Neutrino Phenomenology and Theory

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INSS
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Lecture 1

NASA Hubble Photo

What Are Neutrinos Good For?

Energy generation in the sun starts with the reaction —

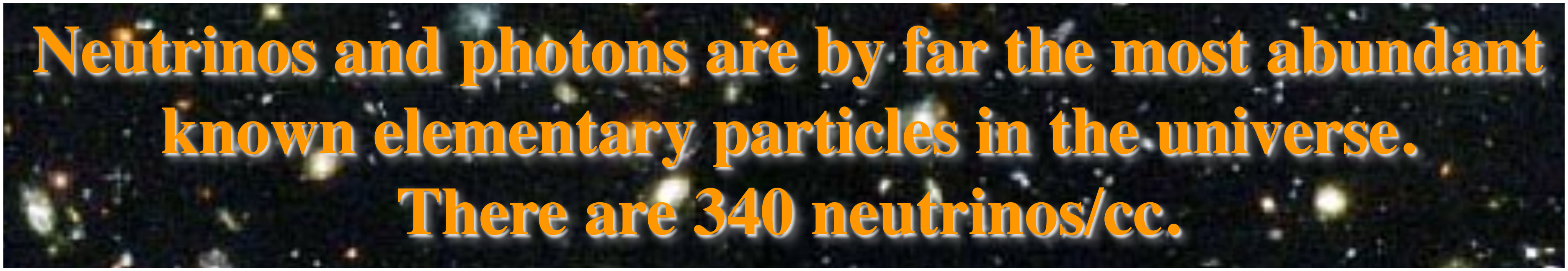
$$p + p \longrightarrow d + e^{+} + \nu$$

$$\text{Spin:} \quad \frac{1}{2} \quad \frac{1}{2} \qquad 1 \quad \frac{1}{2} \quad \frac{1}{2}$$

Without the neutrino, angular momentum would not be conserved.

Uh, oh ……

The Neutrinos



The neutrinos are spin – 1/2, electrically neutral, leptons.

The only known forces they experience are the weak force and gravity.

This means that they do not interact with other matter very much at all. Thus, neutrinos are difficult to detect and study.

Their weak interactions are successfully described by the Standard Model.

The Neutrino Revolution
(1998 – …)

Neutrinos have nonzero masses!

Leptons mix!

The 2015 Nobel Prize in Physics went to **Takaaki Kajita** and **Art McDonald** for the experiments that proved this.

Super-Kamiokande, Japan

Sudbury Neutrino Observatory, Canada

The Origin of Neutrino Mass

One of the most fundamental questions we ask in elementary particle physics is:

What is the origin of mass?

The fundamental constituents of matter are the ***quarks***, the ***charged leptons***, and the ***neutrinos***.

The discovery and study of the ***Higgs boson*** at CERN's Large Hadron Collider has provided strong evidence that the ***quarks*** and ***charged leptons*** derive their masses from an interaction with the ***Higgs field***.

Most theorists strongly suspect that the origin of the neutrino masses is different from the origin of the quark and charged lepton masses.

The Standard-Model ***Higgs field*** is probably still involved, but there is probably something more — something way outside the Standard Model —

Majorana masses.

More later ……..

The discovery that neutrinos have masses and leptons mix comes from the observation of ***neutrino flavor change (neutrino oscillation)***.

The Physics of Neutrino Oscillation

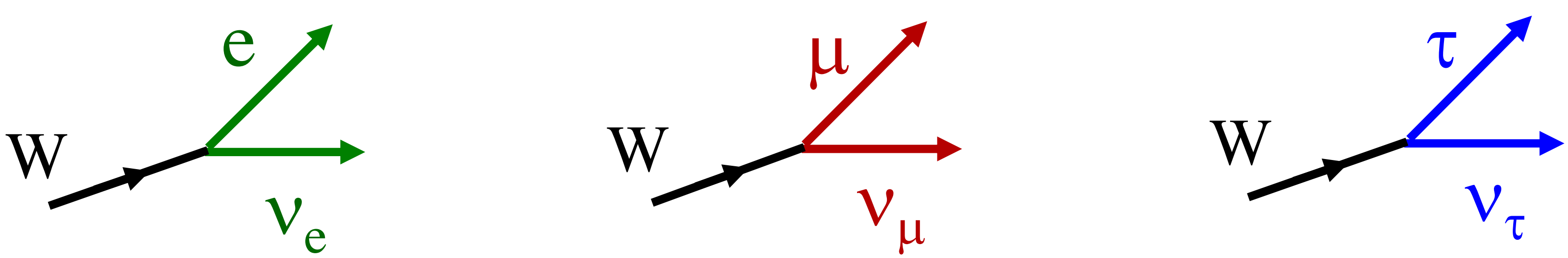
— Preliminaries

The Neutrino Flavors

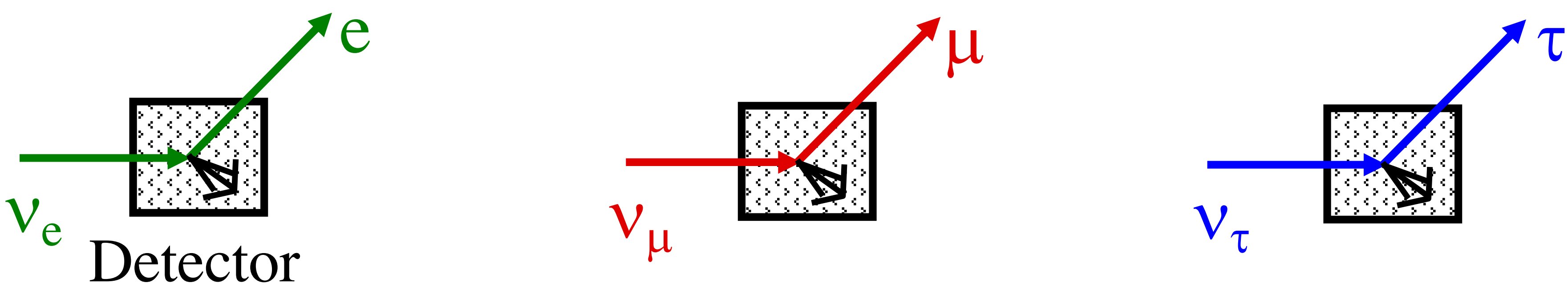
There are three flavors of charged leptons: e , μ , τ

