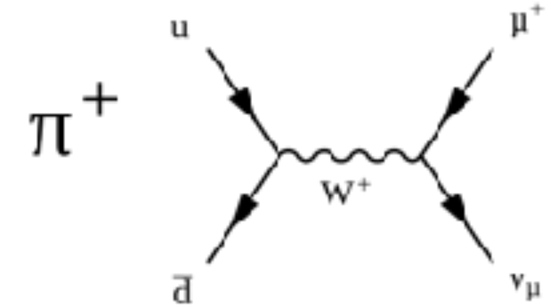
Coherence in Neutrino Oscillations

Johachim Kopp
Tyler Alion
Clément Bordereau
Hichem Tedjditi
Anupam Ray
Soumya Bhattacaryya

Pion Decay At Rest



Center of Mom.: $$p_\mu = p_\pi - p_\nu$$

Square both sides: $$p_\mu^2 = p_\pi^2 + p_\nu^2 - 2p_\pi \cdot p_\nu$$

$\mathbf{p}_\pi$ = 0: $$m_\mu^2 = m_\pi^2 + m_\nu^2 - 2m_\pi E_\nu$$

$$E_\nu = \frac{m_\pi^2 + m_\nu^2 - m_\mu^2}{2m_\pi}$$

So there is an *intrinsic uncertainty* on neutrino energy corresponding to the difference in mass eigenstates

…but then how are these ever coherent?

$$\left\langle e^{-i(E_1 - E_2)t} \right\rangle_t = 0 \quad \textit{unless } E_2 = E_1$$

"Only neutrino mass eigenstates with a common energy E are coherent" —Stodolsky

Intrinsic Momentum Uncertainty

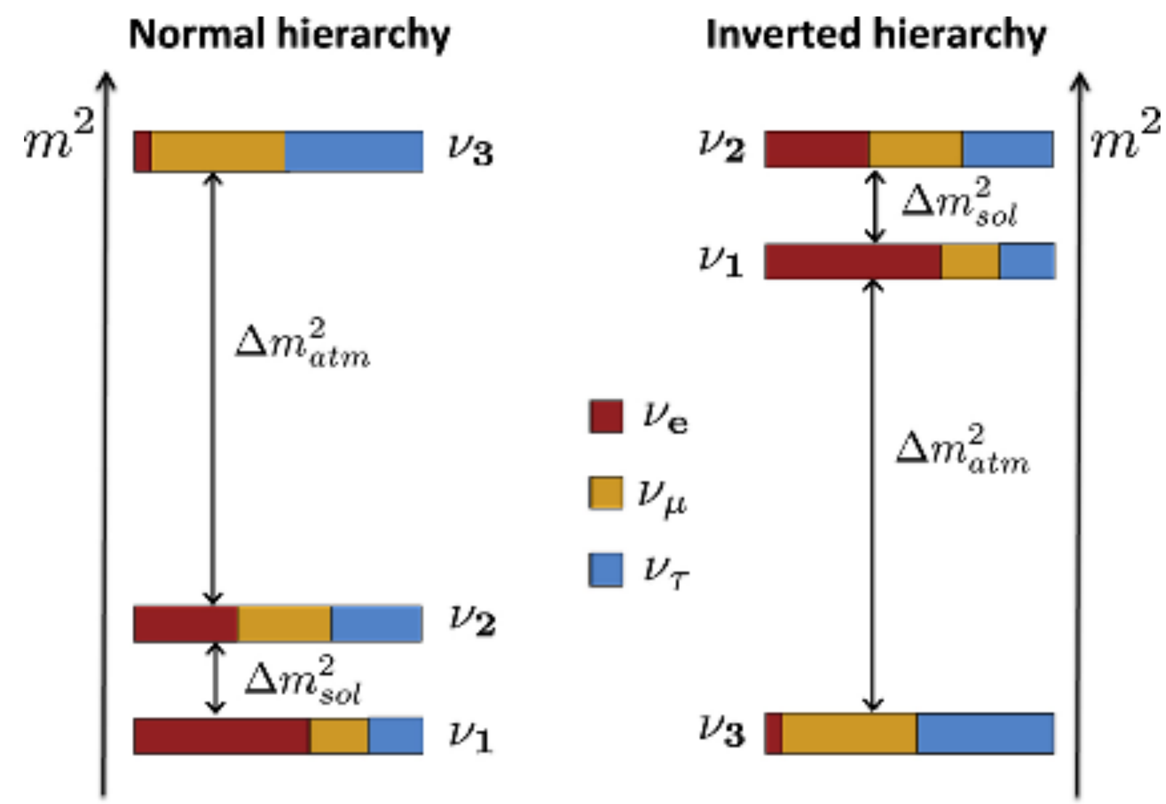
of a superposition of mass-split eigenstates

