Proton radius and Rydberg constant from electronic and muonic atoms

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before: Max-Planck Institute of Quantum Optics

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Outline

- Muonic atoms

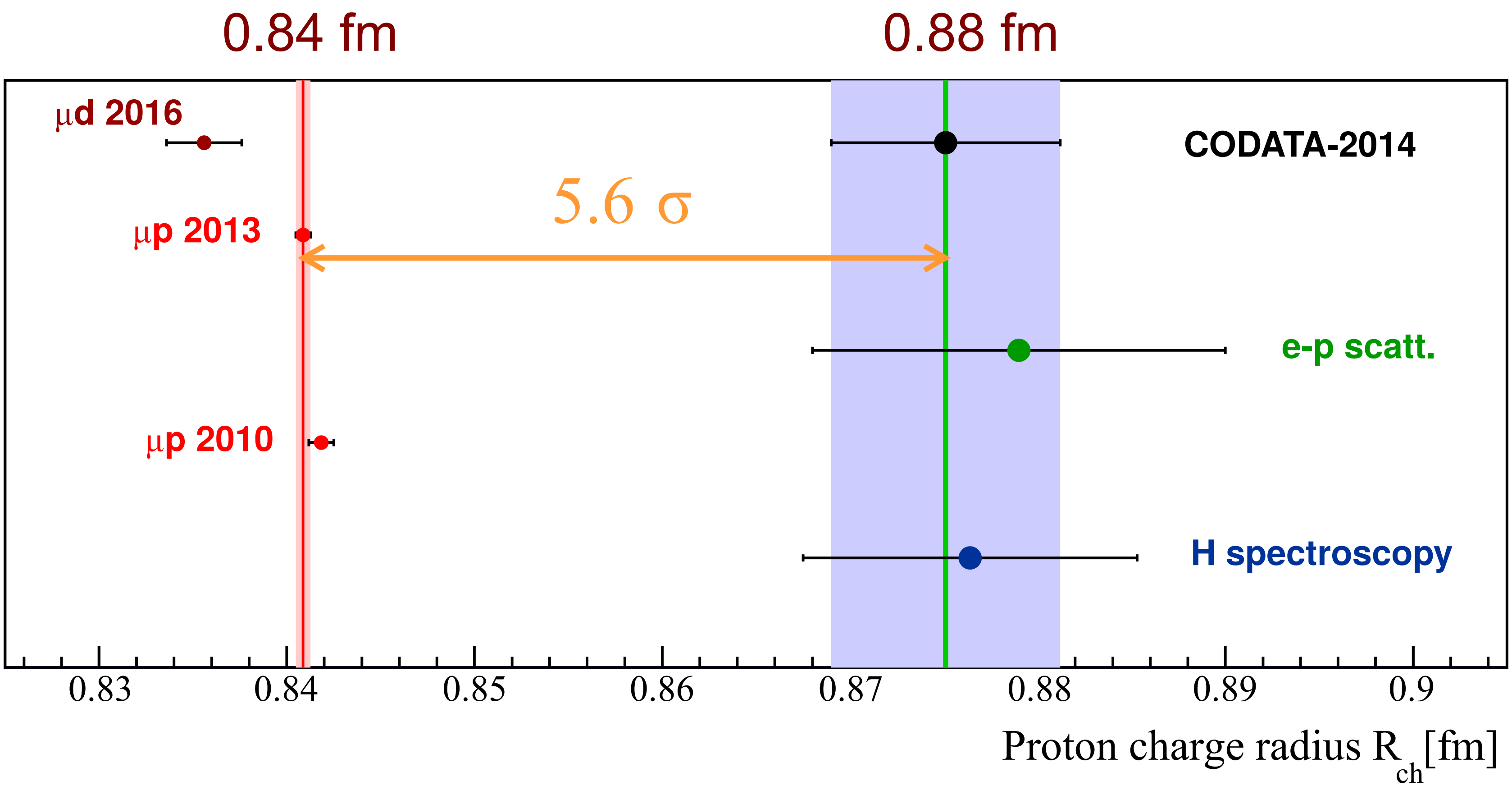
  as a probe of nuclear physics (**charge radii**, magnetization radii, polarizabilities, …)
- The “Proton Radius Puzzle”
- Rydberg constant

  key parameter to check **atomic physics** part of the discrepancy
- Muonic helium, later Li, Be, T?

The “Proton Radius Puzzle”

Measuring R_p using electrons: 0.88 fm (+- 0.7%)

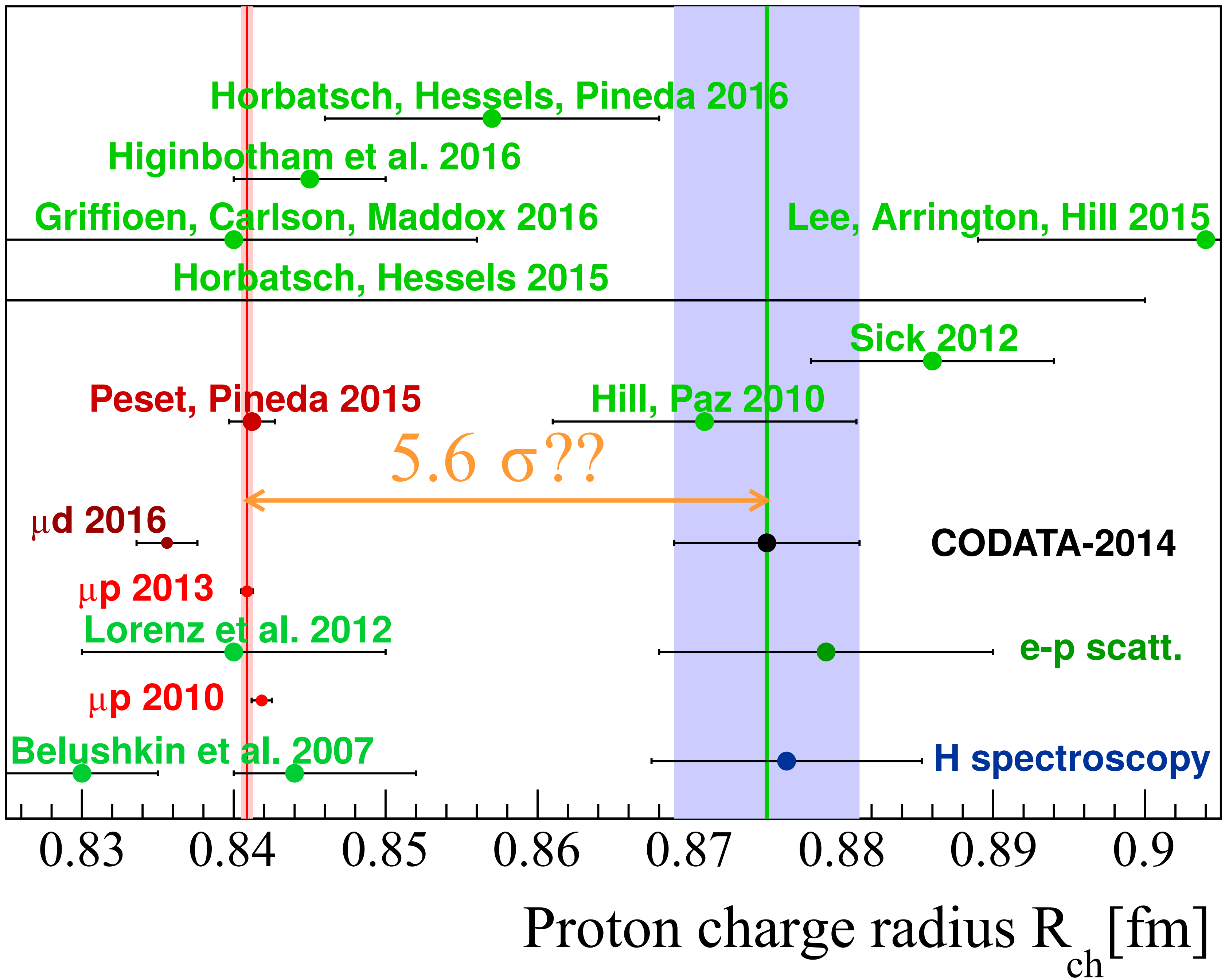
using muons: 0.84 fm (+- 0.05%)



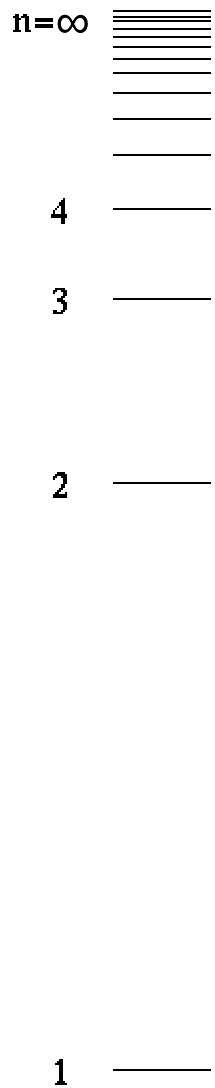
μd 2016: RP et al (CREMA Coll.) Science 353, 669 (2016)

μp 2013: A. Antognini, RP et al (CREMA Coll.) Science 339, 417 (2013)

A “Proton Radius **Puzzle**” ??



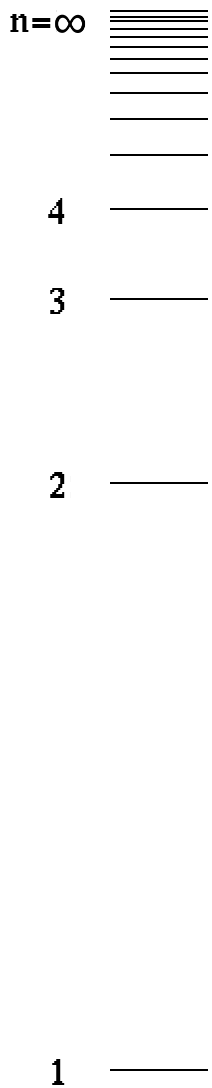
Energy levels of hydrogen



$$E_n \approx -\frac{R_\infty}{n^2}$$

Bohr formula

Energy levels of hydrogen



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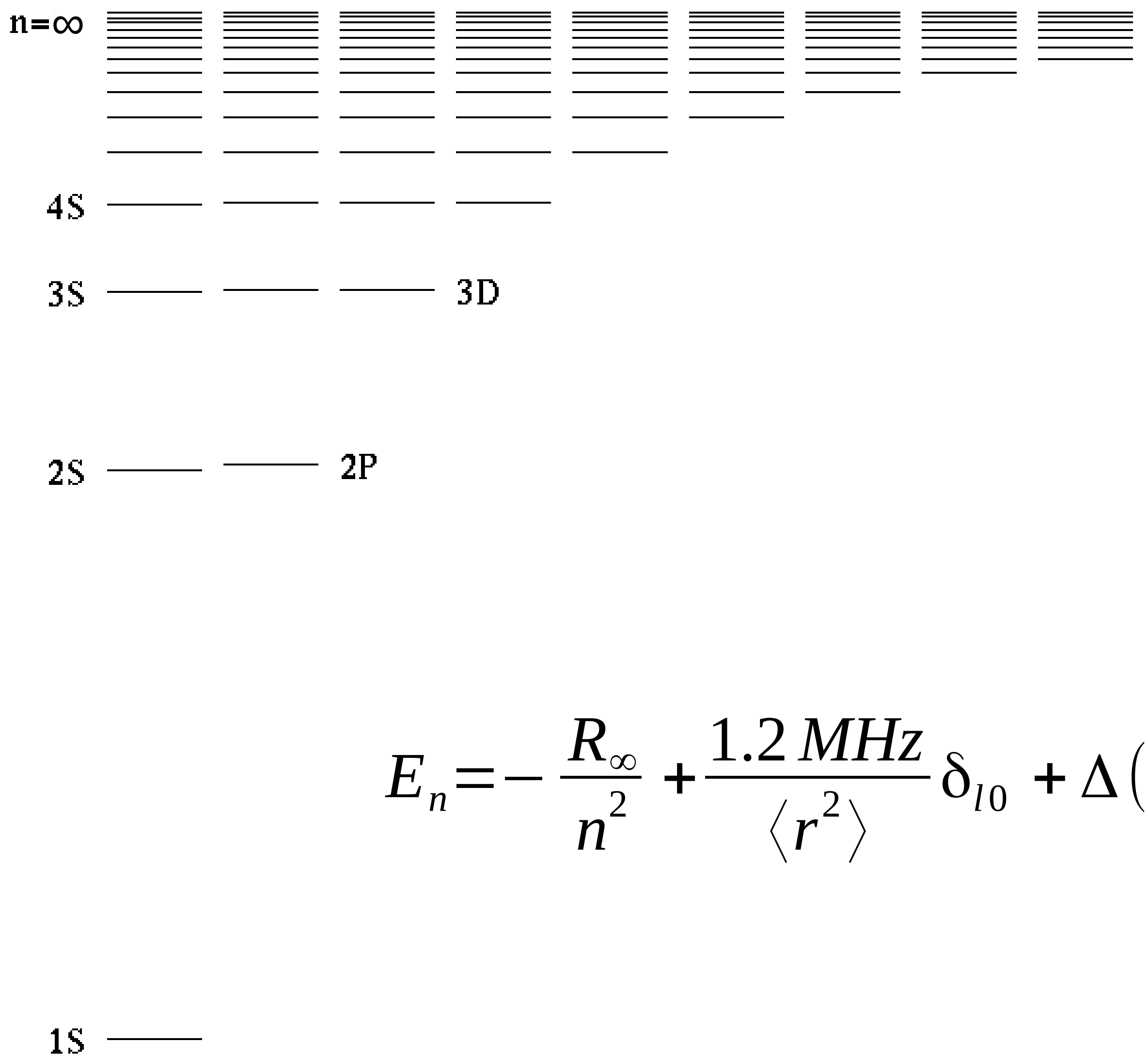
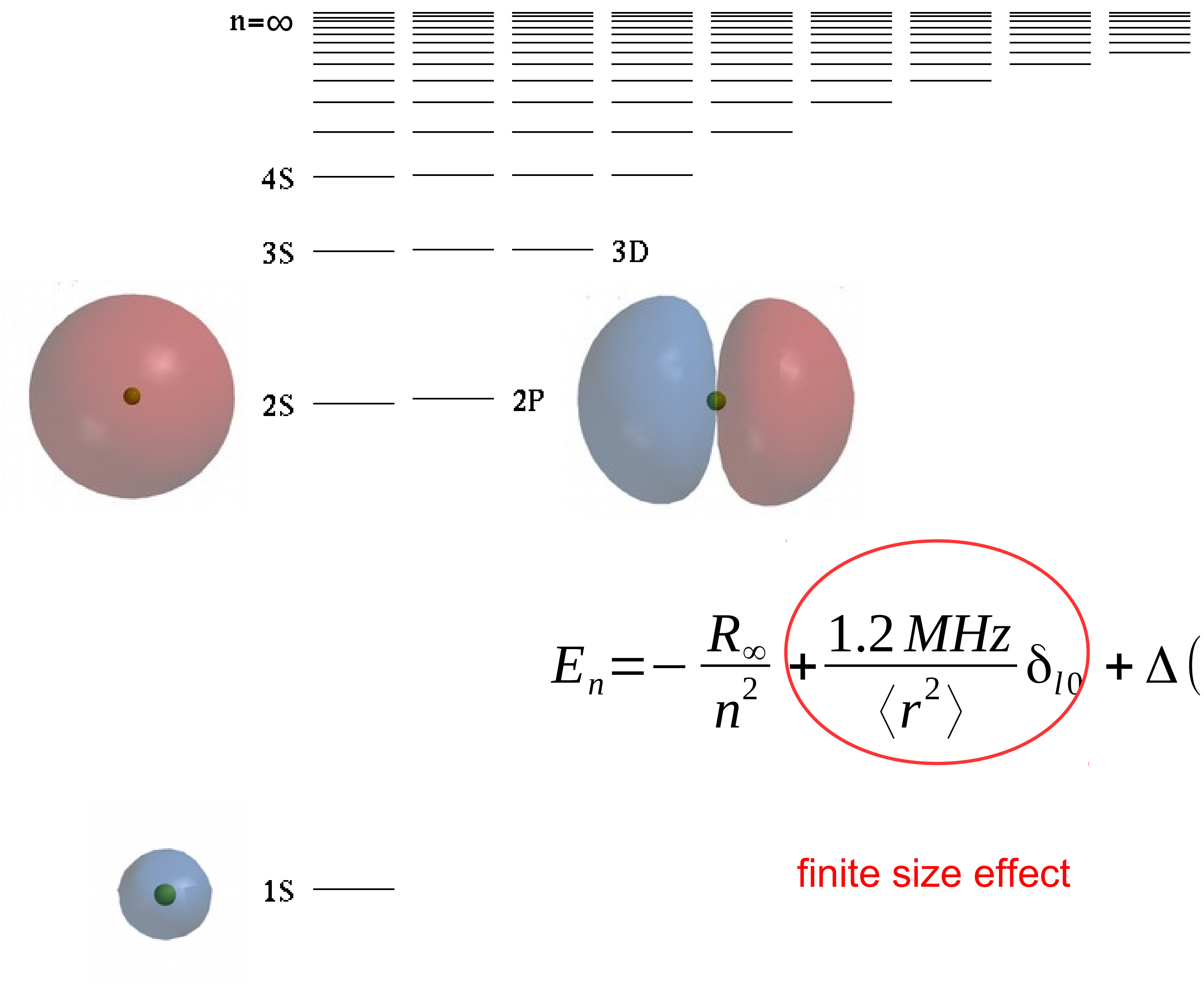


$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{\langle r^2 \rangle}\delta_{l0} + \Delta(n, l, j)$$

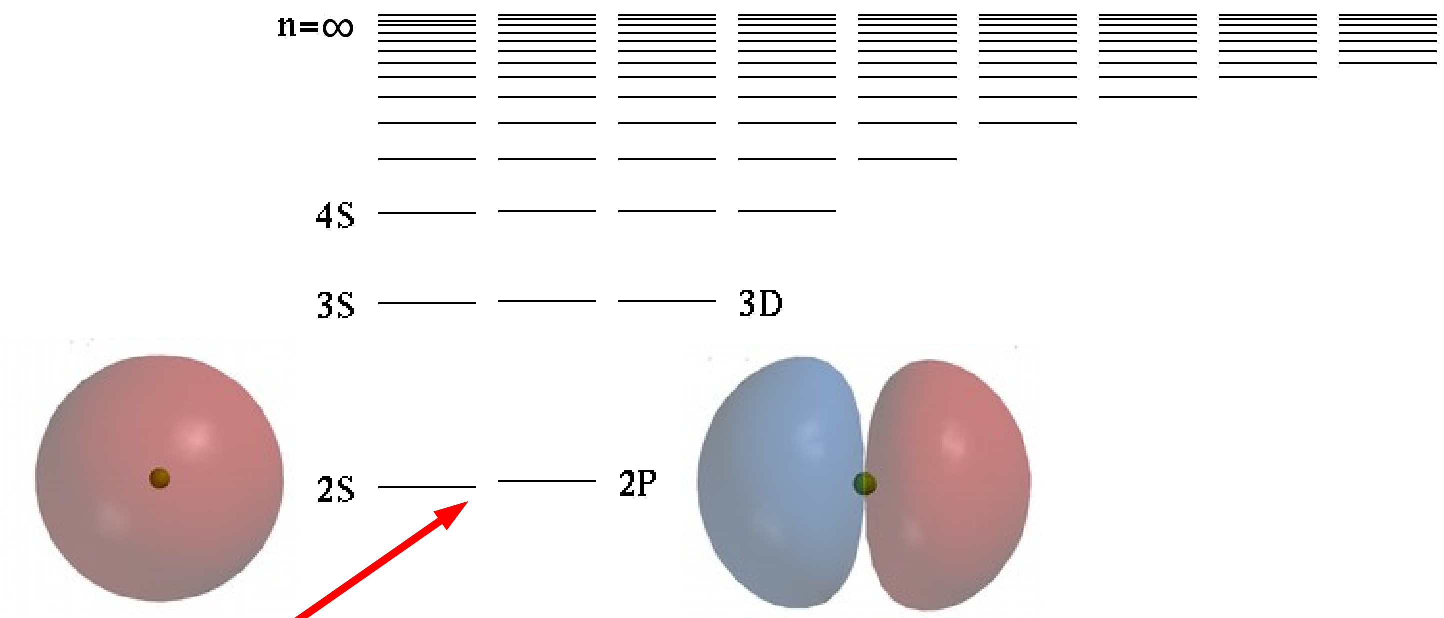
Energy levels of hydrogen



$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{\langle r^2 \rangle}\delta_{l0} + \Delta(n,l,j)$$

finite size effect

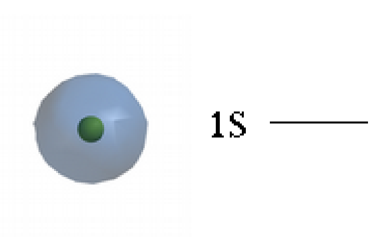
Energy levels of hydrogen



2S-2P Lamb shift

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{\langle r^2 \rangle}\delta_{l0} + \Delta(n,l,j)$$

finite size effect



Part 1: Muonic atoms

A nucleus, orbited by one negative muon

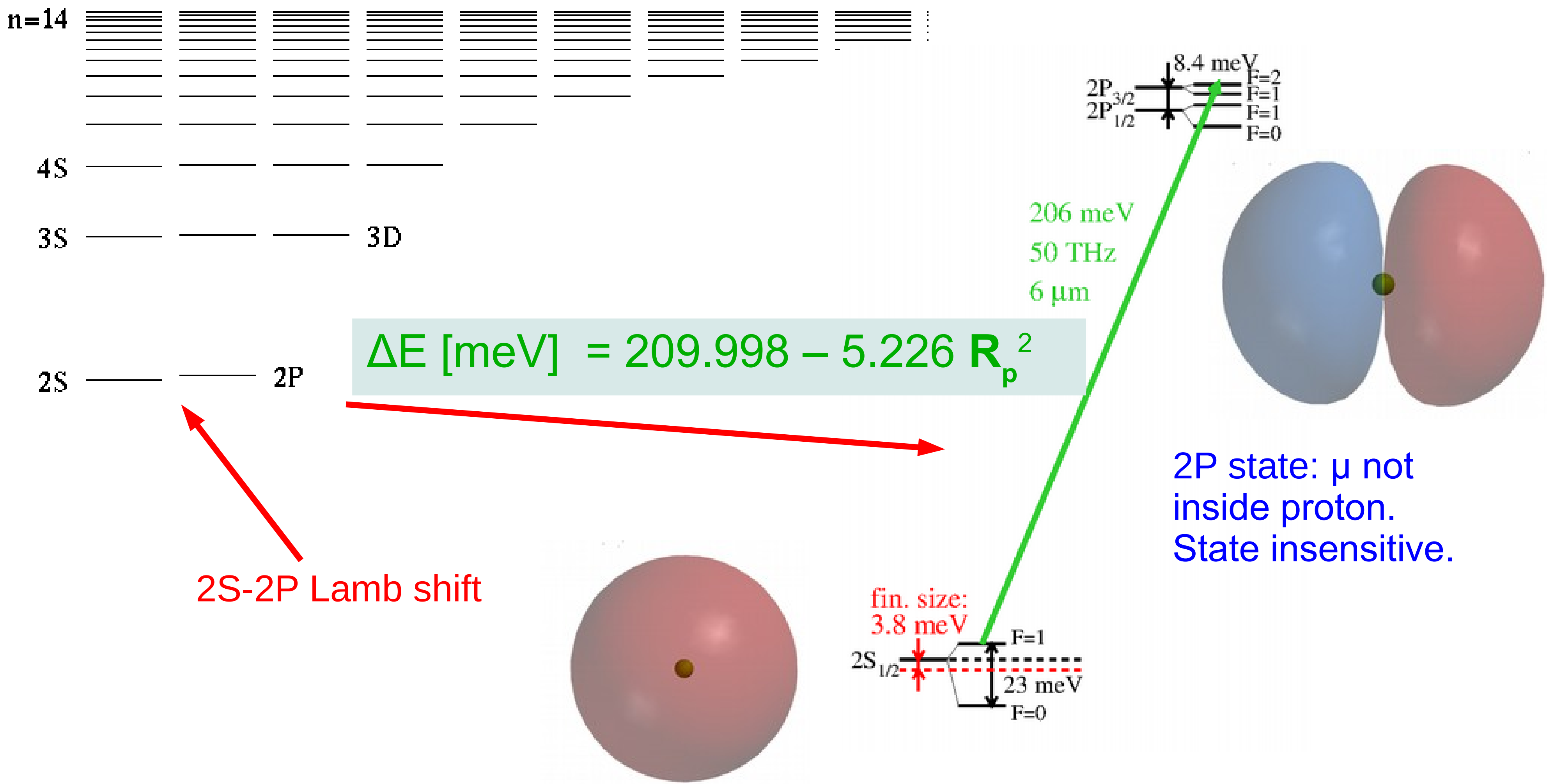
Muon mass = 200 x electron mass

muonic Bohr radius = 1/200 electronic Bohr radius

wave function overlap = 200^3 = **10 million times larger**

muon = very sensitive probe of nuclear properties

Muonic Hydrogen

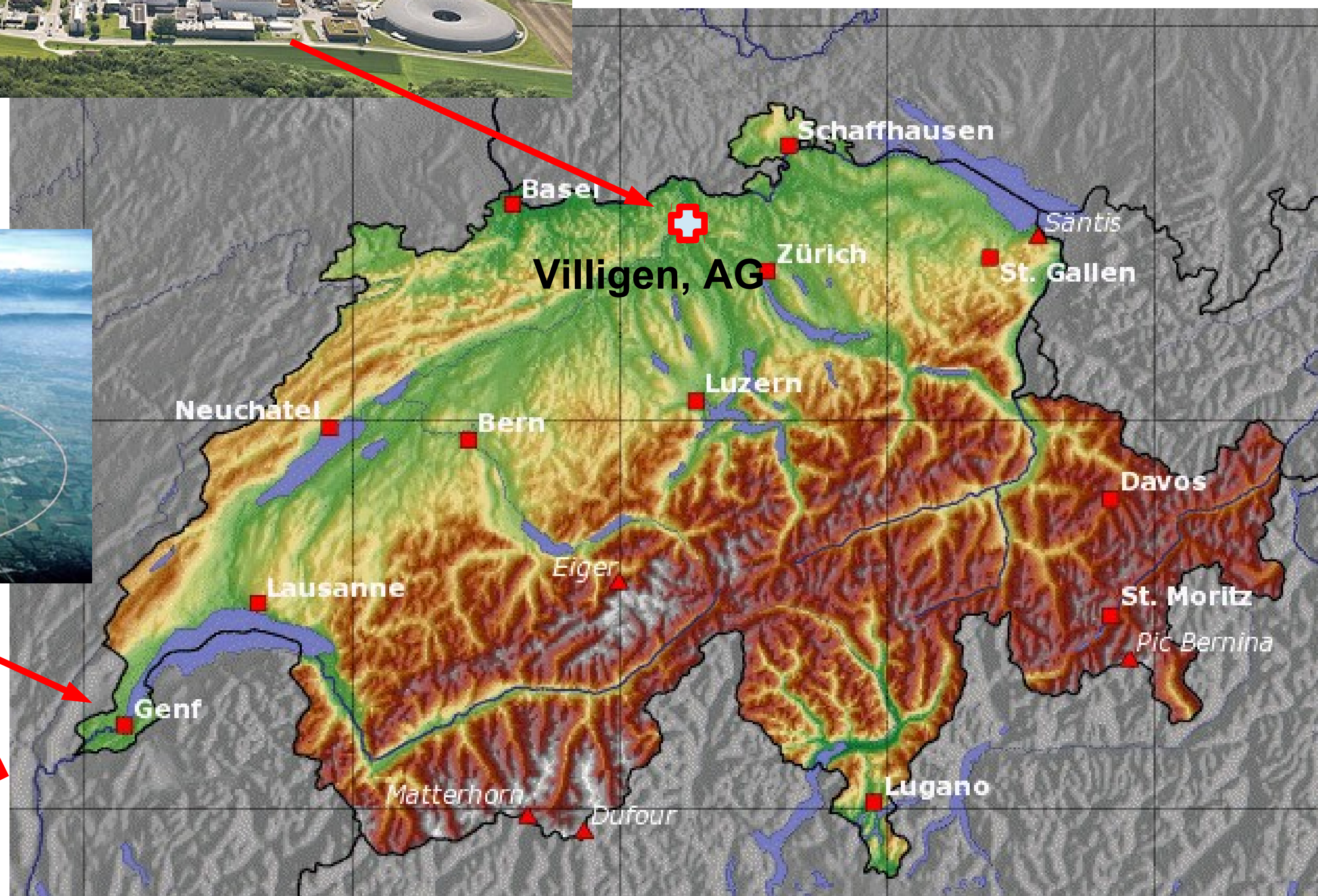


2S state: µ spends some time **inside** the proton!
State is sensitive to the proton size.

