Primordial GWs and PBHs from axion inflation

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Outline

Inflation: the basic picture

Inflation is an early phase of nearly exponential expansion.

The metric of $g_{\mu\nu}$ of an homogeneous and isotropic Universe ($k = 0$) is:

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2$$

Einstein Equations ($\Lambda = 0, \kappa^2 \equiv 8\pi G_N$) read:

$$3\left(\frac{\dot{a}}{a}\right)^2 \equiv 3H^2 = \rho\kappa^2, \qquad -2\dot{H} = (p+\rho)\kappa^2$$

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Homogeneous scalar field ϕ in a homogeneous and isotropic universe:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} + \frac{\dot{\phi}^2}{2} - V(\phi)\right)$$

In this case the pressure and energy density are:

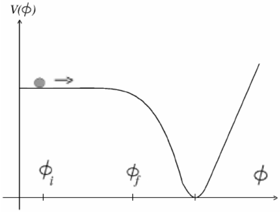
$$p = \frac{\dot{\phi}^2}{2} - V \qquad \rho = \frac{\dot{\phi}^2}{2} + V$$

Slow-roll inflation

Slow-roll inflation:
$$\left|\dot{\phi}^2/2\right| \ll |V(\phi)|.$$

The evolution is fixed by:

$$\begin{aligned} 3H^2 &= \rho\kappa^2 \simeq V\kappa^2 \\ -2\dot{H} &= (p+\rho)\kappa^2 = \dot{\phi}^2\kappa^2 \\ 3H\dot{\phi} &\simeq -V_{,\phi} \end{aligned}$$



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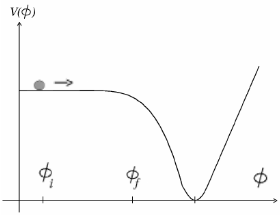

To discuss this problem it is useful to introduce the slow-roll parameters:

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2} \simeq \frac{1}{2\kappa^2}\left(\frac{V_{,\phi}}{V}\right)^2 \equiv \epsilon_V$$

$$\epsilon_2 \equiv \frac{\mathrm{d}\ln(\epsilon_1)}{\mathrm{d}\ln a} \simeq -\frac{2}{\kappa^2}\frac{V_{,\phi\phi}}{V} + \frac{2}{\kappa^2}\left(\frac{V_{,\phi}}{V}\right)^2 \equiv -2\eta_V + 4\epsilon_V$$

and the number of e-foldings (from the end of inflation):

$$N(t) \equiv -\int_{a_f}^{a} \mathrm{d}\ln\hat{a} = -\int_{t_f}^{t} H(\hat{t})\mathrm{d}\hat{t} \simeq \int_{\phi_f}^{\phi} \kappa^2 \frac{V(\hat{\phi})}{V_{,\phi}(\hat{\phi})}\mathrm{d}\hat{\phi}$$

Model classification

Inflationary models can be classified using:

Mukhanov 2013, Roest 2014, Garcia-Bellido and Roest 2014, Binetruy, Kiritsis, Mabillard, Pieroni and Rosset 2015

$$\epsilon_1 \simeq \frac{\beta}{(1+N)^p}$$

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$$\epsilon_1 \simeq \frac{\beta}{(1+N)^p}$$

- (p=1) $\longrightarrow$ Chaotic models:

$$V(\phi) = V_0\ \phi^q$$

- (p=2) $\longrightarrow$ Starobinsky-like models:

$$V(\phi) \simeq V_0\left(1 - \exp\{-\gamma\phi\}\right)^2$$

- (p=3) and (p=4) $\longrightarrow$ Hilltop models:

$$V(\phi) \simeq V_0\left[1 - \left(\frac{\phi}{v}\right)^4\right]^2, \qquad V(\phi) \simeq V_0\left[1 - \left(\frac{\phi}{v}\right)^3\right]^2$$

- $(1 < p < 2)$ $\longrightarrow$ Inverse power law models:

$$V(\phi) \simeq V_0\left(1 - \frac{\alpha}{\phi^\gamma}\right)^2$$

Axion inflation

Inflaton coupled to some Abelian gauge fields:

$$\mathcal{L} = \frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4\Lambda}\phi\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Turner, Widrow '88,
Garretson, Field, Caroll '92,
Anber, Sorbo '06./'10/'12,
Barnaby, Namba, Peloso '11,
Barnaby, Pajer, Peloso '12 ,
…..