Momentum of a neutrino given its energy E

$$p = (E^2 - m^2)^{\frac{1}{2}} \approx E - \frac{m^2}{2E}$$

Taylor Expand around small m²/E²

Heisenberg: $$\Delta x \Delta p \geq \frac{1}{2}$$

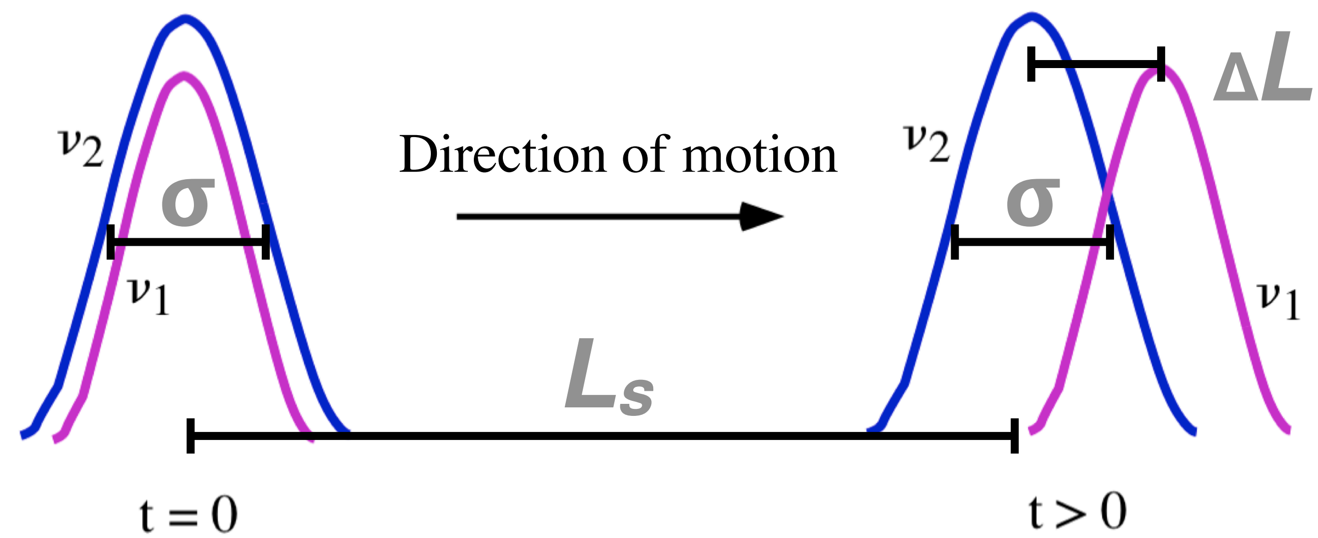


— Plug in Taylor-approximated momentum
— Atmospheric mass splitting dominates

$$\Delta p = \frac{m^2_{\mathrm{max}} - m^2_{\mathrm{min}}}{2E} \approx \frac{\Delta m^2_{\mathrm{atm}}}{2E}$$

$$\Delta x \geq \frac{E}{\Delta m^2_{\mathrm{atm}}}$$

Decoherence



$$\Delta L = t\Delta v = \frac{L_s}{v_{\mathrm{min}}}\Delta v$$

$$v = \beta = \frac{p}{E} = \left(1 - \frac{m^2}{E^2}\right)^{\frac{1}{2}} \approx 1 - \frac{m^2}{2E^2}$$

$$\Delta L = L_s \left(\frac{2E^2}{2E^2 - m_j^2}\right)\left(\frac{m_j^2 - m_i^2}{2E^2}\right)$$