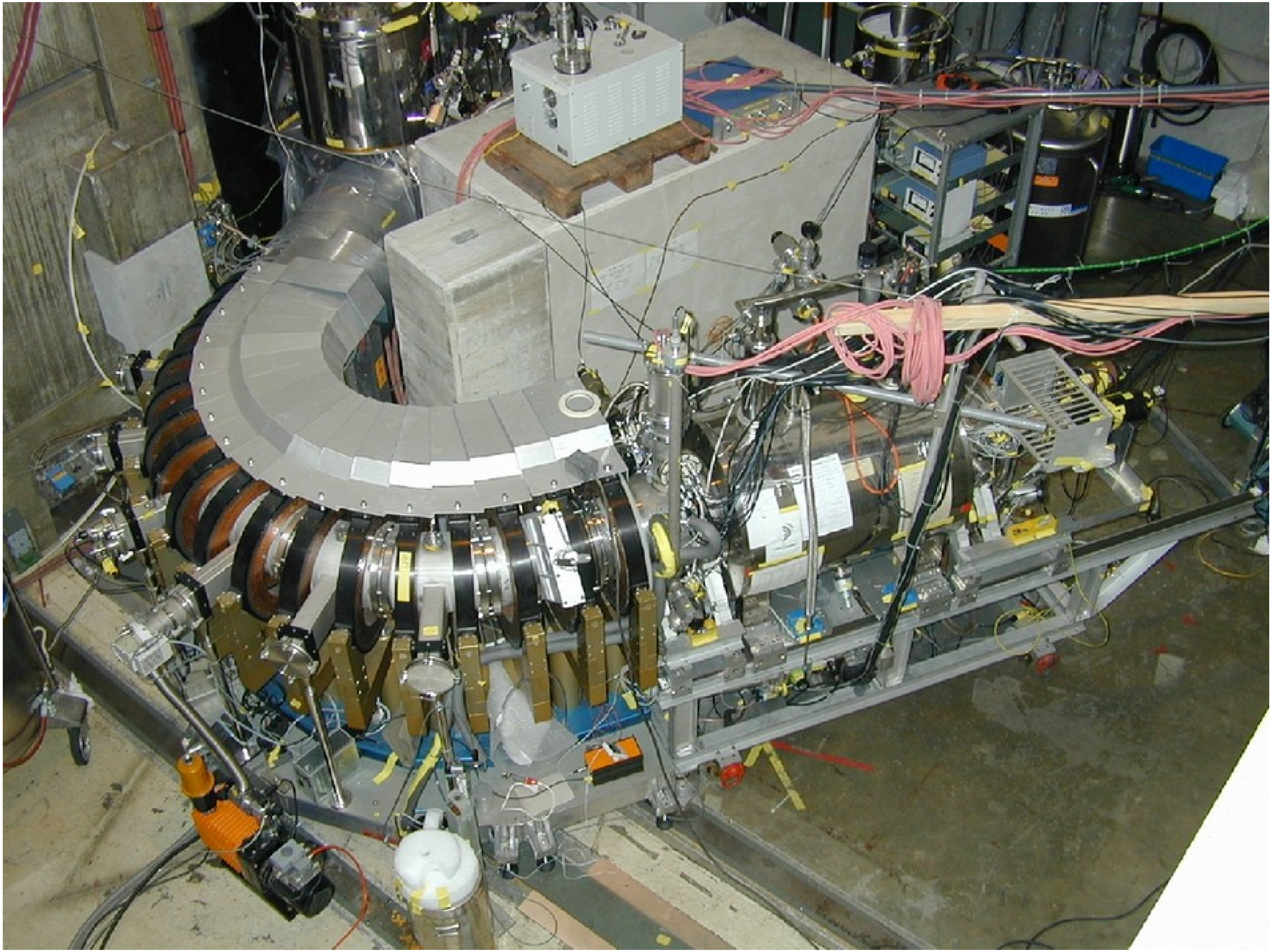
The accelerator at PSI

PAUL SCHERRER INSTITUT

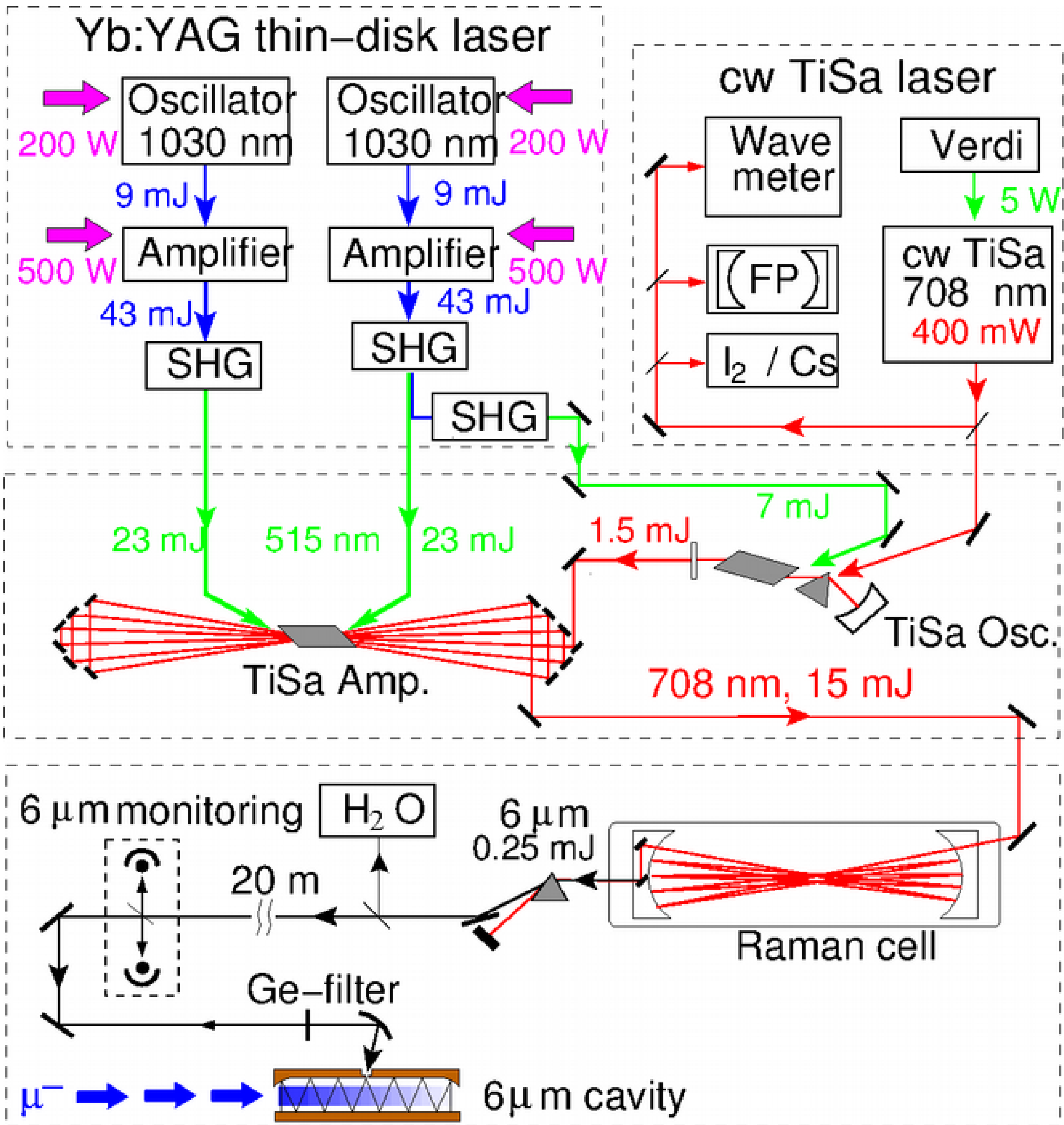
PSI



The muon beam line in πE5



The laser system



Yb:YAG Disk laser
→ fast response on μ

Frequency doubling (SHG)
→ green light to pump
Ti:sapphire laser

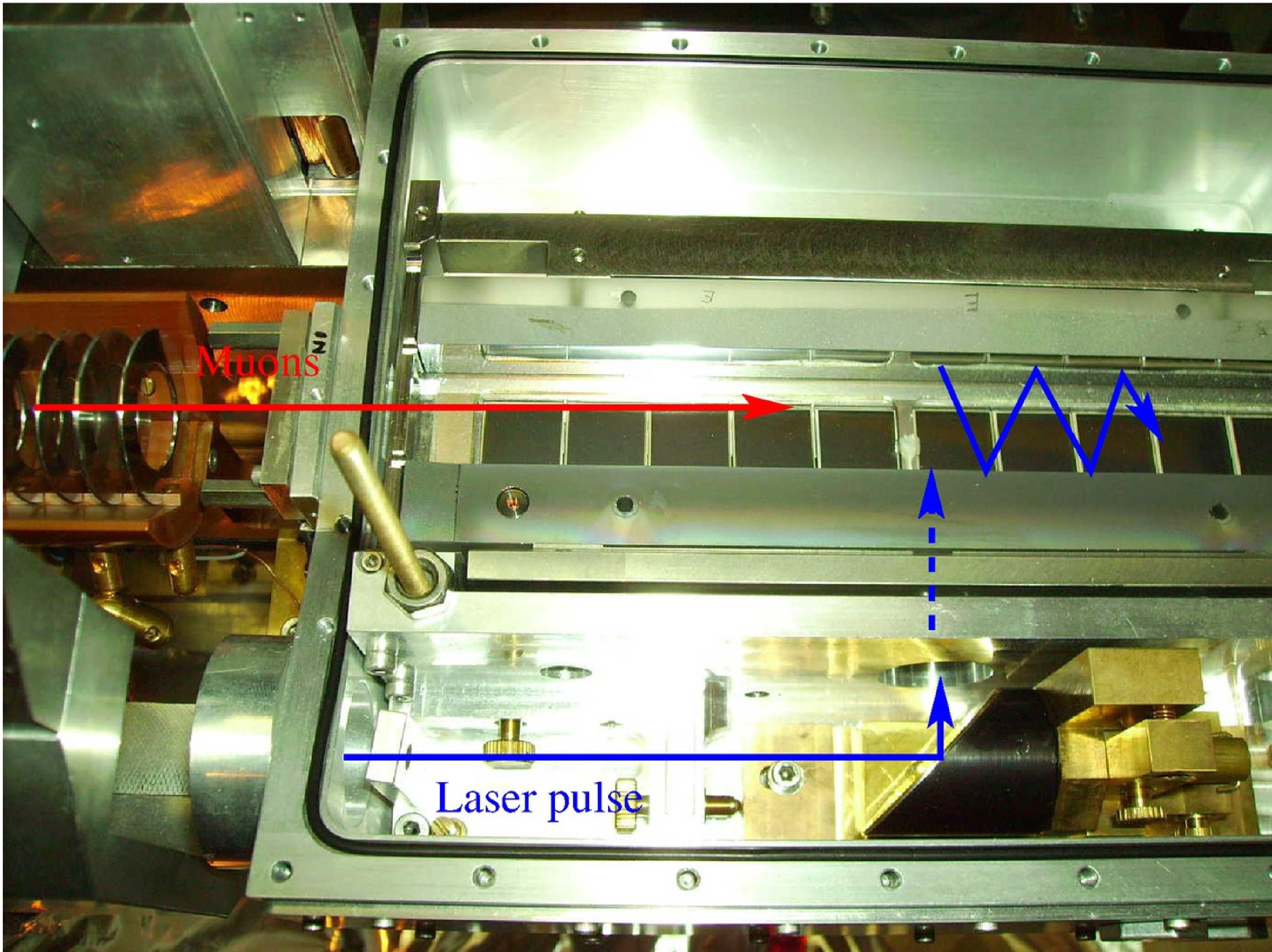
Ti:sapphire cw laser
→ determines laser frequency

Ti:sapphire MOPA
→ high pulse energy (15 mJ)

Raman cell
→ 3 sequential stimulated
Raman Stokes shifts
Laser wave length → 6 μm

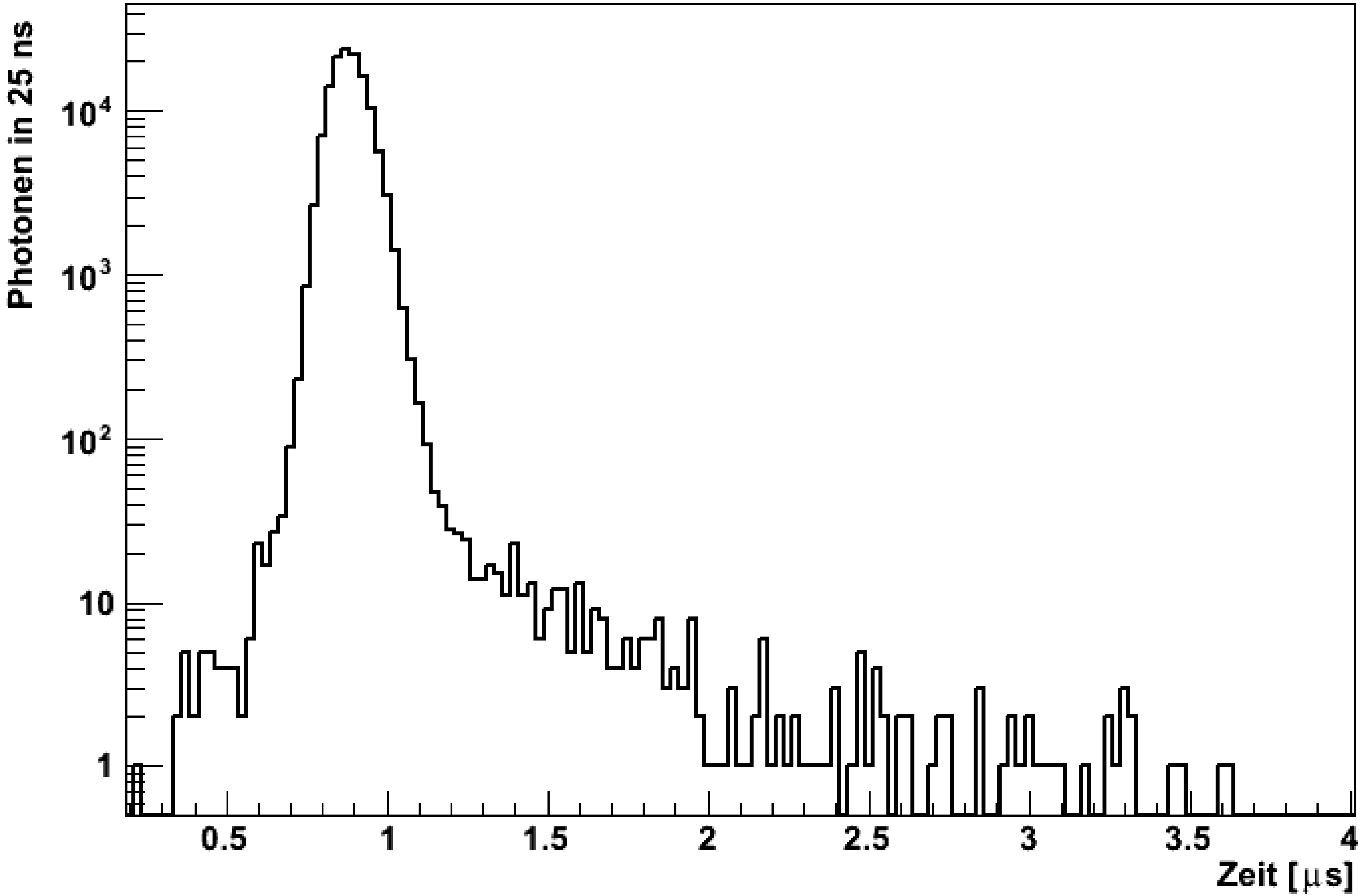
Target Cavity
→ Mirror system to fill the
muon stop volume (H_2)

The hydrogen target



Time Spectra

13 hours of data



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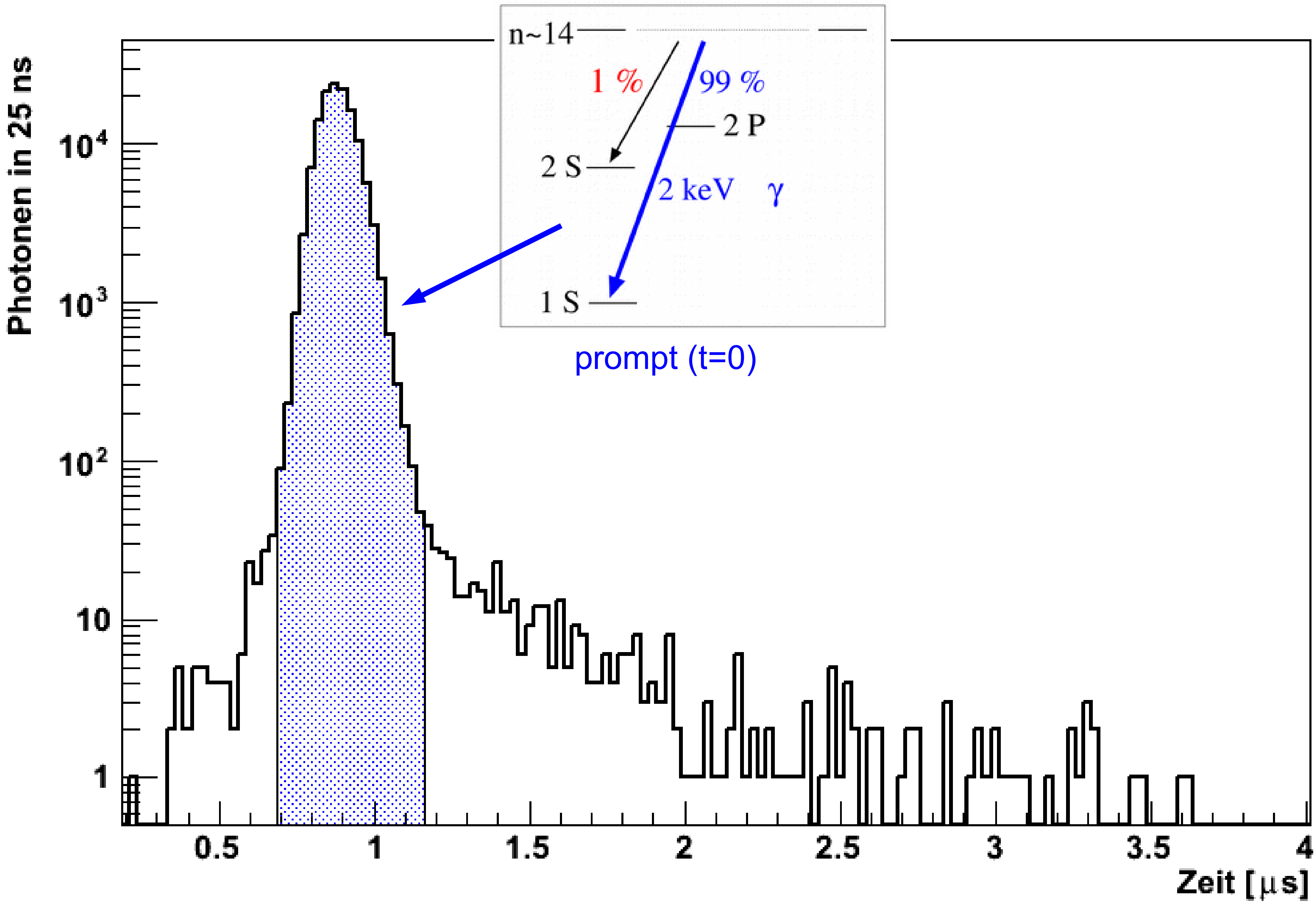

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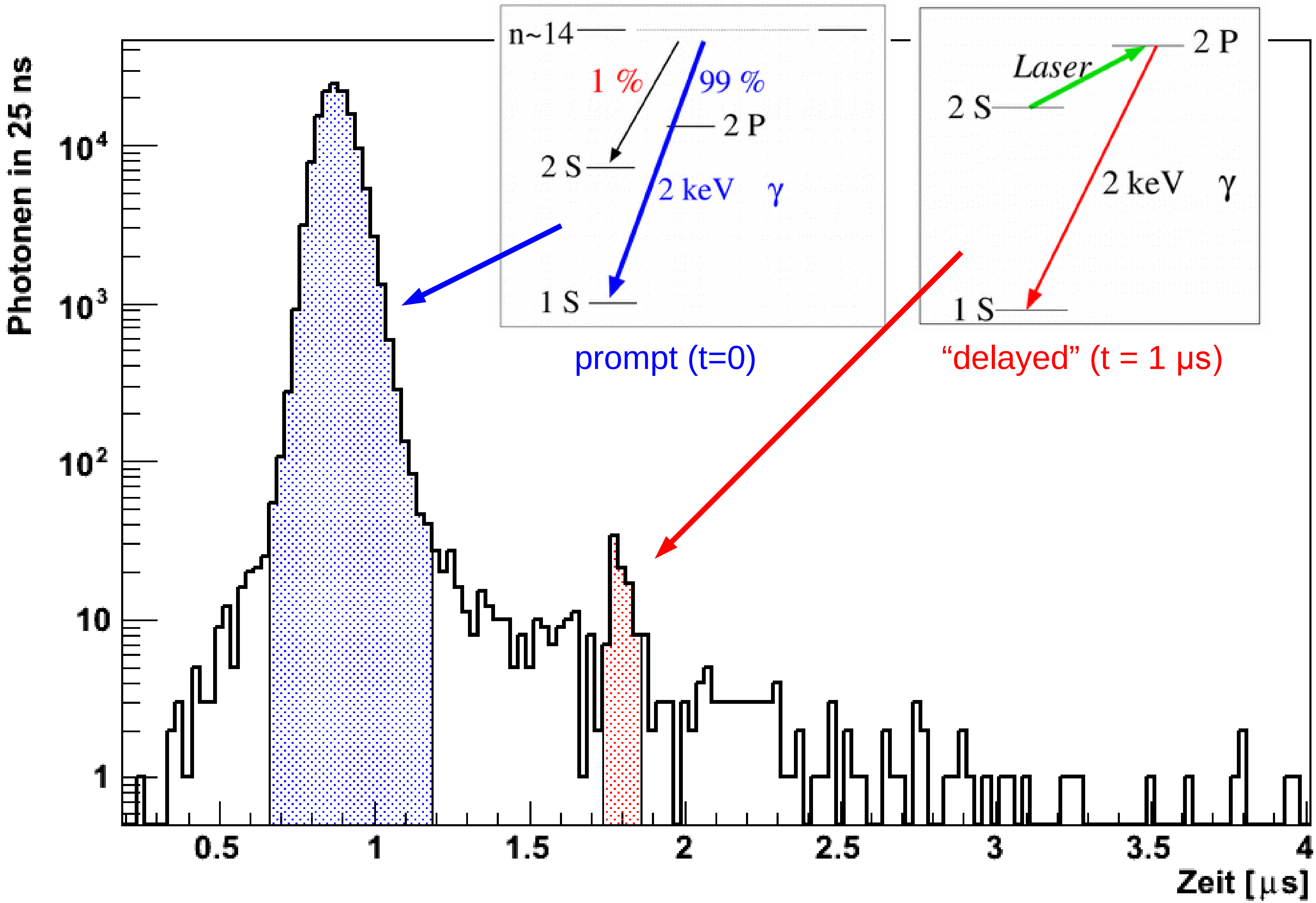

