Neutrino Phenomenology and Theory

Boris Kayser
INSS

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Lecture 3


Looking to the Future
Open
Questions

- Is the physics behind the masses of neutrinos different from that behind the masses of all other known particles?
- Are neutrinos their own antiparticles?

- What is the absolute scale of neutrino mass?

- Is the spectrum like  or  ?

• Is θ_{23} maximal?

• Do neutrino interactions violate CP?

Is $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$?

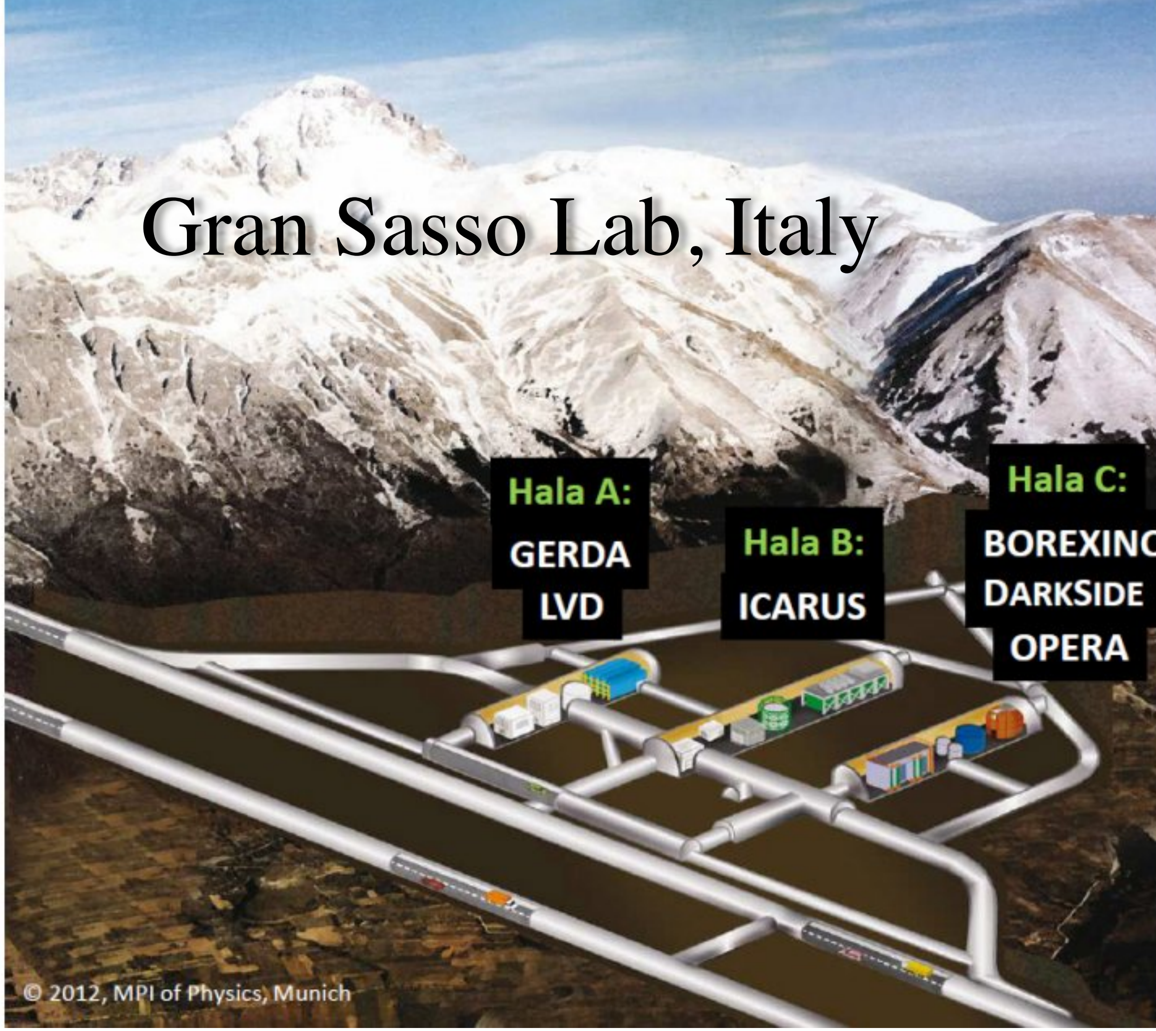
• Is CP violation involving neutrinos the key to understanding the matter – antimatter asymmetry of the universe?

- What can neutrinos and the universe tell us about one another?

- Are there *more* than 3 mass eigenstates?
  - Are there “sterile” neutrinos that don’t couple to the W or Z?

- Do neutrinos have Non-Standard-Model interactions?

- Do neutrinos break the rules?
    - Violation of Lorentz invariance?
    - Violation of CPT invariance?
    - Departures from quantum mechanics?



Is the Origin of Neutrino Mass Different?

The question is whether neutrinos have **"Majorana masses"**. Quarks and charged leptons cannot have such masses. The masses of the quarks and charged leptons are **"Dirac masses"**.

So what are Dirac and Majorana masses?

Dirac and Majorana Masses — In Pictures

We will describe what the quantum field theory does, but first without equations.

For simplicity, let us treat a world with just one flavor, and correspondingly, just one neutrino mass eigenstate.