There are three known flavors of neutrinos: ν_e, ν_μ, ν_τ

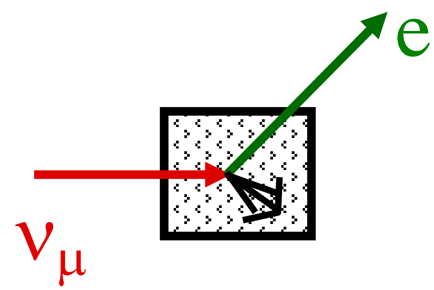
We ***define*** the neutrinos of specific flavor, ν_e, ν_μ, ν_τ, by W boson decays:



As far as we know, when a neutrino of given flavor interacts and turns into a charged lepton, that charged lepton will always be of the same flavor as the neutrino.



but not

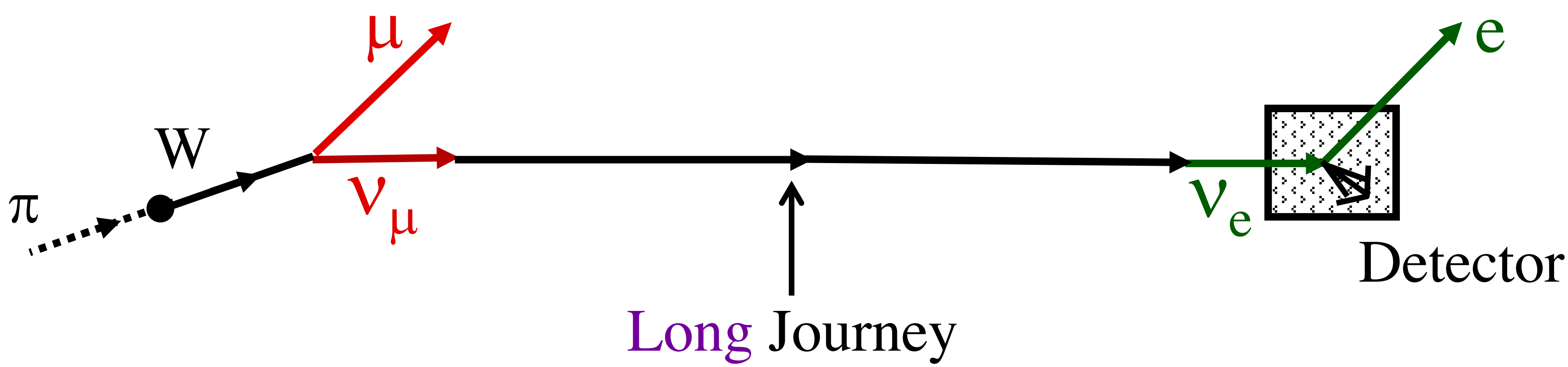


Lederman
Schartz
Steinberger

The weak interaction couples the neutrino of a given flavor only to the charged lepton of the same flavor.

Neutrino Flavor Change ("Oscillation")

If neutrinos have masses, and leptons mix, we can have —



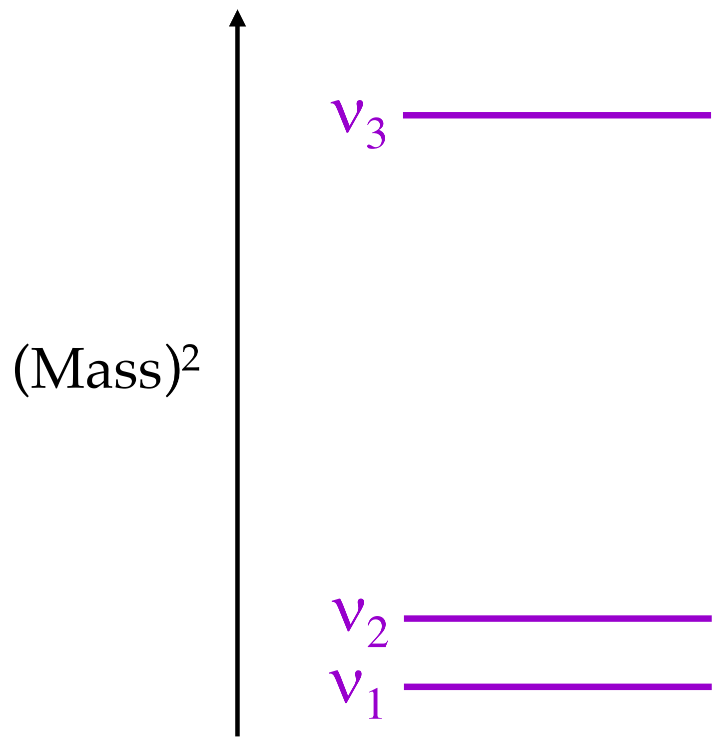
Give a ν time to change character, and you can have

for example: $\nu_\mu \longrightarrow \nu_e$

The last 20 years have brought us compelling evidence that such flavor changes actually occur.

Flavor Change Requires ***Neutrino Masses***

There must be some spectrum of neutrino mass eigenstates ν_i:



Mass $(\nu_i) \equiv m_i$

Flavor Change Requires ***Leptonic Mixing***

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor

$(W \to e\,\nu_e$ or $\mu\nu_\mu$ or $\tau\nu_\tau)$

must be superpositions of the mass eigenstates:

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\,|\nu_i\rangle \;.$$

Neutrino of flavor

$\alpha = e, \mu$, or τ

Neutrino of definite mass m_i

"PMNS" Leptonic Mixing Matrix

Note: The charged leptons of definite flavor are mass eigenstates, but the neutrinos of definite flavor are not mass eigenstates.

Notation: ℓ denotes a charged lepton. $\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$.

Since the only charged lepton ν_α couples to is ℓ_α ,
the 3 ν_α must be orthogonal.

To make up 3 orthogonal ν_α , we must have at least 3 ν_i .
Unless some ν_i masses are degenerate,
all ν_i will be orthogonal.

Then —

$$\delta_{\alpha\beta} = \langle \nu_\alpha | \nu_\beta \rangle = \left\langle \sum_i U_{\alpha i}^* \nu_i \middle| \sum_j U_{\beta j}^* \nu_j \right\rangle$$

$$= \sum_{i,j} U_{\alpha i} U_{\beta j}^* \langle \nu_i | \nu_j \rangle = \sum_i U_{\alpha i} U_{\beta i}^*$$

This says that
U is unitary,
but note the
unitary U may
not be 3 x 3.

