Different regimes of dilepton production[1]

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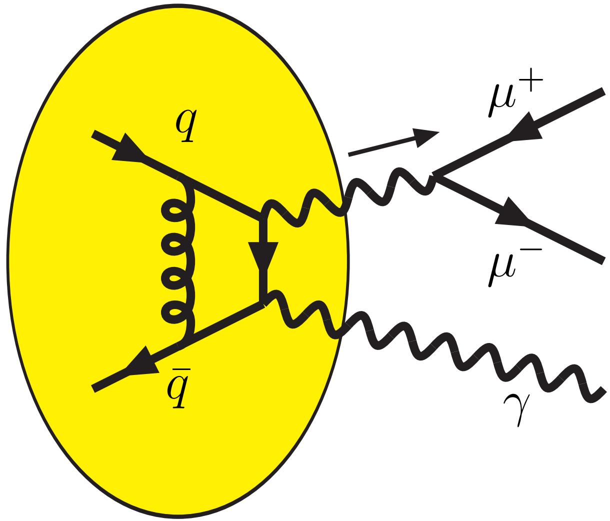
[1] Based on: *Thermal 2-loop master spectral function at finite momentum,* JHEP 05 (2013) 083 [1304.0202]; *NLO thermal dilepton rate at non-zero momentum,* JHEP 11 (2013) 120 [1310.0164]; *Interpolation of hard and soft dilepton rates,* 1407.7955 (with Ioan Ghisoiu).

Background

Observable



Consider thermally produced $\mu^+\mu^-$ or e^+e^- pairs at non-zero momentum ($k \equiv |\mathbf{k}| \sim 1$ GeV), with an invariant mass

$$50 \text{ MeV} < M \equiv \sqrt{\mathcal{K}^2} < 3 \text{ GeV} .$$

Perhaps backgrounds are smaller than for on-shell photons?

Basic formulae[2]

Dilepton rate:

$$\frac{\mathrm{d}N_{\mu^-\mu^+}}{\mathrm{d}^4\mathcal{X}\mathrm{d}^4\mathcal{K}} \overset{4m_\mu^2 \ll \mathcal{K}^2 \ll m_Z^2}{=} -\frac{n_{\mathrm{B}}(k_0)}{3\pi^3\mathcal{K}^2}\,\alpha_e^2 \sum_{i=1}^{N_\mathrm{f}} Q_i^2 \;\; \mathrm{Im}\,\Pi_\mathrm{R} \; .$$

Spectral function:

$$\mathrm{Im}\,\Pi_\mathrm{R} \equiv \int_{\mathcal{X}} e^{i\mathcal{K}\cdot\mathcal{X}} \left\langle \frac{1}{2}\left[\hat{\mathcal{J}}^\mu(\mathcal{X}), \hat{\mathcal{J}}_\mu(0)\right]\right\rangle_\mathsf{c} \; .$$

Electromagnetic current:

$$\hat{\mathcal{J}}^\mu \equiv \hat{\bar{\psi}}\gamma^\mu\hat{\psi} \; .$$

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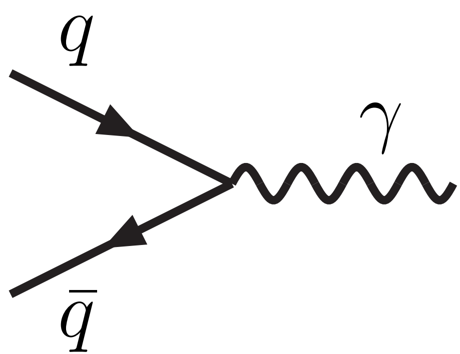
[2] L.D. McLerran and T. Toimela, *Photon and Dilepton Emission from the Quark-Gluon Plasma: Some General Considerations*, Phys. Rev. D 31 (1985) 545; H.A. Weldon, *Reformulation of Finite Temperature Dilepton Production*, Phys. Rev. D 42 (1990) 2384; C. Gale and J.I. Kapusta, *Vector dominance model at finite temperature*, Nucl. Phys. B 357 (1991) 65.

Leading-order result (Drell-Yan):

$$k_\pm \equiv \frac{k_0 \pm k}{2} > 0 \quad \Rightarrow$$

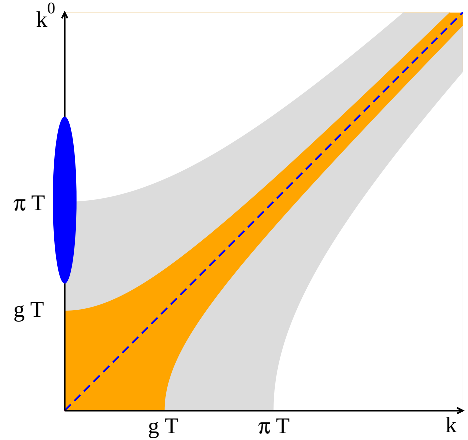
$$-\,\mathrm{Im}\,\Pi_\mathrm{R} = \frac{N_\mathrm{c} T M^2}{2\pi k} \ln\left\{ \frac{\cosh(\frac{k_+}{2T})}{\cosh(\frac{k_-}{2T})} \right\} .$$



Corrections to Drell-Yan: some history (1988-2013)

NLO-result at vanishing momentum ($k = 0$):[3]

etc



$\Rightarrow$ only a small correction.