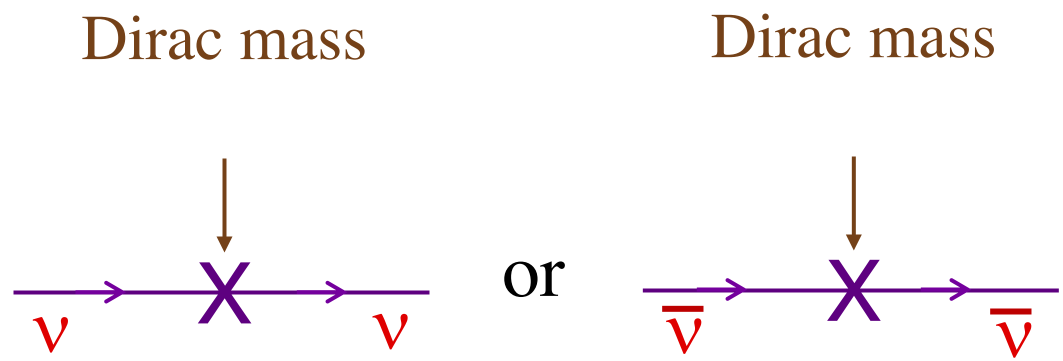
We start with underlying neutrino states ν and $\bar{\nu}$ that are distinct from each other, like other familiar fermions, and are not the mass eigenstates.

We will have to see what the mass eigenstates are later.

We can have two types of masses:

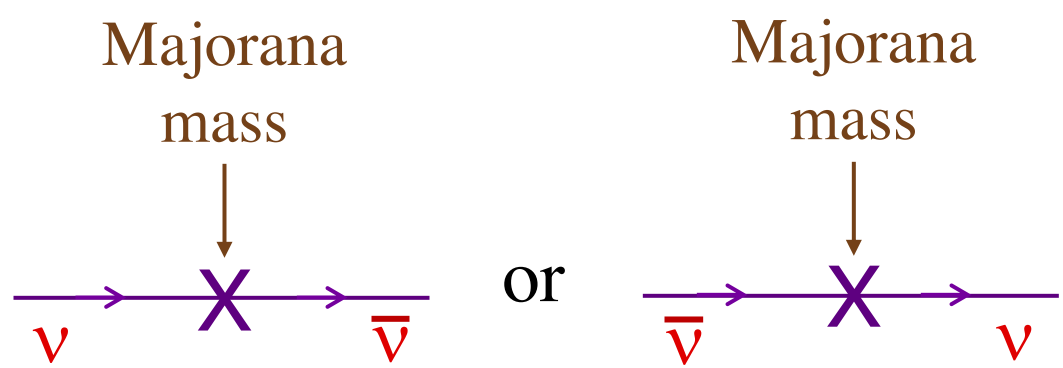
Dirac Mass

A Dirac mass has the effect:



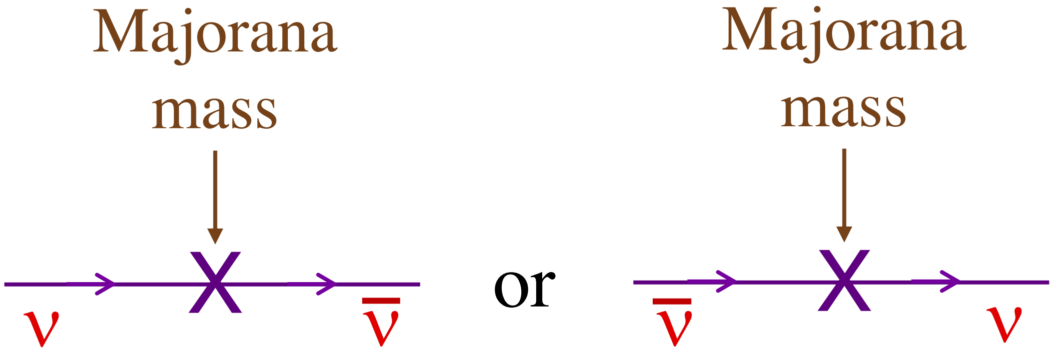
Majorana Mass

A Majorana mass has the effect:



Majorana Mass

A Majorana mass has the effect:



Majorana masses mix ν and $\bar{\nu}$, so they do not conserve the lepton number L that distinguishes leptons from antileptons:

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1$$

***The SM weak interactions and Dirac masses do conserve the lepton number* L.**

If there are no visibly large non-SM interactions that violate lepton number L, any violation of L that we might discover would have to come from Majorana neutrino masses.

A Majorana mass for any fermion f causes $f \longleftrightarrow \bar{f}$.

Quark and ***charged-lepton*** Majorana masses are **forbidden** by electric charge conservation.

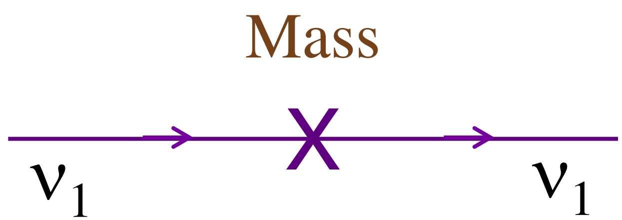
But ***neutrinos*** are electrically neutral, so they **can** have Majorana masses.

Neutrino Majorana masses would make the neutrinos *very* distinctive, because —

Majorana neutrino masses have a different origin than the quark and charged-lepton masses.