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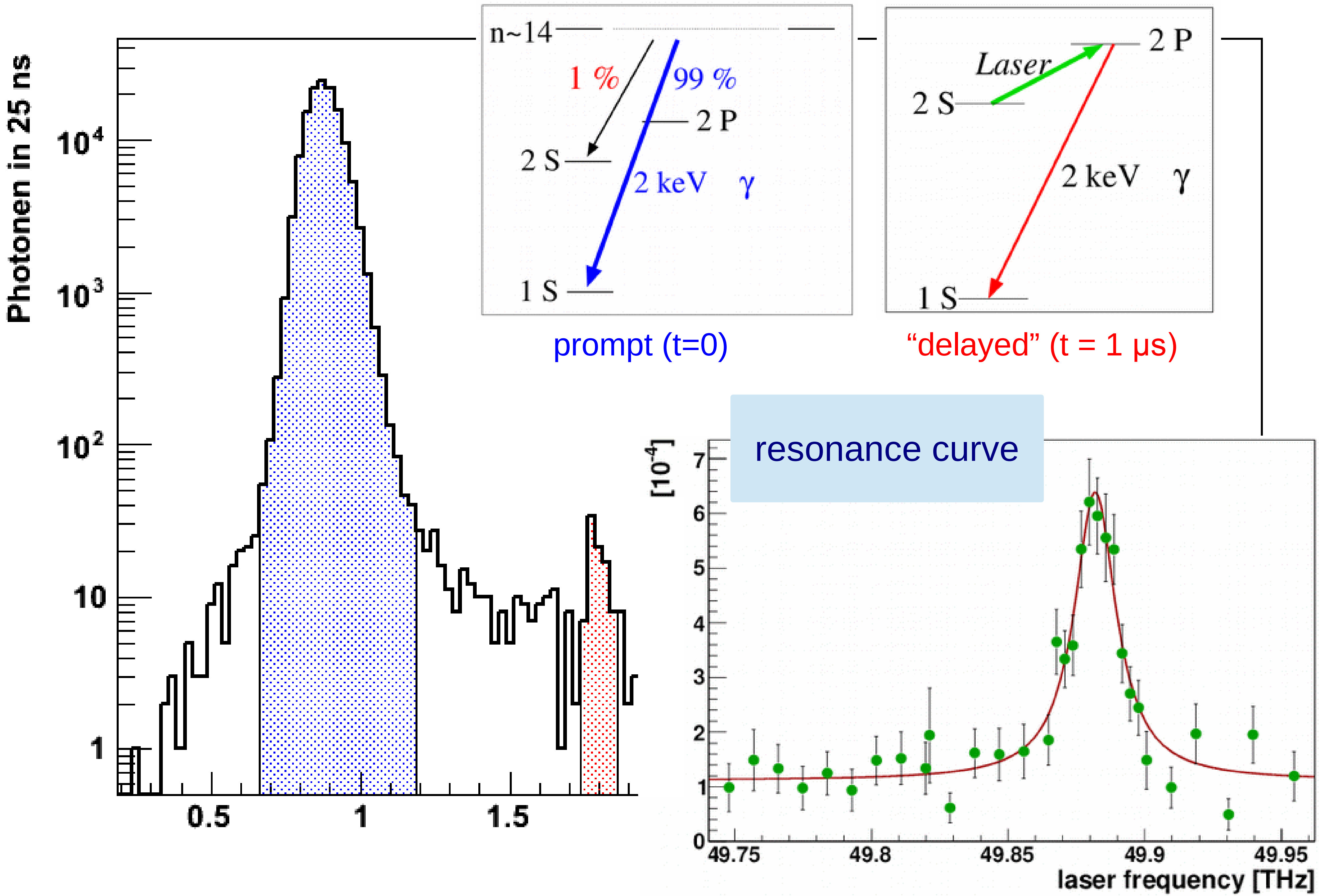
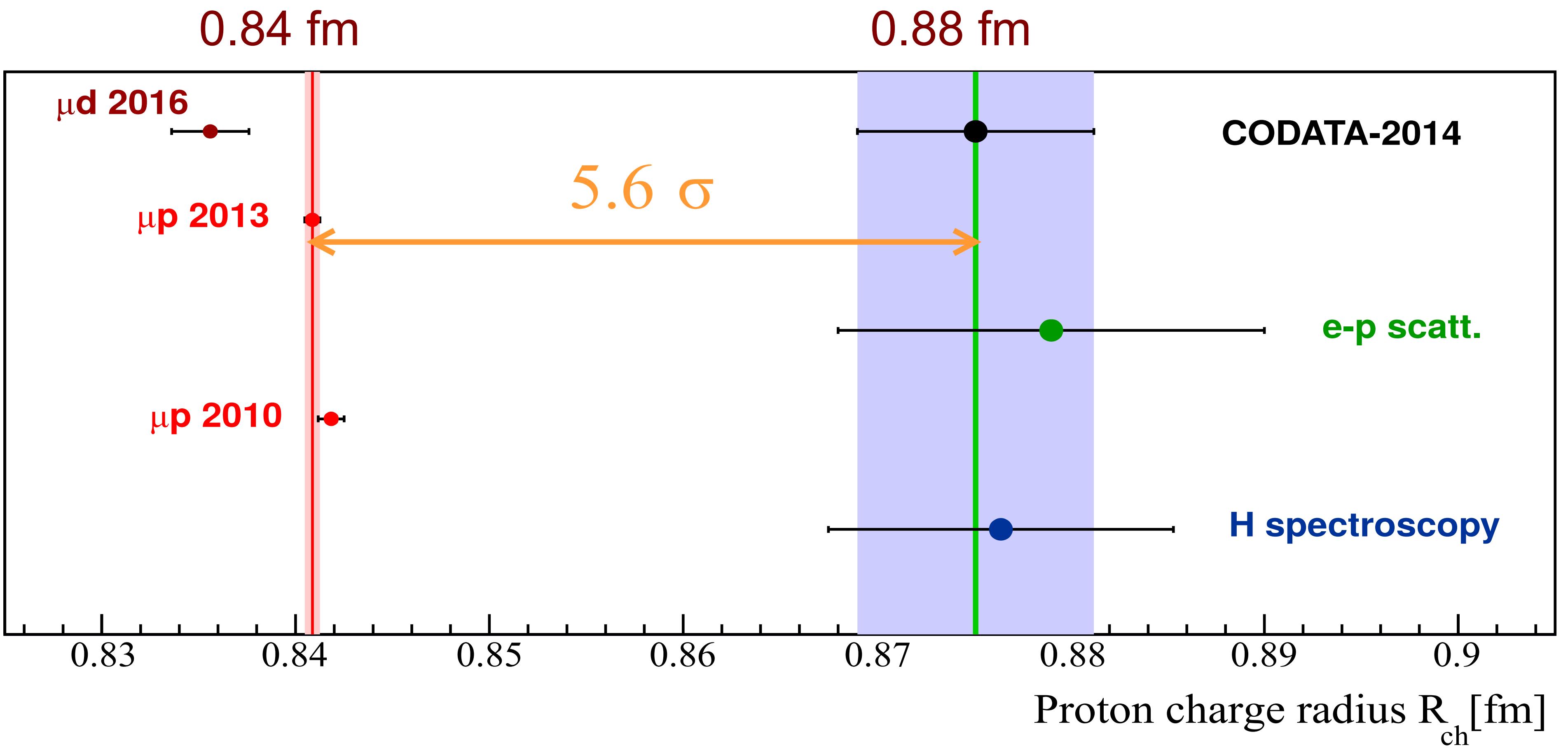


Muonic Hydrogen



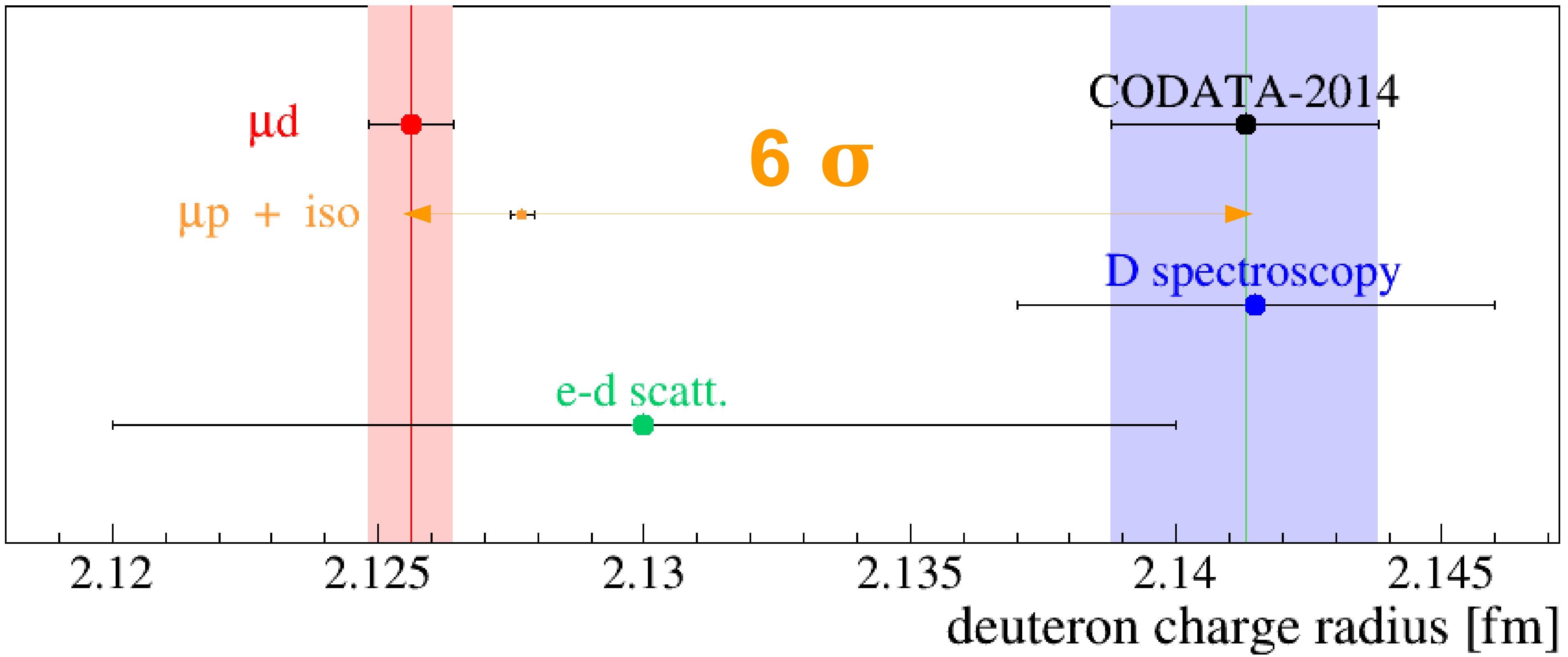
muonic hydrogen: 0.8409 ± 0.0004 fm

electronic hydrogen: 0.876 ± 0.008 fm

electron scattering 0.879 ± 0.011 fm

20x more accurate

Muonic Deuterium



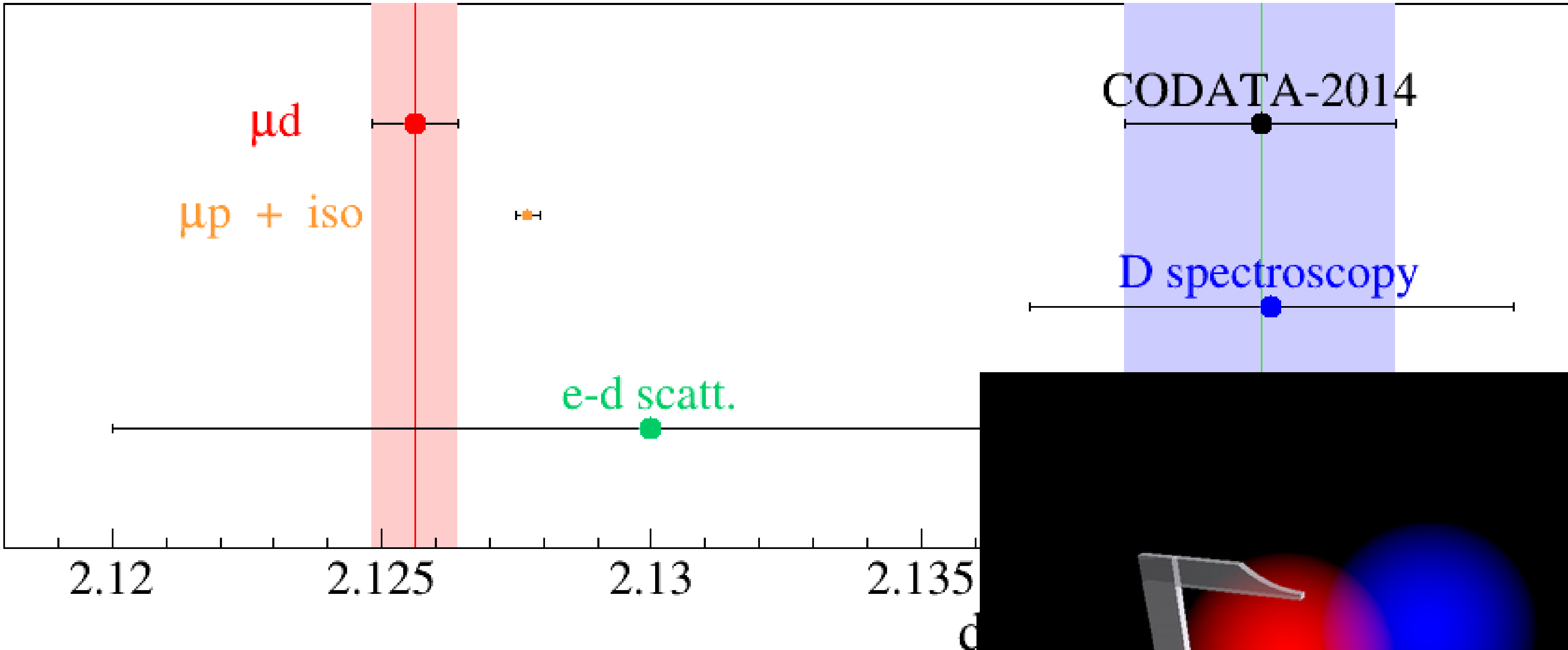
μD: 2.12562 $(13)_{exp}$ $(77)_{theo}$ fm (nucl. polarizability)

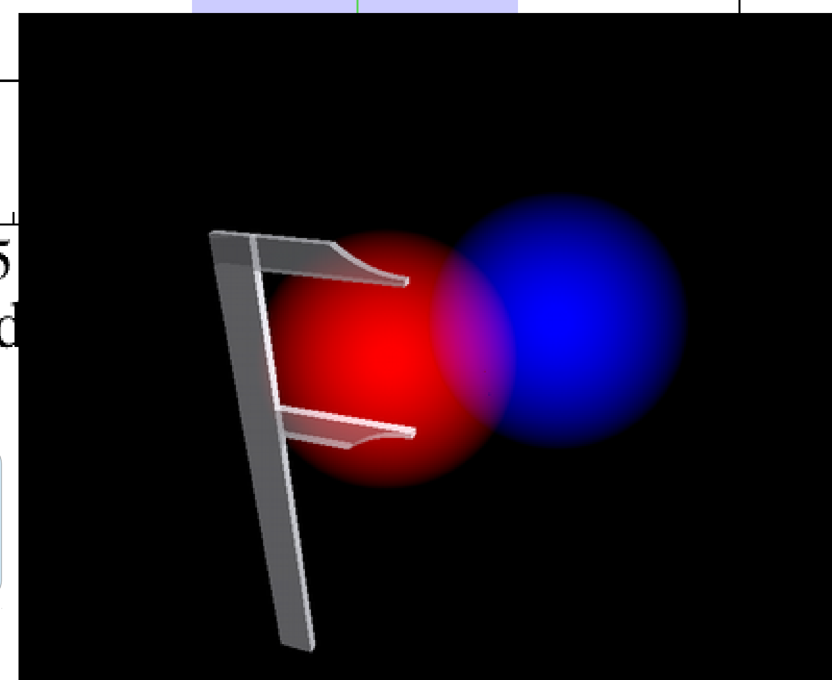
μH + H/D(1S-2S): 2.12771 (22) fm

CODATA-2014: 2.1**4**130 (250) fm

RP et al. (CREMA Coll.), Science 353, 559 (2016)

Deuteron radius



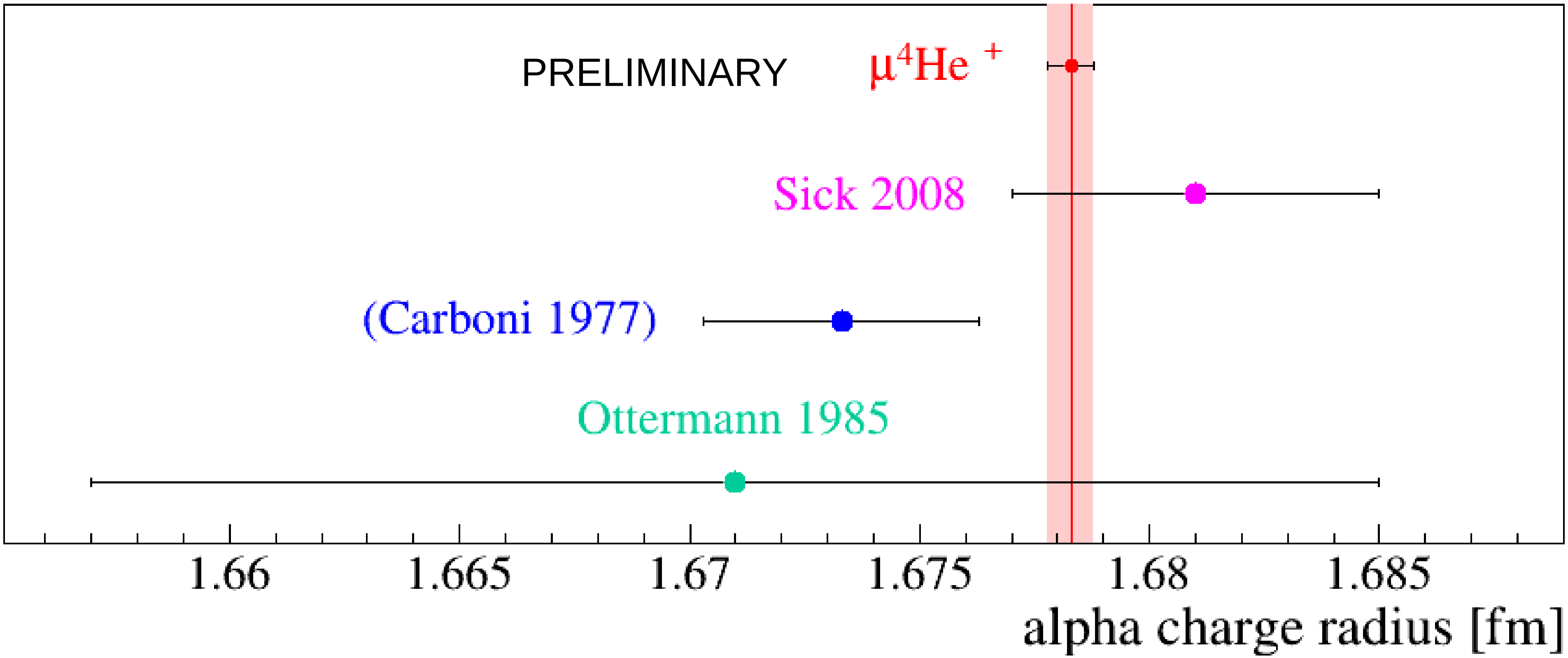


Deuteron is CONSISTENTLY smaller!

$$R_d^2 = R^2_{struct} + R_p^2 + R_n^2 \ (+\ DF)$$

Pohl et al. (CREMA), Science 353, 669 (2016)

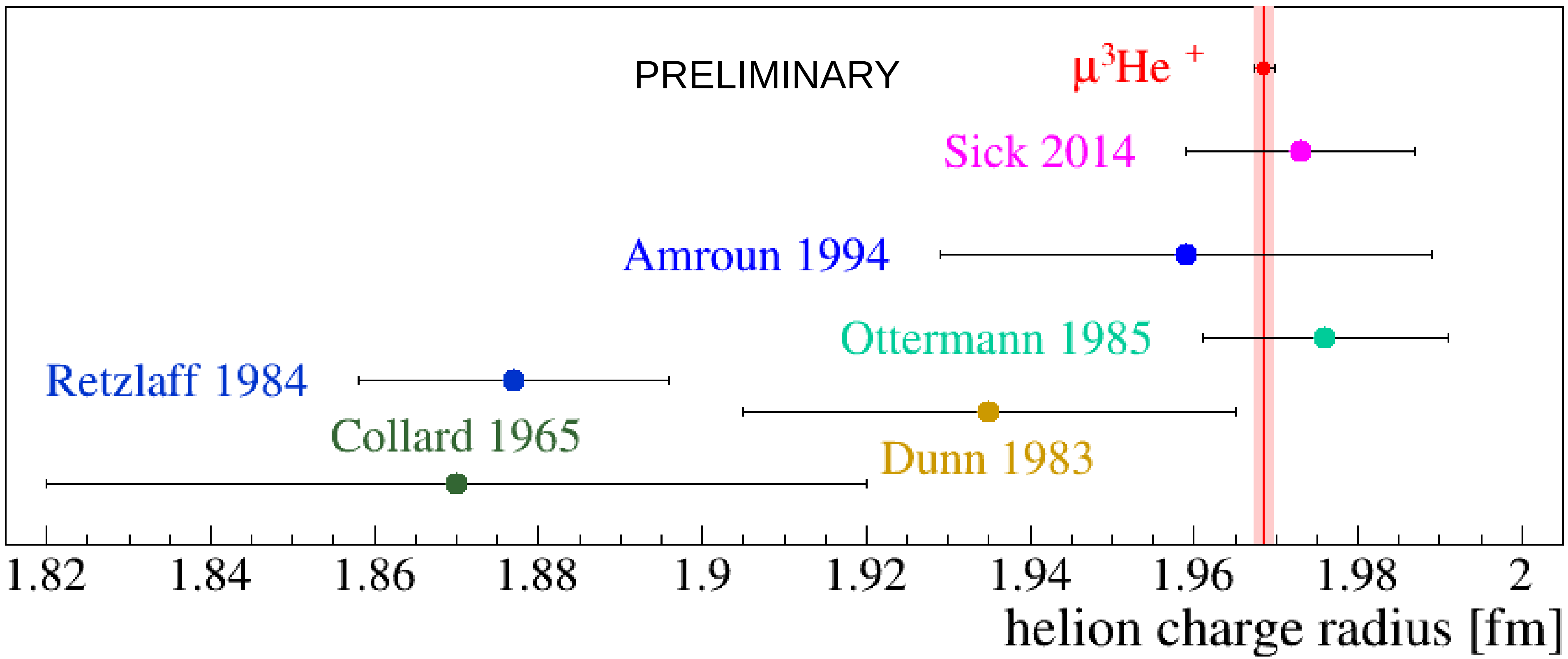
Muonic Helium-4



prel. accuracy: exp +- 0.00019 fm, theo +- 0.00058 fm (nucl. polarizability)

Theory: see Diepold et al. arxiv 1606.05231

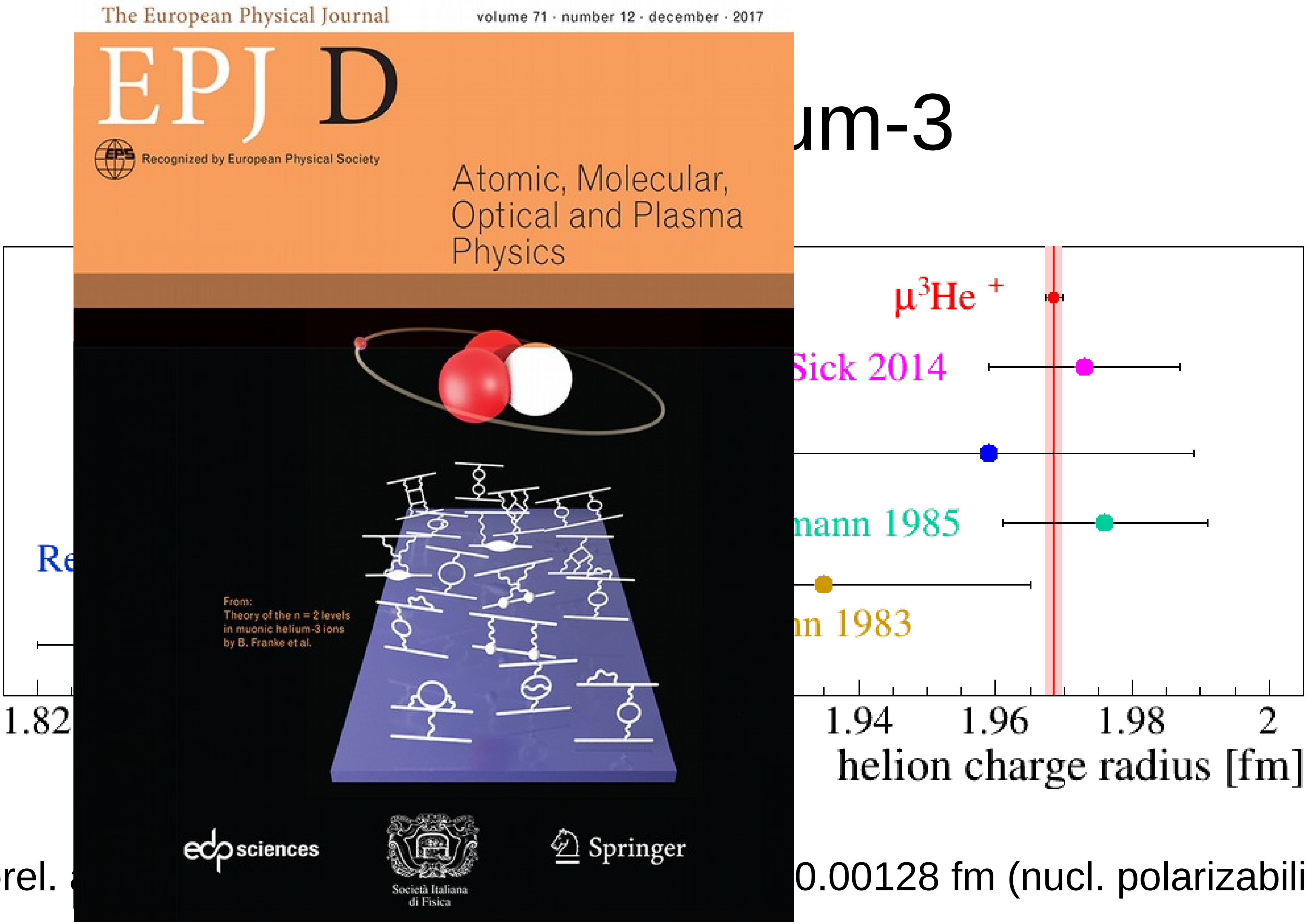
Muonic Helium-3



prel. accuracy: exp +- 0.00012 fm, theo +- 0.00128 fm (nucl. polarizability)

Theory: see Franke et al. EPJ D 71, 341 (2017) [1705.00352]

um-3



prel. a… 0.00128 fm (nucl. polarizability)

