Reactor Neutrinos II: Fluxes

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P. Huber – VT CNP – p. 1

1956



They report a cross section (!) of $6 \times 10^{-44} \,\mathrm{cm}^{-2} \rightarrow$ to measure a cross section one needs to know the flux.

Neutrinos from fission



β -branches



A priori calculations



Updated β -feeding functions from total absorption γ spectroscopy (safe from pandemonium) for the isotopes: ^{102,104,105,106,107}Tc, ¹⁰⁵Mo and ¹⁰²Nb

The calculation for ²³⁸U agrees within 10% with measurement of Haag *et al.*

Still a 10-20% discrepancy with the measured total β -spectra.

Fallot *et al.*, 2012

 β -decay – Fermi theory

$$N_{\beta}(W) = K \underbrace{p^2(W - W_0)^2}_{\text{phase space}} F(Z, W) ,$$

where $W = E/(m_e c^2) + 1$ and W_0 is the value of Wat the endpoint. K is a normalization constant. F(Z, W) is the so called Fermi function and given by

 $F(Z,W) = 2(\gamma+1)(2pR)^{2(\gamma-1)}e^{\pi\alpha ZW/p}\frac{|\Gamma(\gamma+i\alpha ZW/p)|^2}{\Gamma(2\gamma+1)^2}$

 $\gamma = \sqrt{1 - (\alpha Z)^2}$

The Fermi function is the modulus square of the electron wave function at the origin.

Corrections to Fermi theory

 $N_{\beta}(W) = K p^{2} (W - W_{0})^{2} F(Z, W) L_{0}(Z, W) C(Z, W) S(Z, W)$ $\times G_{\beta}(Z, W) (1 + \delta_{WM} W).$

The neutrino spectrum is obtained by the replacements $W \to W_0 - W$ and $G_\beta \to G_\nu$.

 L_0 and S have been recently re-evaluated for fission fragments Wang, Friar, Hayes, 2016.

The whole set of corrections has been critically examined McCutchan, Sonzogni, Hayes, 2017.

 \Rightarrow all well under control for allowed decays!

Finite size corrections – I

Finite size of charge distribution affects outgoing electron wave function

$$L_0(Z,W) = 1 + 13\frac{(\alpha Z)^2}{60} - WR\alpha Z \frac{41 - 26\gamma}{15(2\gamma - 1)} - \alpha ZR\gamma \frac{17 - 2\gamma}{30W(2\gamma - 1)} \dots$$

Parameterization of numerical solutions, only small associated error. This expression is effectively very close to the Mueller *et al.* one.

Finite size corrections – II

Convolution of electron wave function with nucleon wave function over the volume of the nucleus

 $C(Z,W) = 1 + C_0 + C_1 W + C_2 W^2 \text{ with}$ $C_0 = -\frac{233}{630} (\alpha Z)^2 - \frac{(W_0 R)^2}{5} + \frac{2}{35} W_0 R \alpha Z,$ $C_1 = -\frac{21}{35} R \alpha Z + \frac{4}{9} W_0 R^2,$ $C_2 = -\frac{4}{9} R^2.$

Small associated theory error. This expression is not taken into account by Mueller *et al.*, quantitatively largest β -shape difference.

Screening correction

All of the atomic bound state electrons screen the charge of the nucleus – correction to Fermi function

$$\bar{W} = W - V_0, \quad \bar{p} = \sqrt{\bar{W}^2 - 1}, \quad y = \frac{\alpha Z W}{p} \quad \bar{y} = \frac{\alpha Z W}{\bar{p}} \quad \tilde{Z} = Z - 1.$$

 V_0 is the so called screening potential

$$V_0 = \alpha^2 \tilde{Z}^{4/3} N(\tilde{Z}) \,,$$

and $N(\tilde{Z})$ is taken from numerics.

$$S(Z,W) = \frac{\overline{W}}{W} \left(\frac{\overline{p}}{p}\right)^{(2\gamma-1)} e^{\pi(\overline{y}-y)} \frac{\left|\Gamma(\gamma+i\overline{y})\right|^2}{\Gamma(2\gamma+1)^2} \quad \text{for} \quad W > V_0 \,,$$

Small associated theory error. This expression is not taken into account by Mueller *et al*..

