Bubble wall runaways?

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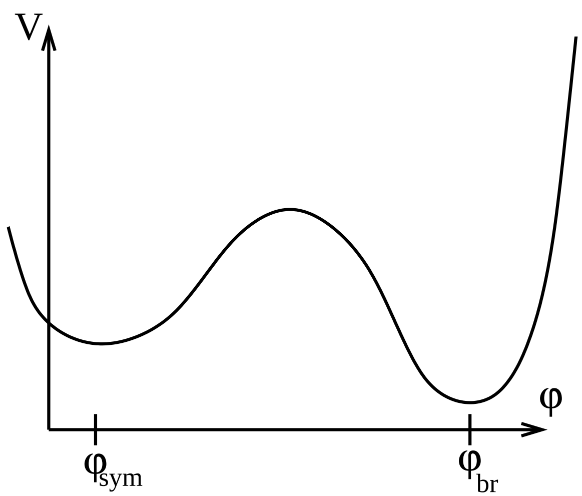
- Scalar in Vacuum
- Thermal, equilibrium
- Finite, and high, velocity
- Radiation corrections
- Implications and Conclusions

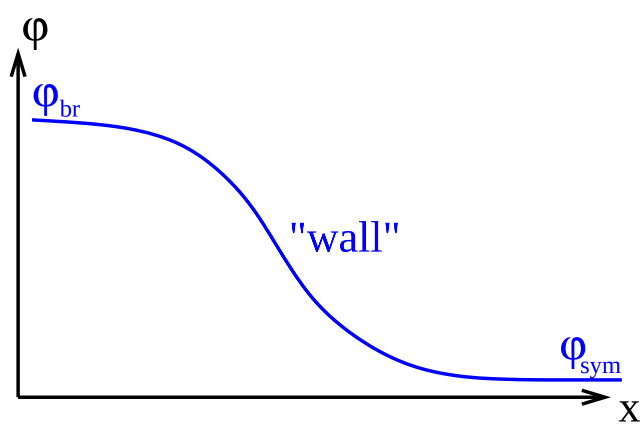
Phase interface at 0-temperature

Real scalar field with $\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + V(\varphi)$

Consider potential $V(\varphi)$ with two local minima

leading to bubble-wall interfaces between two possible phases





Wall accelerates. Field equation of motion:

$$\ddot{\varphi} = \nabla^2 \varphi - \frac{\partial V}{\partial \varphi}$$

Momentum density and its time derivative:

$$T_{0i} = \dot{\varphi}\partial_i \varphi \,, \qquad \dot{T}_{0i} = \ddot{\varphi}\partial_i\varphi + \dot{\varphi}\partial_i\dot{\varphi}$$

Total force per area (net pressure) for wall at rest ($\dot{\varphi} = 0$)

$$\begin{aligned} \int dx \dot{T}_{0i}(x) &= \int \left(\dot{\varphi}\partial_i\dot{\varphi} + \ddot{\varphi}\partial_i\varphi\right) dx \\ &= \int \partial_i\varphi\partial_i^2\varphi dx - \int \frac{\partial V}{\partial\varphi}\frac{\partial\varphi}{\partial x} dx \\ &= V_{\mathrm{sym}} - V_{\mathrm{br}} \end{aligned}$$

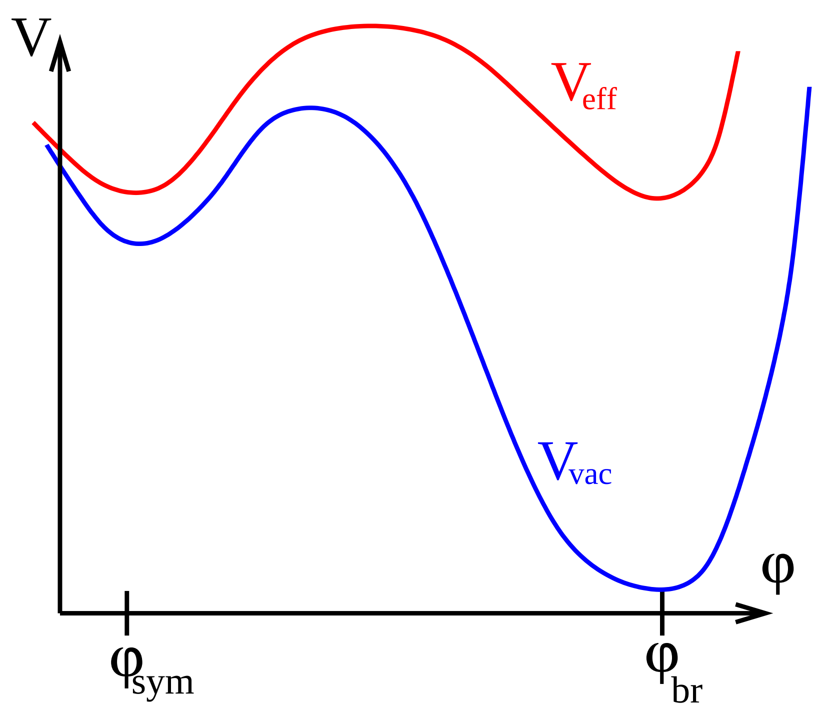
Pressure on wall determined by potential difference.