The Mass Eigenstates When the Mass Terms Are Majorana Masses

For any fermion mass eigenstate, e.g. ν_1, the action of its mass must be —

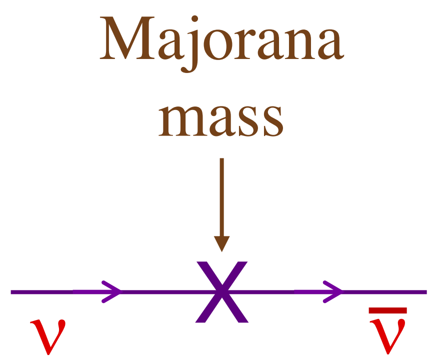


The mass eigenstate must be sent back into itself:

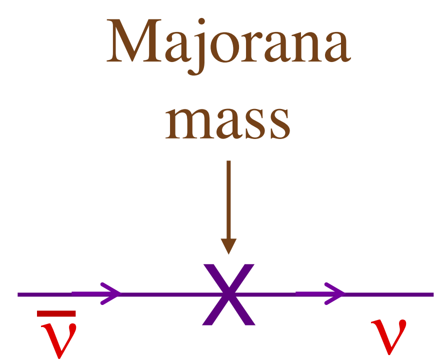
$$H\left|\nu_1\right\rangle = m_1\left|\nu_1\right\rangle$$

Recall that —

A ***Majorana*** mass has the effect:



or



Then the mass eigenstate neutrino ν_1 must be —

$$\nu_1 = \nu + \bar{\nu} \ ,$$

since this is the neutrino that the Majorana mass term sends back into itself, as required for any mass eigenstate particle:

Consequence: The neutrino mass eigenstates ν_1, ν_2, ν_3 are their own antiparticles.

$\bar{\nu}_i = \nu_i$ For given helicity

The Terminology

Suppose ν_i is a mass eigenstate,

with given helicty h.

- $\overline{\nu_i}(h) = \nu_i(h)$ *Majorana neutrino*

- $\overline{\nu_i}(h) \neq \nu_i(h)$ *Dirac neutrino*

These particles have opposite L.

We have just seen that if the underlying neutrino masses are *Majorana masses*, then the mass eigenstates are *Majorana neutrinos*.

Unlike the SM interaction and Dirac mass terms, Majorana mass terms do not conserve the Lepton Number L.

Hence, so long as Majorana mass terms are present, there is no conserved quantum number to distinguish ***neutrinos*** from ***antineutrinos***.

Thus, even if Dirac mass terms are also present,

$$\overline{\nu_i} = \nu_i$$

Majorana neutrinos

(Bilenky and Petcov)

Dirac and Majorana Masses — In Equations

A Little Background

The plane wave expansion of the free field of a Dirac (non-self-conjugate) fermion ψ is —

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \Big(\overset{\text{Absorbs particle}}{f_{p,\,s}}\, u_{p,\,s}\, e^{-ip\cdot x} + \overbrace{\bar{f}^{\dagger}_{p,\,s}}^{\text{Creates antiparticle}} v_{p,\,s}\, e^{ip\cdot x} \Big)$$

By comparison, the charge conjugate $\psi^c(x)$ of the field $\psi(x)$ is —

$$\psi^c(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \Big(\bar{f}_{p,\,s}\, u_{p,\,s}\, e^{-ip\cdot x} + f^{\dagger}_{p,\,s}\, v_{p,\,s}\, e^{ip\cdot x} \Big)$$

$$\boxed{\psi^c(x) = \psi(x;\ \text{particle} \longleftrightarrow \text{antiparticle})}$$

The plane wave expansion of the free field of a Majorana (self-conjugate) fermion ψ is —

$$\psi(x) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \Big(\overset{\text{Absorbs particle}}{f_{p,\,s}} u_{p,\,s} e^{-ip\cdot x} + \overbrace{f^{\dagger}_{p,\,s}}^{\text{Creates particle}} v_{p,\,s} e^{ip\cdot x} \Big)$$

This field is charge-self-conjugate:

$$\boxed{\psi^c(x) = \psi(x;\ \text{particle} \longleftrightarrow \text{antiparticle}) = \psi(x)}$$

Technically, $\psi^c(x)$ = (phase factor) $\psi(x)$ is allowed, but we can, and will, take this phase factor = 1.

Dirac Mass Term

This is the kind of mass term a quark has.

Suppose ν_L and ν_R are underlying chiral fields in terms of which we will construct a mass term for one neutrino. The antineutrinos these fields create are distinct from the neutrinos they absorb.

The ***Dirac*** mass term in the Lagrangian density is —

$$L_D = -m_D \bar{\nu}_R \nu_L + \text{ h.c. } = -m_D (\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R)$$

In terms of $\nu_1 \equiv \nu_L + \nu_R$, $L_D = -m_D \bar{\nu}_1 \nu_1$, since

$$\bar{\nu}_1 \nu_1 = \overline{(\nu_L + \nu_R)} (\nu_L + \nu_R) = \bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R$$

$L_D = -m_D \bar{\nu}_1 \nu_1$ is precisely the mass term in the Lagrangian density for a ***Dirac*** neutrino ***mass eigenstate*** ν_1 of mass m_D, since then the mass term H_m in the Hamiltonian has —

$$\langle \nu_1 \text{ at rest } | H_m | \nu_1 \text{ at rest} \rangle = \langle \nu_1 \text{ at rest } \left| m_D \int d^3x\, \bar{\nu}_1 \nu_1 \right| \nu_1 \text{ at rest} \rangle = m_D$$

As desired, the mass term has the effect —

Majorana Mass Term

In terms of the underlying field ν_R introduced earlier, a ***right-handed Majorana*** mass term in the Lagrangian density is —

Note: Charge conjugating a chiral field reverses its chirality.

$$L_R = -\frac{m_R}{2}\overline{(\nu_R)^c}\,\nu_R + \text{ h.c. } = -\frac{m_R}{2}\left[\overline{(\nu_R)^c}\,\nu_R + \overline{\nu_R}(\nu_R)^c\right]$$

In terms of $\nu_1 \equiv \nu_R + (\nu_R)^c$, $\boxed{L_R = -\frac{m_R}{2}\bar{\nu}_1\nu_1}$, since

$$\bar{\nu}_1\nu_1 = \overline{\left[\nu_R + (\nu_R)^c\right]}\left[\nu_R + (\nu_R)^c\right] = \overline{(\nu_R)^c}\,\nu_R + \overline{\nu_R}(\nu_R)^c$$

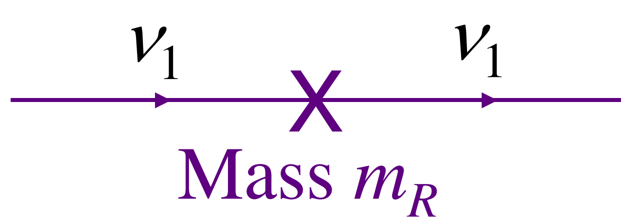
Notice that $(\nu_1)^c = \nu_1$. ***Antineutrino = neutrino***.