The equations of motion for the fields are:

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} &= \frac{\alpha}{\Lambda}\langle \vec{E}\cdot\vec{B}\rangle \\ \frac{\mathrm{d}^2\,\vec{A}^a(\tau,\vec{k})}{\mathrm{d}\tau^2} - \vec{\nabla}^2\vec{A}^a &= \frac{\alpha}{\Lambda}\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\vec{\nabla}\times\vec{A}^a \end{aligned}$$

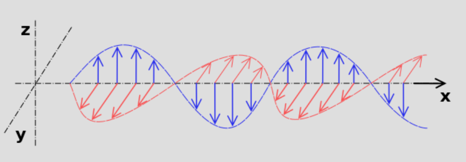
$$\begin{aligned} \mathrm{d}t &\equiv a\,\mathrm{d}\tau \\ N &\equiv -\int H\,\mathrm{d}t \end{aligned}$$

Friedmann equation reads:

$$3H^2\kappa^{-2} = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}\langle \vec{E}^2 + \vec{B}^2\rangle$$

Gauge field amplification

Assuming
$\vec{k}$ parallel to $\hat{x}$



$$\vec{e}_\pm \equiv (\hat{y} \pm i\hat{z})/\sqrt{2}$$
$$\vec{A}^a \equiv \vec{e}_\pm A^a_\pm$$

The equations of motion for the gauge fields (in Fourier transform) read:

$$\frac{\mathrm{d}^2\ A^a_\pm(\tau,\vec{k})}{\mathrm{d}\tau^2} + \left[k^2 \pm 2k\frac{\xi}{\tau}\right] A^a_\pm(\tau,\vec{k}) = 0$$

$$\boxed{\xi \equiv \frac{\alpha}{2\Lambda}\left|\frac{\dot{\phi}}{H}\right| \propto \sqrt{\epsilon_1}}$$

If ξ is nearly constant one mode (A^a_+) is exponentially growing with ξ.

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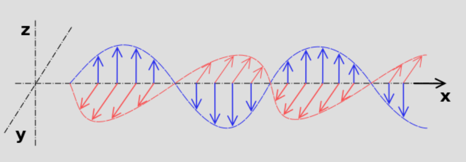

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Substituting $\langle \vec{E} \cdot \vec{B} \rangle$ into the equation of motion for ϕ we get:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = \frac{\alpha}{\Lambda}\langle \vec{E} \cdot \vec{B} \rangle \simeq \frac{\alpha}{\Lambda} 2.4 \cdot 10^{-4} \mathcal{N} \left(\frac{H}{\xi}\right)^4 e^{2\pi\xi}$$

Friction term that dominates the last part of the evolution.

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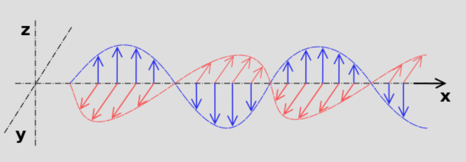

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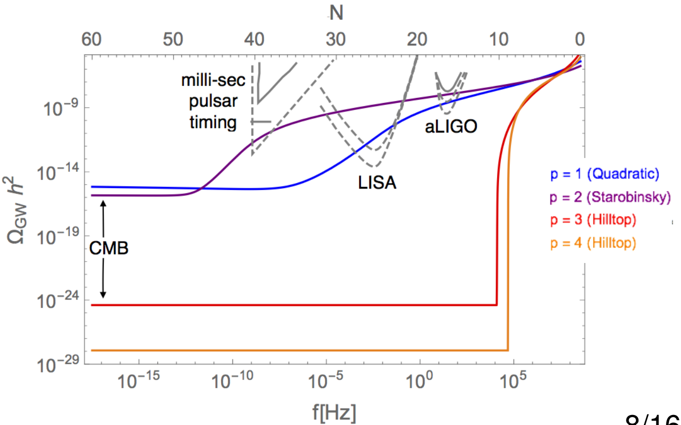
Modified dynamics also affects the scalar and tensor power spectra!