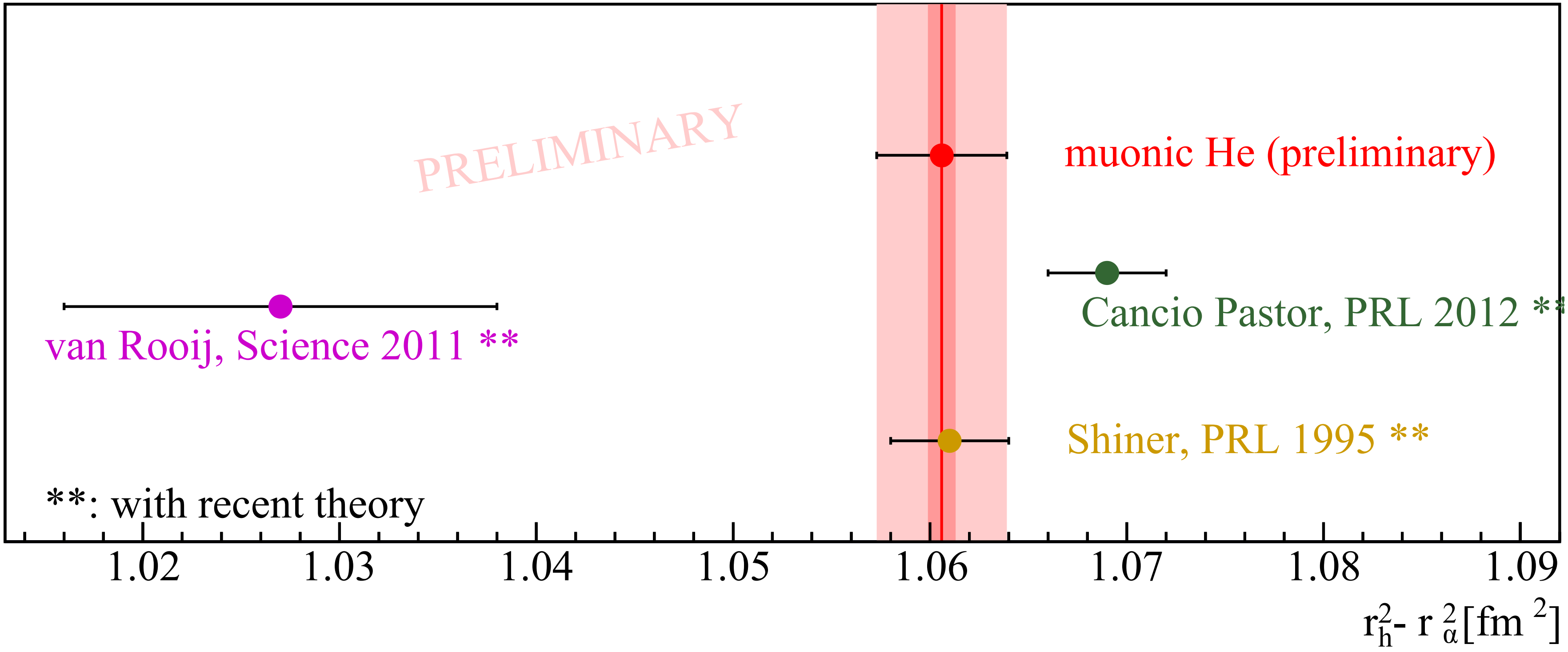
Theory: see Franke et al. EPJ D 71, 341 (2017) [1705.00352]

Muonic conclusions

- The proton radius is 0.84087 $(26)_{exp}$ $(29)_{theo}$ fm
- The deuteron radius is 2.12771 (22) fm
- both are **>5σ smaller** than CODATA values
- No discrepancy for the **absolute radii** of the helion and alpha particle
(limited by e-scattering accuracy)
- **BUT: The helium isotope shift!!!**

The ^{3}He – ^{4}He isotope shift

^{3}He / ^{4}He (squared) charge radius difference



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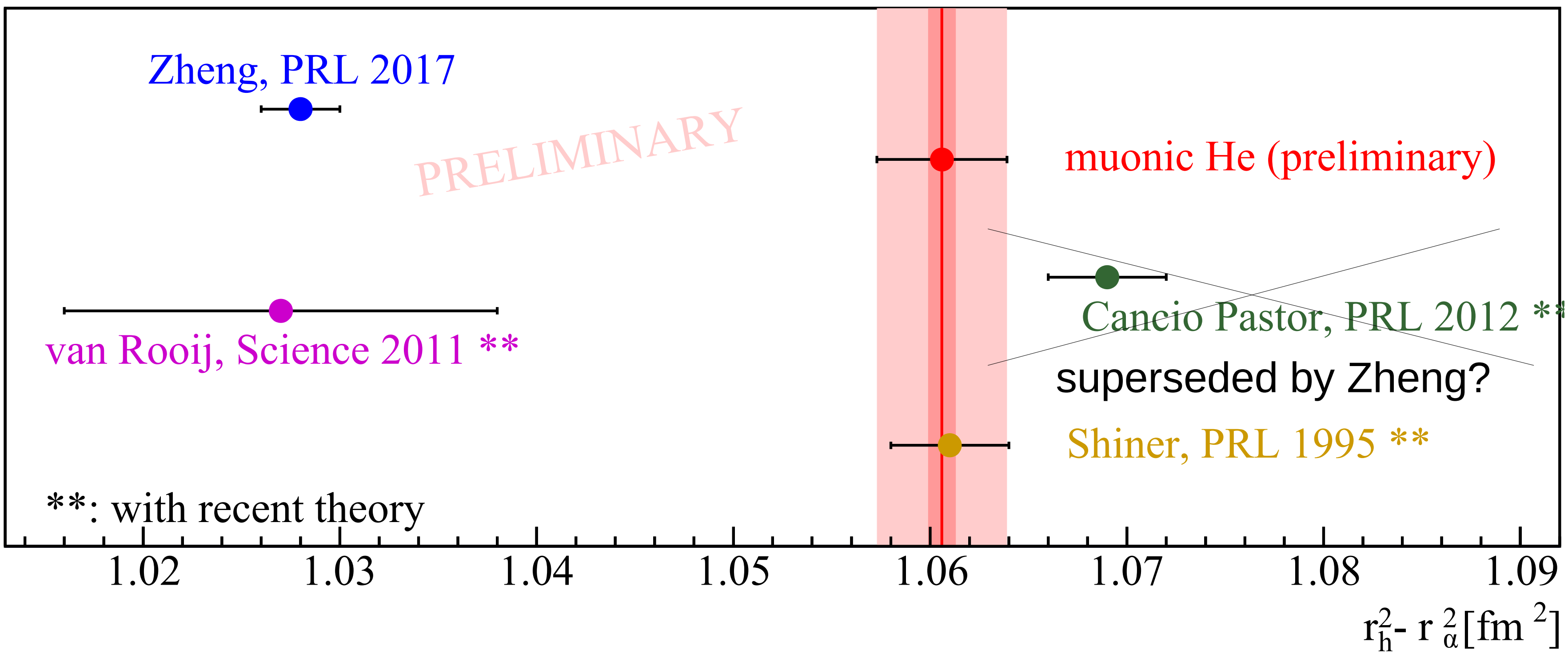


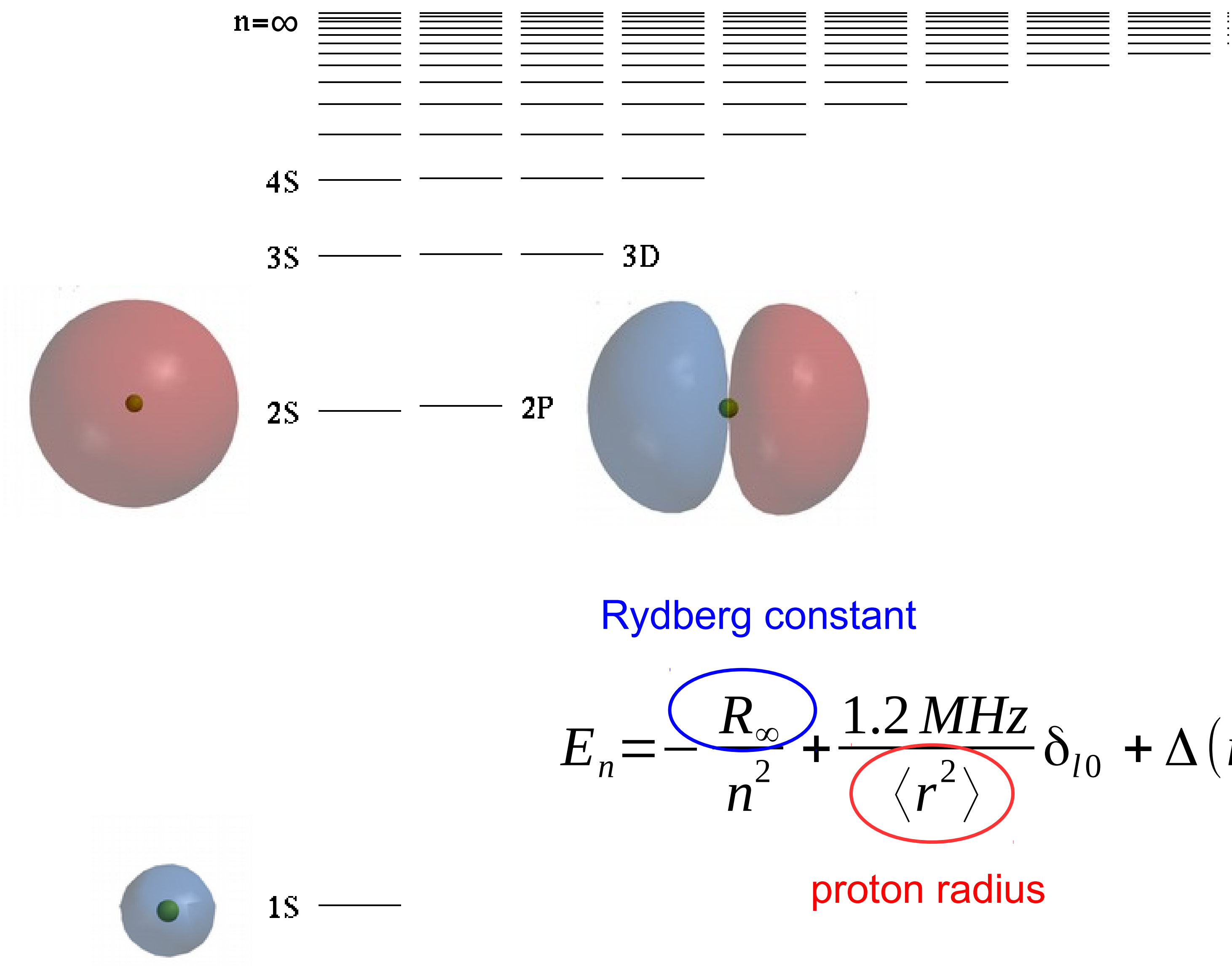
Another >5σ discrepancy?!

Part 2: The Rydberg constant

$$R_\infty = \frac{\alpha^2 m_e c}{2h}$$

- most accurately determined fundamental constant $u_r = 5.9 * 10^{-12}$
- corner stone of the CODATA LSA of fundamental constants
  links fine structure constant α, electron mass m_e, velocity of light c and Planck's constant *h*
- correlation coefficient with proton radius: 0.9891
  → The "proton radius puzzle" could be a "Rydberg puzzle"
- R_∞ is a "unit converter": atomic units → SI (Hertz)

Energy levels of hydrogen



Rydberg constant

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{\langle r^2 \rangle}\delta_{l0} + \Delta(n, l, j)$$

proton radius

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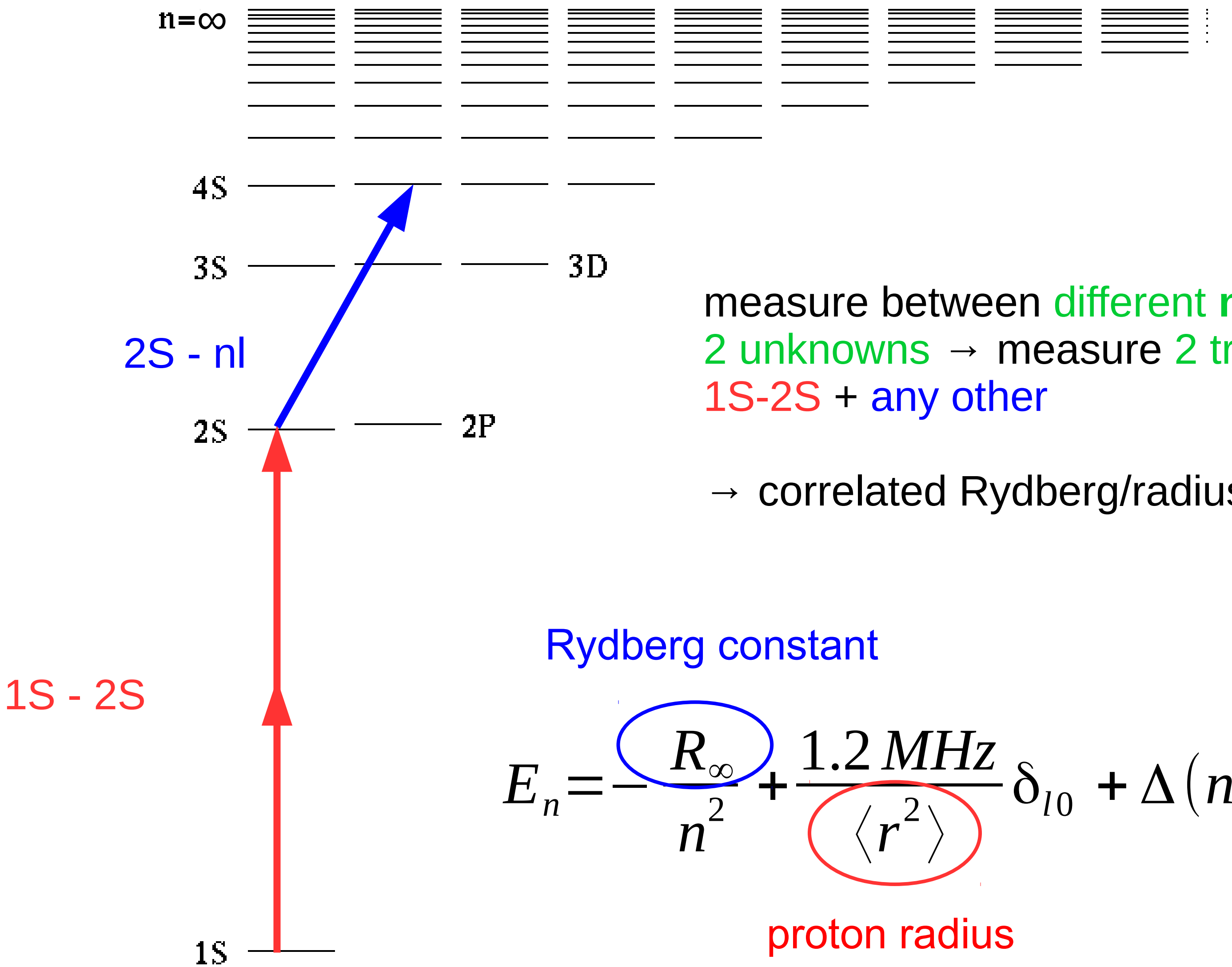


measure between different **n**
2 unknowns → measure 2 transitions:
1S-2S + any other

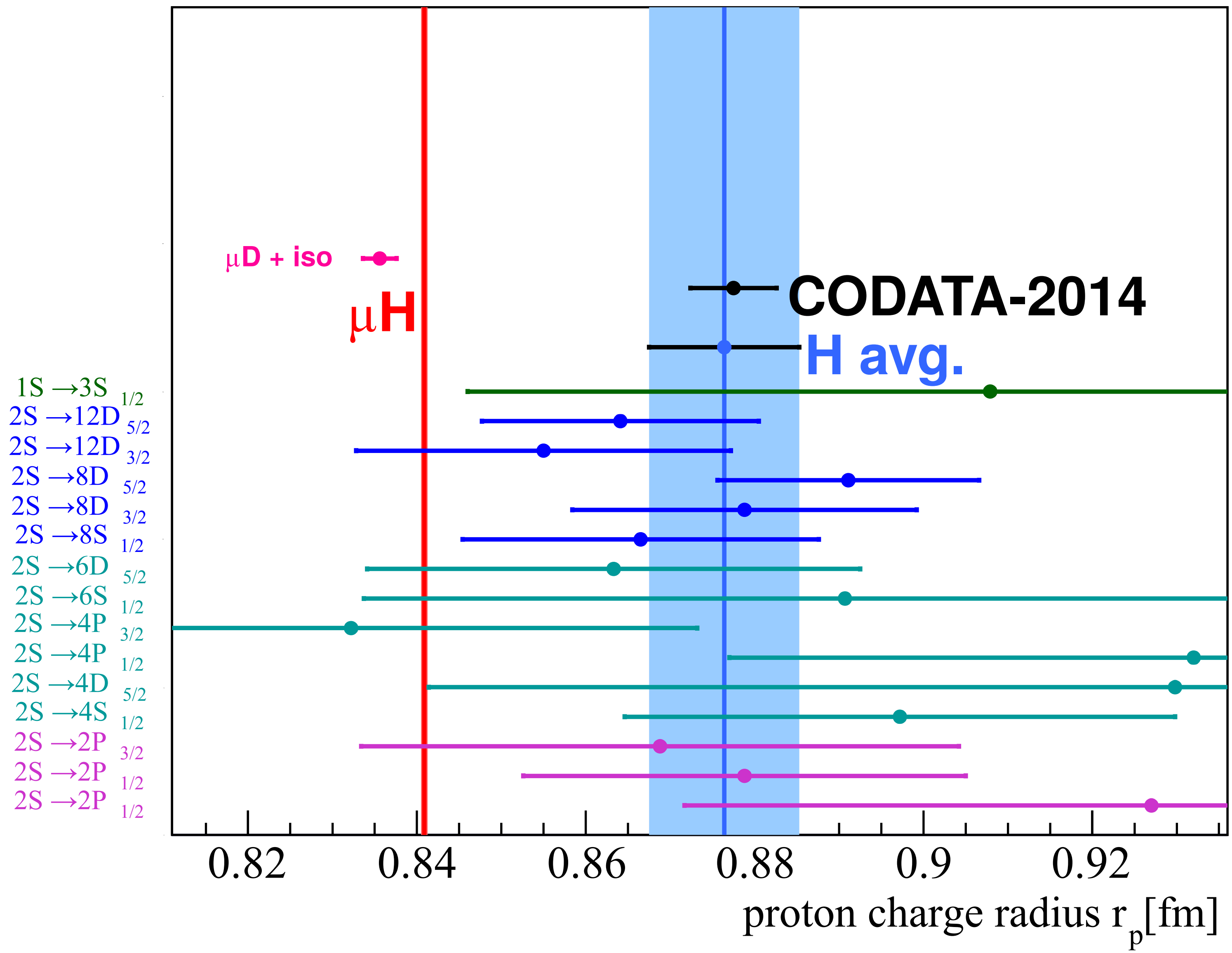
→ correlated Rydberg/radius pairs

Rydberg constant

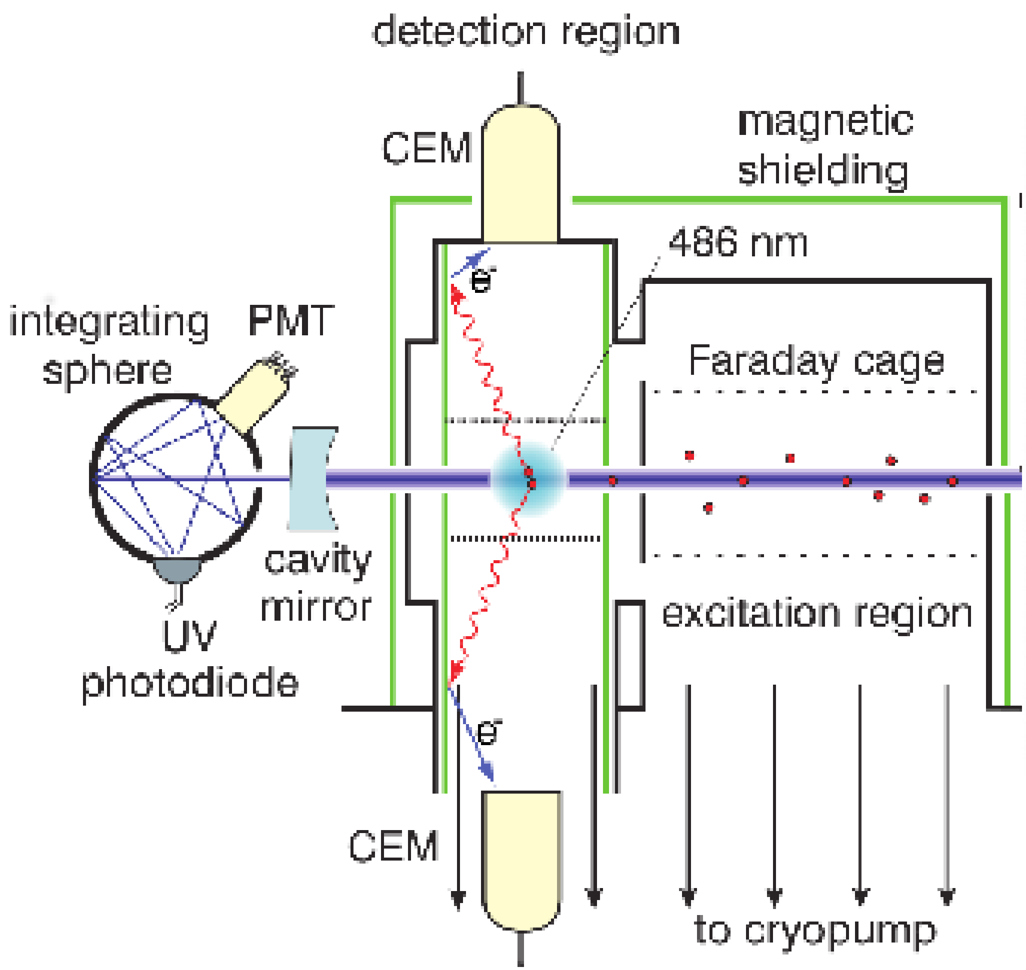
$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{\langle r^2 \rangle}\delta_{l0} + \Delta(n, l, j)$$

proton radius

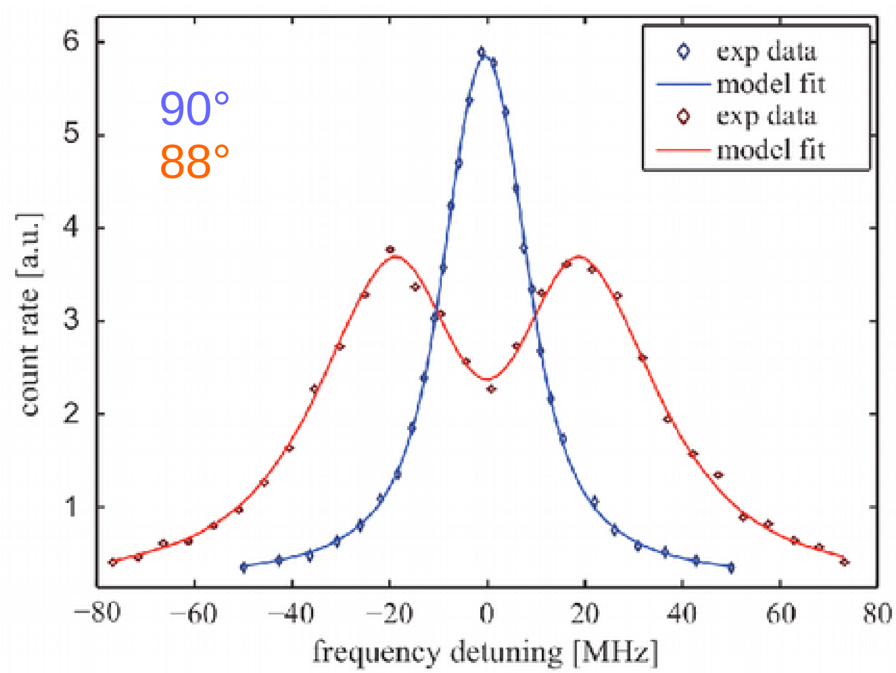
Rp from H spectroscopy



Garching H(2S-4P)



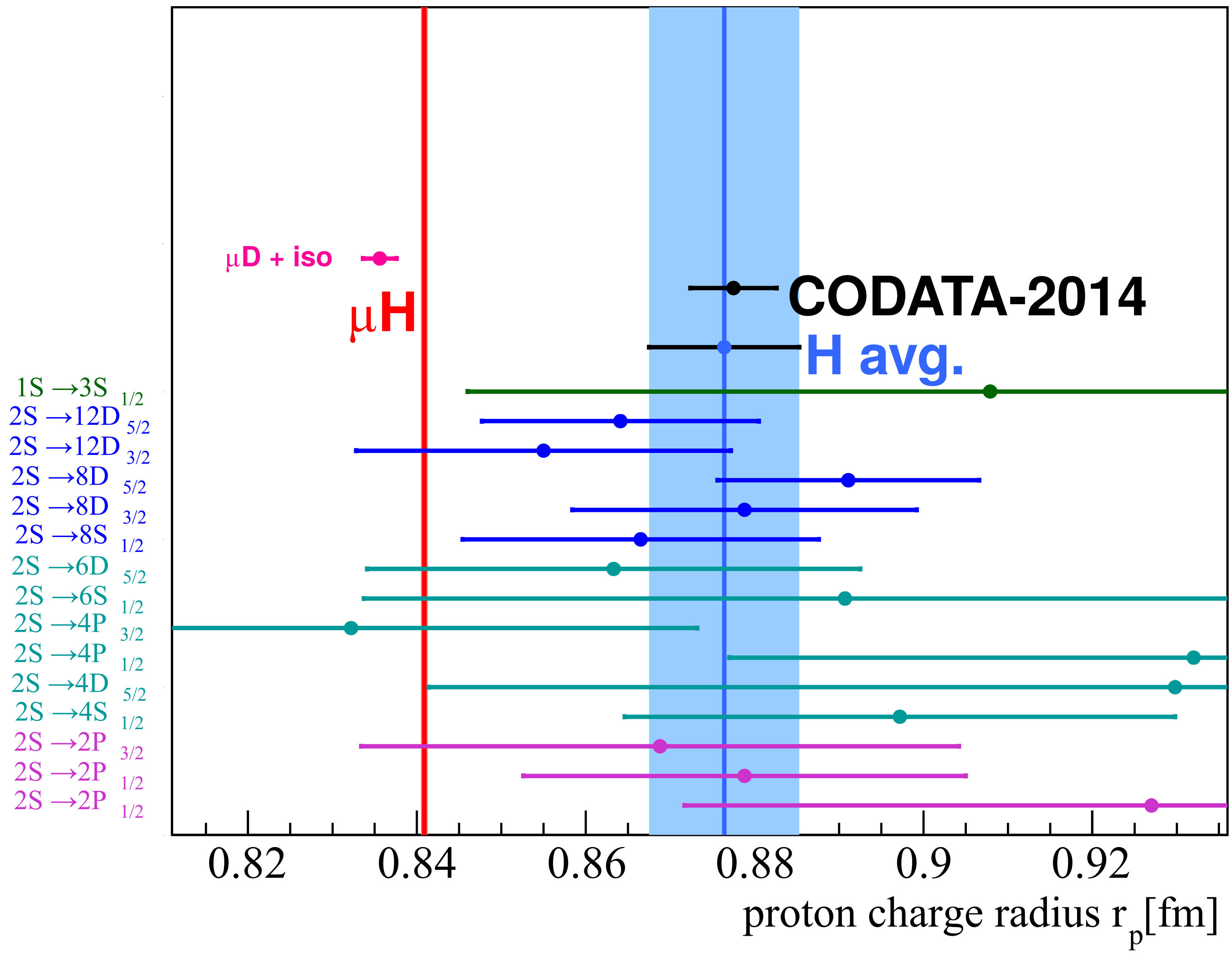
1st order Doppler cancellation



- cryogenic H beam (6 K)
- optical 1S-2S excitation (2S, F=0)
- 2S-4P transition is 1-photon: retroreflector
- split line to 10^{-4} !!!
- 2.3 kHz vs. 9 kHz PRP
- large systematics

Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

Rp from H spectroscopy



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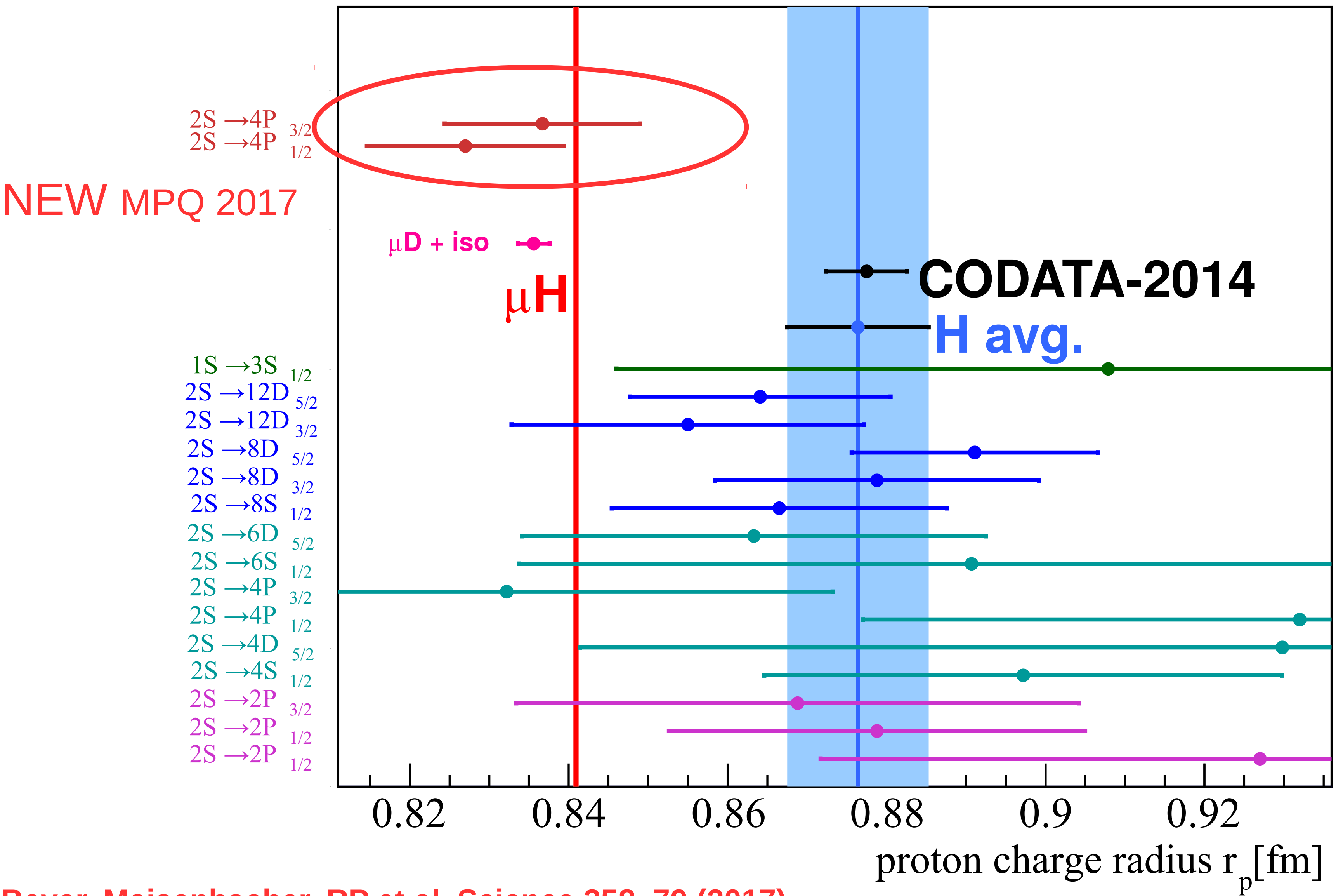

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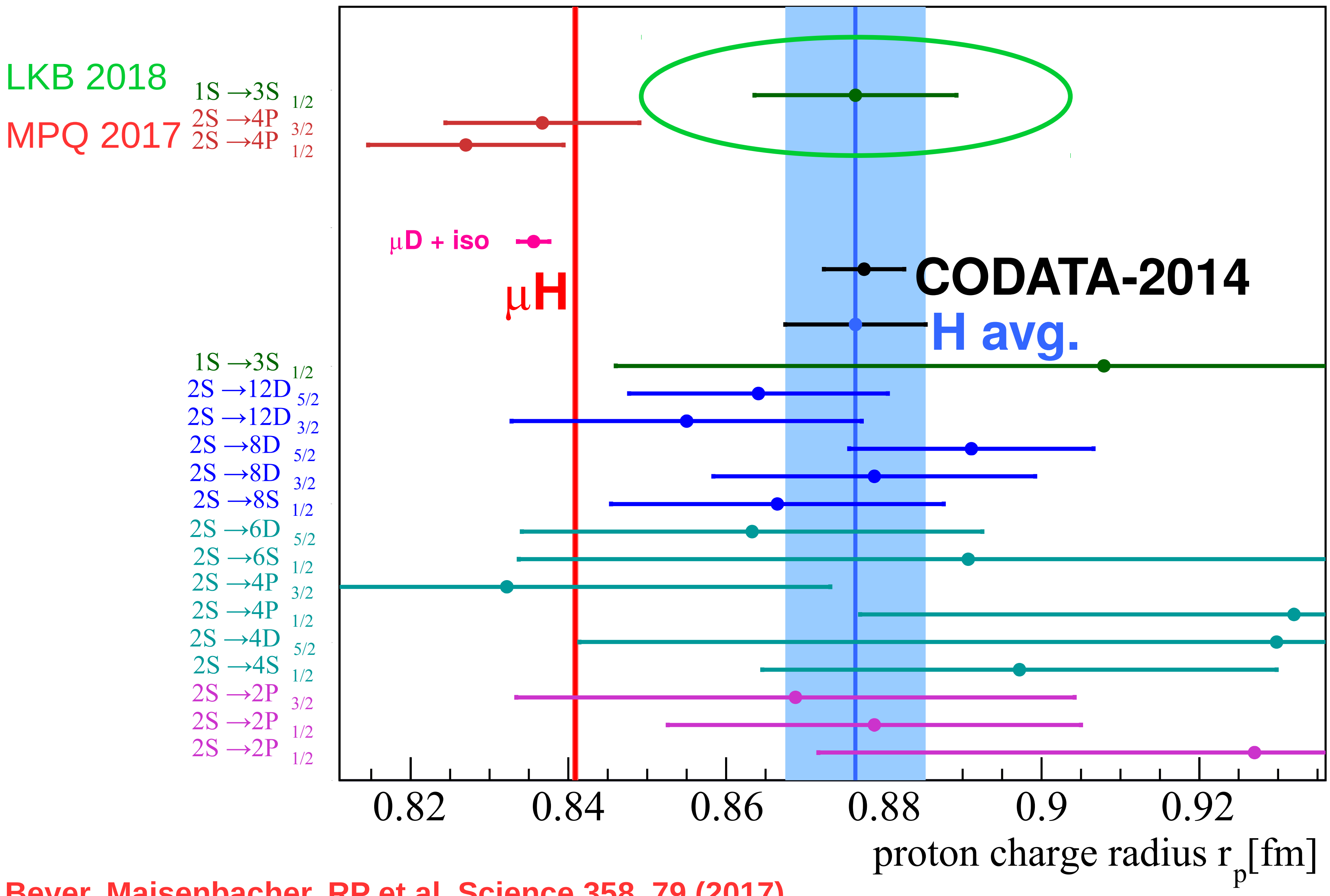

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Fleurbaey , PhD thesis (2017)

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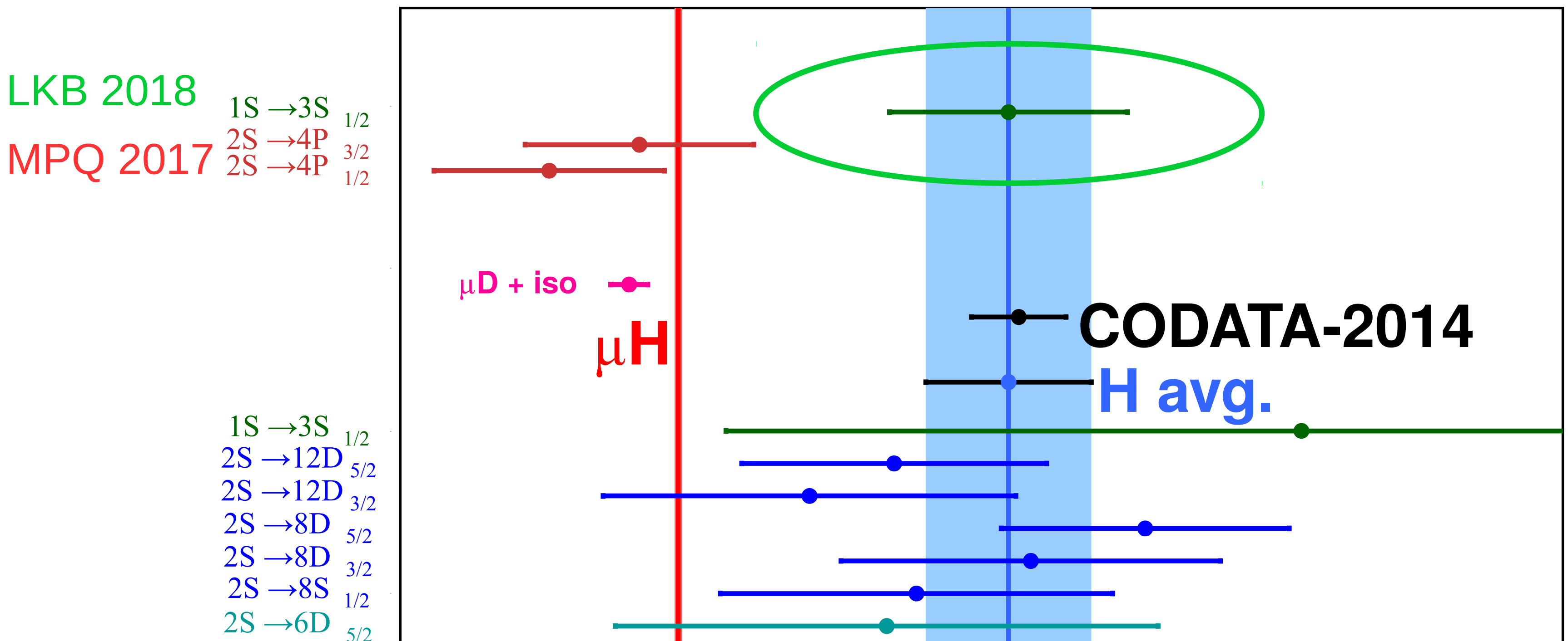


Proton Radius Puzzle is NOT “solved” !!

We have a “Rydberg problem” now → need more data!

proton charge radius r_p[fm]

Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)
Fleurbaey , PhD thesis (2017), paper submitted

Conclusions

- smaller radii from muonic **hydrogen** and **deuterium** imply a smaller Rydberg constant
- new H(2S-4P) gives small Rydberg constant in agreement with muonic values
- new H(2S-4P) gives thus a smaller proton radius, too
- new H(1S-3S) however confirms large proton radius

More data needed:

- H(2S – 6P, 8P, 9P, …) and D(2S-nl) underway in Garching and Colorado
- H(1S – 3S, 4S, ..) underway in Paris and Garching
- H(2S-2P) in Toronto (Hessels)
- Muonium
- Positronium (Cassidy, Crivelli)
- He^+(1S-2S) underway in Garching (Udem) and Amsterdam (Eikema)
- HD^+, H_2, etc.

- new low-Q^2 electron scattering at MAMI, JLab, MESA
- muon scattering: MUSE @ PSI, COMPASS @ CERN

Up next: Hyperfine structure in μp

The 21 cm line in hydrogen (1S hyperfine splitting) has been **measured** to 12 digits (0.001 Hz) in 1971:

$$\nu_{exp} = 1\,420\,405.\,751\,766\,7 \;\pm 0.000\,001\ \text{kHz}$$

Essen et al., Nature 229, 110 (1971)

QED test is limited to 6 digits (800 Hz) because of proton structure effects:

$$\nu_{theo} = 1\,420\,403.\,1 \;\pm 0.6_{\text{proton size}} \;\pm 0.4_{\text{polarizability}}\ \text{kHz}$$

Eides et al., Springer Tracts 222, 217 (2007)

Proton Zemach radius

HFS depends on “Zemach” radius:

$$\Delta E = -2(Z\alpha) m \langle r \rangle_{(2)} E_F$$

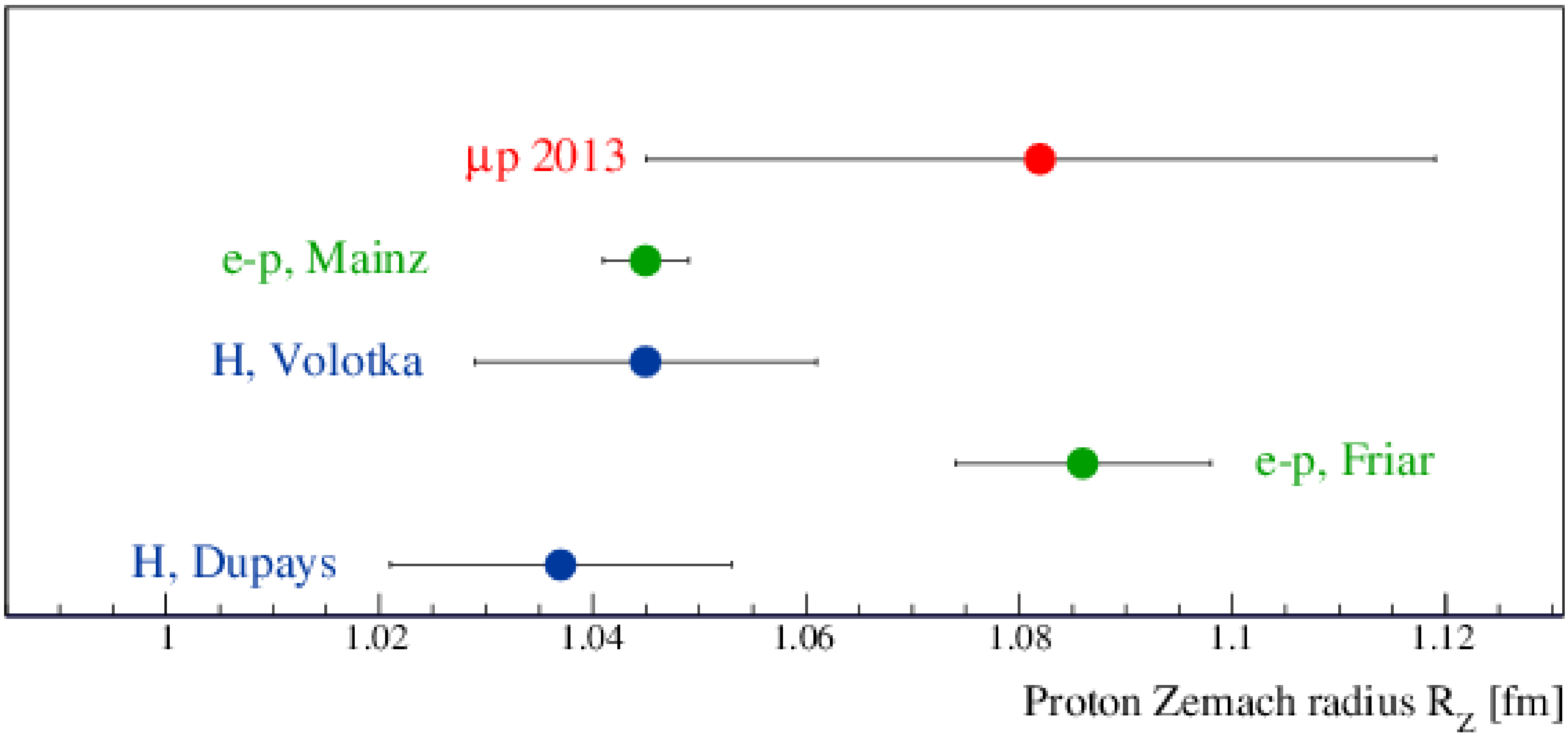
$$\langle r \rangle_{(2)} = \int d^3 r \, d^3 r' \, \rho_E(r) \rho_M(r') |r - r'|$$

Zemach, Phys. Rev. 104, 1771 (1956)

Form factors and momentum space

$$\Delta E = \frac{8(Z\alpha)m}{\pi n^3} E_F \int_0^\infty \frac{dk}{k^2} \left[\frac{G_E(-k^2) G_M(-k^2)}{1+\kappa} \right]$$

Proton Zemach radius from μp



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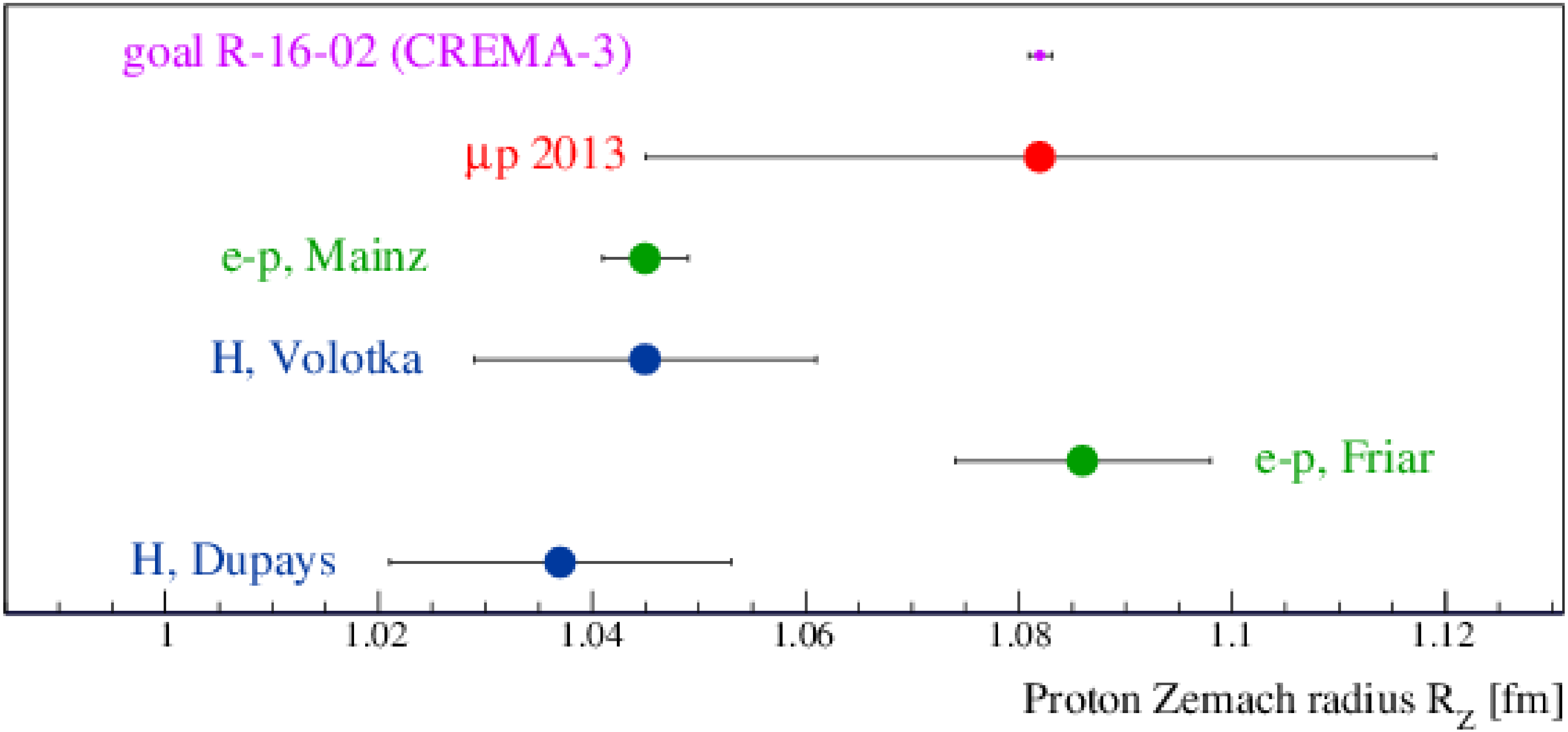
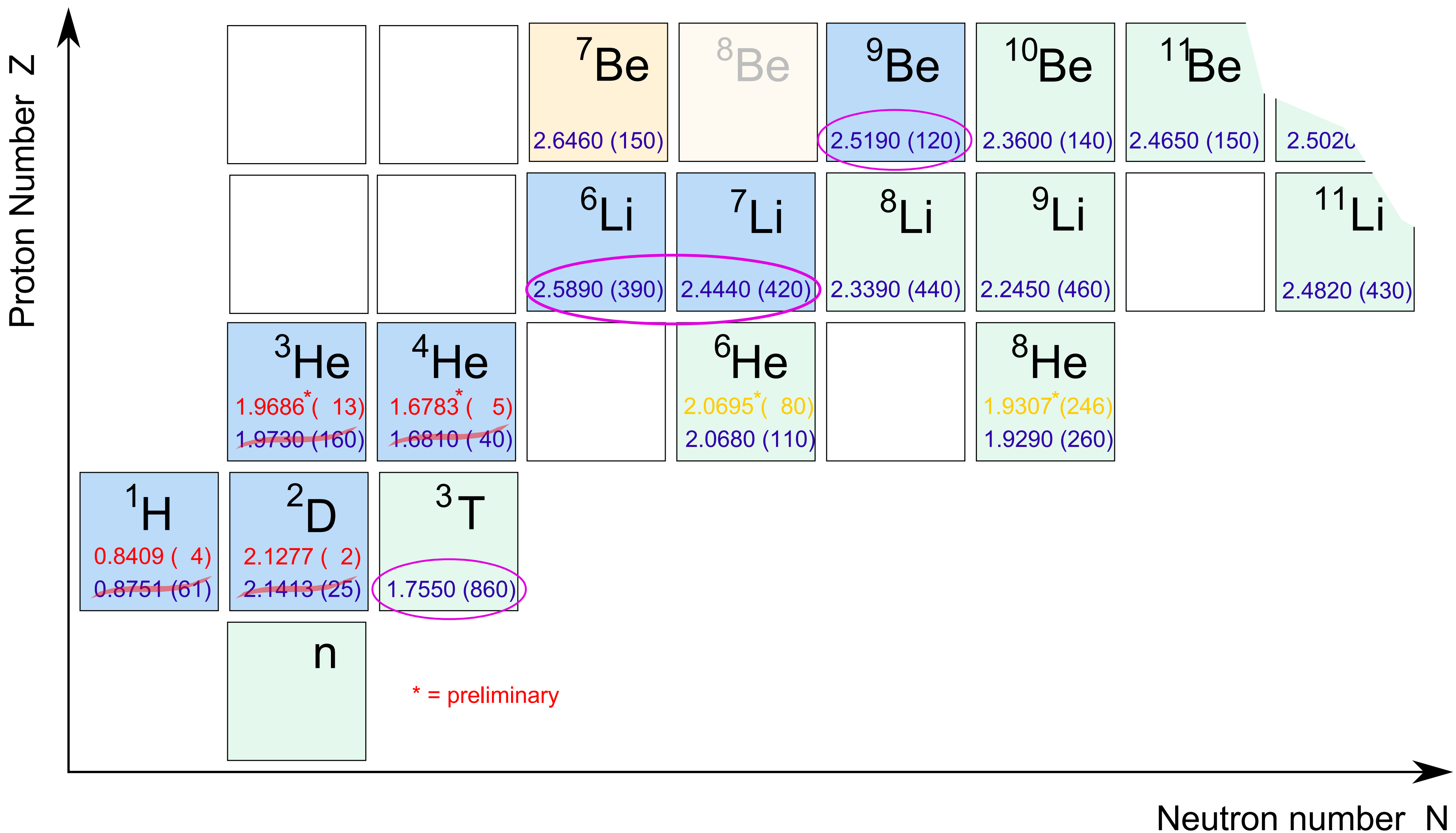


PSI Exp. R-16-02: Antognini, RP et al. (CREMA-3 / HyperMu)