$L_R = -\frac{m_R}{2}\bar{\nu}_1\nu_1$ is precisely the mass term in the Lagrangian density for a ***Majorana*** neutrino ***mass eigenstate*** ν_1 of mass m_R, since then the mass term H_m in the Hamiltonian has —

$$\langle \nu_1 \text{ at rest} \,|H_m|\, \nu_1 \text{ at rest}\rangle = \langle \nu_1 \text{ at rest} \left| \frac{m_R}{2}\int d^3x\, \bar{\nu}_1\nu_1 \right| \nu_1 \text{ at rest} \rangle = m_R$$

The matrix element of $\bar{\nu}_1\nu_1$ is doubled in the Majorana case, because ν_1 can either absorb or create a neutrino, and so can $\bar{\nu}_1$. {The particles ν_1 and $\bar{\nu}_1$ are identical.}

As desired, the mass term has the effect —



The Standard Model Higgs mechanism is the origin of *Dirac* quark and neutrino masses

$$-\mathcal{L}_{SM} = \overset{\text{Coupling constant}}{y}\,\underset{\text{SM Higgs field}}{H_{SM}}\bar{q}_R q_L \Rightarrow y\underbrace{\langle H_{SM}\rangle}_{\text{vev}}\bar{q}_R q_L \equiv m_q \bar{q}_R q_L$$

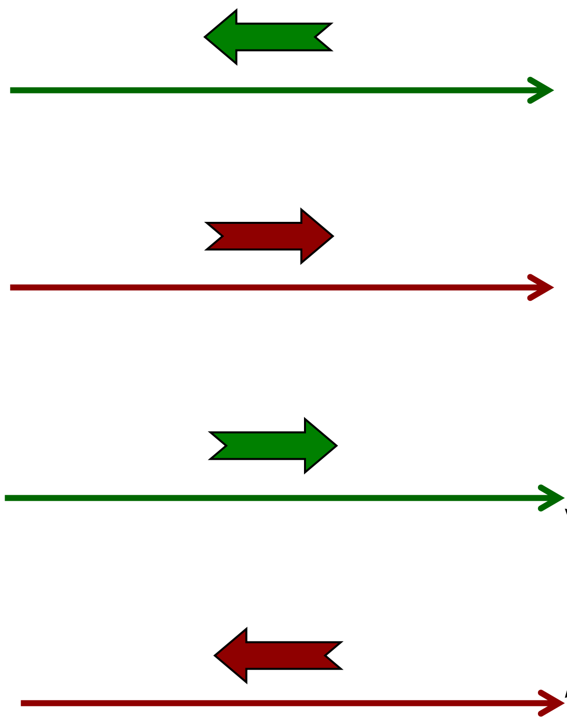
Possible (Weak-Isospin-Conserving) progenitors of *Majorana* masses:

$$\underbrace{H_{SM}H_{SM}\overline{\nu_L^c}\nu_L}_{\text{Not renormalizable}},\quad \underbrace{H_{I_W=1}}_{\text{This Higgs not in SM}}\overline{\nu_L^c}\nu_L,\quad \underbrace{m_R\overline{\nu_R^c}\nu_R}_{\text{No Higgs}}$$

The Interactions of Dirac, and Especially Majorana, Ultra-Relativistic Neutrinos

SM Interactions Of A Dirac Neutrino

We have 4 mass-degenerate states:

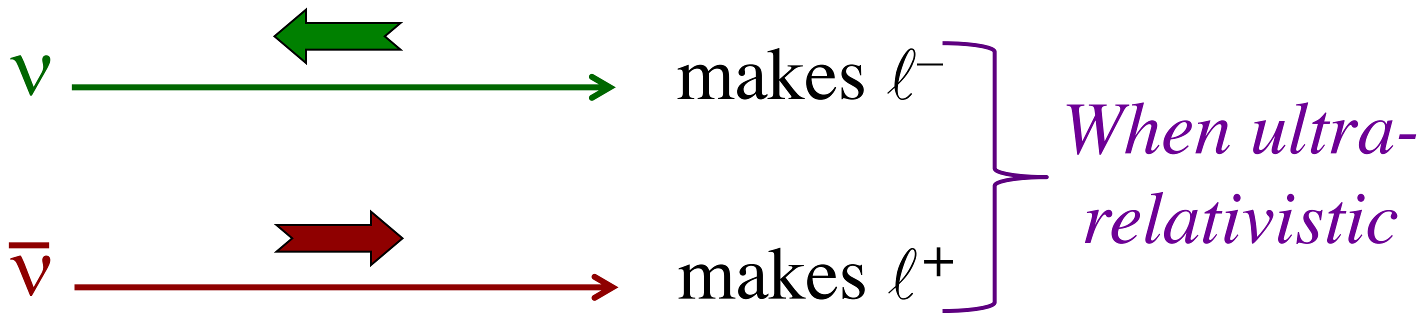


		Conserved L
ν	makes ℓ^-	+1
$\bar{\nu}$	makes ℓ^+	–1
ν		
$\bar{\nu}$		

These states, *when Ultra Rel.*, do not interact.
(The weak interaction is Left Handed.)

