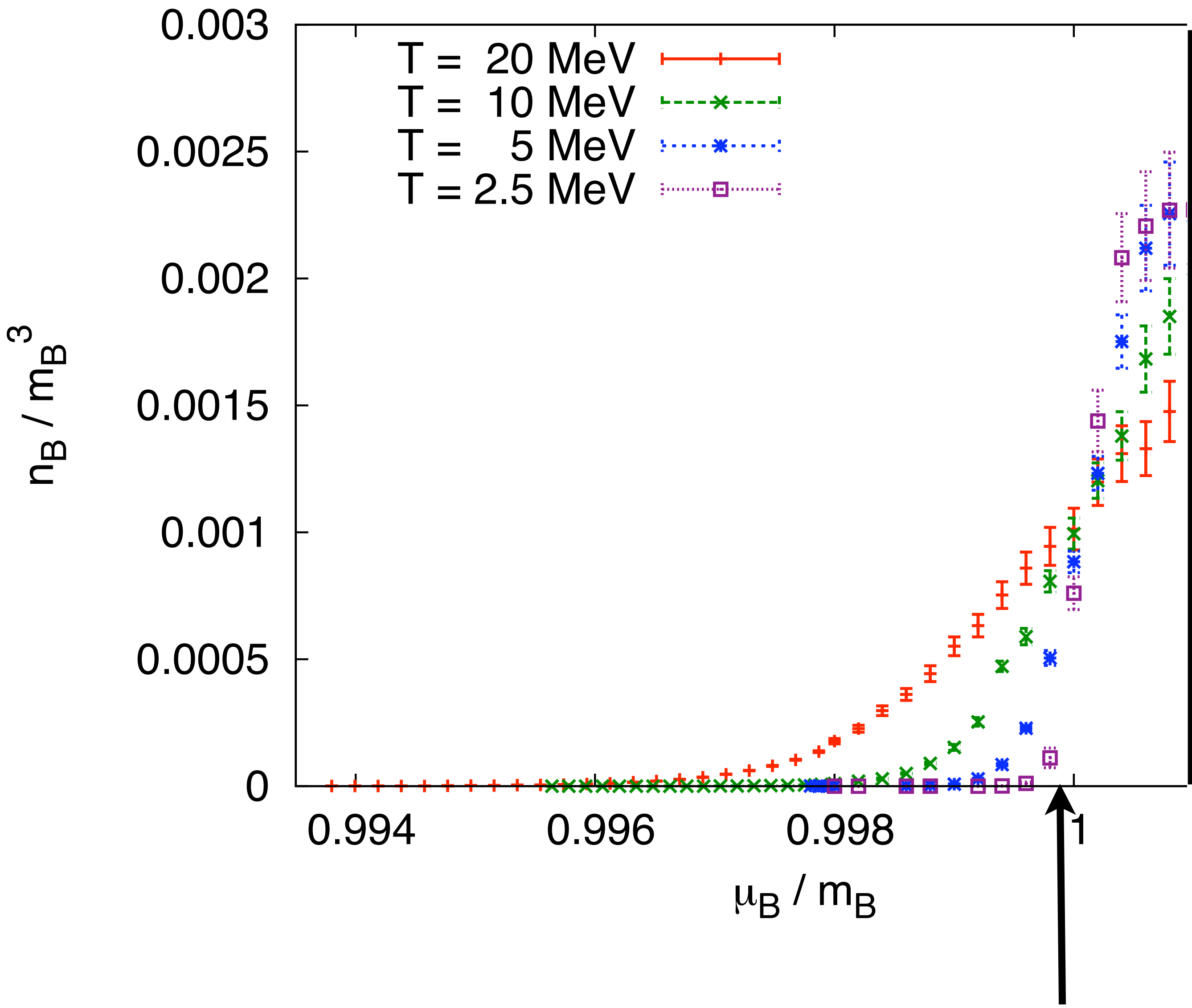


- Continuum approach ~a as expected for Wilson fermions
- Cut-off effects grow rapidly beyond onset transition
- Finer lattice necessary for larger density to avoid saturation

