Correlations, Fluctuations and the QCD Phase diagram

A. Bzdak, VK, N. Strodthoff: arXiv:1607.07375
A. Bzdak, VK, V. Skokov: arXiv:1612.05128
A. Bzdak, VK: arXiv:1707.02640
J. Steinheimer, A. Bzdak, D. Oliinychenko, and VK: in preparation
The phases of matter: An old question

Fermi 1953
discussed for many years ....
gets more colorful ...
What we know about the Phase Diagram

Lattice QCD:

\( T_c \approx 155 \text{ MeV} \)

pseudo-critical line up to \( O(\mu^2) \)

pressure (EoS) up to \( O(\mu^4) \)

Nuclear Liquid-Gas

\( \sim 920 \text{ MeV} \)

Theory, Measurements

\( \mu \)
What we “hope” for

Cross over transition

Nuclear Liquid-Gas

155 MeV

\( \mu \approx 920 \text{ MeV} \)
Is there a critical point?
Nothing you cannot find in LA...
Cumulants and phase structure

What we always see....

What it really means....

“$T_c$” ~ 160 MeV
Derivatives

$0^{\text{th}}$ order

$1^{\text{st}}$ order

$3^{\text{rd}}$ order

$5^{\text{th}}$ order
How to measure derivatives

At $\mu = 0$:

$$Z = \text{tr} \, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z} \text{tr} \, \hat{E} \, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left( -\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left( -\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left( -\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS
How to measure derivatives

At $\mu = 0$:

$$Z = \text{tr} \, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

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$$\langle (\delta E)^n \rangle = \left( -\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS
Cumulants of Baryon number measure the chem. pot. derivatives of the EOS
Cumulants of (Baryon) Number

\[ K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle \]

\[ K_1 = \langle N \rangle, \quad K_2 = \langle N - \langle N \rangle \rangle^2, \quad K_3 = \langle N - \langle N \rangle \rangle^3 \]

Cumulants scale with volume (extensive): \( K_n \sim V \)

Volume not well controlled in heavy ion collisions

Cumulant Ratios: \( \frac{K_2}{\langle N \rangle}, \frac{K_3}{K_2}, \frac{K_4}{K_2} \)
Simple model

Change degrees of freedom at phase transition

\[ \langle N \rangle = \text{dof}(\mu) e^{\mu/T} \int d^3 p e^{-E/T} \]

Degrees of freedom

\[ \frac{K_2}{\langle N \rangle} \]

\[ \frac{K_3}{K_2} \]

\[ \frac{K_4}{K_2} \]
What to expect from experiment?

Below “$T_c$”  
Above “$T_c$”

$\frac{K_3}{K_2}$  
$\frac{K_4}{K_2}$

$\langle N \rangle$  
$\frac{K_2}{\langle N \rangle}$

Freeze out line  
Baseline

Beam Energy

$\mu - \mu_c$

$T$

$\mu$
Latest STAR result on net-proton cumulants

Xiaofeng Luo

Figure 3: (Color online) Energy dependence of efficiency corrected cumulant ratios $\kappa_2/\sigma$ and $S_\sigma/\kappa_2$ of net-proton distributions in Au+Au collisions at different centralities (0 ∼ 5%, 5 ∼ 10%, 30 ∼ 40%, 70 ∼ 80%). For peripheral (70 ∼ 80%) and mid-central (30 ∼ 40%) collisions, the $\kappa_2/\sigma$ values are close to unity and the $S_\sigma$ show strong monotonic increase when the energy decreases. For 0 ∼ 5% most-central collisions, the values of $\kappa_2/\sigma$ are close to unity at energies above 39 GeV, while below 39 GeV, they start to deviate from unity and show significant deviation below unity around 19.6 and 27 GeV. Finally, they show a strong increase and stay above unity at 7.7 GeV. The $S_\sigma$ at 0 ∼ 5% centrality bin shows a large drop at 7.7 GeV. One may note that we only have statistical errors shown in the figure, which are still large due to limited statistics. The systematical errors, which are dominated by the efficiency correction and the particle identification, are being studied.