SM Interactions Of A Majorana Neutrino

We have only 2 mass-degenerate states:



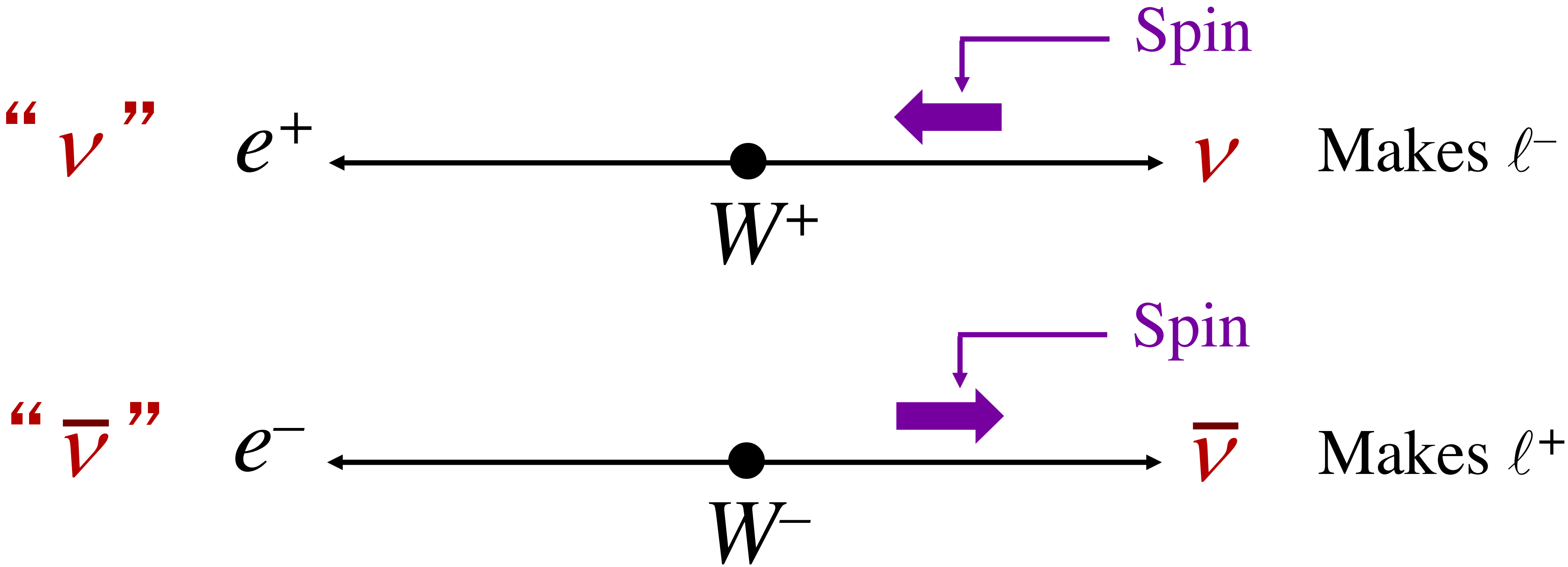
The SM weak interactions violate *parity*.

(They can tell *Left* from *Right*.)

An incoming left-handed neutral lepton makes ℓ^-.

An incoming right-handed neutral lepton makes ℓ^+.

Note: " ν " and " $\bar{\nu}$ " are ***produced*** with opposite helicity.



The weak interactions violate ***Parity***. ***Particles with left-handed and right-handed helicity can behave differently.***

For ***ultra-relativistic Majorana*** neutrinos,
helicity is a "substitute" for lepton number.

Majorana neutrinos behave
indistiguishably from Dirac neutrinos.

We have to find an exception, in order to
determine whether the neutrinos are
Majorana or Dirac particles.

Or, we have to work with ***non-relativistic*** neutrinos,
where there can be a big difference between the
behavior of Majorana neutrinos and Dirac neutrinos.

To Determine Whether Majorana Masses Occur in Nature, So That $\bar{\nu} = \nu$

The Promising Approach — Seek

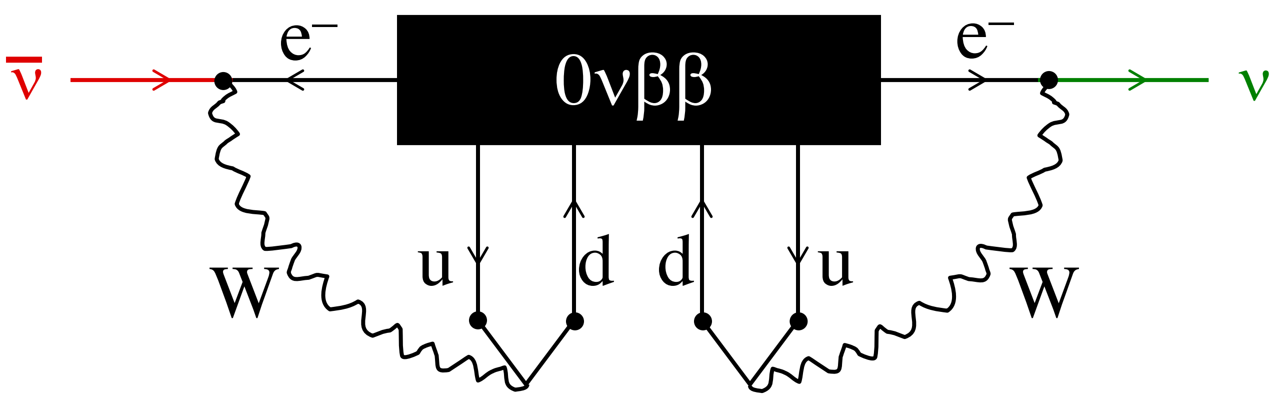
Neutrinoless Double Beta Decay $[0\nu\beta\beta]$

