# Correlations, Fluctuations and the QCD Phase diagram

A. Bzdak, VK, N. Strodthoff: arXiv:1607.07375

A. Bzdak, VK, V. Skokov: arXiv:1612.05128

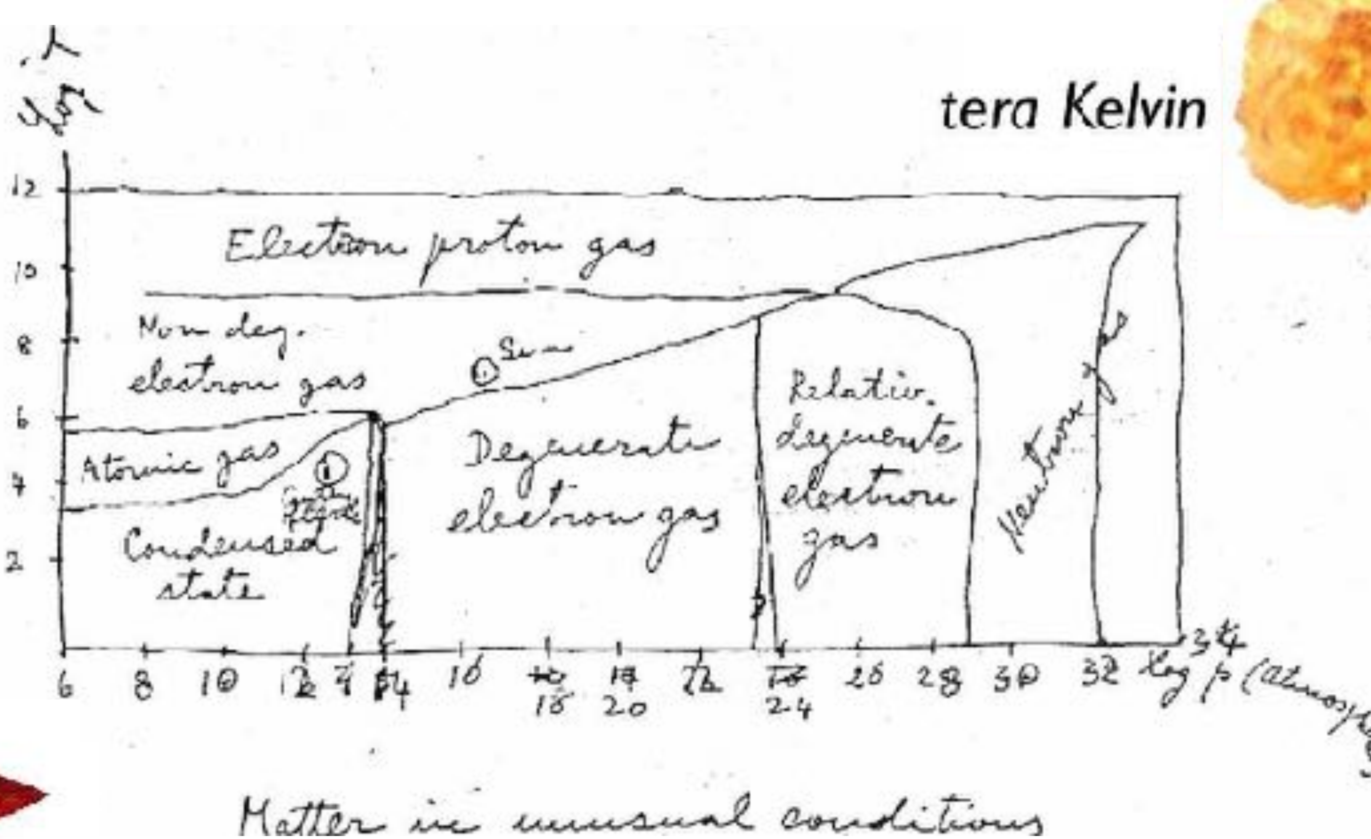
A. Bzdak, VK: arXiv:1707.02640

J. Steinheimer, A. Bzdak, D. Oliinychenko, and VK: in preparation

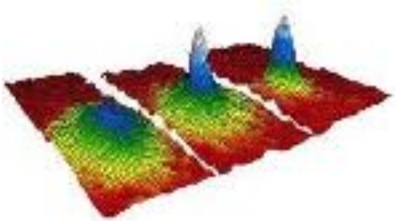
BEST COLLABORATION

# The phases of matter: An old question

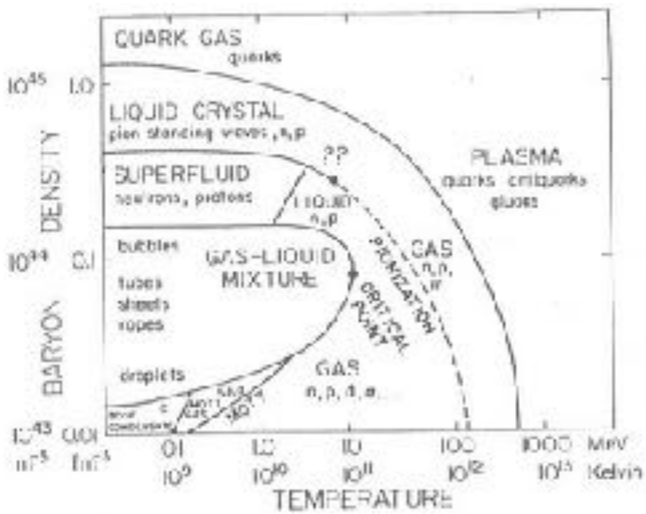
Fermi 1953

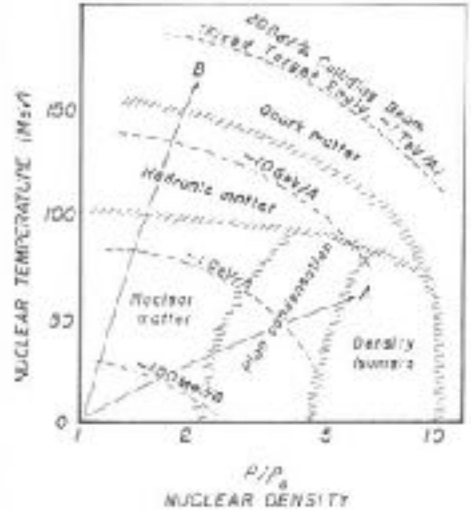


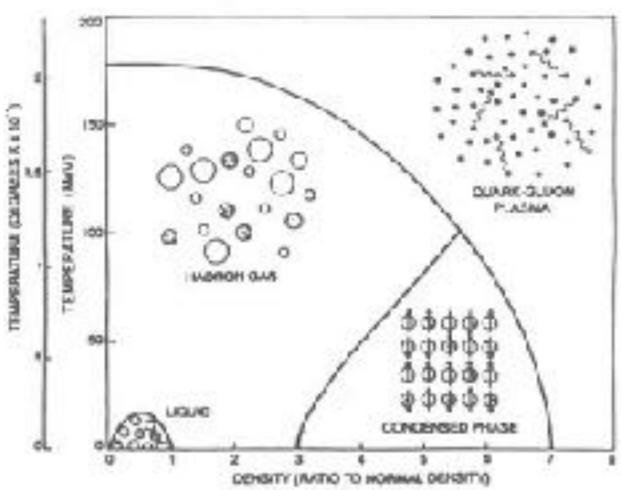

tera Kelvin

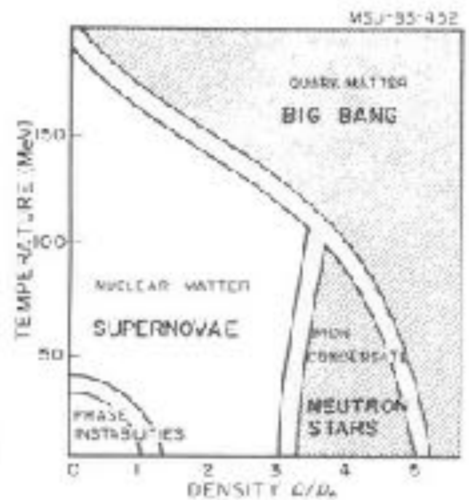


# discussed for many years ....

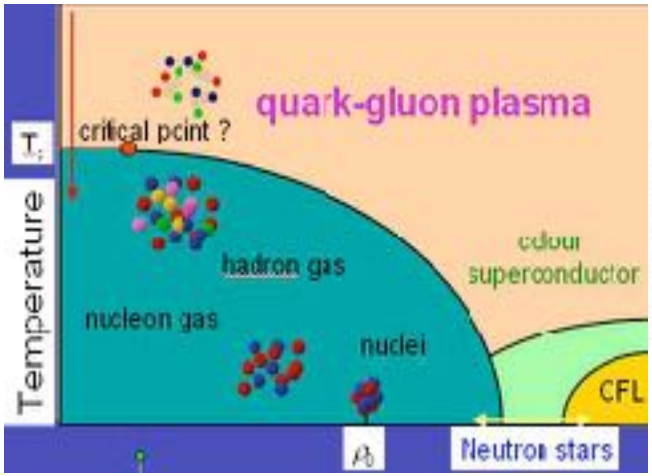


