SIMULATING A VACUUM PHASE TRANSITION

LISA Cosmology Working Group

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Based on ongoing work with M. Hindmarsh and D. Weir

MOTIVATION

- Understand scalar only first order phase transitions.
  - Hidden sector. *[Garcia, Krippendorf and March-Russell, 2016]*
  - Highly supercooled thermal transition.
- Test envelope approximation. *[Kosowsky et al, 1992] [Huber and Konstandin, 2008] [Weir, 2016]*

- Previous work used few bubbles and low wall speeds. *[Child and Giblin, 2012]*

OVERVIEW

- Toy Model
- Initial conditions
- Evolution equations
- Envelope approximation
- Gravitational wave power spectra

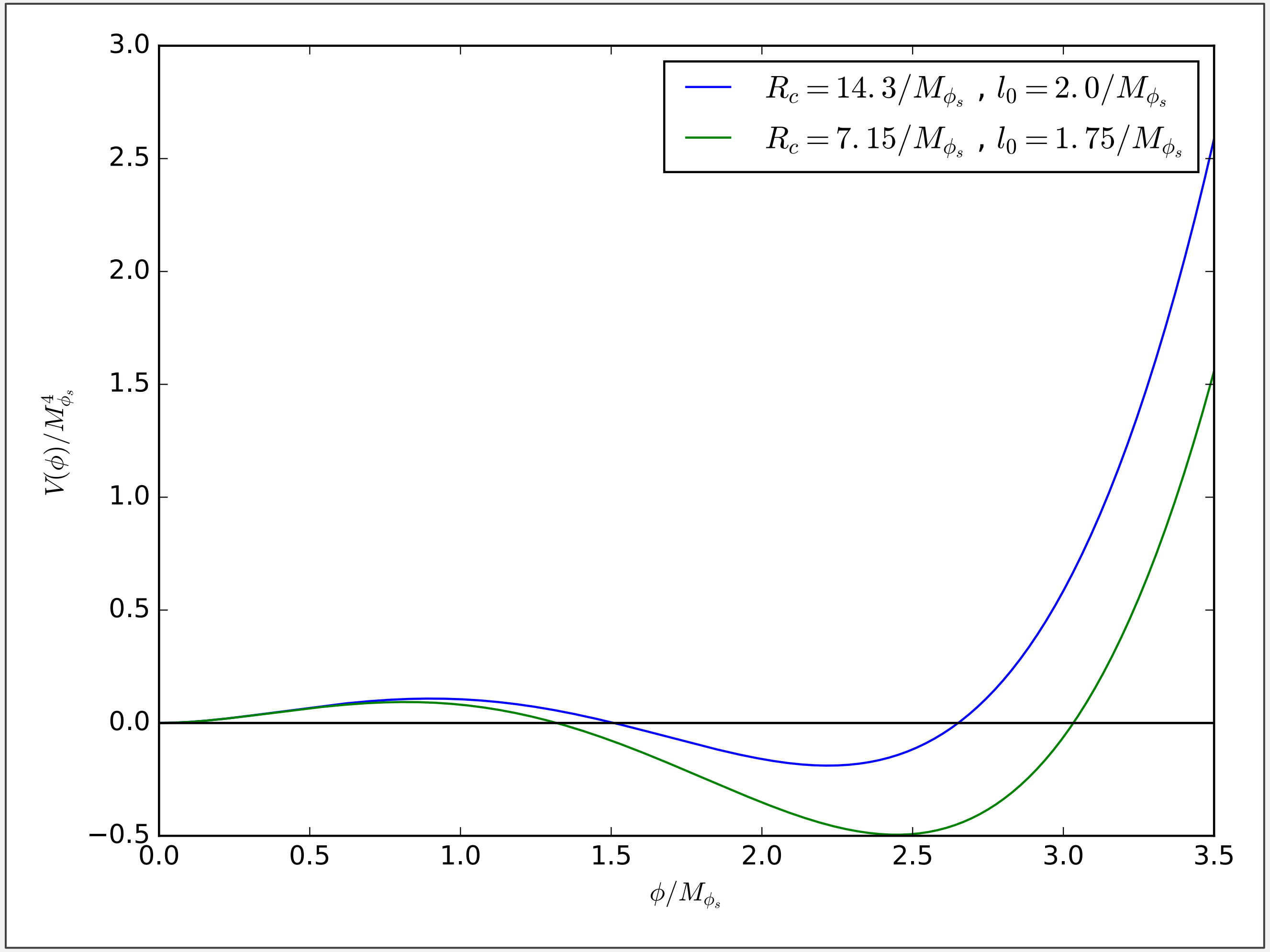
TOY MODEL

- Single real scalar field $\phi(x,t)$ with potential as follows:

$$V(\phi) = \frac{M_{\phi_s}^2 \phi^2}{2} - \frac{\delta\phi^3}{3} + \frac{\lambda\phi^4}{4} \; .$$

- Fix $M_{\phi_s}^2$, λ and vary δ in order to change the critical bubble radius, R_c, and wall width l_0.

- Use shooting method to find bubble profile.



