What can lattice teach us about heavy quarks ?

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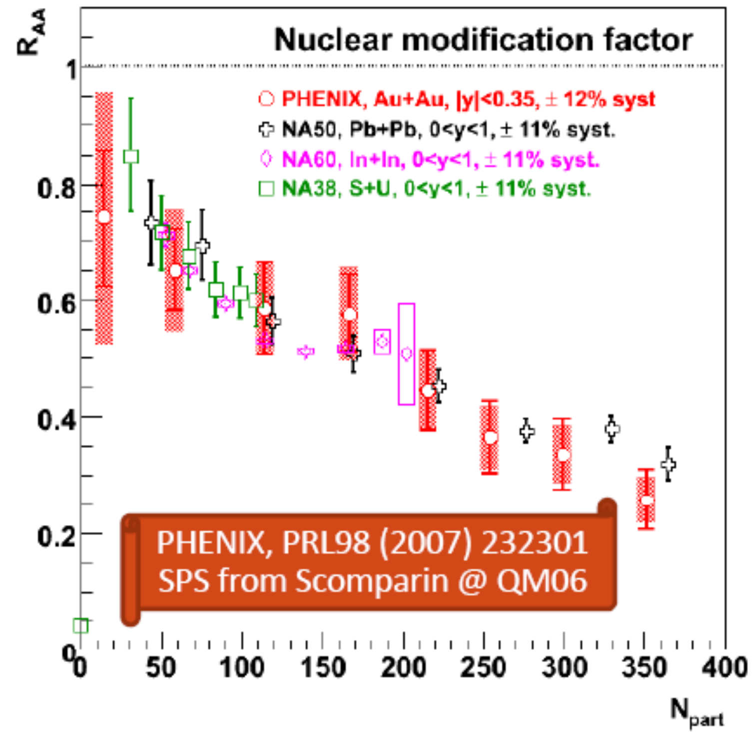
Heavy quarks and bound states of heavy quarks (quarkonia) are probes of the matter created in heavy ion collisions
Relation between color screening (deconfinement) and quarkonium suppression ?

Outline:

1) Deconfinement and chiral transition in QCD
2) Meson spectral functions at *T>0* and imaginary time correlation functions
3) Static QQbar pair at *T>0* and color screening
4) Spatial correlation functions and meson spectral functions
5) Charm susceptibilities

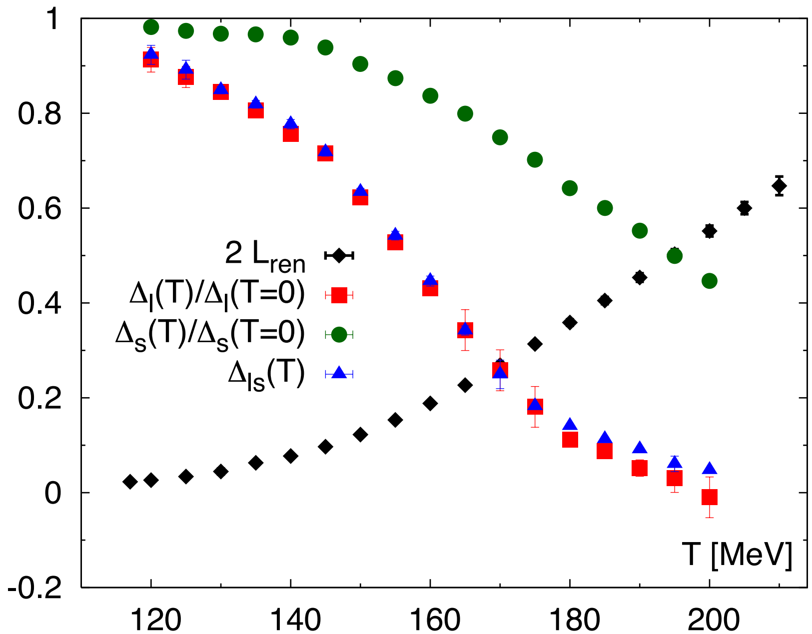


MITP, Mainz, July 28 - August 8, 2014

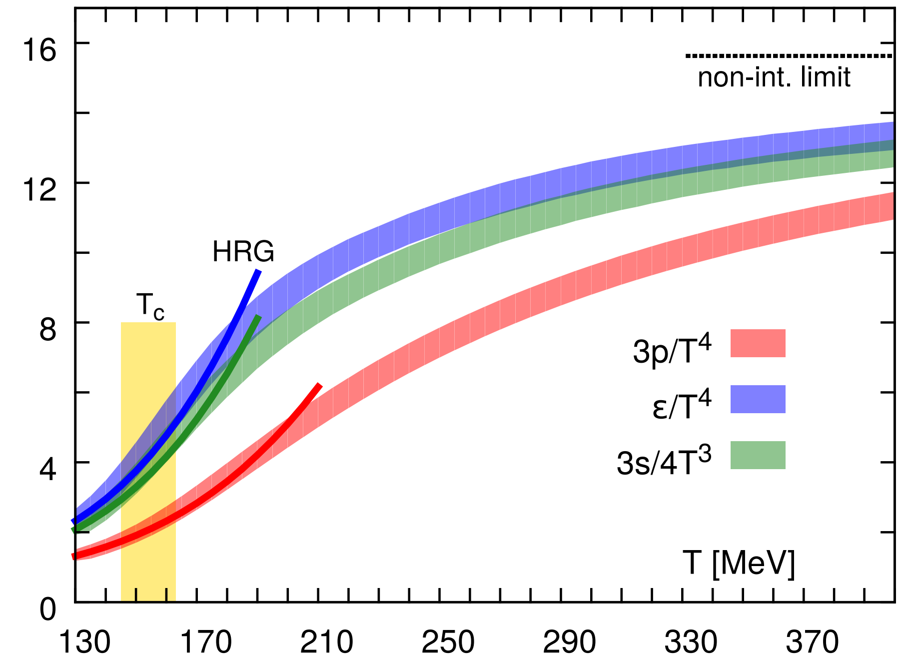
Deconfinement and chiral transition in lattice QCD

HISQ action = Highly Improved Staggered Quark action → Continuum results

Bazavov, Petreczky PRD87 (2013) 094505



HotQCD, arXiv:1407.6387



T_c=154(9) MeV is chiral transition temperature that is related to true chiral phase transition in the limit of zero u and d quark masses (agrees with Wuppertal-Budapest results)

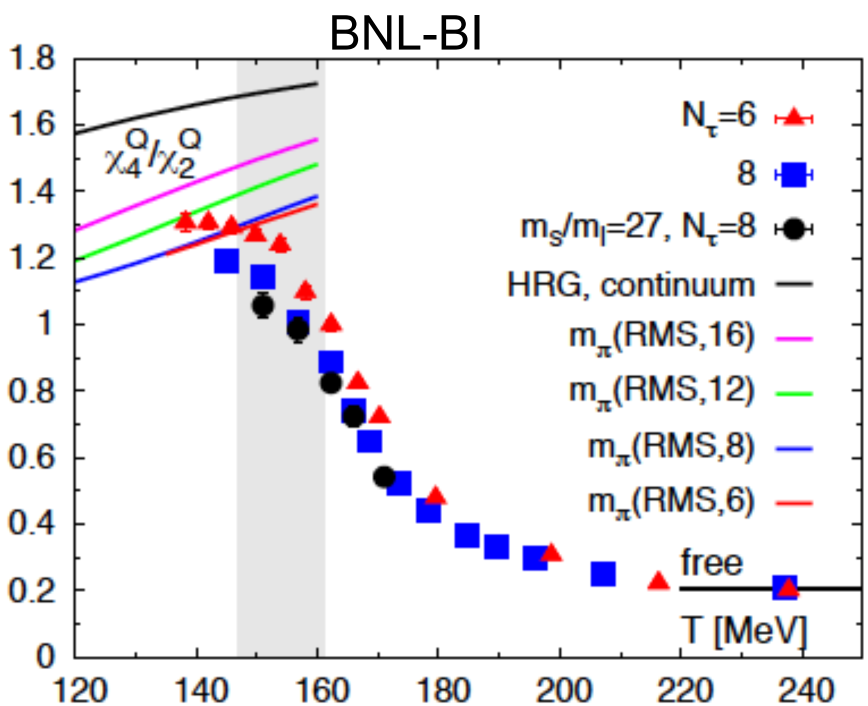
HotQCD and WB results agree in the Continuum limit !

Hadron Resonance Gas (HRG) works for $T<T_c$; for $T>300$ MeV weakly interaction quark-gluon gas ?