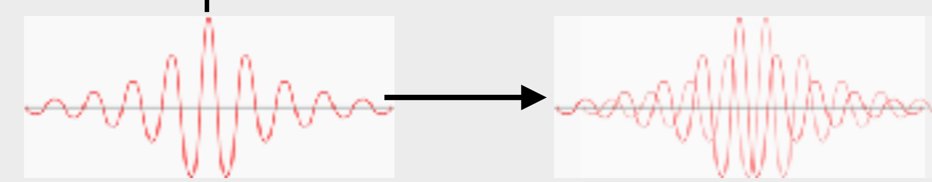
$$L_s = \Delta L \left(\frac{2E^2 - m_j^2}{\Delta m_{ij}^2}\right)$$

Define width σ

$$L_{\mathrm{coh}} = \sigma \left(\frac{2E^2}{\Delta m_{ij}^2}\right)$$

It's much more subtle…

Wave packets are more like:



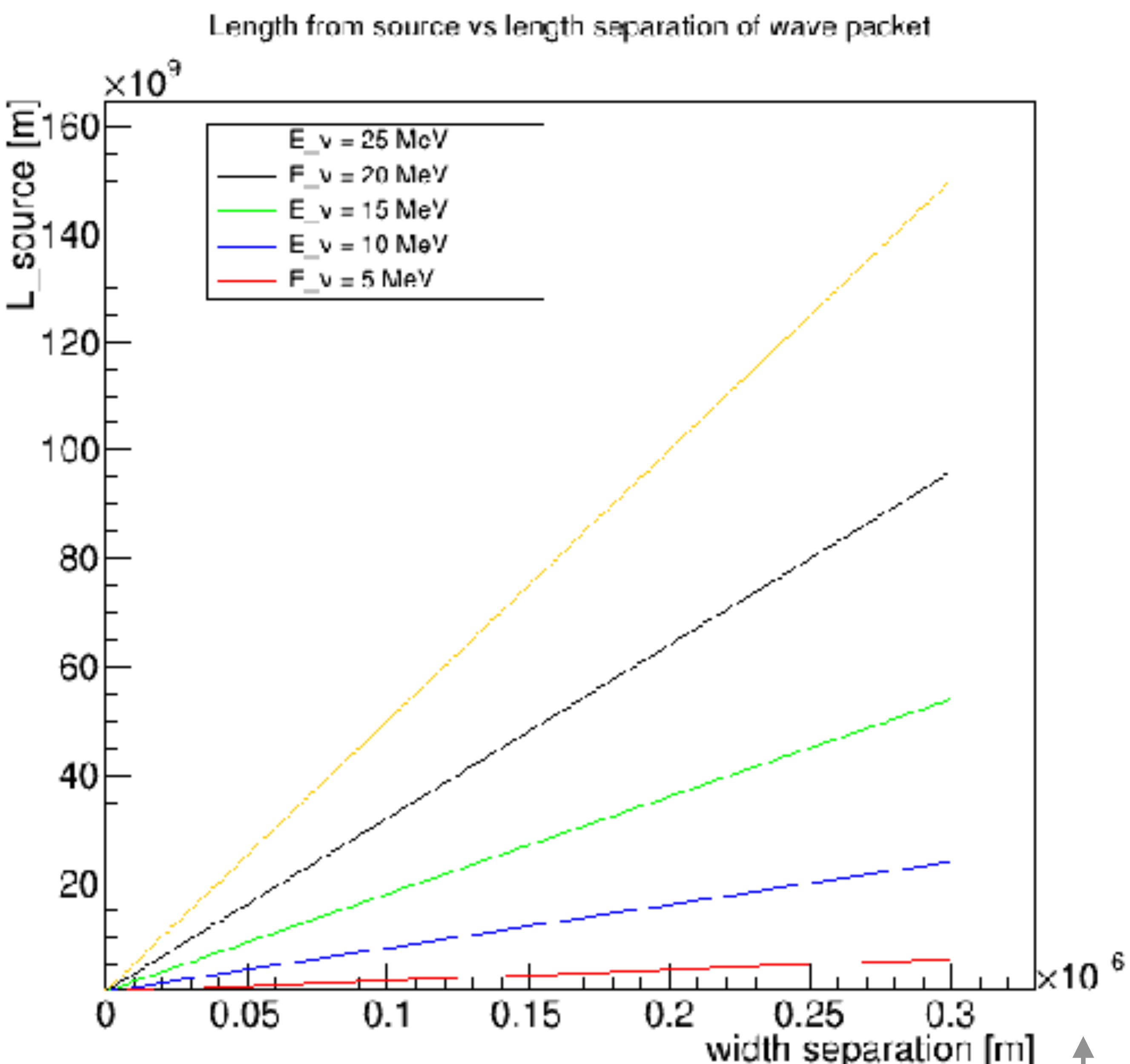
What happens when $\boldsymbol{\nu}_3$ is incoherent while $\boldsymbol{\nu}_1$ and $\boldsymbol{\nu}_2$ are coherent? 1 and 2 still interfere, so they still oscillate. Writing that down is a matter of wave packet formalism.