Now add plasma.

At $T = T_c$,

$V_{\text{eff}} = V_{\text{vac}} + V_{\text{therm}}$

balance to give equal minima



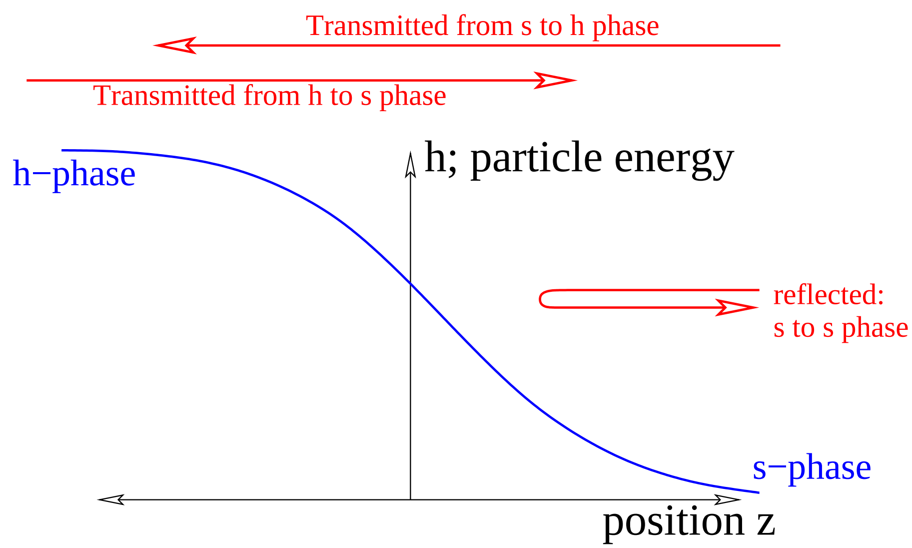
Vacuum still gives strong forward pressure on wall.

Must somehow be balanced by thermal effect. How?

Assume a thick wall, equilibrium

If $L_{\rm wall} \gg 1/T$, particle (WKB) approx for excitations.

E conserved (in bubble frame = plasma frame). But particle *mass* is φ dependent:



$$E = \sqrt{k^2 + m^2} \quad \text{or} \quad k = \sqrt{E^2 - m^2(\varphi)}$$

Momentum transfer of particles

Momentum density delivered by particles to wall:

$$\dot{T}_{0i} = \int \frac{d^3k}{(2\pi)^3} f(E(k)) \frac{dk_i}{dt}$$

$$\frac{dk}{dt} = \frac{dk}{dx}\frac{dx}{dt} = \frac{k_x}{E}\frac{dk}{dx}$$

$$\frac{dk}{dx} = \frac{d\sqrt{E^2 - m^2}}{dx} = \frac{-1}{k}\frac{dm^2}{d\varphi}\frac{d\varphi}{dx}$$

Put it together:

$$\dot{T}_{0i} = \int \frac{d^3k}{E(2\pi)^3} f(E) \frac{dm^2}{d\varphi}\, \frac{d\varphi}{dx}\,.$$

The thermal potential

Integrate it up for force/area on wall:

$$\int dx \dot{T}_{0i} = \int dx \frac{d\varphi}{dx}\frac{dm^2}{d\varphi}\int \frac{d^3k}{E(2\pi)^3} f(E)$$
$$= \int_{\varphi_{\rm sym}}^{\varphi_{\rm br}} d\varphi \left(\int \frac{d^3k}{E(2\pi)^3} f(E) \frac{dm^2}{d\varphi} \right)$$

Quantity in parenthesis is $dV_{\rm therm}/d\varphi$ at 1-loop level.

Force/area of particles on wall is precisely the difference in thermal effective potential.

Turok, Khlebnikov, Arnold early 1990s

Nonequilibrium: moving wall

By same arguments, force on moving wall (in wall-frame) is:

$$\int \dot{T}_{0i}\, dx = \int dx\, \frac{d\varphi}{dx}\, \frac{dm^2}{d\varphi} \int \frac{d^3k}{(2\pi)^3 E} f(\vec{k}, x)$$

but now distribution function $f(\vec{k}, x)$ is .. what?

Not determined by E – in equilibrium

$$f_{\rm eq}(\vec{k}) = f_{\rm b,f}(\gamma E - \gamma v k_x)$$

with v the wall velocity, $\gamma = 1/\sqrt{1-v^2}$.
Combination $\gamma E - \gamma v k_x$ not conserved.

Particles leave equilibrium in crossing wall.

General case: a mess!

Particle leave equilibrium as they cross wall.

