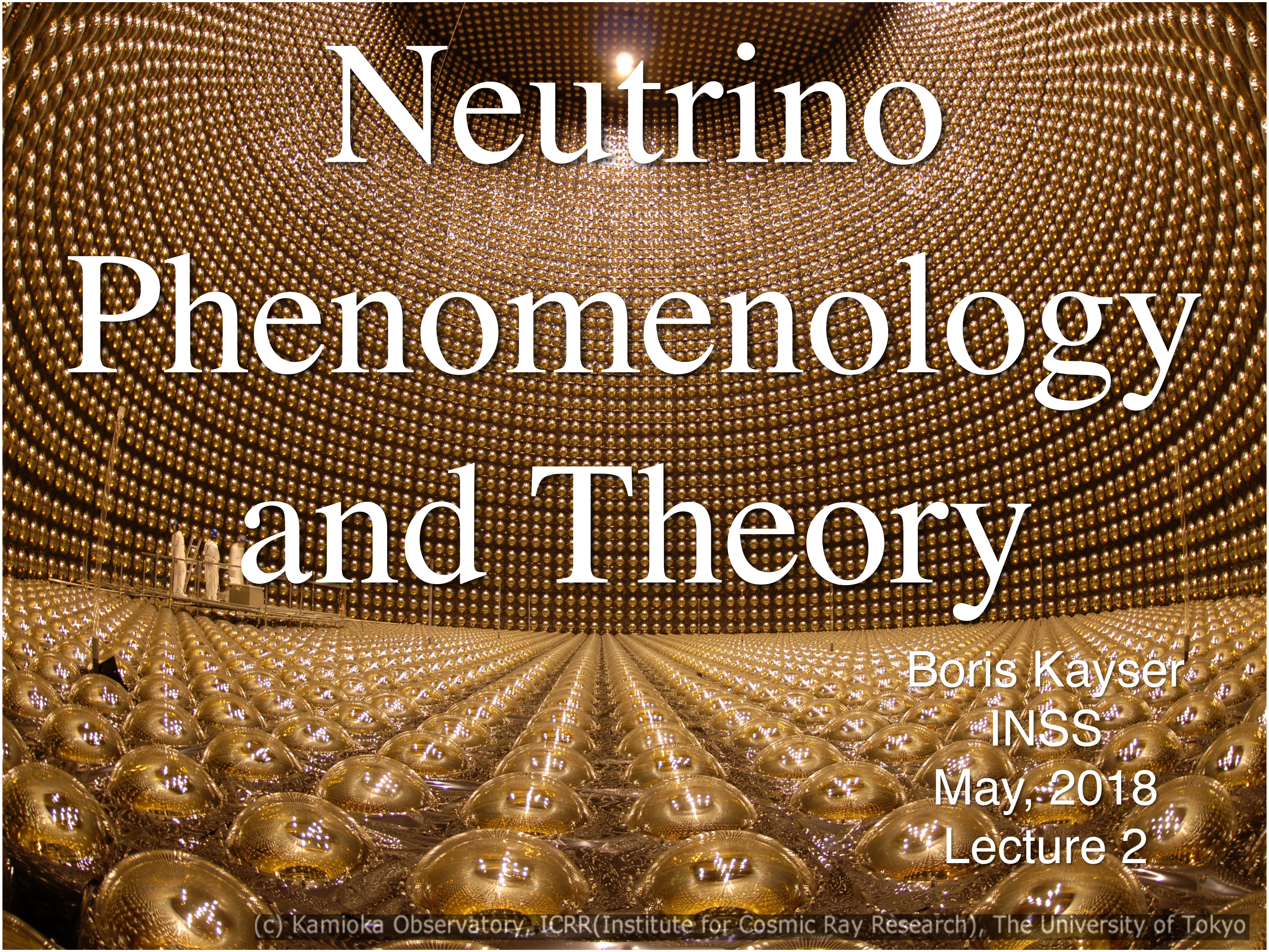
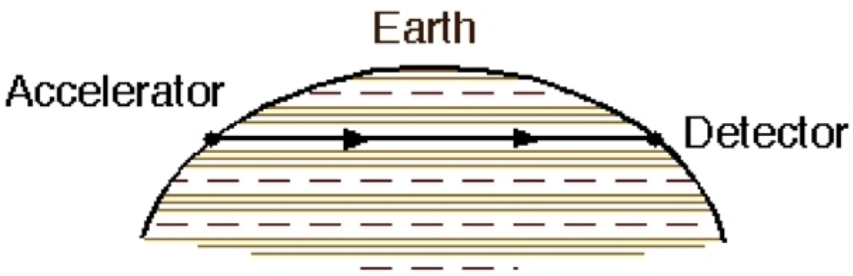


Neutrino Phenomenology and Theory

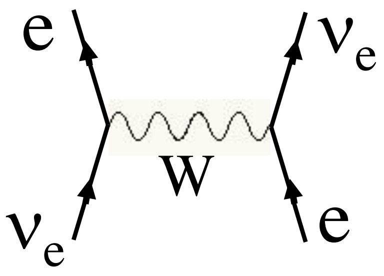
Boris Kayser
INSS
May, 2018
Lecture 2

(c) Kamioka Observatory, ICRR(Institute for Cosmic Ray Research), The University of Tokyo

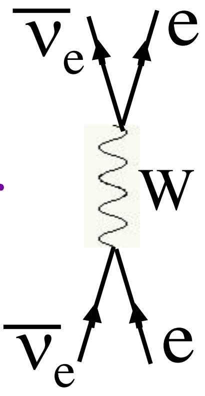
Neutrino Flavor Change In Matter



involves



or



Coherent forward scattering via this
W-exchange interaction leads to
an extra interaction potential energy —

$$V_W = \begin{cases} +\sqrt{2}G_F N_e, & \nu_e \\ -\sqrt{2}G_F N_e, & \overline{\nu_e} \end{cases}$$

Fermi constant

Electron density

This raises the effective mass of ν_e, and lowers that of $\overline{\nu_e}$.