Cold and dense, interacting: onset to nuclear matter

continuum extrapolated

$$m_\pi = 20\ \mathrm{GeV}$$



Effect of binding between baryons: $\mu_c < m_B$

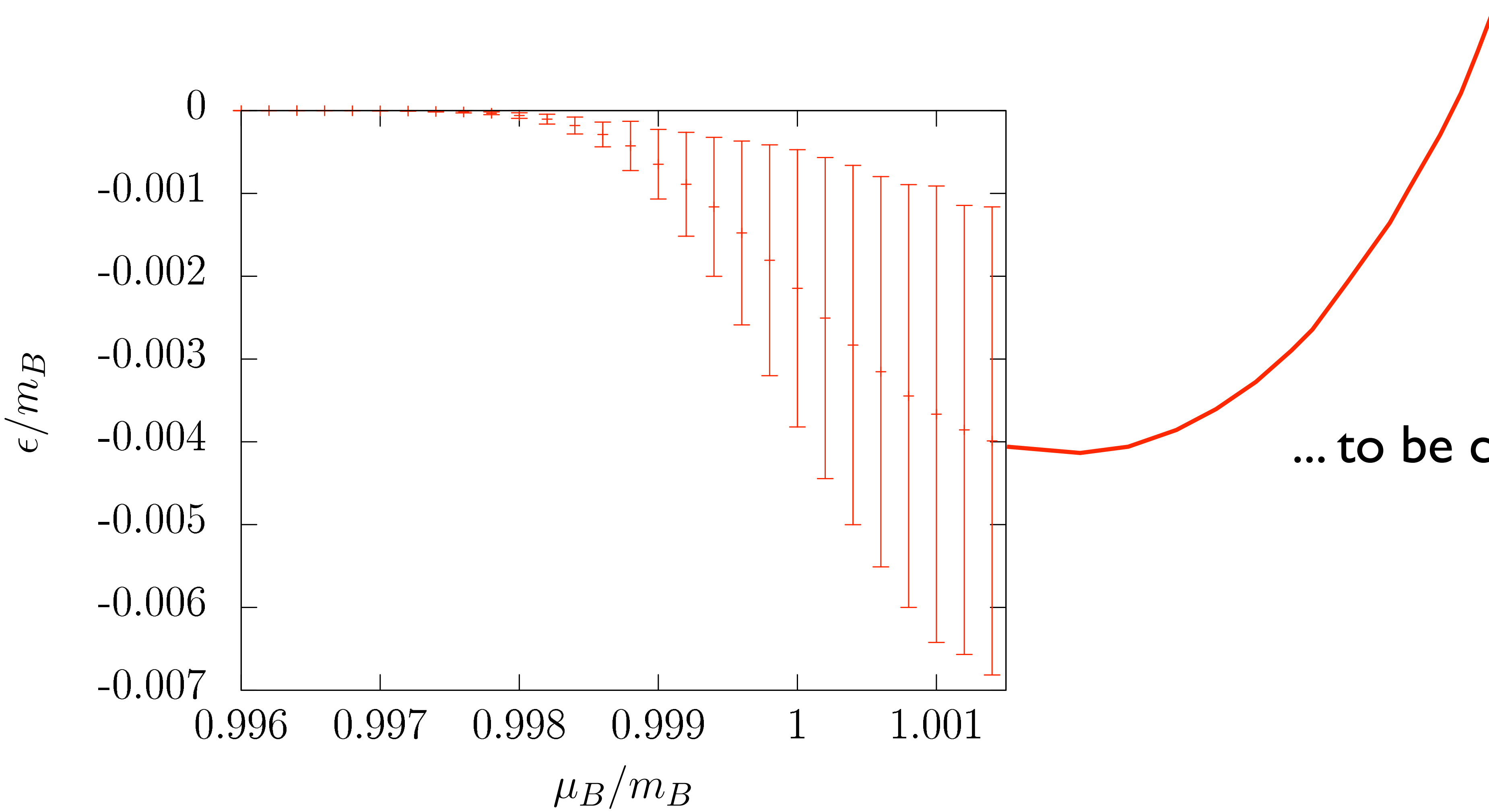
$$\frac{\mu}{T} \sim 4000$$

Binding energy per nucleon: $\epsilon = \dfrac{\mu_c - m_B}{m_B} \sim 10^{-3}$

Transition is smooth crossover: $T > T_c \sim \epsilon\, m_B$

Binding energy per nucleon

$$\epsilon = \frac{e - n_B m_B}{n_B m_B} = \frac{e}{n_B m_B} - 1$$

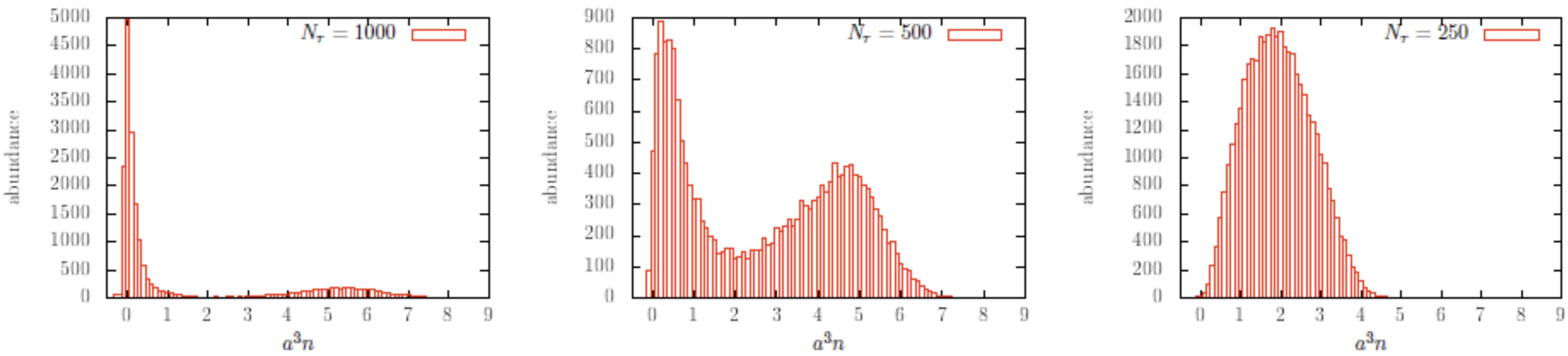


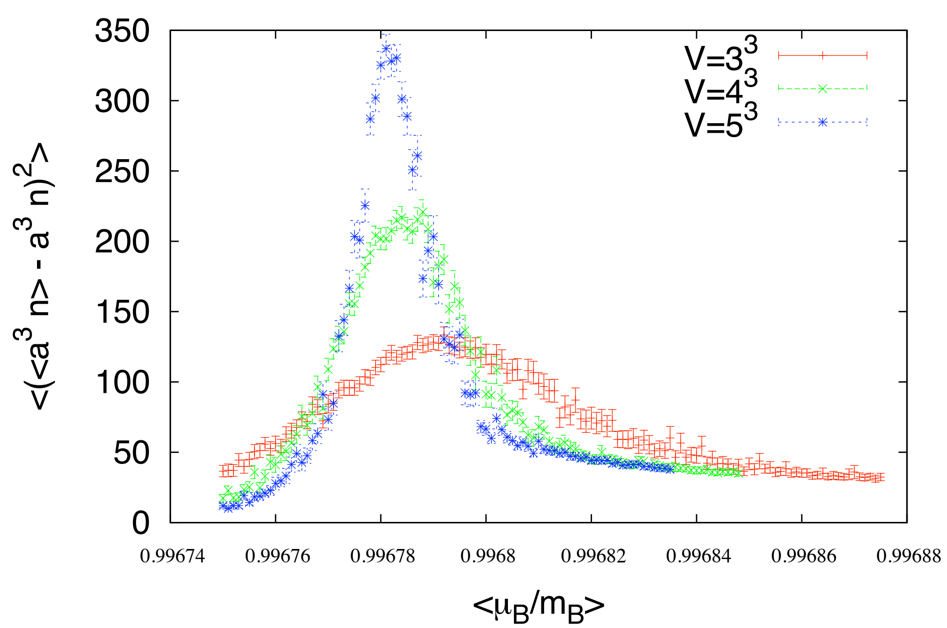
... to be continued...

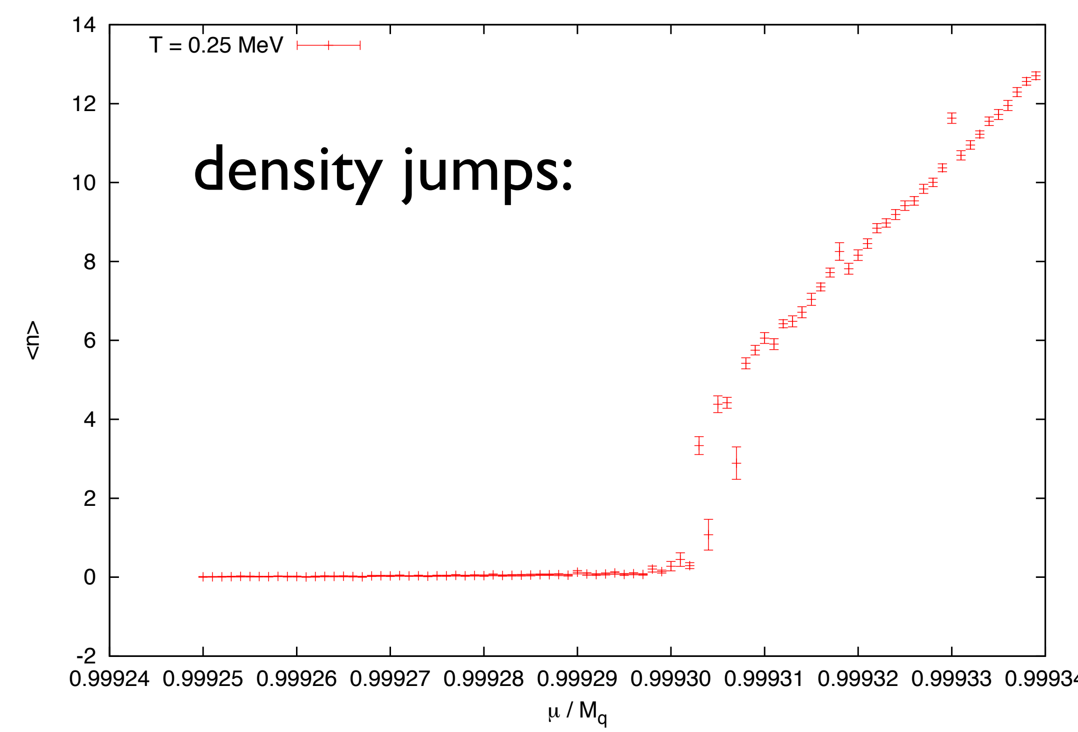
Minimum: access to nucl. binding energy, nucl. saturation density!