INITIAL CONDITIONS

- Mean bubble separation:

$$R_* = \left(\frac{V}{N_b}\right)^{1/3}$$

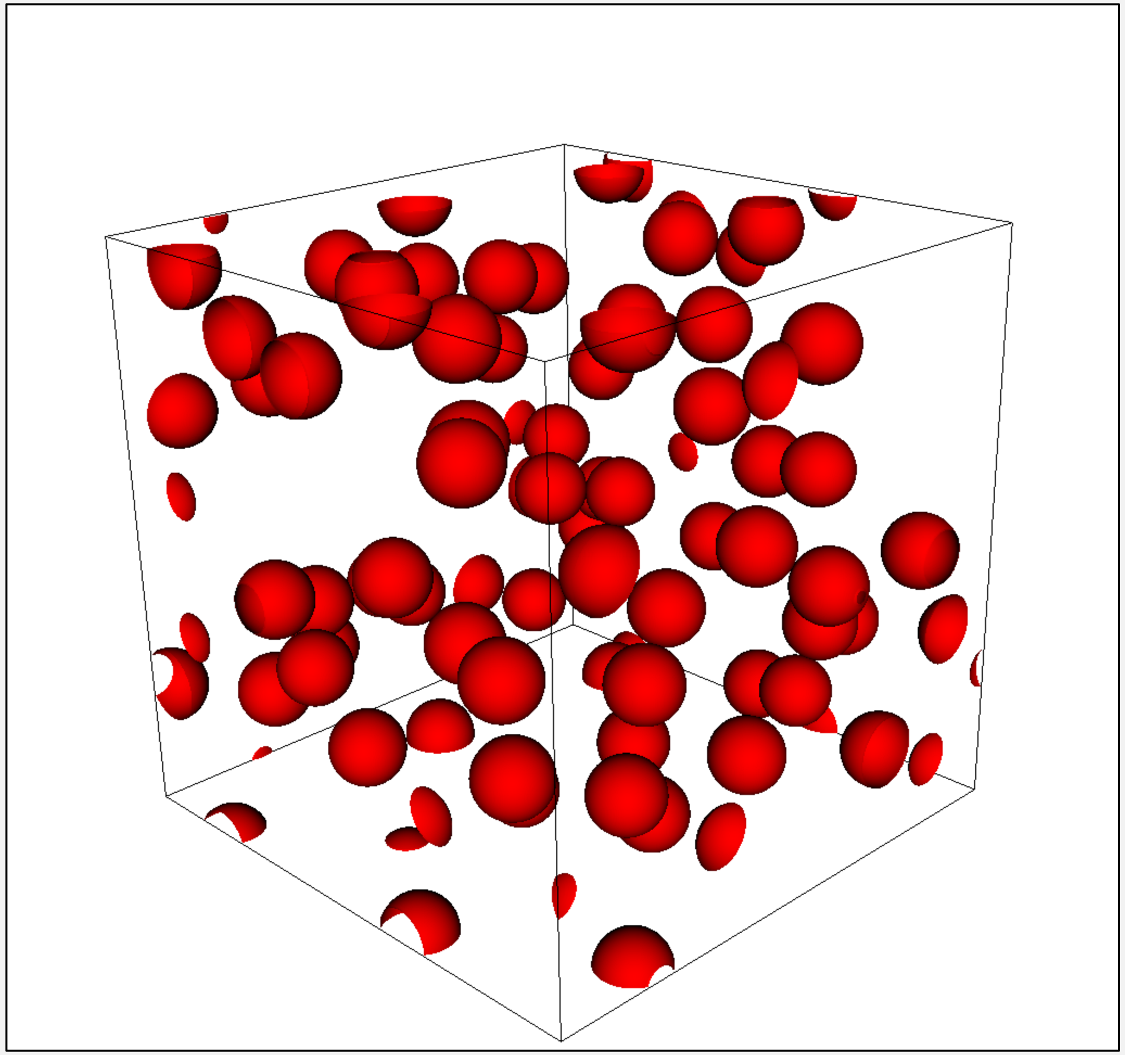
- Can nucleate critical bubbles simultaneously.
- Alternatively nucleate critical bubbles with exponentially increasing nucleation rate:

$$p(t) = p_0 \exp(\beta t),$$

$$\beta \approx \frac{(8\pi)^{1/3}}{R_*} \qquad \text{for} \qquad R_* \gg R_c.$$

- Lorentz factor of bubble wall at collision given by:

$$\gamma_* = \frac{1}{2}\frac{R_*}{R_c}.$$



EVOLUTION

- Neglect expansion of the universe:

$$\Box\phi(\mathbf{x},t) + V'(\phi(\mathbf{x},t)) = 0.$$

- Track auxiliary tensor $u_{ij}(x,t)$ for convenience:

$$\Box u_{ij}(\mathbf{x},t) = 16\pi G \partial_i\phi\partial_j\phi.$$

- Obtain the metric perturbations $h_{ij}(k,t)$ by projecting in k-space:

$$h_{ij}(\mathbf{k},t) = \Lambda_{ij,lm}(\mathbf{k}) u_{lm}(\mathbf{k},t),$$

$$\Lambda_{ij,lm}(\mathbf{k}) = P_{im}(\mathbf{k})P_{jl}(\mathbf{k}) - \frac{1}{2}P_{ij}(\mathbf{k})P_{lm}(\mathbf{k}),$$

