Neutrino Phenomenology and Theory

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Lecture 4

How Non-Relativistic Must Neutrinos Be For Us To Be Sensitive To Whether They Are Dirac Or Majorana Particles?

The insensitivity when the neutrinos are relativistic is due to their ***helicity h*** substituting for a conserved lepton number.

Example: (Anti)Neutrinos from π^- decay are of ~ 100 % right-handed (RH) helicity, whether Dirac or Majorana.

Suppose they interact with some target
to make either an electron or a positron.

The interaction is described by —

$$\mathcal{L}_{\mathrm{CC}} \propto \bar{e}\gamma^{\lambda}\frac{\left(1-\gamma_5\right)}{2}\nu J_{\lambda}^{\mathrm{Target}} + \bar{\nu}\gamma^{\lambda}\frac{\left(1-\gamma_5\right)}{2} e J_{\lambda}^{\mathrm{Target}\,\dagger}$$

Creates e^-
Not e^+

Creates e^+
Not e^-

Owing to the LH chiral projection operator $(1 - \gamma_5)/2$, the amplitude for a Majorana neutrino to make an e^-, which comes from the first term alone, contains a factor that, in the rest frame of the target, is —

$$1-2h\sqrt{\left(E-m\right)/\left(E+m\right)} \equiv F\left(h,E\right)$$

The factor is —

$$1-2h\sqrt{\left(E-m\right)/\left(E+m\right)}\equiv F\left(h,E\right)$$

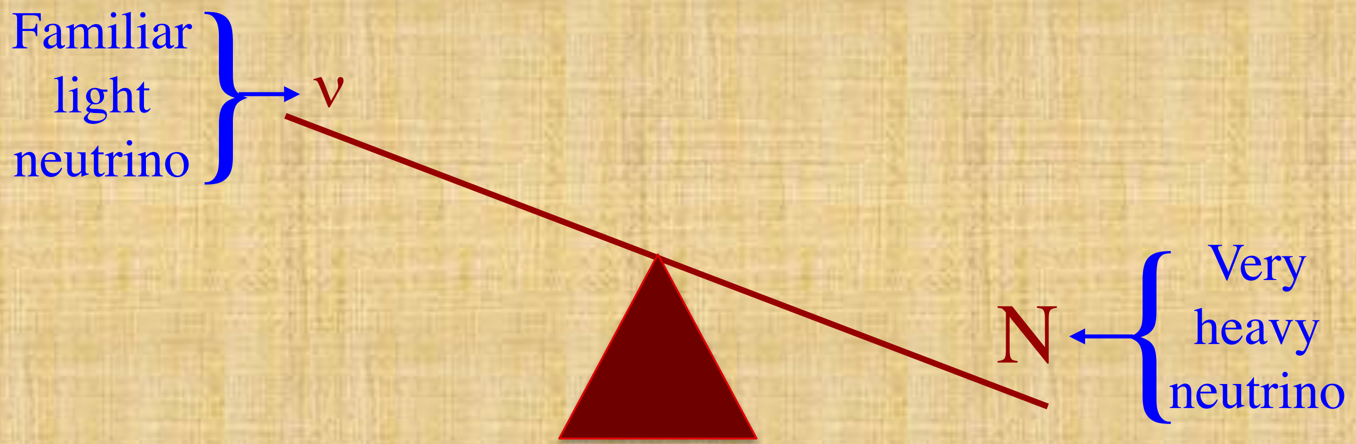
When the neutrino is highly relativistic in the target rest frame, $E >> m$, and $F(h, E) = 0$.

An e^- will not produced, even in the Majorana case.

When E is smaller, but still far from m, $F\left(h,E\right)\simeq m/E$.
A Majorana neutrino can now produce an e^-.

When $E\simeq m, F\left(h,E\right)\simeq 1$, and a Majorana neutrino can produce an e^- with no suppression at all.

The See-Saw Mechanism



(Gell-Mann, Ramond, and Slansky; Yanagida;
Mohapatra and Senjanovic; Minkowski)

The See-Saw Mechanism is the most popular hypothesis for explaining how the neutrino masses, athough not zero, can be so small.

We saw that when an underlying ν has only a Majorana mass, the resulting mass eigenstate is a Majorana neutrino.

We will now see, illustrated by the See-Saw picture, that when there are ***both*** Dirac and Majorana masses, ***two*** Majorana mass eigenstates result.

The See-Saw

We include *both* Dirac and Majorana mass terms:

We assume that, in terms of underlying fields, the mass sector is —

$$\mathcal{L}_m = -m_D \bar{N}_R \nu_L - \frac{m_R}{2} \overline{(N_R)^c} N_R + \text{h.c.}$$

$$= -\frac{1}{2} \left[\overline{(\nu_L)^c}, \bar{N}_R \right] \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} \nu_L \\ (N_R)^c \end{bmatrix} + \text{h.c.}$$

We have used $\overline{(\nu_L)^c} m_D (N_R)^c = \bar{N}_R m_D \nu_L$.

$M_\nu = \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix}$ is called the neutrino mass matrix.

No SM principle prevents m_R from being extremely large.

But we expect m_D to be of the same order as the masses of the quarks and charged leptons.

Thus, we assume that $m_R >> m_D$.