Observation at any non-zero level would imply —

- Lepton number L is not conserved ($\Delta L = 2$)
- Neutrinos have Majorana masses
- Neutrinos are Majorana particles (self-conjugate)

Whatever diagrams cause $0\nu\beta\beta$, its observation would imply the existence of a Majorana mass term:

(Schechter and Valle)

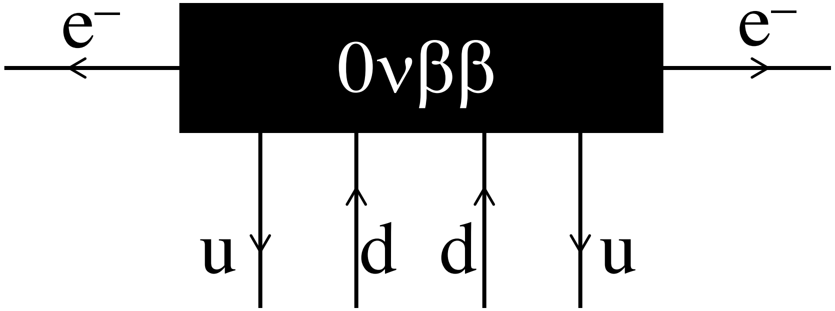


$\bar{\nu} \rightarrow \nu$: A (tiny) Majorana mass term

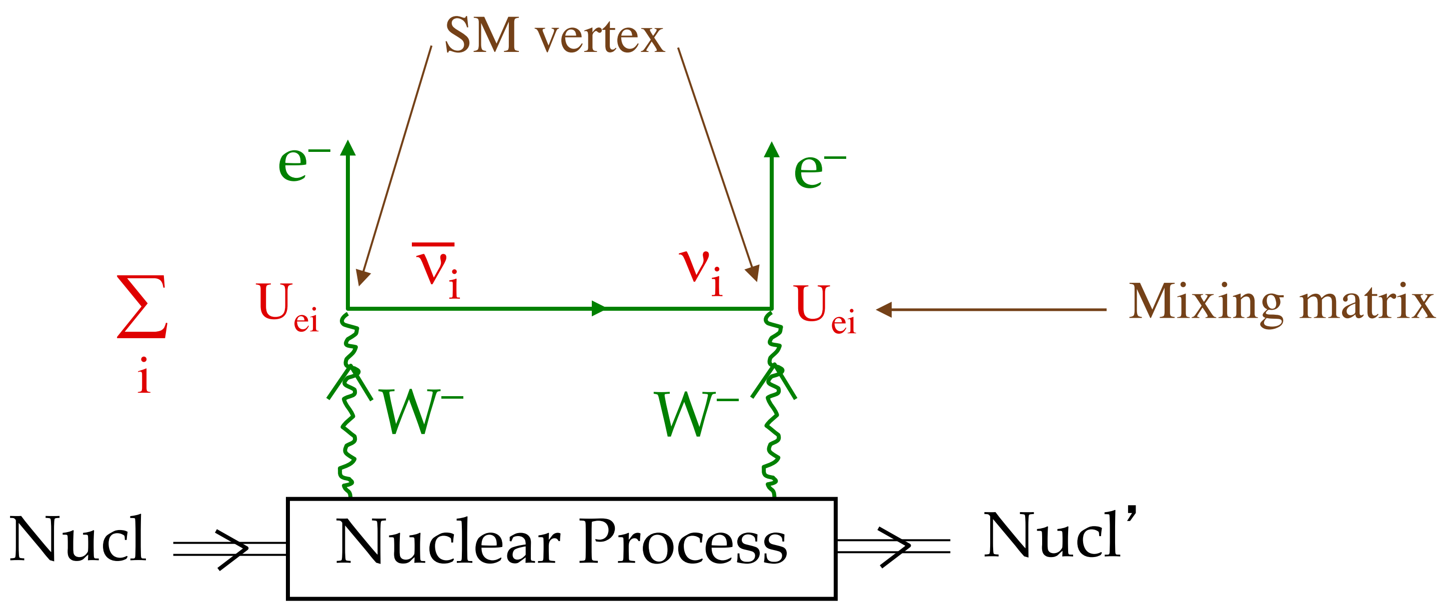
(Duerr, Lindner, Merle)

$\therefore$ $0\nu\beta\beta$ $\Rightarrow$ $\bar{\nu}_i = \nu_i$

What is inside?



We anticipate that 0νββ is dominated by a diagram with light neutrino exchange and Standard Model vertices:

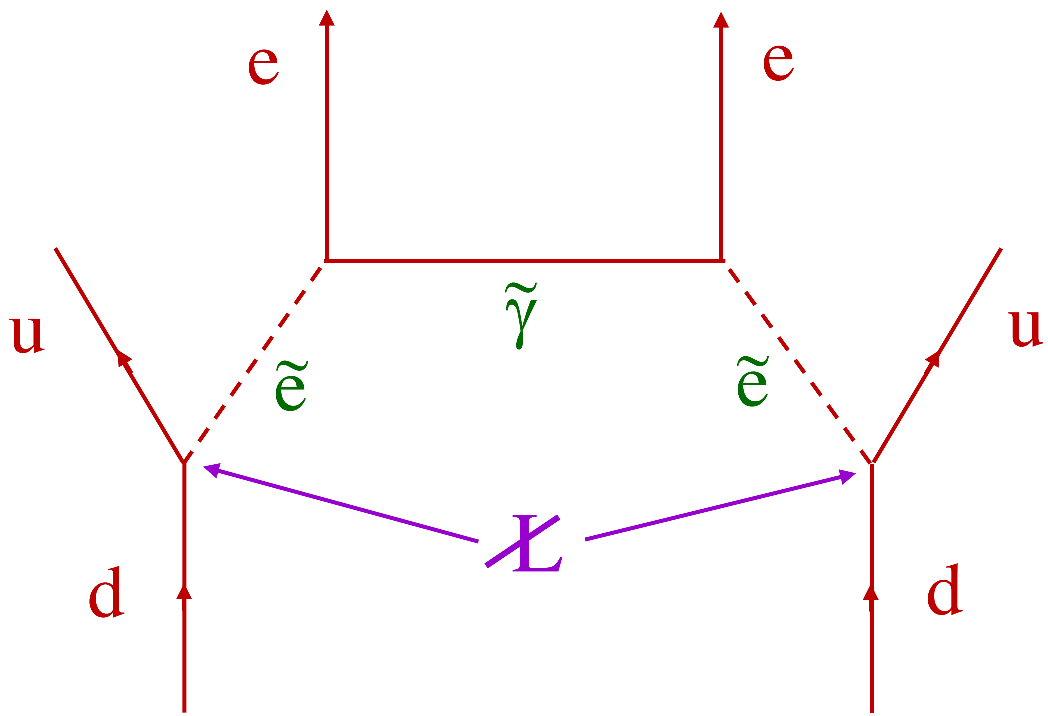


"The Standard Mechanism"

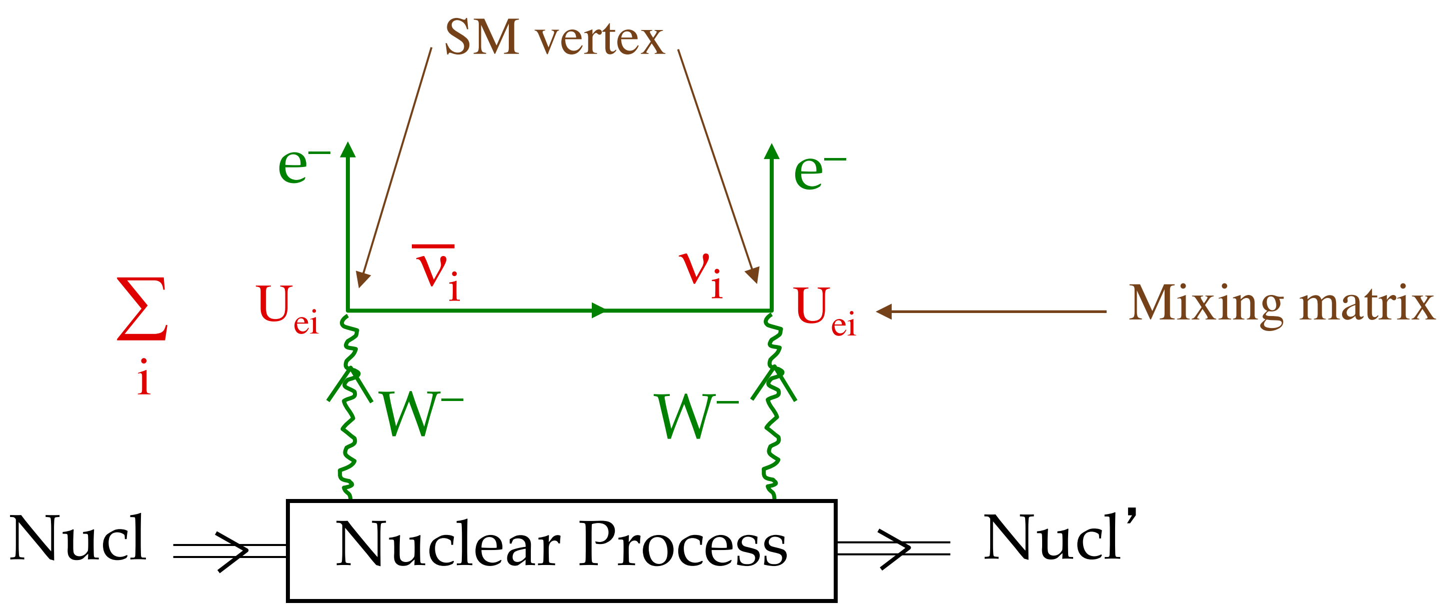
But there could be other contributions to $0\nu\beta\beta$, which at the quark level is the process

dd $\rightarrow$ uuee.

An example from Supersymmetry:



If the dominant mechanism is —



Then —

Mass (ν_i)

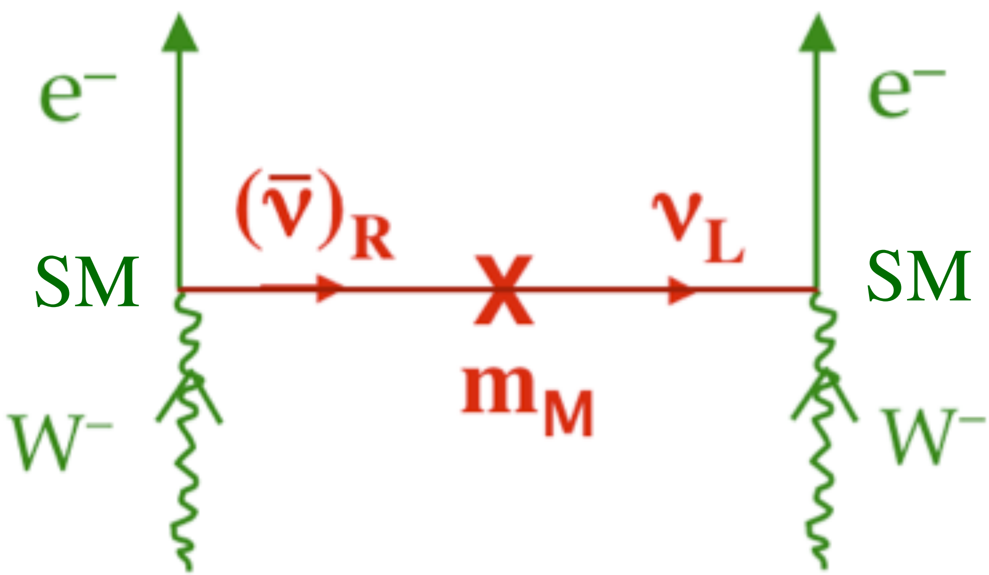
$$\mathrm{Amp}[0\nu\beta\beta] \propto \left| \sum m_i U_{ei}^2 \right| \equiv m_{\beta\beta}$$

(In the phase convention where $\nu_i^c = $ (no phase factor) ν_i.)

