Gravitational waves from phase transitions

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Outline

Recap: transition via growth and merger of bubbles

- Below critical temperature T_c, thermal fluctuations in symmetric phase produce bubbles at rate/volume

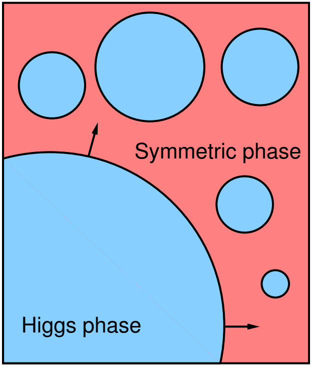
$$\Gamma(T) = \Gamma_0(T)\exp(-S(T))$$

with $\Gamma_0(T) \simeq T^4$, $S(T) = A/(T/T_c - 1)^2$

- Bubble wall speed v_w
- Fraction in symmetric (+) phase

$$h(t) = \exp\left(-I(t)\right)$$

with $I(t) = \frac{4\pi}{3}\int_{t_c}^{t} dt' \Gamma(t') v_w^3 (t - t')^3$



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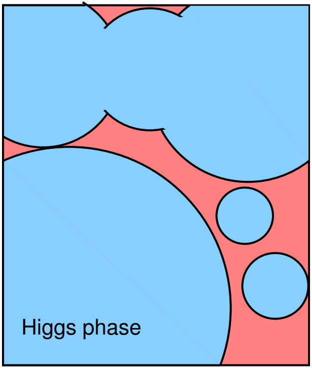

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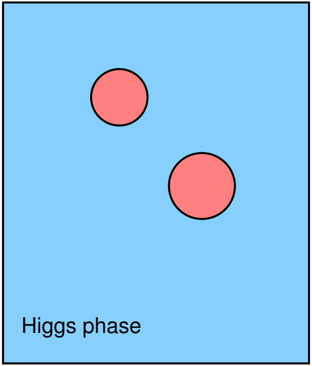


Recap of basics

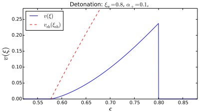
- Saddle-point solution:

$$h(t) = \exp\left(-e^{\beta(t-t_f)}\right)$$

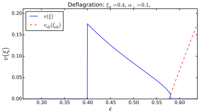
with $\beta = -S'(t_f)$ and $8\pi(v_\mathrm{w}/\beta)^3\Gamma_f/\beta = 1$

- Time of peak nucleation rate t_f
- 'Nucleation temperature' $T_\mathrm{n} = T_f$
- Trace anomaly $\theta(T) = e(T) - 3p(T)$, difference $\Delta\theta = \theta_+(T_\mathrm{n}) - \theta_-(T_\mathrm{n})$
- Transition strength $\alpha_\theta = \frac{\Delta\theta}{3w(T_\mathrm{n})}$
- Wall speed v_w