$$P_{ij}(\mathbf{k}) = \delta_{ij} - \hat{k}_i\hat{k}_j.$$

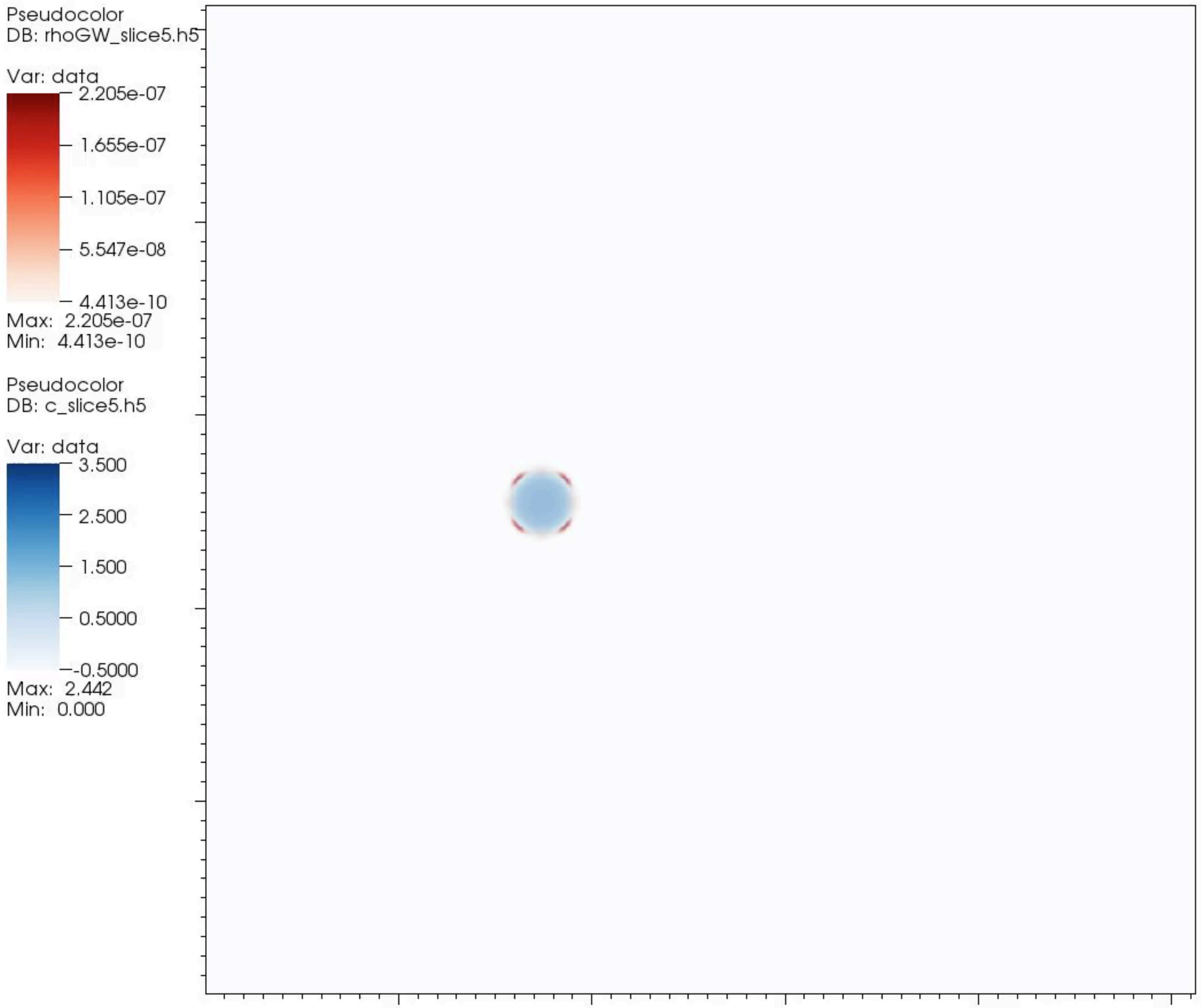
POWER SPECTRUM

- Define spectral density of the time derivative of the metric perturbation $P_{\dot{h}}(\boldsymbol{k}, t)$ as

$$\left\langle \dot{h}_{ij}(\mathbf{k}, t)\dot{h}_{ij}(\mathbf{k}', t) \right\rangle = P_{\dot{h}}(\mathbf{k}, t)(2\pi)^3\delta(\mathbf{k} + \mathbf{k}')\,.$$

- Then the gravitational wave density parameter power spectrum is

$$\frac{d\Omega_{GW}}{d\ln(k)} = \frac{1}{32\pi G\rho_c}\frac{k^3}{2\pi^2}P_{\dot{h}}(\mathbf{k}, t)\,.$$



Pseudocolor
DB: rhoGW_slice5.h5
Var: data
2.205e-07
1.655e-07
1.105e-07
5.547e-08
4.413e-10
Max: 2.205e-07
Min: 4.413e-10
Pseudocolor
DB: c_slice5.h5
Var: data
3.500
2.500
1.500
0.5000
-0.5000
Max: 2.442
Min: 0.000

ENVELOPE APPROXIMATION

- Assumptions:
  - Stress-energy concentrated in infinitesimal thin shell.
  - Neglect any region where bubbles overlap.
- Numeric fit:

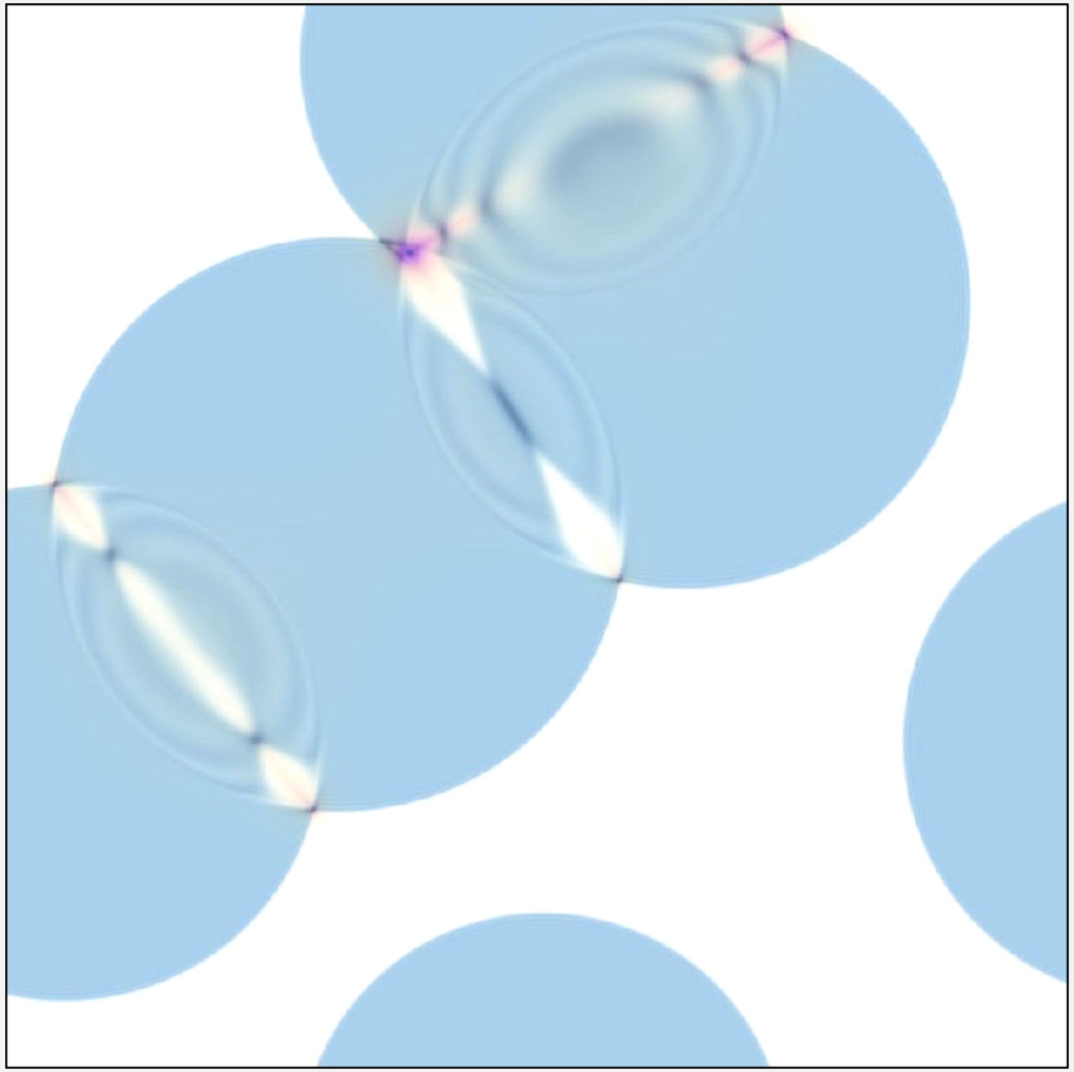
  [Huber and Konstandin, 2008] [Weir, 2016]

  - Power spectrum fit given by

$$\frac{d\Omega_{GW}^{env}}{d\ln(\omega)} = \tilde{\Omega}_{GW}^{env} \frac{(a+b)\tilde{\omega}^b \omega^a}{b\tilde{\omega}^{(a+b)} + a\omega^{(a+b)}},$$