Charge radii: The future



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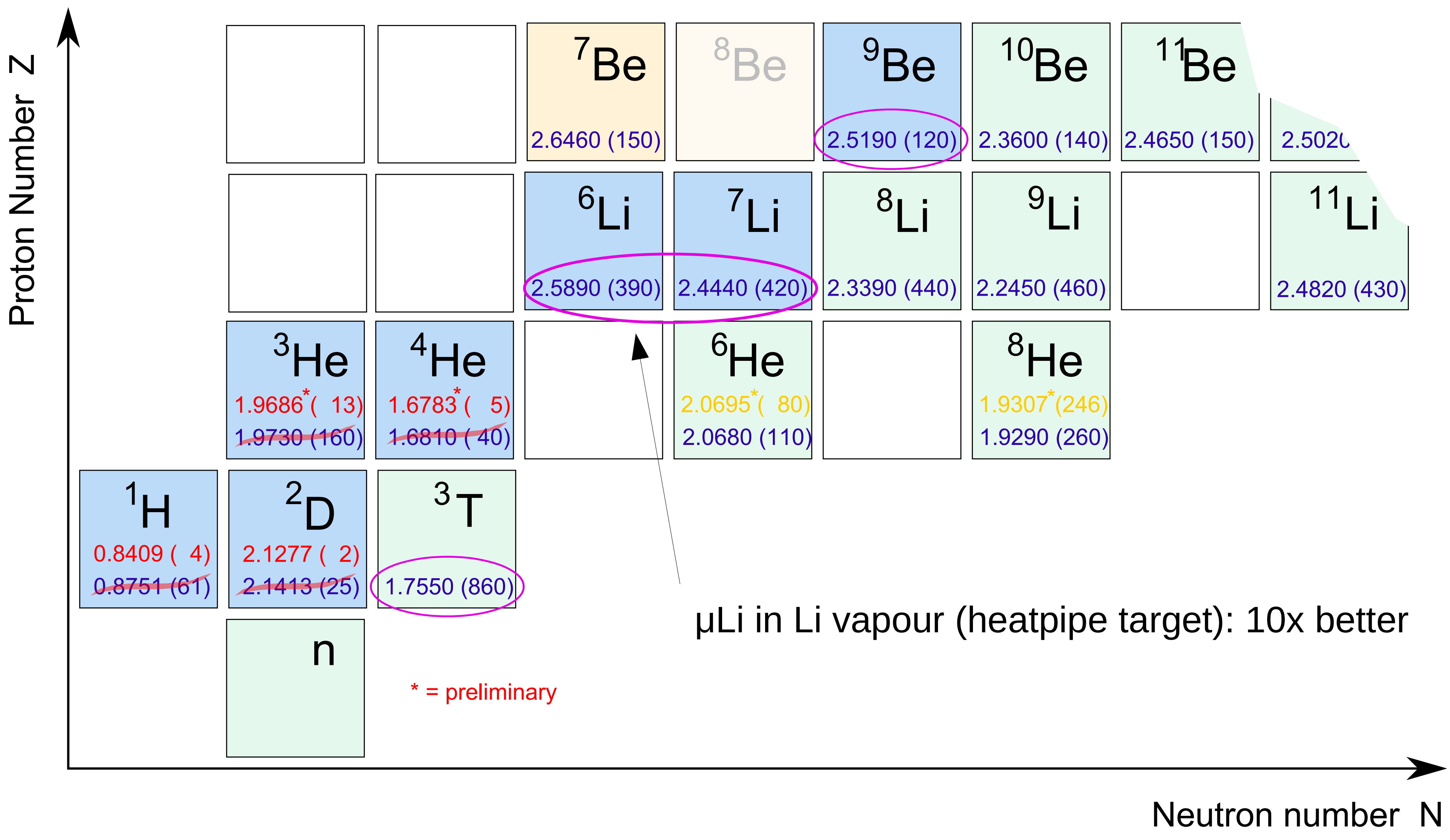

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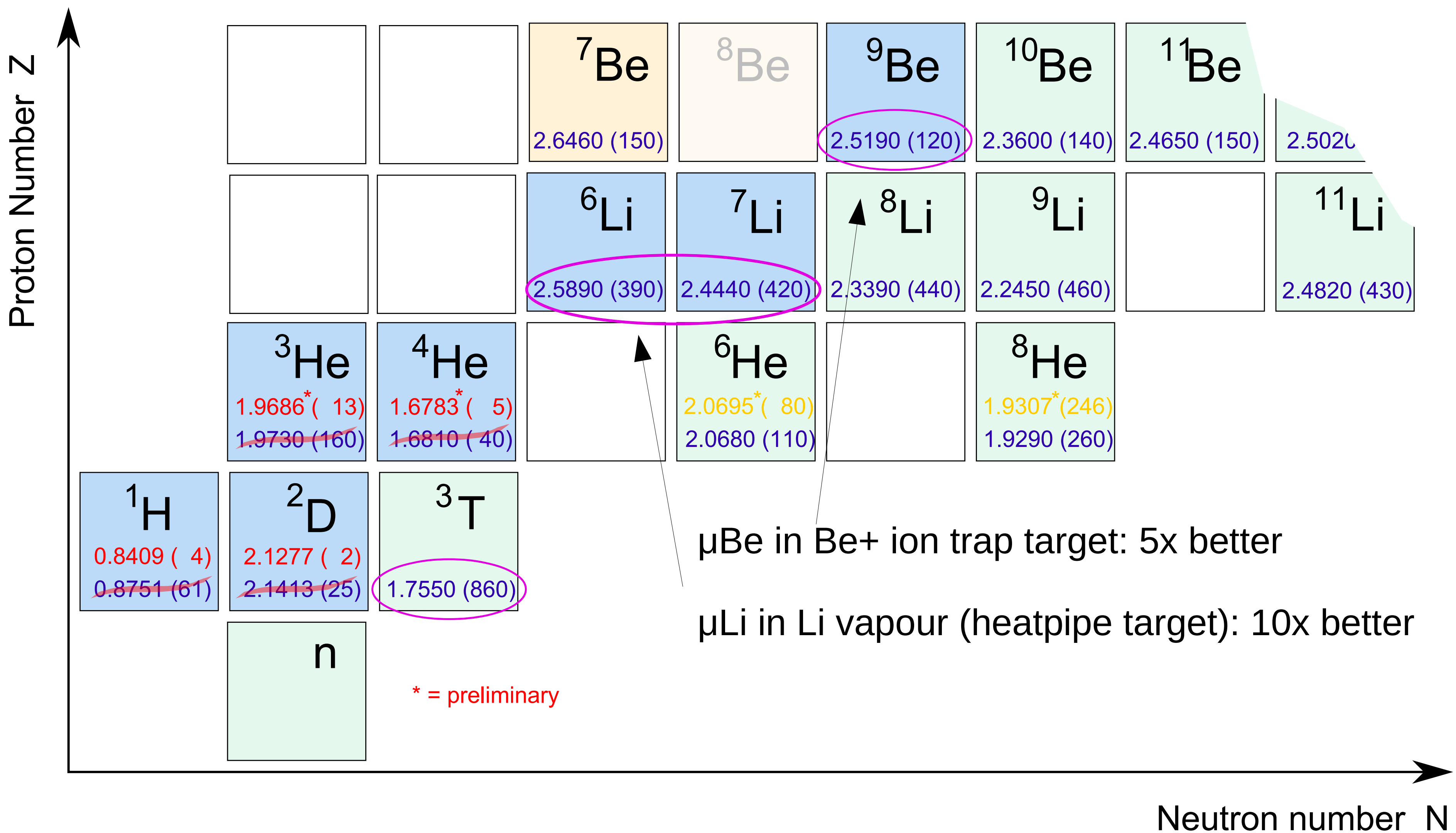

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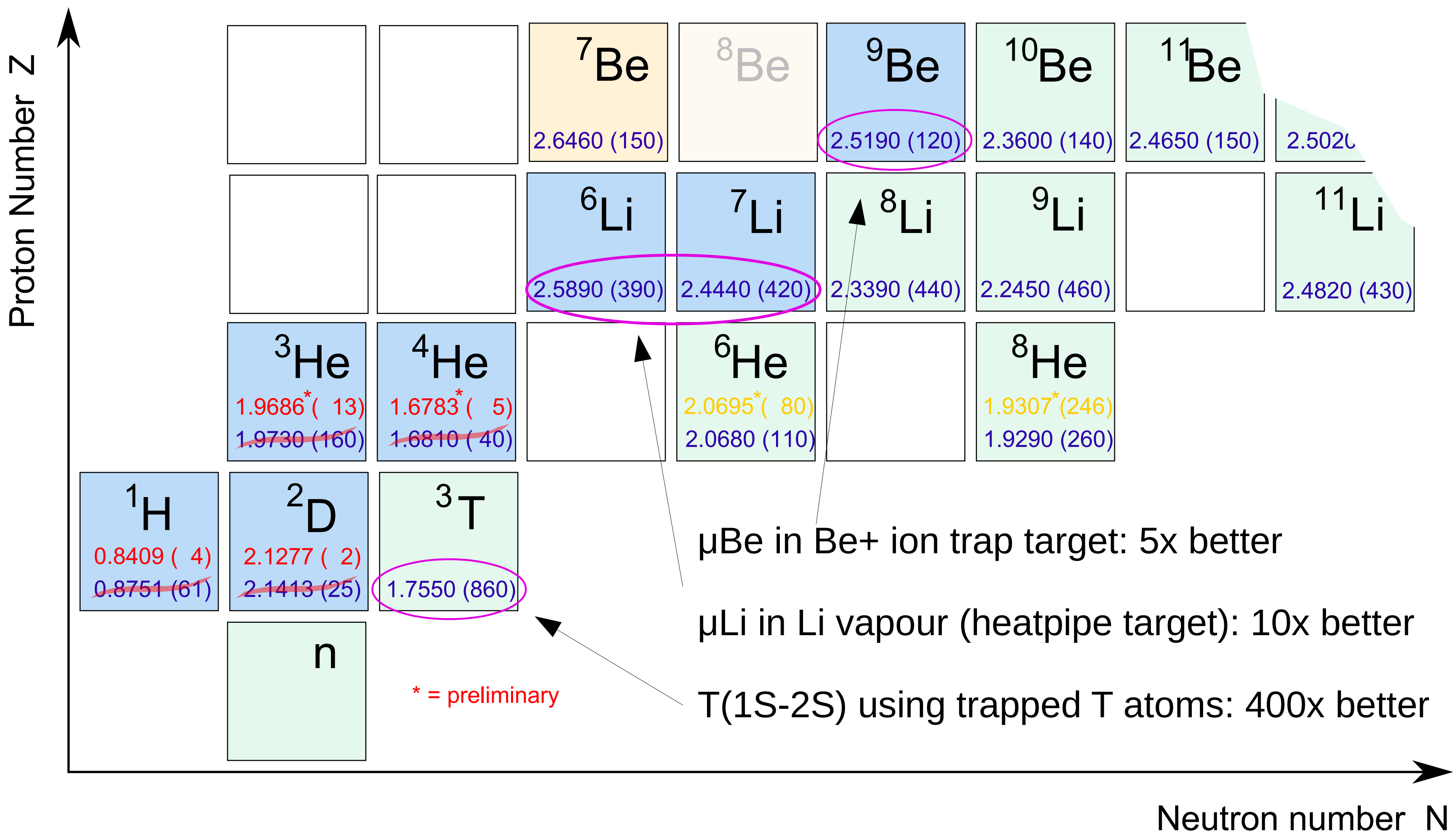

Thanks a lot for your attention

The Garching Hydrogen Team:

Axel Beyer, Lothar Maisenbacher, Arthur Matveev, RP,
Ksenia Khabarova, Alexey Grinin, Tobias Lamour, Dylan C. Yost,
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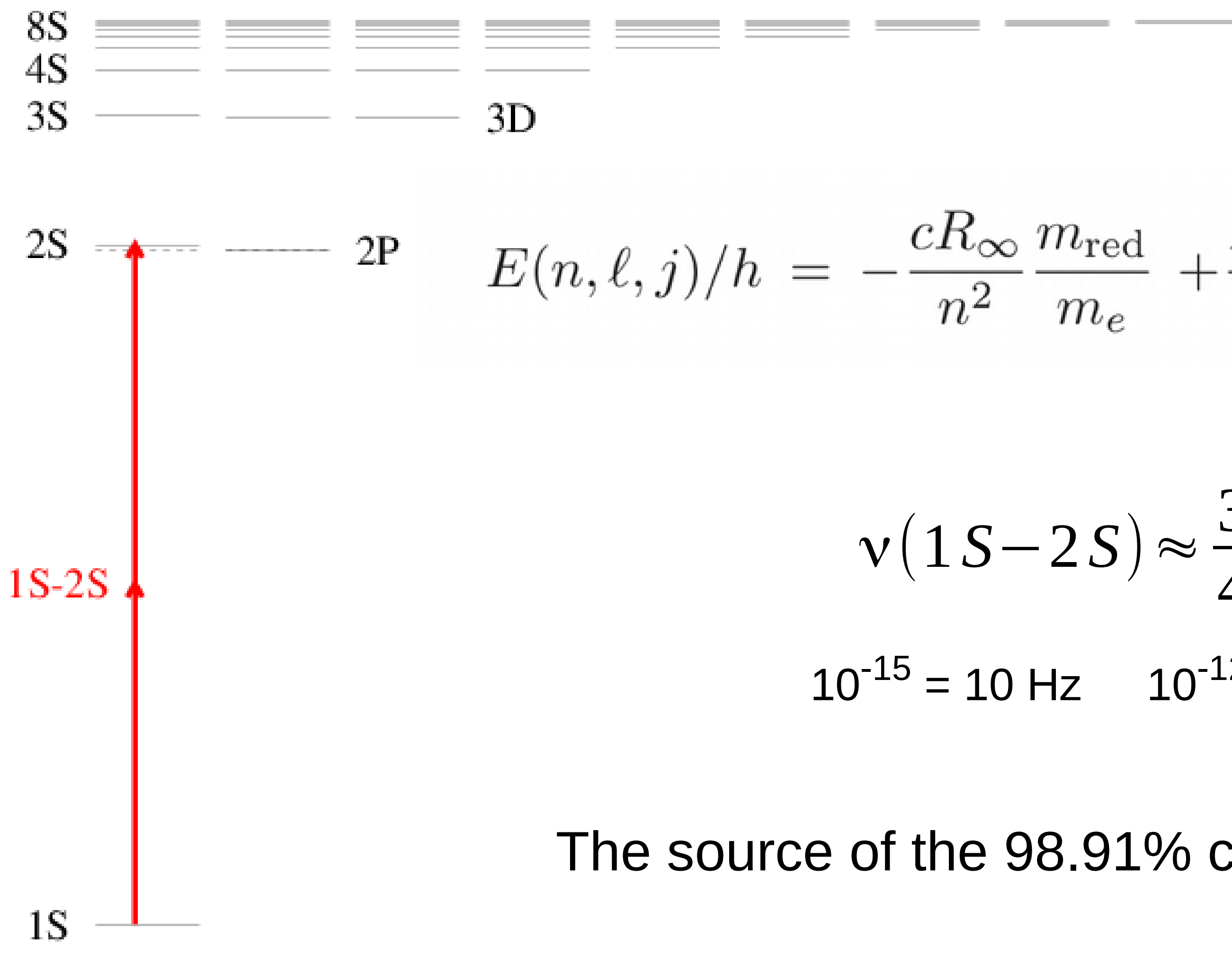
My new Mainz group:

Jan Haack, Julian J. Krauth, Stefan Schmidt, Marcel Willig, Rishi Horn

...

Correlation between R_∞ and R_p / R_d



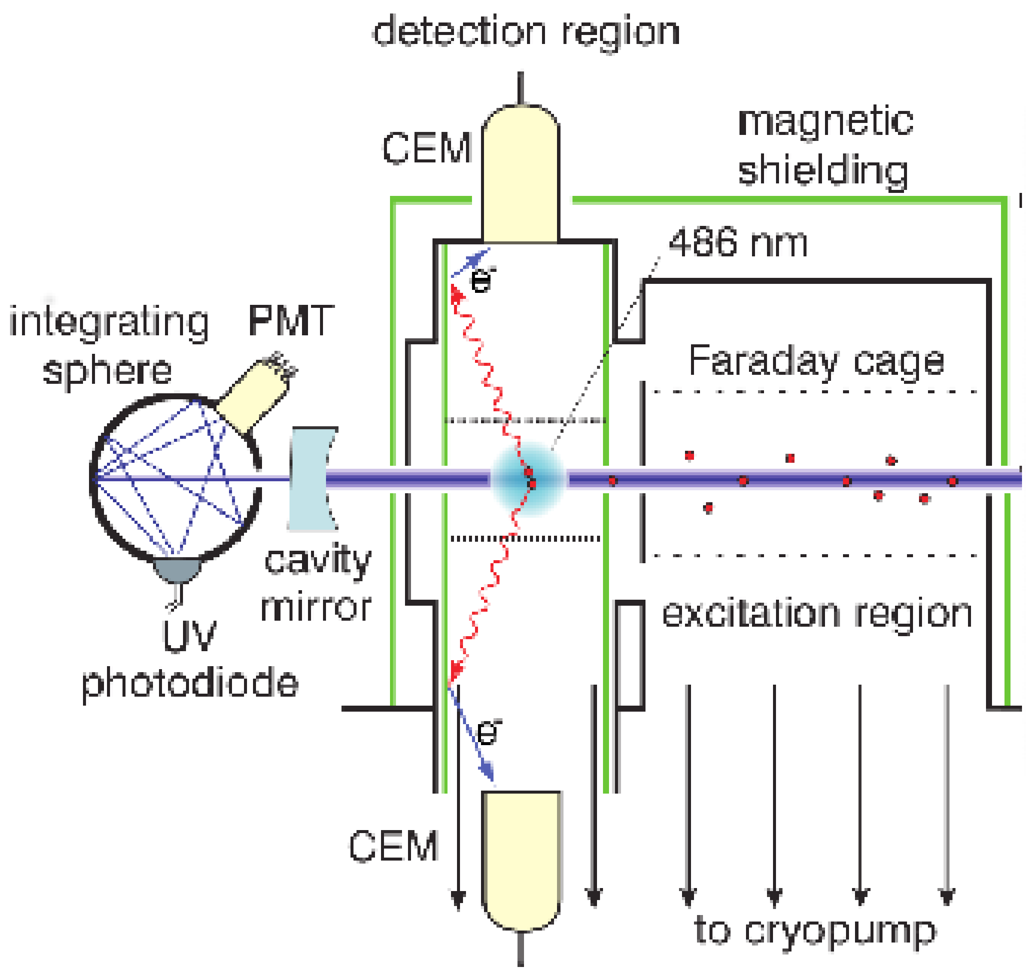
$$E(n,\ell,j)/h = -\frac{cR_\infty}{n^2}\frac{m_{\rm red}}{m_e} + \frac{E_{NS}}{n^3}\delta_{\ell 0} + \Delta(n,\ell,j). \quad (7)$$

$$\nu(1S-2S) \approx \frac{3}{4}R_\infty - \frac{7}{8}E_{NS}$$

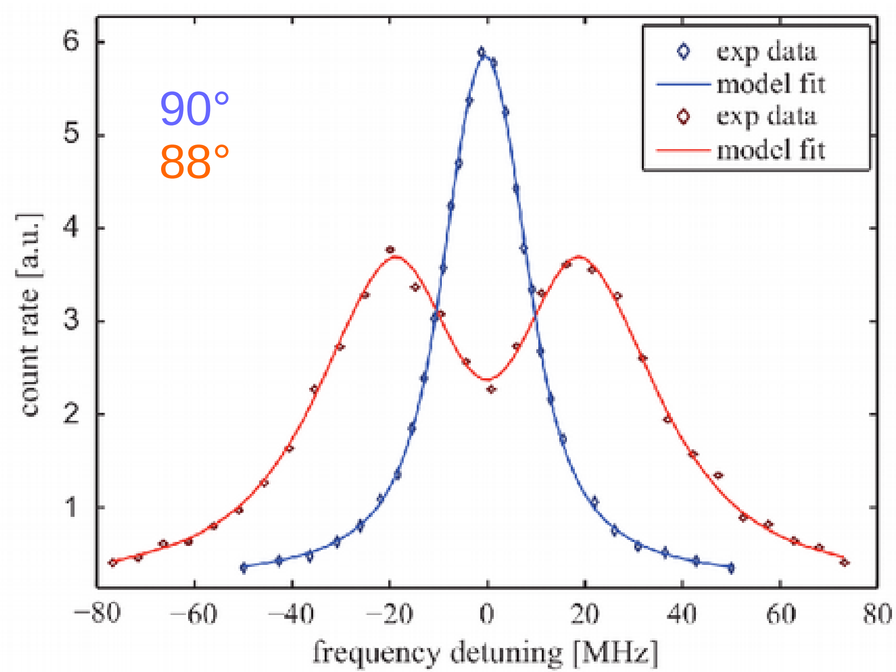
10^{-15} = 10 Hz $\quad$ 10^{-12} = 20 kHz

The source of the 98.91% correlation of R_∞ and R_p

Garching H(2S-4P)



1st order Doppler cancellation



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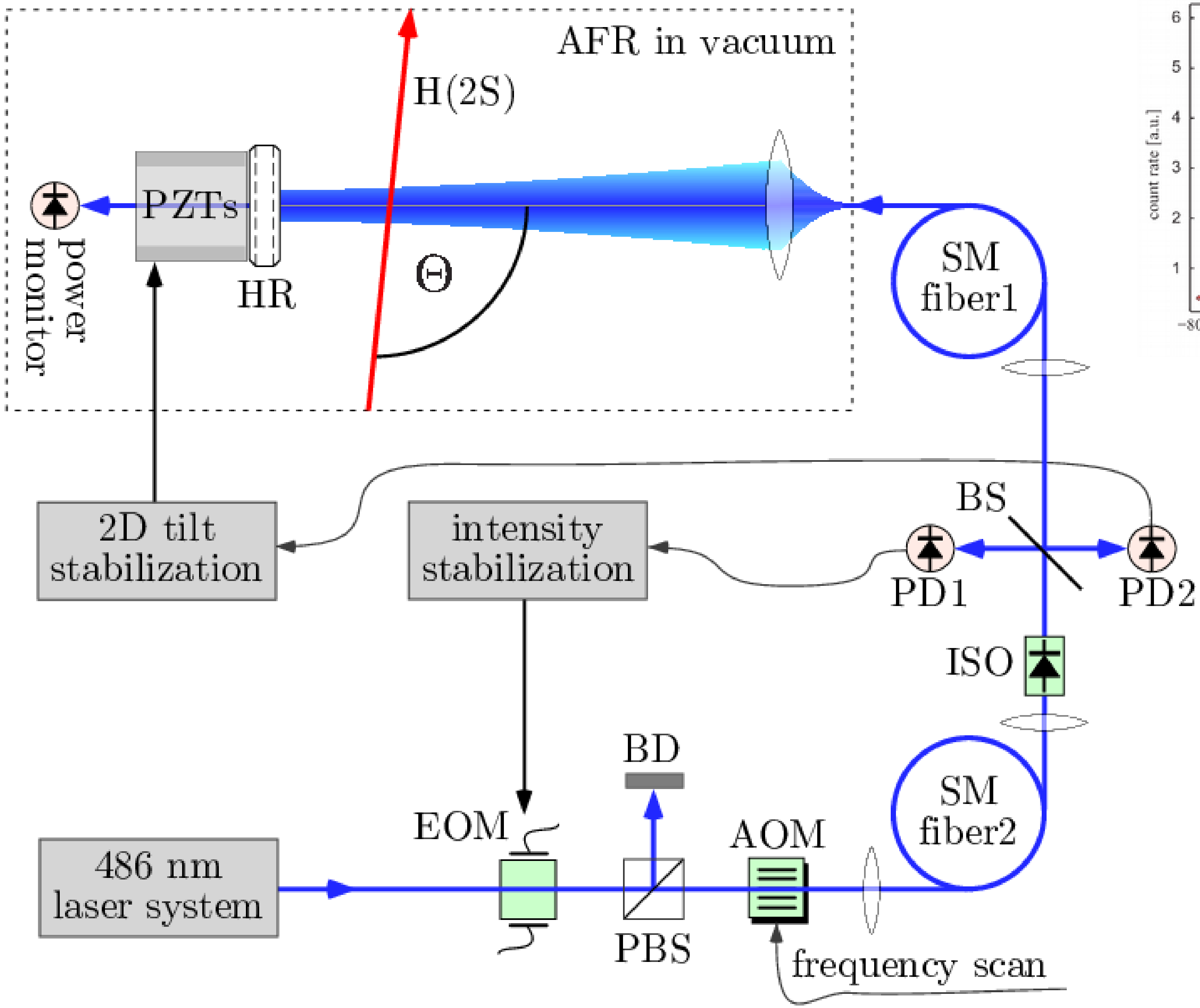
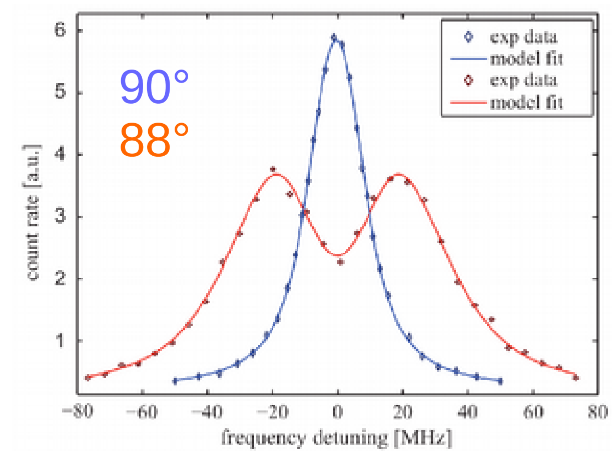
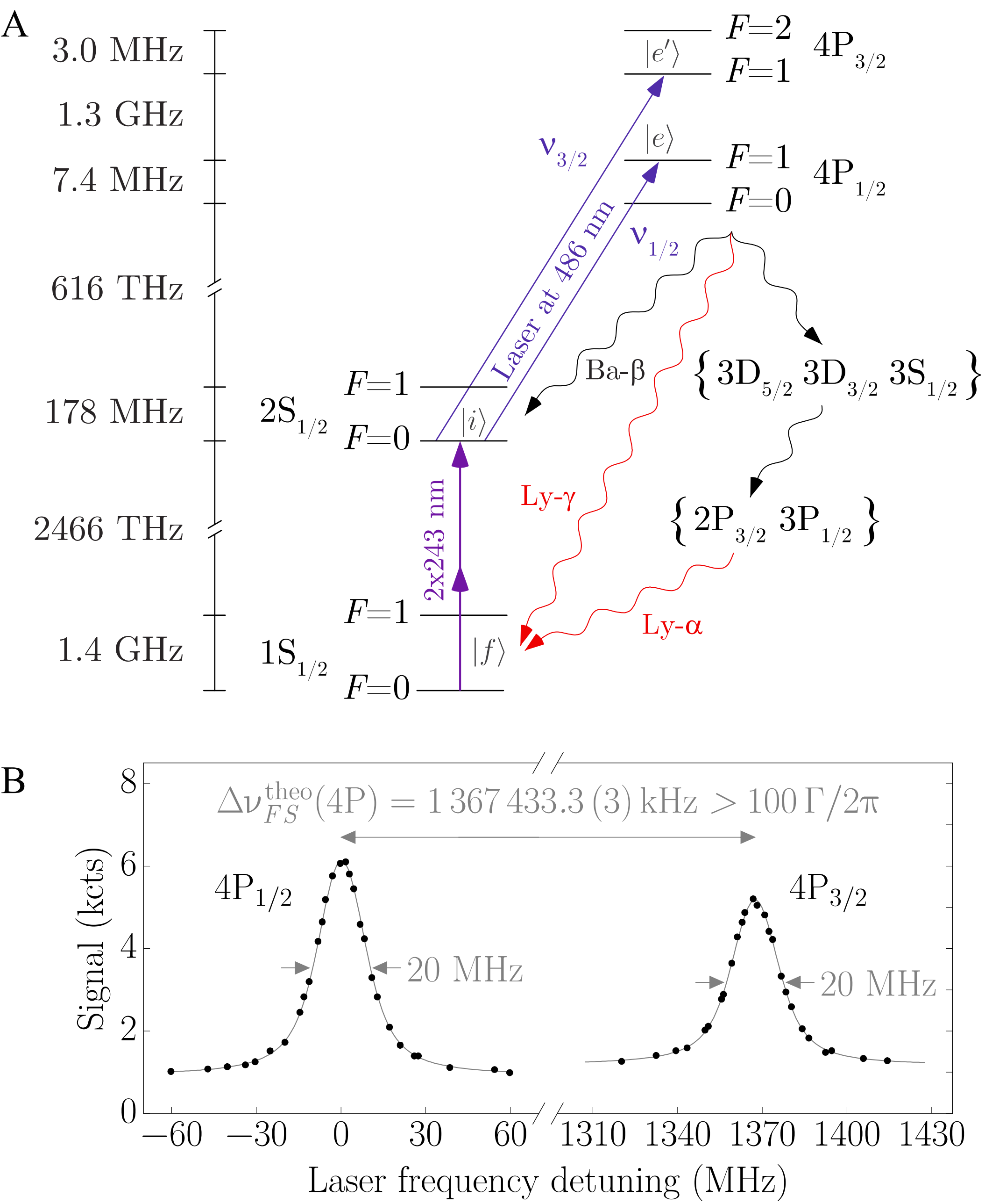
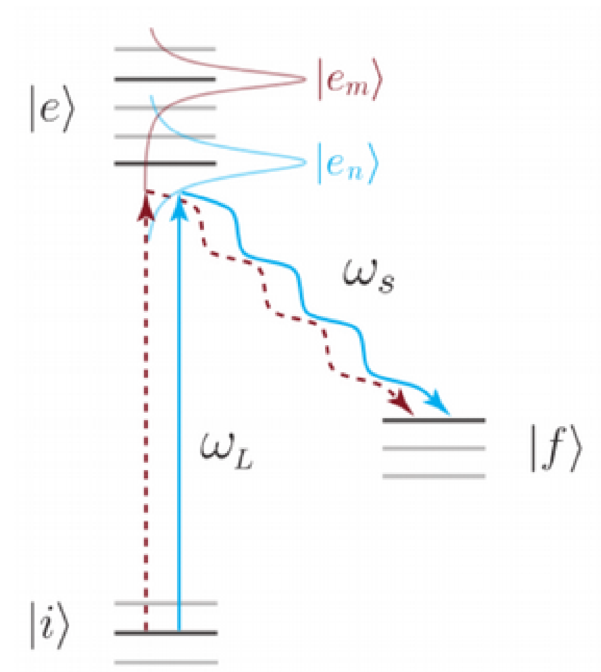