The fractional importance of matter effects on an oscillation involving a vacuum splitting Δm^2 is —

$$\overbrace{[\sqrt{2}G_F N_e]}^{\text{Interaction energy}} \; / \; \overbrace{[\Delta m^2/2E]}^{\text{Vacuum energy}} \; .$$

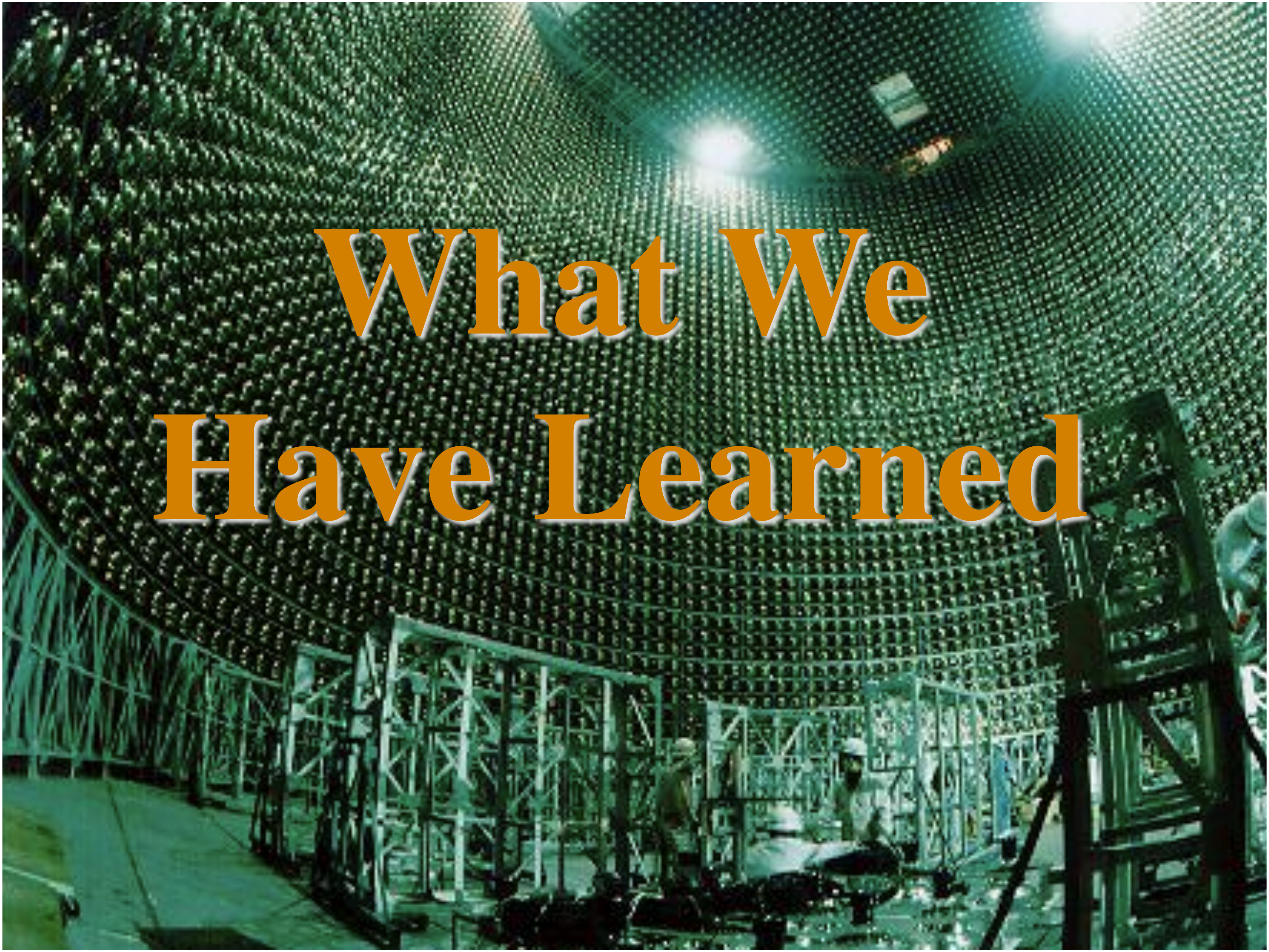
The matter effect —

- — Grows with neutrino energy E
- — Is sensitive to Sign(Δm^2)
- — Reverses when ν is replaced by $\bar{\nu}$

This last is a "fake CP violation" that has to be taken into account in searches for genuine CP violation.