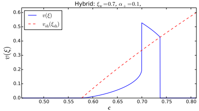
Similarity solutions:
detonation



deflagration



hybrid

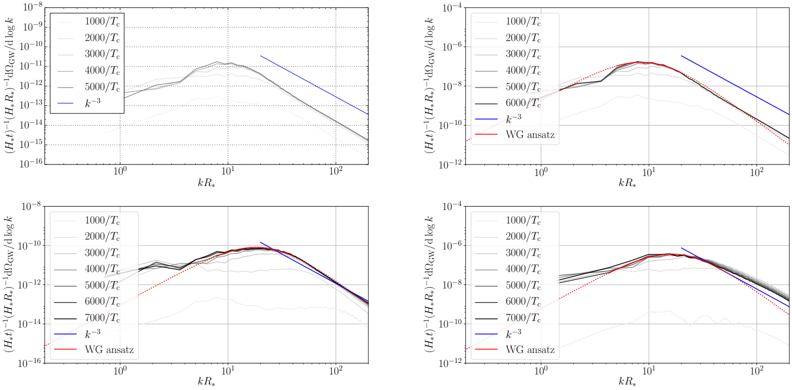


Method

- Scalar field
$$-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta W(\dot{\phi} + V^i \partial_i \phi)\,,$$
W Lorentz factor, V^i is fluid 3-velocity, $U^i = WV^i$.
- Fluid energy variable $E = We$,
$$\dot{E} + \partial_i(EV^i) + p[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi} W(\dot{\phi} + V^i \partial_i \phi) = \eta_T(\phi) W^2 (\dot{\phi} + V^i \partial_i \phi)^2.$$
- Momentum density $Z_i = W(e+p)U_i$:
$$\dot{Z}_i + \partial_j(Z_i V^j) + \partial_i p + \frac{\partial V}{\partial \phi}\partial_i \phi = -\eta W(\dot{\phi} + V^j \partial_j \phi)\partial_i \phi.$$
- Numerical solution with GWEPT (D. Weir), see arXiv:1504.03291.

Gravitational wave power spectrum: detonations

α_θ	0.0046 ('weak')	0.050 ('intermediate')
v_w		
0.92		
0.7		

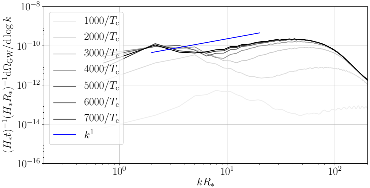


Gravitational wave power spectrum: deflagrations

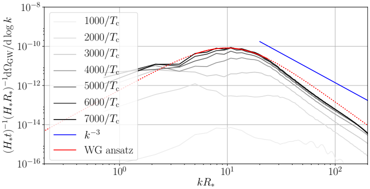
α_θ — 0.0046 ('weak') — 0.050 ('intermediate')

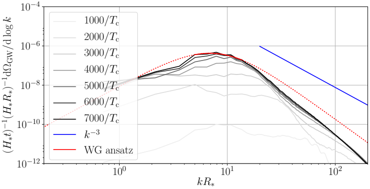
v_w

0.56



0.44





Phenomenological fit ($|v_\mathrm{w} - v_\mathrm{CJ}| \gtrsim 0.1$)

Wall speeds away from Chapman-Jouguet speed[(1)]

$$\frac{d\Omega_{\mathrm{gw},0}}{d\ln(f)} = 0.68 F_{\mathrm{gw},0} (\Gamma \overline{U}_\mathrm{f}^2)^2 (H_\mathrm{n} R_*) \tilde{\Omega}_\mathrm{gw} C\left(\frac{f}{f_{\mathrm{p},0}}\right). \quad (1)$$

$$C(s) = s^3 \left(\frac{7}{4+3s^2}\right)^{\frac{7}{2}}, \; \Gamma\overline{U}_\mathrm{f}^2 = \frac{\langle w\gamma^2 v^2\rangle}{\bar{e}}, \; R_* = (8\pi)^{\frac{1}{3}} \left(\frac{v_\mathrm{w}}{\beta}\right) \quad (2)$$

$$f_{\mathrm{p},0} \simeq 26 \left(\frac{1}{H_\mathrm{n} R_*}\right) \left(\frac{z_\mathrm{p}}{10}\right) \left(\frac{T_\mathrm{n}}{10^2\,\mathrm{GeV}}\right) \left(\frac{h_*}{100}\right)^{\frac{1}{6}} \mu\mathrm{Hz}, \quad (3)$$

with numerically obtained values: $\tilde{\Omega}_\mathrm{gw} \simeq 0.12$, $z_\mathrm{p} \simeq 10$ (peak value of kR_*).
Standard GW matter era dilution factor (with Planck value for Hubble rate)

$$F_{\mathrm{gw},0} = (3.57 \pm 0.05) \times 10^{-5} \left(\frac{100}{h_*}\right)^{\frac{1}{3}}. \quad (4)$$

[(1)] Deflagration: $v_\mathrm{CJ} = c_\mathrm{s}$, detonation $v_\mathrm{CJ} \simeq c_\mathrm{s}(1+\sqrt{2\alpha_\theta})$.