Quantum interference shifts





$$P(\omega)\propto\left|\frac{(\vec{d}_1\vec{E}_0)\vec{d}_1}{\omega_1-\omega_L+i\gamma_1/2}+\frac{(\vec{d}_2\vec{E}_0)\vec{d}_2 e^{i\Delta\Phi}}{\omega_2-\omega_L+i\gamma_2/2}\right|^2$$

= Lorentzian(1) + Lorentzian(2)

+ cross-term (QI)

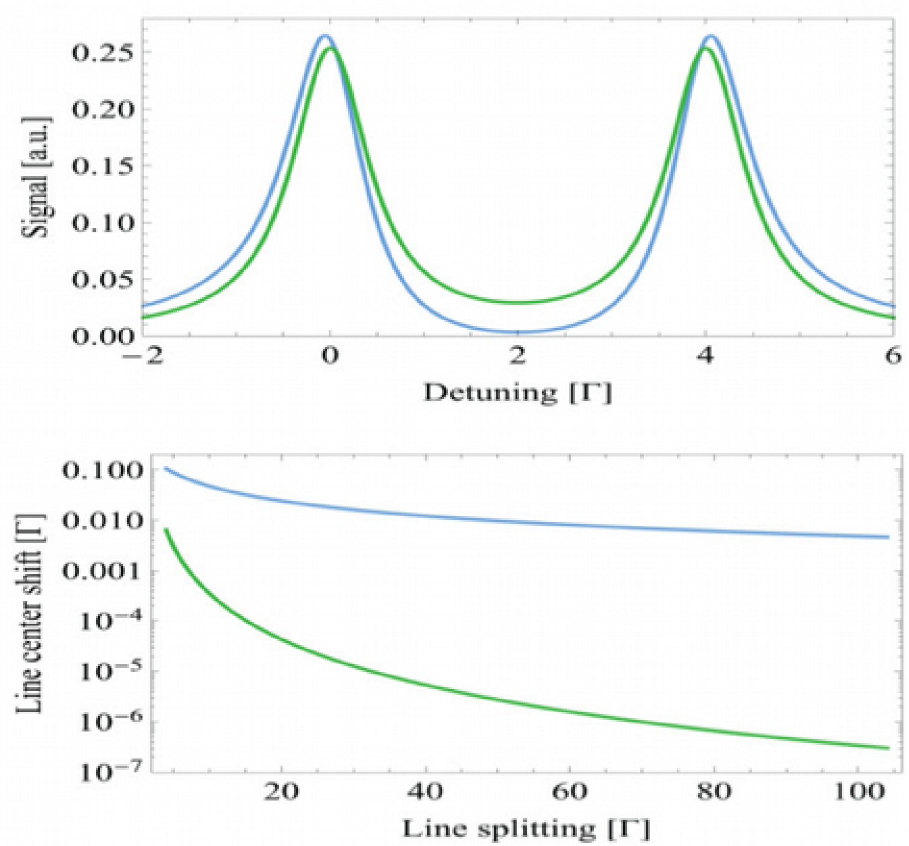
see

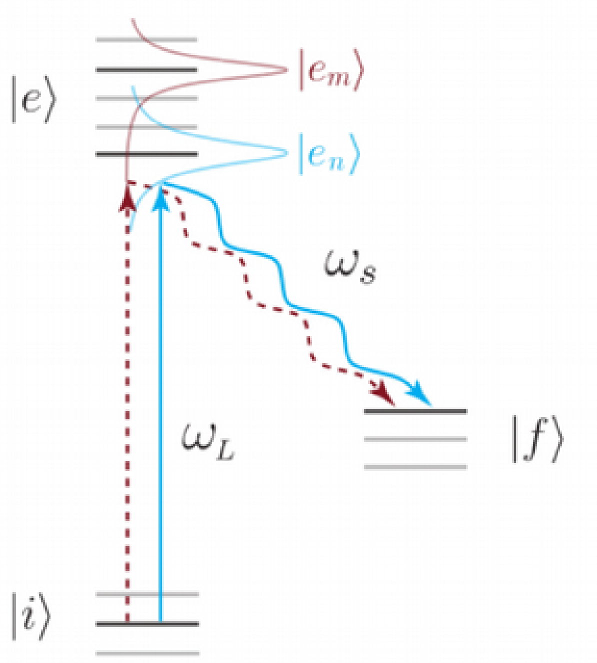
Horbatsch, Hessels, PRA 82, 052519 (2010); PRA 84, 032508 (2011); PRA 86 040501 (2012)

Sansonetti et al., PRL 107, 021001 (2011)

Brown et al., PRA 87, 032504 (2013)

Quantum interference shifts





$$P(\omega) \propto \left| \frac{(\vec{d}_1 \vec{E}_0)\vec{d}_1}{\omega_1 - \omega_L + i\gamma_1/2} + \frac{(\vec{d}_2 \vec{E}_0)\vec{d}_2 e^{i\Delta\Phi}}{\omega_2 - \omega_L + i\gamma_2/2} \right|^2$$

= Lorentzian(1) + Lorentzian(2)

+ cross-term (QI)

Fitting this with 2 Lorentzians creates

line shifts

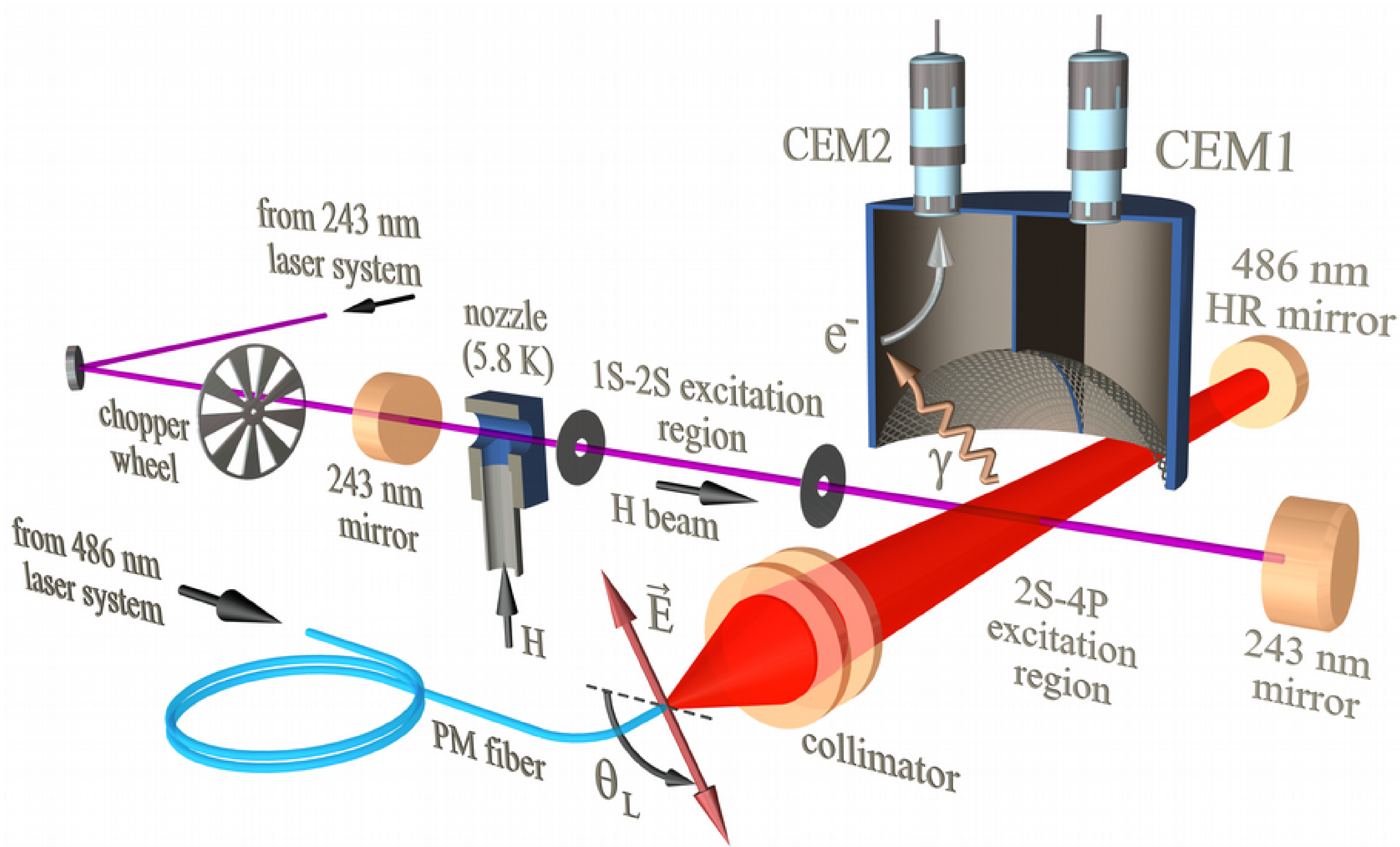
see

Horbatsch, Hessels, PRA 82, 052519 (2010); PRA 84, 032508 (2011); PRA 86 040501 (2012)

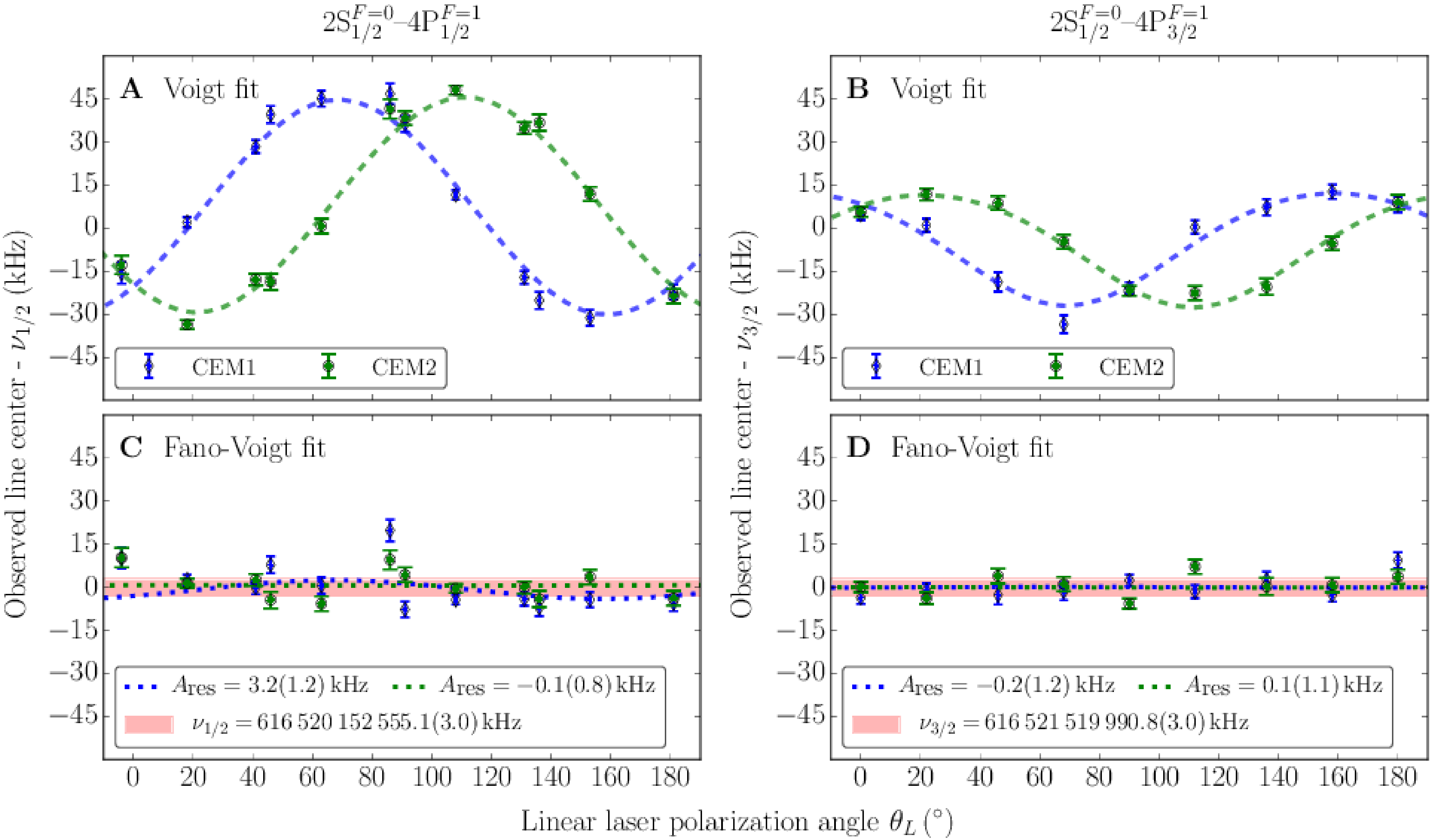
Sansonetti et al., PRL 107, 021001 (2011)

Brown et al., PRA 87, 032504 (2013)

Studying QI in 2S-4P



QI in hydrogen   (Δ = 100 Γ)



Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

Systematics

	$\Delta\nu$ (kHz)	σ (kHz)
Statistics	0.0	0.40
First-order Doppler shift	0.0	2.13
Quantum interference shift	0.0	0.20
Light force shift	-0.32	0.30
Model corrections	0.11	0.06
Sampling bias	0.44	0.49
Second-order Doppler shift	0.22	0.05
DC Stark shift	0.0	0.20
Zeeman shift	0.0	0.22
Pressure shift	0.0	0.008
Laser spectrum	0.0	0.1
Laser frequency determination	0.0	0.1
Frequency standard (H maser)	0.0	0.06
Recoil shift	-837.23	0.00
Hyperfine structure (HFS) corrections	-132552.092	0.075
Total	-133388.9	2.3

CODATA “sub-adjustments”

Adj. 3: “The Adjustment” (all data) Rp = 0.8775(51) fm, Rd = 2.1424(21) fm
Adj. 8: H spectroscopy only Rp = 0.8764(89) fm
Adj. 10: D spectroscopy only Rd = 2.1210(250) fm

TABLE XXXVIII. Summary of the results of some of the least-squares adjustments used to analyze the input data related to R_∞. The values of R_∞, r_p, and r_d are those obtained in the indicated adjustment, N is the number of input data, M is the number of adjusted constants, $\nu = N - M$ is the degrees of freedom, and $R_B = \sqrt{\chi^2/\nu}$ is the Birge ratio. See the text for an explanation and discussion of each adjustment. In brief, adjustment 6 is 3 but the scattering data for the nuclear radii are omitted; 7 is 3, but with only the hydrogen data included (no isotope shift); 8 is 7 with the r_p data deleted; 9 and 10 are similar to 7 and 8, but for the deuterium data; 11 is 3 with the muonic Lamb-shift value of r_p included; and 12 is 11, but without the scattering values of r_p and r_d.

Adj.	N	M	ν	χ^2	R_B	R_∞ (m^{-1})	$u_r(R_\infty)$	r_p (fm)	r_d (fm)
3	149	82	67	58.1	0.93	10 973 731.568 539(55)	5.0×10^{-12}	0.8775(51)	2.1424(21)
6	146	82	64	55.5	0.93	10 973 731.568 521(82)	7.4×10^{-12}	0.8758(77)	2.1417(31)
7	131	72	59	53.4	0.95	10 973 731.568 561(60)	5.5×10^{-12}	0.8796(56)	
8	129	72	57	52.5	0.96	10 973 731.568 528(94)	8.6×10^{-12}	0.8764(89)	
9	114	65	49	46.9	0.98	10 973 731.568 37(13)	1.1×10^{-11}		2.1288(93)
10	113	65	48	46.8	0.99	10 973 731.568 28(30)	2.7×10^{-11}		2.121(25)
11	150	82	68	104.9	1.24	10 973 731.568 175(12)	1.1×10^{-12}	0.842 25(65)	2.128 24(28)
12	147	82	65	74.3	1.07	10 973 731.568 171(12)	1.1×10^{-12}	0.841 93(66)	2.128 11(28)

Spectroscopy data in CODATA

TABLE XI. Summary of measured transition frequencies ν considered in the present work for the determination of the Rydberg constant R_∞ (H is hydrogen and D is deuterium).

Authors	Laboratory[a]	Frequency interval(s)	Reported value ν (kHz)	Rel. stand. uncert. u_r
(Fischer *et al.*, 2004)	MPQ	$\nu_H(1S_{1/2}-2S_{1/2})$	2 466 061 413 187.080(34)	1.4×10^{-14}
(Weitz *et al.*, 1995)	MPQ	$\nu_H(2S_{1/2}-4S_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	4 797 338(10)	2.1×10^{-6}
		$\nu_H(2S_{1/2}-4D_{5/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	6 490 144(24)	3.7×10^{-6}
		$\nu_D(2S_{1/2}-4S_{1/2})-\frac{1}{4}\nu_D(1S_{1/2}-2S_{1/2})$	4 801 693(20)	4.2×10^{-6}
		$\nu_D(2S_{1/2}-4D_{5/2})-\frac{1}{4}\nu_D(1S_{1/2}-2S_{1/2})$	6 494 841(41)	6.3×10^{-6}
(Parthey *et al.*, 2010)	MPQ	$\nu_D(1S_{1/2}-2S_{1/2})-\nu_H(1S_{1/2}-2S_{1/2})$	670 994 334.606(15)	2.2×10^{-11}
(de Beauvoir *et al.*, 1997)	LKB/SYRTE	$\nu_H(2S_{1/2}-8S_{1/2})$	770 649 350 012.0(8.6)	1.1×10^{-11}
		$\nu_H(2S_{1/2}-8D_{3/2})$	770 649 504 450.0(8.3)	1.1×10^{-11}
		$\nu_H(2S_{1/2}-8D_{5/2})$	770 649 561 584.2(6.4)	8.3×10^{-12}
		$\nu_D(2S_{1/2}-8S_{1/2})$	770 859 041 245.7(6.9)	8.9×10^{-12}
		$\nu_D(2S_{1/2}-8D_{3/2})$	770 859 195 701.8(6.3)	8.2×10^{-12}
		$\nu_D(2S_{1/2}-8D_{5/2})$	770 859 252 849.5(5.9)	7.7×10^{-12}
(Schwob *et al.*, 1999)	LKB/SYRTE	$\nu_H(2S_{1/2}-12D_{3/2})$	799 191 710 472.7(9.4)	1.2×10^{-11}
		$\nu_H(2S_{1/2}-12D_{5/2})$	799 191 727 403.7(7.0)	8.7×10^{-12}
		$\nu_D(2S_{1/2}-12D_{3/2})$	799 409 168 038.0(8.6)	1.1×10^{-11}
		$\nu_D(2S_{1/2}-12D_{5/2})$	799 409 184 966.8(6.8)	8.5×10^{-12}
(Arnoult *et al.*, 2010)	LKB	$\nu_H(1S_{1/2}-3S_{1/2})$	2 922 743 278 678(13)	4.4×10^{-12}
(Bourzeix *et al.*, 1996)	LKB	$\nu_H(2S_{1/2}-6S_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-3S_{1/2})$	4 197 604(21)	4.9×10^{-6}
		$\nu_H(2S_{1/2}-6D_{5/2})-\frac{1}{4}\nu_H(1S_{1/2}-3S_{1/2})$	4 699 099(10)	2.2×10^{-6}
(Berkeland, Hinds, and Boshier, 1995)	Yale	$\nu_H(2S_{1/2}-4P_{1/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	4 664 269(15)	3.2×10^{-6}
		$\nu_H(2S_{1/2}-4P_{3/2})-\frac{1}{4}\nu_H(1S_{1/2}-2S_{1/2})$	6 035 373(10)	1.7×10^{-6}
(Hagley and Pipkin, 1994)	Harvard	$\nu_H(2S_{1/2}-2P_{3/2})$	9 911 200(12)	1.2×10^{-6}
(Lundeen and Pipkin, 1986)	Harvard	$\nu_H(2P_{1/2}-2S_{1/2})$	1 057 845.0(9.0)	8.5×10^{-6}
(Newton, Andrews, and Unsworth, 1979)	U. Sussex	$\nu_H(2P_{1/2}-2S_{1/2})$	1 057 862(20)	1.9×10^{-5}

[a] MPQ: Max-Planck-Institut für Quantenoptik, Garching. LKB: Laboratoire Kastler-Brossel, Paris. SYRTE: Systèmes de référence Temps Espace, Paris, formerly Laboratoire Primaire du Temps et des Fréquences (LPTF).

H(1S-2S)

D(1S-2S) –
H(1S-2S)

(iso shift)

Spectroscopy data: H

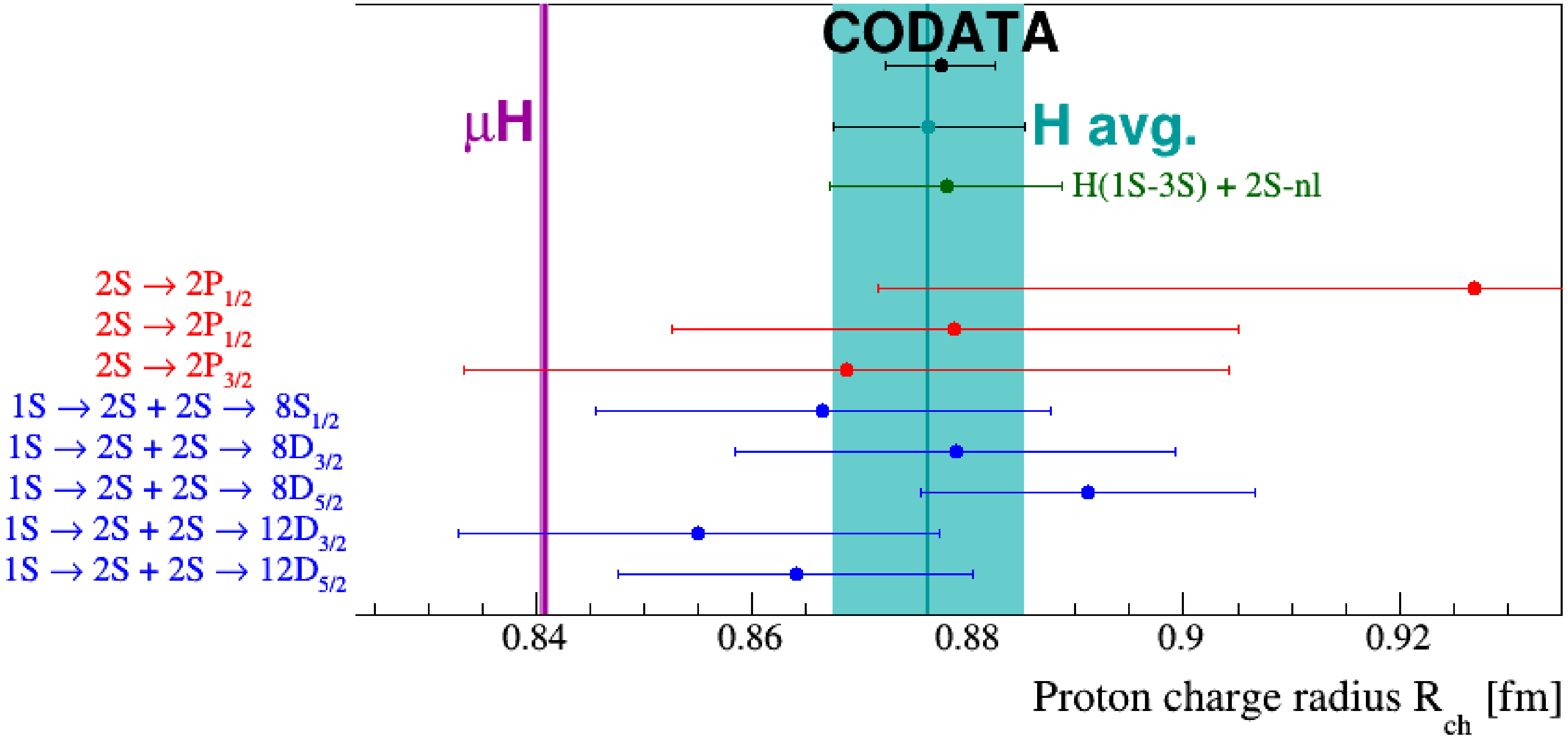
TABLE III: Some recent measurements in atomic hydrogen. An asterisk following the reference denotes items considered in the most recent CODATA-2010 report. Following our nomenclature, the $2S \to 2P_{1/2}$ transition must be assigned a negative frequency, because the final state $(n', \ell', j') = 2P_{1/2}$ is *lower* than the initial $(n, \ell, j) = 2S_{1/2}$ state.