Why the neutrino mass is involved

$0\nu\beta\beta$ has $\Delta L = 2$. This ΔL must come from a ***Majorana mass***, since the SM interactions conserve L.

Viewed as a perturbation in a small Majorana mass m_M, what happens is —



The Search for CP Violation When It Might Be That

$$\overline{\nu} = \nu$$

A major open question is whether neutrino interactions violate CP invariance.

The experimental approach to testing for this violation is almost always described as the attempt to see whether —

$$P(\overline{\nu}_\mu \to \overline{\nu}_e) \neq P(\nu_\mu \to \nu_e).$$

This description is valid if $\overline{\nu} \neq \nu$, but not if $\overline{\nu} = \nu$.

However, the present and future experimental probes of leptonic CP-invariance violation are valid probes of this violation whether $\overline{\nu} \neq \nu$ *or* $\overline{\nu} = \nu$.

These experiments are ***completely insensitive*** to whether $\overline{\nu} \neq \nu$ or $\overline{\nu} = \nu$.

For any process $i \to f$, and its CP-mirror image $\mathrm{CP}(i) \to \mathrm{CP}(f)$, CP invariance means that —

$$\left|\langle f|T|i\rangle\right|^2 = \left|\langle \mathrm{CP}(f)|T|\mathrm{CP}(i)\rangle\right|^2.$$

So, compare two CP-mirror-image processes.

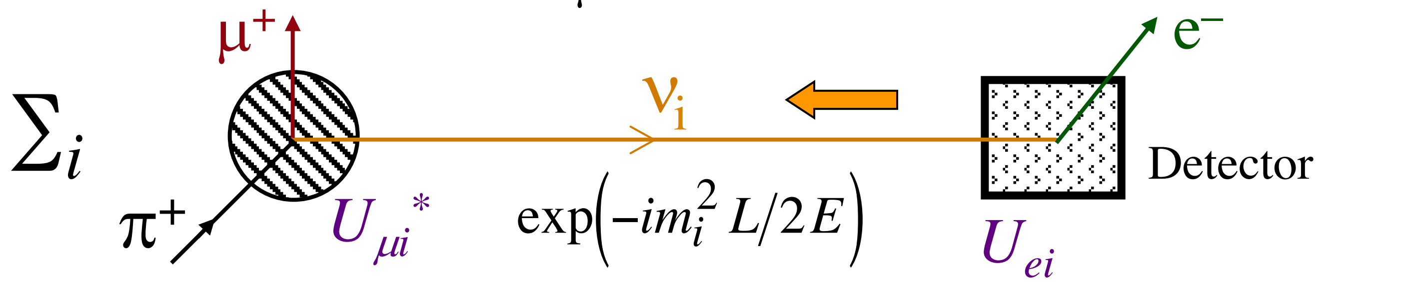
If they have different rates, CP invariance is violated.

Acting on a particle ψ with momentum $\vec{p}$ and helicity h,

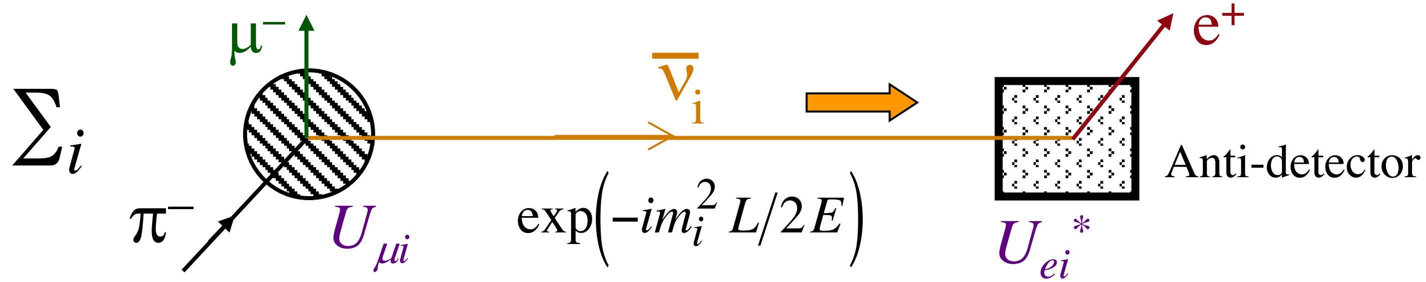
$$\mathrm{Rotation}(\pi)\mathrm{CP}\left|\psi(\vec{p},h)\right\rangle = \eta\left|\bar{\psi}(\vec{p},-h)\right\rangle$$

Irrelevant phase factor

Compare $\nu_\mu \to \nu_e$



with "$\bar{\nu}_\mu \to \bar{\nu}_e$"



If these two CP-mirror-image processes have different rates, CP invariance is violated.

Important Notice

To correct for our not using an anti-detector, we must know how the cross sections for left-handed and right-handed neutrinos to interact in a detector compare.

Experiments to determine these cross sections are very important.

(Lectures by Minerba Betancourt)