$\epsilon \sim 10^{-3}$ consistent with the location of the onset transition

Liquid gas transition: first order + endpoint



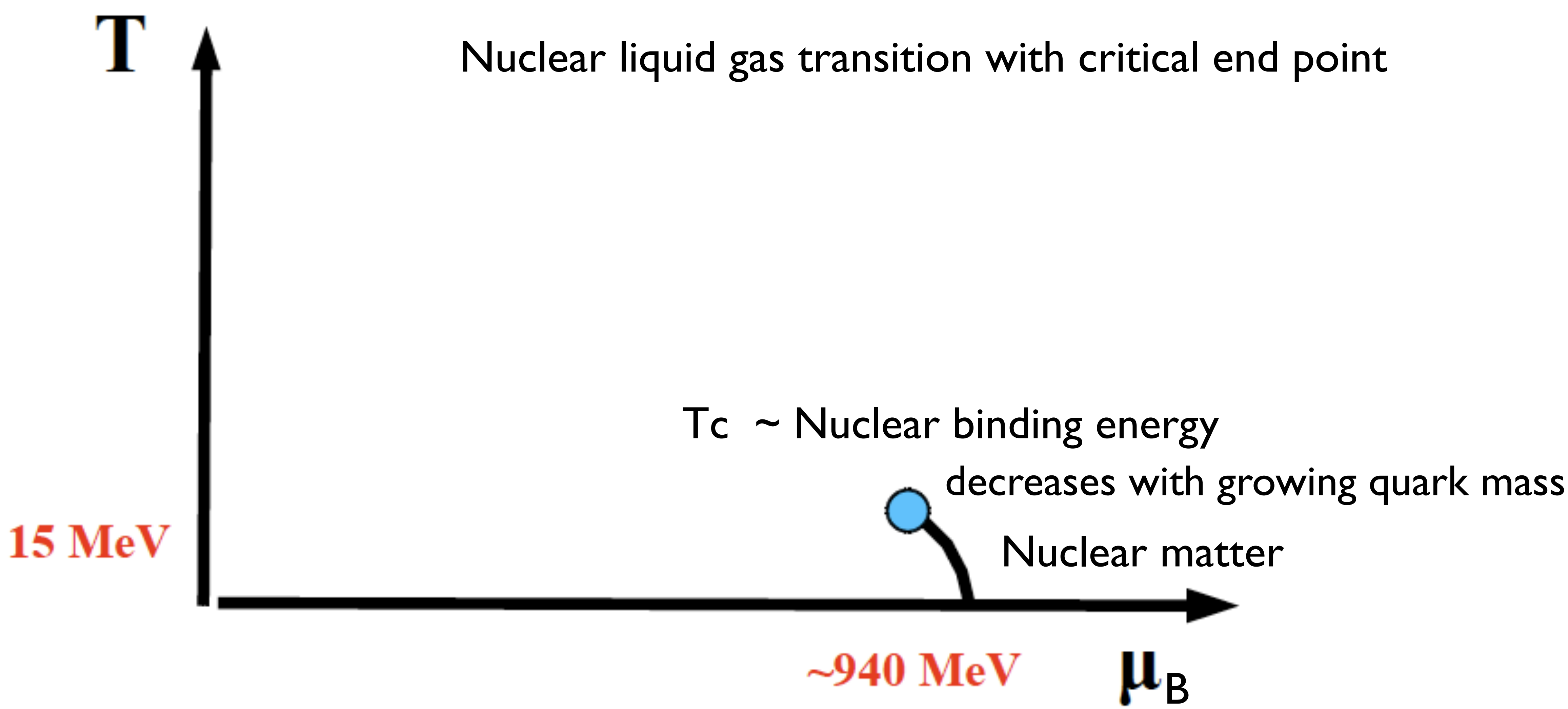




For sufficiently light quarks: $\kappa \sim 0.1$

- Coexistence of vacuum and finite density phase: 1st order
- If the temperature $T = \frac{1}{aN_\tau}$ or the quark mass is raised this changes to a crossover

nuclear liquid gas transition!!!


T
Nuclear liquid gas transition with critical end point
Tc ~ Nuclear binding energy
decreases with growing quark mass
15 MeV
Nuclear matter
~940 MeV
μ_B

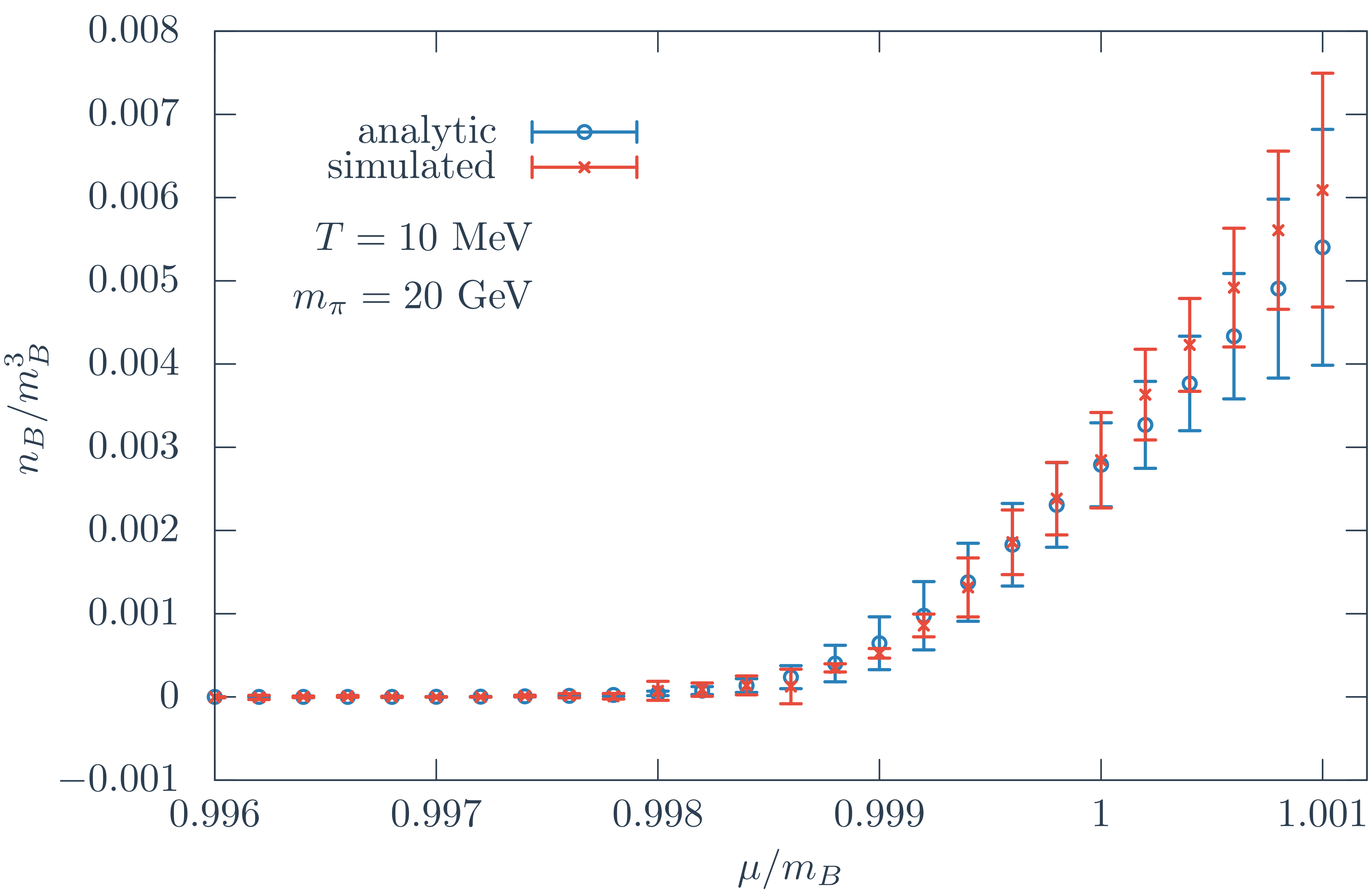
Linked cluster expansion of effective theory

$$\mathcal{Z} = \int \mathcal{D}\phi \, e^{-S_0[\phi]+\frac{1}{2}\sum v_{ij}(x,y)\phi_i(x)\phi_j(y)+\frac{1}{3!}\sum u_{ijk}(x,y,z)\phi_i(x)\phi_j(y)\phi_k(z)+\ldots}$$