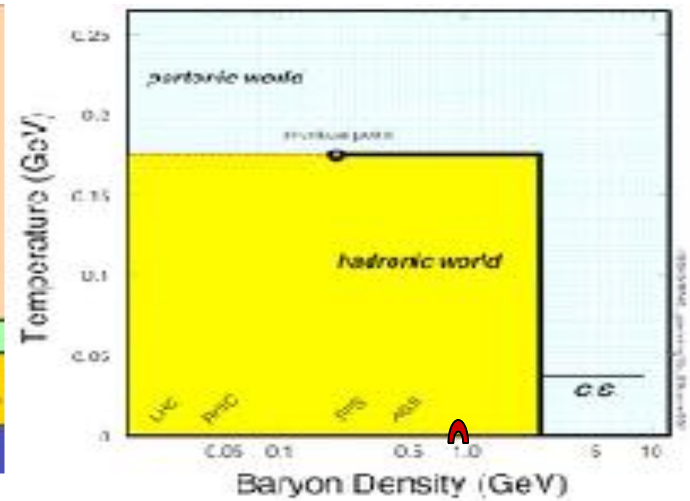



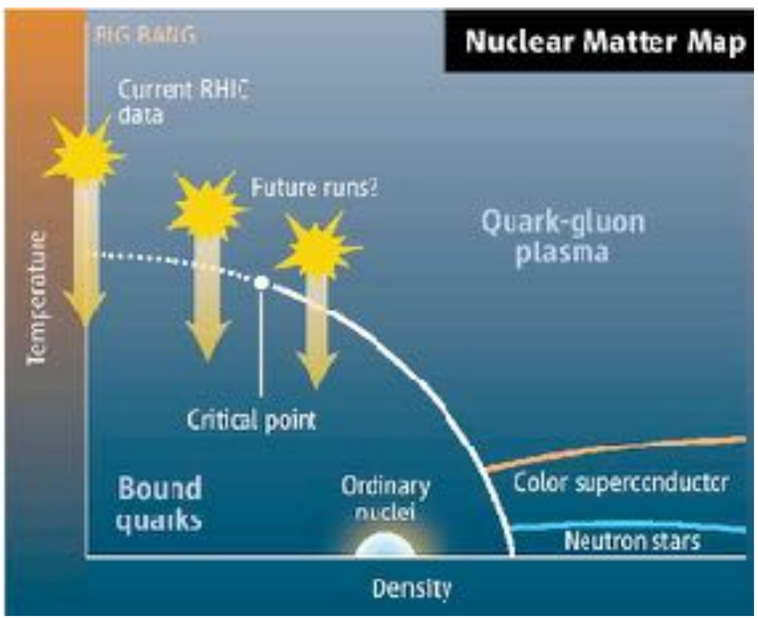



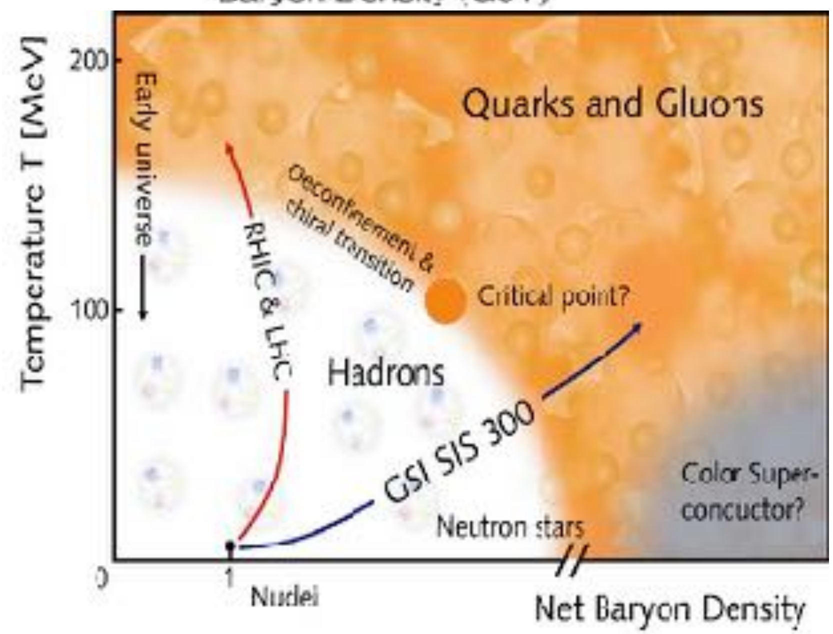



# gets more colorful ...

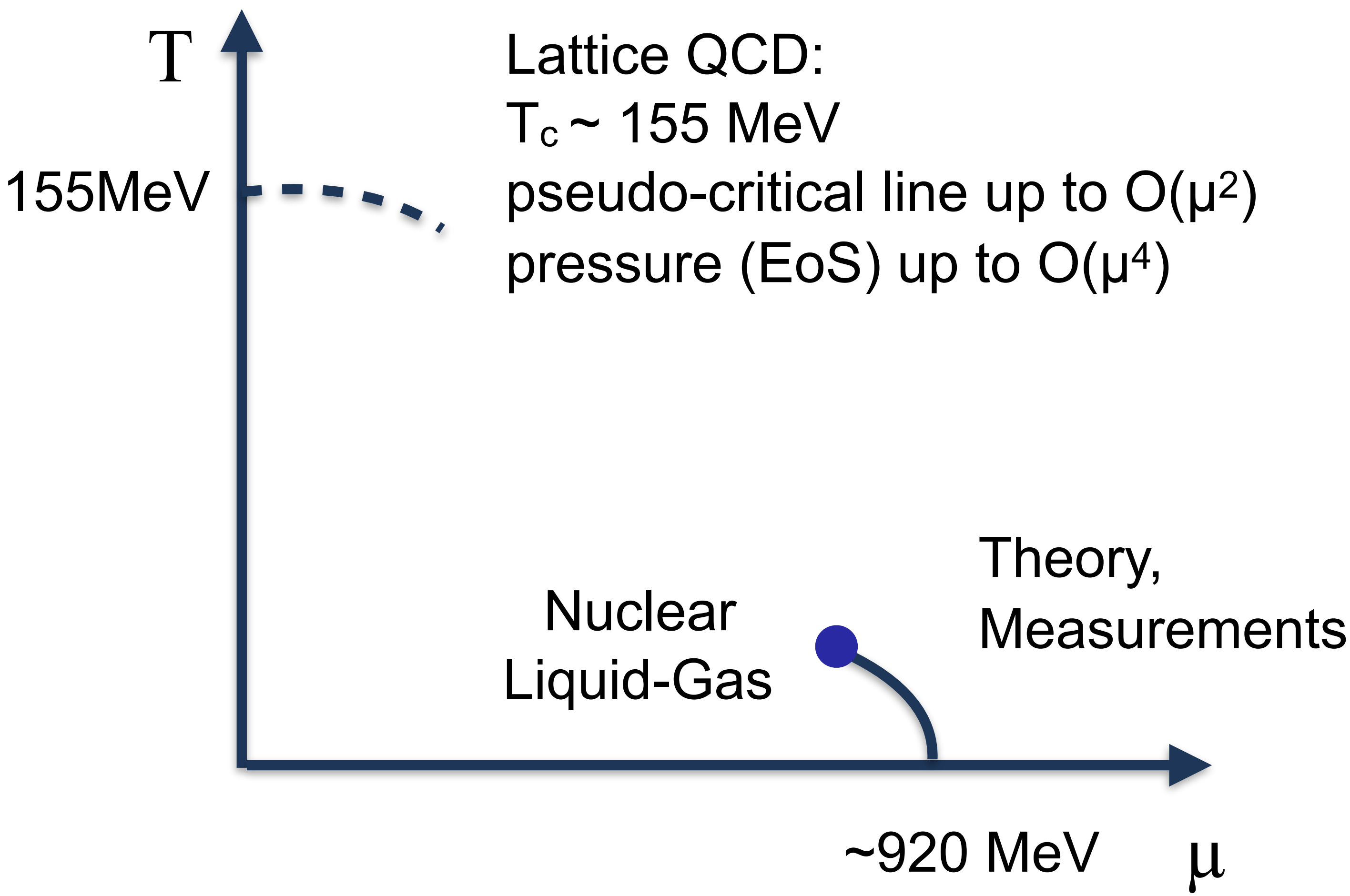


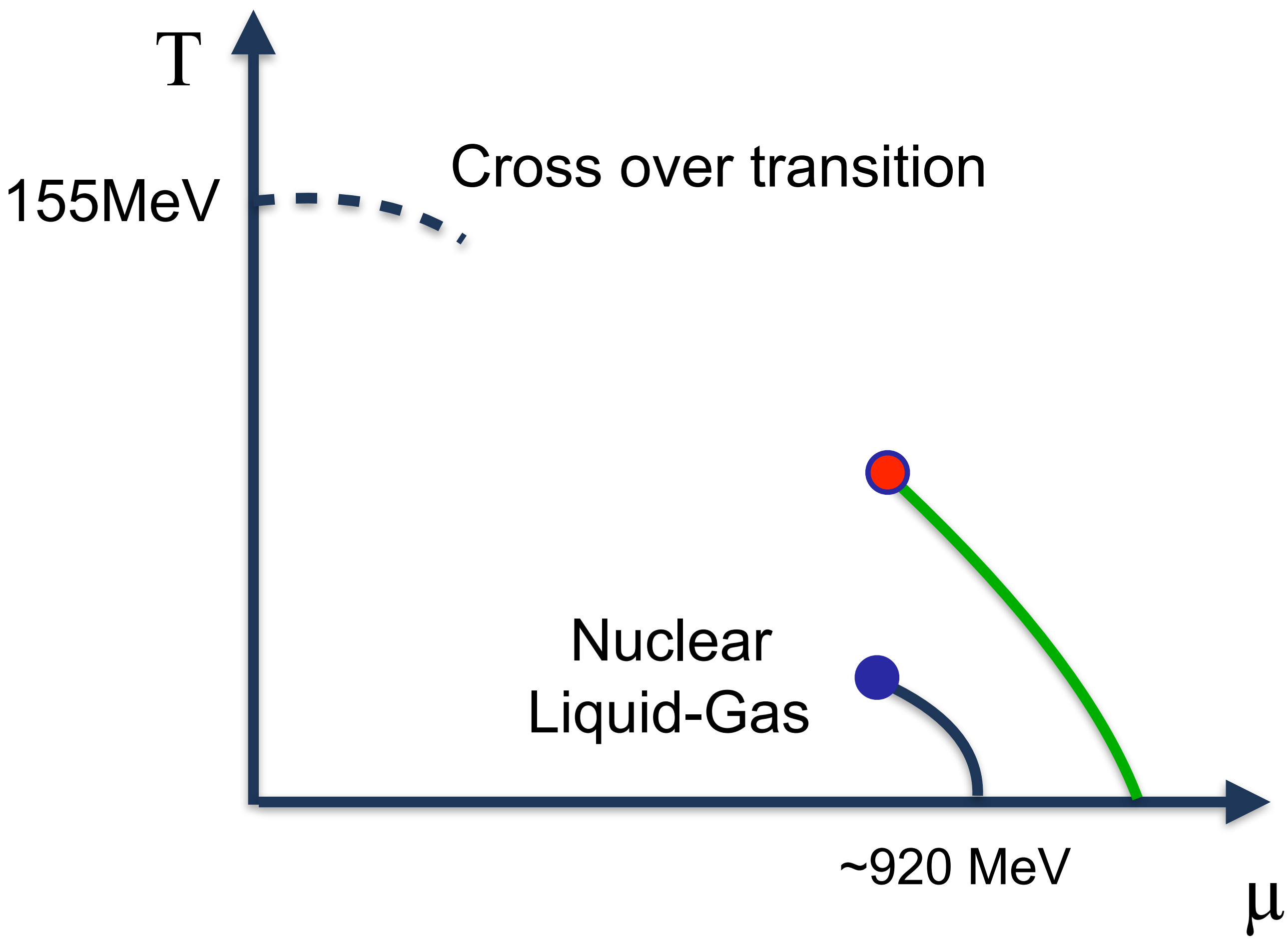



# What we know about the Phase Diagram

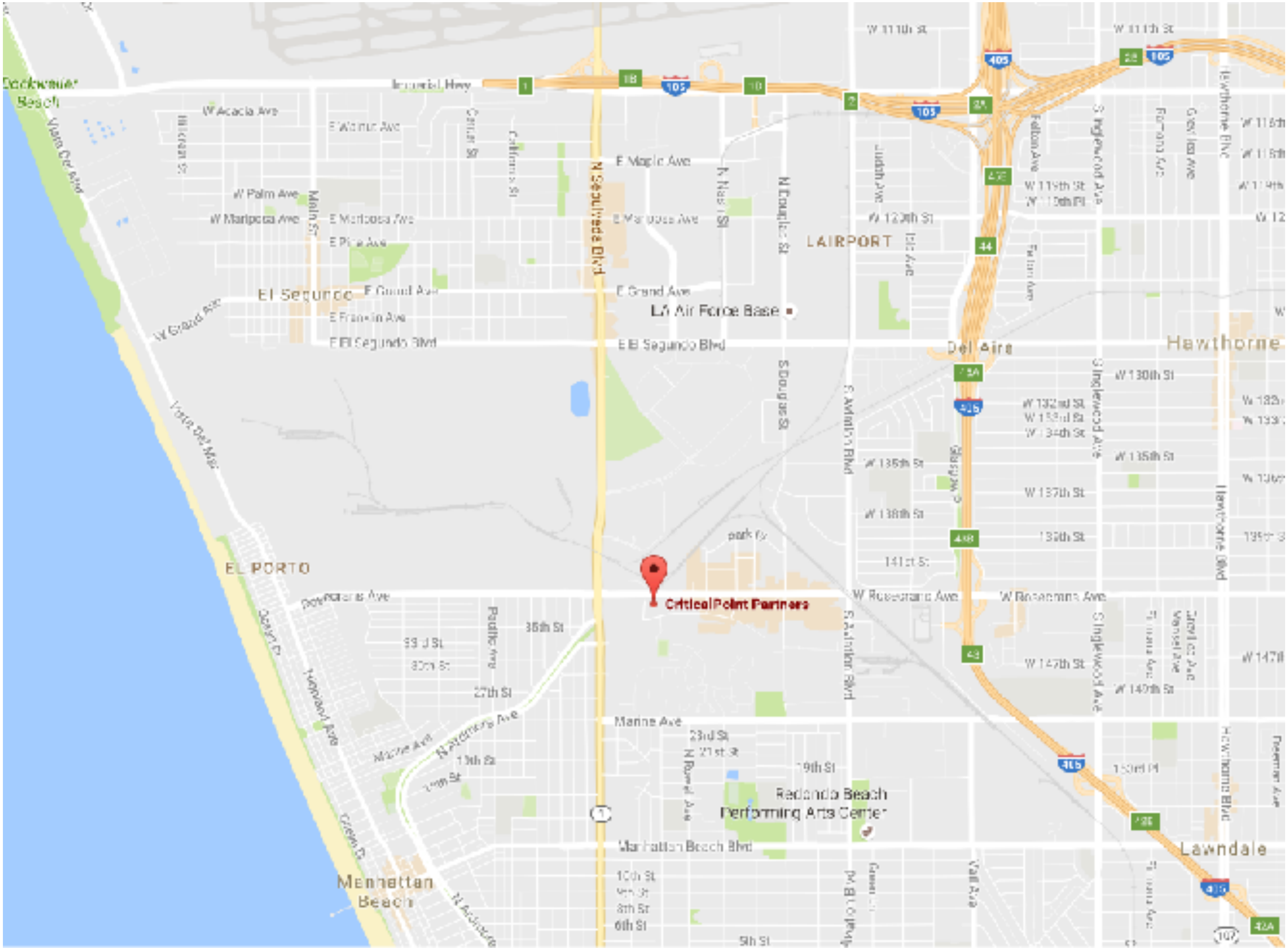


# What we “hope” for

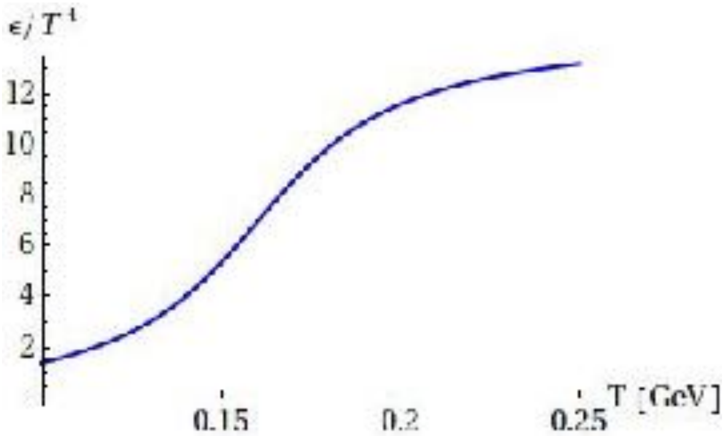


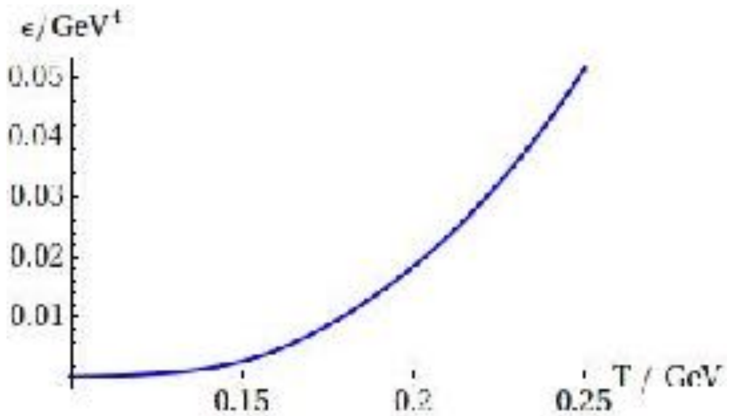
# Is there a critical point?

# Nothing you cannot find in LA…



# Cumulants and phase structure



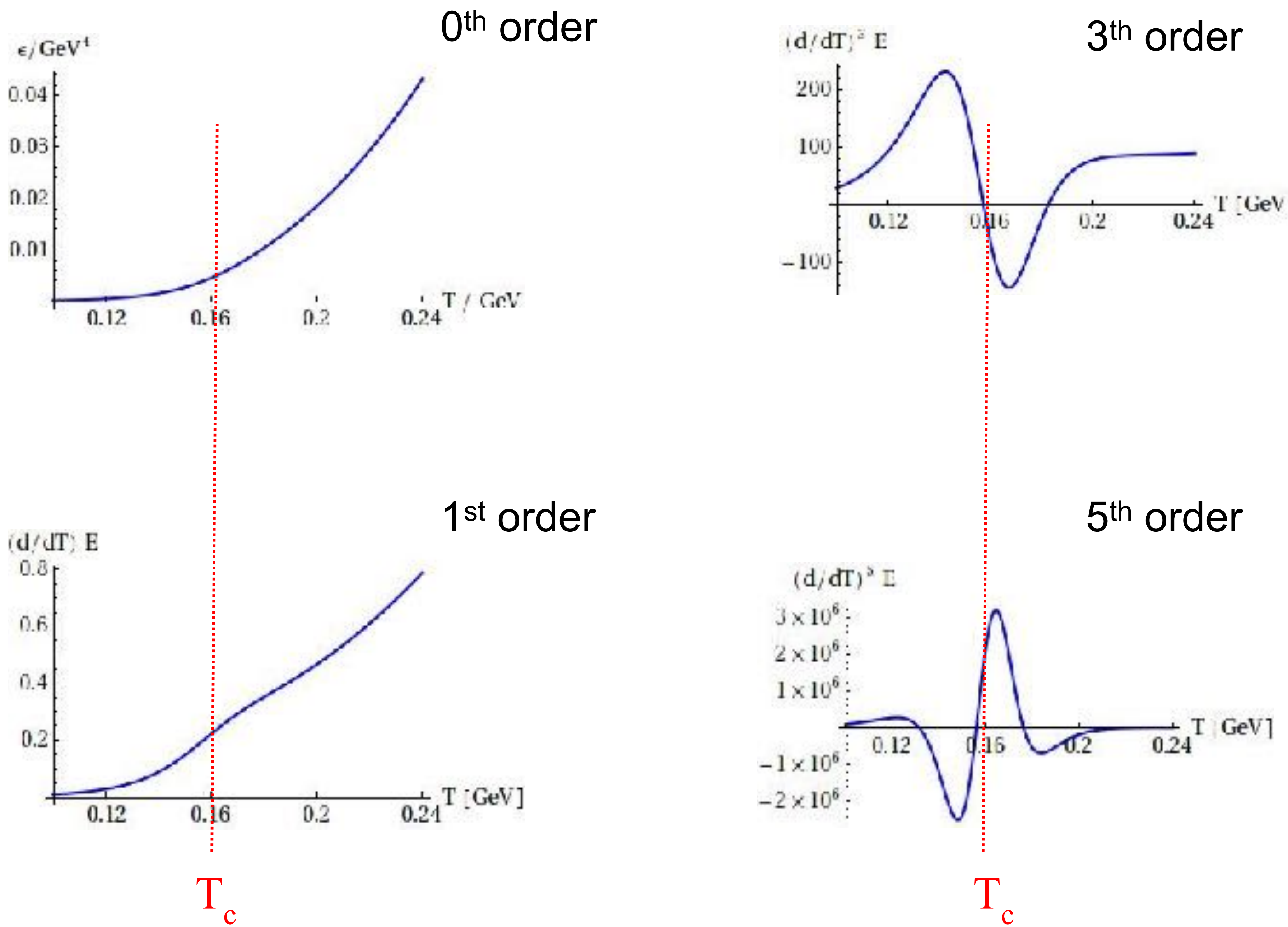




What we always see....

What it really means....