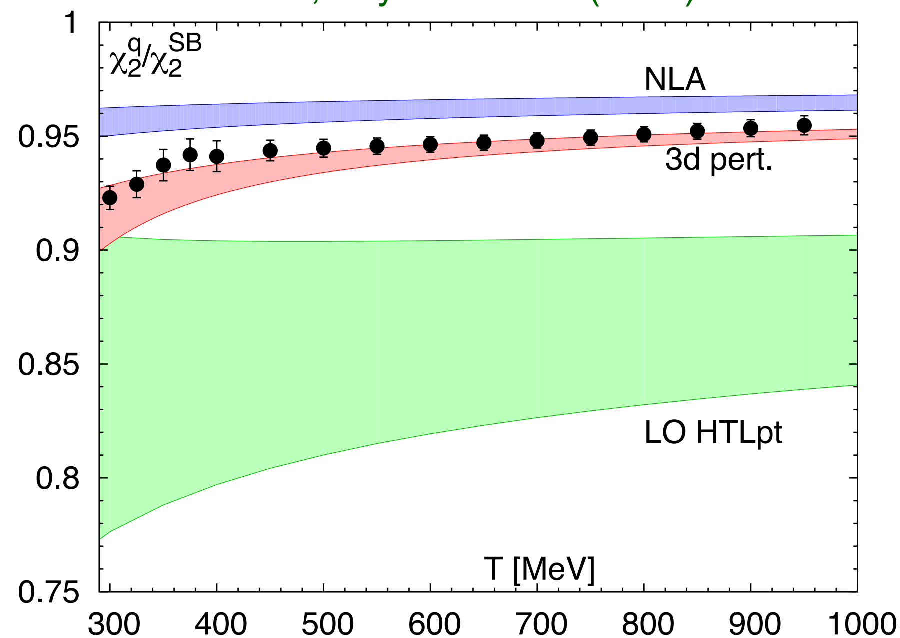
Deconfinement and fluctuations of conserved charges

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!l!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_Q}{T}\right)^k$$

$$\frac{p(T,\mu_u,\mu_d,\mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k$$



Bazavov et al, Phys.Rev. D88 (2013) 094021



Deconfinement: hadronic description (HRG) works below T_c, but breaks down around the transition region

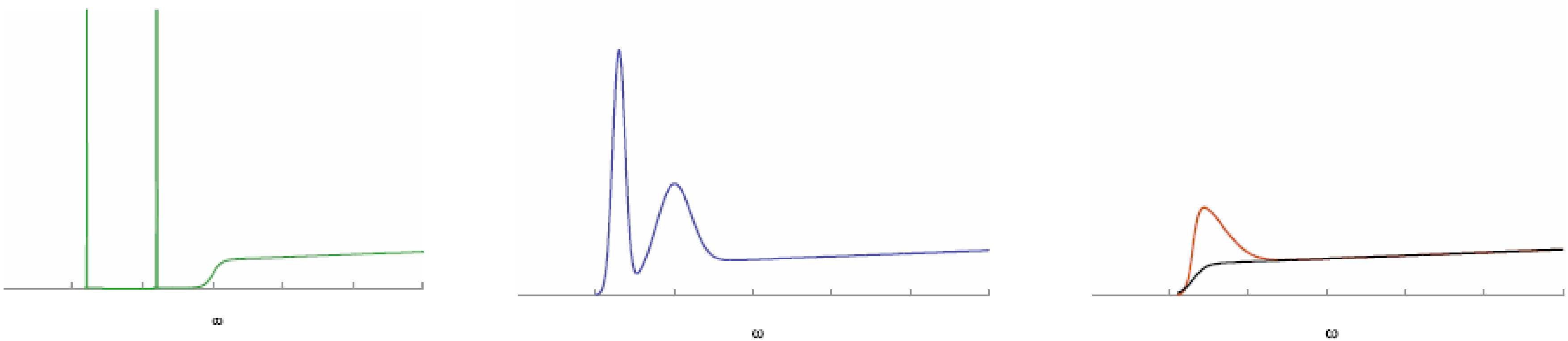
Weak coupling results (3d EFT) describes the lattice data quantitatively for $T>300$ MeV

Static quark anti-quark pair in T>0 QCD

In-medium properties and/or dissolution of quarkonium states are encoded in the spectral functions

$$\sigma(\omega,p,T)=\frac{1}{2\pi}\,\mathrm{Im}\int_{-\infty}^{\infty}dt e^{i\omega t}\int d^3x e^{ipx}\langle[J(x,t),J(x,0)]\rangle_T$$

Melting is see as progressive broadening and disappearance of the bound state peaks



Due to analytic continuation spectral functions are related to Euclidean time quarkonium correlators that can be calculated on the lattice

$$G(\tau,p,T)=\int d^3x e^{ipx}\langle J(x,-i\tau),J(x,0)\rangle_T$$

MEM

$$G(\tau,p,T)=\int_0^{\infty}d\omega\sigma(\omega,p,T)\frac{\cosh(\omega\cdot(\tau-\frac{1}{2T}))}{\sinh(\omega/(2T))}$$

$\sigma(\omega,p,T)$

1S charmonium survives to *1.6*T_c??

Umeda et al, EPJ C39S1 (05) 9, Asakawa, Hatsuda, PRL 92 (2004) 01200, Datta, et al, PRD 69 (04) 094507, ...

Charmonium correlators T>0

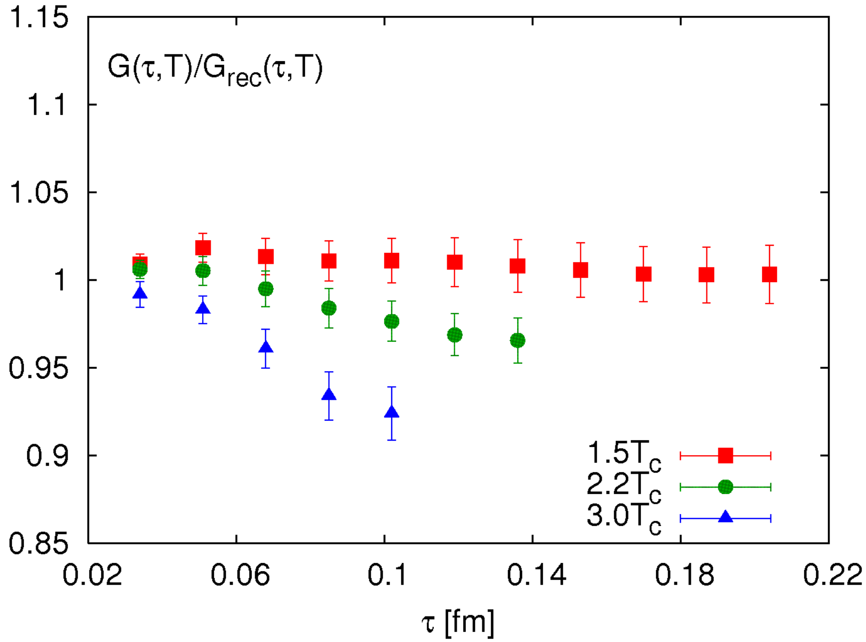
temperature dependene of $G(\tau,T)$

$$G(\tau,T)=\int_0^\infty d\omega\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T))}{\sinh(\omega/(2T))}$$

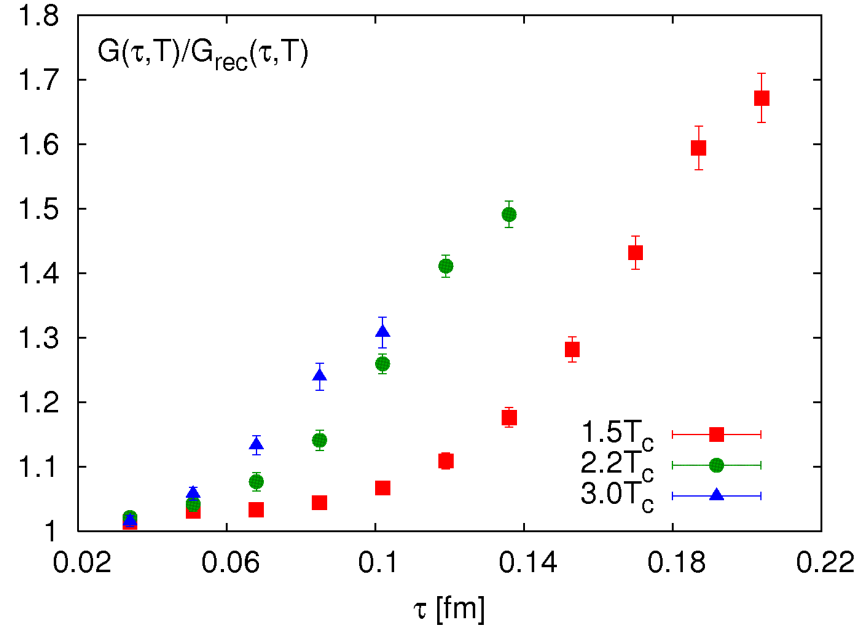
If there is no T-dependence in the spectral function, $G(\tau,T)/G_{rec}(\tau,T)=1$

$$G_{rec}(\tau,T)=\int_0^\infty d\omega\sigma(\omega,T=0)\frac{\cosh(\omega\cdot(\tau-\frac{1}{2T}))}{\sinh(\omega/(2T))}$$