M_ν can be diagonalized by the transformation —

$$Z^T M_\nu Z = D_\nu$$

With $\rho \equiv m_D / m_R << 1$,

$$Z \cong \begin{bmatrix} 1 & \rho \\ -\rho & 1 \end{bmatrix} \overbrace{\begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}}^{\text{Makes eigenvalues positive}}$$

and

$$D_\nu \cong \begin{bmatrix} m_D{}^2 / m_R & 0 \\ 0 & m_R \end{bmatrix}$$

Define $\begin{bmatrix} \nu_{1L} \\ N_{1L} \end{bmatrix} \equiv Z^{-1} \begin{bmatrix} \nu_L \\ (N_R)^c \end{bmatrix}$ and $\begin{bmatrix} \nu_1 \\ N_1 \end{bmatrix} \equiv \begin{bmatrix} \nu_{1L} + (\nu_{1L})^c \\ N_{1L} + (N_{1L})^c \end{bmatrix}$.

Majorana neutrinos

Then —

$$\mathcal{L}_m = -\frac{1}{2}\frac{m_D^2}{m_R}\bar{\nu}_1 \nu_1 - \frac{1}{2} m_R \bar{N}_1 N_1$$

Mass of ν_1

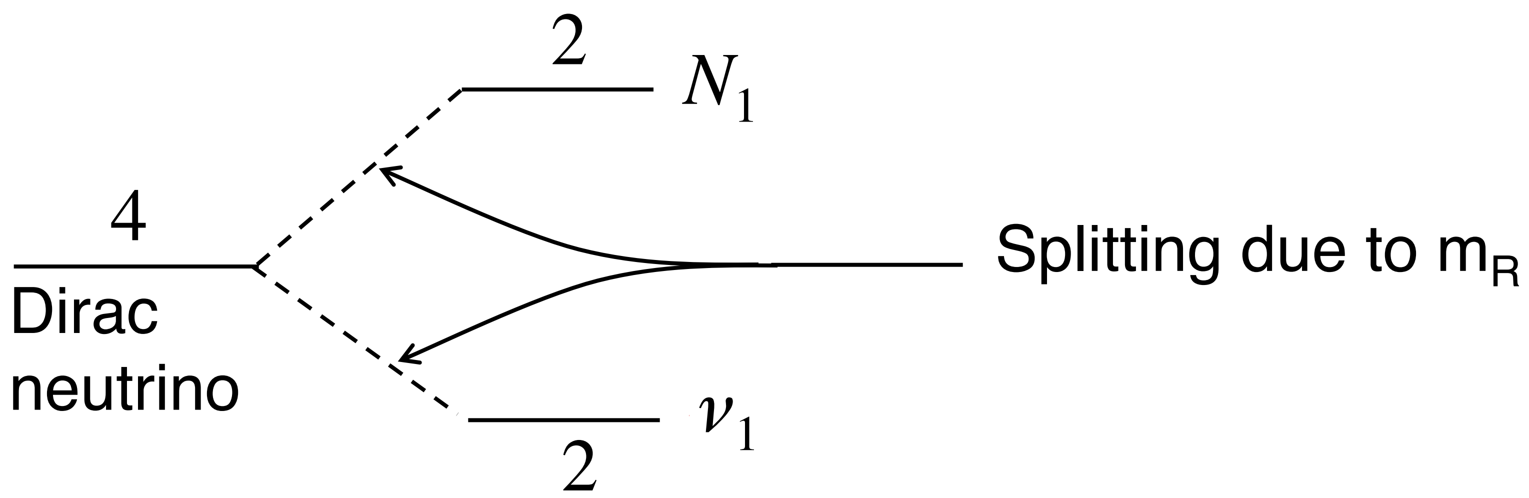
Mass of N_1

$$M_{\nu_1} = \frac{m_D^2}{M_{N_1}} \sim \frac{m^2_{\text{quark or charged lepton}}}{M_{N_1}}$$