Modified tensor spectrum

GW spectrum $\longrightarrow$ $$\Delta_t^2(k) = \frac{1}{12}\left(\frac{\kappa H}{\pi}\right)^2\left(1 + 4.3\cdot 10^{-7}\frac{\kappa^2 H^2}{\xi^6}e^{4\pi\xi}\right)$$

N-frequency relation $\longrightarrow$ $$N = N_{\mathrm{CMB}} + \ln\frac{k_{\mathrm{CMB}}}{0.002\ \mathrm{Mpc}^{-1}} - 44.9 - \ln\frac{f}{10^2\ \mathrm{Hz}}$$

- Spectra asymptote to an universal value at small scales
- Low scale models ($p = 3, 4$) have a stronger increase
- Some models produce GW in the observable range of direct GW detectors

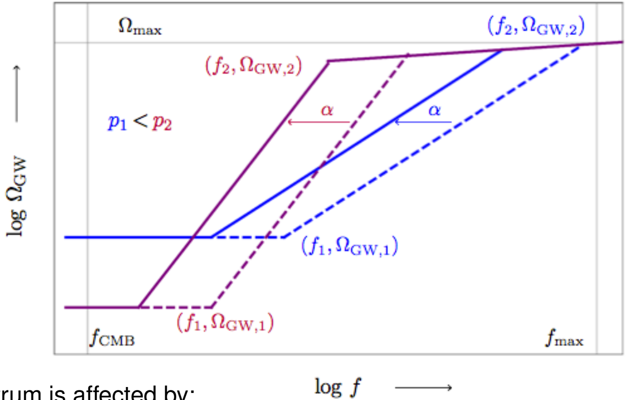


Domcke, Pieroni and Binetruy 2016

General features of the GW spectrum

Notice that:

- Gauge fields take over at f_1
- Gauge fields' friction dominates after f_2
- Ω_{GW}^{CMB} is fixed by COBE and r.
- Ω_{GW}^{Max} is fixed by $\epsilon_1 \lesssim 1$.



The shape of the spectrum is affected by:

- p : the slope and the vacuum amplitude
- β : vacuum amplitude
- α/Λ : shifts the spectrum horizontally

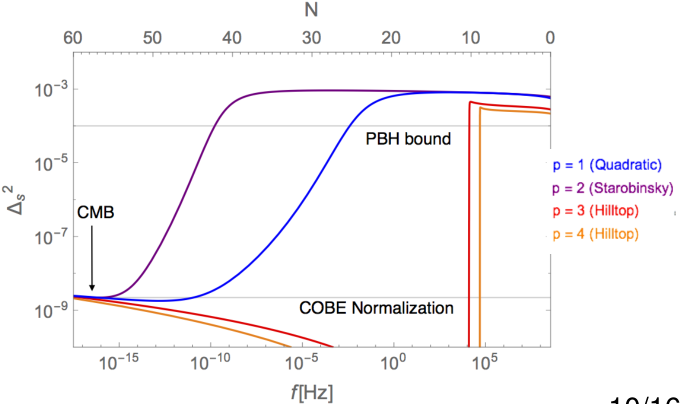
$$\epsilon_1 \simeq r \simeq \beta/(1+N)^p$$
$$\mathcal{L}_{int} = \frac{\alpha}{4\Lambda}\phi F_{\mu\nu}\tilde{F}_{\mu\nu}$$

Modified scalar spectrum

Scalar spectrum $\longrightarrow$ $$\Delta_s^2(k) = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 + \left(\frac{\alpha\langle\vec{E}\cdot\vec{B}\rangle}{3bH\dot{\phi}}\right)^2$$

where: $$b \equiv 1 - 2\pi\xi\frac{\alpha\langle\vec{E}\cdot\vec{B}\rangle}{3\Lambda H\dot{\phi}}$$