"$T_c$" ~ 160 MeV

# Derivatives



# How to measure derivatives

At μ = 0:

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z}\, tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left( -\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left( -\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left( -\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

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Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

# Cumulants of (Baryon) Number

$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

$$K_1 = \langle N \rangle\,, \quad K_2 = \langle N - \langle N \rangle \rangle^2\,, \quad K_3 = \langle N - \langle N \rangle \rangle^3$$

Cumulants scale with volume (extensive): $K_n \sim V$

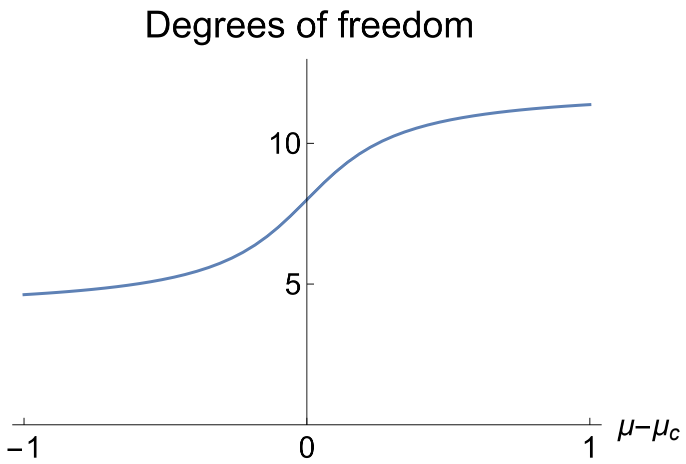
Volume not well controlled in heavy ion collisions

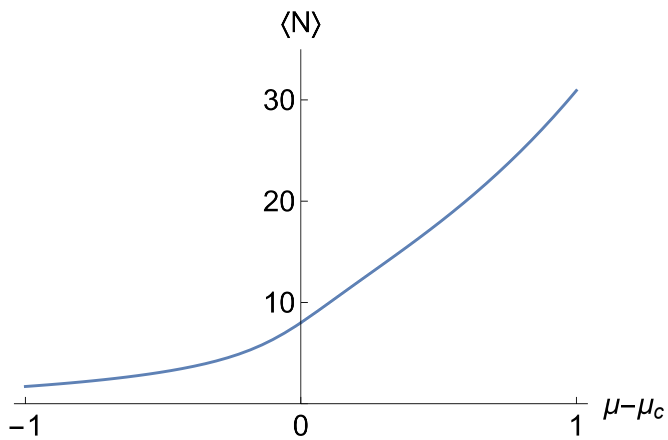
Cumulant Ratios: $\frac{K_2}{\langle N \rangle},\ \frac{K_3}{K_2},\ \frac{K_4}{K_2}$

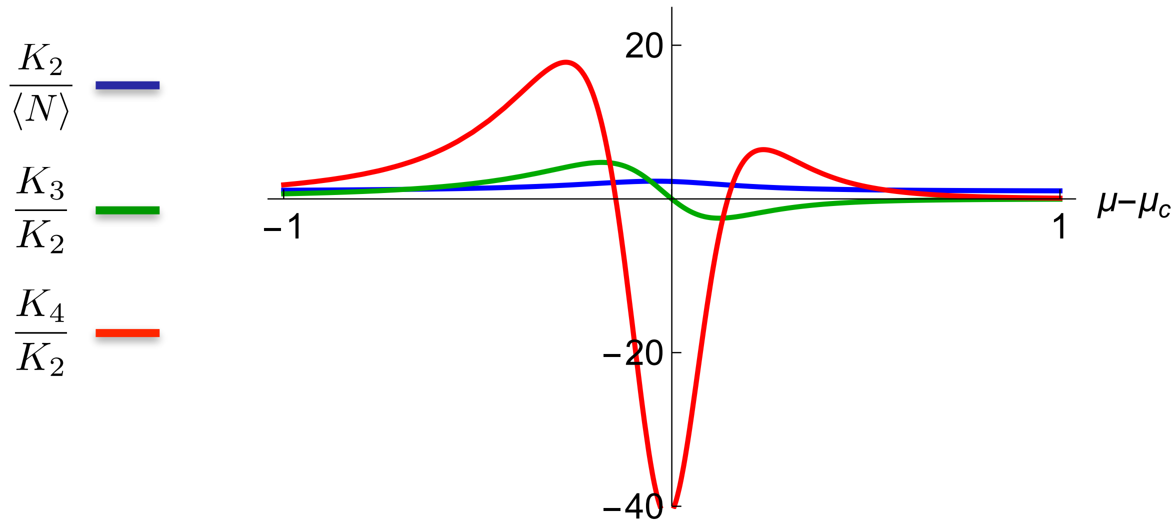
# Simple model

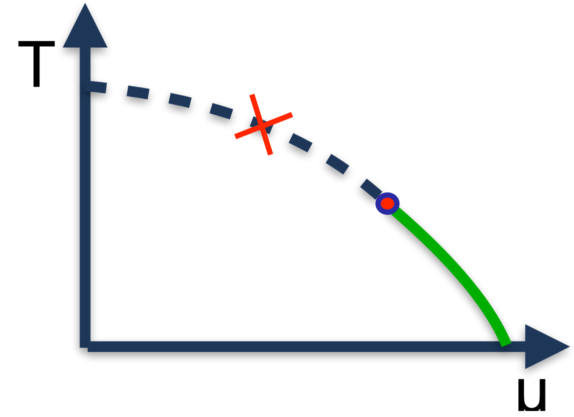
Change degrees of freedom at phase transition

$$\langle N \rangle = \color{red}{dof(\mu)}\, e^{\mu/T} \int d^3p e^{-E/T}$$





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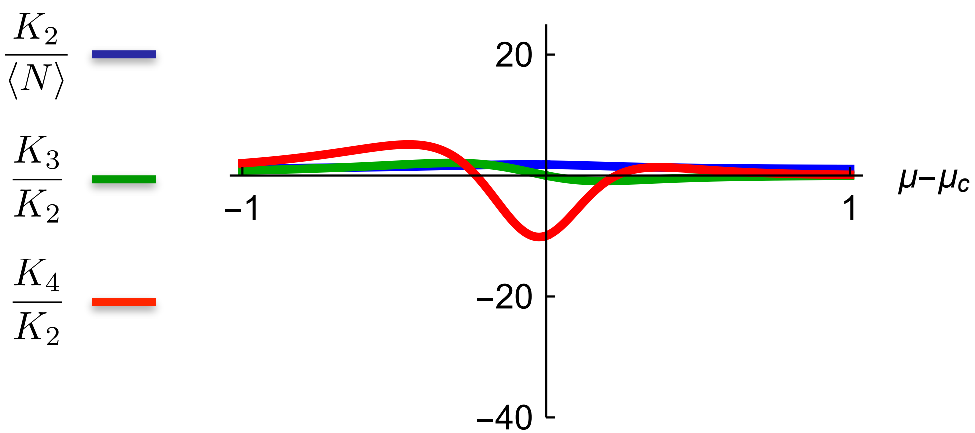
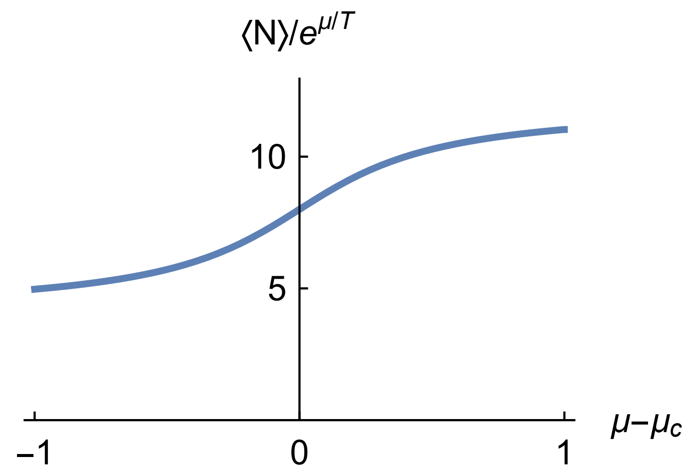
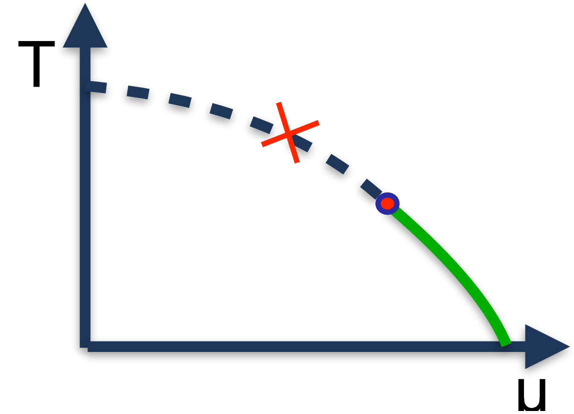


$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
20
−20
−40
−1
1
$\mu-\mu_c$

$\langle N \rangle / e^{\mu/T}$
10
5
−1
0
1
$\mu-\mu_c$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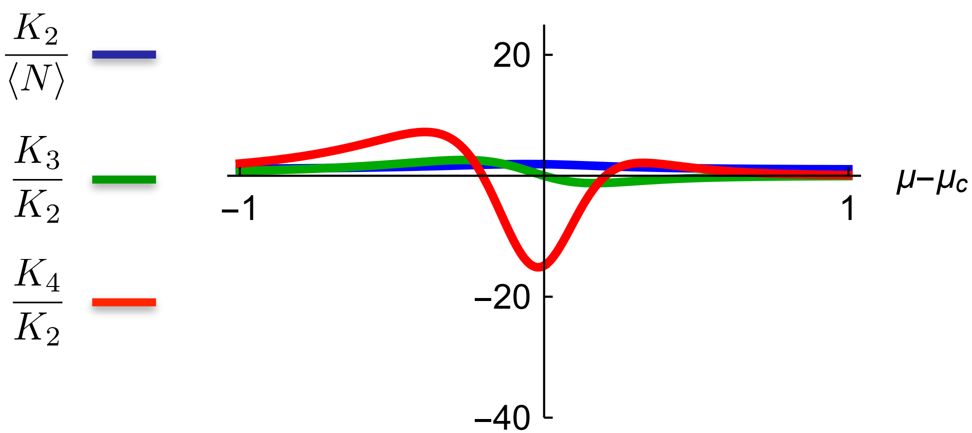
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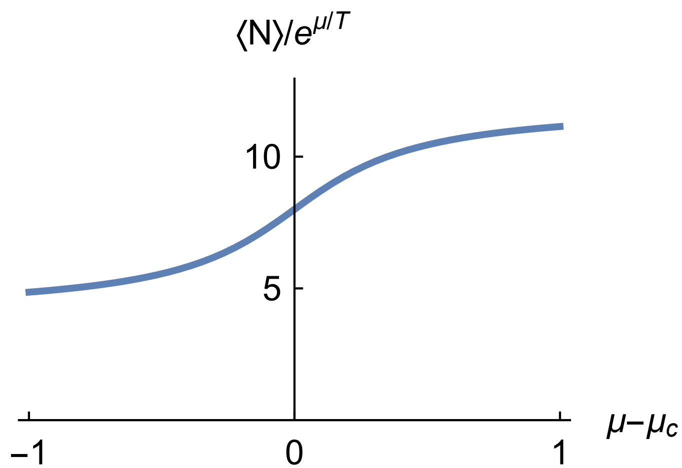
⟨N⟩/$e^{\mu/T}$
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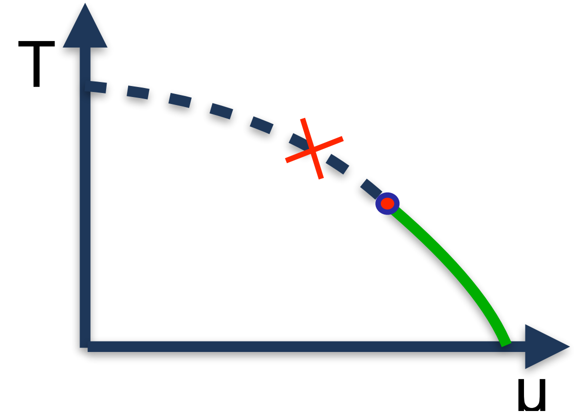
5
−1
0
1
μ−μc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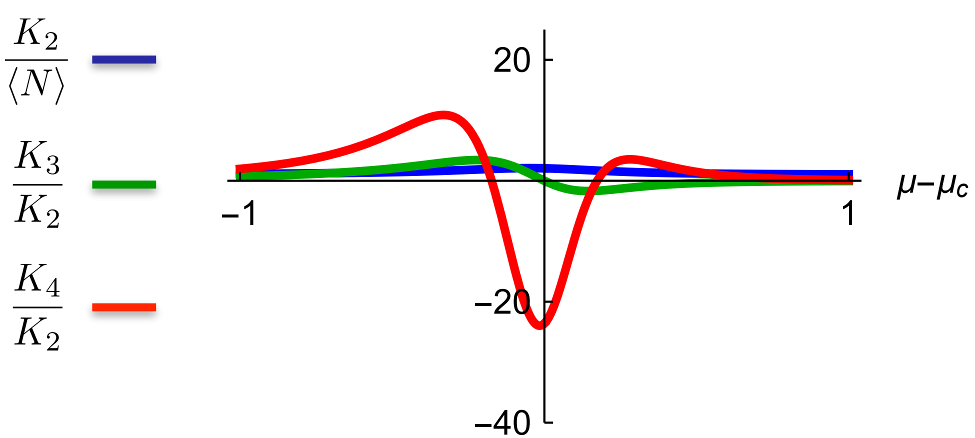
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$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
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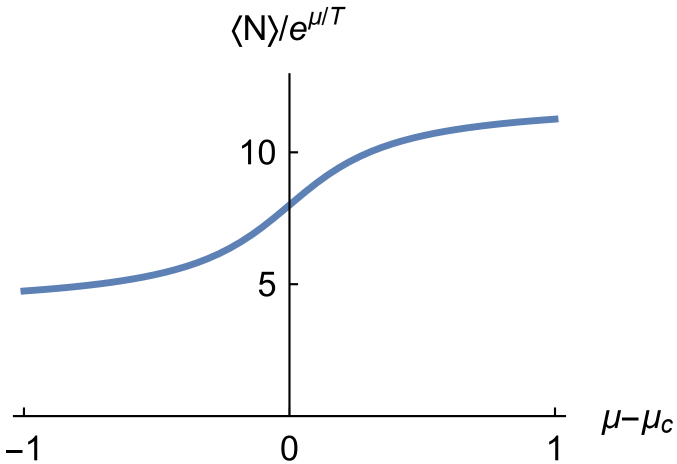
⟨N⟩/e^{μ/T}
10
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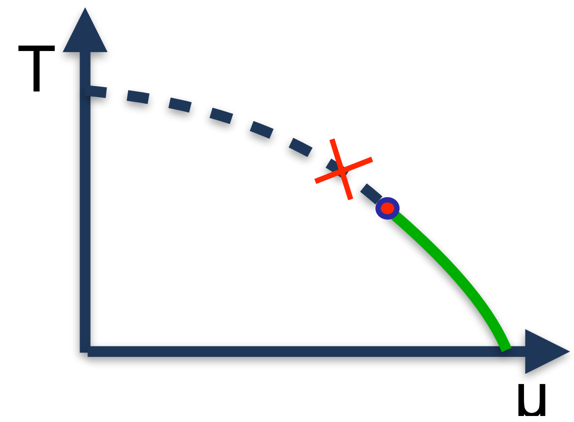

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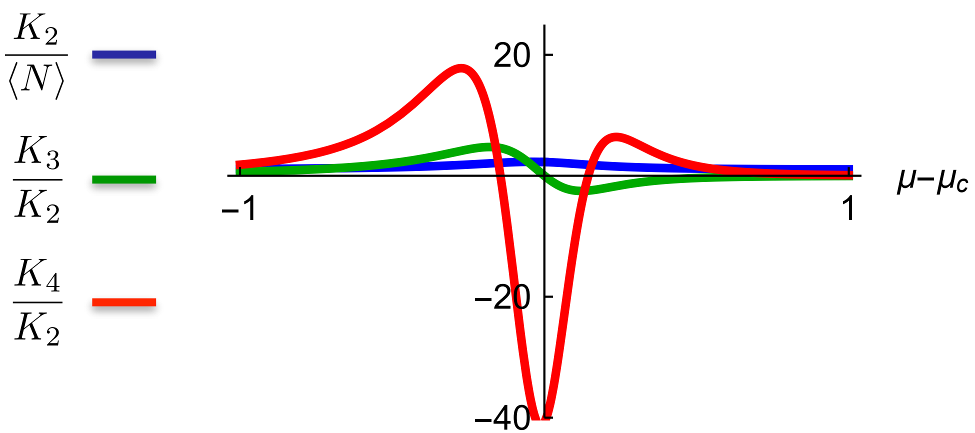