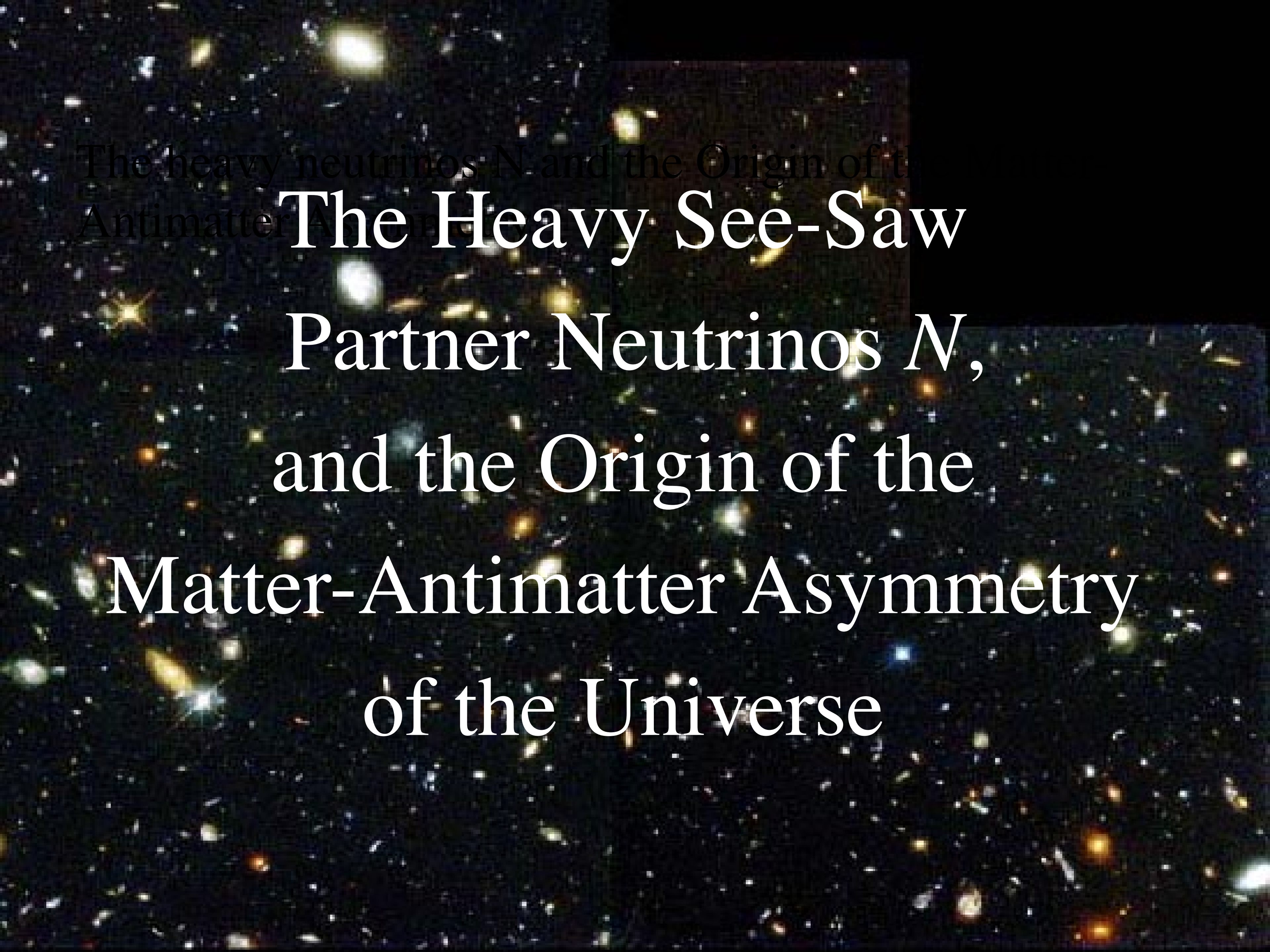
The See-Saw Relation

What Happened?

The Majorana mass term split a Dirac neutrino into two Majorana neutrinos.



$4 = 2 + 2$


The heavy neutrinos N and the Origin of t
Antimatter
The Heavy See-Saw
Partner Neutrinos *N*,
and the Origin of the
Matter-Antimatter Asymmetry
of the Universe

The Cosmic Puzzle

Today: $B \equiv \#(\text{Baryons}) - \#(\text{Antibaryons}) \neq 0.$

Standard cosmology: Right after the Big Bang, $B = 0.$

Also, $L \equiv \#(\text{Leptons}) - \#(\text{Antileptons}) = 0.$

How did $B = 0 \Longrightarrow B \neq 0$?

Sakharov: $B = 0 \Longrightarrow B \neq 0$ requires $\cancel{C}$ and $\cancel{CP}$.

$\not{C}$ is easy to achieve, but the required degree and kind of $\not{CP}$ is harder.

The $\not{CP}$ in the quark mixing matrix, seen in B and K decays, leads to much too small a $B-\overline{B}$ asymmetry.

If ***quark*** $\not{CP}$ cannot generate the observed $B-\overline{B}$ asymmetry, can some scenario involving ***leptons*** do it?

The candidate scenario: ***Leptogenesis***, a very natural consequence of the See-Saw picture.

(Fukugita, Yanagida)

We will now need the See-Saw picture with ***several*** neutrinos.

The straightforward (type-I) See-Saw model adds to the SM 3 heavy neutrinos N_i, with —

$$\mathcal{L}_{\text{new}} = -\frac{1}{2}\sum_i \underbrace{m_{N_i}}_{\text{Large Majorana masses}} \underbrace{\overline{N^c_{iR}}}_{\text{Charge conjugate}} N_{iR} + \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \underbrace{y_{\alpha i}}_{\text{Yukawa coupling matrix}} \left[\overline{\nu}_{\alpha L} \overline{H^0} - \overline{\ell}_{\alpha L} H^- \right] N_{iR} + h.c.$$

SM lepton doublet: $\overline{\nu}_{\alpha L}$, $\overline{\ell}_{\alpha L}$

SM Higgs doublet: $\overline{H^0}$, H^-

The Yukawa interaction causes the decays —

$$N \to \ell^- + H^+,\ N \to \ell^+ + H^-,$$
$$N \to \nu + H^0,\ N \to \overline{\nu} + \overline{H^0}.$$

(*$\overline{N} = N$, so the decays in each line are C and CP mirror images.*)

Note: Today, there is only a ***neutral*** SM Higgs particle.

However, we are going to use the SM
+ See-Saw model in the early universe.

It has not yet cooled to the temperature at which
$\langle H^0 \rangle_0$ turns on, and the charged Higgs particles
are "eaten" by the W bosons.

At this early time, H^+ and H^-
are physical particles.

The N_i are heavy, but they would have been made during the ***hot*** Big Bang.

They would then have quickly decayed via the decay modes we just identified.

Phases in the Yukawa coupling matrix y would have led to $\cancel{C}$ and $\cancel{CP}$ effects.

In particular, such phases would have led to —

$$\Gamma\left(N \to \ell^- + H^+\right) \neq \Gamma\left(N \to \ell^+ + H^-\right)$$

and

$$\Gamma\left(N \to \nu + H^0\right) \neq \Gamma\left(N \to \bar{\nu} + \overline{H^0}\right)$$

$\cancel{C}$ and $\cancel{CP}$

How Phases Lead To CP Non-Invariance

$\not{CP}$ ***always*** comes from ***phases***.