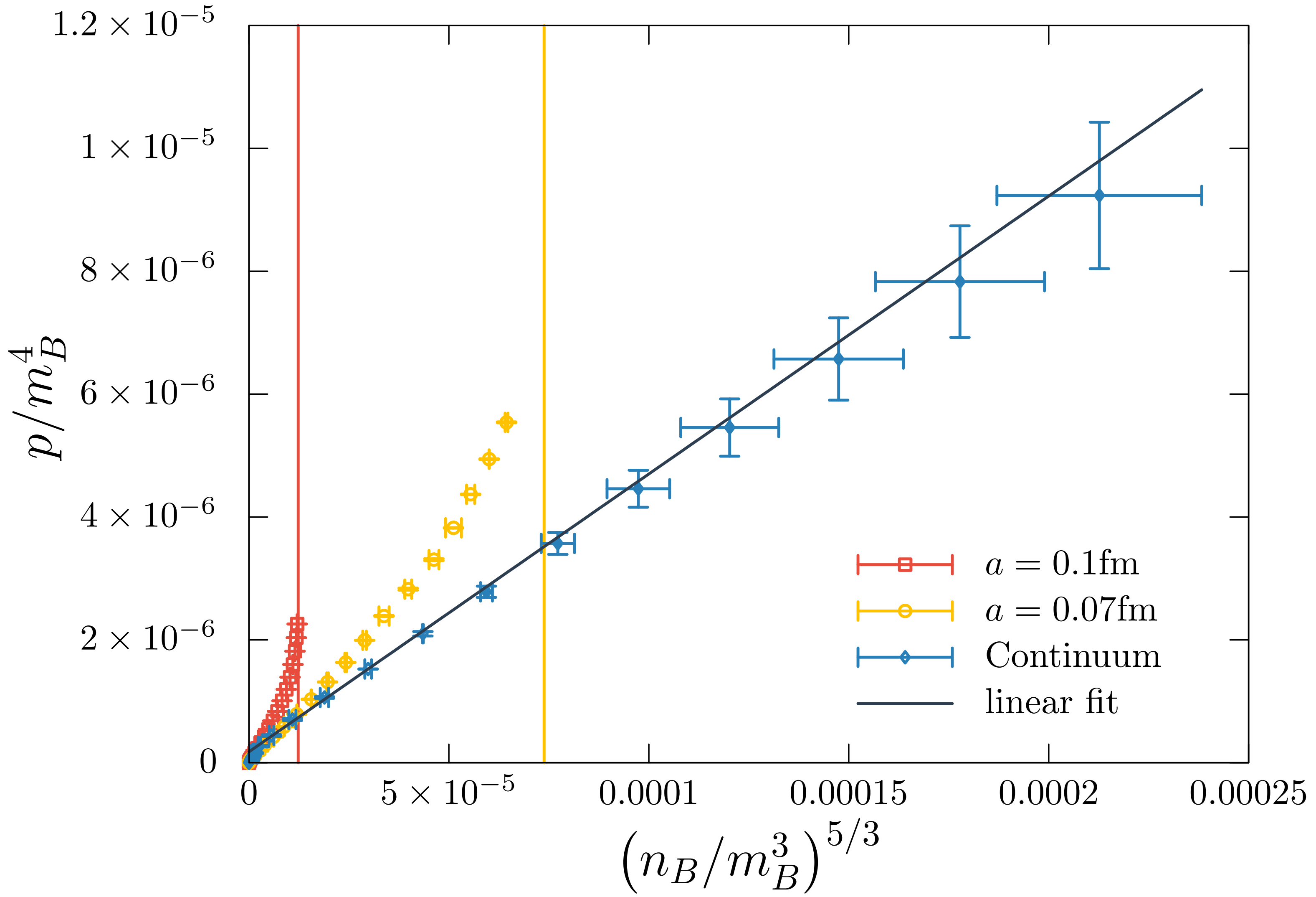
"perturbation theory" in effective couplings

Glesaaen, Neuman, O.P. 15

through $u^5\kappa^8$



Equation of state of heavy nuclear matter, continuum



- EoS fitted by polytrope, non-relativistic fermions!
- Can we understand the pre-factor? Interactions, mass-dependence…

The effective lattice theory approach II

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: integrate over gauge links in strong coupling expansion, leave fermions (staggered)

$$Z_{\mathrm{QCD}} = \int d\psi d\bar{\psi} dU e^{S_F+S_G} = \int d\psi d\bar{\psi} Z_F \left\langle e^{S_G} \right\rangle_{Z_F}$$

$$\left\langle e^{S_G} \right\rangle_{Z_F} \simeq 1+\langle S_G\rangle_{Z_F} = 1+\frac{\beta}{2N_c}\sum_P \left\langle \mathrm{tr}[U_P+U_P^\dagger] \right\rangle_{Z_F} \qquad Z_F(\psi,\bar{\psi}) = \int dU e^{S_F}$$

- Result: 4d "polymer" model of QCD (hadronic degrees of freedom!)
  Valid for all quark masses (also m=0!), at strong coupling (very coarse lattices)

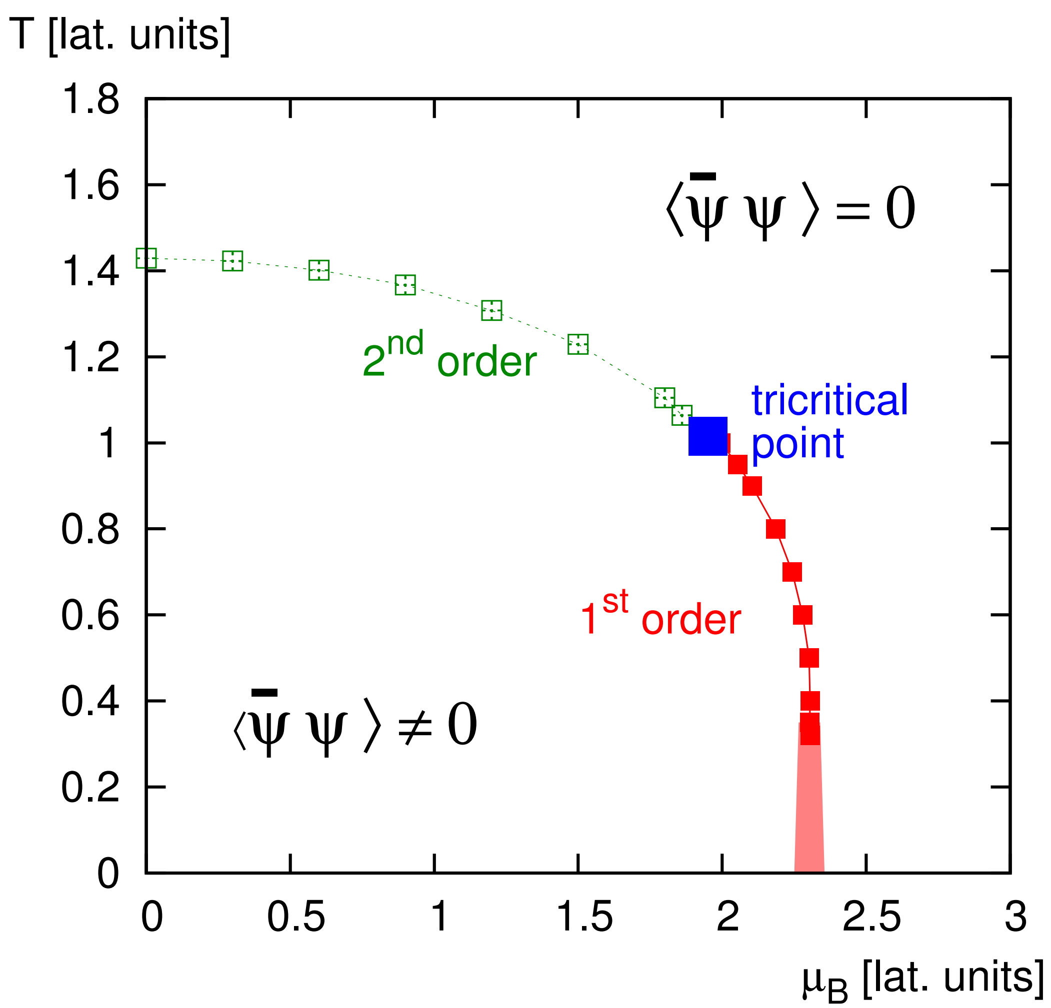
- Step II: sign problem milder: Monte Carlo with worm algorithm