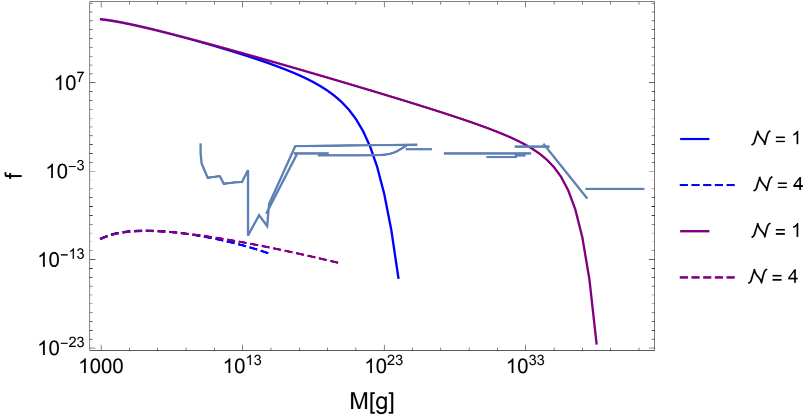
- COBE normalization fixes V_0
- Nearly universal behavior at large scales
- $\Delta_s^2(k) \simeq \frac{1}{\mathcal{N}(2\pi\xi)^2}$ at small scales (Linde, Mooij, Pajer 2013)
- Strong increase at small scales $\rightarrow$ PBHs



Domcke, Pieroni and Binetruy 2016

Comparison with PBH bounds

The PBHs can account for part (or all) of the DM in our Universe!
Predictions must be consistent with all the bounds:



A generalized case

Assuming the inflaton to be non-minimally coupled with gravity:

$$\mathcal{L} = \Omega(\phi)\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{4\Lambda}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

The equations of motion in Einstein frame read:

$$\ddot{\phi} + \dot{\phi}\frac{\dot{K}(\phi)}{K(\phi)} + 3H\dot{\phi} + \frac{1}{K(\phi)}\frac{\partial V}{\partial\phi} = \frac{\alpha}{\Lambda}\frac{\langle\vec{E}\cdot\vec{B}\rangle}{K(\phi)}$$

$$\frac{\mathrm{d}^2\,\vec{A}^a(\tau,\vec{k})}{\mathrm{d}\tau^2} - \vec{\nabla}^2\vec{A}^a = \frac{2\xi}{aH}\vec{\nabla}\times\vec{A}^a$$

$$\mathrm{d}t \equiv a\,\mathrm{d}\tau$$

$$N \equiv -\int H\,\mathrm{d}t$$

$$\xi \equiv \frac{\dot{\phi}}{H} = \sqrt{2\epsilon/K(\phi)}$$

Friedmann equation reads:

$$3H^2\kappa^{-2} = \frac{K(\phi)}{2}\dot{\phi}^2 + V(\phi) + \frac{1}{2}\langle\vec{E}^2 + \vec{B}^2\rangle$$

where the kinetic function is defined as:

$$K(\phi) = \frac{1}{\Omega(\phi)} + \frac{3}{2}\left(\frac{\mathrm{d}\ln\Omega}{\mathrm{d}\phi}\right)^2 \qquad (1)$$

Modified spectra

As $\xi \propto \sqrt{2\epsilon/K} \rightarrow K(\phi)$ growing at small scales shuts off the instability!

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For example: $\Omega(\phi) = 1 + \varsigma g(\phi)$ and $V(\phi) = V_0 g^2(\phi)$.