Radiative correction - I

Order α QED correction to electron spectrum, by Sirlin, 1967

$$g_{\beta} = 3\log M_N - \frac{3}{4} + 4\left(\frac{\tanh^{-1}\beta}{\beta}\right)\left(\frac{W_0 - W}{3W} - \frac{3}{2} + \log\left[2(W_0 - W)\right]\right) + \frac{4}{\beta}L\left(\frac{2\beta}{1+\beta}\right) + \frac{1}{\beta}\tanh^{-1}\beta\left(2(1+\beta^2) + \frac{(W_0 - W)^2}{6W^2} - 4\tanh^{-1}\beta\right)$$

where L(x) is the Spence function, The complete correction is then given by

$$G_{\beta}(Z, W) = 1 + \frac{\alpha}{2\pi} g_{\beta} \,.$$

Small associated theory error.

Radiative correction - II

Order α QED correction to neutrino spectrum, recent calculation by Sirlin, Phys. Rev. **D84**, 014021 (2011).

$$h_{\nu} = 3\ln M_N + \frac{23}{4} - \frac{8}{\hat{\beta}}L\left(\frac{2\hat{\beta}}{1+\hat{\beta}}\right) + 8\left(\frac{\tanh^{-1}\hat{\beta}}{\hat{\beta}} - 1\right)\ln(2\hat{W}\hat{\beta}) + 4\frac{\tanh^{-1}\hat{\beta}}{\hat{\beta}}\left(\frac{7+3\hat{\beta}^2}{8} - 2\tanh^{-1}\hat{\beta}\right)$$

$$G_{\nu}(Z,W) = 1 + \frac{\alpha}{2\pi}h_{\nu}.$$

Very small correction.

Weak currents

In the following we assume $q^2 \ll M_W$ and hence charged current weak interactions can be described by a current-current interaction.

$$-rac{G_F}{\sqrt{2}}V_{ud}J^h_\mu J^l_\mu$$

where

$$J_{\mu}^{h} = \bar{\psi}_{u} \gamma_{\mu} (1 + \gamma_{5}) \psi_{d} = V_{\mu}^{h} + A_{\mu}^{h}$$

However, we are not dealing with free quarks ...

Induced currents

Describe protons and neutrons as spinors which are solutions to the free Dirac equation, but which are **not** point-like, we obtain for the hadronic current

$$V_{\mu}^{h} = i\bar{\psi}_{p} \left[g_{V}(q^{2})\gamma_{\mu} + \frac{g_{M}(q^{2})}{8M}\sigma_{\mu\nu}q_{\nu} + ig_{S}(q^{2})q_{\mu} \right]\psi_{n}$$
$$A_{\mu}^{h} = i\bar{\psi}_{p} \left[g_{A}(q^{2})\gamma_{\mu}\gamma_{5} + \frac{g_{T}(q^{2})}{8M}\sigma_{\mu\nu}q_{\nu}\gamma_{5} + ig_{P}(q^{2})q_{\mu}\gamma_{5} \right]\psi_{n}$$

In the limit $q^2 \rightarrow 0$ the form factors $g_X(q^2) \rightarrow g_X$, *i.e.* new induced couplings, which are not present in the SM Lagrangian, but are induced by the bound state QCD dynamics.

Isospin

Proton and neutron can be regarded as a two state system in the same way a spin 1/2 system has two states \Rightarrow isospin.

In complete analogy we chose the Pauli matrices as basis, but call them τ to avoid confusion with regular spin $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$, we define the new 8-component spinor

$$\Psi = \left(\begin{array}{c} \psi_p \\ \psi_n \end{array}\right)$$

and we define the isospin ladder operators as $\tau^a = \tau^{\pm} = \tau_1 \pm i\tau_2$, with τ^+ corresponding to β^- -decay and τ^- to β^+ -decay.