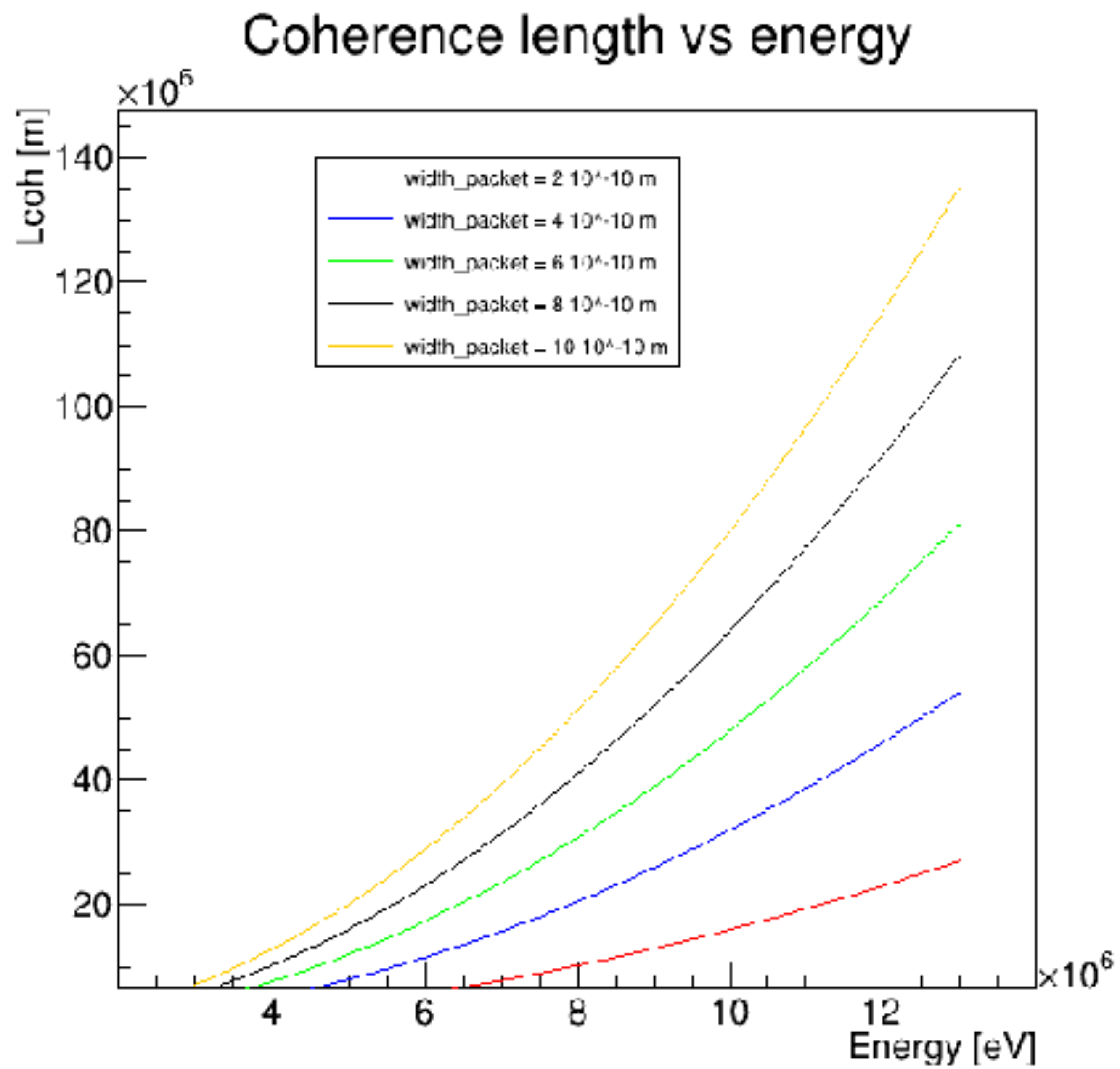
https://arxiv.org/abs/hep-ph/0109119

https://arxiv.org/abs/1001.4815

Estimating the width $\boldsymbol{\sigma}$ of the wave packet depends on the source. How precisely *can* you know the position the neutrino was created ($\boldsymbol{\Delta}$x). Not how well *do* you know