Leptonic Mixing In the Extended Standard Model

The Standard Model (SM) of the weak and electromagnetic interactions, by **Glashow, Salam, and Weinberg**, neglects any nonzero neutrino mass.

For the moment, let us neglect **all** lepton masses.

Then we can think of the Left-Handed and Right-Handed leptons as different particles.

In the SM, these massless particles are put into multiplets of a **Weak Isospin** I_W that is conserved until the Higgs field develops a nonzero vacuum expectation value.

Weak-Isospin eigenstate

$$\begin{array}{c} \underline{I_{W3}} \\ +1/2 \\ -1/2 \end{array} \begin{pmatrix} \nu_{Le}^0 \\ e_L^0 \end{pmatrix} \quad \begin{pmatrix} \nu_{L\mu}^0 \\ \mu_L^0 \end{pmatrix} \quad \begin{pmatrix} \nu_{L\tau}^0 \\ \tau_L^0 \end{pmatrix} \qquad \begin{array}{c} \underline{I_{W3}} \\ 0 \end{array} \; e_R^0 \quad \mu_R^0 \quad \tau_R^0 \qquad \begin{array}{c} \underline{I_{W3}} \\ +1 \\ 0 \\ -1 \end{array} \begin{pmatrix} W^+ \\ W^0 \\ W^- \end{pmatrix}$$

Left-handed

There are no RH neutrinos

With —

$$\ell_L^0 \equiv \begin{bmatrix} e_L^0 \\ \mu_L^0 \\ \tau_L^0 \end{bmatrix} \equiv \begin{bmatrix} \ell_{Le}^0 \\ \ell_{L\mu}^0 \\ \ell_{L\tau}^0 \end{bmatrix} \quad \text{and} \quad \nu_L^0 \equiv \begin{bmatrix} \nu_{Le}^0 \\ \nu_{L\mu}^0 \\ \nu_{L\tau}^0 \end{bmatrix},$$

the lepton – W coupling, which conserves $\boldsymbol{I_W}$, is —

Semi-weak coupling

$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \left(\bar{\ell}_L^0 \gamma^\lambda \nu_L^0 \right) W_\lambda^- + h.c.$$

$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}}\left(\bar{\ell}_L^0 \gamma^\lambda \nu_L^0\right) W_\lambda^- + h.c.$$

When the lepton masses are turned on,
the charged lepton weak-isospin eigenstates are linear combinations of the charged lepton mass eigenstates:

3 x 3 matrices

$$\ell_{L,R}^0 = A_{L,R}\,\ell_{L,R}$$

Column vectors including the 3 generations

$$\ell \equiv \ell_L + \ell_R = \begin{bmatrix} e \\ \mu \\ \tau \end{bmatrix}$$

These are the familiar mass eigenstates.

$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}}\left(\bar{\ell}^0_L \gamma^\lambda \nu^0_L\right) W_\lambda^- + h.c. = -\frac{g}{\sqrt{2}}\left(\overline{A_L \ell_L}\,\gamma^\lambda \nu^0_L\right) W_\lambda^- + h.c.$$

$$= -\frac{g}{\sqrt{2}}\left(\underbrace{\bar{\ell}_L}_{\text{Mass eigenstates}}\gamma^\lambda \underbrace{A_L^\dagger \nu^0_L}_{=\begin{bmatrix}\nu_{Le}\\ \nu_{L\mu}\\ \nu_{L\tau}\end{bmatrix}}\right) W_\lambda^- + h.c.$$

These are the neutrinos of definite "flavor".

Note that this way of writing the interaction does not treat charged and neutral leptons on a par.

The interaction is written in terms of the charged lepton mass eigenstates, but not the neutrino mass eigenstates.

When the lepton masses are turned on, the neutrino weak-isospin eigenstates are linear combinations of the neutrino mass eigenstates:

3 x 3 matrix

$$\nu_L^0 = B_L \nu_L$$

Column vectors including the 3 generations

This could be $\left(\nu_L\right)^c$

$$\nu \equiv \nu_L + \nu_R = \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

This depends on the model

These are the neutrino mass eigenstates.

All mass eigenstates

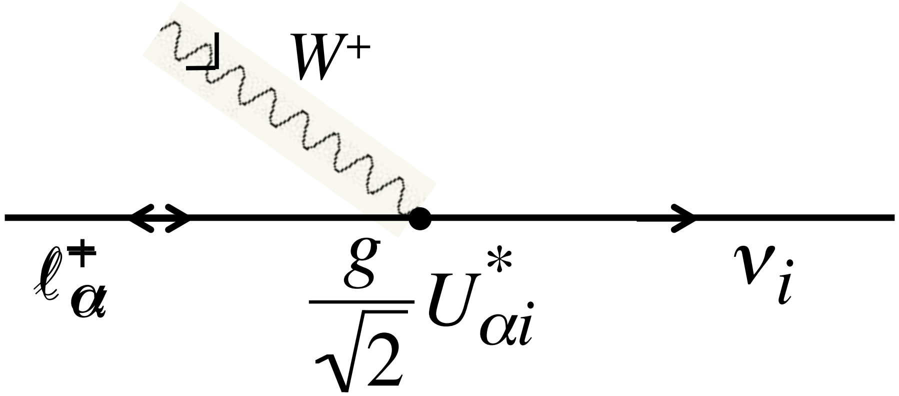
$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}}\left(\bar{\ell}_L \gamma^\lambda A_L^\dagger \nu_L^0\right) W_\lambda^- + h.c. = -\frac{g}{\sqrt{2}}\left(\bar{\ell}_L \gamma^\lambda \underbrace{A_L^\dagger B_L} \nu_L\right) W_\lambda^- + h.c.$$