$\frac{K_2}{\langle N \rangle}$
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−20
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1
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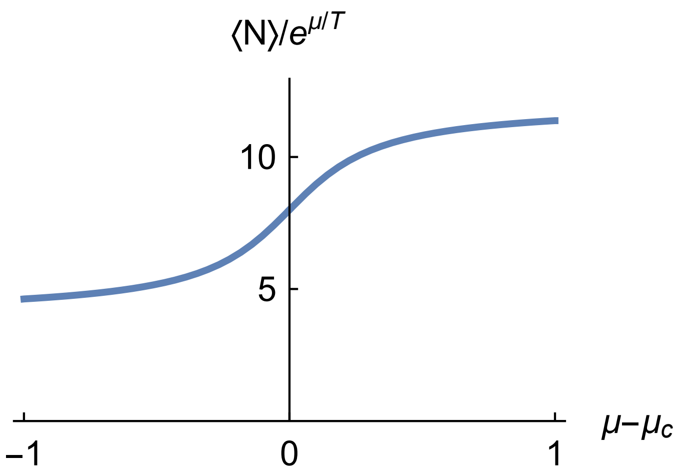
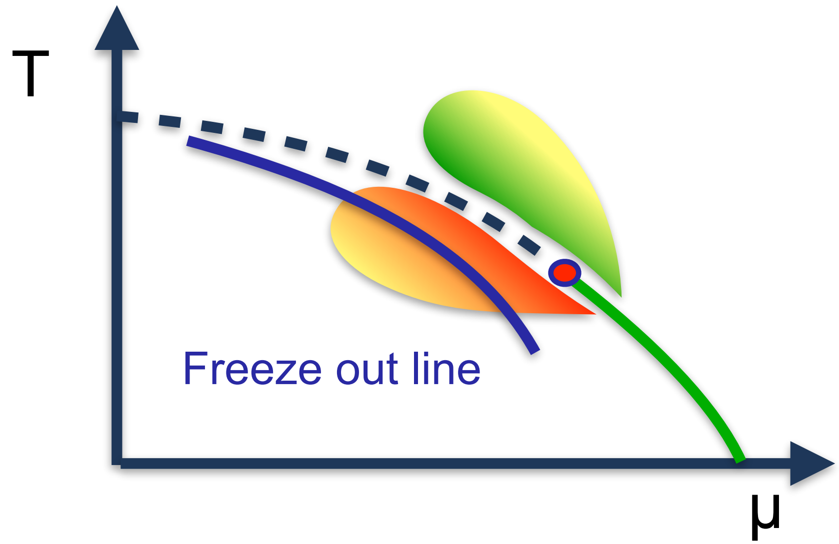
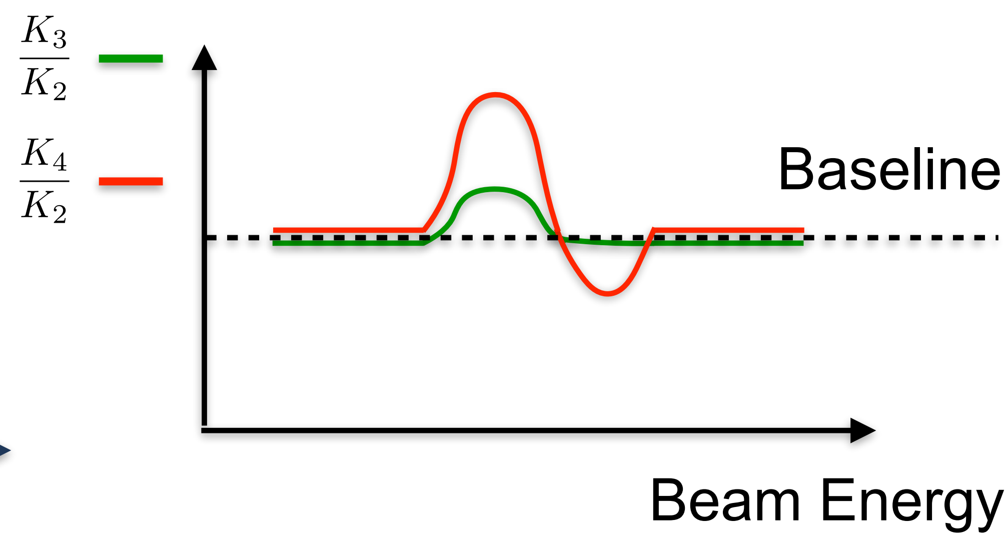
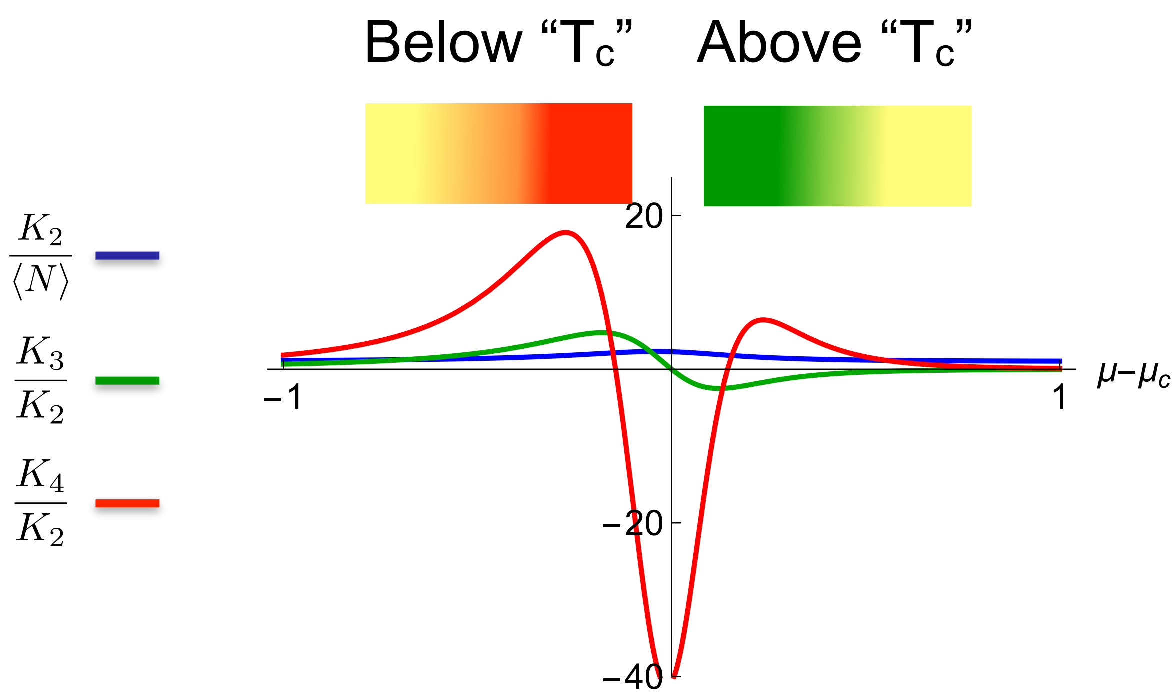


⟨N⟩/e^{μ/T}
10
5
−1
0
1
μ−μc

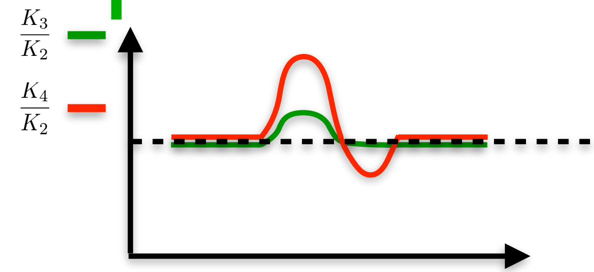
# What to expect from experiment?



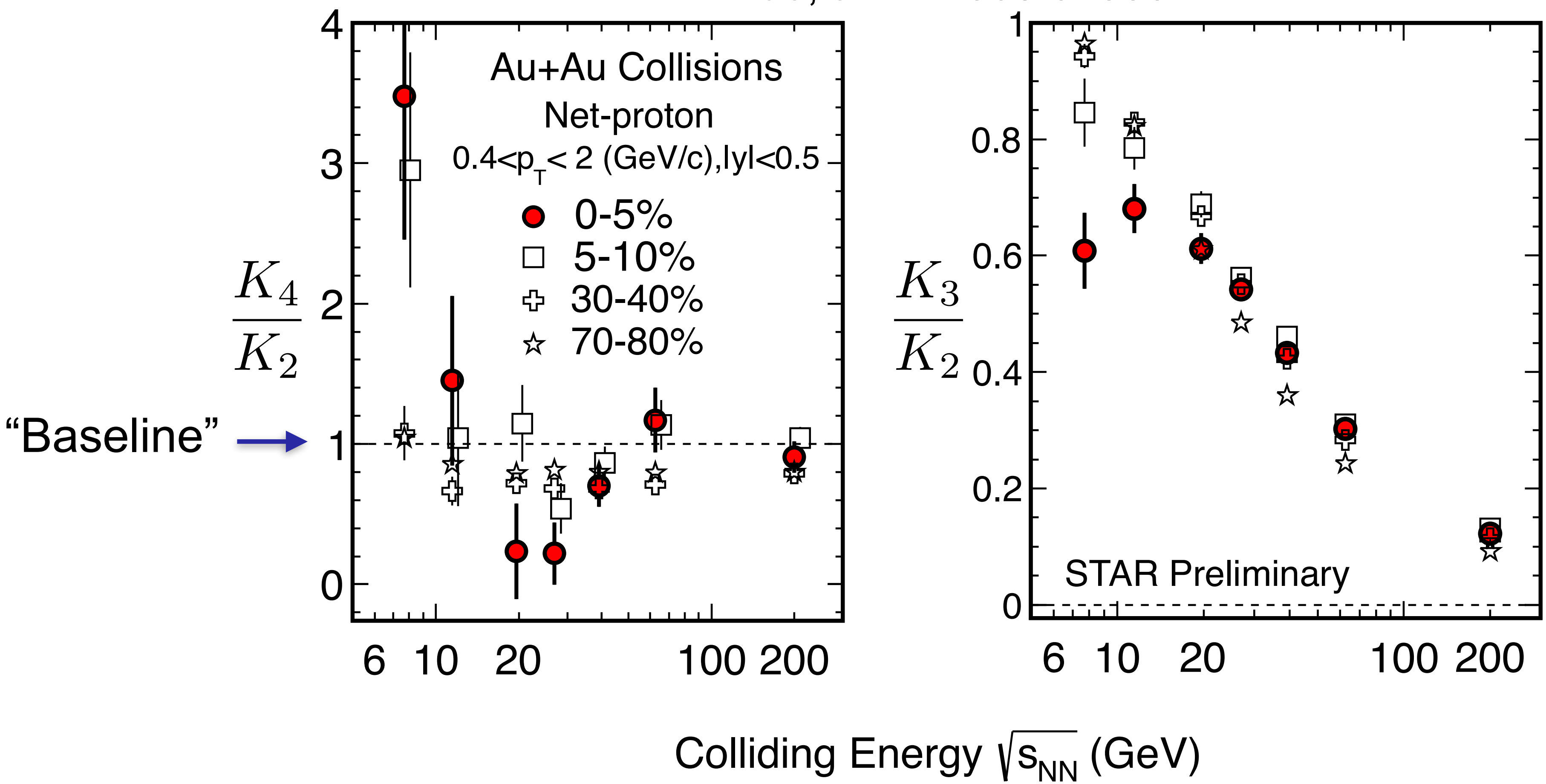



# Latest STAR result on net-proton cumulants




X. Luo, arXiv:1503.02558




$K_4/K_2$ follows expectation, $K_3/K_2$ no so much…..

Let’s take the preliminary STAR data at face value

# Further insights: Correlations

Cumulants $\quad K_n = \frac{\partial^n}{\partial \hat{\mu}^n} P/T^4$

$$K_2 = \langle N - \langle N \rangle \rangle^2 = \langle (\delta N)^2 \rangle$$

$$\rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + C_2(p_1, p_2),$$

$C_2$: Correlation Function

$$K_3 = \langle (\delta N)^3 \rangle$$

$$\rho_3(p_1, p_2, p_3) = \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) + \rho_1(p_1)\underline{C_2(p_2, p_3)} + \rho_1(p_2)\underline{C_2(p_1, p_3)} + \rho_1(p_3)\underline{C_2(p_1, p_2)} + \underline{C_3(p_1, p_2, p_3)}$$

# From Cumulants to Correlations (no anti-protons)

Defining integrated correlations function

$$C_n = \int dp_1 \dots dp_n C_n(p_1, \dots, p_n)$$

Simple Algebra leads to relation between correlations $C_n$ and $K_n$

$$C_2 = -K_1 + K_2,$$
$$C_3 = 2K_1 - 3K_2 + K_3,$$
$$C_4 = -6K_1 + 11K_2 - 6K_3 + K_4, .$$

or vice versa

$$K_2 = \langle N \rangle + C_2$$
$$K_3 = \langle N \rangle + 3C_2 + C_3$$
$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

# Correlations near the critical point

M. Stephanov, 0809.3450, PRL 102

Scaling of Cumulants $K_n$ with correlation length $\xi$

$$K_2 \sim \xi^2, \;\; K_3 \sim \xi^{4.5}, \;\; K_4 \sim \xi^7$$

Cumulants from Correlations

$$K_2 = \langle N \rangle + C_2$$
$$K_3 = \langle N \rangle + 3C_2 + C_3$$
$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

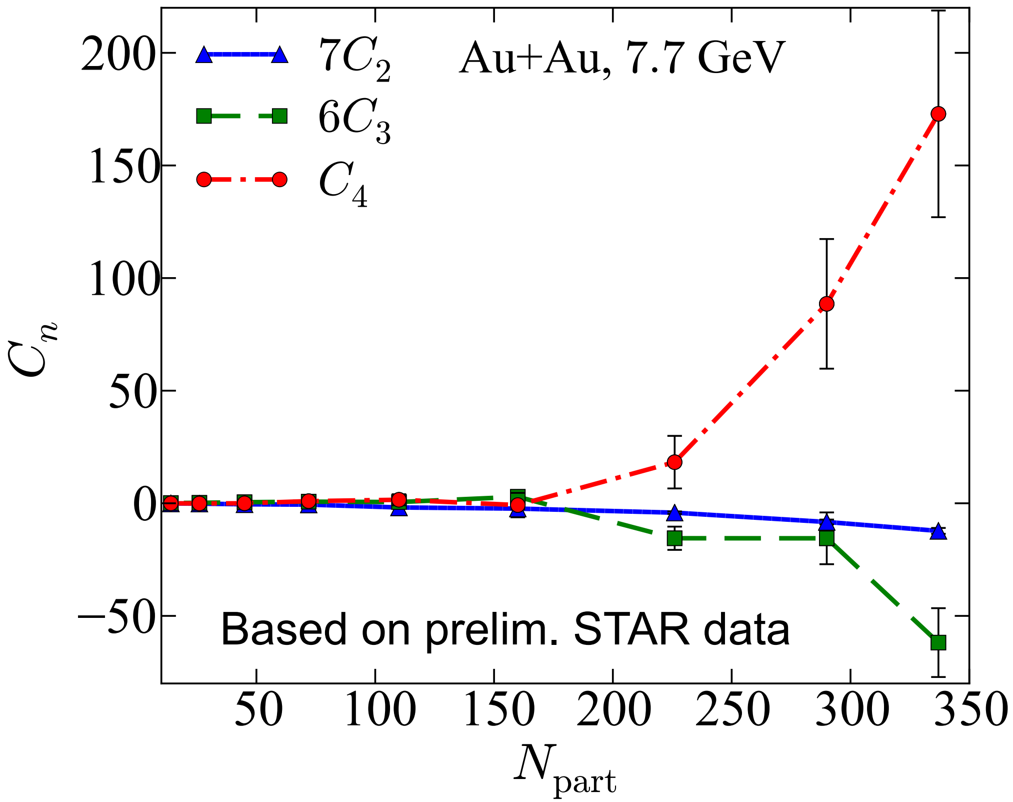
Consequently:

$$C_2 \sim \xi^2, \;\; C_3 \sim \xi^{4.5}, \;\; C_4 \sim \xi^7$$

Correlations $C_n$ pick up the most divergent pieces of cumulants $K_n$!

# Preliminary Star Data

(X. Luo, PoS Cpod 2014 (019))




Significant four particle correlations!