#	$(n, \ell, j) - (n', \ell', j')$	ν_{meas} (kHz)	rel. unc.	Source	Ref.
H1	$2S_{1/2} \to 2P_{1/2}$	-1 057 862(20)	1.9×10^{-5}	Sussex 1979	[25] *
H2		-1 057 845.0(9.0)	8.5×10^{-6}	Harvard 1986	[26] *
H3	$2S_{1/2} \to 2P_{3/2}$	9 911 200(12)	1.2×10^{-6}	Harvard 1994	[27] *
H4	$2S_{1/2} \to 8S_{1/2}$	770 649 350 012.0(8.6)	1.1×10^{-11}	LKB 1997	[28] *
H5	$2S_{1/2} \to 8D_{3/2}$	770 649 504 450.0(8.3)	1.1×10^{-11}	LKB 1997	[28] *
H6	$2S_{1/2} \to 8D_{5/2}$	770 649 561 584.2(6.4)	8.3×10^{-12}	LKB 1997	[28] *
H7	$2S_{1/2} \to 12D_{3/2}$	799 191 710 472.7(9.4)	1.1×10^{-11}	LKB 1999	[29] *
H8	$2S_{1/2} \to 12D_{5/2}$	799 191 727 403.7(7.0)	8.7×10^{-12}	LKB 1999	[29] *
H9	$1S_{1/2} \to 2S_{1/2}$	2 466 061 413 187.103(46)	1.9×10^{-14}	MPQ 2000	[30]
H10		2 466 061 413 187.080(34)	1.4×10^{-14}	MPQ 2004	[31] *
H11		2 466 061 413 187.035(10)	4.2×10^{-15}	MPQ 2011	[32]
H12		2 466 061 413 187.018(11)	4.5×10^{-15}	MPQ 2013	[33]
H13	$1S_{1/2} \to 3S_{1/2}$	2 922 743 278 678(13)	4.4×10^{-12}	LKB 2010	[34] *
H14		2 922 743 278 659(17)	5.8×10^{-12}	MPQ 2016	[35]

Rp from H spectroscopy

#	Transition	r_{p} [fm]	
H1	$2S \to 2P_{1/2}$	0.9270 ± 0.0553	
H2	$2S \to 2P_{1/2}$	0.8788 ± 0.0262	
H3	$2S \to 2P_{3/2}$	0.8688 ± 0.0354	
H10 + H4	$1S \to 2S + 2S \to 8S_{1/2}$	0.8666 ± 0.0211	
H10 + H5	$1S \to 2S + 2S \to 8D_{3/2}$	0.8789 ± 0.0204	
H10 + H6	$1S \to 2S + 2S \to 8D_{5/2}$	0.8911 ± 0.0155	
H10 + H7	$1S \to 2S + 2S \to 12D_{3/2}$	0.8551 ± 0.0222	
H10 + H8	$1S \to 2S + 2S \to 12D_{5/2}$	0.8641 ± 0.0164	
$1S \to 2S$ (H10) +	all H($2S \to n\ell$)	0.8747 ± 0.0091	avg.
$1S \to 3S$ (H13+H14) + all H($2S \to n\ell$)		0.8780 ± 0.0108	
CODATA Adj. 8		0.8764 ± 0.0089	Eq. (18)

Rp from H spectroscopy



Spectroscopy data: D

TABLE V: Some recent measurements of the H-D isotope shift. An asterisk following the reference denotes items considered in the most recent CODATA-2010 report.

#	Transition	Frequency (kHz)	rel. unc.	Source	Ref.
I1	$1S_{1/2} \to 2S_{1/2}$	670 994 334.64(15)	2.2×10^{-10}	MPQ 1998	[7]
I2		670 994 334.606(15)	2.2×10^{-11}	MPQ 2010	[8] *

TABLE VI: Some recent measurements in atomic deuterium. An asterisk following the reference denotes items considered in the most recent CODATA-2010 report. Items D9 and D10 are direct measurements, while D11 and D12 have been constructed as justified in the text.

#	$(n, \ell, j) - (n', \ell', j')$	$\nu_{\rm meas}$ (kHz)	rel. unc.	Source	Ref.
D4	$2S_{1/2} \to 8S_{1/2}$	770 859 041 245.7(6.9)	8.9×10^{-12}	LKB 1997	[28] *
D5	$2S_{1/2} \to 8D_{3/2}$	770 859 195 701.8(6.3)	8.2×10^{-12}	LKB 1997	[28] *
D6	$2S_{1/2} \to 8D_{5/2}$	770 859 252 849.5(5.9)	7.7×10^{-12}	LKB 1997	[28] *
D7	$2S_{1/2} \to 12D_{3/2}$	799 409 168 038.0(8.6)	1.1×10^{-11}	LKB 1999	[29] *
D8	$2S_{1/2} \to 12D_{5/2}$	799 409 184 966.8(6.8)	8.5×10^{-12}	LKB 1999	[29] *
D9	$1S_{1/2} \to 2S_{1/2}$	2 466 732 407 521.8(1.5)	6.1×10^{-13}	MPQ 1997	[36]
D10		2 466 732 407 522.88(91)	3.7×10^{-13}	MPQ 1997	[36]
D11		2 466 732 407 521.74(20)	7.9×10^{-14}	MPQ 1998/2000	H9 +I1
D12		2 466 732 407 521.641(25)	1.0×10^{-14}	MPQ 2010/2011	H11+I2

D only

H + iso

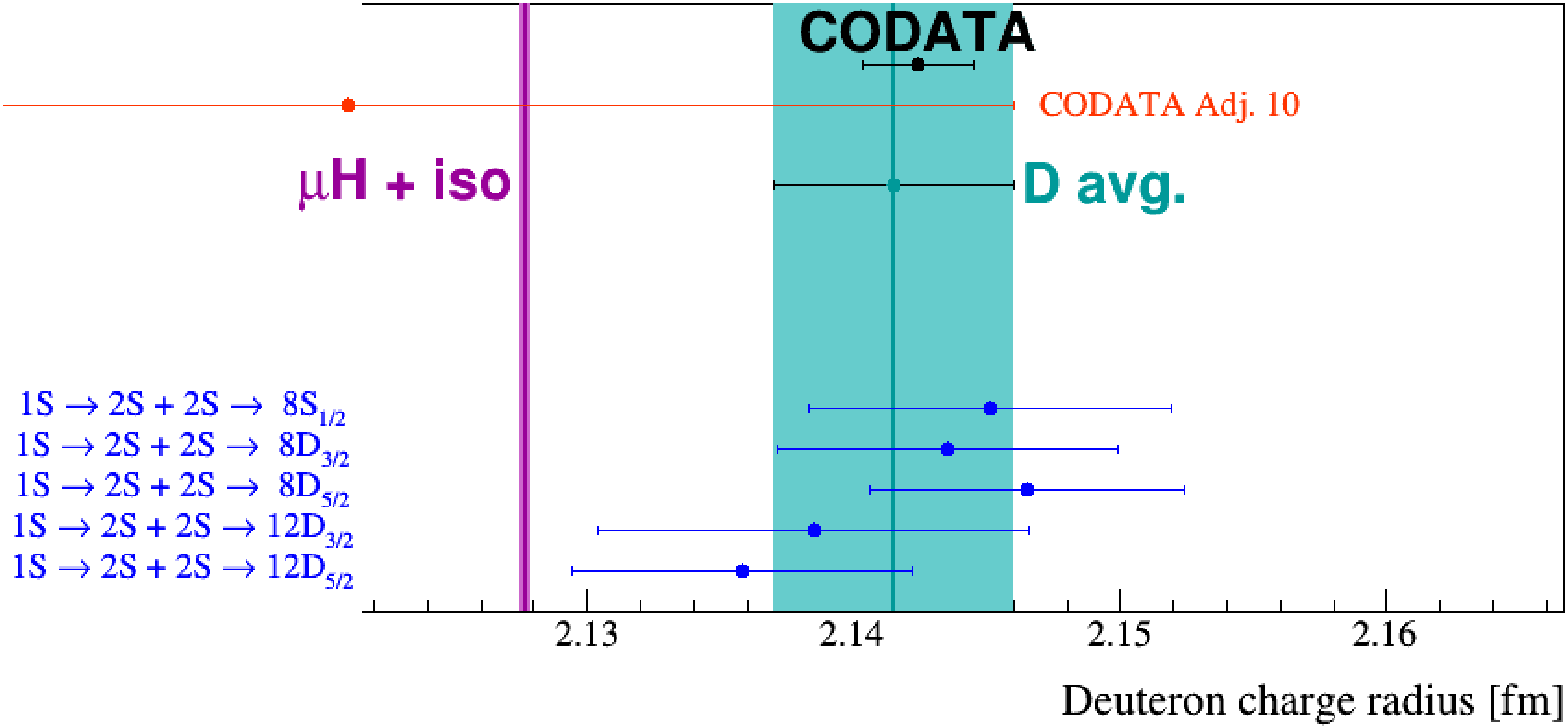
Rd from D spectroscopy

TABLE VII: Deuteron charge radii from deuterium. The value labelled "Eq. (19)" is our result. It is the average of the individual values above it, taking into account the known correlations between the $2S \to n\ell$ measurements. The next 2 values use items D9 and D10, which have not been measured using atomic hydrogen as a transfer oscillator (see text).

#	Transition	r_d [fm]	
D12 + D4	$1S \to 2S + 2S \to 8S_{1/2}$	2.1451 ± 0.0068	
D12 + D5	$1S \to 2S + 2S \to 8D_{3/2}$	2.1435 ± 0.0064	
D12 + D6	$1S \to 2S + 2S \to 8D_{5/2}$	2.1465 ± 0.0059	
D12 + D7	$1S \to 2S + 2S \to 12D_{3/2}$	2.1385 ± 0.0081	
D12 + D8	$1S \to 2S + 2S \to 12D_{5/2}$	2.1358 ± 0.0064	
D12 + all D($2S \to n\ell$)		2.1415 ± 0.0045	Eq. (19)
D9 + all D($2S \to n\ell$)		2.1414 ± 0.0045	
D10 + all D($2S \to n\ell$)		2.1411 ± 0.0045	
CODATA Adj. 10:		2.1214 ± 0.0253	

5.6 times more accurate!

Rd from D spectroscopy



Rd from D spectroscopy

WHICH 1S-2S we choose is IRRELEVANT!

#	Transition	r_d [fm]	
D12 + D4	$1S \to 2S + 2S \to 8S_{1/2}$	2.1451 ± 0.0068	
D12 + D5	$1S \to 2S + 2S \to 8D_{3/2}$	2.1435 ± 0.0064	
D12 + D6	$1S \to 2S + 2S \to 8D_{5/2}$	2.1465 ± 0.0059	
D12 + D7	$1S \to 2S + 2S \to 12D_{3/2}$	2.1385 ± 0.0081	
D12 + D8	$1S \to 2S + 2S \to 12D_{5/2}$	2.1358 ± 0.0064	
D12 + all D($2S \to n\ell$)		2.1415 ± 0.0045	Eq. (19)
D9 + all D($2S \to n\ell$)		2.1414 ± 0.0045	
D10 + all D($2S \to n\ell$)		2.1411 ± 0.0045	
CODATA Adj. 10:		2.1214 ± 0.0253	

Deuteron charge radius from spectroscopy data in atomic deuterium

Randolf Pohl,[1,2,*] François Nez,[3] Thomas Udem,[1] Aldo Antognini,[4,5] Axel Beyer,[1] Hélène Fleurbaey,[3] Alexey Grinin,[1] Theodor W. Hänsch,[1,6] Lucile Julien,[3] Franz Kottmann,[4] Julian J. Krauth,[1] Lothar Maisenbacher,[1] Arthur Matveev,[1] and François Biraben[3]

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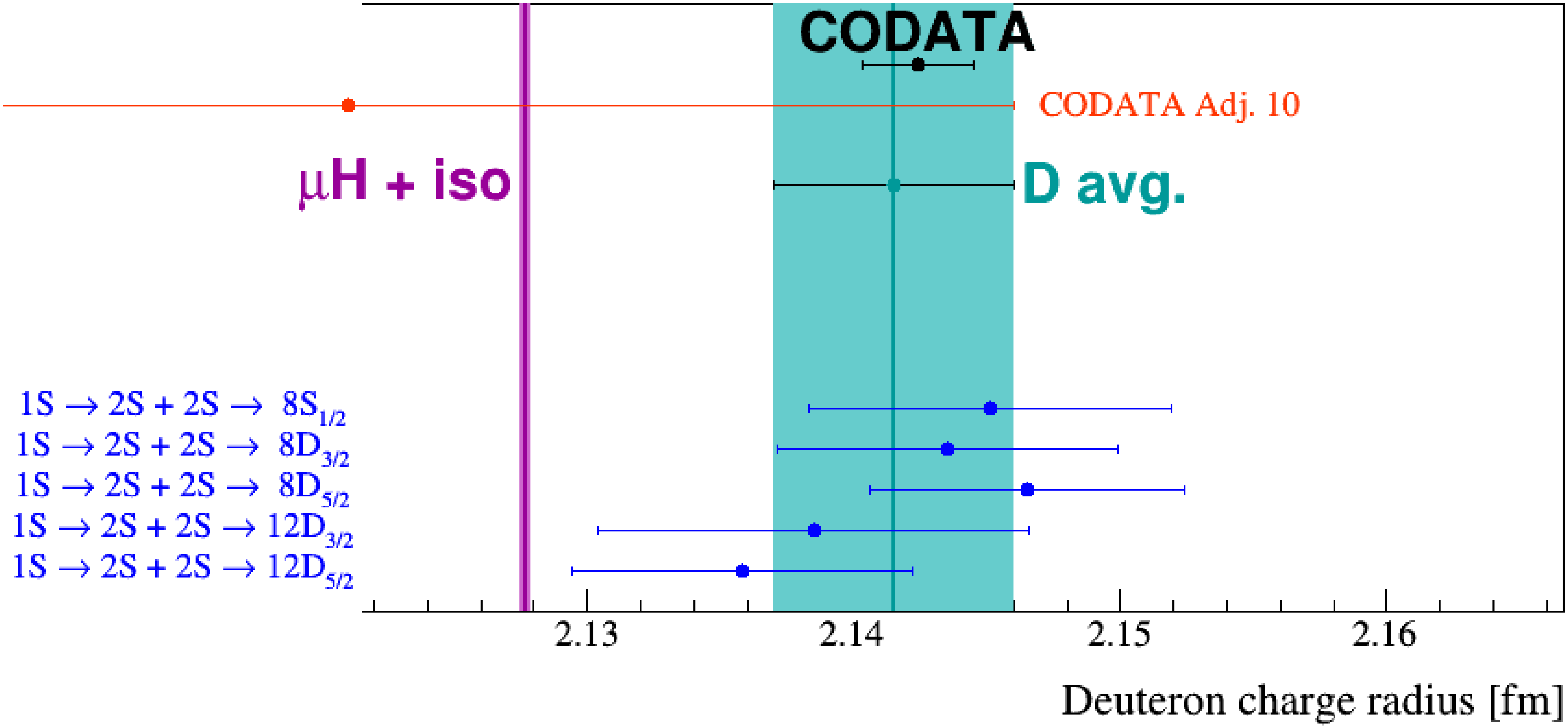
(Dated: July 11, 2016)

We give a pedagogical description of the method to extract the charge radii and Rydberg constant from laser spectroscopy in regular hydrogen (H) and deuterium (D) atoms, that is part of the CODATA least-squares adjustment (LSA) of the fundamental physical constants. We give a deuteron charge radius r_d from D spectroscopy alone of 2.1415(45) fm. This value is independent of the proton charge radius, and five times more accurate than the value found in the CODATA Adjustment 10.

arXiv 1607.03165

Related work:
* Horbatsch, Hessels, “Tabulation of bound-state energies of atomic hydrogen”,
  PRA 93, 022513 (2016) [1601.01057]
  (see Talk Wed.)

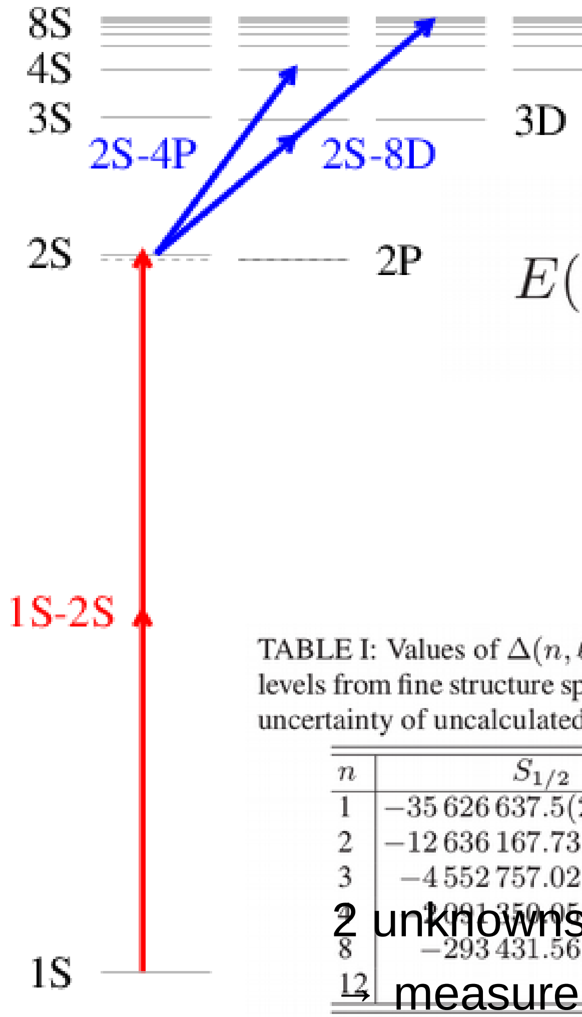
Rd from D spectroscopy



Summary

- Rp = 0.8775(51) fm CODATA-2010
  0.8747(91) fm H(1S-2S) + 2S-nl (*) uncorrel.
  0.8780(108) fm H(1S-3S) + 2S-nl
  0.8764(89) fm CODATA Adj. 8
  0.8409(4) fm muH 4.0 sigma
- Rd = 2.1424(21) fm CODATA-2010
  2.1415(45) fm Deuterium only (*) uncorrel.
  2.1XXX(8) fm muD → next talk

Rp and R_∞ from H spectroscopy



Bohr | Finite size | Dirac, fine struct., QED, ..

$$E(n,\ell,j)/h = -\frac{cR_\infty}{n^2}\frac{m_{\rm red}}{m_e} + \frac{E_{NS}}{n^3}\delta_{\ell 0} + \Delta(n,\ell,j). \quad (7)$$

$$m_{\rm red} = \frac{m_e\, m_N}{m_e + m_N} = \frac{m_e}{1+\frac{m_e}{m_N}}. \quad (8)$$

TABLE I: Values of $\Delta(n,\ell,j)$ in kHz for relevant energy levels in atomic hydrogen. $\Delta(n,\ell,j)$ includes all relevant corrections to the energy levels from fine structure splittings and QED effects. The uncertainties are taken from Ref. [3], Tab.XVIII. They arise mostly from the estimated uncertainty of uncalculated two-loop corrections [14]. An uncertainty of "(0)" denotes "negligibly small".

n	$S_{1/2}$	$P_{1/2}$	$P_{3/2}$	$D_{3/2}$	$D_{5/2}$
1	−35 626 637.5(2.5)				
2	−12 636 167.73(31)	−13 693 861.67(3)	−2 724 820.10(3)		
3	−4 552 757.02(9)			−1 623 042.90(0)	−539 705.73(0)
[illegible]	[illegible]	[illegible]	−853 278.87(0)	−855 566.49(0)	−398 533.10(0)
8	−293 431.56(1)			−138 996.24(0)	−81 867.09(0)
12				−44 349.61(0)	−27 422.46(0)

2 unknowns: R_∞ and **Rp** (E_{NS})

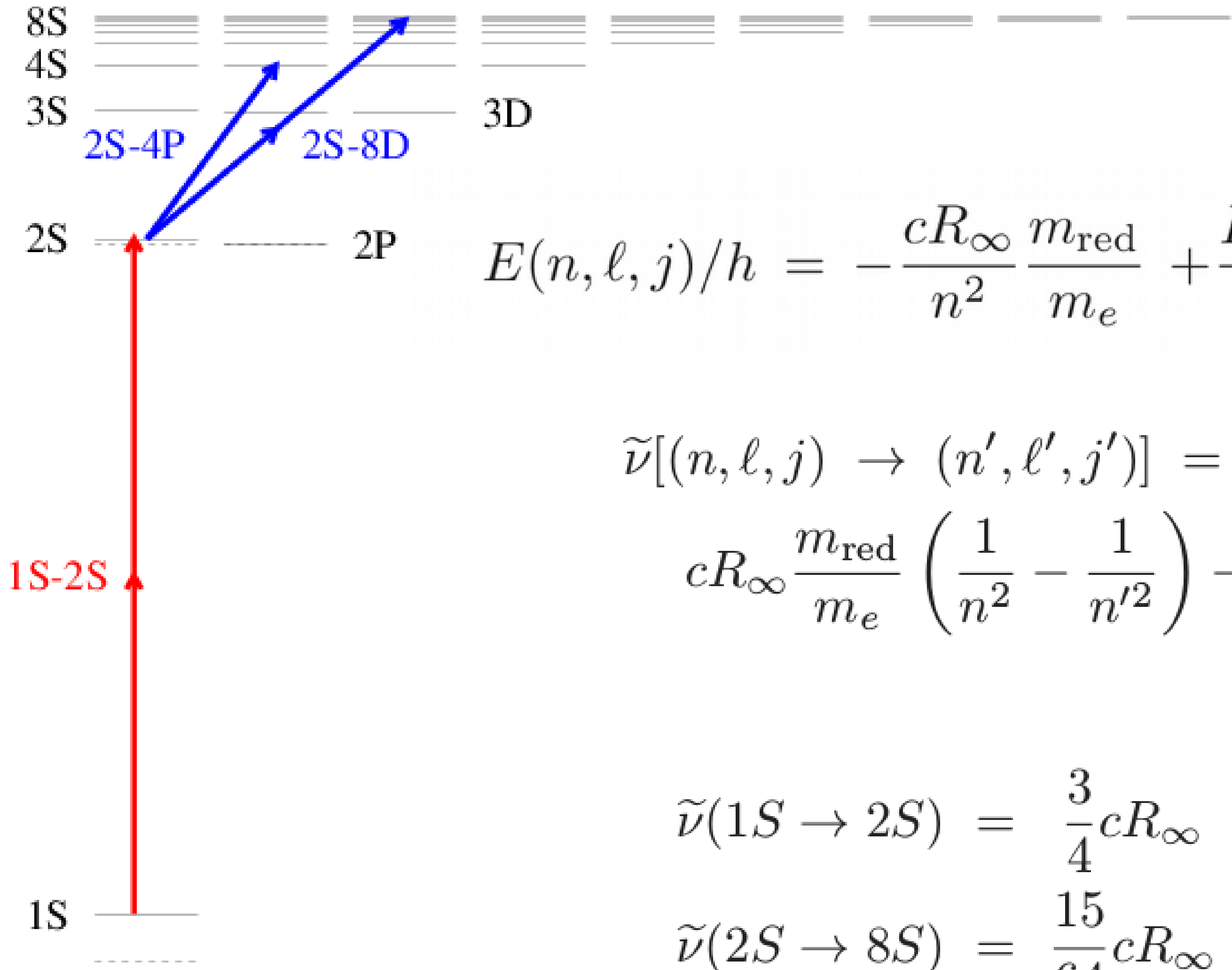
→ measure 2 transitions (1S-2S and 2S-nl)

Sum of all terms from CODATA report
(3 independent calculations, cross-checked with P. Mohr's code)

vs. 1S-2S accuracy 10 Hz (**0.01 kHz**)
Parthey, RP et al., PRL 107, 203001 (2011)

RP et al., Metrologia 54, L1 (2017) [1607.03165]

Combine transition frequencies



$$E(n,\ell,j)/h = -\frac{cR_\infty}{n^2}\frac{m_{\rm red}}{m_e} + \frac{E_{NS}}{n^3}\,\delta_{\ell 0} + \Delta(n,\ell,j). \quad (7)$$

$$\widetilde{\nu}[(n,\ell,j) \to (n',\ell',j')] = cR_\infty \frac{m_{\rm red}}{m_e}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) - E_{NS}\left(\frac{\delta_{\ell 0}}{n^3} - \frac{\delta_{\ell' 0}}{n'^3}\right), \quad (14)$$

$$\widetilde{\nu}(1S \to 2S) = \frac{3}{4}cR_\infty - \frac{7}{8}E_{NS} \quad (16)$$

$$\widetilde{\nu}(2S \to 8S) = \frac{15}{64}cR_\infty - \frac{63}{512}E_{NS}. \quad (17)$$

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