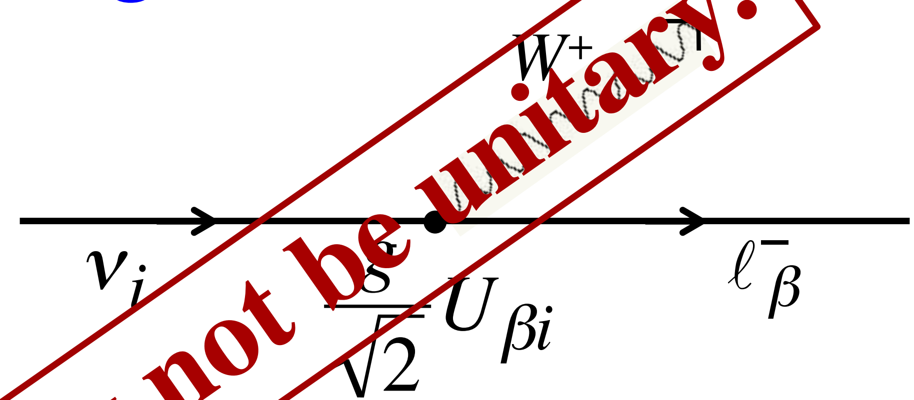
This is the leptonic mixing matrix ***U***

Explicitly —

$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha = e,\mu,\tau \\ i = 1,2,3}} \left(\bar{\ell}_{L\alpha} \gamma^\lambda U_{\alpha i} \nu_{Li} W_\lambda^- + \bar{\nu}_{Li} \gamma^\lambda U_{\alpha i}^{\;*} \ell_{L\alpha} W_\lambda^+\right)$$

The Meaning of *U*





Caution: The 3 x 3 *U* may not be unitary.

$$U = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \end{array}$$

Please talk about *leptonic* mixing, not neutrino mixing.

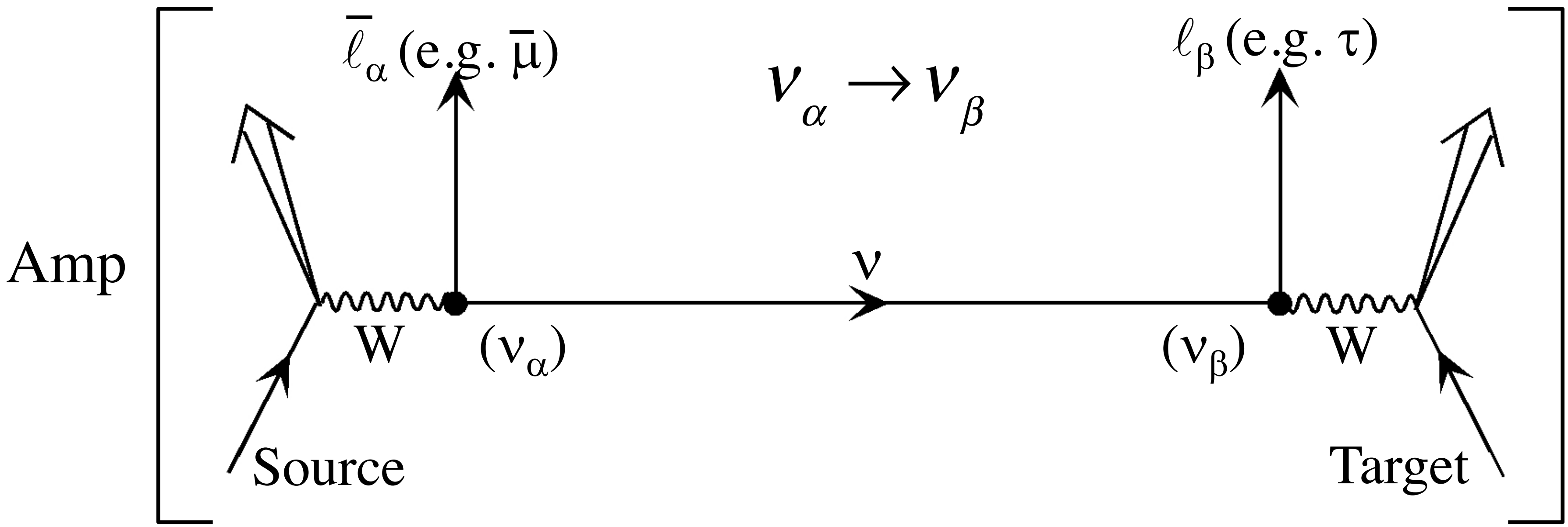
The *e* row of *U*: The linear combination of neutrino mass eigenstates that couples to *e*.

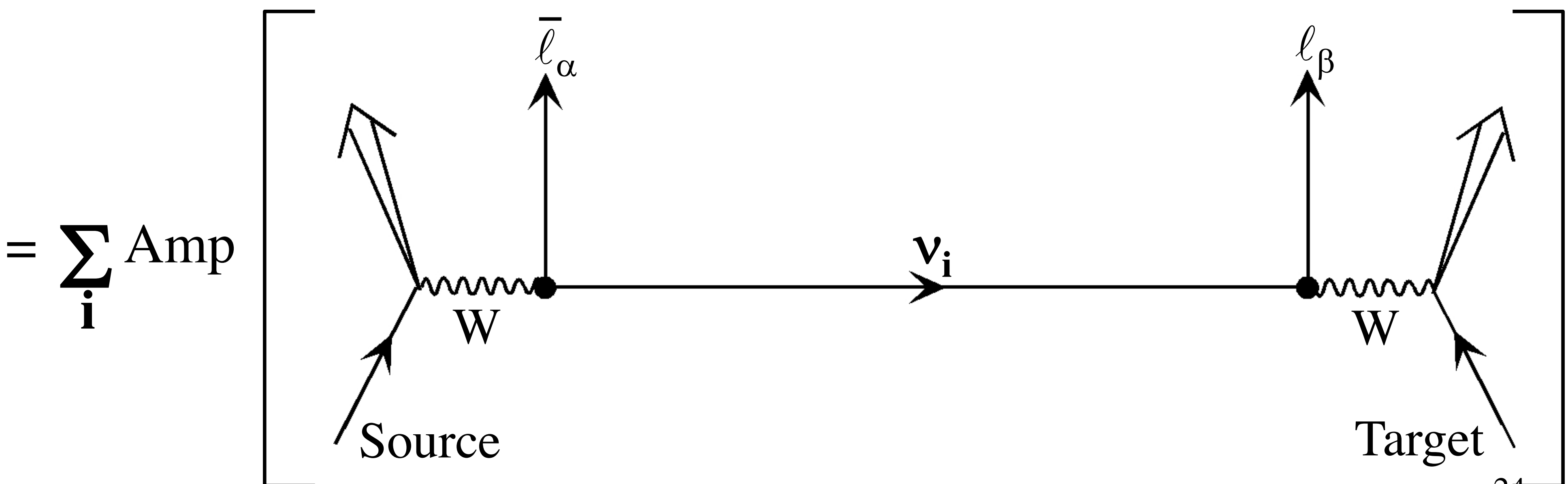
The ν_1 column of *U*: The linear combination of charged-lepton mass eigenstates that couples to ν_1 .