Therefore, $\not{CP}$ always requires an ***interference*** between (at least) two amplitudes.

For example, an interference between two Feynman diagrams.

Let us consider how a CP-violating rate difference between two CP-mirror-image processes arises.

Suppose some process P has the amplitude —

$$A = M_1 e^{i\theta_1} e^{i\delta_1} + M_2 e^{i\theta_2} e^{i\delta_2}$$

CP-invariant magnitude

CP-odd “weak” phase from constants

CP-even “strong” phase

Then the CP-mirror-image process $\overline{\mathrm{P}}$ has the amplitude —

$$\overline{A} = M_1 e^{i\theta_1} e^{-i\delta_1} + M_2 e^{i\theta_2} e^{-i\delta_2}$$

Then the rates for $\overline{\mathrm{P}}$ and P differ by —

$$\overline{\Gamma} - \Gamma = \left|\overline{A}\right|^2 - \left|A\right|^2 = 4 M_1 M_2 \sin(\theta_1 - \theta_2) \sin(\delta_1 - \delta_2)$$

$$\overline{\Gamma} - \Gamma = \left|\overline{A}\right|^2 - \left|A\right|^2 = 4 M_1 M_2 \sin\left(\theta_1 - \theta_2\right)\sin\left(\delta_1 - \delta_2\right)$$

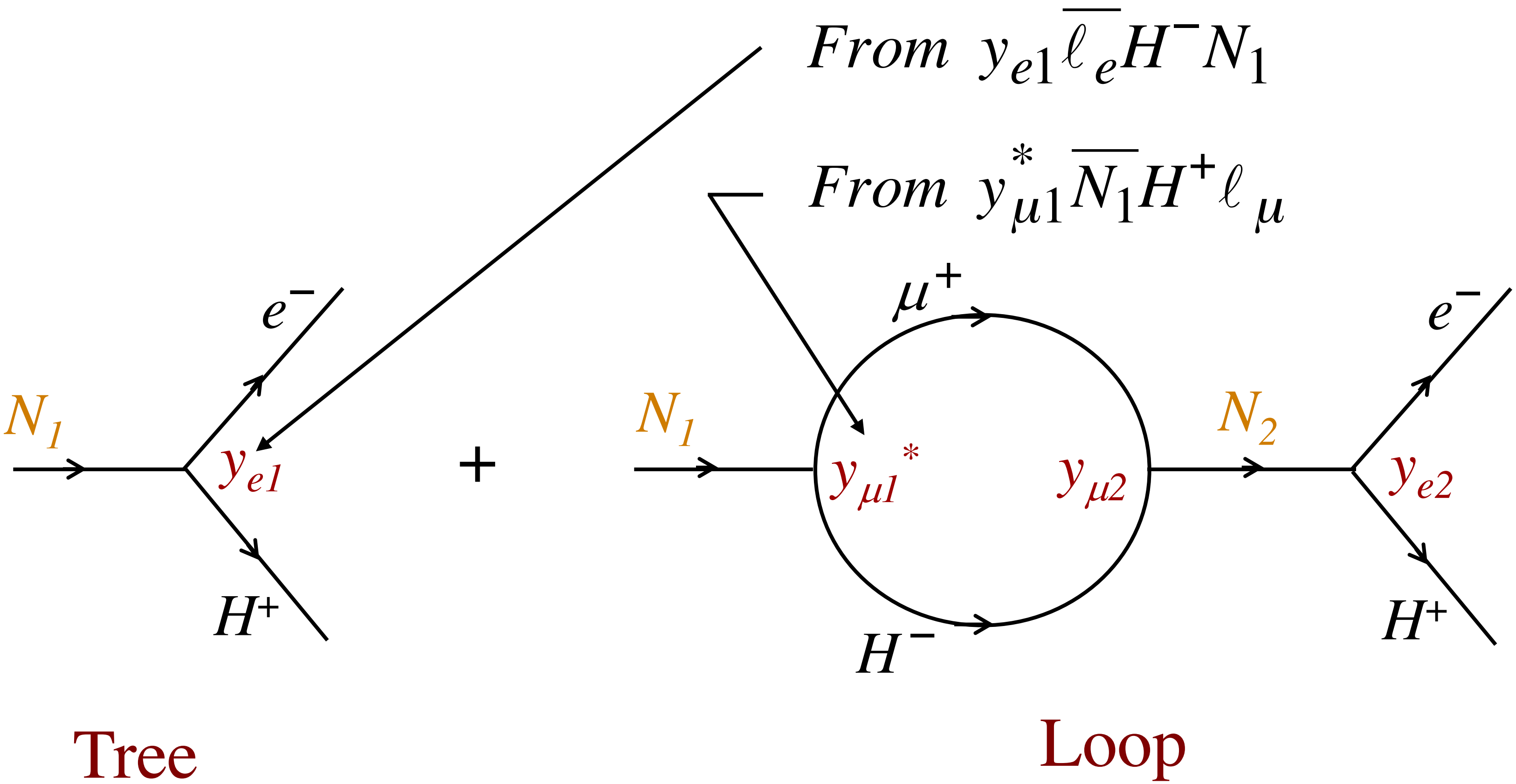
A CP-violating rate difference
requires 3 ingredients:

- Two interfering amplitudes
- These two amplitudes must have different CP-even phases
- These two amplitudes must have different CP-odd phases

How Do $\cancel{CP}$ Inequalities Between *N* Decay Rates Come About?