Weak isovector current

Using isospin notation we can write the Lorentz vector part of the weak charged current as

$$V^{h}_{\mu} = i\bar{\Psi} \left[g_{V}(q^{2})\gamma_{\mu} + \frac{g_{M}(q^{2})}{8M} \sigma_{\mu\nu}q_{\nu} + ig_{S}(q^{2})q_{\mu} \right] \frac{1}{2}\tau^{a}\Psi$$

and see that it transform as a vector in isospin space, therefore this together with the corresponding Lorentz axial vector A^h_{μ} part, which has the same isospin structure, is also called the weak isovector current.

EM isovector current

The fundamental EM current is given by

$$V^{EM}_{\mu} = i\frac{2}{3}\bar{\psi}_{u}\gamma_{\mu}\psi_{u} - i\frac{1}{3}\bar{\psi}\gamma_{\mu}\psi_{d}$$

which transforms as Lorentz vector. How does it transform under isospin?

$$V_{\mu}^{EM} = \underbrace{iQ_{+}\bar{\Psi}_{q}\gamma_{\mu}\Psi_{q}1}_{\text{isoscalar}} + \underbrace{iQ_{-}\bar{\Psi}_{q}\gamma_{\mu}\Psi_{q}\tau^{3}}_{\text{isovector}}$$

with $Q_{\pm} = \frac{1}{2} \left(\frac{2}{3} \mp \frac{1}{3} \right)$.

A triplet of isovector currents

Next, we can dress up the isovector part of V_{μ}^{EM} , v_{μ}^{EM} to account for nucleon structure

$$v_{\mu}^{EM} = i\bar{\Psi} \left[F_1^V(q^2)\gamma_{\mu} + \frac{F_2^V(q^2)}{2M}\sigma_{\mu\nu}q_{\nu} + iF_3^V(q^2)q_{\mu} \right] Q_-\tau_3\Psi$$

Compare with the Lorentz vector part of the weak isovector current

$$V^{h}_{\mu} = i\bar{\Psi} \left[g_{V}(q^{2})\gamma_{\mu} + \frac{g_{M}(q^{2})}{8M} \sigma_{\mu\nu}q_{\nu} + ig_{S}(q^{2})q_{\mu} \right] \frac{1}{2}\tau^{a}\Psi$$

These three currents form a triplet of isovector currents and this observation was made by Feynman and Gell-Mann in 1958.

Conserved vector currents

We know that V_{μ}^{EM} is a conserved quantity which is a direct consequence of U(1) gauge invariance in the SM.

- This implies that all components of the triplet are conserved.
- This is termed the Conserved Vector Current (CVC), which in the SM is a result not an input.

$$g_V(q^2) = F_1^V(q^2) \xrightarrow{q^2 \to 0} 1$$

$$g_M(q^2) = F_2^V(q^2)$$

$$g_S(q^2) = F_3^V(q^2) = 0$$

Weak magnetism & β -spectra

 g_M is call weak magnetism and the question is how it manifests itself in nuclear β -decay. Nuclear structure effects can be summarized by the use of appropriate form factors F_X^N .

The weak magnetic nuclear, F_M^N form factor by virtue of CVC is given in terms of the analog EM form factor as

$$F_M^N(0) = \sqrt{2}\mu(0)$$

The effect on the β decay spectrum is given by

$$1 + \delta_{WM} W \simeq 1 + \frac{4}{3M} \frac{F_M^N(0)}{F_A^N(0)} W$$

Impulse approximation

In the impulse approximation nuclear β -decay is described as the decay of a free nucleon inside the nucleus. The sole effect of the nucleus is to modify the initial and final state densities.

In impulse approximation

$$F_M^N(0) = \mu_p - \mu_n \simeq 4.7$$
 and $F_A^N(0) = C_A \simeq 1.27$,
and thus

 $\delta_{WM} \simeq 0.5\% \,\mathrm{MeV}^{-1}$

This value, in impulse approximation, is universal for all β -decays since it relies only on free nucleon parameters.