Pseudo-scalar $\Leftrightarrow 1S$



Scalar $\Leftrightarrow 1P$



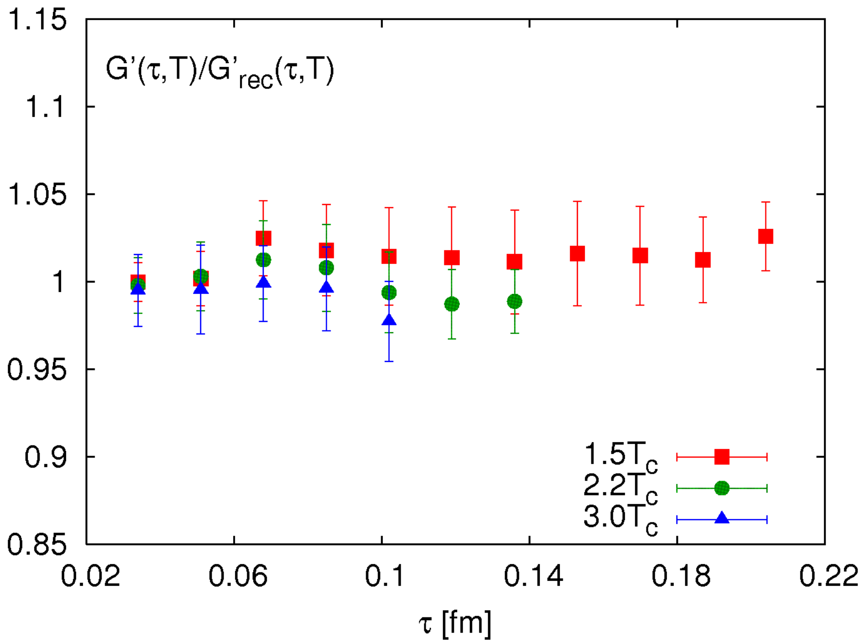
Datta, Karsch, P.P., Wetzorke, PRD 69 (04) 094507

Charmonium correlators T>0

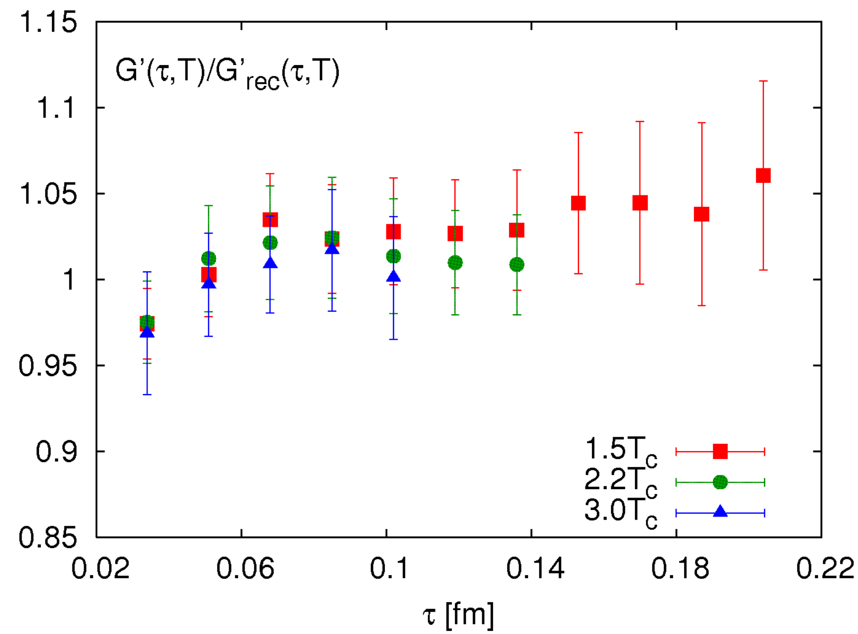
zero mode contribution is not present in the time derivative of the correlator

Umeda, PRD 75 (2007) 094502

Pseudo-scalar $\Leftrightarrow 1S$



Scalar $\Leftrightarrow 1P$



P.P., EPJC 62 (09) 85

the derivative of the scalar correlators does not change up to *$3T_c$*, all the T-dependence was due to zero mode

⇓

either the 1P state (χ_c) with binding energy of *300*MeV can survive in the medium with ε=*100*GeV/fm^3

or temporal quarkonium correlators are not very sensitive to the changes in the spectral functions due to the limited $\tau_{max}=1/(2\,T)$

Static quark anti-quark pair in T>0 QCD

Consider correlation functions of statice meson operators

$$G_1(t,x,y,T) = \langle O(x,y;t)O(x,y;0)\rangle, \quad G_8(t,x,y,T) = \langle O^\alpha(x,y;t)O^\alpha(x,y;0)\rangle$$

for color singlet and adjoint represenation at time $t = 1/T$ with

$$O(x,y;t) = \bar\psi(x,t)U(x,y;t)\psi(y,t)$$
$$O^\alpha(x,y;t) = \bar\psi(x,t)U(x,x_0;t)T^\alpha U(x_0,y;t)\psi(y,t)$$

After integration out the static quarks we get

$$G_1(r,T) = \frac{1}{N}\langle \mathrm{Tr} L^\dagger(x)U(x,y;0)L(y)U^\dagger(x,y,1/T)\rangle,$$
$$G_8(r,T) = \frac{1}{N^2-1}\langle \mathrm{Tr} L^\dagger(x)\mathrm{Tr} L(y)\rangle$$
$$-\tfrac{1}{N(N^2-1)}\langle \mathrm{Tr} L^\dagger(x)U(x,y;0)L(y)U^\dagger(x,y,1/T)\rangle, r = |x-y|.$$

$L(x)$ is the temporal Wilson line; on the lattice $L(x) = \prod_{\tau=0}^{N_\tau - 1} U_0(x,\tau)$

Alternative choice of $O(x,y;t)$: fix the Coulomb gauge and omit $U(x,y;t)$
For $t \to \infty$ there the choice of meson operators is not important but $t \leq 1/T$

Static quark anti-quark pair in T>0 QCD (cont'd)

The color averaged correlator gives the ratio of the partition of partition function of QCD at $T > 0$ with static $Q\bar{Q}$ to partition function without static sources:

$$G(r,T) = \frac{1}{N^2}\langle \mathrm{Tr} L(r) \mathrm{Tr} L^\dagger(0)\rangle = \frac{Z_{QQ}(r,T)}{Z(T)} = e^{-F(r,T)/T}$$

McLerran, Svetitsky 1981

$\rightarrow -T \ln G(r,T) \equiv F(r,T)$ is the excess free energy due to static sources.

$$G(r,T) = \frac{1}{N^2} G_1(r,T) + \frac{N^2-1}{N^2} G_a(r,T) \equiv \frac{1}{N^2} e^{-F_1(r,T)/T} + \frac{N^2-1}{N^2} e^{-F_8(r,T)/T}$$

The spectral represenation of singlet and averaged correlators $(T < T_c)$:

$$G_1(r,T) = \sum_{n=1}^{\infty} c_n(r) e^{-E_n(r)/T}, \qquad G(r,T) = \frac{1}{N^2} \sum_{n=1}^{\infty} e^{-E_n(r)/T}$$

Jahn, Philipsen, 2004

Perturbation theory $(T \gg T_c)$:

$$F_1(r,T) = -\frac{N^2-1}{2N}\frac{\alpha_s}{r} e^{-m_D r} - \frac{(N^2-1)\alpha_s m_D}{2N},$$

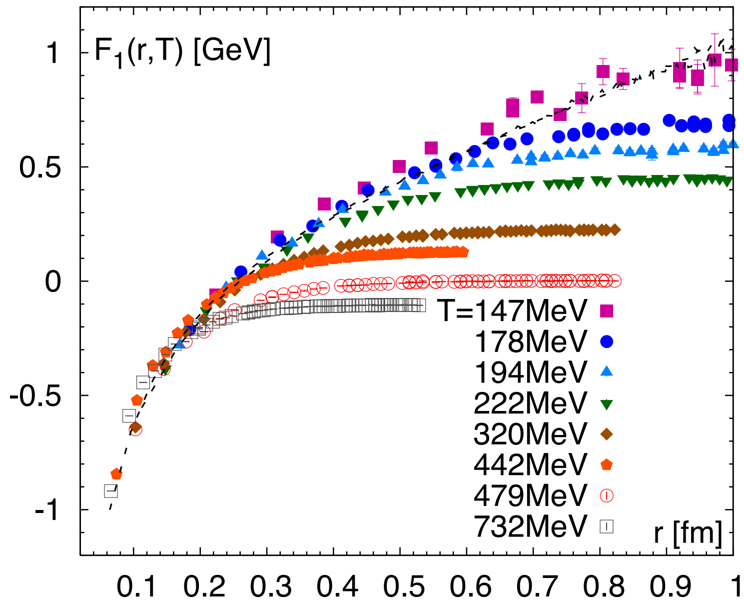
$$F_8(r,T) = +\frac{1}{2N}\frac{\alpha_s}{r} e^{-m_D r} - \frac{(N^2-1)\alpha_s m_D}{2N},$$

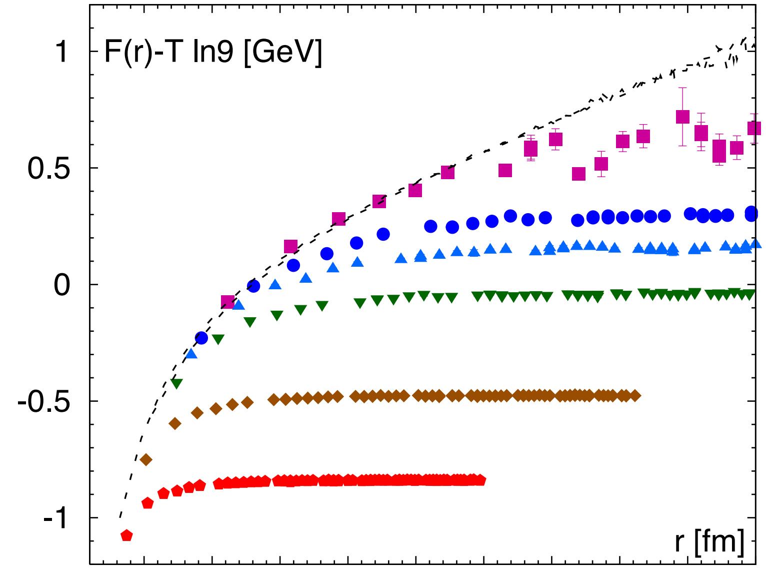
decomposition in terms of F_1 and F_8 can be extended to any order for $rT \ll 1$