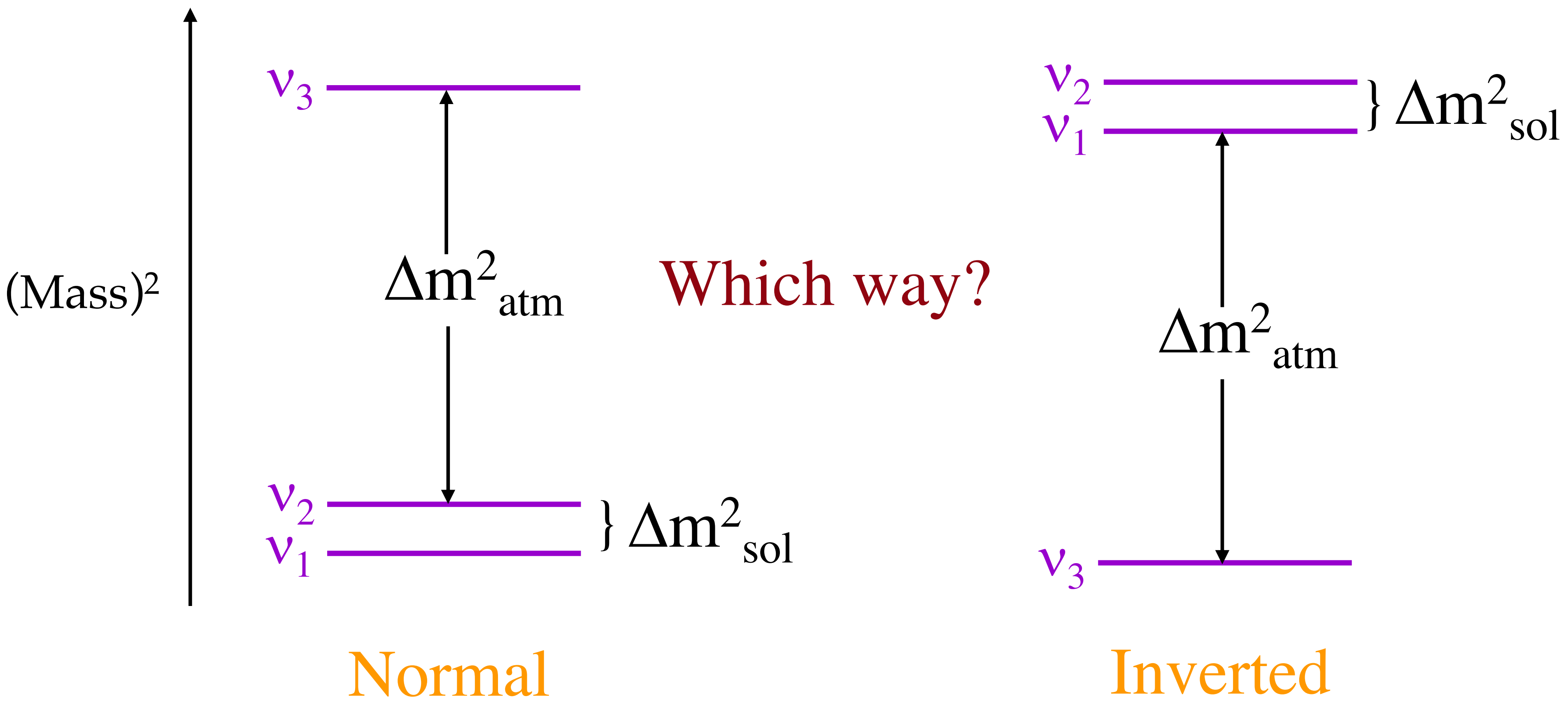
Evidence For Flavor Change

Neutrinos	Evidence of Flavor Change
Solar	Compelling
Reactor (Long-Baseline)	Compelling
Atmospheric	Compelling
Accelerator (Long-Baseline)	Compelling
Accelerator, Reactor, and Radioactive Sources (Short-Baseline)	“Interesting”



What We Have Learned

The (Mass)^2 Spectrum



(Mass)^2

ν_3

Δm^2_{atm}

ν_2

ν_1

} Δm^2_{sol}

Which way?

ν_2

ν_1

} Δm^2_{sol}

Δm^2_{atm}

ν_3

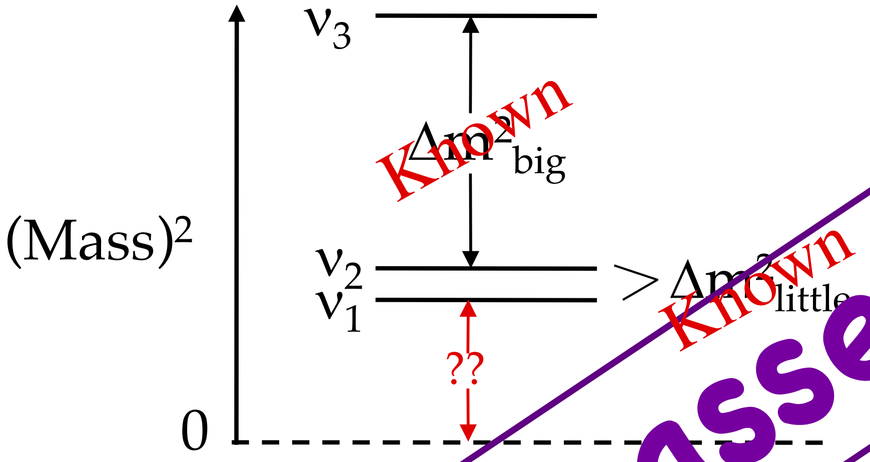
Normal

Inverted

$\Delta m^2_{sol} \cong 7.3 \times 10^{-5}$ eV^2, $\Delta m^2_{atm} \cong 2.5 \times 10^{-3}$ eV^2

Are there ***more*** mass eigenstates?

Constraints On the Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Neutrino masses are tiny

Cosmology, *under certain assumptions* → $\sum_{\text{All } i} m(\nu_i) < 0.2$ eV

Tritium beta decay → $\sqrt{0.69m^2(\nu_1)+0.29m^2(\nu_2)+0.02m^2(\nu_3)} < 2$ eV

Oscillation → Mass[Heaviest ν_i] > $\sqrt{\Delta m^2_{\text{big}}}$ > 0.05 eV

What Tritium *β* Decay Measures

Tritium decay: $^{3}H \rightarrow {}^{3}He + e^{-} + \bar{\nu}_i\ ;\ i = 1, 2, \text{ or } 3$

There are 3 distinct final states.

The amplitudes for the production of these 3 distinct final states contribute *incoherently*.

$$BR\left({}^{3}H \rightarrow {}^{3}He + e^{-} + \bar{\nu}_i\right) \propto \left|U_{ei}\right|^2$$

In $^{3}H \rightarrow {}^{3}He + e^{-} + \bar{\nu}_i$, the bigger m_i is, the smaller the maximum electron energy is.

There are 3 separate thresholds in the β energy spectrum.