Isospin analog γ **-decays**



$$\Gamma(C^{12*} - C^{12})_{M1} = \frac{\alpha E_{\gamma}^3}{3M^2} \left| \sqrt{2}\mu(0) \right|^2$$

$$b := \sqrt{2}\mu(0) = F_M^N(0)$$

Gamow-Teller matrix element c

$$c = F_A^N(0) = \sqrt{\frac{2ft_{\rm Fermi}}{ft}}$$

and thanks to CVC $ft_{\text{Fermi}} \simeq 3080 \,\text{s}$ is universal $t_{\text{CNP-p.22}}$

What is the value of δ_{WM} ?

Three ways to determine δ_{WM}

- impulse approximation universal value $0.5\% \,\mathrm{MeV}^{-1}$
- using $CVC F_M$ from analog M1 γ -decay width, F_A from ft value
- direct measurement in β-spectrum only very few, light nuclei have been studied. In those cases the CVC predictions are confirmed within (sizable) errors.

In the following, we will compare the results from CVC with the ones from the impulse approximation.

CVC at work

Collect all nuclei for which we

- can identify the isospin analog energy level
- and know Γ_{M1}

then, compute the resulting δ_{WM} . This exercise has been done in Calaprice, Holstein, Nucl. Phys. A273 (1976) 301. and they find for nuclei with $ft < 10^6$

$$\delta_{WM} = 0.82 \pm 0.4\% \, {
m MeV^{-1}}$$

which is in reasonable agreement with the impulse approximated value of $\delta_{WM} = 0.5\% \text{MeV}^{-1}$. Our result for $ft < 10^6$ is $\delta_{WM} = (0.67 \pm 0.26)\% \text{MeV}^{-1}$.

CVC at work

D	ecay	$J_i \rightarrow J_f$	$E_{oldsymbol{\gamma}}$	Γ_{M1}	b_γ	ft	c	b_{γ}/Ac	dN/dE
			(keV)	(eV)		(s)			$(\% {\rm MeV}^{-1})$
⁶ He	$ ightarrow ^{6}$ Li	$0^+ \rightarrow 1^+$	3563	8.2	71.8	805.2	2.76	4.33	0.646
^{12}B	\rightarrow^{12} C	$1^+ \rightarrow 0^+$	15110	43.6	37.9	11640.	0.726	4.35	0.62
12 N	\rightarrow^{12} C	$1^+ \rightarrow 0^+$	15110	43.6	37.9	13120.	0.684	4.62	0.6
¹⁸ Ne	$ ightarrow^{18}$ F	$0^+ \rightarrow 1^+$	1042	0.258	242.	1233.	2.23	6.02	0.8
²⁰ F -	$ ightarrow^{20}$ Ne	$2^+ \rightarrow 2^+$	8640	4.26	45.7	93260.	0.257	8.9	1.23
^{22}Mg	$ ightarrow^{22}$ Na	$0^+ \rightarrow 1^+$	74	0.0000233	148.	4365.	1.19	5.67	0.757
24 Al -	$ ightarrow^{24}$ Mg	$4^+ \rightarrow 4^+$	1077	0.046	129.	8511.	0.85	6.35	0.85
²⁶ Si	$ ightarrow^{26}$ Al	$0^+ \rightarrow 1^+$	829	0.018	130.	3548.	1.32	3.79	0.503
²⁸ Al	$ ightarrow^{28}$ Si	$3^+ \rightarrow 2^+$	7537	0.3	20.8	73280.	0.29	2.57	0.362
²⁸ P -	$ ightarrow^{28}$ Si	$3^+ \rightarrow 2^+$	7537	0.3	20.8	70790.	0.295	2.53	0.331
¹⁴ C	\rightarrow^{14} N	$0^+ \rightarrow 1^+$	2313	0.0067	9.16	1.096×10^9	0.00237	276.	37.6
¹⁴ 0	\rightarrow^{14} N	$0^+ \rightarrow 1^+$	2313	0.0067	9.16	1.901×10^{7}	0.018	36.4	4.92
³² P	\rightarrow^{32} S	$1^+ \rightarrow 0^+$	7002	0.3	26.6	7.943×10^{7}	0.00879	94.4	12.9