Let us look at an example.

This example illustrates that $\cancel{CP}$ in ***any decay*** always involves amplitudes ***beyond*** those of lowest order in the Hamiltonian.



$$\Gamma\left(N_1 \to e^- + H^+\right) = \left| y_{e1} K_{\text{Tree}} + y^*_{\mu 1} y_{\mu 2} y_{e2} K_{\text{Loop}} \right|^2$$

Kinematical factors

$$\Gamma\left(N_1 \to e^- + H^+\right) = \left|y_{e1}K_{\text{Tree}} + y_{\mu 1}^* y_{\mu 2} y_{e2} K_{\text{Loop}}\right|^2$$

When we go to the CP-mirror-image decay, $N_1 \to e^+ + H^-$, all the coupling constants get complex conjugated, but the kinematical factors do not change.

From Hermiticity of H

$$\Gamma\left(N_1 \to e^+ + H^-\right) = \left|y_{e1}^* K_{\text{Tree}} + y_{\mu 1} y_{\mu 2}^* y_{e2}^* K_{\text{Loop}}\right|^2$$

Then —

$$\boxed{\begin{aligned}&\Gamma\left(N_1 \to e^- + H^+\right) - \Gamma\left(N_1 \to e^+ + H^-\right) \\ &= 4\,\text{Im}\left(y_{e1}^* y_{\mu 1}^* y_{e2} y_{\mu 2}\right)\text{Im}\left(K_{\text{Tree}} K_{\text{Loop}}^*\right)\end{aligned}}$$

The inequalities —

$$\Gamma\left(N \to \ell^- + H^+\right) \neq \Gamma\left(N \to \ell^+ + H^-\right)$$

and

$$\Gamma\left(N \to \nu + H^0\right) \neq \Gamma\left(N \to \bar{\nu} + \overline{H^0}\right)$$

violate CP in the leptonic sector, and violate lepton number L.

Starting with a universe with $L = 0$, these decays would have produced one with $L \neq 0$, containing unequal numbers of SM leptons and antileptons.

Next —

The Standard-Model ***Sphaleron*** process,
which does not conserve Baryon Number B,
or Lepton Number L, but does conserve $B - L$, acts.

$$B_i = 0$$
$$L_i \neq 0$$

Initial state from N decays

→ Sphaleron Process →

$$B_f \cong -\frac{1}{3} L_i$$
$$L_f \cong \frac{2}{3} L_i \cong -2B_f$$

Final state

There is now a nonzero *Baryon Number* $\mathcal{B}$**.**

Eventually, there are baryons, but ~ no antibaryons.

Reasonable couplings y give the observed value of $\mathcal{B}$**.**

How Heavy Must the N_i Be?

Once the Higgs vev turns on,

$$y\bar{\nu}_L \overline{H^0} N_R \Rightarrow y\bar{\nu}_L \left\langle \overline{H^0} \right\rangle_0 N_R \equiv m_D \bar{\nu}_L N_R$$

This is the Dirac mass term in the See-Saw model.

SM Higgs vev

The see-saw relation is then $M_\nu \sim \dfrac{v^2 y^2}{M_N}$.

Yukawa coupling

Light neutrino

This relation, the light ν masses, and the $y^2 \sim 10^{-5}$ called for by the observed cosmic baryon asymmetry,

$$\Rightarrow \quad M_N \gtrsim 10^{(9-10)} \text{ GeV}.$$

Such heavy particles cannot be produced at the LHC or any foreseen accelerator.

So how can we get evidence that leptogenesis occurred?

Generically, leptogenesis and light-neutrino ~~CP~~ imply each other.

They both come from phases in the same Yukawa coupling matrix y.

We've already seen that the phases in y are the origin of CP violation in heavy N decays.

Now we will see that in the See-Saw model, they are also the origin of the phases in the mixing matrix U.

The Oscillation — Leptogenesis Connection

(B.K.)

The See-Saw Relation

Leptonic mixing matrix

Heavy N mass eigenvalues

$$UM_{\nu}U^{T} = -v^{2}\left(yM_{N}^{-1}y^{T}\right)$$

Light ν mass eigenvalues

The Higgs vev, a real number

$$\left(\underbrace{UM_{\nu}U^{T}}_{\text{Outputs}} = -v^{2}\underbrace{\left(yM_{N}^{-1}y^{T}\right)}_{\text{Inputs, in } \mathcal{L}}\right)$$

Through ***U***, the phases in ***y*** lead to $\not{CP}$ in light neutrino oscillation.

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$\underset{(-)}{+} 2 \sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