The β energy spectrum is modified according to —

$$(E_0 - E)^2 \Theta[E_0 - E] \Rightarrow \sum_i |U_{ei}|^2 (E_0 - E)\sqrt{(E_0 - E)^2 - m_i^2}\, \Theta[(E_0 - m_i) - E]$$

Maximum β energy when there is no neutrino mass

β energy

Present experimental energy resolution is insufficient to separate the thresholds.

Measurements of the spectrum bound the average neutrino mass —

$$\langle m_\beta \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

$$\boxed{\text{Presently: } \langle m_\beta \rangle < 2 \text{ eV}}$$

Mainz & Troitzk

Leptonic Mixing

Mixing means that —

$$|\nu_\alpha> = \sum_i U^*_{\alpha i} \, |\nu_i> \quad .$$

Neutrino of flavor

α = e, μ, or τ

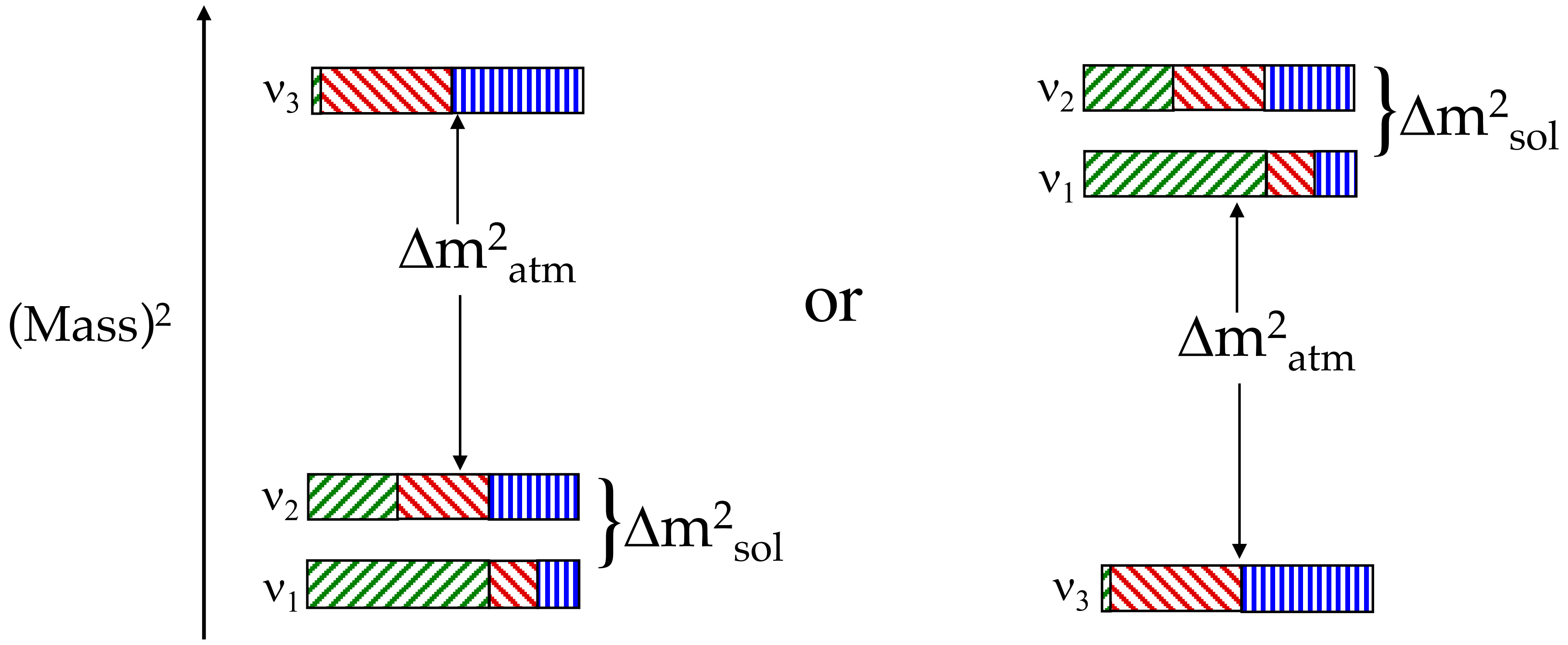
Neutrino of definite mass m_i

Inversely, $|\nu_i> = \sum_\alpha U_{\alpha i} \, |\nu_\alpha> \, .$ (***if*** *U* is unitary)

Flavor-α fraction of $\nu_i = |U_{\alpha i}|^2$.

When a ν_i interacts and produces a charged lepton,
the probability that this charged lepton
will be of flavor α is $|U_{\alpha i}|^2$.