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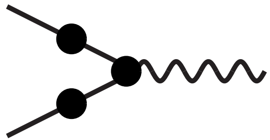
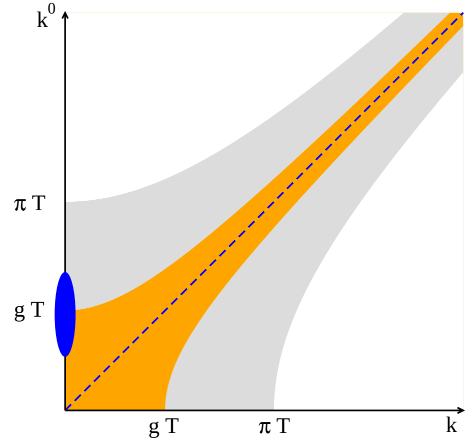
[3] R. Baier, B. Pire and D. Schiff, *Dilepton production at finite temperature: Perturbative treatment at order* α_s, Phys. Rev. D 38 (1988) 2814; Y. Gabellini, T. Grandou and D. Poizat, *Electron-positron annihilation in thermal QCD*, Annals Phys. 202 (1990) 436; T. Altherr and P. Aurenche, *Finite temperature QCD corrections to lepton-pair formation in a quark-gluon plasma*, Z. Phys. C 45 (1989) 99.

HTL resummation in a soft regime ($k = 0$):[4]

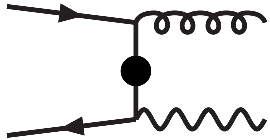




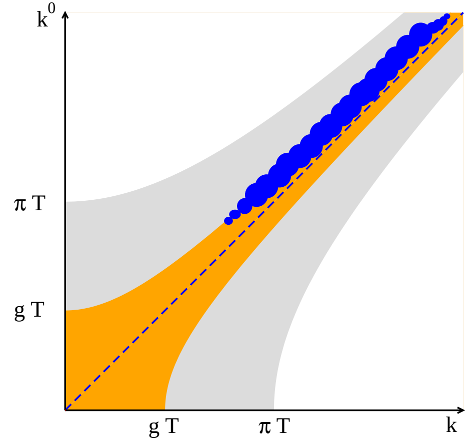
⇒ a large enhancement.

[4] E. Braaten, R.D. Pisarski and T.-C. Yuan, *Production of soft dileptons in the quark–gluon plasma*, Phys. Rev. Lett. 64 (1990) 2242.

HTL resummation for hard momenta ($k \sim \pi T$):[5]



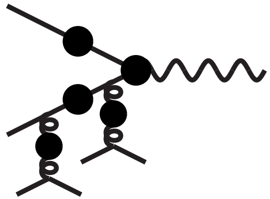
If $M \to 0$ there is an IR singularity from soft t-channel exchange, which is regulated by Landau damping:



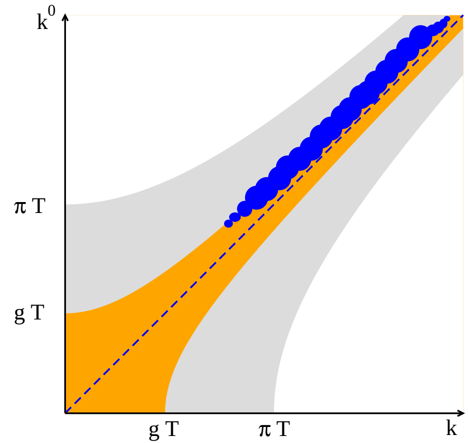
$$-\,\mathrm{Im}\,\Pi_{\mathrm{R}} = \ldots + \frac{\alpha_s N_{\mathrm{c}} C_{\mathrm{F}} T^2}{4} \ln\left(\frac{T^2}{M^2 \to \alpha_s T^2}\right)[1 - 2n_{\mathsf{F}}(k)]$$

[5] J.I. Kapusta, P. Lichard and D. Seibert, *High-energy photons from quark-gluon plasma versus hot hadronic gas*, Phys. Rev. D 44 (1991) 2774 [Erratum-ibid. D 47 (1993) 4171]; R. Baier, H. Nakkagawa, A. Niégawa and K. Redlich, *Production rate of hard thermal photons and screening of quark mass singularity*, Z. Phys. C 53 (1992) 433.

LPM resummation for hard momenta ($k \sim \pi T$):[6]



Removing the divergence is not enough: there are terms of similar magnitude from multiple scatterings with collinear enhancement.