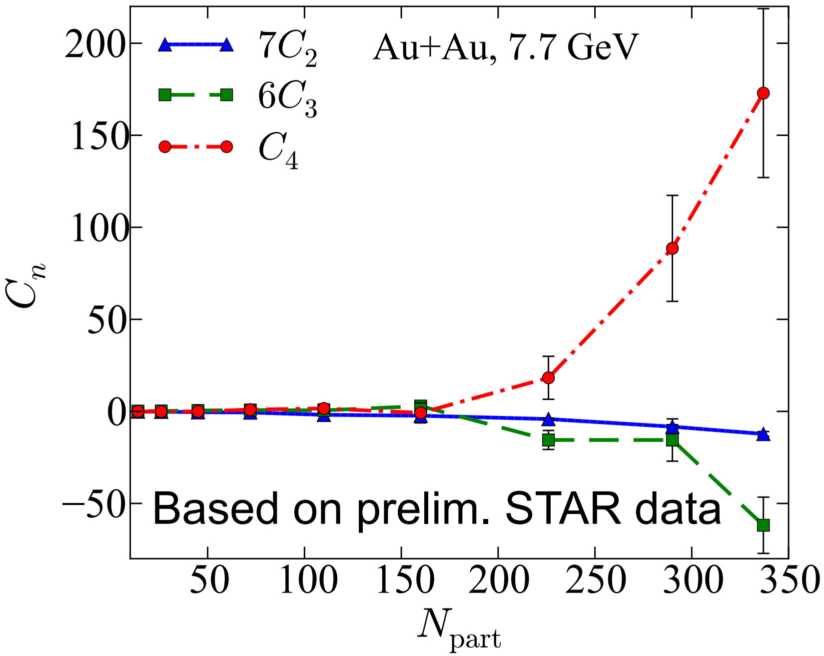
Four particle correlation dominate $K_4$ for central collisions at 7.7 GeV

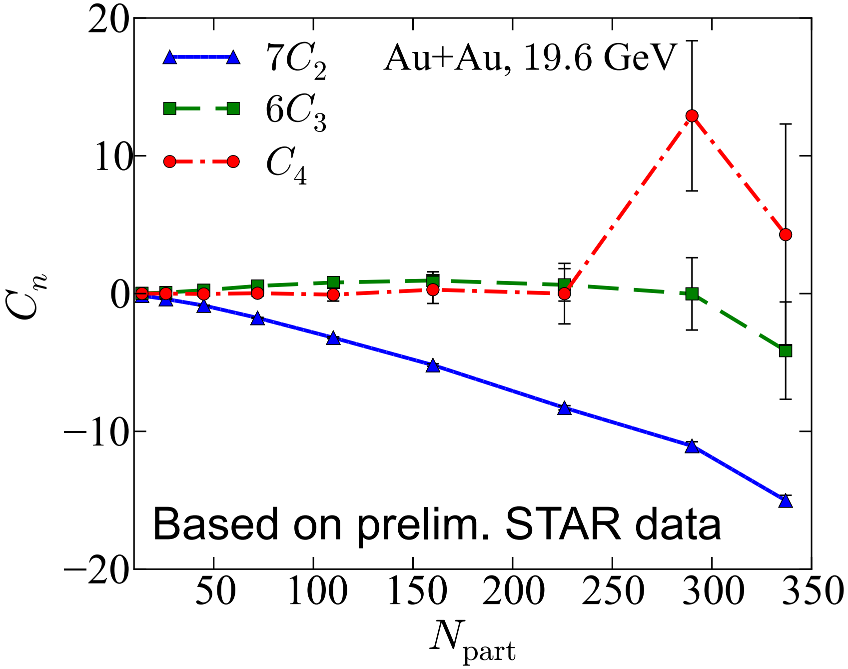
$$K_2 = \langle N \rangle + C_2$$

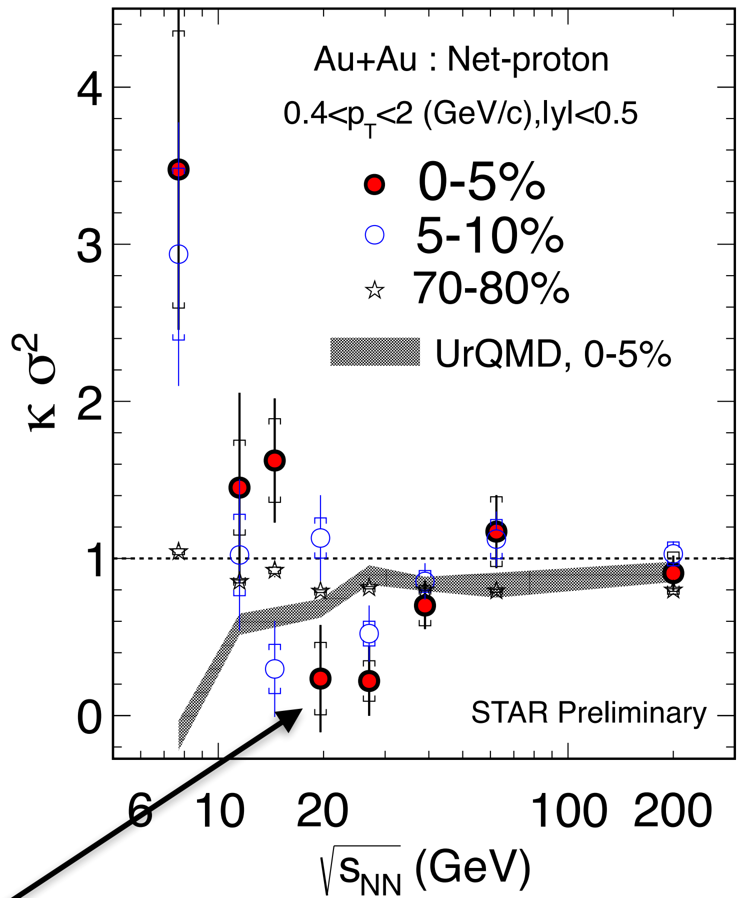
$$K_3 = \langle N \rangle + 3C_2 + C_3$$

$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

# Correlations




Dip at 19.6 GeV from NEGATIVE $C_2$ !

# Reduced correlation function

Reduced correlation function

$$c_k = \frac{\int \rho_1(y_1)\cdots\rho_1(y_k)\, c_k(y_1,...,y_k)\, dy_1\cdots dy_k}{\int \rho_1(y_1)\cdots\rho_1(y_k)\, dy_1\cdots dy_k} \qquad C_k = \langle N\rangle^k c_k$$

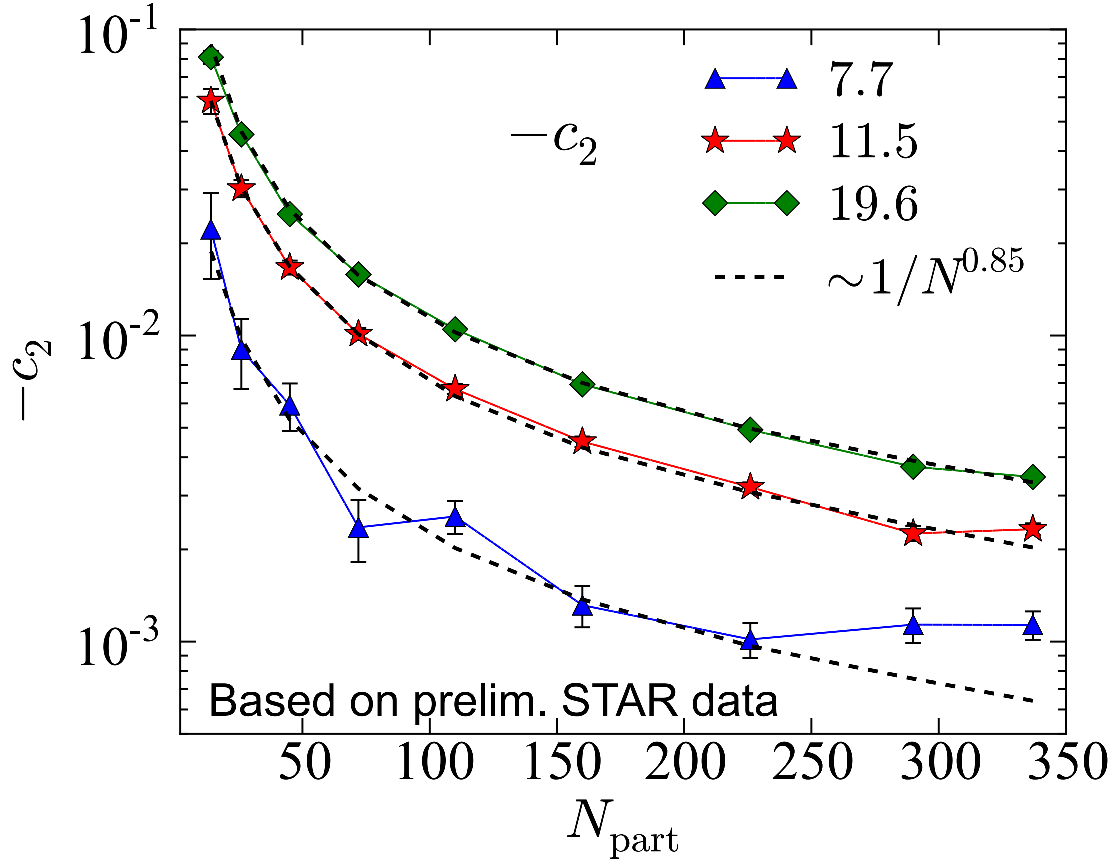
Independent sources such as resonances, cluster, p+p:

$$c_k \sim \frac{\langle N_s\rangle}{\langle N\rangle^k} \sim \frac{1}{\langle N\rangle^{k-1}}$$

For example two particle correlations:

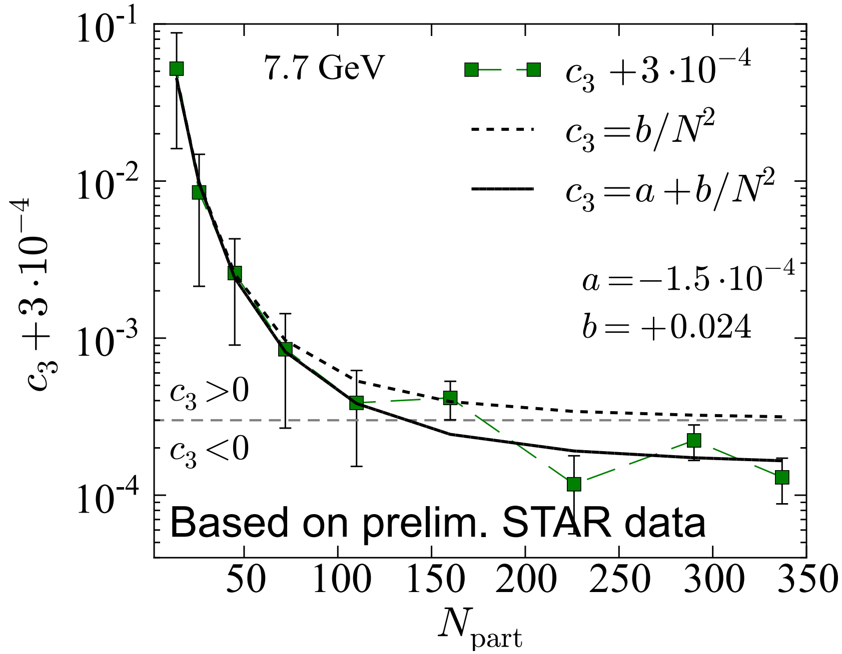
$$c_2 \sim \frac{\text{Number of sources}}{\text{Number of all pairs}} = \frac{\text{Number of correlated pairs}}{\text{Number of all pairs}} = \frac{1}{\langle N\rangle}$$

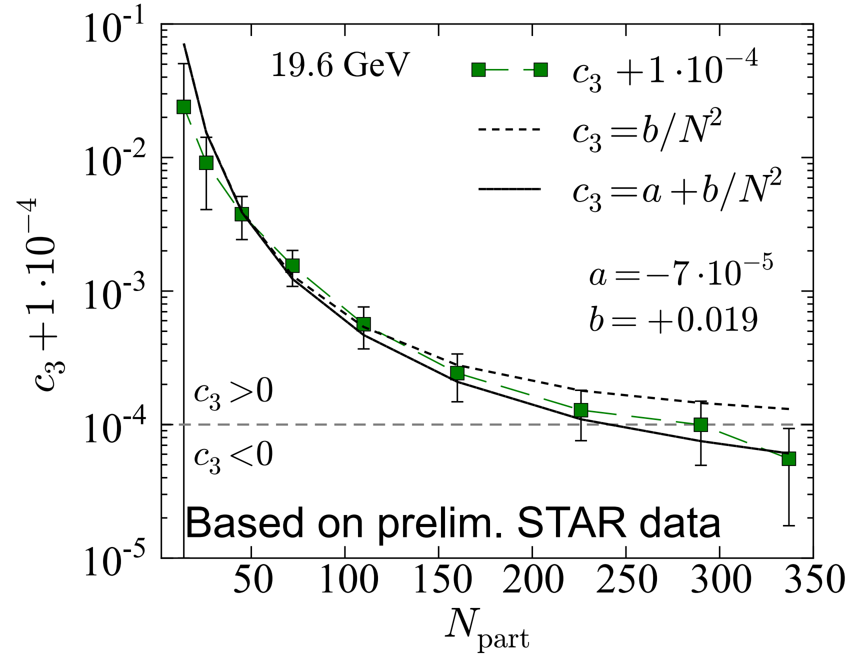
# Centrality dependence



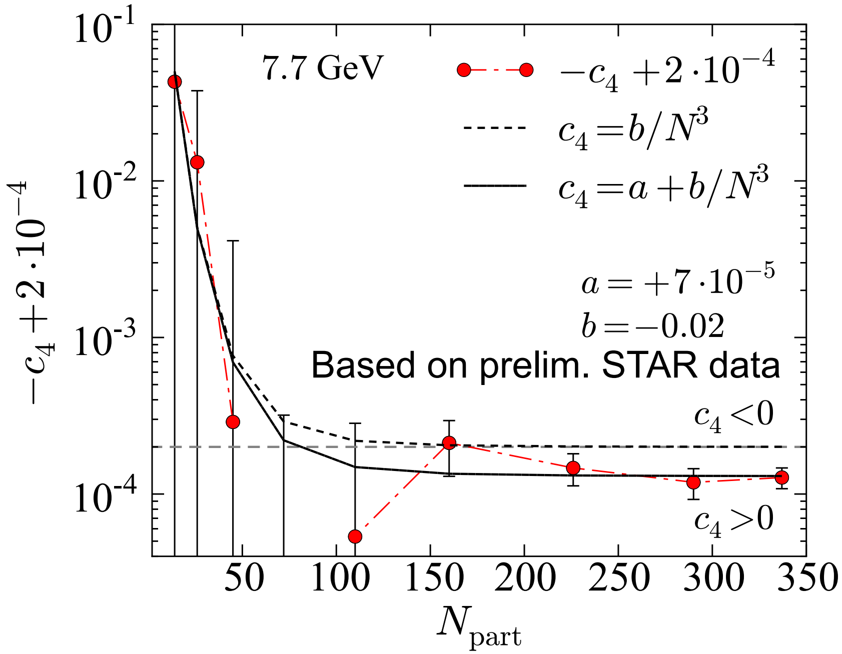
# Centrality dependence

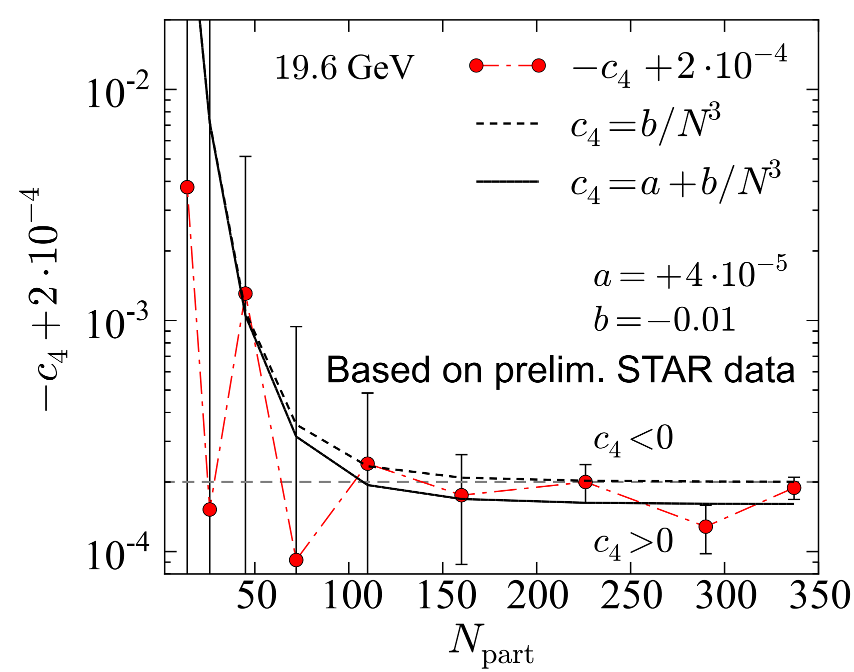
$C_3$




$C_4$




7.7 GeV

19.6 GeV

# Rapidity dependence

$$C_k(\Delta Y) = \int_{\Delta Y} dy_1 \ldots dy_k \rho_1(y_1) \ldots \rho_1(y_k) c_k(y_1, \ldots, y_k)$$