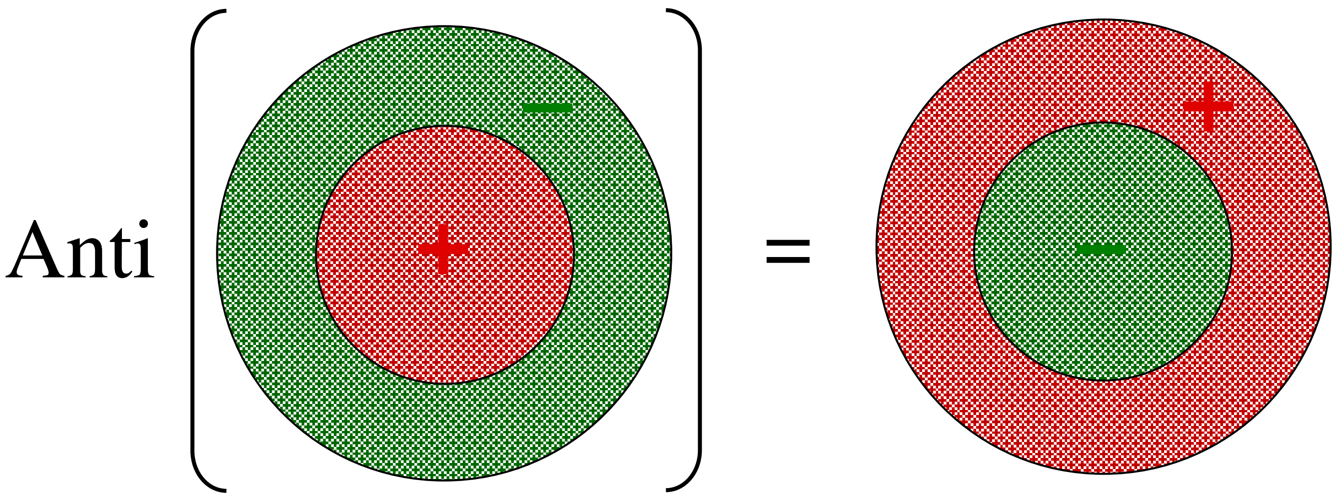
If the oscillation CP phase δ proves to be large, it could explain almost the entire Baryon – Antibaryon asymmetry by itself.

(Pascoli, Petcov, Riotto)

Electromagnetic Properties of Neutrinos

Can a Majorana Neutrino Have an Electric Charge *Distribution*?

No!

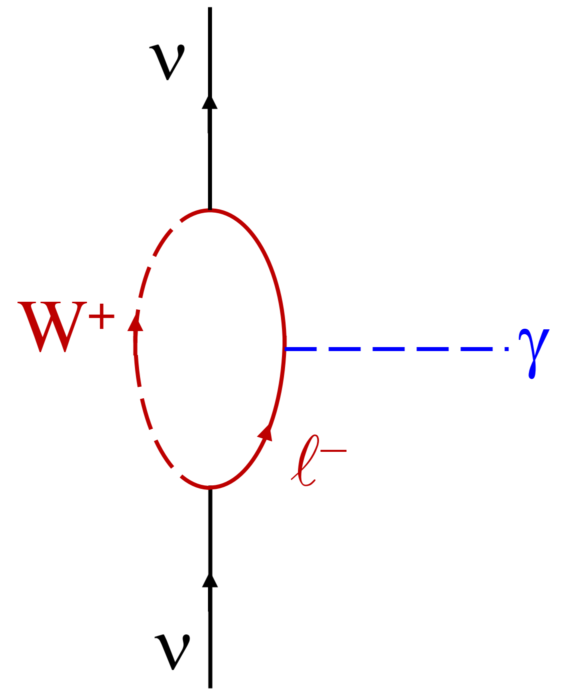


But for a Majorana neutrino —

Anti $(\nu) = \nu$

Dipole Moments

In the Standard Model,
loop diagrams like —



Why doesn't this diagram produce a dipole moment for a *Majorana* neutrino??

produce, for a *Dirac* neutrino of mass m_ν,
a magnetic dipole moment —

$$\mu_\nu = 3 \times 10^{-19} (m_\nu/1\text{eV})\ \mu_B$$

(Marciano, Sanda; Lee, Shrock; Fujikawa, Shrock)

A *Majorana* neutrino cannot have a magnetic or electric dipole moment:

$$\vec{\mu}\left[\ \underset{e^+}{\Uparrow}\ \right] = -\,\vec{\mu}\left[\ \underset{e^-}{\Uparrow}\ \right]$$