How Neutrino Oscillation In Vacuum Works

Neutrino Oscillation

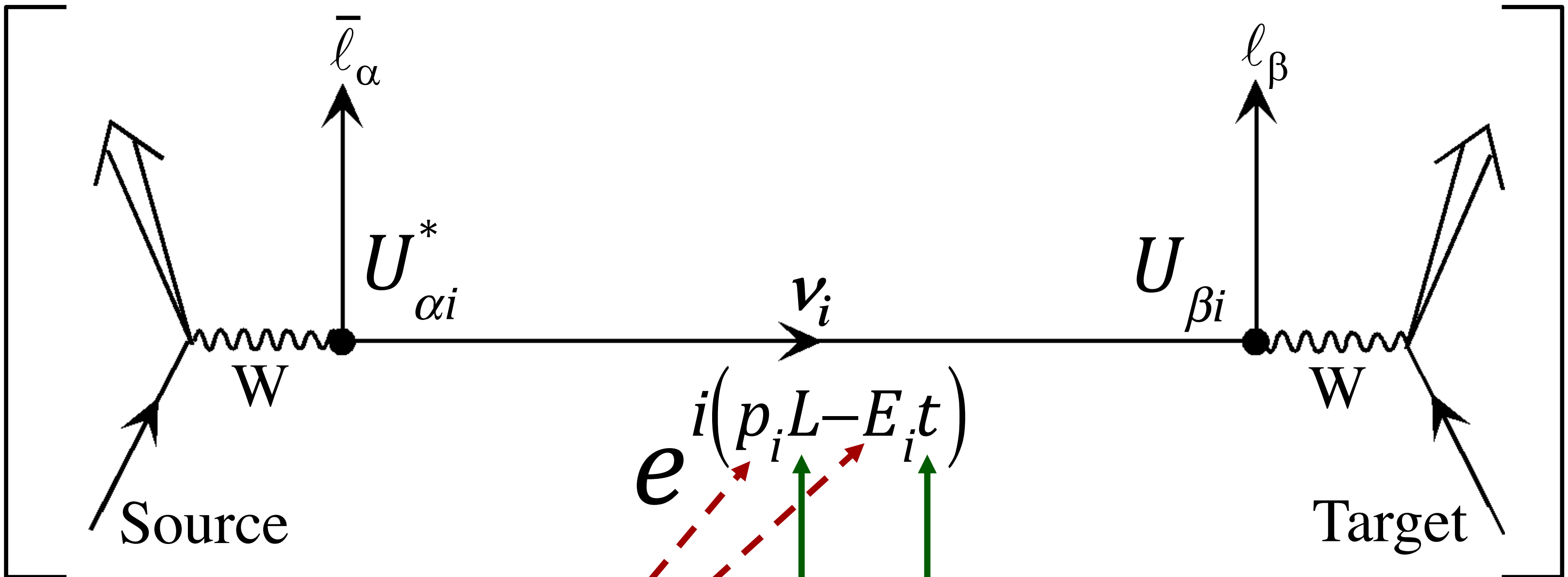
(Approach of B.K. and Stodolsky)





$$\mathrm{Amp}\left(\nu_\alpha \rightarrow \nu_\beta\right)$$

$$= \sum_{\mathbf{i}} \mathrm{Amp} \left[\quad \right]$$



Momentum and energy of ν_i

Coordinates of detection point, taking source point as (0, 0)

Neutrino sources are ~ constant in time.

Averaged over time, the

$$e^{-iE_1 t} - e^{-iE_2 t} \quad \text{interference}$$

is —

$$\left\langle e^{-i(E_1 - E_2)t} \right\rangle_t = 0 \quad \textit{unless } E_2 = E_1.$$

Only neutrino mass eigenstates with a common energy E are coherent.

(Stodolsky)

For each mass eigenstate ν_i,

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} \; .$$

Then the plane-wave factor $e^{i(p_i L - E_i t)}$ is —

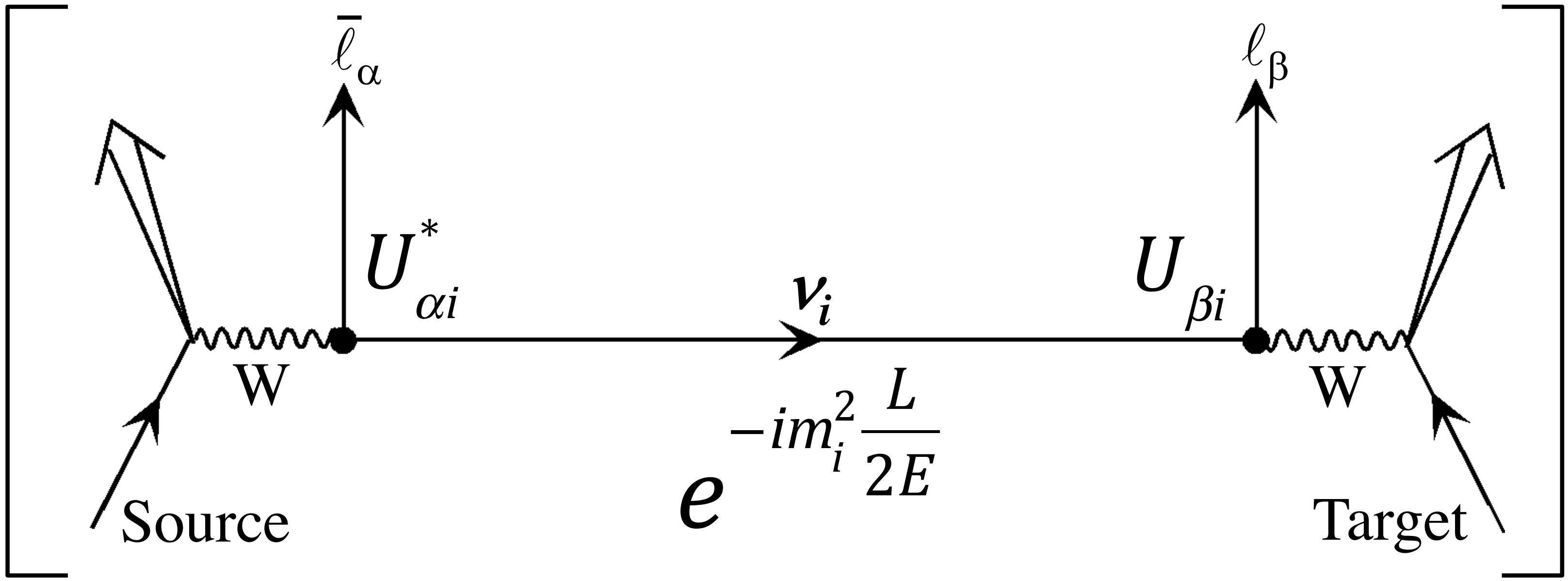
$$e^{i(p_i L - E_i t)} \cong e^{i\left\{\left(E - \frac{m_i^2}{2E}\right)L - Et\right\}} = e^{iE(L-t)} e^{-im_i^2 \frac{L}{2E}}$$