Relative sizes of acoustic and turbulent sources

- General: RMS velocity $\overline{U}_{\rm f}$, length scale $L_{\rm f}$, Hubble parameter $H_{\rm n}$:

$$\Omega_{\rm gw}^{\rm ac} \sim (\Gamma \overline{U}_{\rm f}^2)^2 (H_{\rm n} L_{\rm f}),$$

- Turbulent phase starts when $H = \tau_{\rm sh}^{-1}$, similar RMS velocity, length scale:

$$\Omega_{\rm gw}^{\rm tu} \sim (\Gamma \overline{U}_{\rm f}^2)^2 (L_{\rm f}/\tau_{\rm sh}),$$

- Ratio: $\dfrac{\Omega_{\rm gw}^{\rm tu}}{\Omega_{\rm gw}^{\rm ac}} \sim \dfrac{1}{\tau_{\rm sh} H_{\rm n}}$
- Strong phase transition: both phases last for time O($\tau_{\rm sh}$)

$$\Omega_{\rm gw}^{\rm tu} \sim \Omega_{\rm gw}^{\rm ac} \sim \Gamma^2 \overline{U}_{\rm f}^3 (L_{\rm f} H_{\rm n})^2$$

- Very strong transition: $\overline{U}_{\rm f} \sim 1$, back to envelope model ($L_{\rm f} \sim v_{\rm w}/\beta$)

Shapes of acoustic and turbulent sources (from CWG paper)

- Acoustic production (based on numerical simulations):

$$C^{\mathrm{ac}}(s) = s^3 \left(\frac{7}{4 + 3s^2} \right)^{\frac{7}{2}}$$

$s = f/f_{\mathrm{ac}}$, $f_{\mathrm{ac}} \simeq 3(v_{\mathrm{w}}/R_*)$ (New, factor 2 larger than CWG paper)

- Turbulent production (model of Caprini, Durrer, Servant 2009)

$$C^{\mathrm{tu}}(s) = s^3 \frac{1}{(1+s)^{\frac{11}{3}}(1 + 8\pi f/H_*)}$$

$s = f/f_{\mathrm{tu}}$, $f_{\mathrm{tu}} \simeq 2 f_{\mathrm{ac}}$

Sensitivity in kinetic energy - bubble separation space

- Kinetic energy fraction

$$\Gamma \overline{U}_{\mathrm{f}}^2 = \frac{\langle w \gamma^2 v^2 \rangle}{\bar{e}}$$

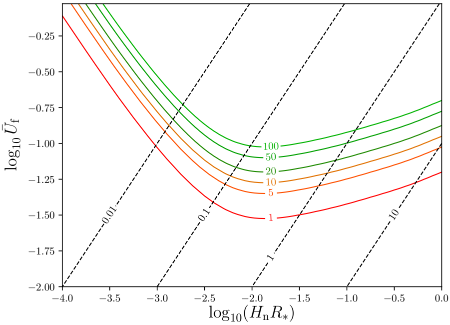
  Adiabatic index $\Gamma = \bar{w}/\bar{e}$
  Mean square fluid velocity
  $\overline{U}_{\mathrm{f}}^2 = \langle w \gamma^2 v^2 \rangle / \bar{w}$.

- "Efficiency parameter" κ

$$\frac{\kappa \alpha_\theta}{1 + \alpha_\theta} = \Gamma \overline{U}_{\mathrm{f}}^2$$

- Shock appearance (eddy turn-over) time:

$$\tau_{\mathrm{sh}} = \frac{R_*}{\overline{U}_{\mathrm{f}}}$$



Coloured contours: signal-to-noise ratio (eLISA config C2, L6A2M5N2M28)
Dashed contours: $\tau_{\mathrm{sh}} H_{\mathrm{n}}$