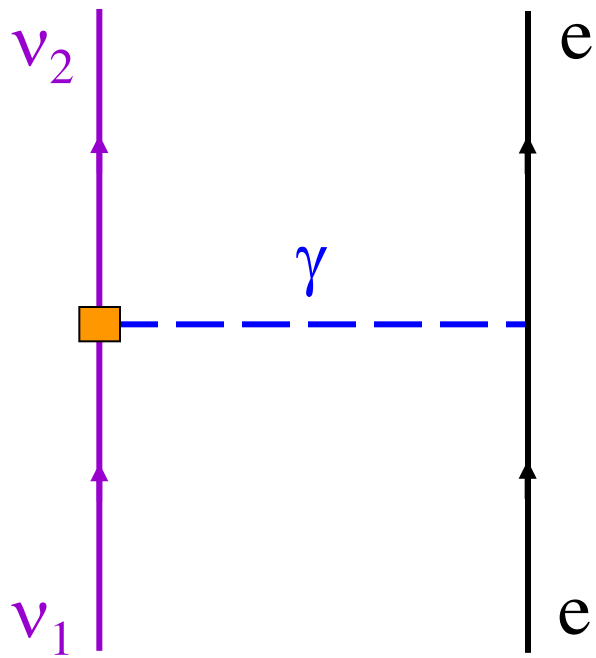
But for a Majorana neutrino,

$$\overline{\nu_i} = \nu_i$$

Therefore,

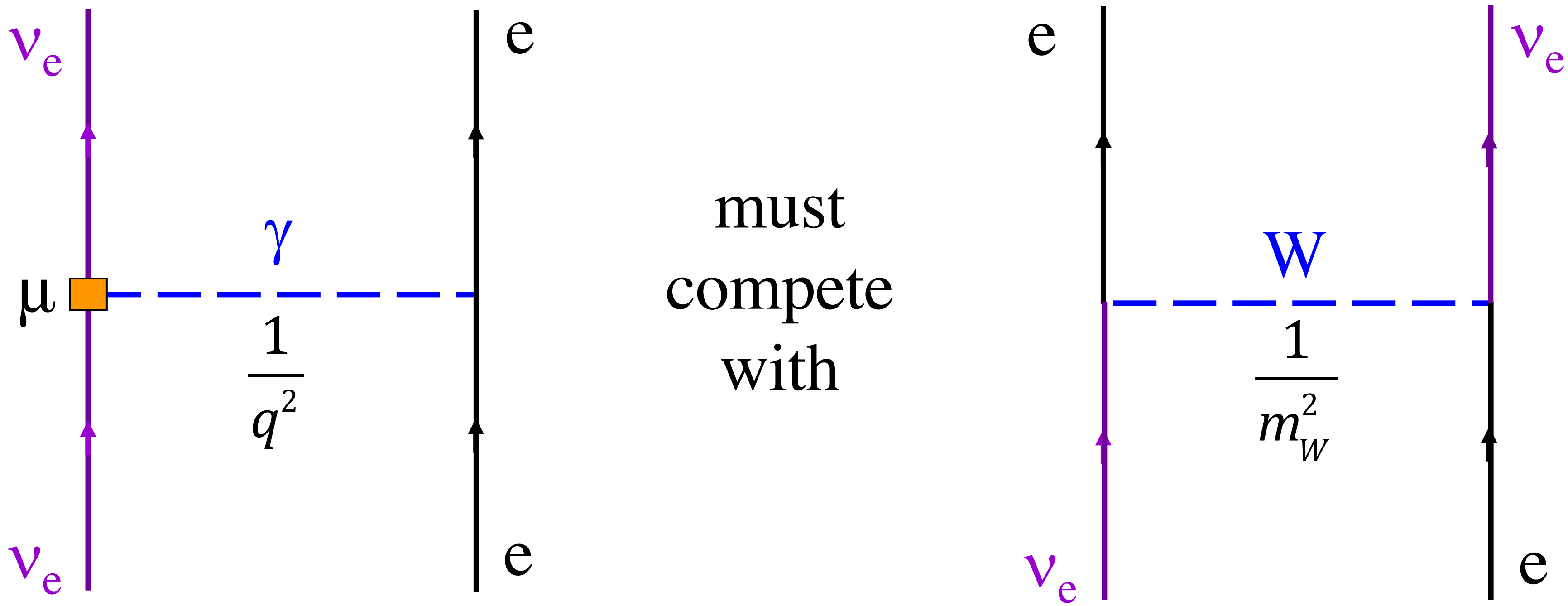
$$\vec{\mu}\,[\overline{\nu_i}] = \vec{\mu}\,[\nu_i] = 0$$

Both *Dirac* and *Majorana* neutrinos can have ***transition*** dipole moments, leading to —



One can look for the dipole moments this way.

How big must they be to be visible?