Irrelevant overall phase factor

Then —

$$\mathrm{Amp}\left(\nu_\alpha \to \nu_\beta\right)$$

$$= \sum_{\mathbf{i}} \mathrm{Amp}$$



$$= \sum_i U^*_{\alpha i} e^{-im_i^2 \frac{L}{2E}} U_{\beta i}$$

Probability of Neutrino Oscillation in Vacuum

$$P\left(\nu_\alpha \to \nu_\beta\right) = \left|\mathrm{Amp}\left(\nu_\alpha \to \nu_\beta\right)\right|^2 =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*\right) \sin^2\left(\Delta m_{ij}^2 \frac{L}{4E}\right)$$

$$+ 2\sum_{i>j} \mathrm{Im}\left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*\right) \sin\left(\Delta m_{ij}^2 \frac{L}{2E}\right)$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$.

Neutrino flavor change implies neutrino mass!

Neutrinos vs. Antineutrinos

$$\left[\bar{\nu}_\alpha(\mathrm{RH}) \to \bar{\nu}_\beta(\mathrm{RH})\right] = \mathrm{CP}\left[\nu_\alpha(\mathrm{LH}) \to \nu_\beta(\mathrm{LH})\right]$$

A difference between the probabilities of these two oscillations in vacuum would be a leptonic violation of CP invariance.

Assuming CPT invariance —

$$P\left[\bar{\nu}_\alpha(\mathrm{RH}) \to \bar{\nu}_\beta(\mathrm{RH})\right] = P\left[\nu_\beta(\mathrm{LH}) \to \nu_\alpha(\mathrm{LH})\right]$$

Probability

$$P\left(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta\right) =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\Delta m^2_{ij} \frac{L}{4E}\right)$$

$$\overset{+}{\scriptstyle(-)} 2\sum_{i>j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin\left(\Delta m^2_{ij} \frac{L}{2E}\right)$$

In neutrino oscillation, CP non-invariance comes from phases in the leptonic mixing matrix U.

Must we assume all mass eigenstates
have the same E?

No, we can take entanglement into account,
and use momentum-energy conservation.

The oscillation probabilities are still the same.

(B.K.)

For Some Applications, the Plane Wave Treatment of Neutrino Oscillation Is Wrong.

The probability of neutrino oscillation depends on the distance L between the neutrino source and the point of detection.

To determine L, we must know where the neutrino started, and where it was detected.

A plane wave has a definite, precise momentum p.

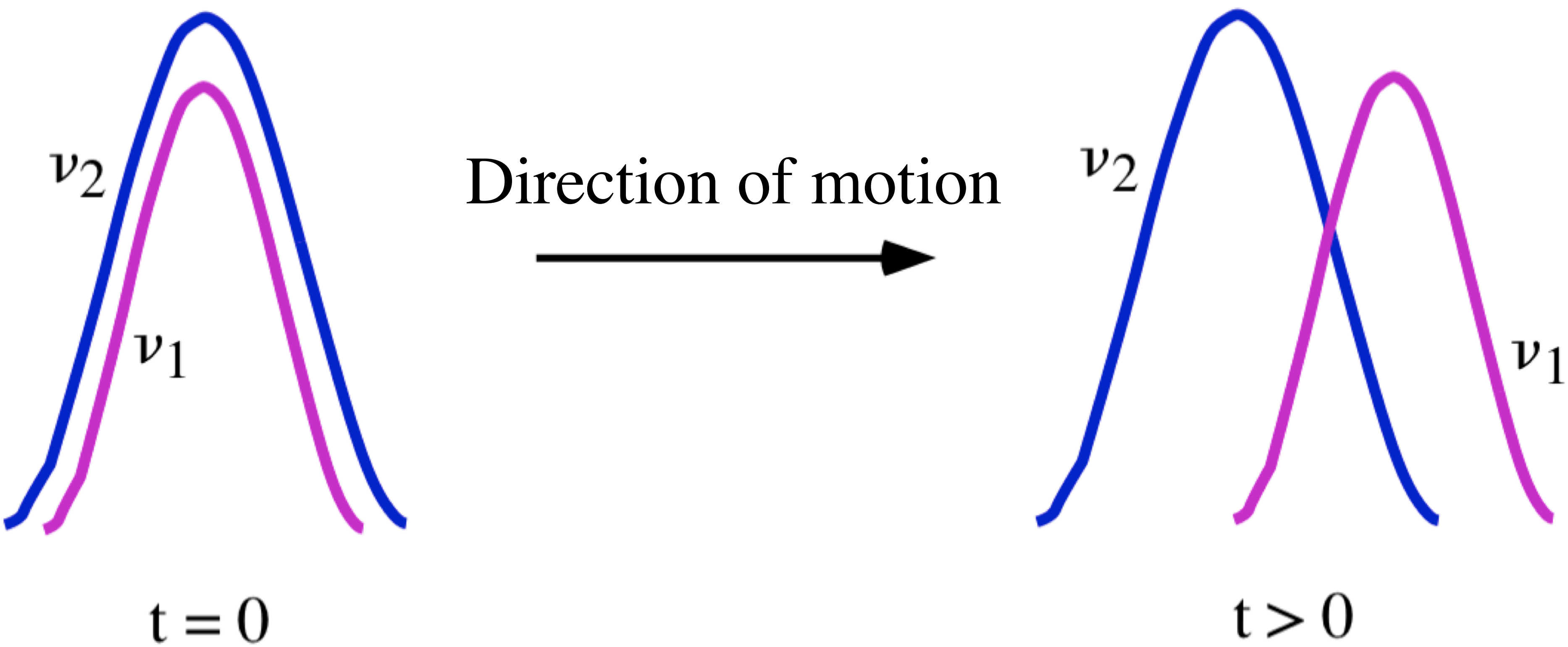
Heisenberg: $\Delta x \Delta p \geq \hbar/2$.