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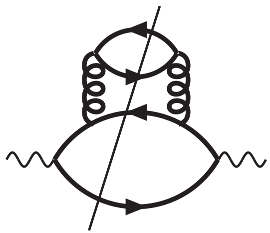
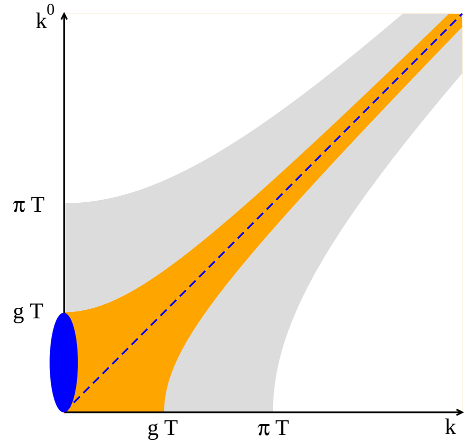
[6] P. Aurenche, F. Gelis, G.D. Moore and H. Zaraket, *Landau-Pomeranchuk-Migdal resummation for dilepton production*, JHEP 12 (2002) 006 [hep-ph/0211036]; M.E. Carrington, A. Gynther and P. Aurenche, *Energetic di-leptons from the Quark Gluon Plasma*, Phys. Rev. D 77 (2008) 045035 [0711.3943].

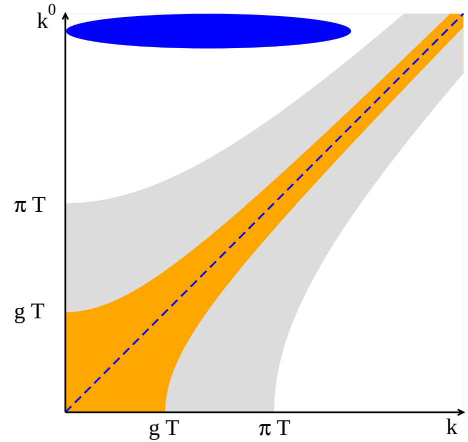
HTL is not enough even at zero momentum.[7]





[7] G.D. Moore and J.-M. Robert, *Dileptons, spectral weights, and conductivity in the quark-gluon plasma*, hep-ph/0607172.

Analytic results exist for $M \gg \pi T$ from OPE.[8]



$$-\,\mathrm{Im}\,\Pi_{\mathrm{R}} = \frac{N_{\mathrm{c}} M^2}{4\pi}\left(1 + \frac{3\alpha_s C_{\mathrm{F}}}{4\pi}\right) + \frac{4\alpha_s N_{\mathrm{c}} C_{\mathrm{F}}}{9}\left(1 + \frac{4k^2}{3M^2}\right)\frac{\pi^2 T^4}{M^2} + \mathcal{O}\Big(\frac{\alpha_s T^6}{M^4}\Big)\ .$$

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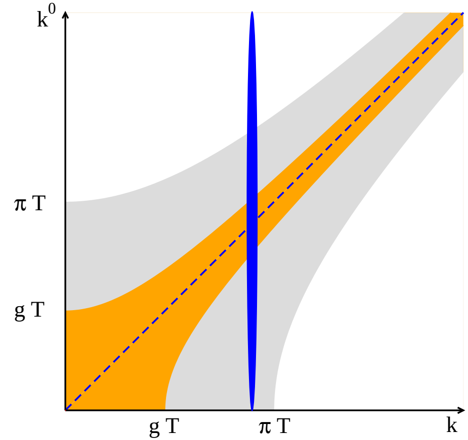
[8] S. Caron-Huot, *Asymptotics of thermal spectral functions*, Phys. Rev. D 79 (2009) 125009 [0903.3958].

Lattice results can also help but only partly.[9]



$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}k_0}{\pi}\, \mathrm{Im}\,\Pi_{\mathrm{R}}(k_0, \mathbf{k}) \frac{\cosh(\frac{1}{2T} - \tau)k_0}{\sinh(\frac{k_0}{2T})} \, .$$

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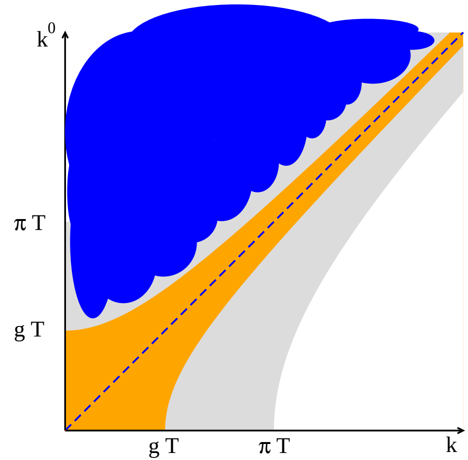
9 G. Cuniberti, E. De Micheli and G.A. Viano, *Reconstructing the thermal Green functions at real times from those at imaginary times*, Commun. Math. Phys. 216 (2001) 59 [cond-mat/0109175].