Decoherence





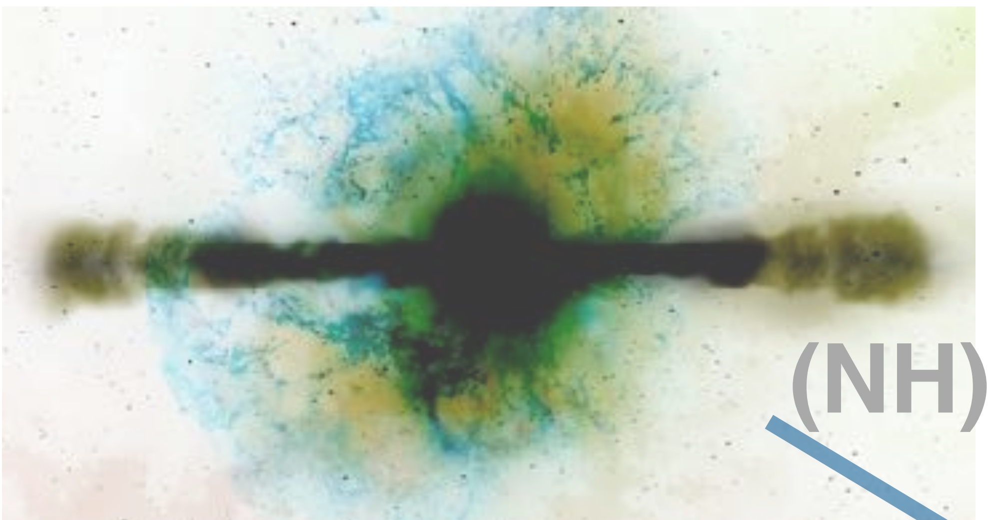
$$L_s = \Delta L \left(\frac{2E^2 - m_j^2}{\Delta m_{ij}^2} \right)$$

Nucleon density
Atomic widths
Length of decay pipe
  but can still pinpoint nu
  creation much more closely