If we know precisely the momentum with which a neutrino was born, we know nothing about <u>*where*</u> *it was born.*

The Wave Packet Picture

Each mass eigenstate is described by a wave packet.

Suppose ν_2 is heavier than ν_1.



Eventually the wave packets will separate.

No more oscillation.

How soon do the wave packets separate??

For accelerator neutrinos with energy $E = 1$ GeV, and a wave packet width equal to the length of the pion decay region where the neutrinos are born, the bigger $\Delta m^2 = 2.4 \times 10^{-3}$ eV^2 leads to wave packet separation in

10^{20} km.

This separation may be safely ignored!

However, for supernova neutrinos from SN 1987A, with energy $E \sim 10$ MeV, and a wave packet width equal to an *estimated* inter-nucleon distance within the star, separation occurs in

10^3 km.

Supernova neutrinos are no longer oscillating when they reach us.

Different mass eigenstates <u>produced at the same instant</u> arrive at separate times, depending on their individual speeds.

The arrival time difference for the SN 1987 A neutrinos could have been $\sim 10^{-4}$ sec.

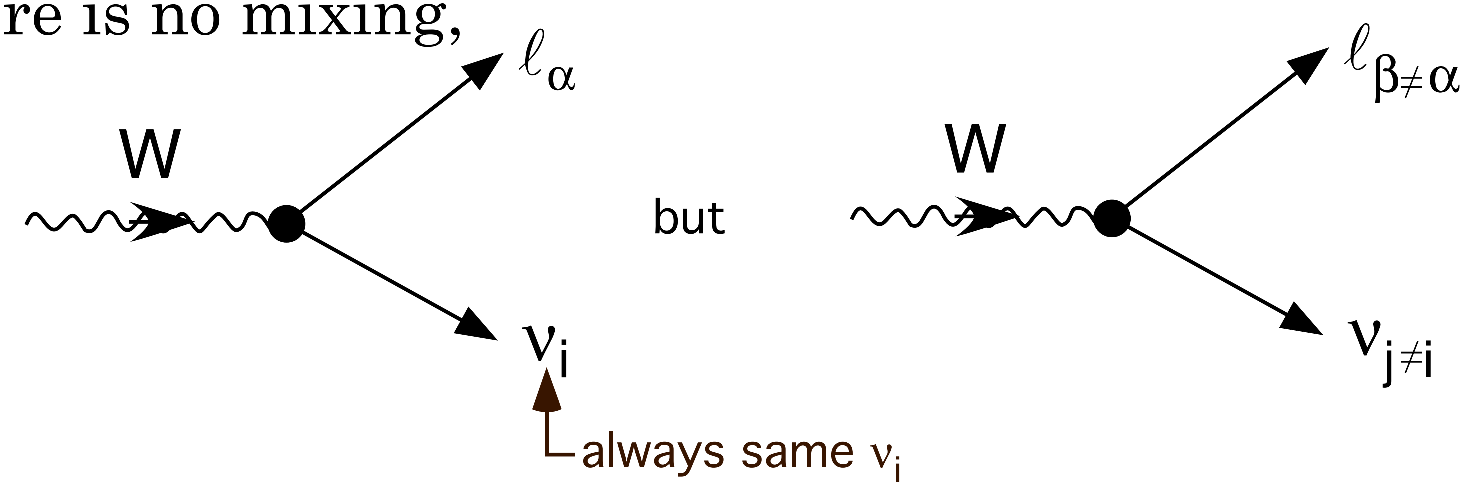
— Comments —

1. If all $m_i = 0$, so that all $\Delta m_{ij}^2 = 0$,

$$P\,(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}$$

Flavor *change* $\Rightarrow \nu$ Mass

2. If there is no mixing,



$$\Rightarrow U_{\alpha i} U_{\beta\neq\alpha, i} = 0, \text{ so that } P\,(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = \delta_{\alpha\beta}.$$

Flavor *change* $\Rightarrow$ Mixing

3. One can detect ($\nu_\alpha \rightarrow \nu_\beta$) in two ways:

See $\nu_{\beta\neq\alpha}$ in a ν_α beam (Appearance)

See some of known ν_α flux disappear (Disappearance)

4. Including $\hbar$ and c

$$\Delta m^2 \frac{L}{4E} = 1.27 \Delta m^2(\mathrm{eV}^2) \frac{L(\mathrm{km})}{E(\mathrm{GeV})}$$

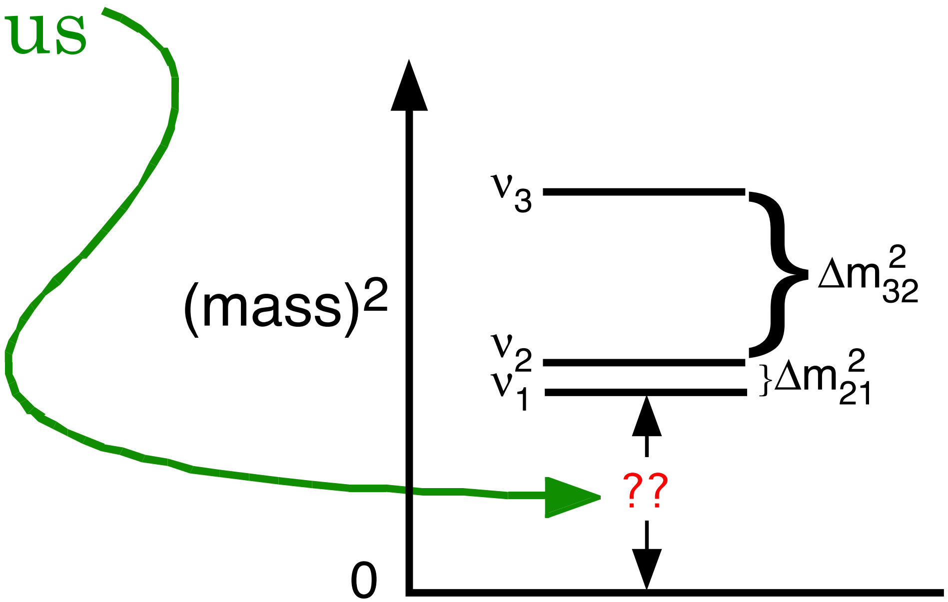
$\sin^2[1.27 \Delta m^2 (\mathrm{eV})^2 \frac{L(\mathrm{km})}{E(\mathrm{GeV})}]$ becomes appreciable when its argument reaches $\mathcal{O}(1)$.