Kallosh and Linde 2010, Kallosh, Linde and Roest 2014, . . .

with $g(\phi) = 1 - 1/\phi$ (which for $\varsigma = \alpha = 0$ corresponds to $\epsilon_V \simeq 1/N^{4/3}$):

Domcke, Muia, Pieroni and Witkowski 2017

$$K(N) \simeq \text{const (large scales)}\,, \qquad K(N) \simeq e^{-N} \text{ (small scales)}\,.$$

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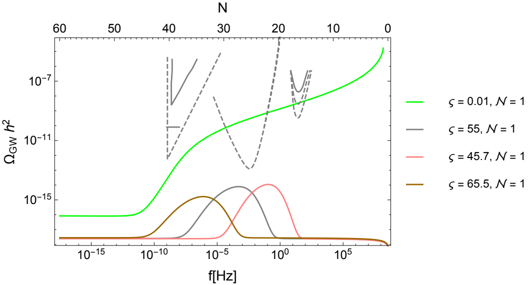
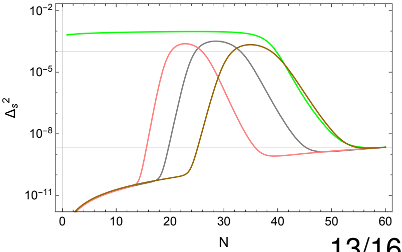
$$K(N) \simeq \text{const (large scales)} , \qquad K(N) \simeq e^{-N} \text{ (small scales)} .$$

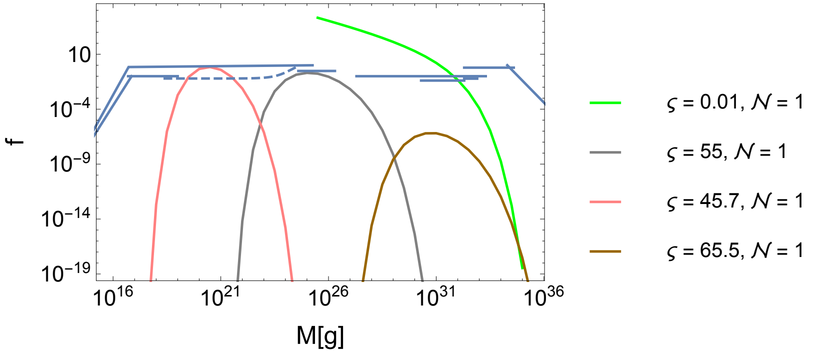
Which leads to the modified spectra ($\alpha/\Lambda = \{97, 63, 71, 74\}$):





Comparison with PBH bounds

The corresponding PBH distributions are:



Leading to :

$$f_{\rm tot}^{\varsigma=45.7} = 98.6\%, \quad f_{\rm tot}^{\varsigma=55} = 39.4\%, \quad f_{\rm tot}^{\varsigma=65.5} = 1.2 \cdot 10^{-4}\,\%\,.$$

Domcke, Muia, Pieroni and Witkowski 2017

Conclusions and future perspectives

Conclusions:

- Possibility of generating of an observable GW background.
- If observed informations on the microphysics of inflation.
- Possibility of generating a distribution of PBHs.

Future perspectives:

- Non-abelian gauge fields
- Reheating
- Embedding in high energy theories
- Model building (using $\epsilon(N)$ and $K(N)$?)

Last Slide

The End

Thank you

Model classification

The system is specified by four parameters: $\alpha/\Lambda, \beta, p, V_0$.

No gauge fields $\longrightarrow$ $n_s \simeq 1 - \dfrac{\mathcal{O}(1)}{N}$ $r \propto \epsilon_V \simeq \dfrac{\mathcal{O}(1)}{(1+N)^p}$

The gauge fields introduce an additional friction term.

- The CMB observables are effectively 'shifted' at a 'later' point N_*:

$$N_* < N_{CMB} \simeq 60 \qquad \longrightarrow \qquad n_s \simeq 1 - \frac{\mathcal{O}(1)}{N_*} \qquad r \propto \epsilon_V \simeq \frac{\mathcal{O}(1)}{(1+N_*)^p}$$

  We get reduced n_s and increased r with respect to the standard case.
- As $\xi \propto \sqrt{\epsilon_1} \simeq \sqrt{\epsilon_V}$, the effects on models with big p will be stronger.

Domcke, Pieroni and Binétruy 2016

CMB constraints

The presence of the gauge fields modifies the scalar and tensor power spectra but CMB constraints should not be violated.