$$L_{\text{coh}} = \sigma \left(\frac{2E^2}{\Delta m_{ij}^2} \right)$$

Supernova Neutrinos

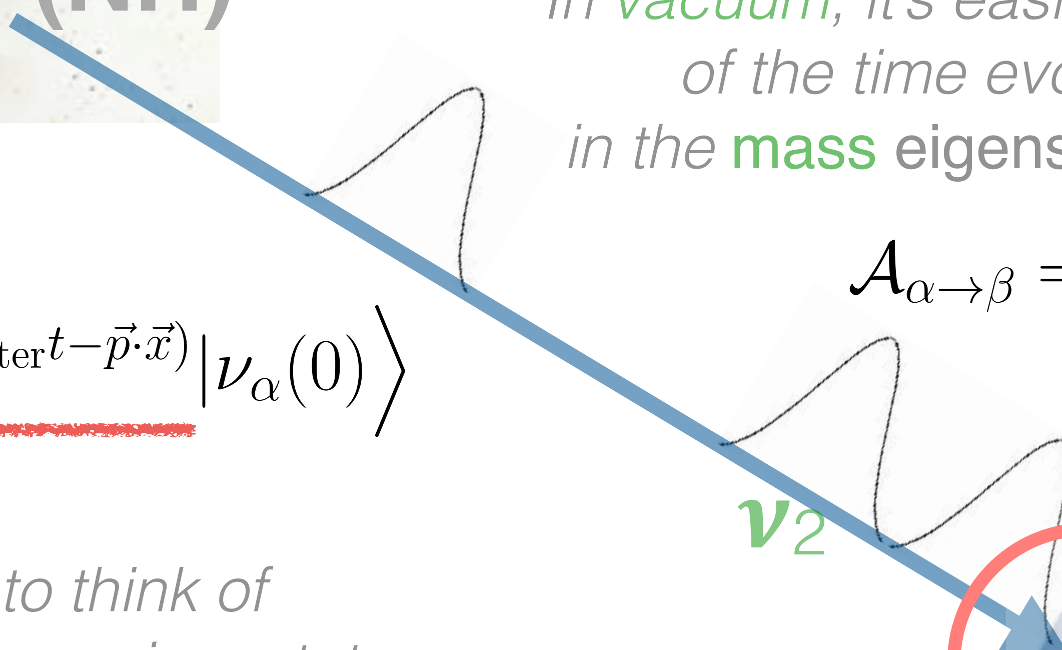
arrive at Earth with incoherent mass states



$$\left|\langle \nu_e \mid \nu_2 \rangle\right|^2 = |U_{e2}|^2 = \frac{1}{3}$$

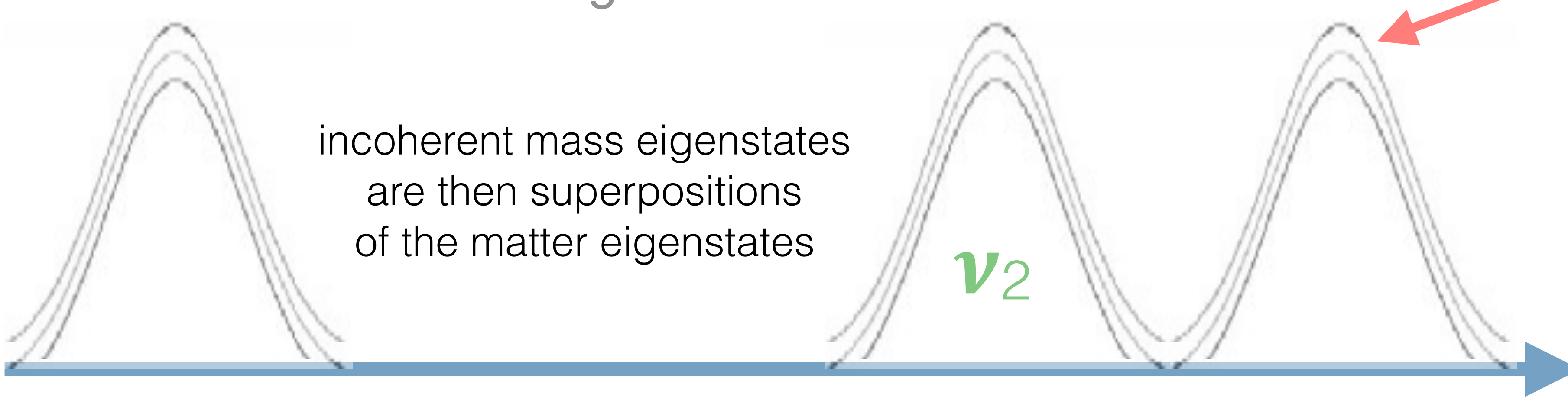
*In vacuum, it's easiest to think of the time evolution in the mass **eigenstate** basis*

$$\mathcal{A}_{\alpha\to\beta} = \Big\langle \nu_\beta(0) \big| \underline{e^{-i(\hat{H}_{\rm vac}t - \vec{p}\cdot\vec{x})}} \big| \nu_\alpha(0) \Big\rangle$$