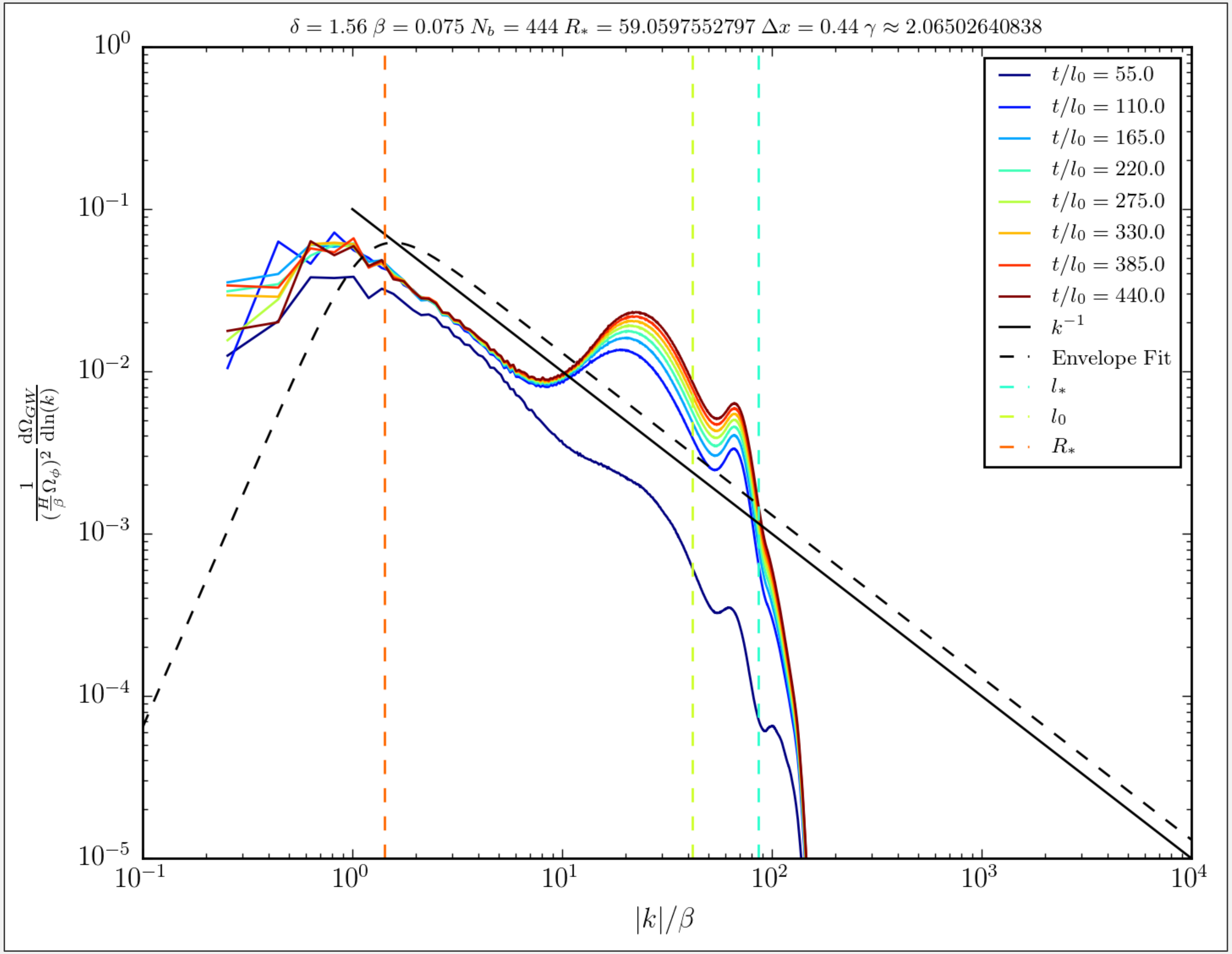
$$\tilde{\Omega}_{GW}^{env} \approx \frac{0.11 v_w^3}{0.42 + v_w^2} \left(\frac{H_*}{\beta}\right)^2 (\kappa_\phi \Omega_\phi)^2,$$