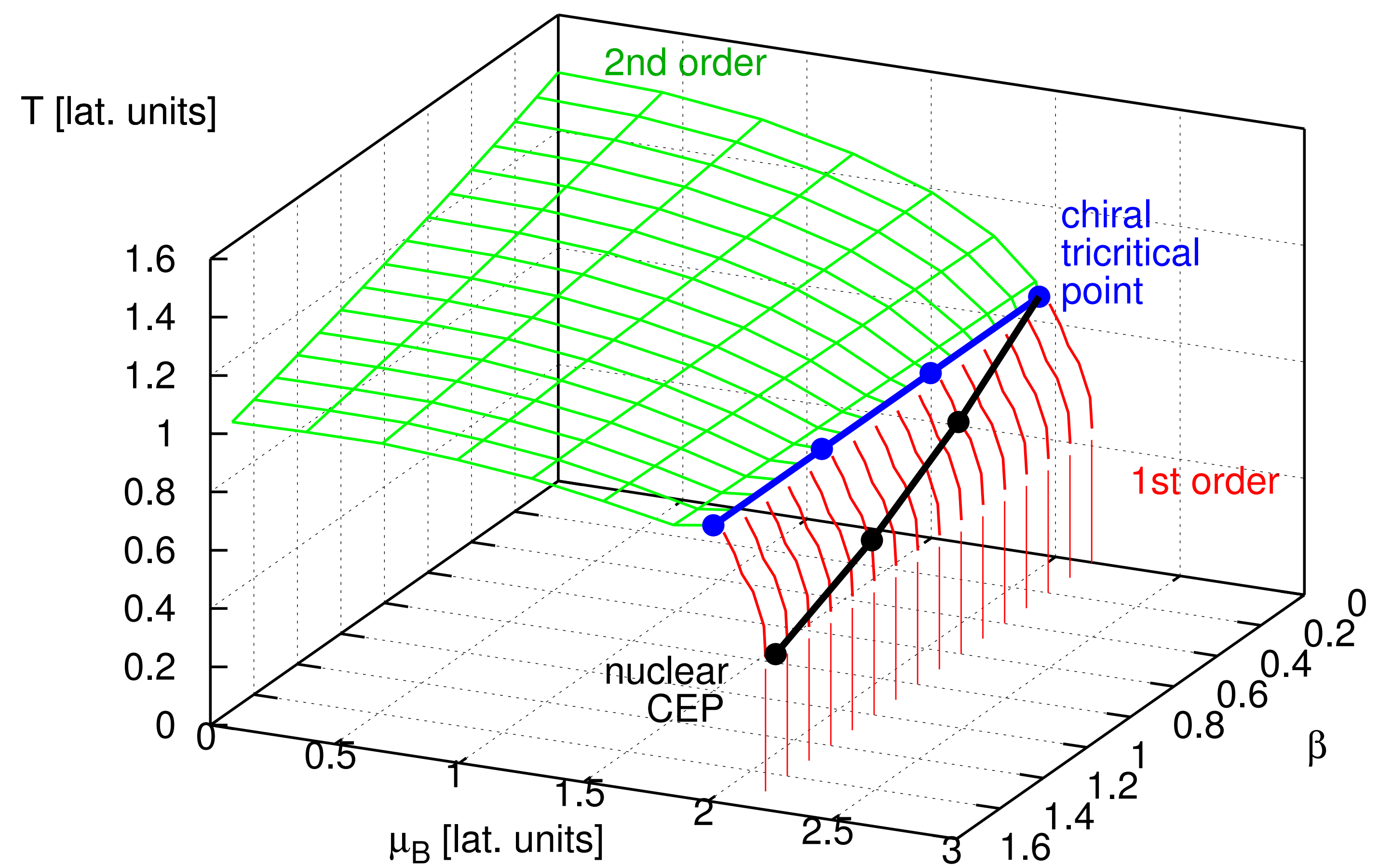
- Numerical simulations without fermion matrix inversion, very cheap!

From strong coupling limit to finite coupling

Unrooted staggered fermions: Nf=4

de Forcrand, Langelage, O.P., Unger 14





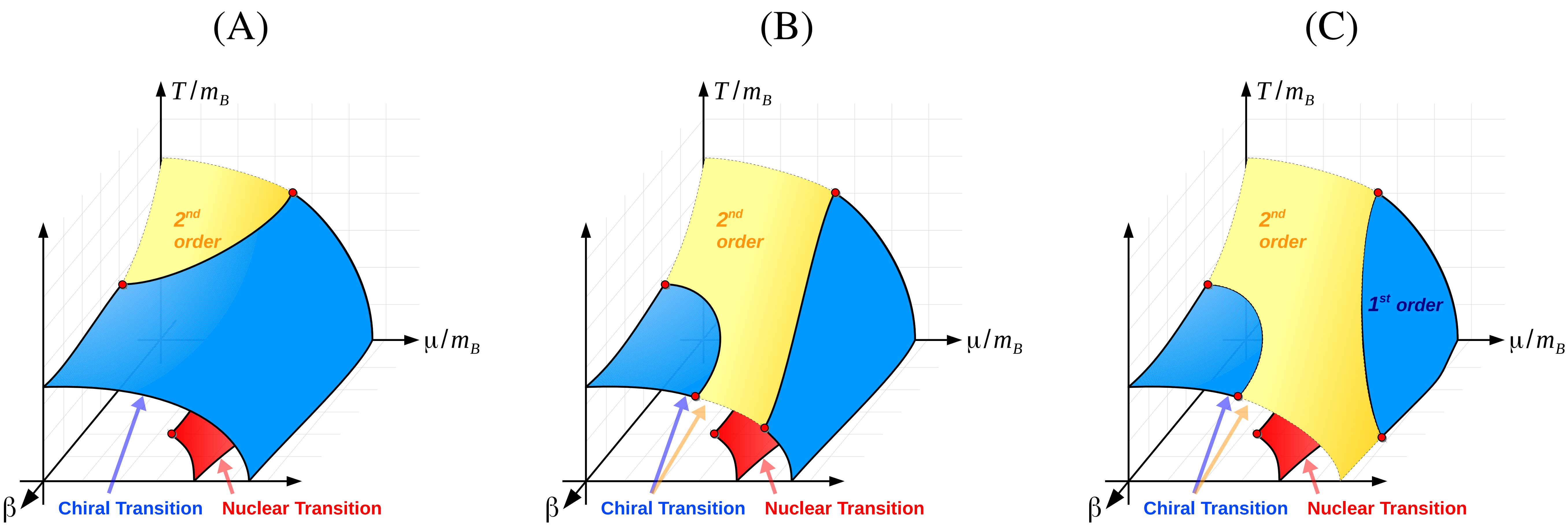
Strong coupling limit: $\beta = 0$

Including leading gauge corrections

Chiral limit: m=0

Nucl. and chiral transition coincide!