Need to consider scatterings (bring back towards equil)

Particles entering phases on either side are *not in equilibrium*

Nonequilibrium on both sides of wall

$\Rightarrow$ particles out of equilibrium when they reach wall

Need to compute nonequilibrium condition everywhere

Hard. Several relevant papers Moore Prokopec Schmidt John etc

Case of very fast wall $\gamma \gg 1$

Typical energy $E \sim \gamma T \gg m$, and exponentially few particles with $k_x > 0$:

- WKB approximation is perfect
- Almost all particles come from one direction
- Almost all particles cross wall (no reflection)
- Plasma in front doesn't know wall is coming $\Rightarrow$ Equilibrium

Makes the problem solvable again! Turok, private communication, 1994

Solving case of large $\gamma \gg T$

Start with previous expression:

$$\int \dot{T}_{0i}\, dx = \int dx \frac{d\varphi}{dx} \frac{dm^2}{d\varphi} \int \frac{d^3k}{(2\pi)^3 E} f(k)$$

Use that $E, k \gg m$: k barely changes, use symmetric-phase value:

$$\int \dot{T}_{0i}\, dx = (m^2_{\rm br} - m^2_{\rm sym}) \int \frac{d^3k}{(2\pi)^3 E} f(k_{\rm sym})$$

Use that integration measure invariant, $f(k)$ covariant, to evaluate the latter in the plasma rest frame!

$$\int \dot{T}_{0i}\, dx = (m^2_{\rm br} - m^2_{\rm sym}) \left. \frac{dV_{\rm therm}}{dm^2} \right|_{m^2 = m^2(\varphi_{\rm sym})}$$

Result

Friction has a finite large-γ limit:

- Density of particles grows as $n \sim \gamma T^3$ but
- Force per particle scales as $\Delta E \sim m^2/E \sim \gamma^{-1}$

Same as replacing $V_{\rm therm}(\varphi)$ with mean-field approx.

- Fluctuation-induced transition: $-E\varphi^3$ term removed. **Wall does not run away.**
- Singlet extensions: use mean-field potential. **Wall generically (not always) does run away.**

What that misses

At next-loop level, particles have virtual “clouds.”

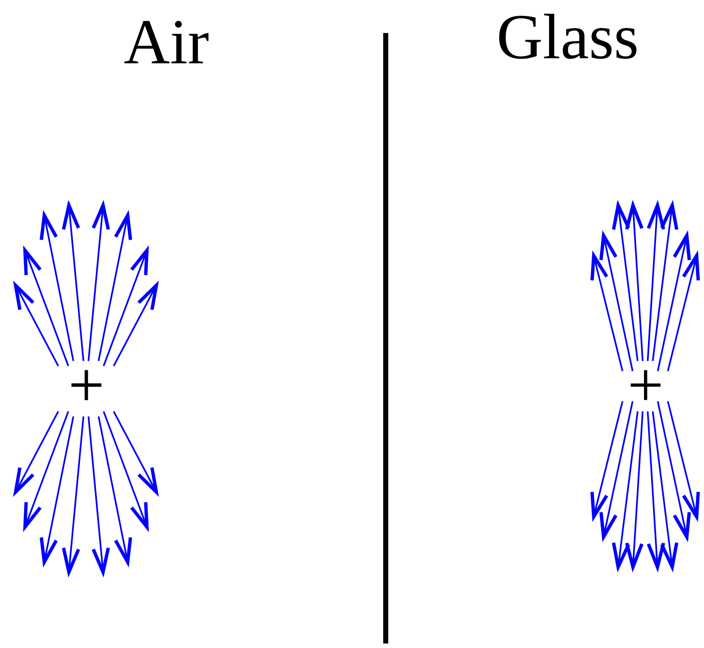
Consider e-charge,

$v < c/n$,

in air or glass.

E-fields in tighter fan in

glass than in air.



E&M fields “mismatched” at boundary.

Difference between fields comes off as **transition radiation**

Notes about transition radiation

- Depends on q, v of charge, n_1, n_2 indices in media.
- Independent of size or energy of charge (p vs e)
- Distinct from Cherenkov radiation.
  Occurs even if $v < c/n$
- Strictly associated with boundary.
  Does not occur in bulk medium.

Transition radiation and bubble walls

- Each incoming particle has W-field "cloud"
- W-boson propagation is different in the two phases
- Radiation of W quantum with probability $\propto \alpha_w$
- W-spectrum depends on wall but not on E of incoming particle

Formalism for computing electroweak transition radiation developed in JCAP 1705 025 (arXiv:1703.08215)

Force from transition radiation

- Force per W depends on W spectrum
- W spectrum depends on wall details, not particle E
- W pressure $\propto$ number of incitent particles

Leading-order and NLO pressures:

$$P_{\rm LO} \;\sim\; \Delta m^2 \int \frac{d^3k}{(2\pi)^3 E} f(k) \sim m^2 T^2$$

$$P_{\rm NLO} \;\sim\; \alpha m \int \frac{d^3k}{(2\pi)^3} f(k) \sim \alpha\gamma m T^3$$

Result: NLO effects

Unlike LO, the NLO transition-radiation contribution grows with increasing wall γ-factor.

- $\gamma \sim 1/\alpha_W$ large enough to balance forward force
- Generically, walls move with $\gamma \sim 1/\alpha_w$
- Practically speed of light, but
- Wall energy infinitesimal vs energies in bulk-phases

Good approximation: wall moves at $v \simeq c$, generating enough entropy that wall does not accumulate energy.