$$\tilde{\omega}/\beta = 2\pi \frac{0.62}{1.8 - 0.1 v_w + v_w^2}.$$



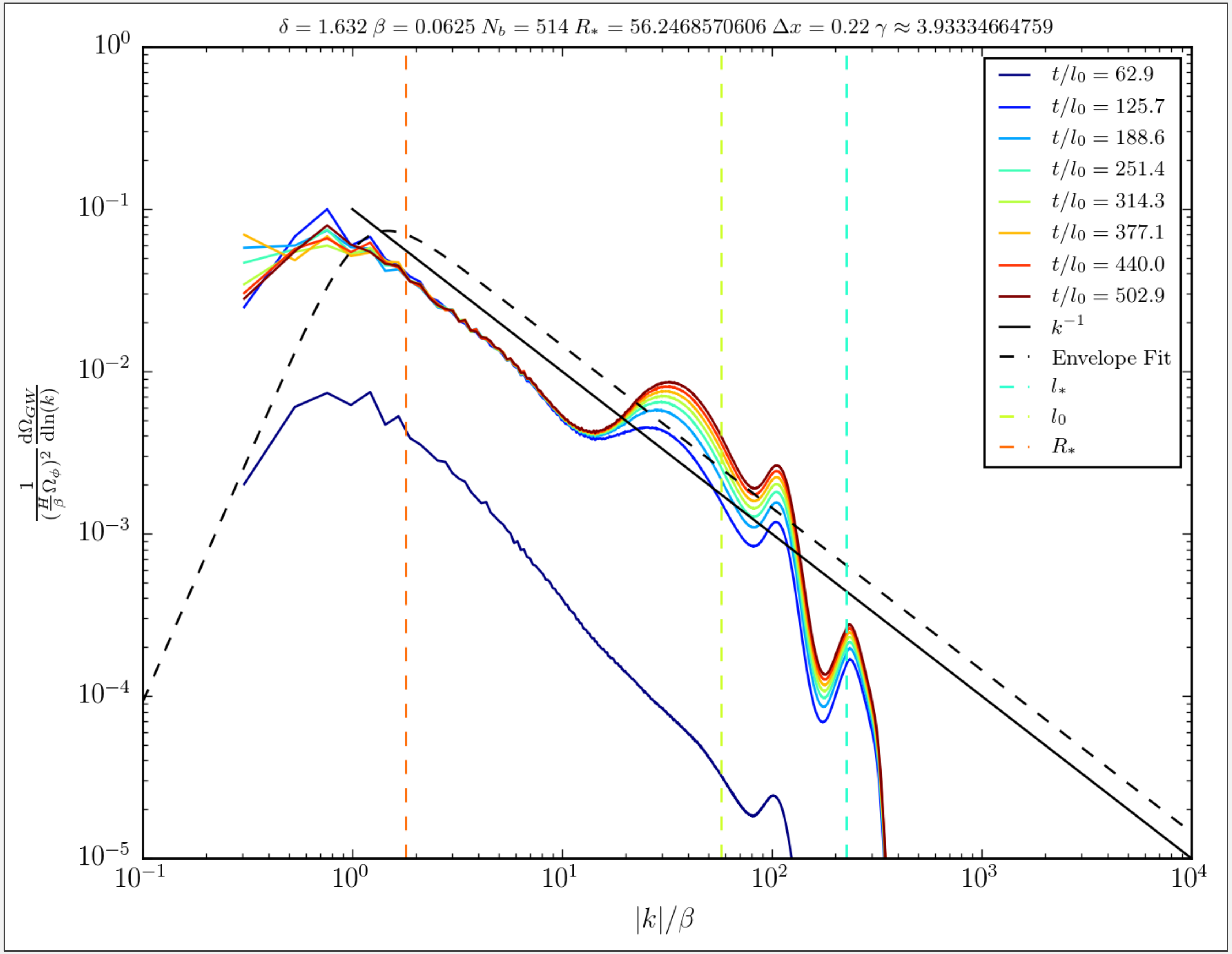
LOW GAMMA SPECTRA

- Lorentz factor $\gamma_* \sim 2$.
- Number of bubbles $N_b = 444$
- $\beta = 0.075$
- Peak amplitude matches envelope.
- Power law matches envelope.
- Extra bump in the UV.