Possibilities for continuum Nf=4 phase diagram:



Nf=4 is known to have first order transition at zero density

Conclusions

- Nuclear matter described by QCD directly in "one-parameter distortions"
- Heavy dense QCD near continuum with fully analytic methods
- Chiral, dense QCD on coarse lattices

Backup slides

Subleading couplings

Subleading contributions for next-to-nearest neighbours:

$$\lambda_2 S_2 \quad \propto \quad u^{2N_\tau+2} \sum_{[kl]}{}' 2\mathrm{Re}(L_k L_l^*) \quad \text{distance} \ = \sqrt{2}$$

$$\lambda_3 S_3 \quad \propto \quad u^{2N_\tau+6} \sum_{\{mn\}}{}'' 2\mathrm{Re}(L_m L_n^*) \quad \text{distance} \ = 2$$

as well as terms from loops in the *adjoint* representation:

$$\lambda_a S_a \propto u^{2N_\tau} \sum_{<ij>} \mathrm{Tr}^{(a)} W_i \mathrm{Tr}^{(a)} W_j \quad ; \quad \mathrm{Tr}^{(a)} W = |L|^2 - 1$$

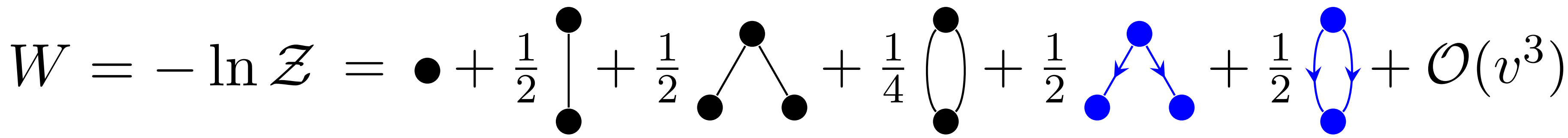
$N_f = 2$

$$z_0 = (1 + 4h_d^3 + h_d^6) + (6h_d^2 + 4h_d^5)h_u + (6h_d + 10h_d^4)h_u^2 + (4 + 20h_d^3 + 4h_d^6)h_u^3 \\ +(10h_d^2 + 6h_d^5)h_u^4 + (4h_d + 6h_d^4)h_u^5 + (1 + 4h_d^3 + h_d^6)h_u^6 \; . \tag{3.11}$$

Free gas of baryons: complete spin flavor structure of vacuum states!

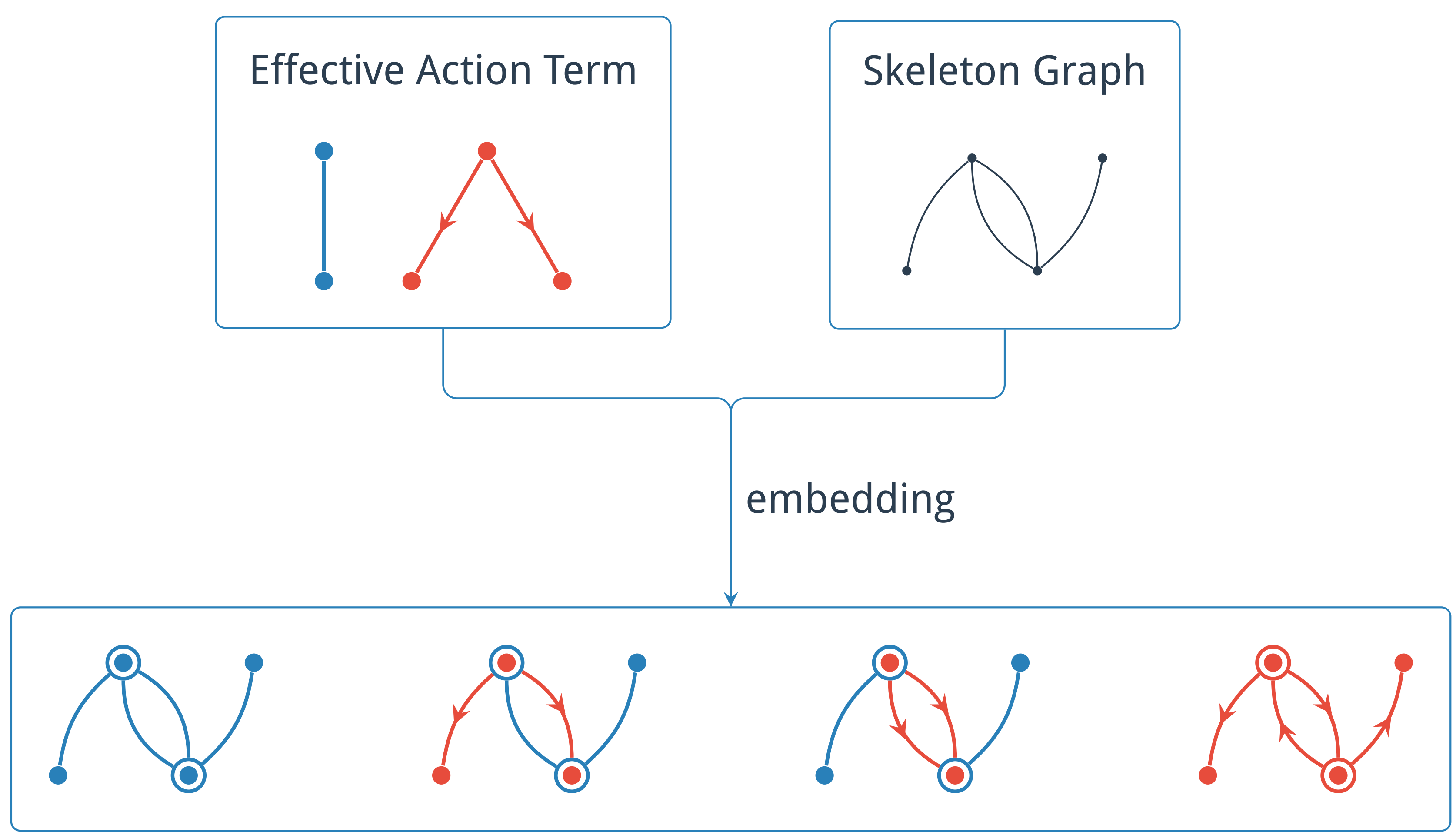
Linked cluster expansion of effective theory

$$\mathcal{Z} = \int \mathcal{D}\phi \, e^{-S_0[\phi] + \frac{1}{2}\sum v_{ij}(x,y)\phi_i(x)\phi_j(y) + \frac{1}{3!}\sum u_{ijk}(x,y,z)\phi_i(x)\phi_j(y)\phi_k(z) + \ldots}$$