Assume: $\rho_1(y) \simeq const.$

short range correlations:

$$c_k(y_1, \ldots, y_k) \sim \delta(y_1 - y_2) \ldots \delta(y_{n-1} - y_k)$$

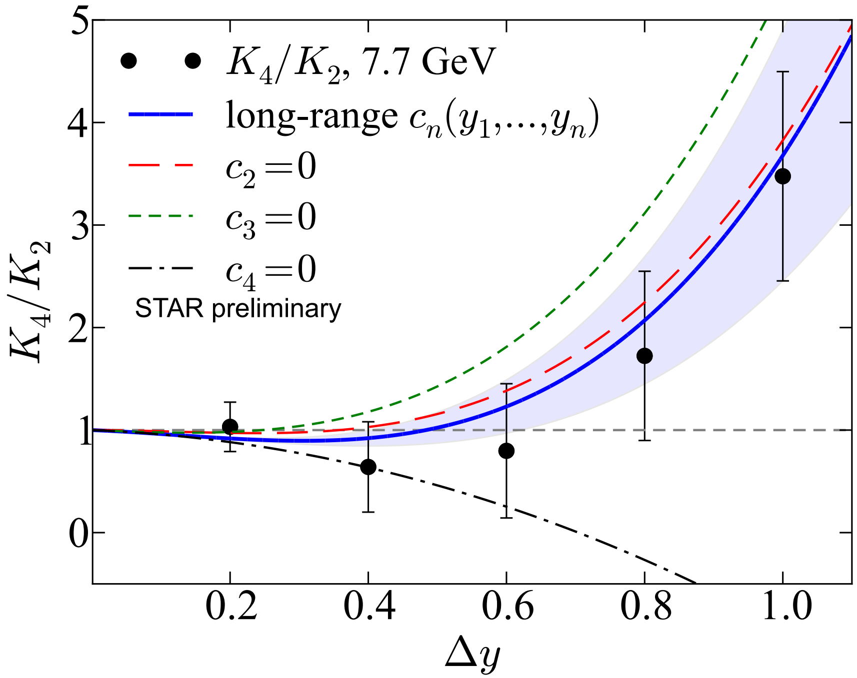
$$C_k(\Delta Y) \sim \Delta Y \to K_k \sim \Delta Y$$

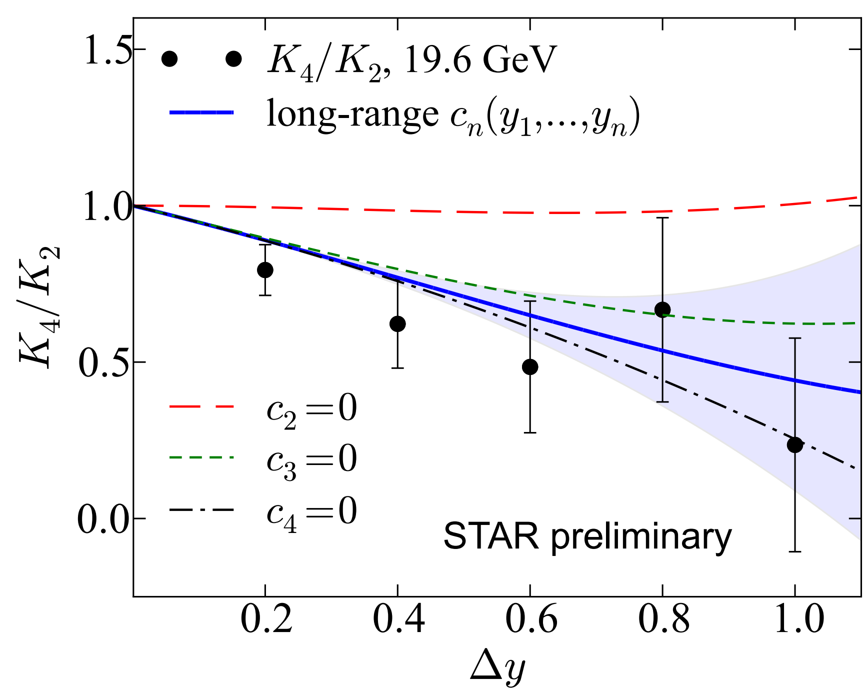
Long range correlations:

$$c_k(y_1, \ldots, y_k) = const.$$

$$C_k(\Delta Y) \sim (\Delta Y)^k$$

# Preliminary Star data are consistent with long range correlations



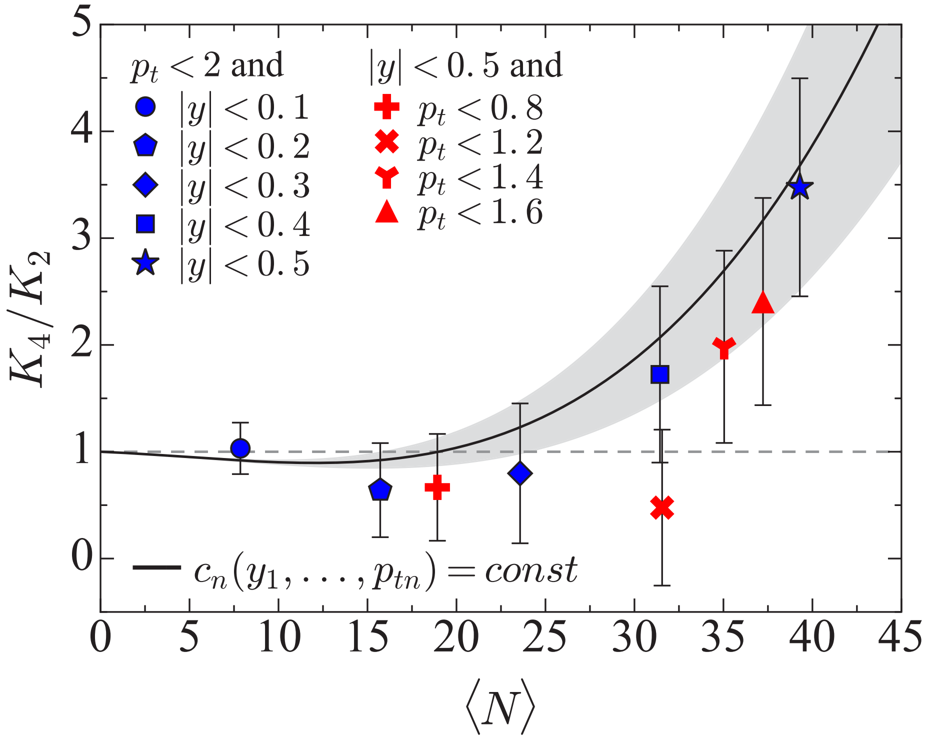

7.7 GeV
central

19.6 GeV
central

# Long range correlations

$$C_k = \langle N \rangle^k c_k$$

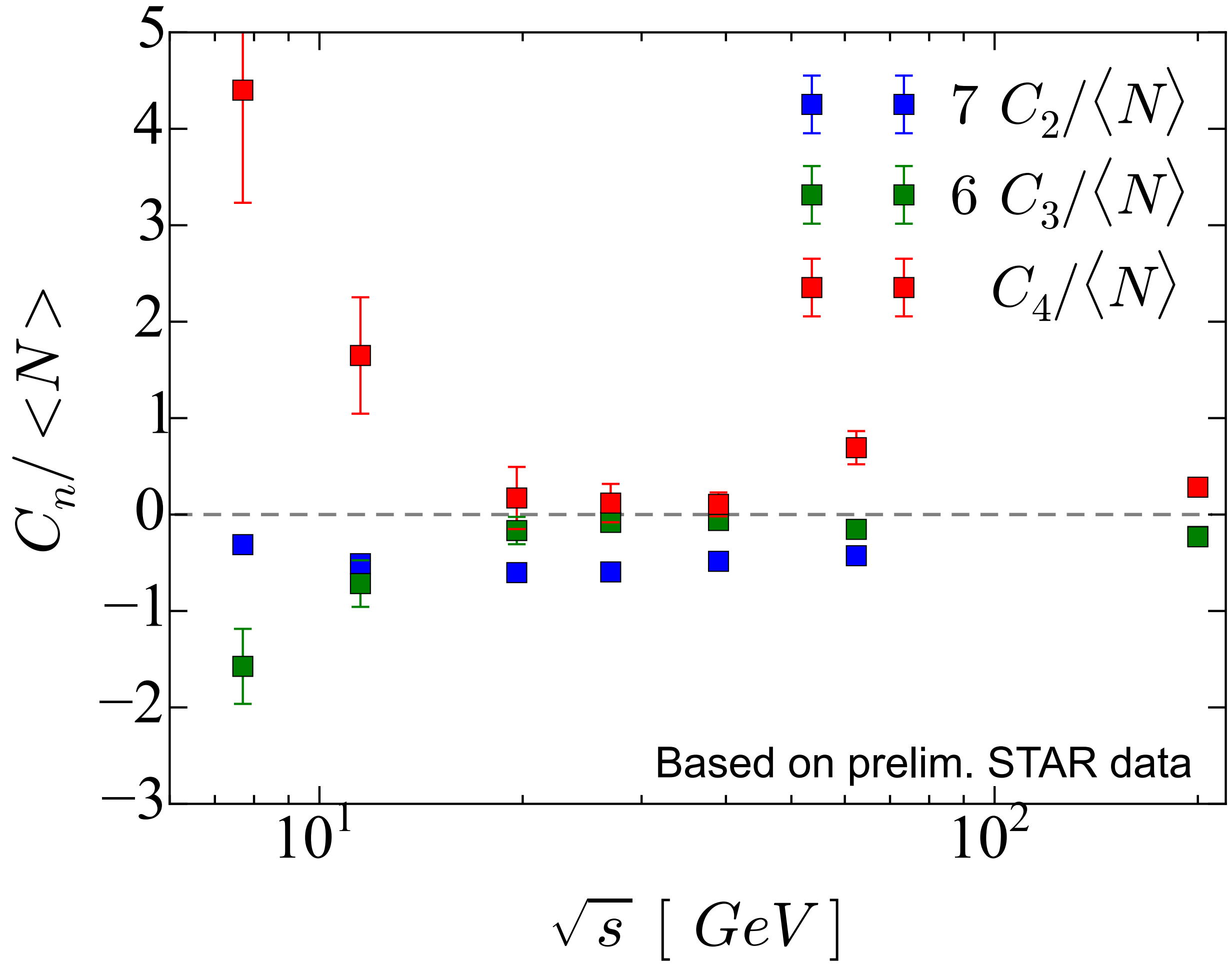
$$c_k = const. \;\Rightarrow K_n = K_n(\langle N \rangle)$$




NB: Data are consistent with small “repulsive” component

$$c_2(y_1, y_2) = c_2^0 + \gamma_2 (y_1 - y_2)^2 \quad \gamma_2 > 0$$

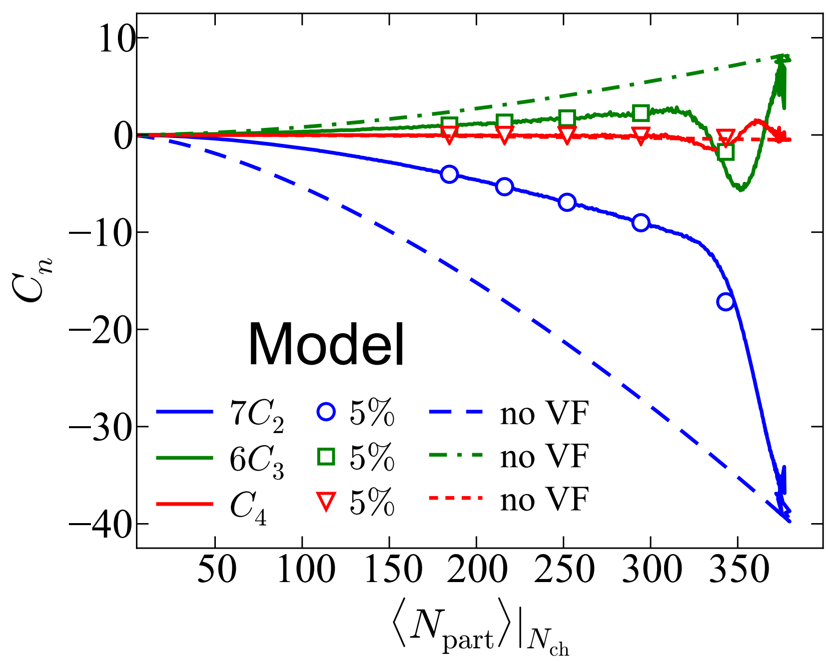
# Energy dependence

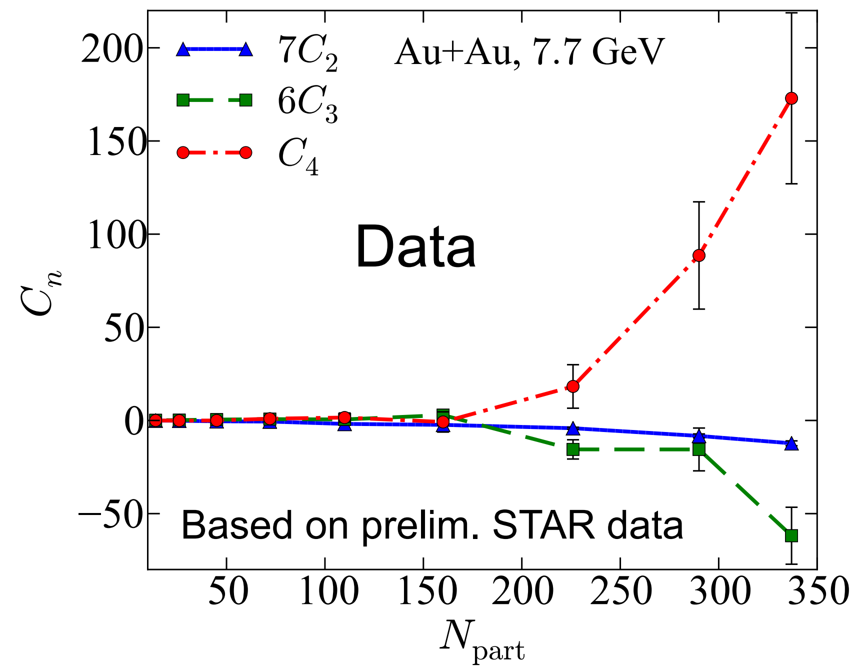



Note: anti-protons are non- negligible above 19.6 GeV
Data are protons only

# Can we understand these correlations?

- Two particle correlations can be understood by simple Glauber model + Baryon number conservation

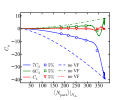


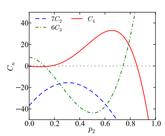

Four particle correlations are orders of magnitudes larger in the data

# Can we understand these correlations?

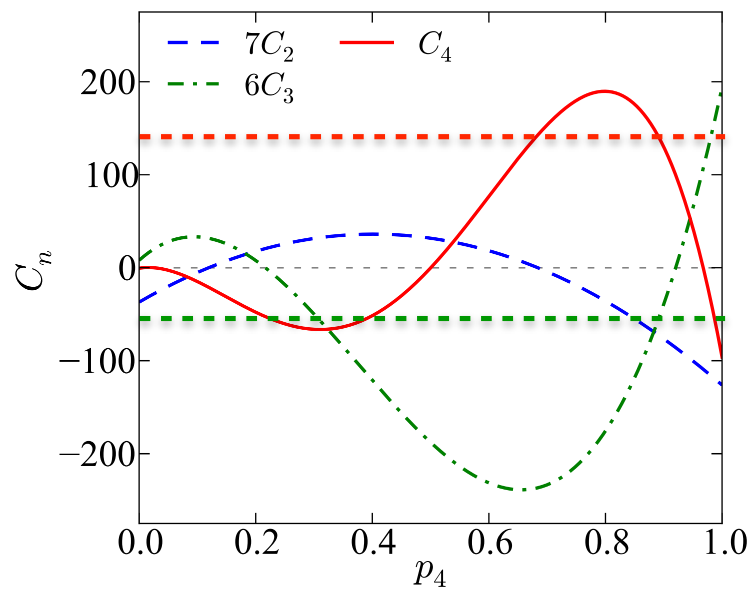
- Three and four particle correlations require lots of “fantasy”…
- For example, if about 40% of the nucleons are come in 8-nucleon clusters one can get near the data…