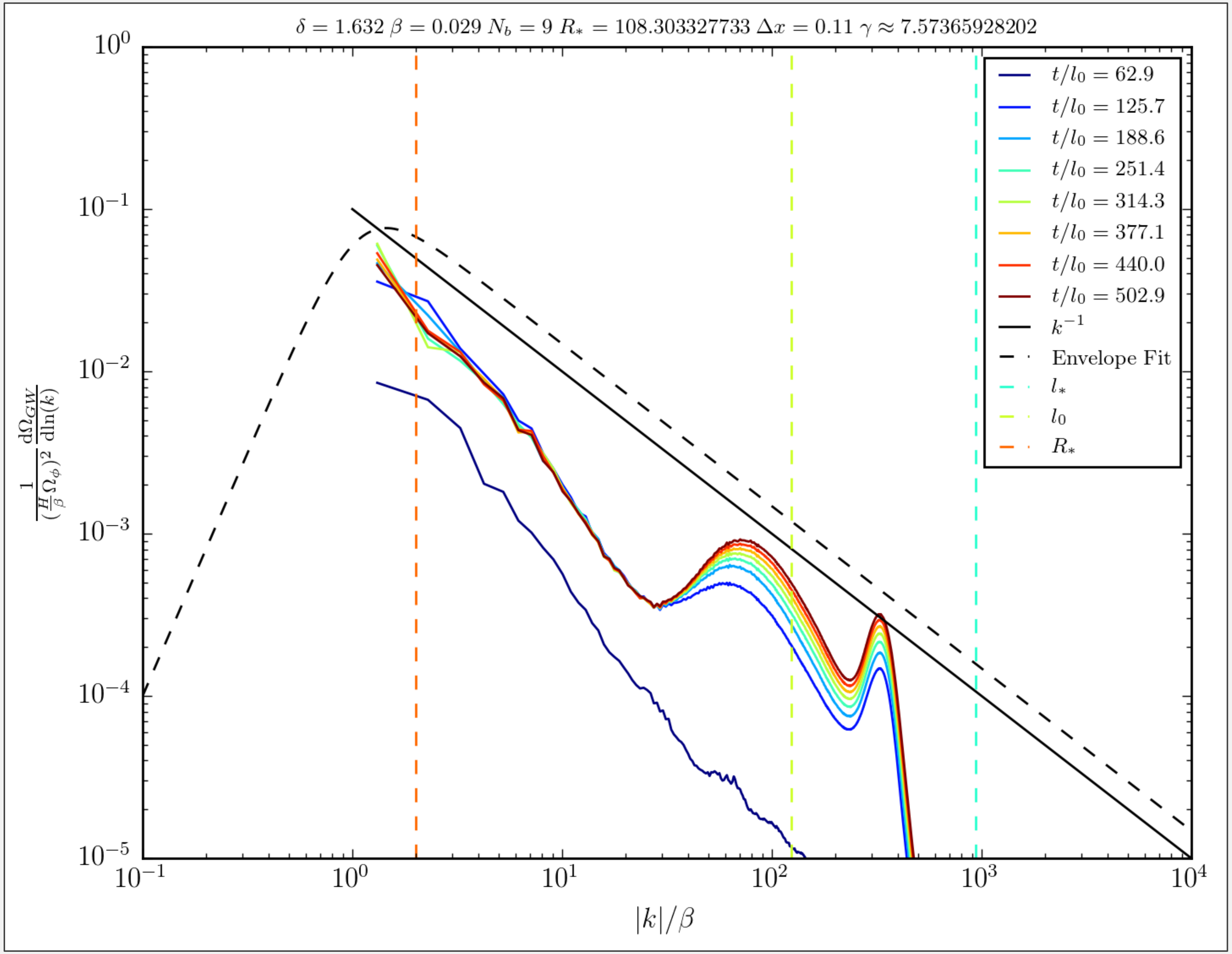
MEDIUM GAMMA SPECTRA

- Lorentz factor $\gamma_* \sim 4$.
- Number of bubbles $N_b = 514$
- $\beta = 0.0625$
- Peak amplitude matches envelope.
- Power law slightly steeper than envelope.
- Extra bump in the UV.



HIGH GAMMA SPECTRA

- Lorentz factor $\gamma_*\sim 8$.
- Number of bubbles $N_b = 9$
- $\beta = 0.029$
- Can't resolve the IR peak.
- Power law steeper than envelope.
- Extra bump in the UV.

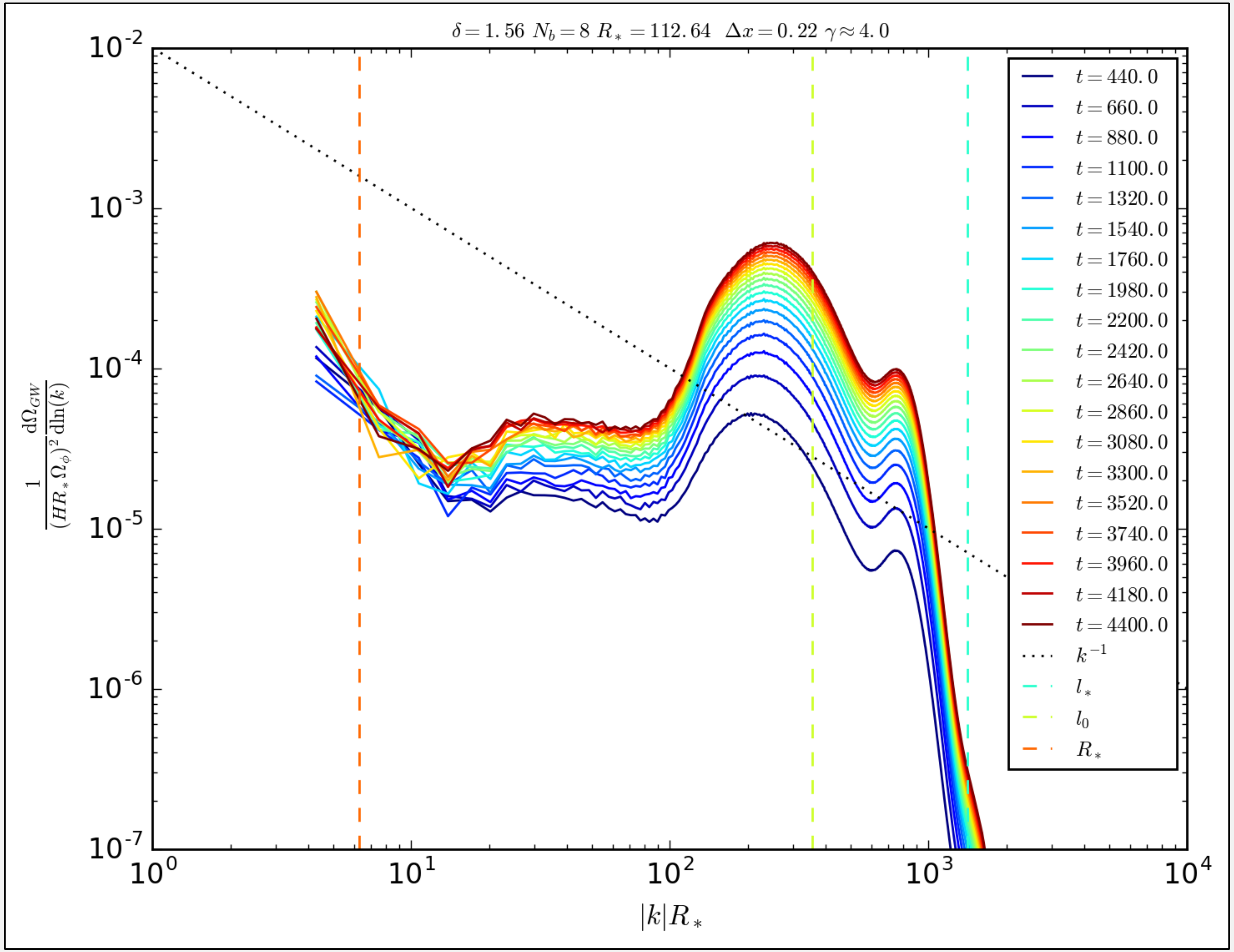


CONCLUSIONS

- We recover the envelope power law for small γ_*, though shifted to the IR.
- At higher γ_* the peak amplitude seems similar but the power law towards the UV is steeper.
- Haven't succeeded in finding asymptotic behavior for $\gamma_* \to \infty$.
- We see a second peak at a scale associated with the initial wall width.
  - We predict the effect of this peak to be negligible for most models.