Large acceptance is crucial for fluctuations of conserved quantities in heavy-ion collisions to probe the QCD phase transition and critical point. The signals for the phase transition and/or CP will be suppressed with small acceptance. In the Fig. 4, we show the energy dependence of efficiency corrected $\kappa_2/\sigma$ and $S_\sigma/\kappa_2$ of net-proton distributions with various $p_T$ and rapidity range for 0 ∼ 5% most central Au+Au collisions. The Skellam baseline assumes the protons and anti-protons distribute as independent Poisson distributions. It is constructed from the efficiency-corrected mean values of the protons and anti-protons. It is expected to represent the thermal statistical fluctuations of the net-proton number \[24\]. The $\kappa_2/\sigma$ and $S_\sigma/\kappa_2$ are to be unity for Skellam baseline as well as in the Hadron Resonance Gas model. In the two upper panels of Fig. 4, when we gradually enlarge the $p_T$ or rapidity acceptance, the values of $\kappa_2/\sigma$ show a small changes close to unity at energies above 39 GeV, while below 39 GeV, more pronounced structure is observed for a larger $p_T$ or rapidity acceptance. In the two lower panels of Fig. 4, when we...
Let’s take the preliminary STAR data at face value
Further insights: Correlations

Cumulants

\[ K_n = \frac{\partial^n}{\partial \mu^n} \frac{P}{T^4} \]

\[ K_2 = \langle N - \langle N \rangle \rangle^2 = \langle (\delta N)^2 \rangle \]
\[ \rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + C_2(p_1, p_2), \quad \text{C}_2: \text{Correlation Function} \]

\[ K_3 = \langle (\delta N)^3 \rangle \]
\[ \rho_3(p_1, p_2, p_3) = \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) + \rho_1(p_1)C_2(p_2, p_3) + \rho_1(p_2)C_2(p_1, p_3) \\
+ \rho_1(p_3)C_2(p_1, p_2) + C_3(p_1, p_2, p_3) \]
From Cumulants to Correlations (no anti-protons)

Defining integrated correlations function

\[ C_n = \int dp_1 \ldots dp_n C_n(p_1, \ldots, p_n) \]

Simple Algebra leads to relation between correlations \( C_n \) and \( K_n \)

\[
\begin{align*}
C_2 &= -K_1 + K_2, \\
C_3 &= 2K_1 - 3K_2 + K_3, \\
C_4 &= -6K_1 + 11K_2 - 6K_3 + K_4,
\end{align*}
\]

or vice versa

\[
\begin{align*}
K_2 &= \langle N \rangle + C_2 \\
K_3 &= \langle N \rangle + 3C_2 + C_3 \\
K_4 &= \langle N \rangle + 7C_2 + 6C_3 + C_4
\end{align*}
\]
Correlations near the critical point

M. Stephanov, 0809.3450, PRL 102

Scaling of Cumulants $K_n$ with correlation length $\xi$

\[ K_2 \sim \xi^2, \quad K_3 \sim \xi^{4.5}, \quad K_4 \sim \xi^7 \]

Cumulants from Correlations

\[ K_2 = \langle N \rangle + C_2 \]
\[ K_3 = \langle N \rangle + 3C_2 + C_3 \]
\[ K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4 \]

Consequently:

\[ C_2 \sim \xi^2, \quad C_3 \sim \xi^{4.5}, \quad C_4 \sim \xi^7 \]

Correlations $C_n$ pick up the most divergent pieces of cumulants $K_n$!
Preliminary Star Data
(X. Luo, PoS Cpod 2014 (019))

Significant four particle correlations!

Four particle correlation dominate $K_4$ for central collisions at 7.7 GeV

\[ K_2 = \langle N \rangle + C_2 \]
\[ K_3 = \langle N \rangle + 3C_2 + C_3 \]
\[ K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4 \]
Based on prelim. STAR data

Correlations

- Figure 6. (Color online) Left: Energy dependence of net-kaon (proxy for net-strangeness) numbers in the Au+Au collisions measured by STAR [31, 32].