Singles



Pairs




STAR $C_4$

STAR $C_3$

Proton-quartets = 8 nucleon clusters

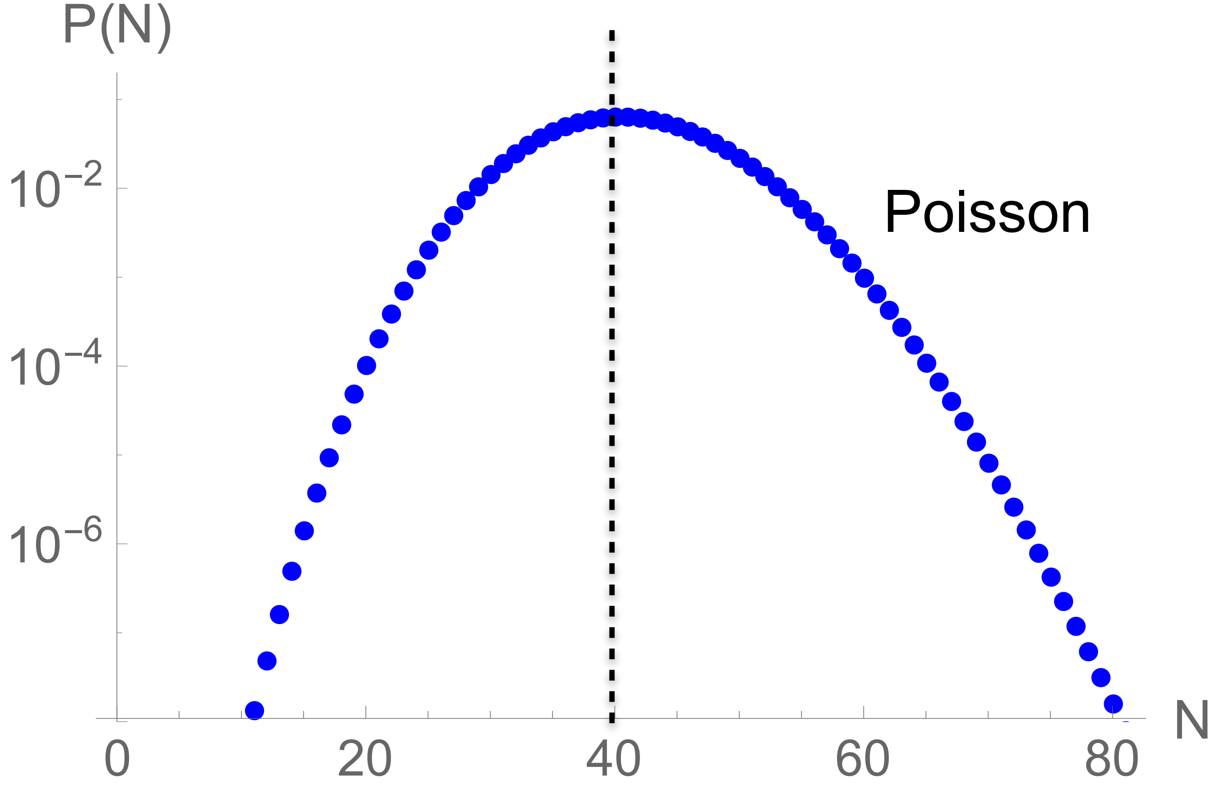
# Shape of probability distribution

$K_3 < \langle N \rangle$

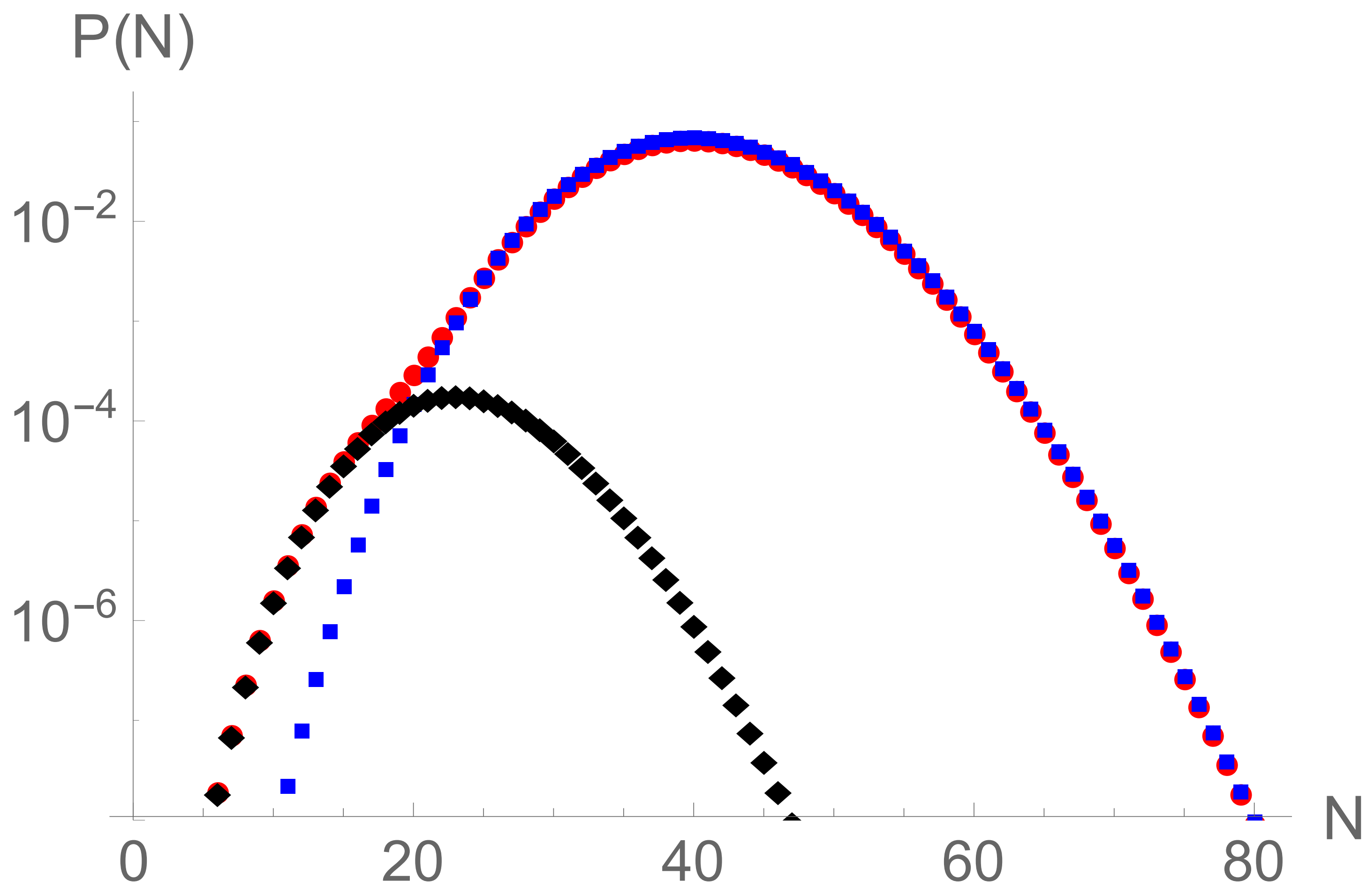
$K_4 > \langle N \rangle$

$$K_3 = \langle N - \langle N \rangle \rangle^3$$

$$K_4 = \langle N - \langle N \rangle \rangle^4 - 3 \langle N - \langle N \rangle \rangle^2$$



# Simple two component model



# Summary

- Fluctuations sensitive to phase structure:
  - measure “derivatives” of EOS
- Measurements are difficult
- Cumulants contain information about correlations
- Preliminary STAR data:
  - Significant four particle correlations at 7.7 and 11.5 GeV
  - Dip in $K_4/K_2$ at 19.6 GeV is due to negative two-particle correlations
  - Centrality dependence (at 7.7 GeV) indicates independent sources for $N_{part} < 150$ and “collective” correlations for $N_{part} > 200$.
  - At about the same centrality three- and four particle correlations change sign!
    - New dynamics?

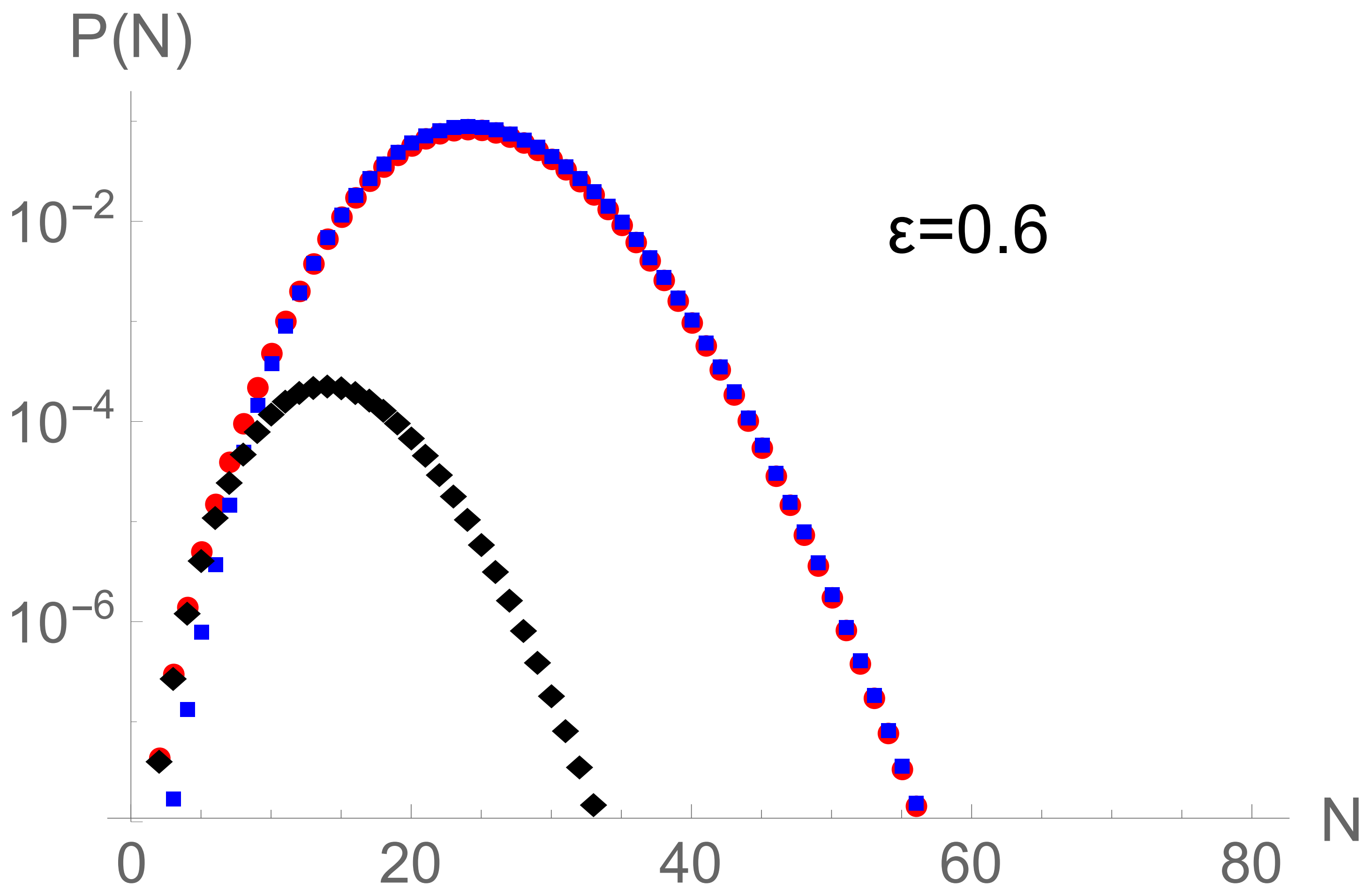
# Summary

- Preliminary STAR data continued:
  - For central 7.7 and 11.5 GeV two and three particle correlations are negative and four particle are positive.
- Other more mundane effects may contribute
  - Fluctuations of system size ($N_{part}$)
    - May explain 2-particle correlations
    - Fail to reproduce the magnitude of 3- and 4- particle correlations
- Understanding 3- and 4 particle correlations requires “desperate measures”!

# Thank You

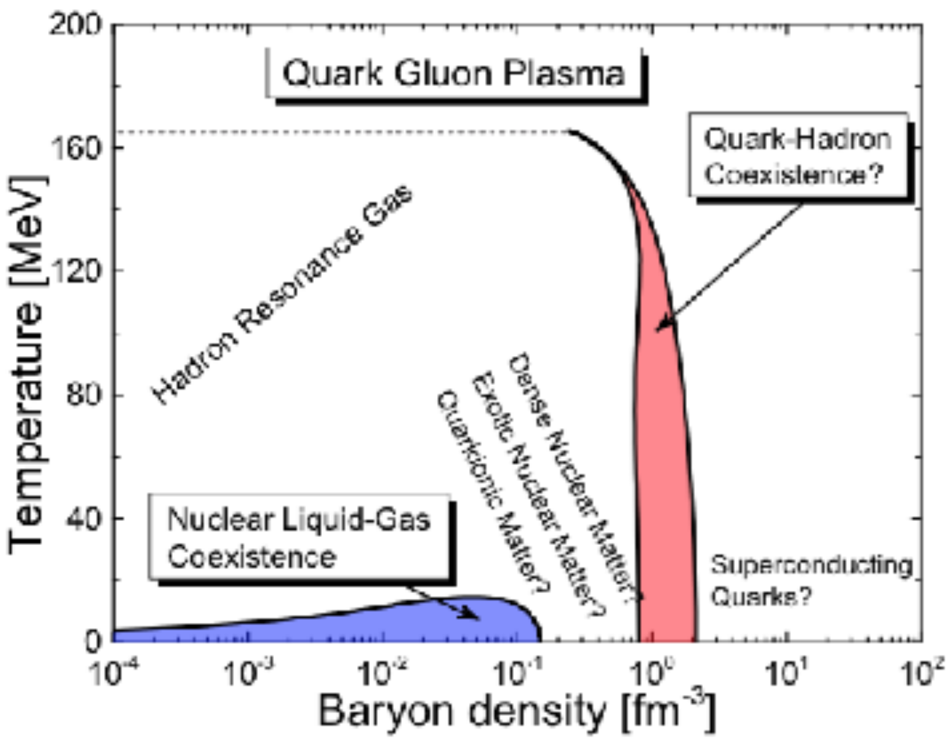
# Simple two component model

Difficult to see in the real data with efficiency ε=0.6

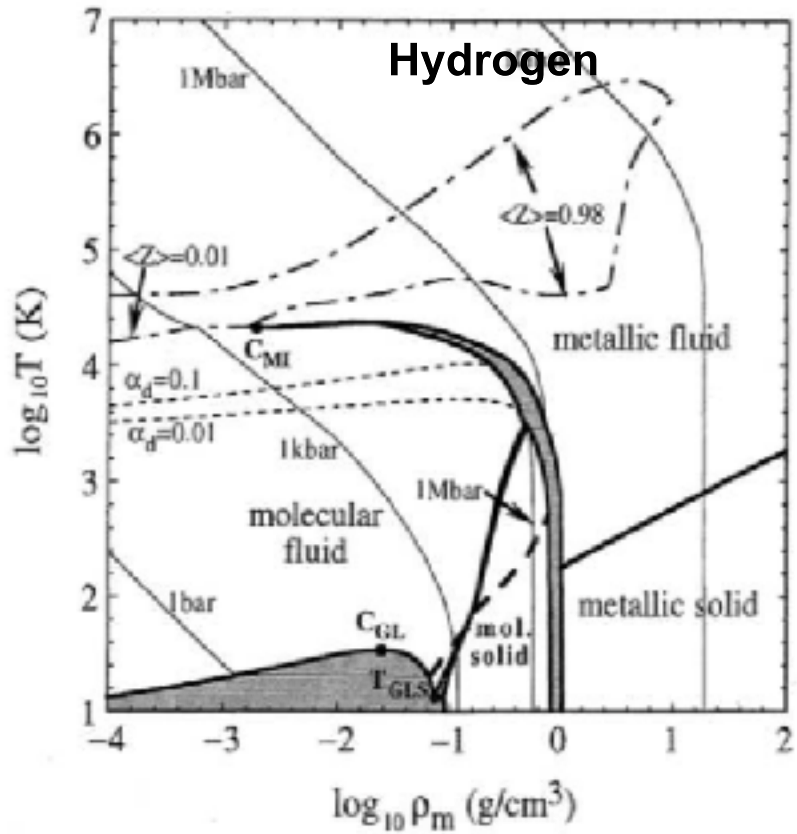


# Phase Diagrams

Maybe it's better to look at the Phase diagram in density.