An experiment with given L/E is sensitive to

$$\Delta m^2(\mathrm{eV}^2) \gtrsim \frac{E(\mathrm{GeV})}{L(\mathrm{km})} \quad .$$

5. Flavor change in vacuum oscillates with L/E. Hence the name "neutrino oscillation". {The L/E is from the proper time τ.}

6. $P\,(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta)$ depends only on squared-mass splittings. Oscillation experiments cannot tell us



7. Neutrino flavor change does not change the total flux in a beam.

It just redistributes it among the flavors.

$$\sum_{\text{All } \beta} P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) = 1$$

But some of the flavors $\beta \neq \alpha$ could be sterile.

Then some of the *active* flux disappears:

$$\phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} < \phi_{\text{Original}}$$

Important Special Cases

Three Flavors

For $\beta \neq \alpha$,

$$e^{-im_1^2 \frac{L}{2E}} \mathrm{Amp}^*(\nu_\alpha \to \nu_\beta) = \sum_i U_{\alpha i} U^*_{\beta i} e^{im_i^2 \frac{L}{2E}} e^{-im_1^2 \frac{L}{2E}}$$

$$= U_{\alpha 3} U^*_{\beta 3} e^{2i\Delta_{31}} + U_{\alpha 2} U^*_{\beta 2} e^{2i\Delta_{21}} \underbrace{- (U_{\alpha 3} U^*_{\beta 3} + U_{\alpha 2} U^*_{\beta 2})}_{\text{Unitarity}}$$

$$= 2i[U_{\alpha 3} U^*_{\beta 3} e^{i\Delta_{31}} \sin \Delta_{31} + U_{\alpha 2} U^*_{\beta 2} e^{i\Delta_{21}} \sin \Delta_{21}]$$

$$\text{where } \Delta_{ij} \equiv \Delta m^2_{ij} \frac{L}{4E} \equiv (m_i^2 - m_j^2) \frac{L}{4E} \quad .$$

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) = \left| e^{-im_1^2 \frac{L}{2E}} \mathrm{Amp}^*(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \right|^2$$

$$= 4[|U_{\alpha 3}U_{\beta 3}|^2 \sin^2 \Delta_{31} + |U_{\alpha 2}U_{\beta 2}|^2 \sin^2 \Delta_{21}$$

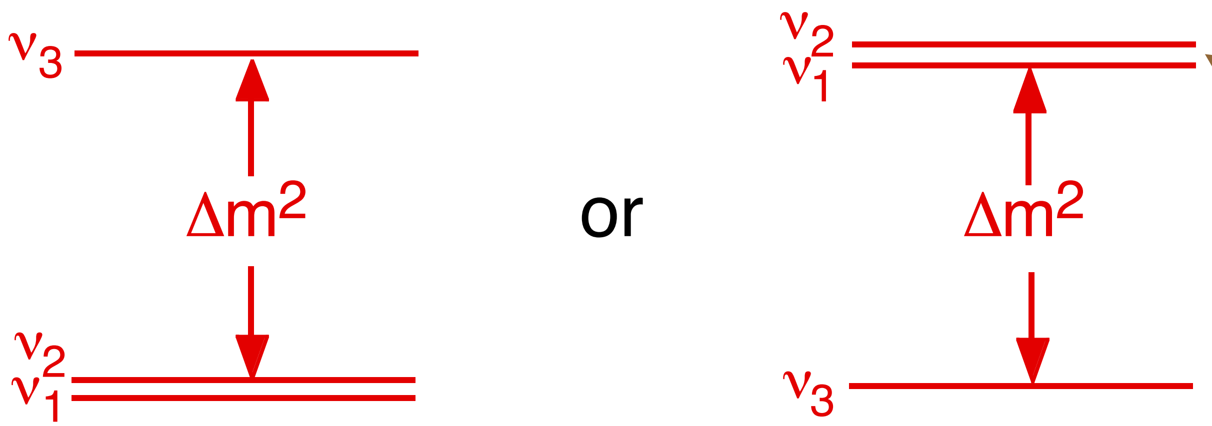
$$+2|U_{\alpha 3}U_{\beta 3}U_{\alpha 2}U_{\beta 2}| \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} \underset{(-)}{+} \delta_{32})] \; .$$

Here $\delta_{32} \equiv \arg(U_{\alpha 3}U^*_{\beta 3}U^*_{\alpha 2}U_{\beta 2})$, a CP – violating phase.

Two waves of different frequencies,

and their ~~CP~~ interference.

When the Spectrum Is—



Invisible if
$\Delta m^2 \dfrac{L}{E} = \mathcal{O}(1)$.

For $\beta \neq \alpha$,

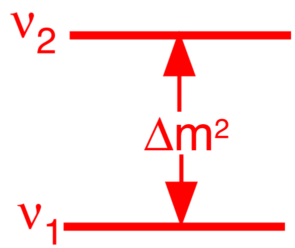
$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) \cong 4|U_{\alpha 3} U_{\beta 3}|^2 \sin^2(\Delta m^2 \tfrac{L}{4E}) \quad .$$

For no flavor change,

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha) \cong 1 - 4|U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2) \sin^2(\Delta m^2 \tfrac{L}{4E}) \quad .$$

Experiments with $\Delta m^2 \frac{L}{E} = \mathcal{O}(1)$ can determine the flavor content of ν_3.

When There are Only Two Flavors and Two Mass Eigenstates



Majorana ~~CP~~ phase

$$U = \begin{bmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} e^{i\xi} & 0 \\ 0 & 1 \end{bmatrix}$$

Mixing angle

For $\beta \neq \alpha$, $\quad P(\overset{(-)}{\nu}_\alpha \leftrightarrow \overset{(-)}{\nu}_\beta) = \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E})$.

For no flavor change, $P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\alpha) = 1 - \sin^2 2\theta \sin^2(\Delta m^2 \frac{L}{4E})$.