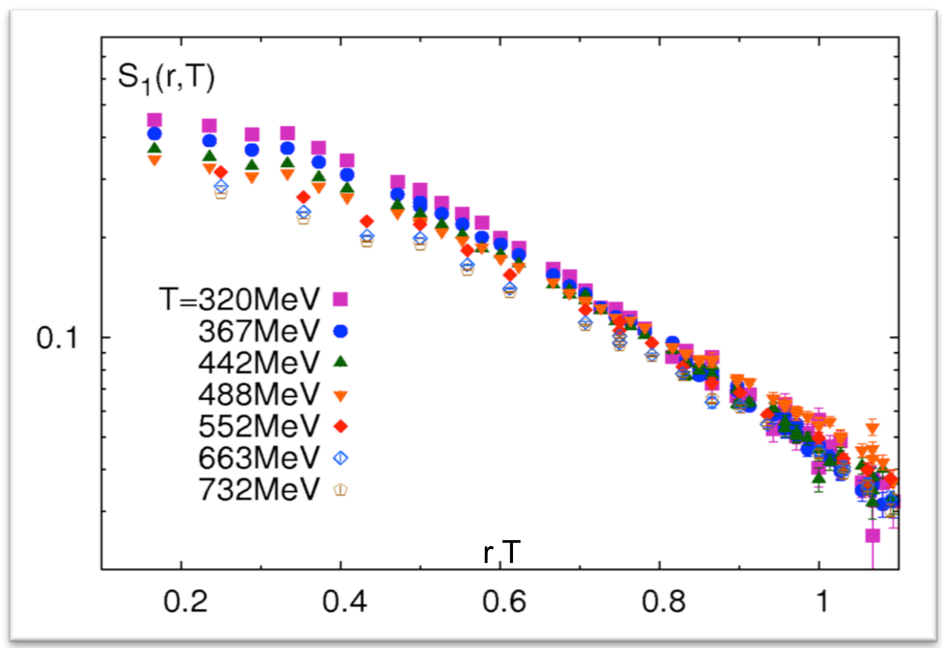
Brambilla, Ghiglieri, PP, Vairo 2010

Static quark anti-quark free energy in 2+1f QCD

HISQ action, $24^3 \times 6$, $16^3 \times 4$ (high T) lattices, $m_\pi \simeq 160$ MeV







- The strong *T*-dependence for *T<200* MeV is not necessarily related to color screening

- The free energy has much stronger *T*-dependence than the singlet free energy due to the octet contribution

- Onset of screening at *r T ~ 0.7*

$$S_1(r,T) = (F_\infty(T) - F_1(r,T) \cdot r$$

$$rT \gg 1, \quad S_1(r,T) \sim \exp(-m_D r)$$

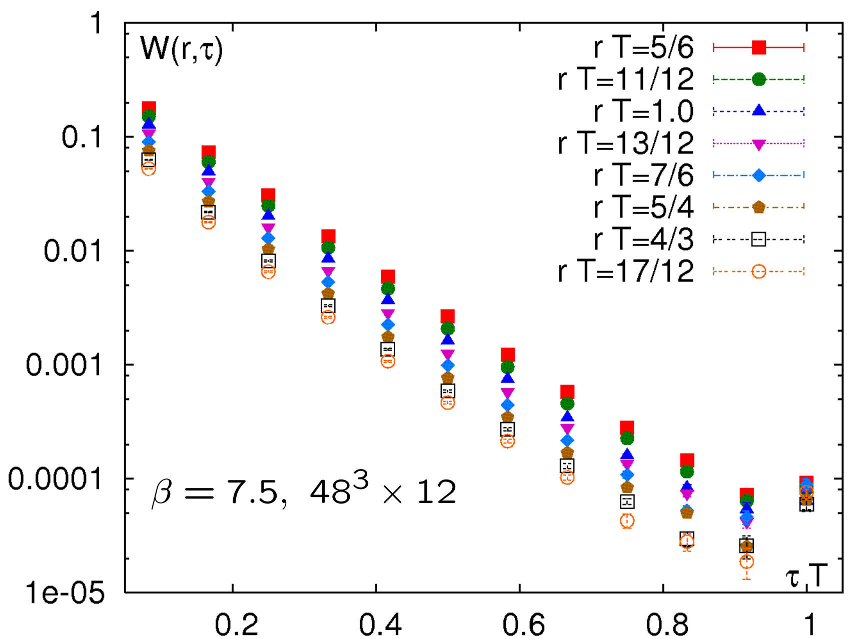
Wilson loops and potential at T>0

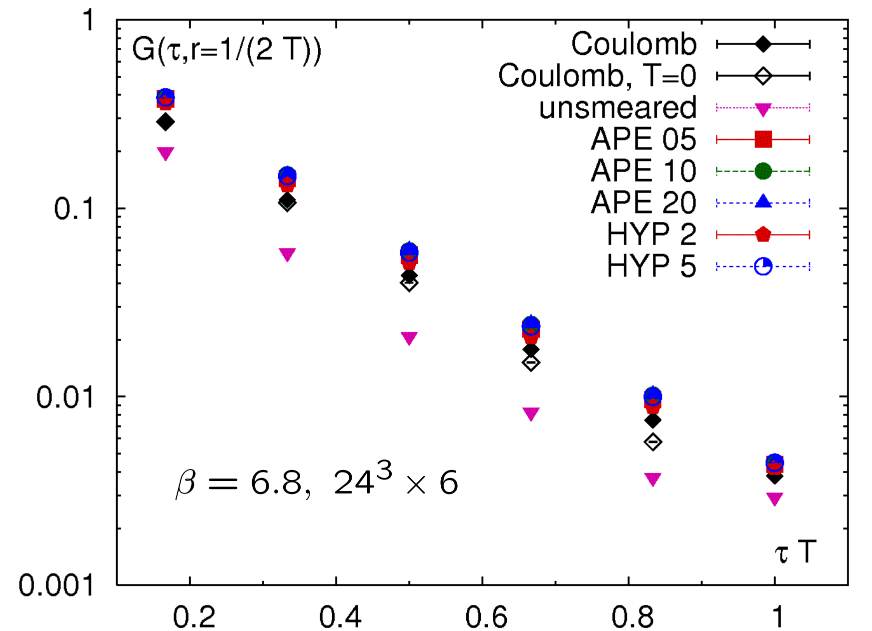
HISQ action, $48^3 \times 12$, $24^3 \times 6$ lattices, $m_\pi \simeq 160$ MeV

$$W(r,\tau) = \int_{-\infty}^{\infty} d\omega \sigma(\omega, T) e^{-\omega\tau}, \tau < 1/T$$

Rothkopf 2009, Hatsuda, Rothkopf, Hatsuda 2011

not related to the free energy !





Choices of the spatial links:

or use Coulomb gauge and *U(x,y)=1*

Un-smeared Wilson loops show non-exponential behavior and are suppressed compared to the smeared Wilson loops and Coulomb gauge corr. which decay exponentially, except for $\tau T \sim 1$

The temperature dependence of the effective potentials

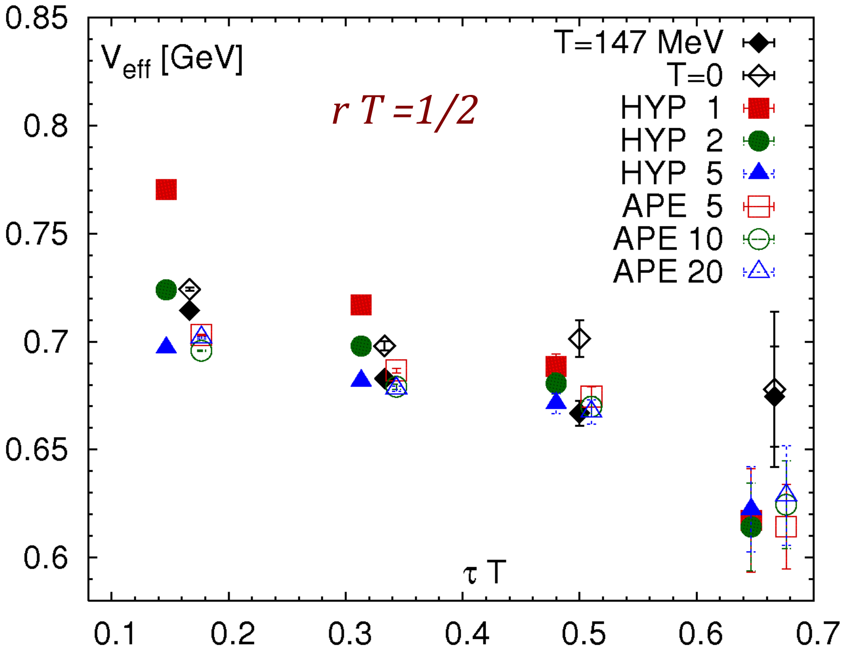
Assume single state dominance
$\sigma(\omega,T) \sim \delta(\omega - V(r,T))$