- Dip at 19.6 GeV from Au+Au : Net-proton
  - 0-5%
  - 5-10%
  - 70-80%
  - UrQMD, 0-5%

- Figure 5 right shows
  - $\Delta S_{PP}/S_{Skellam}$
  - $kO^2$

- Dip at 19.6 GeV from

- $\sqrt{s_{NN}}$ (GeV)
Reduced correlation function

\[ c_k = \frac{\int \rho_1(y_1) \cdots \rho_1(y_k) c_k(y_1, \ldots, y_k) \, dy_1 \cdots dy_k}{\int \rho_1(y_1) \cdots \rho_1(y_k) \, dy_1 \cdots dy_k} \]

\[ C_k = \langle N \rangle^k c_k \]

Independent sources such as resonances, cluster, p+p:

\[ c_k \sim \frac{\langle N_s \rangle}{\langle N \rangle^k} \sim \frac{1}{\langle N \rangle^{k-1}} \]

For example two particle correlations:

\[ c_2 \sim \frac{\text{Number of sources}}{\text{Number of all pairs}} = \frac{\text{Number of correlated pairs}}{\text{Number of all pairs}} = \frac{1}{\langle N \rangle} \]
Centrality dependence

Based on prelim. STAR data
Centrality dependence

Based on prelim. STAR data

$C_3$

$c_3 + 3 \cdot 10^{-4}$
$c_3 = b/N^2$
$c_3 = a + b/N^2$

$a = -1.5 \cdot 10^{-4}$
$b = +0.024$

c_3 > 0

c_3 < 0

$N_{part}$

7.7 GeV

Based on prelim. STAR data

$C_4$

$-c_4 + 2 \cdot 10^{-4}$
$c_4 = b/N^3$
$c_4 = a + b/N^3$

$a = +7 \cdot 10^{-5}$
$b = -0.02$

c_4 < 0

c_4 > 0

$N_{part}$

7.7 GeV

Based on prelim. STAR data

$C_3$

$c_3 + 1 \cdot 10^{-4}$
$c_3 = b/N^2$
$c_3 = a + b/N^2$

$a = -7 \cdot 10^{-5}$
$b = +0.019$

c_3 > 0

c_3 < 0

$N_{part}$

19.6 GeV

Based on prelim. STAR data

$C_4$

$-c_4 + 2 \cdot 10^{-4}$
$c_4 = b/N^3$
$c_4 = a + b/N^3$

$a = +4 \cdot 10^{-5}$
$b = -0.01$

c_4 < 0

c_4 > 0

$N_{part}$

19.6 GeV
Rapidity dependence

\[ C_k(\Delta Y) = \int_{\Delta Y} dy_1 \ldots dy_k \rho_1(y_1) \ldots \rho_1(y_k) c_k(y_1, \ldots, y_k) \]

Assume: \( \rho_1(y) \sim const. \)

short range correlations:

\[ c_k(y_1, \ldots, y_k) \sim \delta(y_1 - y_2) \ldots \delta(y_{n-1} - y_k) \]

\[ C_k(\Delta Y) \sim \Delta Y \rightarrow K_k \sim \Delta Y \]

Long range correlations:

\[ c_k(y_1, \ldots, y_k) = const. \]

\[ C_k(\Delta Y) \sim (\Delta Y)^k \]
Preliminary Star data are consistent with long range correlations

7.7 GeV central

19.6 GeV central
Long range correlations

\[ C_k = \langle N \rangle^k c_k \]
\[ c_k = \text{const.} \Rightarrow K_n = K_n (\langle N \rangle) \]

NB: Data are consistent with small “repulsive” component

\[ c_2(y_1, y_2) = c_2^0 + \gamma_2 (y_1 - y_2)^2 \quad \gamma_2 > 0 \]
Energy dependence

Note: anti-protons are non-negligible above 19.6 GeV
Data are protons only

Based on prelim. STAR data
Can we understand these correlations?