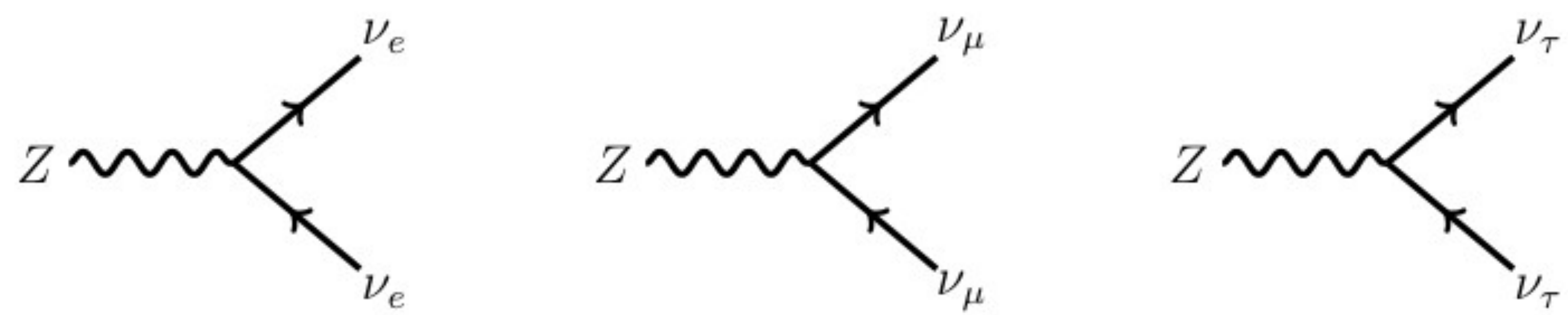
$$\mathcal{A}_{\alpha\to\beta} = \Big\langle \nu_\beta(0) \big| \underline{e^{-i(\hat{H}_{\rm matter}t - \vec{p}\cdot\vec{x})}} \big| \nu_\alpha(0) \Big\rangle$$

*In matter, it's easiest to think of the time evolution of the mass eigenstates in the matter **eigenstate** basis*



Do neutrinos from Z^0 decays oscillate ?

Flavor blind Z^0 decay :



Production flux from Z^0 decay :

$$\phi_e = \phi_\mu = \phi_\tau = \phi^0$$

General misconception : the observed neutrinos are decoherent

Observed flux for a given flavor β :

$$\phi_\beta(r) = \sum_\alpha \phi_\alpha P_{\alpha\beta}(r)$$

$$\Rightarrow \quad \phi_\beta(r) = \phi^0 \sum_\alpha P_{\alpha\beta}(r) = \phi^0$$

Flux for one given flavor independent of distance → no oscillation process observed

https://inspirehep.net/record/321272/

What would it take to observe these oscillation ?

Coherence $\Longrightarrow$ We write the neutrino state as a combination of the mass eigenstates

$$\Rightarrow \quad |\nu_z(r,\bar{r})> = \frac{1}{\sqrt{3}} \sum_{1,2,3} |\nu_i> |\bar{\nu_i}> \boxed{e^{i\phi_i(r,\bar{r})}}$$

Phase factor

Probability to observe a ν_α at distance r and a $\bar{\nu_\beta}$ at distance $\bar{r}$:

$$W_{\alpha\beta} = \frac{1}{3} \sum_i |U_{i\alpha}|^2 |U_{i\beta}|^2 + \underbrace{\frac{2}{3} \sum_{i>j} |U^*_{i\alpha} U_{i\beta} U_{j\alpha} U^*_{j\beta}| cos(\phi_i - \phi_j - \xi_{\alpha\beta ij})}_{\text{Oscillation factor}}$$

Probability to observe a ν_α :

$$W_\alpha = \sum_\beta W_{\alpha\beta} = \frac{1}{3}$$

We need to look both the neutrino and antineutrino to actually observe the oscillations

Thank You

Safe Travels!