For "typical" momenta no resummation is needed at NLO.[10]

Compute the NLO imaginary-time correlator and take the cut:



$$\Pi_{\rm E}(K) \equiv \int_0^{1/T} {\rm d}\tau \int_{\mathbf{x}} e^{iK\cdot X} \left\langle (\bar\psi\gamma^\mu\psi)(\tau,\mathbf{x})\,(\bar\psi\gamma_\mu\psi)(0,\mathbf{0})\right\rangle_T$$

$$\Pi_{\rm R}(\mathcal{K}) = \Pi_{\rm E}\big|_{k_n\to -i[k_0+i0^+]} \;.$$

[10] ML, *Thermal 2-loop master spectral function at finite momentum*, JHEP 05 (2013) 083 [1304.0202]; *NLO thermal dilepton rate at non-zero momentum*, JHEP 11 (2013) 120 [1310.0164].

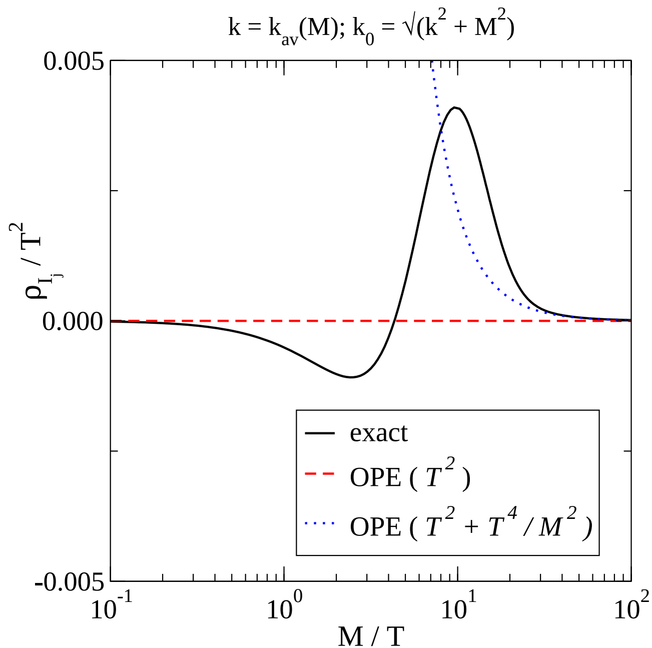
Technical challenge

After analytic continuation $K = (k_n, \mathbf{k}) \to \mathcal{K} = (k_0, \mathbf{k})$ and taking the cut this yields structures which can be identified as

etc

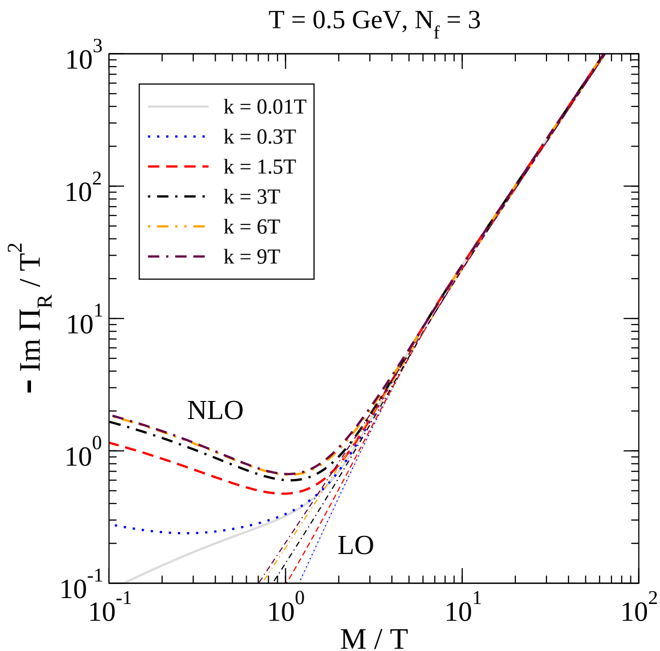
Virtual and real processes contain soft, collinear and thermal divergences, which cancel in the sum if consistently regulated.

A 2d integral remains to be carried out numerically.



(This is for $\lim_{\lambda\to 0} \mathrm{Im} \, \not{\Sigma}\!\!\int_{QR} \frac{K^4}{Q^2R^2[(Q-R)^2+\lambda^2](Q-K)^2(R-K)^2}$.)

But the loop expansion breaks down close to light cone.



News (i): convergence of OPE
$(M \gg \pi T)$

[Precisely the same comments apply to the non-relativistic expansion for treating right-handed neutrino dynamics.]