M. Hempel, V. Dexheimer, S. Schramm and I. Iosilevskiy, Phys. Rev. C 88, no. 1, 014906 (2013)



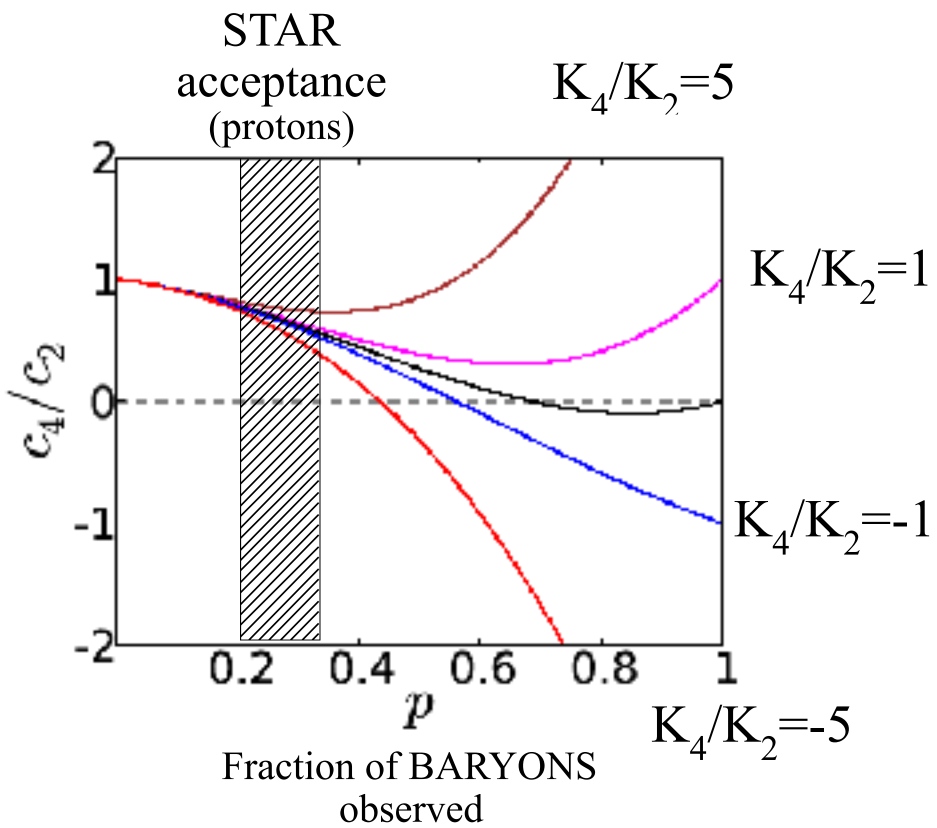

Kitamura H., Ichimaru S., J. Phys. Soc. Japan 67, 950 (1998).

Curious similarity

# Things to consider

- Fluctuations of conserved charges ?!
- Volume Fluctuations
- Net-protons different from net-baryons
  - Isospin fluctuations
- “Stopping” fluctuations
- Higher cumulants probe the tails. Statistics!
- The detector “fluctuates” !
  - Efficiency effects
- …….

# Finite efficiency




Unfolding needed if we want to know what the true cumulants are
Tricky with a real detector

# Compare Data with Lattice QCD

Example: “Charge” susceptibility

$$\chi_Q = \int d^3x < \rho(x)\rho(0) > = \int d^3p < \tilde{\rho}(p)\tilde{\rho}(0) >$$

Equivalence of *integrated* coordinate space and momentum space correlation function

Experiment almost never integrates ALL of momentum space!

Lattice (hopefully) does integrate over all coordinate space

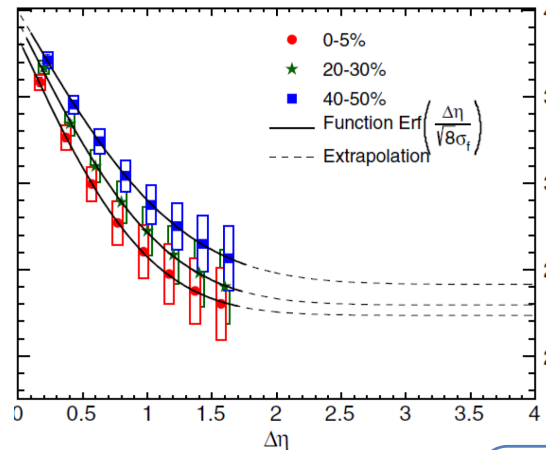
# Correlations: Lattice vs Data

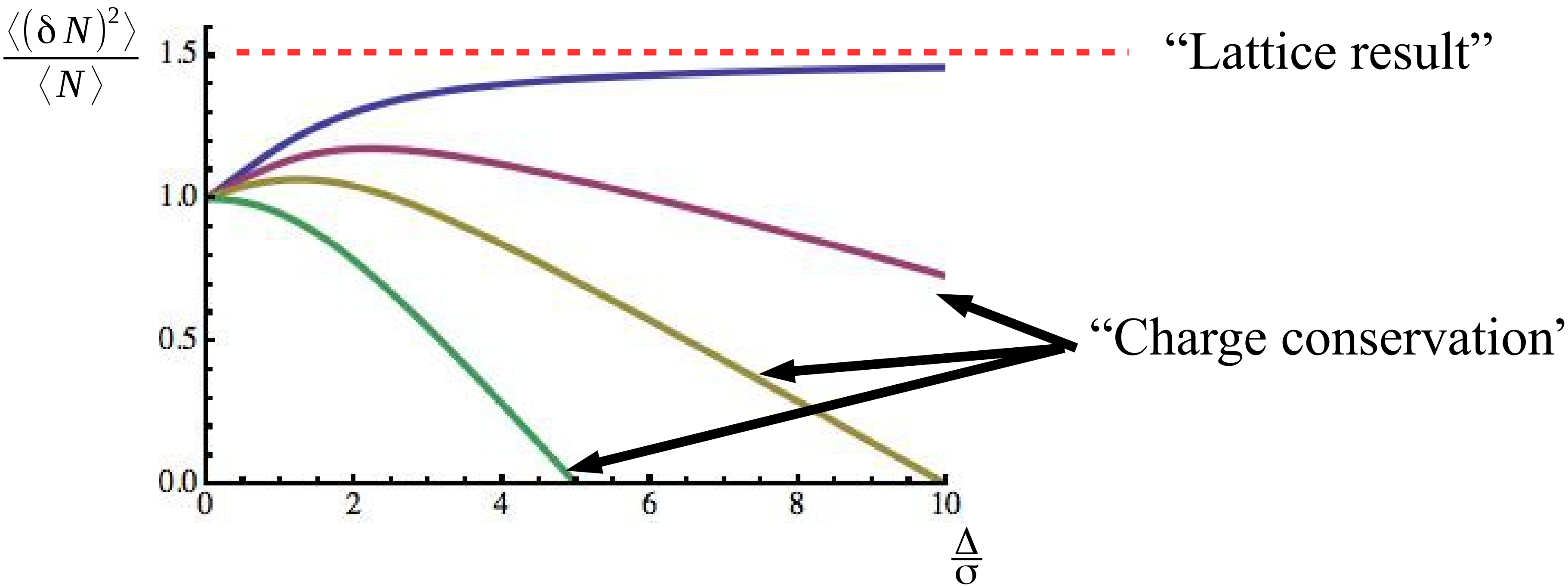
$$\langle n(y_1)(n(y_2)-\delta(y_1-y_2))\rangle=\langle n(y_1)\rangle\langle n(y_2)\rangle\left(1+C(y_1,y_2)\right)$$

$$C(y_1,y_2)\sim\exp\left(\frac{-(y_1-y_2)^2}{2\sigma^2}\right)$$

$$\frac{\langle(\delta N)^2\rangle}{\langle N\rangle}=1+\langle N\rangle\int_{\Delta/2}^{\Delta/2}C(y1,y2)\,dy1\;dy_2$$

Alice Charge Flucts

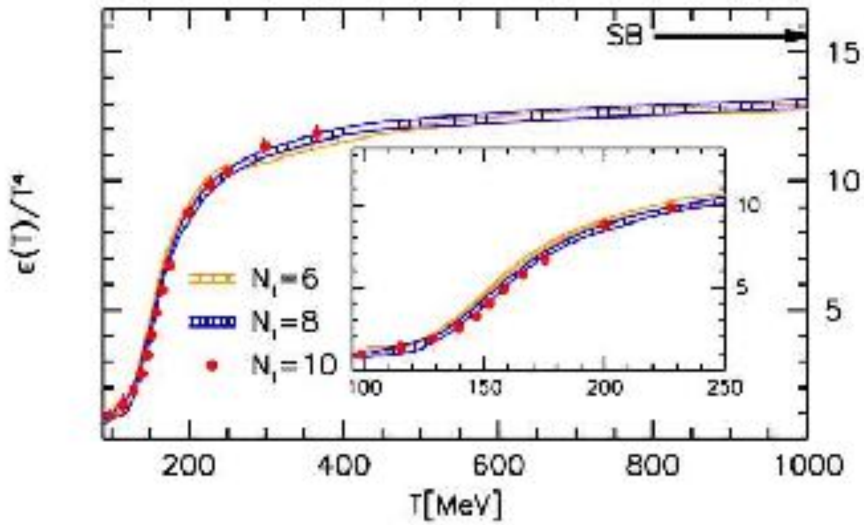


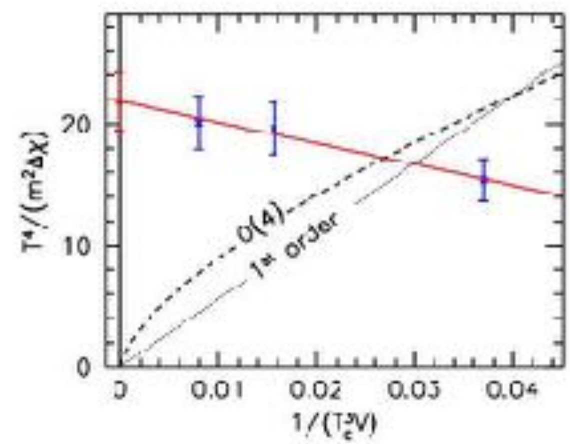
# Lattice QCD

## Equation of state

S. Borsanyi et al, JHEP 1011 (2010) 077

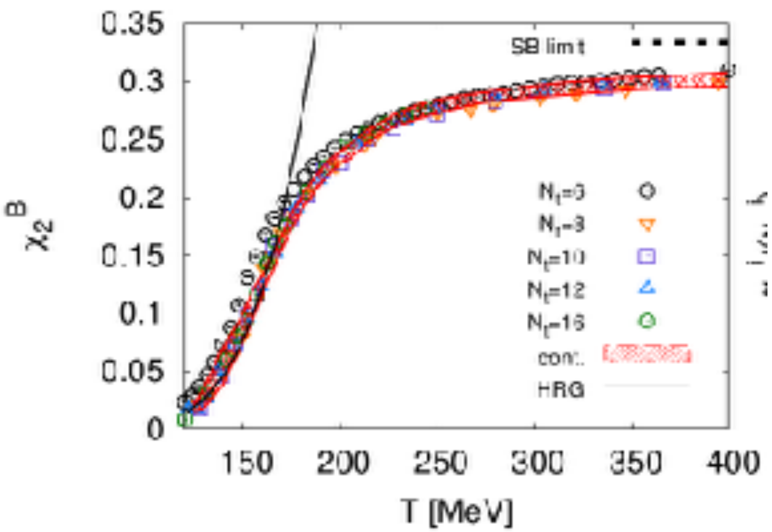



## Cross over transition




Aoki et al, Nature 443:675-678,2006

## Susceptibilities

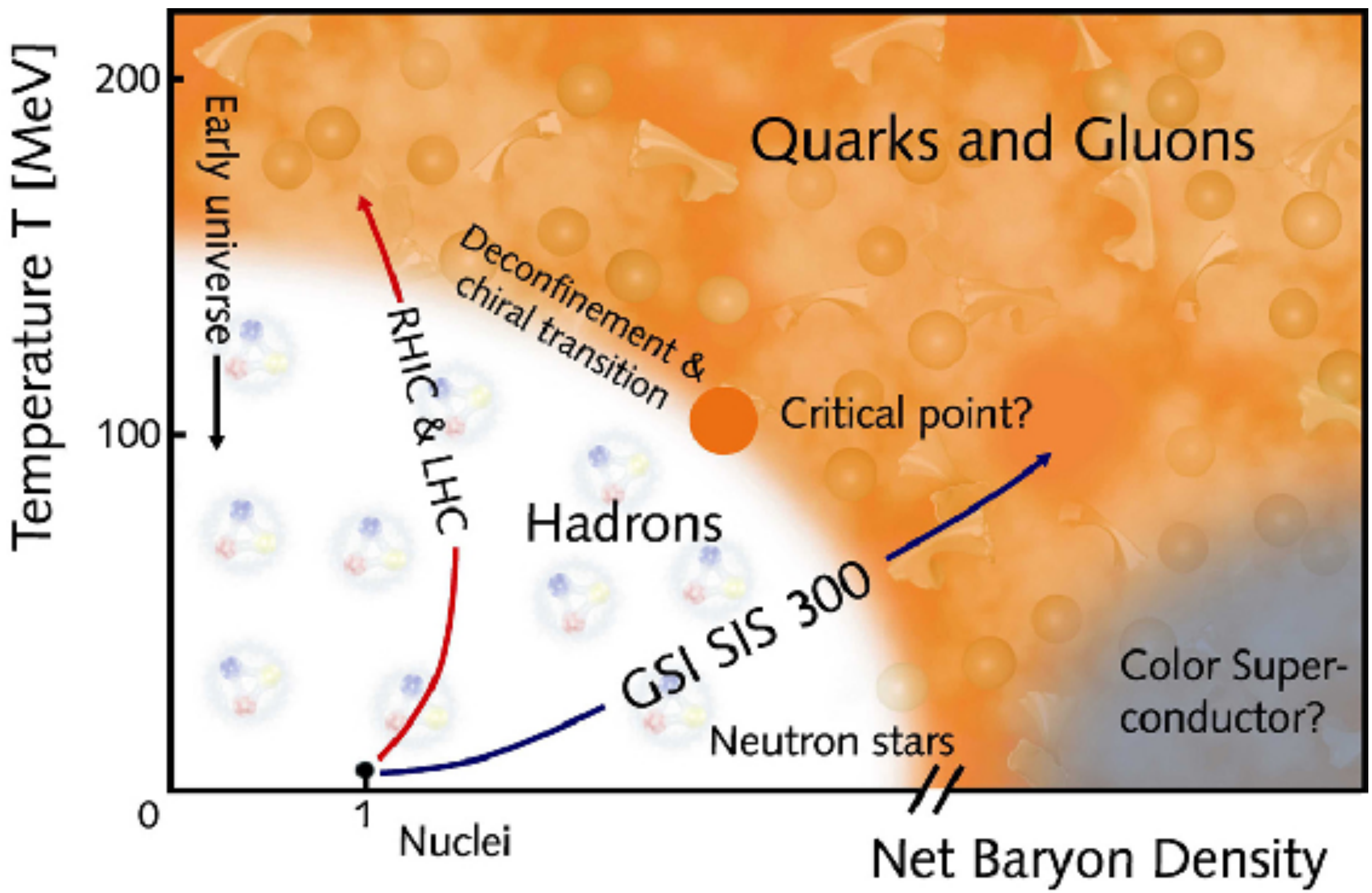



$$\frac{P\,(T,\,\mu_B)}{T^4} = \frac{P\,(T,\,\mu_B=0)}{T^4} + \sum_n c_n\,(\mu/T)^n \qquad c_n = \frac{\chi_n^B}{n!}$$

Susceptibilities provide $\mu_B$-dependence of EOS

AND measure fluctuations

present status: 6th order, 8th in the works



Temperature T [MeV]
200
100
0
1
Nuclei
Net Baryon Density
Early universe
Quarks and Gluons
Deconfinement &
chiral transition
Critical point?
RHIC & LHC
Hadrons
GSI SIS 300
Neutron stars
Color Super-
conductor?