Experimentally, the flavor fractions are —



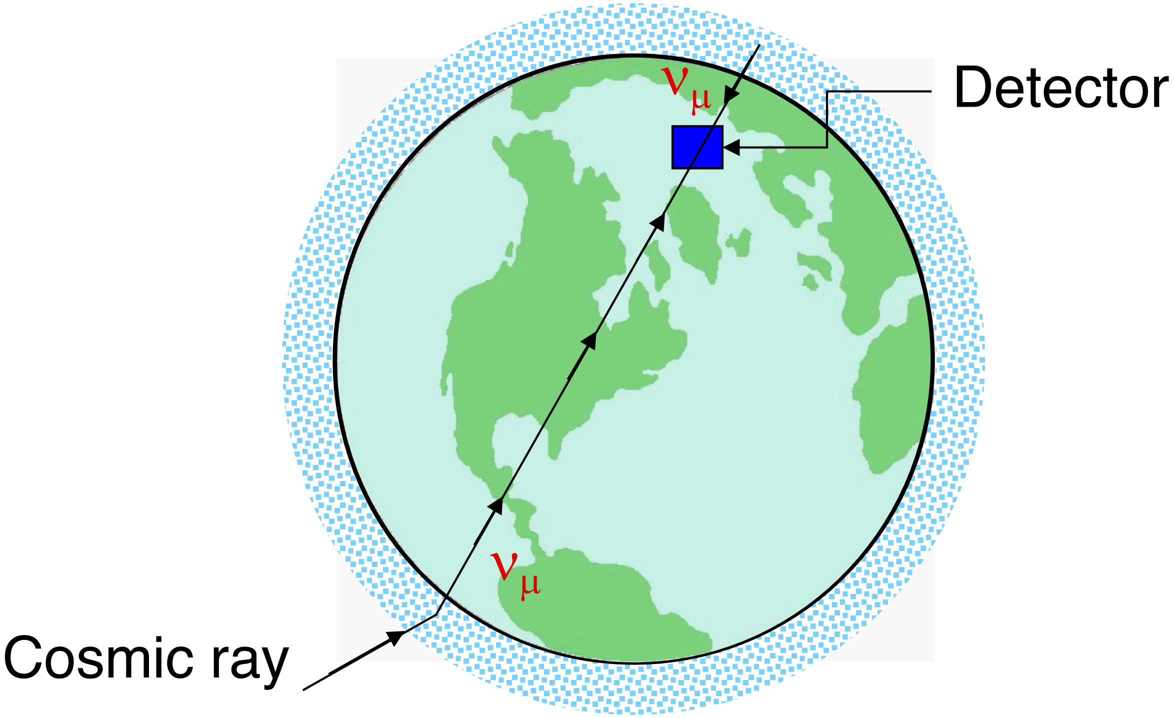
Normal

Inverted

▨ $\nu_e\ [\,|U_{ei}|^2]$ ▨ $\nu_\mu[\,|U_{\mu i}|^2]$ ▥ $\nu_\tau\ [\,|U_{\tau i}|^2]$

Observations We Can Use To Understand The Flavor Fractions

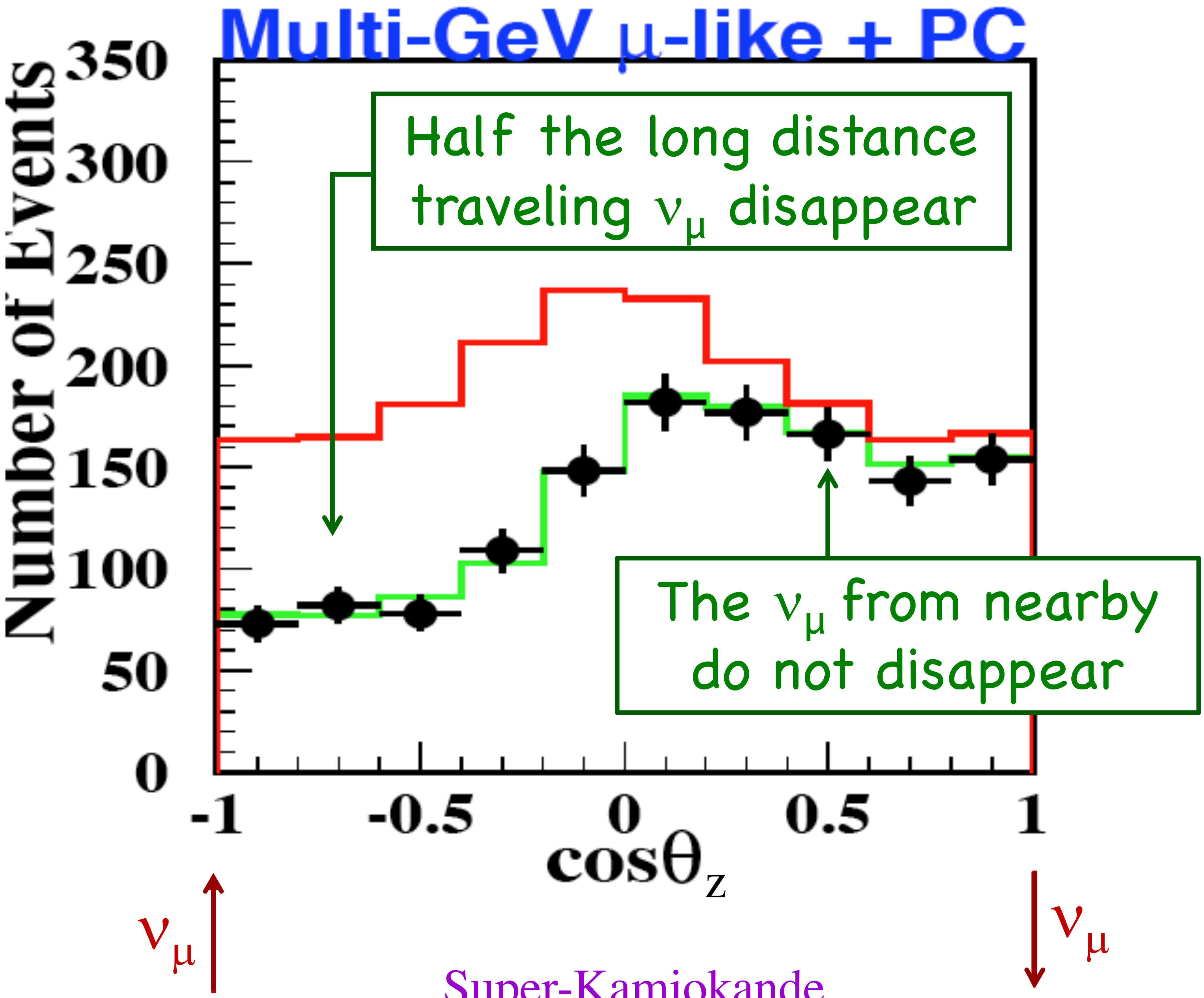
The Disappearance of Atmospheric ν_μ



Isotropy of the $\gtrsim$ 2 GeV cosmic rays + Gauss' Law + No ν_μ disappearance

$$\Rightarrow \quad \frac{\phi_{\nu_\mu}(\mathrm{Up})}{\phi_{\nu_\mu}(\mathrm{Down})} = 1 \ .$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV —

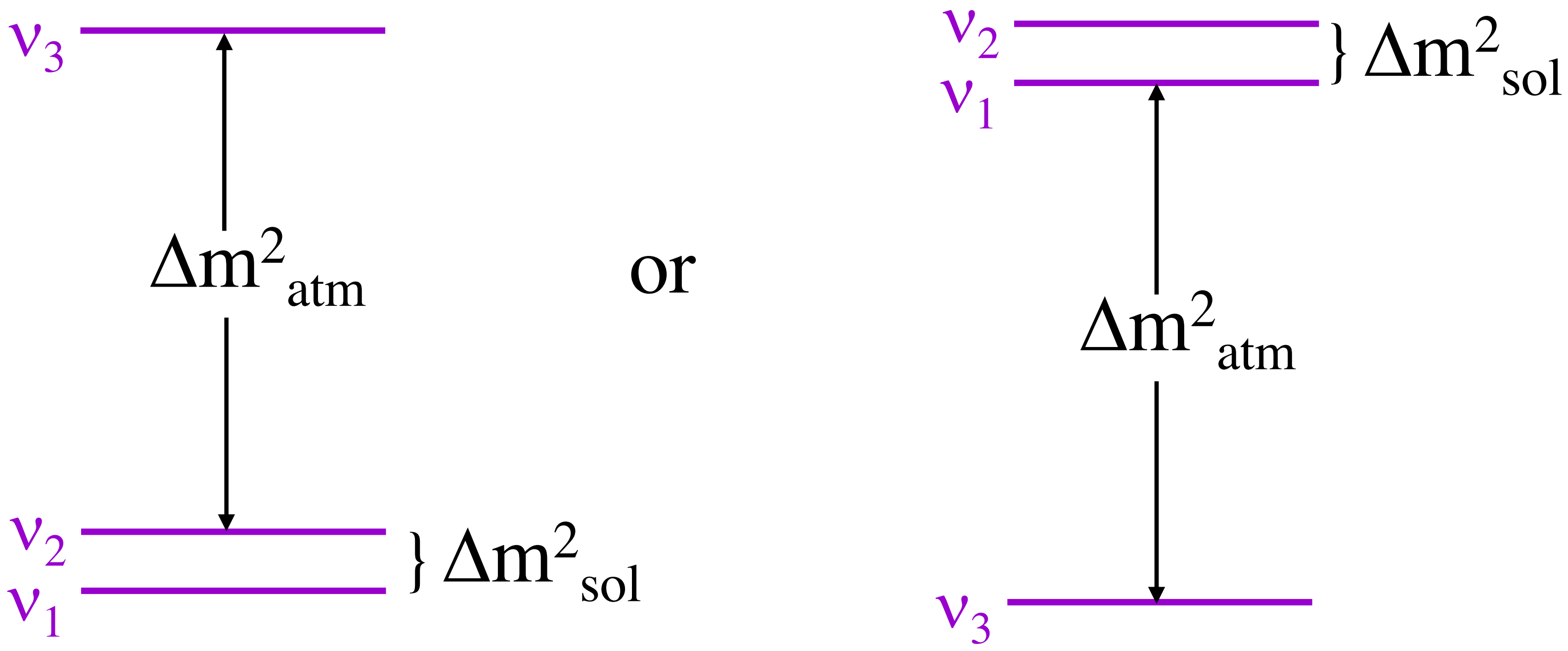


Half the long distance traveling ν_μ disappear

The ν_μ from nearby do not disappear

Super-Kamiokande

At $E_\nu > 1.3$ GeV, in —



the solar splitting is largely invisible. Then—

$$\underbrace{P\left(\nu_\mu \rightarrow \nu_\mu\right)}_{\frac{1}{2}} \cong 1 - \underbrace{4\left|U_{\mu 3}\right|^2\left(1-\left|U_{\mu 3}\right|^2\right)}_{1 \;\Rightarrow\; \left|U_{\mu 3}\right|^2 = \frac{1}{2}} \underbrace{\sin^2\left[1.27\Delta m^2_{\text{atm}}\frac{L(\text{km})}{E(\text{GeV})}\right]}_{\frac{1}{2}}$$

$\boxed{\text{At large } L/E}$

Reactor – Neutrino Experiments and $|U_{e3}|^2 = \sin^2\theta_{13}$

Reactor $\overline{\nu}_e$ have $E \sim 3$ MeV, so if $L \sim 1.5$ km,

$$\sin^2\left[1.27\Delta m^2\left(\mathrm{eV}^2\right)\frac{L(\mathrm{km})}{E(\mathrm{GeV})}\right]$$ will be sensitive to —

$$\Delta m^2 = \Delta m^2_{\mathrm{atm}} = 2.5 \,\mathrm{x}\, 10^{-3}\,\mathrm{eV}^2 = \frac{1}{400}\mathrm{eV}^2$$

but not to —

$$\Delta m^2 = \Delta m^2_{\mathrm{sol}} = 7.5 \,\mathrm{x}\, 10^{-5}\,\mathrm{eV}^2 \approx \frac{1}{13{,}000}\mathrm{eV}^2 \ .$$

Then —

$$P\left(\bar{\nu}_e \rightarrow \bar{\nu}_e\right) \cong 1-4\left|U_{e3}\right|^2\left(1-\left|U_{e3}\right|^2\right)\sin^2\left[1.27\Delta m^2_{\text{atm}}\,\frac{L(\text{km})}{E(\text{GeV})}\right]$$

Measurements by the Daya Bay, RENO,
and Double CHOOZ reactor neutrino experiments,
(and by the T2K accelerator neutrino experiment)

$$\Longrightarrow \; |U_{e3}|^2 \cong 0.02$$

The Change of Flavor of Solar ν_e

Nuclear reactions in the core of the sun produce ν_e. Only ν_e.