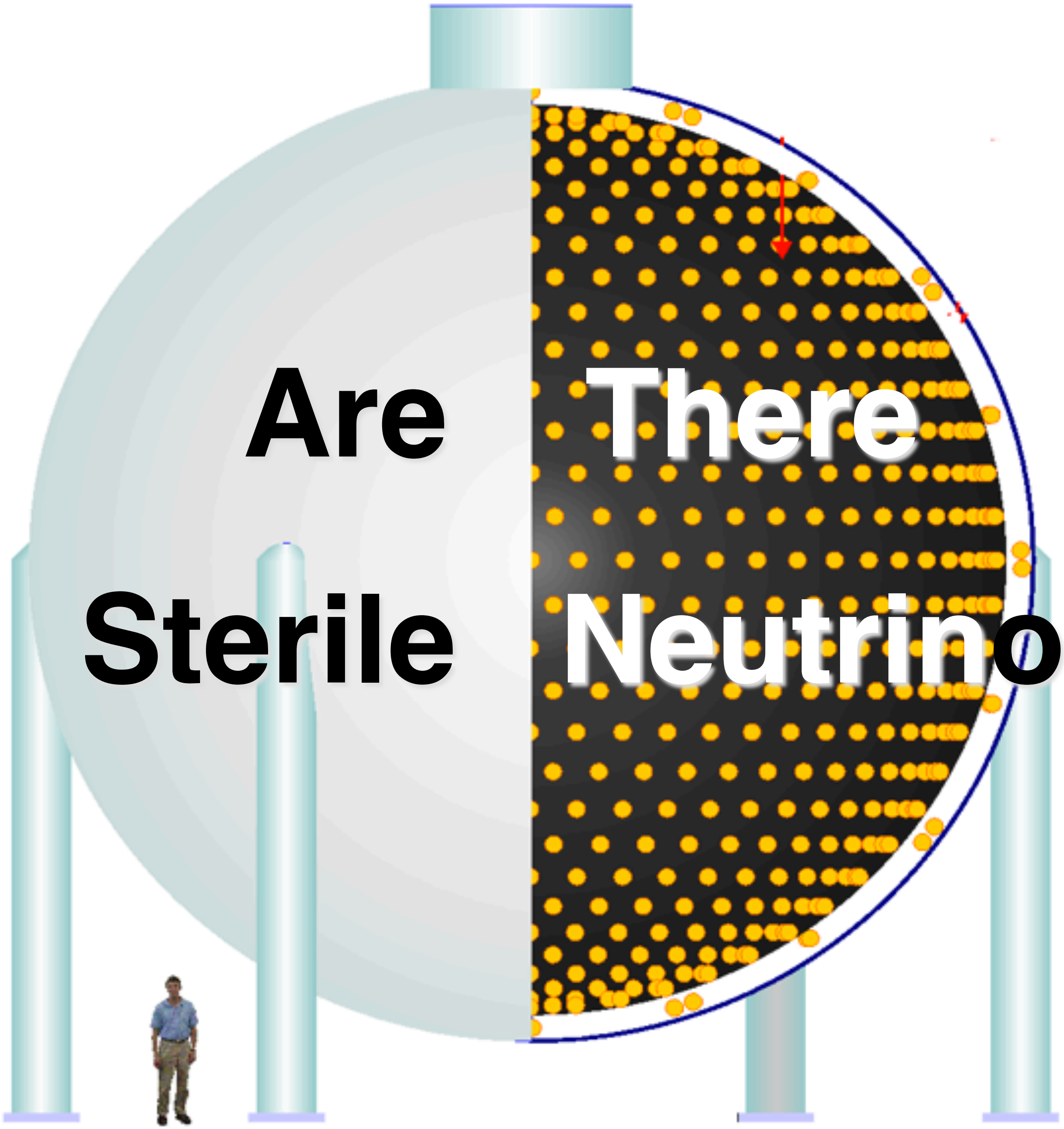


We must seek an excess, beyond the event rate expected from W (and Z) exchange, at low e recoil kinetic energy T.

To be sensitive to an effective dipole moment μ, we must be able to go to —

$$T/m_e < \left(\mu/10^{-10}\mu_B\right)^2 .$$

Are There Sterile Neutrinos?



Sterile Neutrino

One that does not couple to the SM *W* or *Z* boson

A "sterile" neutrino may well couple to some non-SM particles. These particles could perhaps be found at LHC or elsewhere.

The heavy See-Saw partner neutrinos N_i interact with the rest of the world only through the Yukawa coupling —

SM lepton doublet

$$\mathcal{L}_{\text{Yukawa}} = \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} y_{\alpha i} \left[\overline{\nu}_{\alpha L} \overline{H^0} - \overline{\ell}_{\alpha L} H^- \right] N_{iR} + h.c.$$

Yukawa coupling matrix

SM Higgs doublet

The N_i do not couple to the SM W or Z boson.

$\therefore$ The N_i are sterile neutrinos.

Are there also ***light*** sterile neutrinos with masses ~ 1 eV?

I don't know of any compelling models, but they are possible.

The Freedom a 4th Neutrino Flavor Gives to CP Violation

Let $P\left[\nu_\alpha \to \nu_\beta\right] - P\left[\bar{\nu}_\alpha \to \bar{\nu}_\beta\right] \equiv \Delta P_{\alpha\beta}$ be a CP-violating $\nu - \bar{\nu}$ difference in vacuum.

Assuming CPT invariance, $P\left[\nu_\beta \to \nu_\alpha\right] = P\left[\bar{\nu}_\alpha \to \bar{\nu}_\beta\right]$.

Then $\Delta P_{\beta\alpha} = -\Delta P_{\alpha\beta}$. In particular, $\Delta P_{\alpha\beta} = 0$ when $\beta = \alpha$.

Conservation of probability

$$\Rightarrow \sum_{\text{All } \beta} P\left(\nu_\alpha \to \nu_\beta\right) = \sum_{\text{All } \beta} P\left(\bar{\nu}_\alpha \to \bar{\nu}_\beta\right) = 1 .$$

Then $\boxed{\sum_{\text{All } \beta} \Delta P_{\alpha\beta} = \sum_{\beta \neq \alpha} \Delta P_{\alpha\beta} = 0}$.

When there are only 3 neutrino flavors, there are only 3 independent (potentially) nonzero CP-violating differences $\Delta P_{\alpha\beta}$ to be measured: $\Delta P_{e\mu}$, $\Delta P_{\mu\tau}$, and $\Delta P_{\tau e}$.

$$\sum_{\beta\neq\alpha} \Delta P_{\alpha\beta} = 0 \Longrightarrow$$

$$\Delta P_{e\mu} + \Delta P_{e\tau} = 0 \quad \text{and} \quad \Delta P_{\mu e} + \Delta P_{\mu\tau} = 0 \;.$$

Then $\boxed{\Delta P_{e\mu} = \Delta P_{\mu\tau} = \Delta P_{\tau e}}$.

If CP is not violated in $\overset{(-)}{\nu}_\mu \rightarrow \overset{(-)}{\nu}_e$, then it is not violated in ***any*** oscillation channel.

When there are 4 neutrino flavors, assuming CPT invariance, there are 6 independent (potentially) nonzero CP-violating differences $\Delta P_{\alpha\beta}$:

$\Delta P_{e\mu}$, $\Delta P_{\mu\tau}$, $\Delta P_{\tau e}$, ΔP_{es}, $\Delta P_{\mu s}$, and $\Delta P_{\tau s}$.

Sterile flavor

Now $\sum_{\beta\neq\alpha} \Delta P_{\alpha\beta} = 0$ only implies such relations as —

$$\Delta P_{e\mu} = \Delta P_{\mu\tau} + \Delta P_{\mu s}$$

The CP-violating differences $\Delta P_{\alpha\beta}$ in different active-to-active oscillation channels are no longer required to be equal.

Neutrino Physics References

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Good luck!