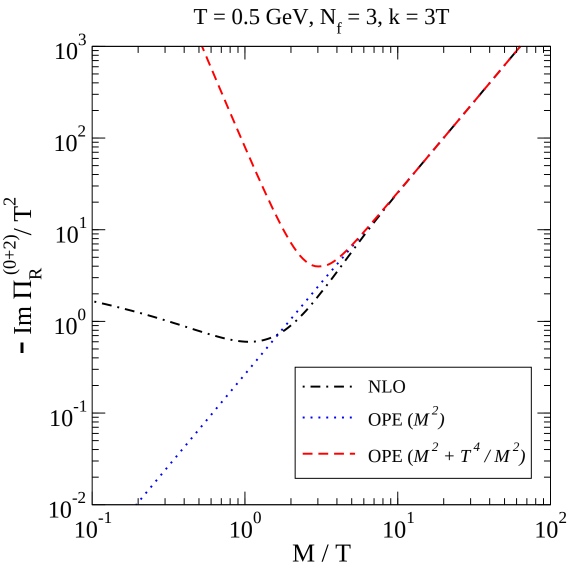
Generic structure of a non-relativistic expansion:[11]

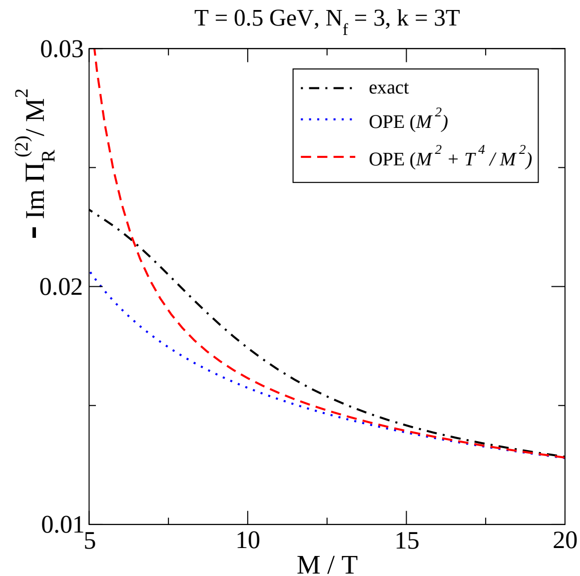
$$
\begin{aligned}
\phi(g^2, T/M) &= \phi^{(0)}(T/M) + g^2\,\phi^{(2)}(T/M) + \ldots \\
&= \phi^{(0)}(0) + O\left(e^{-M/T}\right) \\
&+ g^2\left[\phi^{(2)}(0) + O\left(\frac{T^4}{M^4}\right)\right] \\
&+ \ldots
\end{aligned}
$$

Is $g^2 \times O(T^4/M^4)$ small because of g^2 or $O(T^4/M^4)$?

[11] S. Caron-Huot, *Asymptotics of thermal spectral functions*, Phys. Rev. D 79 (2009) 125009 [0903.3958].

Numerical results: $O(T^4/M^4)$ is *not* small.[12]





The same applies to RH neutrinos; because of a weaker coupling, "apparent" convergence at $M/T \gtrsim 4$, real at $M/T \gtrsim 15$.[13]

12 ML, *NLO thermal dilepton rate...*, JHEP 11(2013)120 [1310.0164].

13 ML, *Thermal right-handed neutrino production...*, JHEP 08 (2013) 138 [1307.4909].

News (ii): interpolation between NLO and LPM

($M \ll \pi T$)

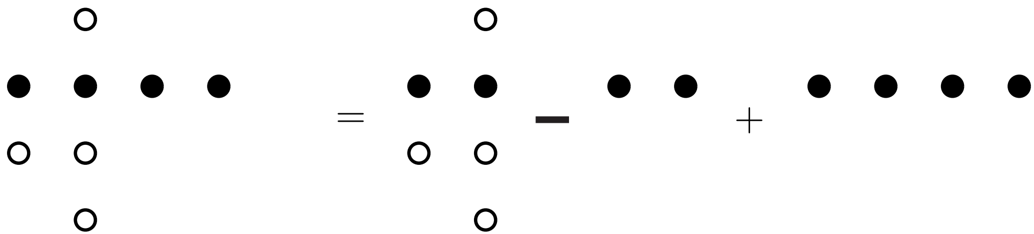
Equation for LPM resummation (longitudinal channel)

$$\left(|\mathbf{k}| - k_0 + \frac{m_\infty^2 - \nabla_\perp^2}{2\omega_r} + iV^+ \right) g(k_0, |\mathbf{k}|; \mathbf{y}) = \delta^{(2)}(\mathbf{y}) \ ,$$

$$\frac{1}{\omega_r} \equiv \frac{1}{\omega_1} + \frac{1}{\omega_2} \ ,$$

$$\mathrm{Im}\,\Pi_{\mathrm{R,L}} = N_\mathrm{c} \int_{-\infty}^{\infty} \mathrm{d}\omega_1 \int_{-\infty}^{\infty} \mathrm{d}\omega_2 \, \delta(k_0 - \omega_1 - \omega_2)$$
$$\times \ [1 - n_{\mathsf{F}}(\omega_1) - n_{\mathsf{F}}(\omega_2)] \ \frac{M^2}{k_0^2} \lim_{\mathbf{y}\to\mathbf{0}} \frac{\mathrm{Im}[g(k_0, |\mathbf{k}|; \mathbf{y})]}{\pi} \ .$$