The Sudbury Neutrino Observatory (SNO) measured, for the high-energy part of the solar neutrino flux:

$$\nu_{sol}\, d \rightarrow e\, p\, p \;\Rightarrow \phi_{\nu e}$$

$$\nu_{sol}\, d \rightarrow \nu\, n\, p \;\Rightarrow \phi_{\nu e} + \phi_{\nu \mu} + \phi_{\nu \tau} \qquad (\nu \text{ remains a } \nu)$$

From the two reactions,

$$\frac{\phi_{\nu e}}{\phi_{\nu e} + \phi_{\nu \mu} + \phi_{\nu \tau}} = 0.301 \pm 0.033$$

For solar neutrinos, $P(\nu_e \rightarrow \nu_e) = 0.3$

The Significance of $P(\nu_e \rightarrow \nu_e)$

For SNO-energy-range solar neutrinos,
there is a very pronounced solar matter effect.

(Mikheyev and Smirnov)

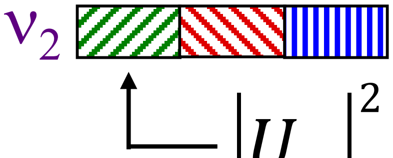
At these energies —

A solar neutrino is born in the core of the sun as a $\boldsymbol{\nu_e}$.

But by the time it emerges from the outer edge
of the sun, with 91% probability it is a $\boldsymbol{\nu_2}$.

(Nunokawa, Parke, Zukanovich-Funchal)

$$\text{Then } P(\nu_e \rightarrow \nu_e) \text{ at earth} = \left|\langle \nu_e | \nu_2 \rangle\right|^2 = \left|U_{e2}\right|^2.$$



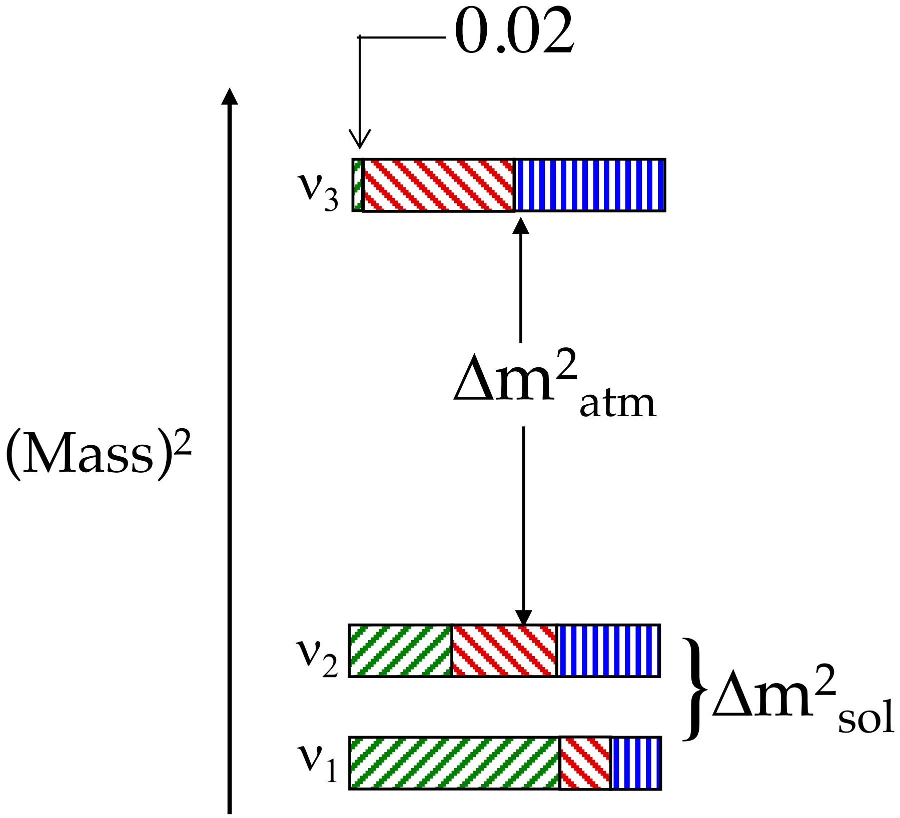
$$\left|U_{e2}\right|^2 = 0.3.$$

Constructing the Approximate Mixing Matrix

(A Blackboard Exercise)

The result —

$$U \approx \begin{array}{cccc} \nu_1 & \nu_2 & \nu_3 & \\ \left[\begin{array}{c} \sqrt{\frac{2}{3}} \\ -\sqrt{\frac{1}{6}} \\ \sqrt{\frac{1}{6}} \end{array}\right. & \begin{array}{c} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} \\ -\sqrt{\frac{1}{3}} \end{array} & \left.\begin{array}{c} 0 \\ \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} \end{array}\right] & \begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \end{array}$$



$$U \approx \begin{array}{cc} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} & \\ \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{bmatrix} & \begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \end{array}$$

▨ ν_e [$|U_{ei}|^2$] ▧ ν_μ [$|U_{\mu i}|^2$] ▥ ν_τ [$|U_{\tau i}|^2$]

Parametrizing the 3 X 3 Unitary Leptonic Mixing Matrix

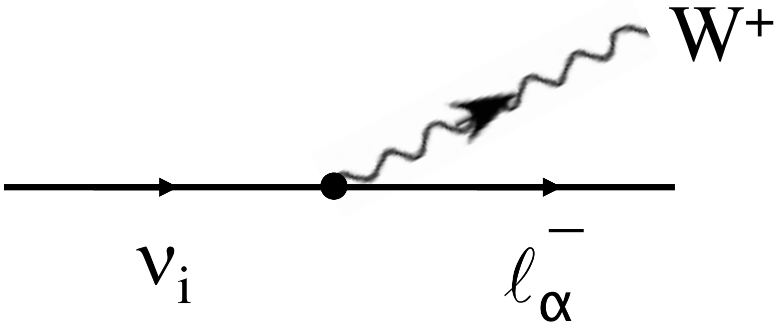
Caution: We are *assuming* the mixing matrix U to be 3 x 3 and unitary.