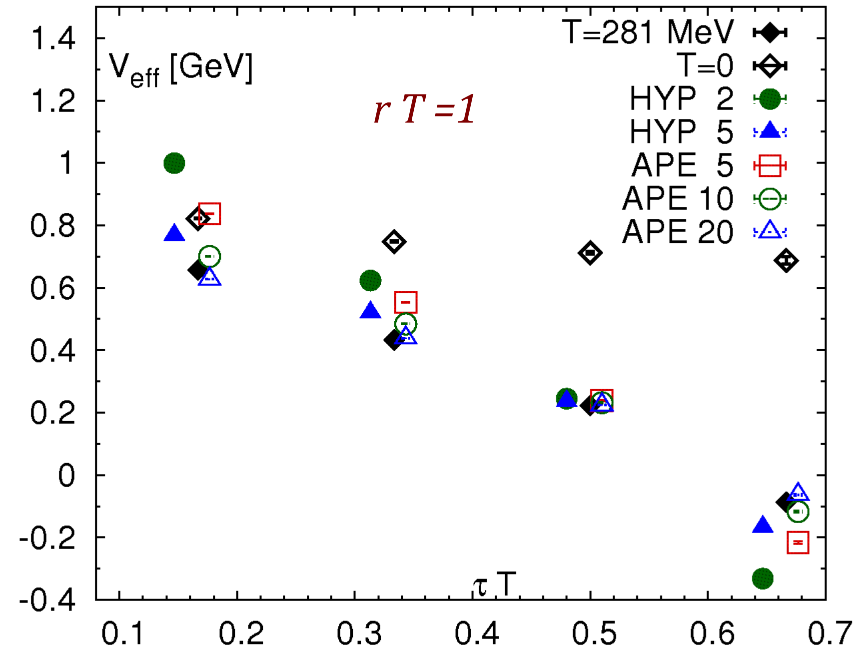
$$V_{eff}(r,\tau) = -\left[\ln W(r,\tau)/\tau\right]$$

$$V_{eff}(r,\tau\to\infty) = V(r)$$

Bazavov, PP, Eur. Phys. J. A49 (2013) 85

Can be seen at *T=0* in the considered range





- At $T>0$ the effective potential V_{eff} does not reach a plateau but shows an approximately linear decrease in τ, and V_{eff} is always smaller than for *T=0*

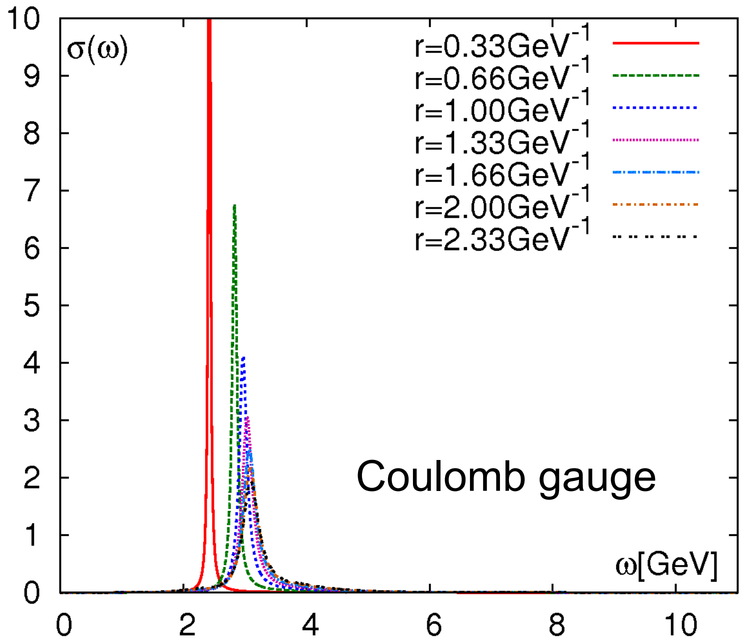
- For large enough τ all smeared Wilson loops and Colomb gauge correlators give the same V_{eff}

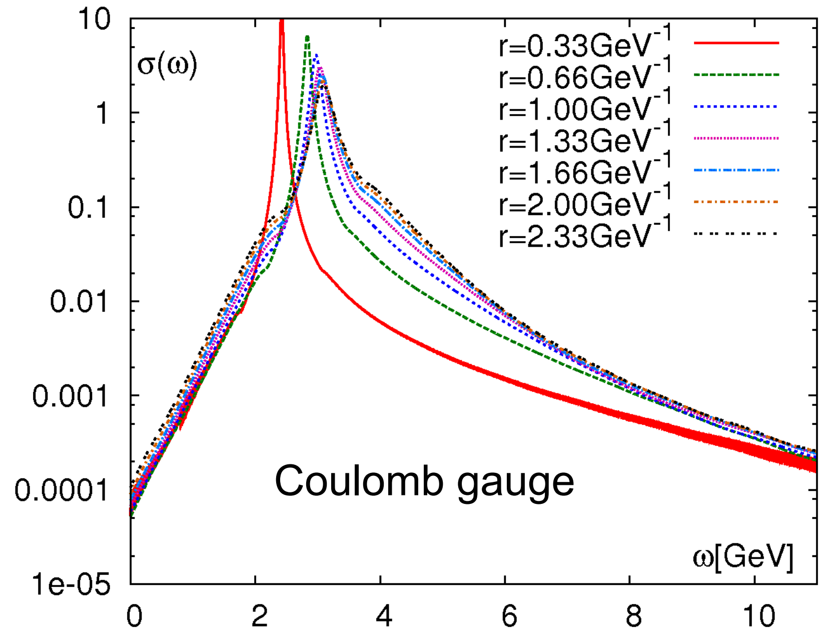
$\Rightarrow$ The observed decrease in τ is a physical effect (width ?) independent of the choice of the correlators

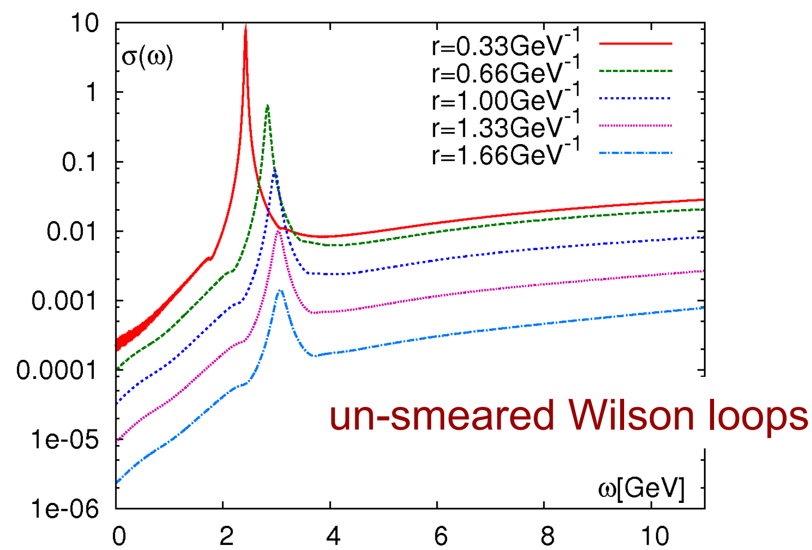
Spectral functions in HTL perturbation theory

Perturbative hard thermal loop (HTL) calculations for $T=2.33\ T_c$, $T_c=270$ MeV $(N_f = 0)$:

Burnier, Rothkopf, 2013





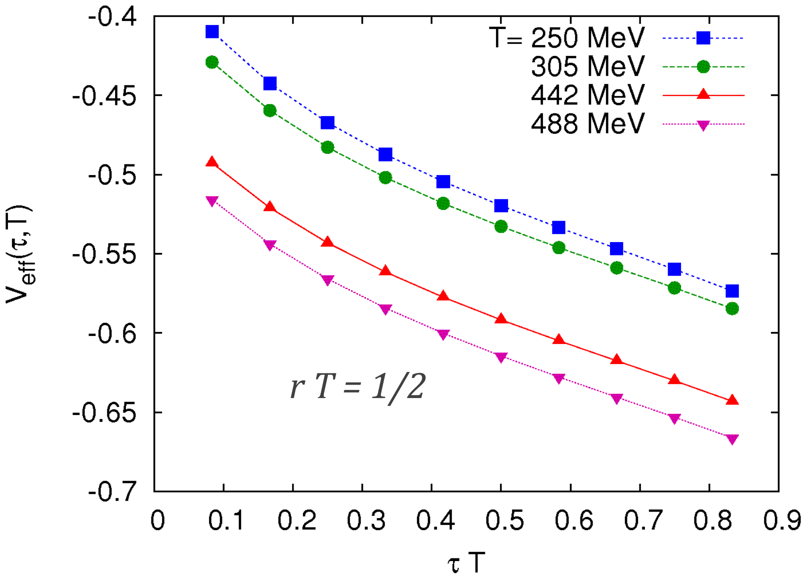


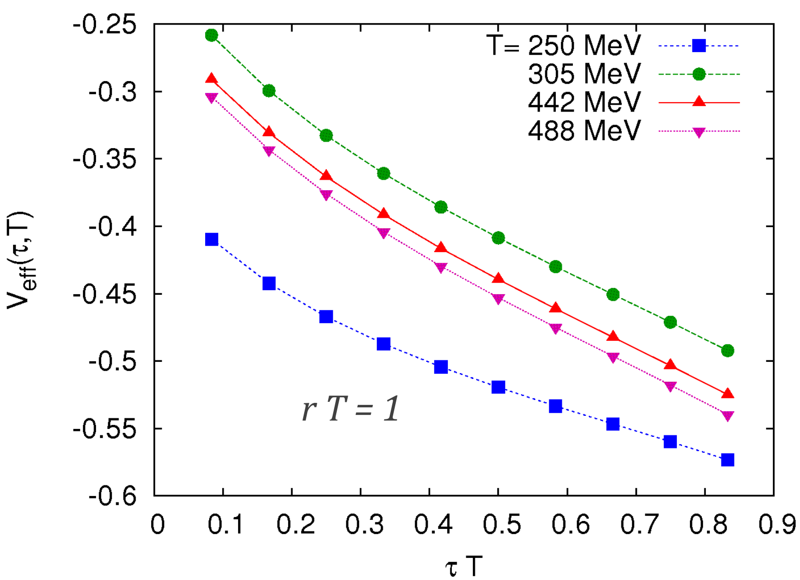
Spectral functions has long tails and non-Lorentzian away from the peak,
⇒ explanation for the behavior of the Wilson loops and V_{eff} at large times

For un-smeared Wilson loops the peak heigh is much suppressed compared to the Coulomb gauge case

Effective potential at high temperatures

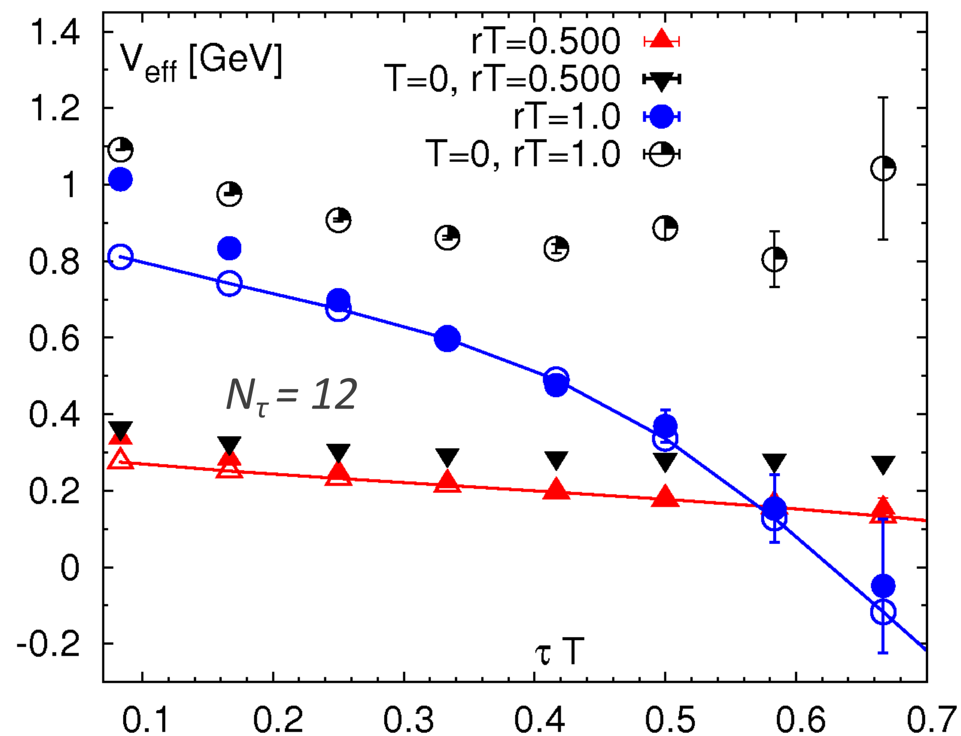
Effective potential V_{eff} in HTL perturbation theory:





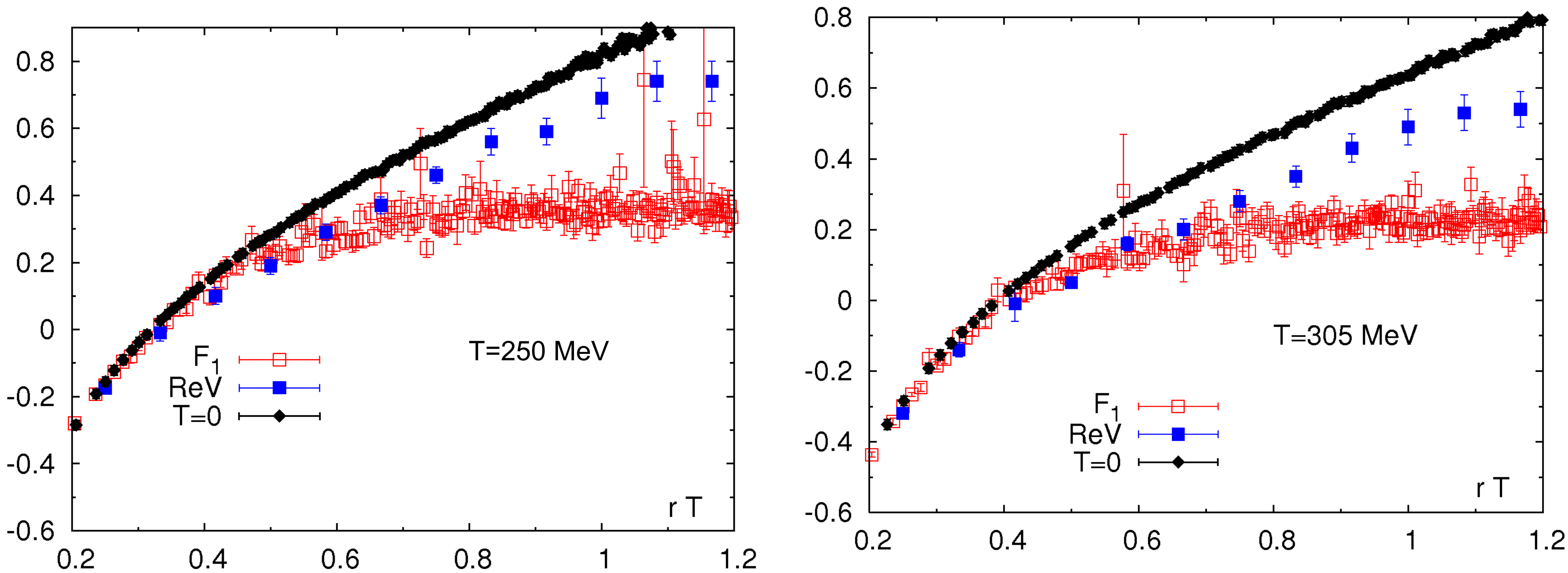
V_{eff} decreases with τ due to the width of the spectral functions, its slope increases with T and the distance r as observed in the lattice calculations

Use $\sigma^{HTL}((\omega-E)\lambda)$ as a 2-parameter fit Ansatz for the lattice results:



Real part of the potential above deconfinement

Results for $N_\tau = 12$ lattices using $\sigma^{HTL}((\omega\text{-}E)\,\lambda)$ as a 2-parameter fit Ansatz:

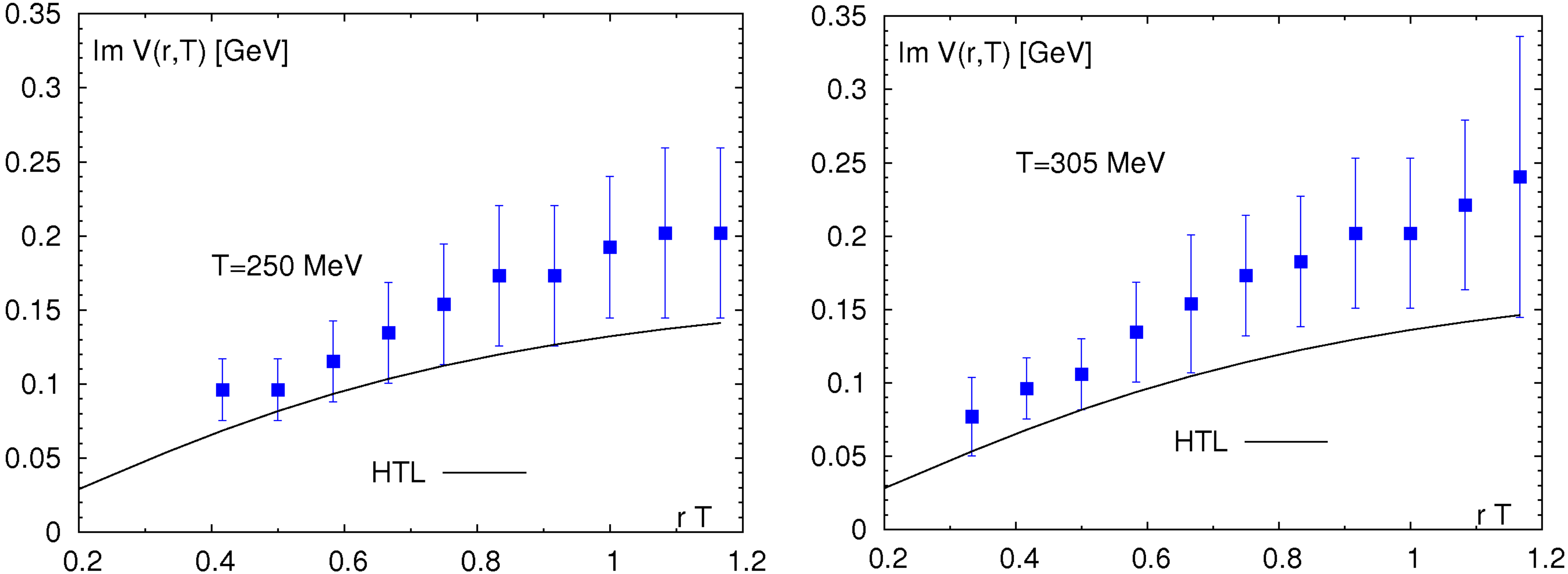


Bazavov, Burnier, PP, arXiv:1404.4267

- For $r\,T < 0.7$ the real part of the potential is roughly equal to the singlet free energy
- At larger distances it is between the singlet free energy and the $T=0$ potential
- The real part of the potential saturates at $r\,T \sim 1$ (screening) at a fairly large value (non-perturbative effects)