To avoid double counting, terms resummed into LPM need to be subtracted from the NLO.[14]



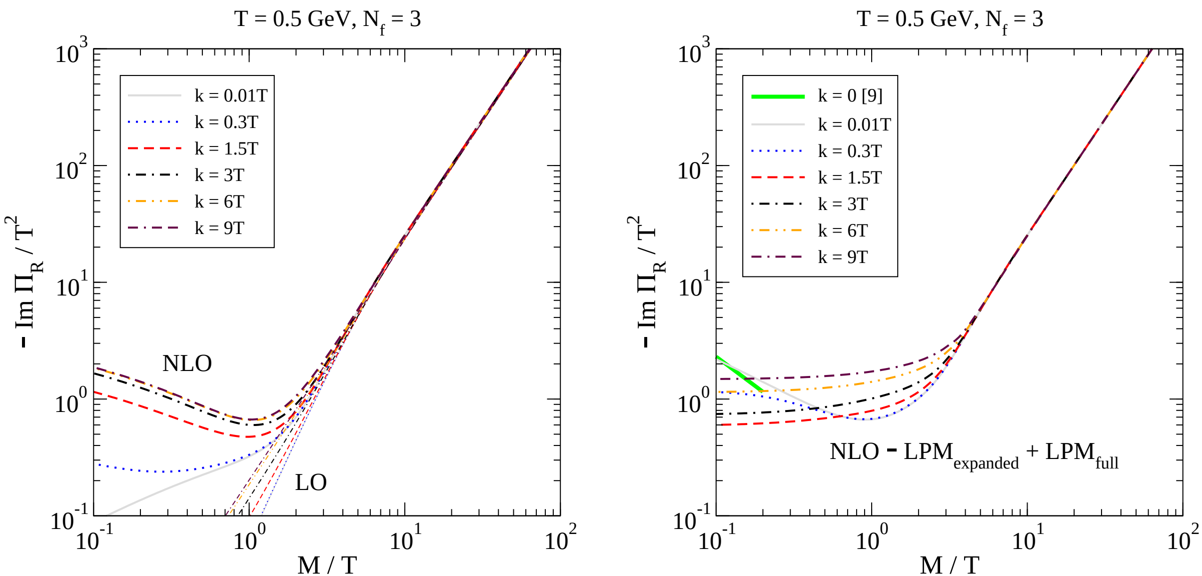
$$\mathrm{Im}\,\Pi_{\mathrm{R}}\big|_{\text{full}} \equiv \mathrm{Im}\,\Pi_{\mathrm{R}}\big|_{\mathrm{NLO}} - \mathrm{Im}\,\Pi_{\mathrm{R}}\big|_{\mathrm{LPM}}^{\text{expanded}} + \mathrm{Im}\,\Pi_{\mathrm{R}}\big|_{\mathrm{LPM}}^{\text{full}} .$$

View from "hard side": $-\ \mathrm{Im}\,\Pi_{\mathrm{R}}\big|_{\mathrm{LPM}}^{\text{expanded}} + \mathrm{Im}\,\Pi_{\mathrm{R}}\big|_{\mathrm{LPM}}^{\text{full}}$ is an addition to the NLO result, consisting of terms of $\mathcal{O}(\alpha_s^2)$.

View from "soft side": $\mathrm{Im}\,\Pi_{\mathrm{R}}\big|_{\mathrm{NLO}} - \mathrm{Im}\,\Pi_{\mathrm{R}}\big|_{\mathrm{LPM}}^{\text{expanded}}$ is an addition to LPM, i.e. a "hard" matching coefficient $(2 \leftrightarrow 2)$.

[14] I. Ghisoiu and ML, *Interpolation of hard and soft dilepton rates*, 1407.7955.