What happens for large *ft***?**

	Decay	$J_i \rightarrow J_f$	E_{γ}	Γ_{M1}	$b_{oldsymbol{\gamma}}$	ft	c	b_{γ}/Ac	dN/dE			
			(keV)	(eV)		(s)			$(\% {\rm MeV}^{-1})$			
14 C	$C \rightarrow^{14} N$	$0^+ \rightarrow 1^+$	2313	0.0067	9.16	1.096×10^{9}	0.00237	276.	37.6			
14 C	$0 \rightarrow^{14} N$	$0^+ \rightarrow 1^+$	2313	0.0067	9.16	1.901×10^7	0.018	36.4	4.92			
32 H	$P \rightarrow^{32} S$	$1^+ \rightarrow 0^+$	7002	0.3	26.6	7.943×10^7	0.00879	94.4	12.9			
	Including these large ft nuclei, we have											

 $\delta_{WM} = (4.78 \pm 10.5) \% \,\mathrm{MeV^{-1}}$

which is about 10 times the impulse approximated value and this are about 3 nuclei out of 10-20...

NB, a shift of δ_{WM} by $1\% \text{MeV}^{-1}$ shifts the total neutrino flux above inverse β -decay threshold by $\sim 2\%$.

Recent work Wang, Hayes, 2017 indicates that this is probably not an issue for allowed decays.

Complete β **-shape**



Computation of Neutrino Spectrum

Extraction of ν **-spectrum**

We can measure the total β -spectrum

$$\mathcal{N}_{\beta}(E_e) = \int dE_0 N_{\beta}(E_e, E_0; \bar{Z}) \eta(E_0) \,. \tag{1}$$

with \overline{Z} effective nuclear charge and try to "fit" the underlying distribution of endpoints, $\eta(E_0)$.

This is a so called Fredholm integral equation of the first kind – mathematically ill-posed, *i.e.* solutions tend to oscillate, needs regulator (typically energy average), however that will introduce a bias.

This approach is know as "virtual branches"

Virtual branches



1 – fit an allowed β -spectrum with free normalization η and endpoint energy E_0 the last s data points

- 2 delete the last s data points
- 3 subtract the fitted spectrum from the data
- 4 goto 1

Invert each virtual branch using energy conservation into a neutrino spectrum and add them all.

β spectrum from fission



²³⁵U foil inside the
High Flux Reactor at
ILL

Electron spectroscopy with a magnetic spectrometer

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Effective nuclear charge

In order to compute all the QED corrections we need to know the nuclear charge Z of the decaying nucleus.

Using virtual branches, the fit itself cannot determine Z since many choices for Z will produce an excellent fit of the β -spectrum

 \Rightarrow use nuclear database to find how the average nuclear charge changes as a function of E_0 , this is what is called effective nuclear charge $\overline{Z}(E_0)$.

Weigh each nucleus by its fission yield and bin the resulting distribution in E_0 and fit a second order polynomial to it.

Effective nuclear charge

The nuclear databases have two fundamental shortcomings

- they are incomplete for the most neutron-rich nuclei we only know the $Q_{gs \rightarrow gs}$, *i.e.* the mass differences
- they are incorrect for many of the neutron-rich nuclei, γ-spectroscopy tends to overlook faint lines and thus too much weight is given to branches with large values of E₀, aka pandemonium effect

Simulation using our synthetic data set: by removing a fraction of the most neutron-rich nuclei and/or by randomly distributing the decays of a given branch onto several branches with $0 < E_0 < Q_{gs \rightarrow gs}$.

Effective nuclear charge



Spread between lines – effect of incompleteness and incorrectness of nuclear database (ENSDF). Only place in this analysis, where database enters directly.

Bias

Use synthetic data sets derived from cumulative fission yields and ENSDF, which represent the real data within 10-20% and compute bias



Approximately 500 nuclei and 8000 β -branches.

Statistical Error

Use synthetic data sets and fluctuate β -spectrum within the variance of the actual data.



Amplification of stat. errors of input data by factor 7.