Imaginary part of the potential above deconfinement

Results for $N_\tau = 12$ lattices using $\sigma^{HTL}((\omega\text{-}E)\,\lambda)$ *as a 2-parameter fit Ansatz:*



Bazavov, Burnier, PP, arXiv:1404.4267

- The imaginary part of the potential has large errors as the width of the spectral functions is difficult to extract from the lattice correlators

- The imaginary part increases with r and saturates at $r\,T \sim 1$

- The central value imaginary part of the potential is *(1.5 - 2.0)* larger than in HTL perturbation theory

EFT, potential models and static energy

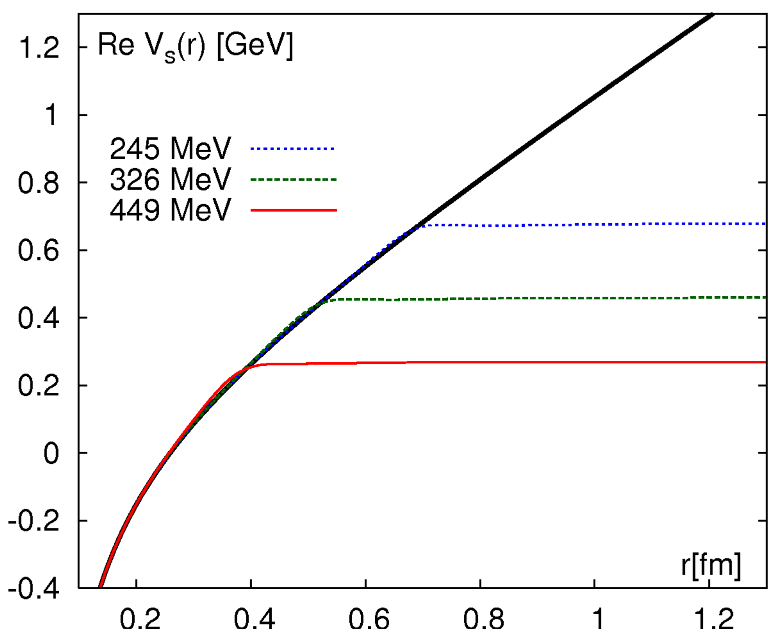
Above deconfinement the binding energy is reduced and eventually $E_{bind} \sim mv^2$ is the smallest scale in the problem (zero binding) $mv^2 << \Lambda_{QCD}, 2\pi T, m_D$ => most of medium effects can be described by a T-dependent potential

Potential = Static Energy

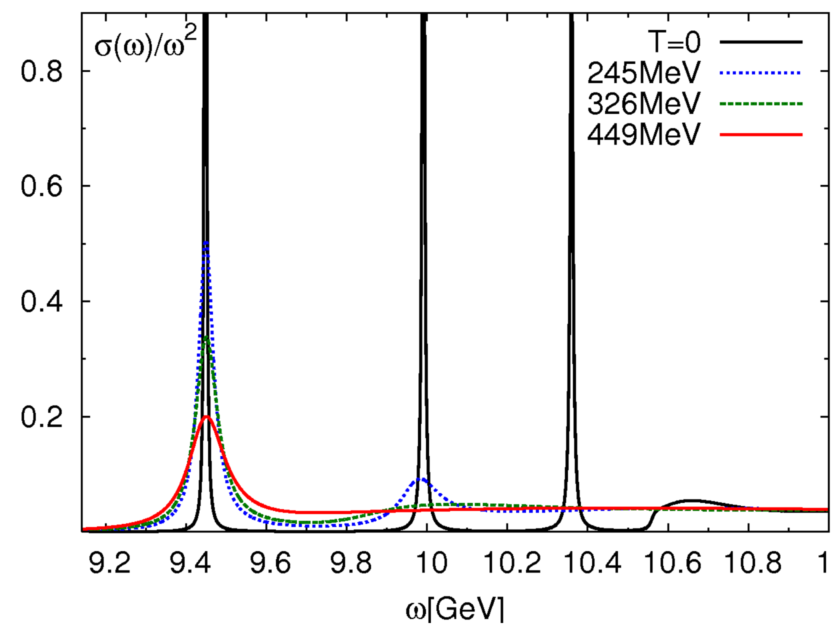
Determine the potential by non-perturbative matching to static quark anti-quark potential calculated on the lattice

Caveat : it is difficult to extract static quark anti-quark energies from lattice correlators => constrain Re $V_s(r)$ by lattice QCD data on the singlet free energy, take Im $V_s(r)$ from pQCD calculations

"Maximal" value for the real part
Mócsy, P.P., 2007



Minimal (perturbative) value for imaginary part
Laine et al, 2007, Brambilla et al, 2008



PP, Miao, Mocsy, Nucl.Phys. A855 (2011) 125

$T_{diss}(Y) > 3T_c$

Spatial meson correlation functions

Spatial correlation functions can be calculated for arbitrarily large separations $z \to \infty$

$$G(z,T) = \int_0^{1/T} d\tau \int dxdy \langle J(\mathbf{x},-i\tau), J(\mathbf{x},0)\rangle_T, \quad G(z\to\infty,T) \simeq A e^{-m_{scr}(T)z}$$

but related to the same spectral functions $G(z,T) = \int_{-\infty}^{\infty} e^{ipz} \int_0^{\infty} d\omega \frac{\sigma(\omega,p,T)}{\omega}$

Low T limit :

$$\sigma(\omega,p,T) \simeq A_{mes}\delta(\omega^2 - p^2 - M_{mes}^2)$$

$$A_{mes} \sim |\psi(0)|^2 \to m_{scr}(T) = M_{mes}$$

$$G(z,T) \simeq |\psi(0)|^2 e^{-M_{mes}(T)z}$$

High T limit :

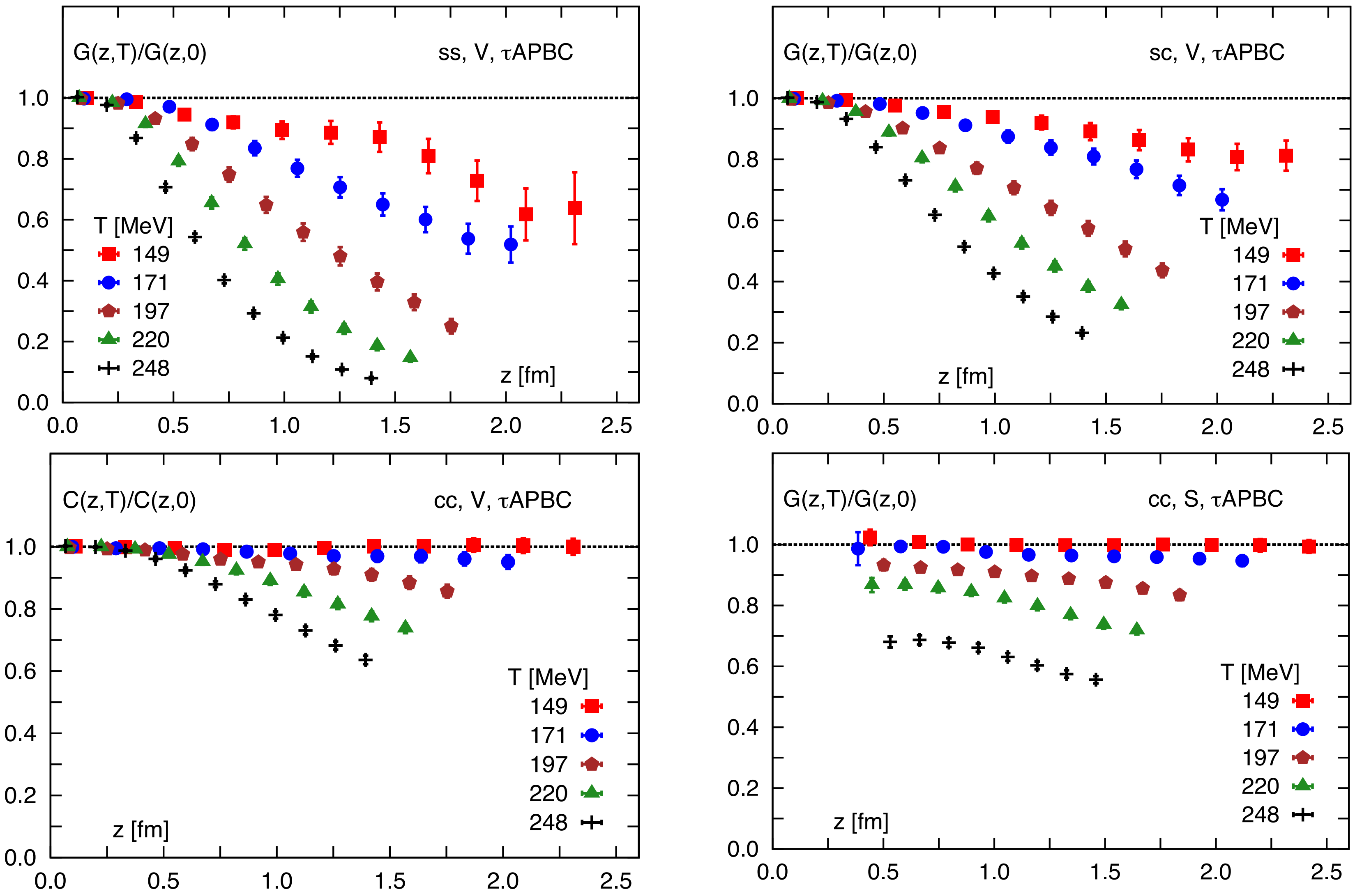
$$m_{scr}(T) \simeq 2\sqrt{m_c^2 + (\pi T)^2}$$