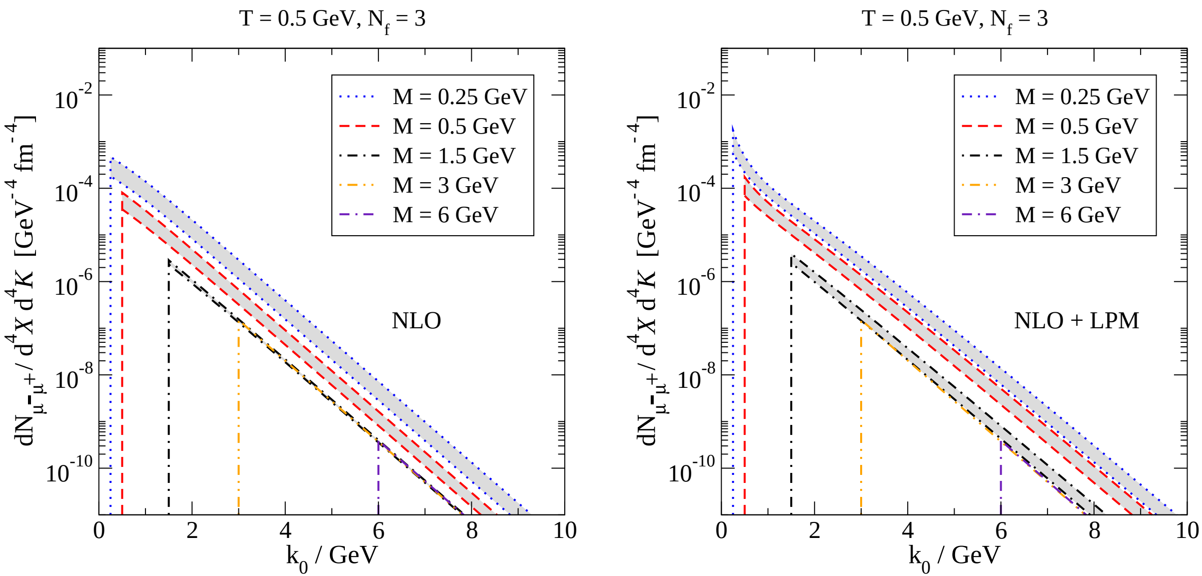
This results in a smoother behaviour:



The green band is an effective kinetic theory result at $k = 0$.[15]

[15] G.D. Moore and J.-M. Robert, *Dileptons, spectral weights, and conductivity in the quark-gluon plasma*, hep-ph/0607172.

Physical dilepton rates (from $N_f = 3$ light quarks):



Data available at: `www.laine.itp.unibe.ch/dilepton-lpm/`.

News (iii): indirect tests with lattice

LPM-type equations play a role for non-static screening masses, although the potential is real rather than a width.[16]

$$G^{(k_n)}_{\mu\nu}(z) \equiv \int_{\mathbf{x}} \int_0^{\frac{1}{T}} \mathrm{d}\tau \, e^{ik_n\tau} \Big\langle V_\mu(\tau,\mathbf{x},z) V_\nu(0) \Big\rangle \overset{\mu=\nu}{=} \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \, e^{-\omega|z|} \, \rho^{(k_n)}_{\mu\nu}(\omega) \; .$$

$$\left(k_n - \omega + \frac{m_\infty^2 - \nabla_\perp^2}{2M_\mathrm{r}} + V^+ - i0^+ \right) g(\omega;\mathbf{y}) = \delta^{(2)}(\mathbf{y}) \; ,$$

$$\frac{1}{M_\mathrm{r}} \equiv \frac{1}{p_n} + \frac{1}{k_n - p_n} \; , \quad 0 < p_n < k_n \; ,$$

$$\rho^{(k_n)}_{00}(\omega) = - \sum_{0<p_n<k_n} 2N_\mathrm{c} T \lim_{\mathbf{y}\to\mathbf{0}} \mathrm{Im}\, g(\omega;\mathbf{y}) \; .$$