- **COBE Normalization**: Sets the value of the scalar power spectrum at the CMB scales:

$$\Delta_s^2\Big|_{N_{CMB}} = (2.21 \pm 0.07) \cdot 10^{-9}$$

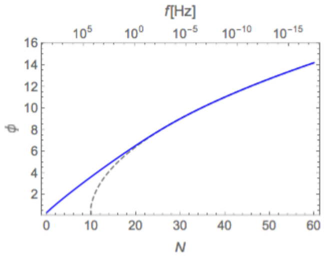
- **Planck constraints on n_s and r**:

$$n_s = 0.9645 \pm 0.0049 \qquad\qquad r < 0.10$$

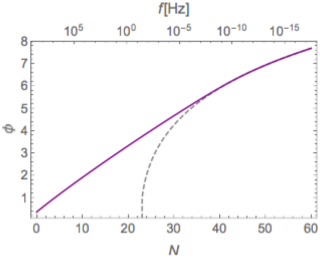
- **Non gaussianities**: The gauge fields induce a non-gaussian component in the power spectrum. To be consistent we need:

$$\xi_{CMB} = \frac{\alpha}{2\Lambda}\left|\frac{\dot{\phi}}{H}\right|_{N=N_{CMB}} \lesssim 2.5$$

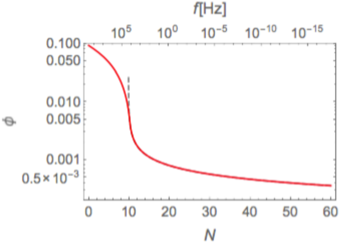
Modified scalar evolution

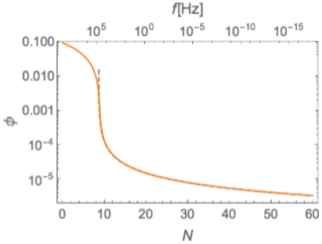


(a) *Quadratic model with* $\alpha/\Lambda = 35$ *and* $V_0 = 1.418 \cdot 10^{-11}$.

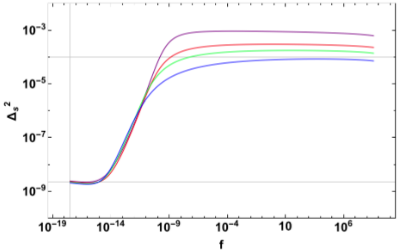


(b) *Starobinsky model with* $\alpha/\Lambda = 75$, $\gamma = 0.3$, $V_0 = 1.17 \cdot 10^{-9}$.

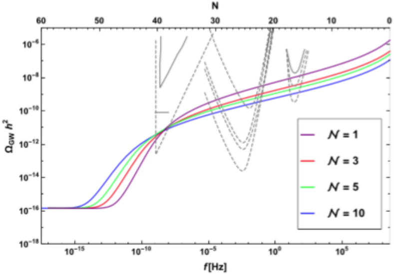




Several gauge fields



(a) *Scalar power spectra.*



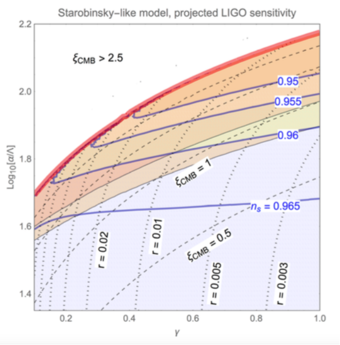
(b) *Tensor power spectra.*

Starobinsky-like model parameter space

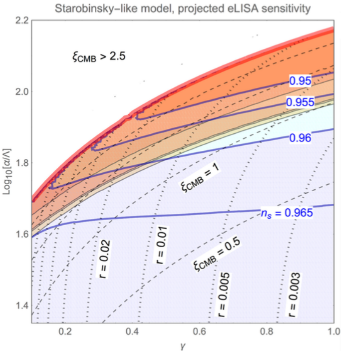
Choosing:

- $p = 2$
- $\beta = 1/(2\gamma)^2$

leads to: $V(\phi) \simeq V_0 \left(1 - \exp\{-\gamma\phi\}\right)^2$



(a) *LIGO plot.*



(b) *eLISA plot.*

The complementarity between different measures can be used to restrict the parameter space!