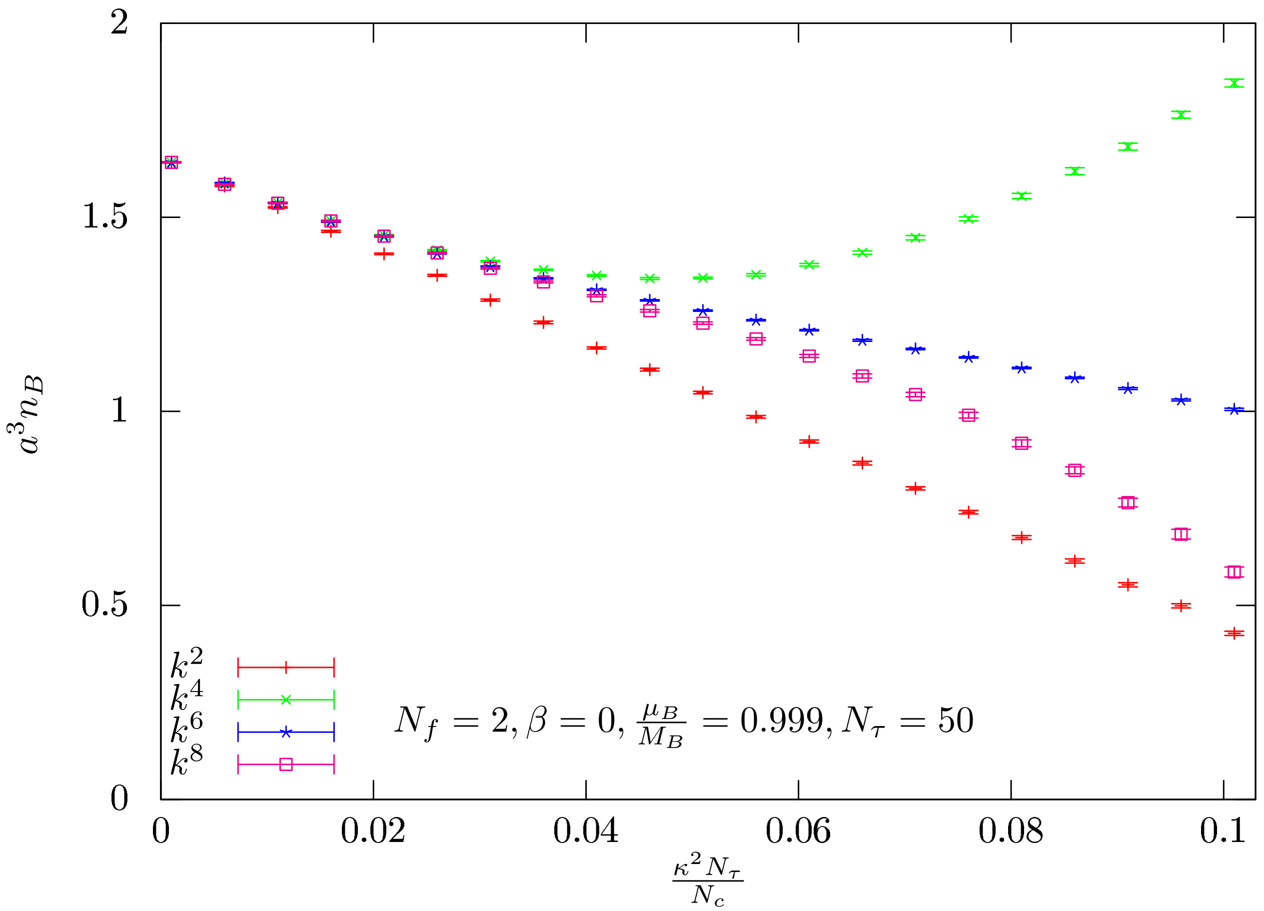


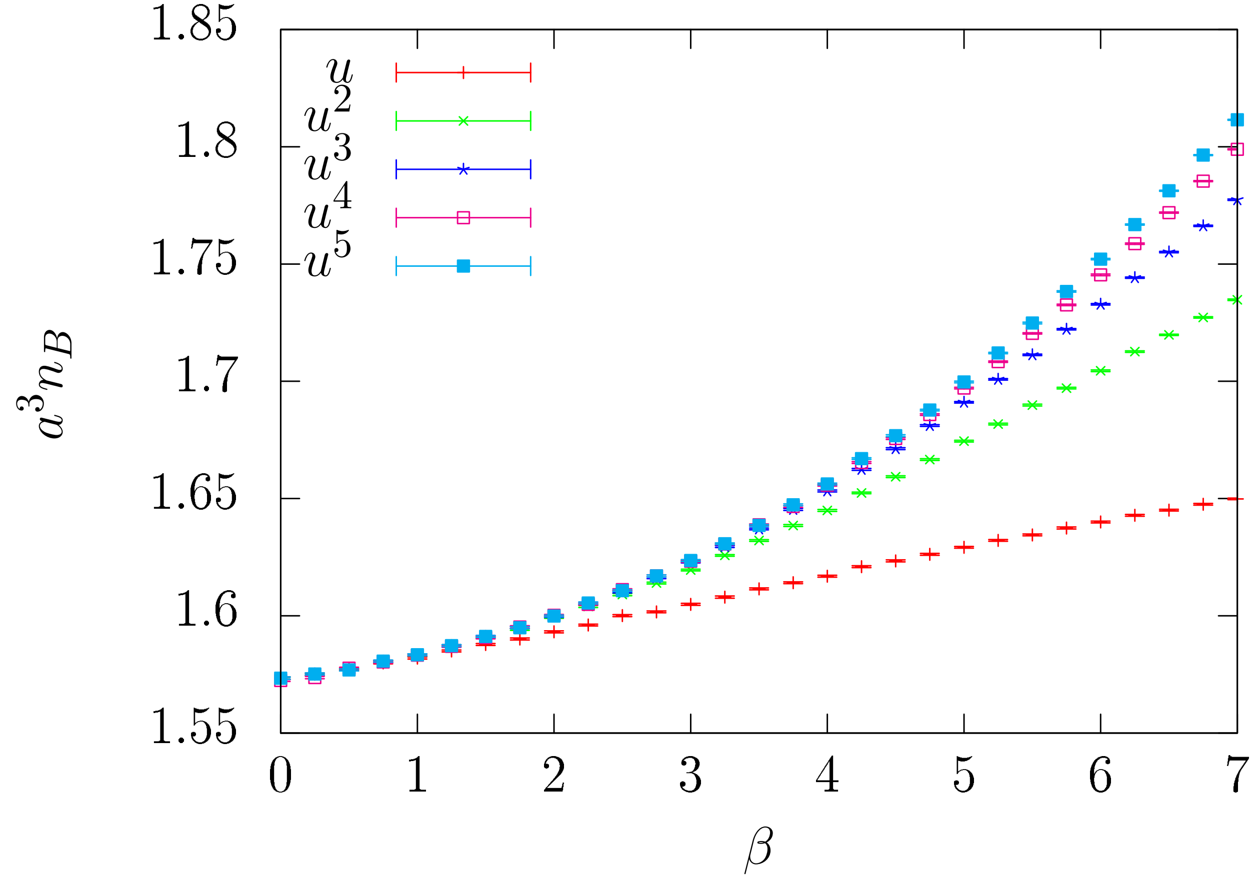
Mapping of the effective theory by embedding:

Glesaaen, Neuman, O.P. 15



Convergence of the effective theory

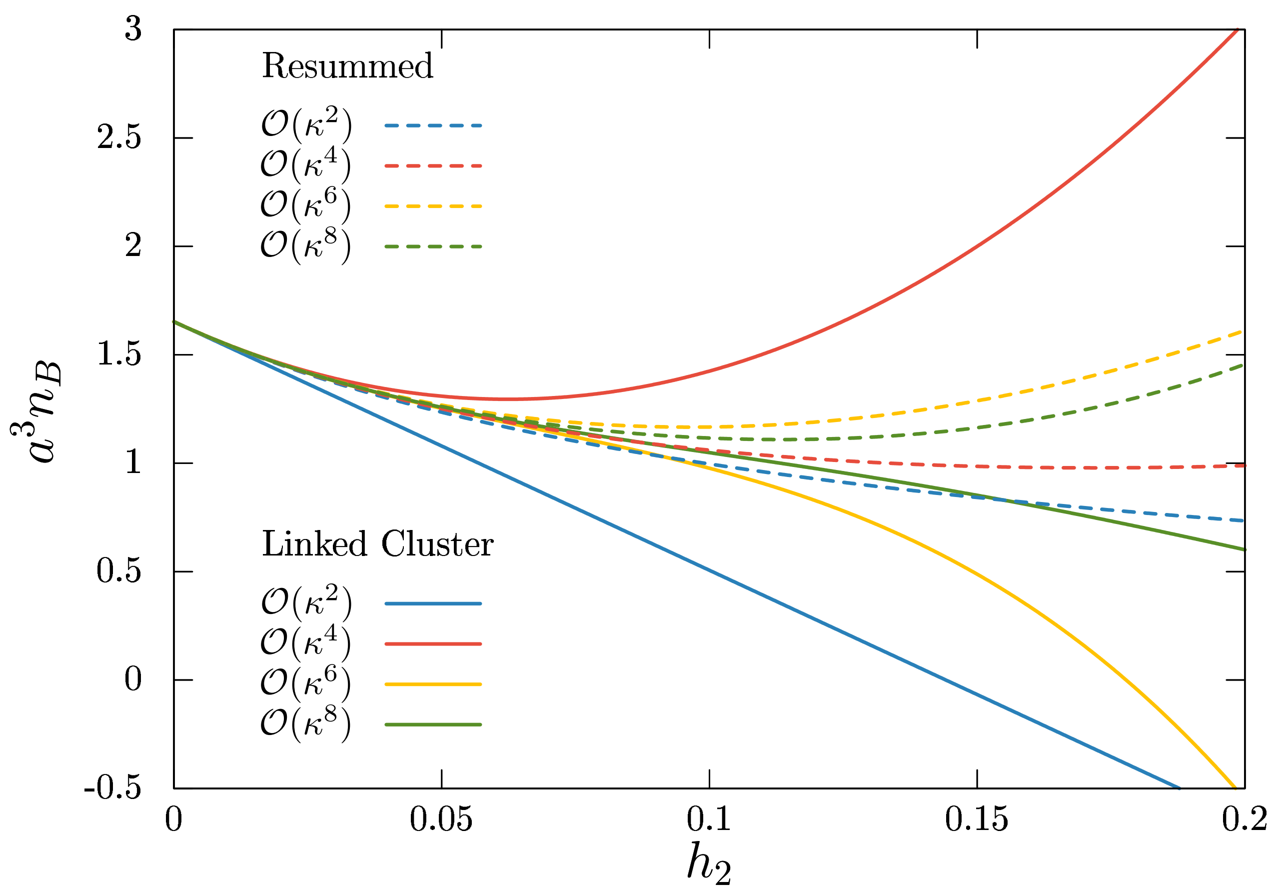


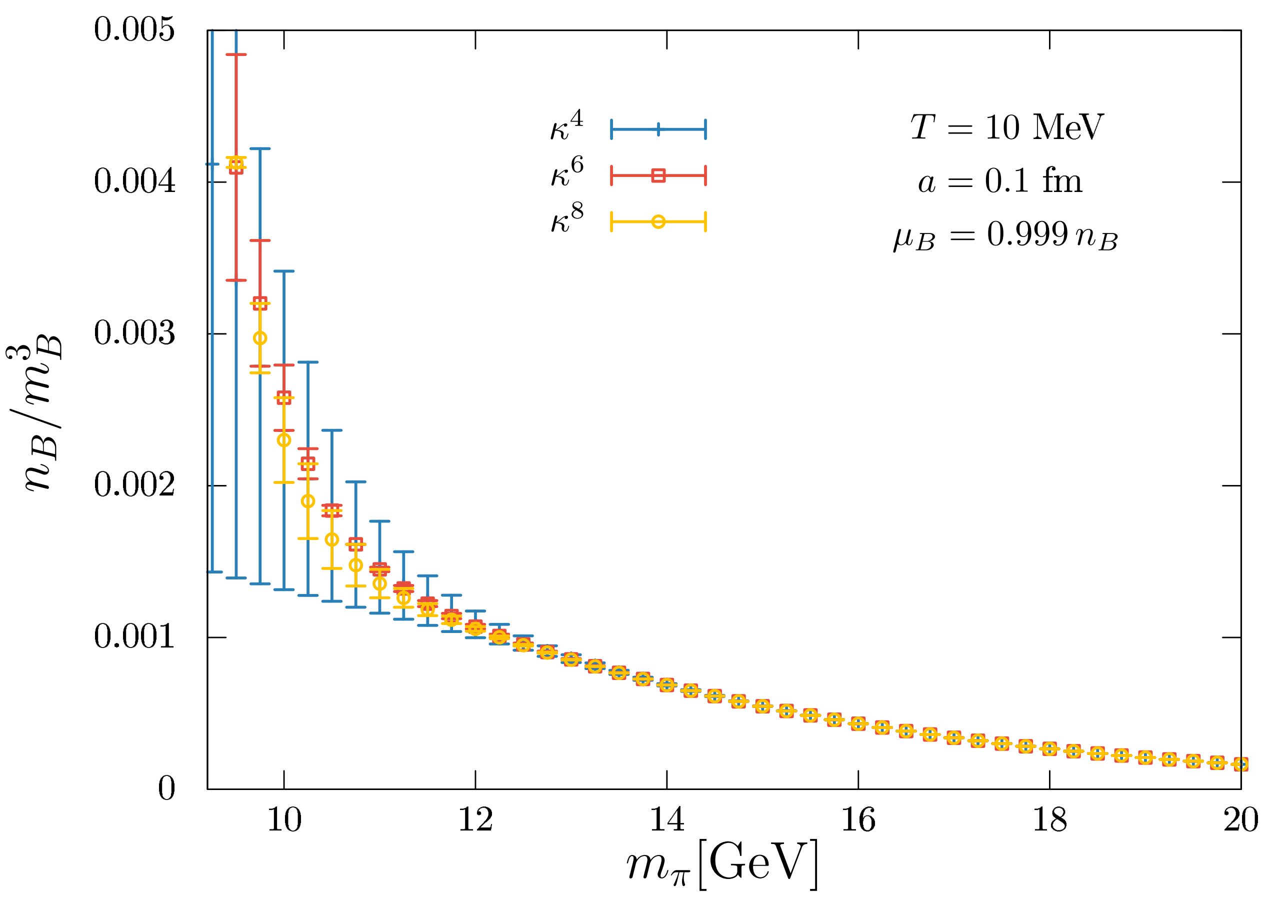


hopping expansion in strong coupling limit

strong coupling expansion at κ^8

Resummations + reach in mass range





Resumming long range non-overlapping chains, gain in mass range "sobering"