Result for ²³⁵U



Shift with respect to ILL results, due toa) different effective nuclear charge distributionb) branch-by-branch application of shape corrections

The reactor anomaly



Daya Bay, 2014

Mueller *et al.*, 2011, 2012 – where are all the neutrinos gone?

Contributors to the anomaly

6% deficit of $\bar{\nu}_e$ from nuclear reactors at short distances

- 3% increase in reactor neutrino fluxes
- decrease in neutron lifetime
- inclusion of long-lived isotopes (non-equilibrium correction), next lecture!

The effects is therefore only partially due to the fluxes, but the error budget is clearly dominated by the fluxes.

Neutron lifetime



Forbidden decays



 $e,\overline{\nu}$ final state can form a singlet or triplet spin state J=0 or J=1

Allowed: s-wave emission (l = 0)Forbidden: p-wave emission (l = 1)or l > 1

Significant dependence on nuclear structure in forbidden decays \rightarrow large uncertainties!

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Forbidden decays



Hayes *et. al*, 2013 point out that in forbidden decays a mixture of different operators are involved.

Large source of uncertainty.

A coincidence?

Based on JEFF fission yields and using ENSDF spin-parity assignments



The 5 MeV bump



Seen by all three reactor experiments Tracks reactor power Seems independent of burn-up





Y. Oh, ICHEP 2016

24m from a large core (power reactor), confirms bump

Explanations?

Dwyer and Lanford, 2014 propose a direct summation. Latest ENSDF database with allowed beta-spectrum shape Sonzogni *et al.*, 2016



This direct summation, as all other direct summations, does not agree with the Schreckenbach measurement.

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What happened?

Fission yield data has been suspected previously Hayes *et al.* 2015 and this what Sonzogni *et al.*, 2016 found:



Who is the odd-one-out?

Fission yields for germanium-86 wrong in ENDF/B but not in JEFF.

Uranium-238?

Hayes and Vogel, 2016 point out that fast neutron fission of 238 U could be responsible for the bump



If true, NO bump should be seen a reactors running on HEU (nearly pure ²³⁵U).

Neutron spectrum?

Hayes and Vogel, 2016 point also out that the neutron spectrum is important



If true, NO bump should be seen a reactors running on HEU (nearly pure 235 U).

Not the neutron spectrum



Fission fragment distributions do depend on incoming neutron energy. Littlejohn, *et al.*, 2018



Enhancement of the high-end of the neutrino spectrum for a realistic neutron spectrum, no bump-like structure and too small.

Different reactors

Optimistic flux errors (per isotope) from Huber, 2011 and bump put by hand to match Daya Bay result



Requires good statistics: 5 ton, 40% efficient, 1 year data taking. Huber, 2016, see also Buck *et al.*, 2015

NEOS vs Daya Bay



Huber, 2017

There is more U235 in NEOS, since core is fresh \Rightarrow 3 - 4 σ evidence against Pu as sole source of bump, but equal bump size is still allowed at better than 2 σ .

BSM explanation for the bump



Berryman, Brdar, PH, 2018

Requires a sterile neutrino consistent with the reactor anomaly and a new vector state X coupling to quarks.

Does it work?



Berryman, Brdar, PH, 2018

Excellent fit

Existence of highenergy neutrino flux is predicted

High energy flux is in agreement with Daya Bay bounds

Position and width of bump entirely determined by SM physics

Is it allowed?



Pb-n scattering and CO-HERENT data are most difficult to satisfy:

Choosing $Y_p = +1$ and $Y_n = -0.65$ exploits the different proton/neutron ration between light nuclei ¹³C and heavy nuclei ²⁰⁸Pb, ¹³⁵Cs and ¹²⁷I.

CONUS bound is avoidable with axial coupling.

Summary

Reactor anti-neutrino fluxes are complex and reliable a priori calculations are elusive.

Measured integrated beta-spectra form the starting point for the most accurate flux predictions.

Forbidden decays introduce very significant(percent-level) nuclear structure related uncertainties.

The 5 MeV bump likely is due to nuclear physics, but no quantitative viable models have been demons rated.

Better understanding will come from neutrino measurements at many different reactors.