BUMP FEATURE

- Turn on metric evolution after bubble collisions.
- IR uptick seems to be left over from bubble collisions.
- Plateau leading into bump.
- Still growing at very late times.



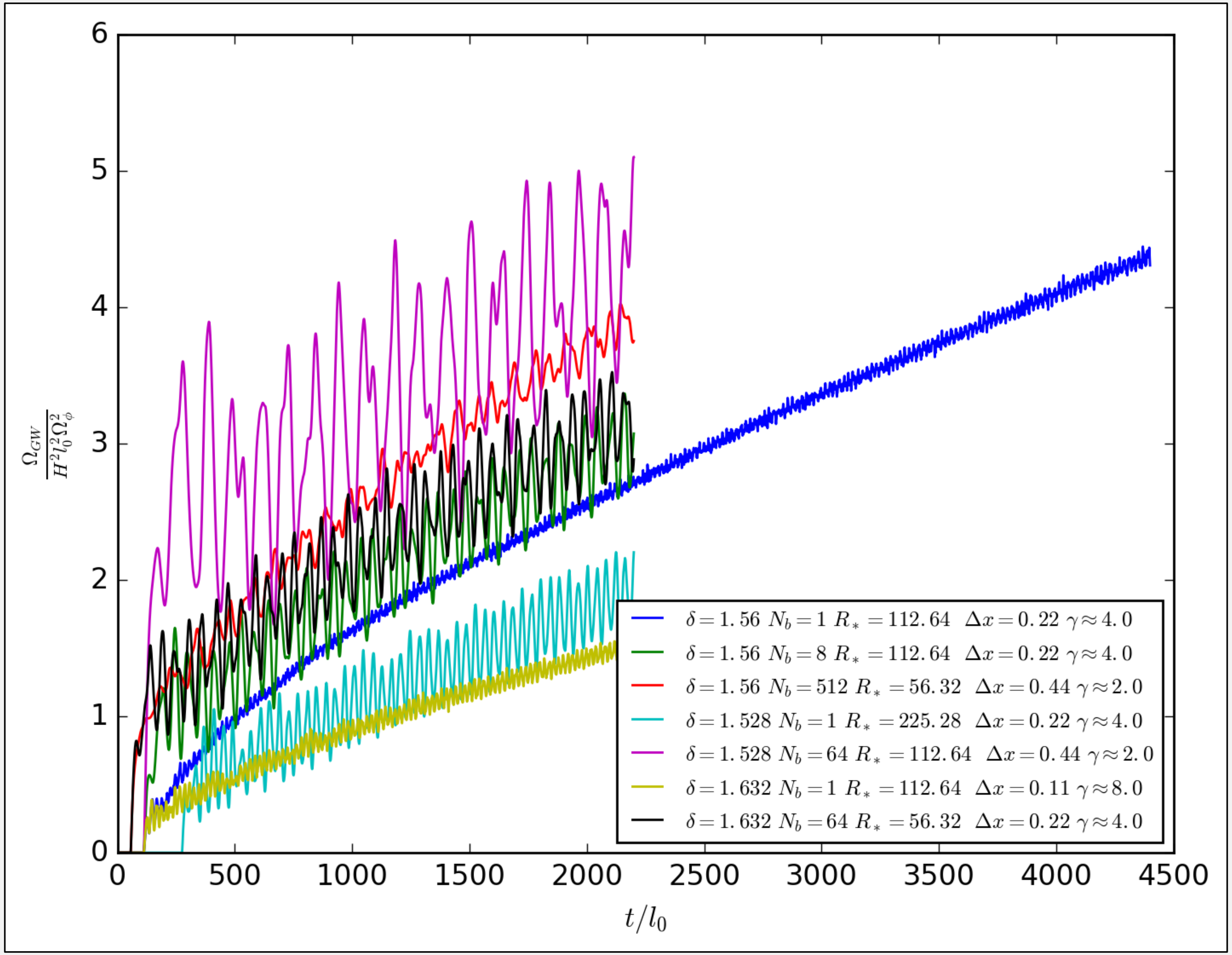
BUMP FEATURE

- Measure total gravitational wave energy density for several simulations.
- Oscillation due to ringing in IR.
- Linear growth:

$$\frac{d\Omega_{GW}^{bump}}{dt} \sim 10^{-3}\, l_0 H^2 \Omega_\phi^2 \,.$$

- Grows at most for H^{-1}.

$$\frac{\Omega_{GW}^{bump}}{\Omega_{GW}^{collision}} \sim 10^{-2} \frac{l_0}{R_*} \frac{1}{H R_*} \,.$$



SIMULATION DETAILS

- 3+1 dimensional classical lattice simulation.
- Built using LATfield2, an open source massively parallel lattice code. *[Daverio, Hindmarsh and Bevis, 2015]*
- Periodic boundary conditions.
- The leapfrog algorithm evolves ϕ and u_{ij}.
  - Calculate Laplacian with 7 point stencil.
  - Take FFT and project u_{ij} to find power spectrum. *[Figueroa, Garcia-Bellido and Rajantie, 2011]*
- Resolve the bubble wall:

$$dx \ll l_* = l_0/\gamma_*$$