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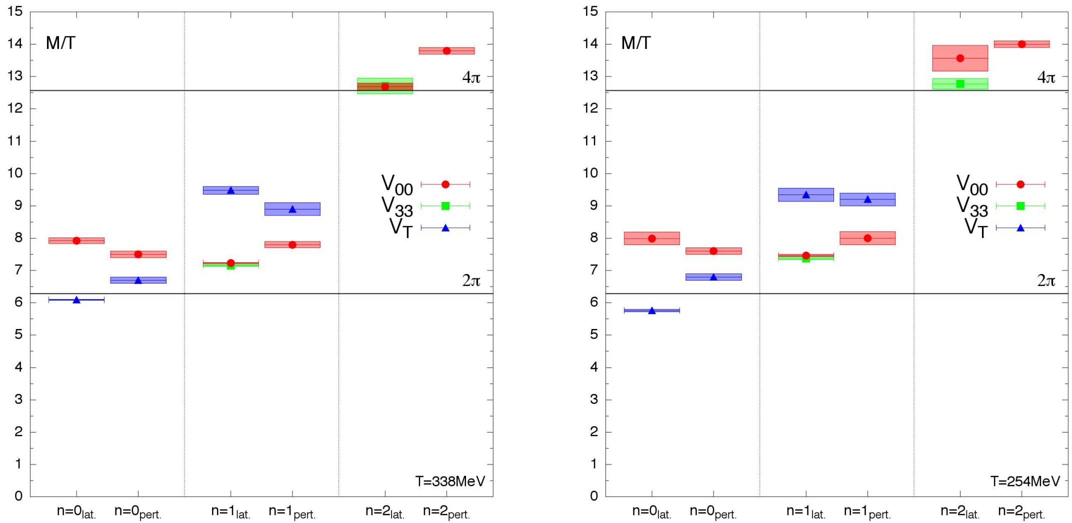
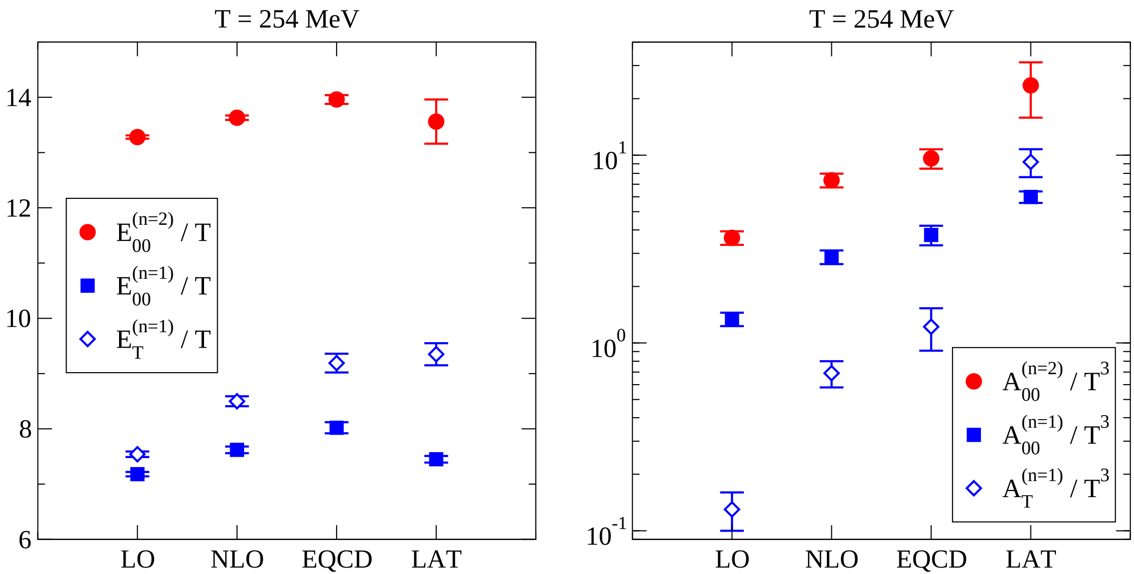
[16] B.B. Brandt, A. Francis, ML, H.B. Meyer, *A relation between screening masses and real-time rates*, JHEP 05 (2014) 117 [1404.2404].

A resulting comparison with $N_{\mathrm{f}} = 2$ lattice data:

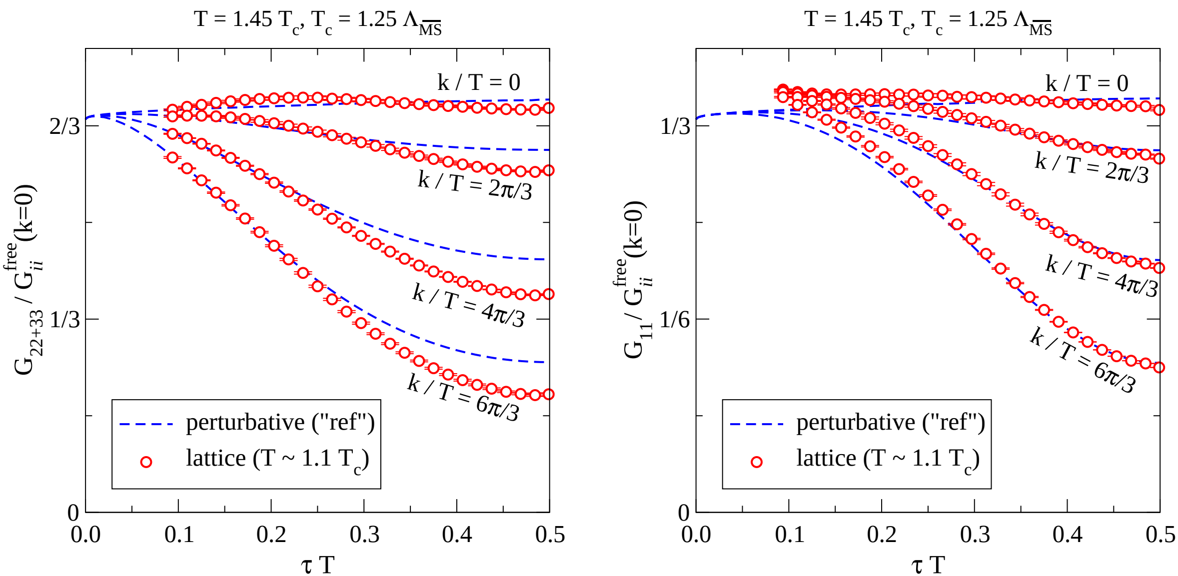


Effect of different potentials V^+:



Here: $G \equiv -A\, e^{-E|z|}$.

Ultimately can also compare with data in time direction:[17]



(The continuum limit is not fully under control yet.)

[17] H.-T. Ding *et al*, *Thermal dilepton rates from quenched lattice QCD*, PoS ConfinementX (2012) 185 [1301.7436].