$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\overline{\ell}_{L\alpha}\gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \overline{\nu}_{Li}\gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

$$(CP)\left(\overline{\ell}_{L\alpha}\gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}{}^{-}\right)(CP)^{-1} = \overline{\nu}_{Li}\gamma^{\lambda} U_{\alpha i} \ell_{L\alpha} W_{\lambda}{}^{+}$$

Phases in U will lead to CP violation, unless they are removable by redefining the leptons.

$U_{\alpha i}$ describes —



$$U_{\alpha i} \sim \langle \ell_\alpha^- W^+ | H | \nu_i \rangle$$

When $|\nu_i\rangle \to |e^{i\varphi} \nu_i\rangle$, $U_{\alpha i} \to e^{i\varphi}\, U_{\alpha i}$, all α

When $|\ell_\alpha^-\rangle \to |e^{i\varphi} \ell_\alpha^-\rangle$, $U_{\alpha i} \to e^{-i\varphi}\, U_{\alpha i}$, all i

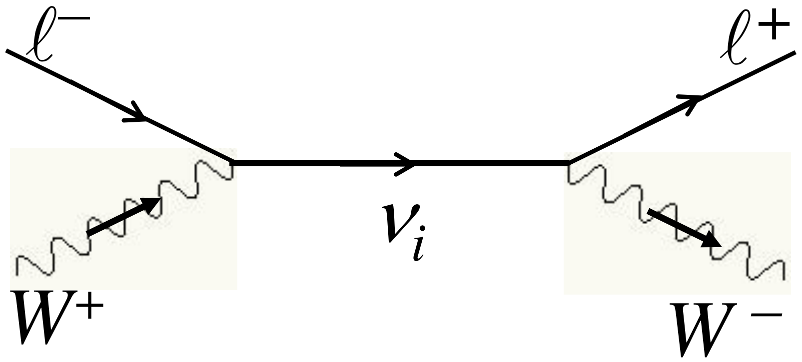
Thus, one may multiply any column, or any row, of U by a complex phase factor without changing the physics.

Some phases may be removed from U in this way.

When the Neutrino Mass Eigenstates Are Their Own Antiparticles

When this is the case, processes that
do not conserve the lepton number
$L \equiv \#(\text{Leptons}) - \#(\text{Antileptons})$ can occur.

Example:



The amplitude for any such L-violating process contains an extra phase factor.

When we phase-redefine ν_i to remove a phase from U, that phase just moves to the extra factor.

It does not disappear from the physics.

Hence, when $\overline{\nu}_i = \nu_i$, U can contain extra physically-significant phases.

These are called Majorana phases.

How Many Mixing Angles and ~~CP~~ Phases Does U Contain?

Real parameters before constraints: 18

Unitarity constraints — $\sum_i U_{\alpha i}{}^{*} U_{\beta i} = \delta_{\alpha\beta}$

Each row is a vector of length unity: – 3

Each two rows are orthogonal vectors: – 6

Rephase the three ℓ_α: – 3

Rephase two ν_i , if $\overline{\nu}_i \neq \nu_i$: – 2

Total physically-significant parameters: 4

Additional (Majorana) ~~CP~~ phases if $\overline{\nu}_i = \nu_i$: 2

How Many Of The Parameters Are Mixing Angles?

The *mixing angles* are the parameters in U when it is *real*.

U is then a three-dimensional rotation matrix.

Everyone knows such a matrix is described in terms of 3 angles.

Thus, U contains 3 mixing angles.

Summary

Mixing angles	$\cancel{CP}$ phases if $\overline{\nu}_i \neq \nu_i$	$\cancel{CP}$ phases if $\overline{\nu}_i = \nu_i$
3	1	3

The Lepton Mixing Matrix *U*

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij}$$
$$s_{ij} \equiv \sin\theta_{ij}$$

$$\times \underbrace{\begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Majorana phases}}$$

Note big mixing!

$$\overbrace{\theta_{12} \approx 33^\circ \;\;, \theta_{23} \approx 41\text{-}51^\circ}^{} \;\;, \;\; \theta_{13} \approx 8.4^\circ \leftarrow$$ *Not very small!*

(Capozzi, Lisi, Marrone, Palazzo)

The phases violate CP. δ would lead to $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \underset{\overset{\uparrow}{\cancel{CP}}}{\neq} P(\nu_\alpha \rightarrow \nu_\beta)$.

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

There is already a 2σ hint of $\cancel{CP}$ ($\sin\delta \neq 0$). (T2K)

$c_{ij} \equiv \cos\theta_{ij}$
$s_{ij} \equiv \sin\theta_{ij}$

The leptonic mixing matrix U is —

$$U = \begin{array}{c} \\ \\ \end{array}\left[\begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \\ c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{array}\right]$$

$$\times \mathrm{diag}\left(\underbrace{\mathrm{e}^{\mathrm{i}\alpha_1/2}, \mathrm{e}^{\mathrm{i}\alpha_2/2}}_{\text{Majorana phases}}, 1\right)$$

The Majorana ~~CP~~ Phases

The phase α_i is associated with neutrino mass eigenstate ν_i:

$$U_{\alpha i} = U^0{}_{\alpha i} \exp(i\alpha_i/2) \text{ for all flavors } \alpha.$$

$$\mathrm{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}{}^* \exp(-\, im_i{}^2L/2E)\, U_{\beta i}$$

is insensitive to the Majorana phases α_i .

Only the phase δ can cause CP violation in neutrino oscillation.

There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for $\cancel{CP}$ in oscillation.

For example —

$$P\left(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}\right) - P\left(\nu_{\mu} \rightarrow \nu_{e}\right) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$

$$\times \sin\left(\Delta m^{2}{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^{2}{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^{2}{}_{21}\frac{L}{4E}\right)$$

In the factored form of U, one can put
δ next to θ_{12} instead of θ_{13}.