- Two particle correlations can be understood by simple Glauber model + Baryon number conservation

Four particle correlations are orders of magnitudes larger in the data.
Can we understand these correlations?

• Three and four particle correlations require lots of “fantasy”…

• For example, if about 40% of the nucleons are come in 8-nucleon clusters one can get near the data…

![Graphs of multi-particle correlations](image)

Proton-quartets = 8 nucleon clusters
Shape of probability distribution

\[ K_3 < \langle N \rangle \]
\[ K_4 > \langle N \rangle \]
\[ K_3 = \langle N - \langle N \rangle \rangle^3 \]
\[ K_4 = \langle N - \langle N \rangle \rangle^4 - 3 \langle N - \langle N \rangle \rangle^2 \]
Simple two component model
Summary

• Fluctuations sensitive to phase structure:
  - measure “derivatives” of EOS
• Measurements are difficult
• Cumulants contain information about correlations
• Preliminary STAR data:
  - Significant four particle correlations at 7.7 and 11.5 GeV
  - Dip in $K_4/K_2$ at 19.6 GeV is due to negative two-particle correlations
  - Centrality dependence (at 7.7 GeV) indicates independent sources for $N_{\text{part}} < 150$ and “collective” correlations for $N_{\text{part}} > 200$.
  - At about the same centrality three- and four particle correlations change sign!
• New dynamics?
Summary

• Preliminary STAR data continued:
  - For central 7.7 and 11.5 GeV two and three particle correlations are negative and four particle are positive.

• Other more mundane effects may contribute
  - Fluctuations of system size ($N_{\text{part}}$)
    • May explain 2-particle correlations
    • Fail to reproduce the magnitude of 3- and 4- particle correlations

• Understanding 3- and 4 particle correlations requires "desperate measures"!
Thank You
Simple two component model

Difficult to see in the real data with efficiency $\varepsilon=0.6$
Phase Diagrams

Maybe it’s better to look at the Phase diagram in density.


Curious similarity
Things to consider

- Fluctuations of conserved charges ?!
- Volume Fluctuations
- Net-protons different from net-baryons
  - Isospin fluctuations
- “Stopping” fluctuations
- Higher cumulants probe the tails. Statistics!
- The detector “fluctuates”!
  - Efficiency effects
- ........
Finite efficiency

Unfolding needed if we want to know what the true cumulants are
Tricky with a real detector
Compare Data with Lattice QCD

Example: “Charge” susceptibility

\[ \chi_Q = \int d^3x \, \langle \rho(x)\rho(0) \rangle = \int d^3p \, \langle \tilde{\rho}(p)\tilde{\rho}(0) \rangle \]

Equivalence of integrated coordinate space and momentum space correlation function

Experiment almost never integrates ALL of momentum space!

Lattice (hopefully) does integrate over all coordinate space
Correlations: Lattice vs Data

\[
\langle n(y_1)(n(y_2) - \delta(y_1 - y_2)) \rangle = \langle n(y_1) \rangle \langle n(y_2) \rangle \left( 1 + C(y_1, y_2) \right)
\]

\[
C(y_1, y_2) \sim \exp \left( \frac{-(y_1 - y_2)^2}{2\sigma^2} \right)
\]

\[
\frac{\langle (\delta N)^2 \rangle}{\langle N \rangle} = 1 + \langle N \rangle \int_{-\Delta/2}^{\Delta/2} C(y_1, y_2) \, dy_1 \, dy_2
\]

“Lattice result”

“Charge conservation”
Lattice QCD

Equation of state
S. Borsanyi et al, JHEP 1011 (2010) 077

Cross over transition

Susceptibilities

\[ \frac{P(T, \mu_B)}{T^4} = \frac{P(T, \mu_B = 0)}{T^4} + \sum_n c_n \left(\frac{\mu}{T}\right)^n \]

\[ c_n = \frac{\chi_n^B}{n!} \]

Susceptibilities provide \( \mu_B \)-dependence of EOS
AND measure fluctuations
present status: 6\(^\text{th}\) order, 8\(^\text{th}\) in the works