Temporal meson correlator only avalaible for $\tau T <$ ½ and thus may not be very sensitive to In-medium modifications of the spectral functions; also require large N_τ (difficult in full QCD)

Spatial correlators can be studied for arbitrarily large separations and thus are more sensitive to the changes in the meson spectral functions; do not require large N_τ (easy in full QCD).

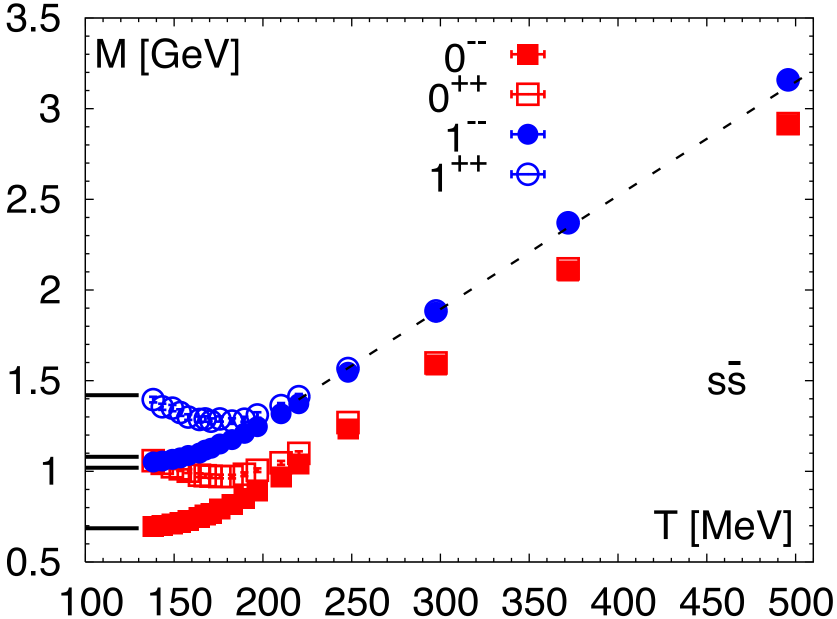
Lattice calculations: spatial meson correlators in 2+1 flavor QCD for $s\bar{s}$, $s\bar{c}$, $c\bar{c}$ sectors using 48^3x12 lattices and highly improved staggered quark (HISQ) action (also suitable for charm quarks) , physical m_s and m_π=160 MeV.

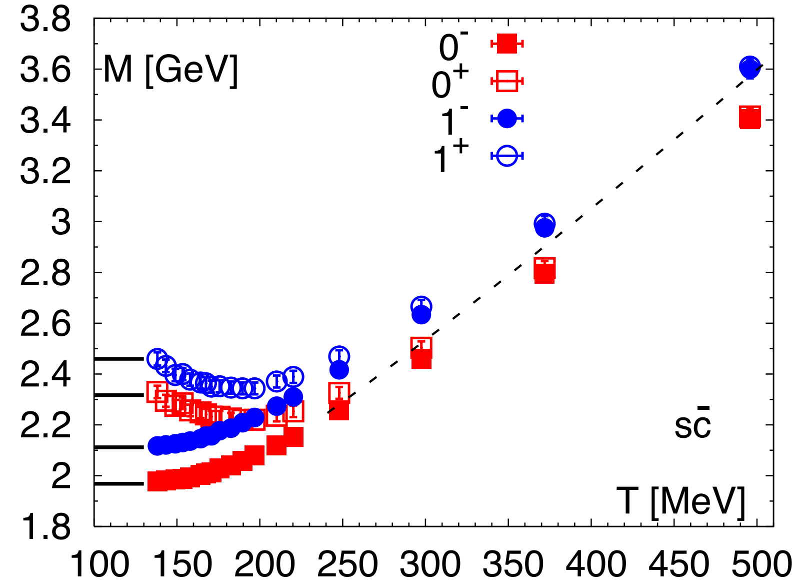
Temperature dependence of spatial meson correlators

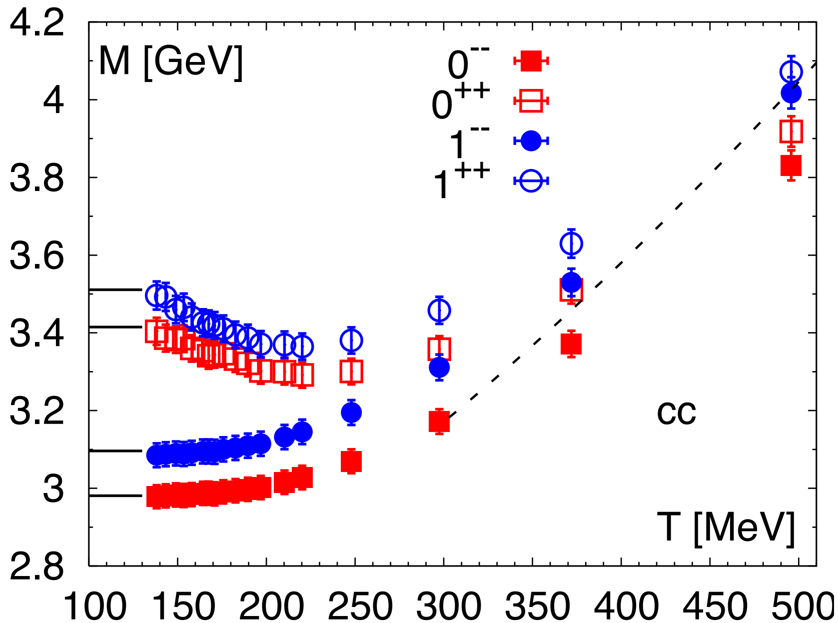


Medium modifications of meson correlators increase with T, but decrease with heavy quark content; larger for *1P* charmonium state than for *1S* charmonium state

Temperature dependence of the meson screening masses





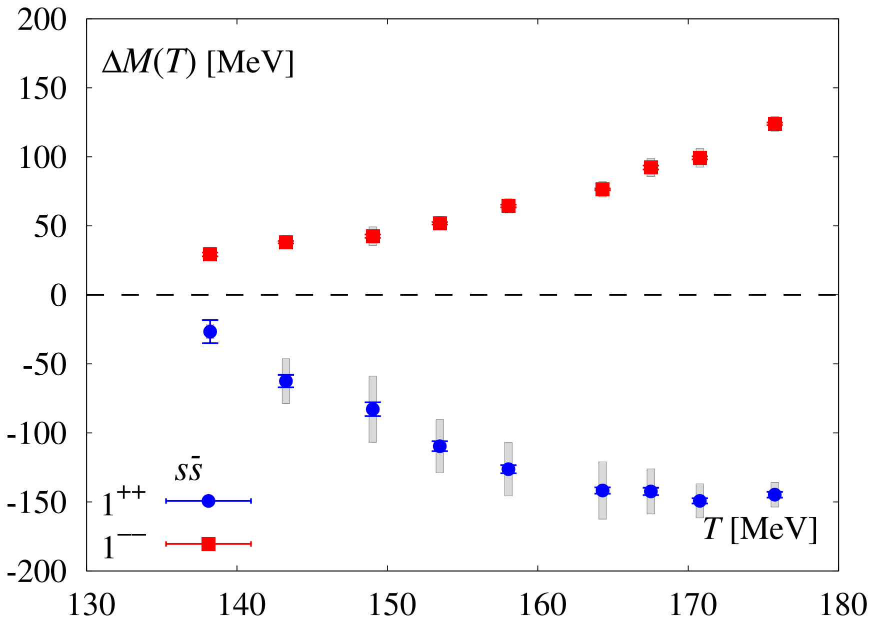


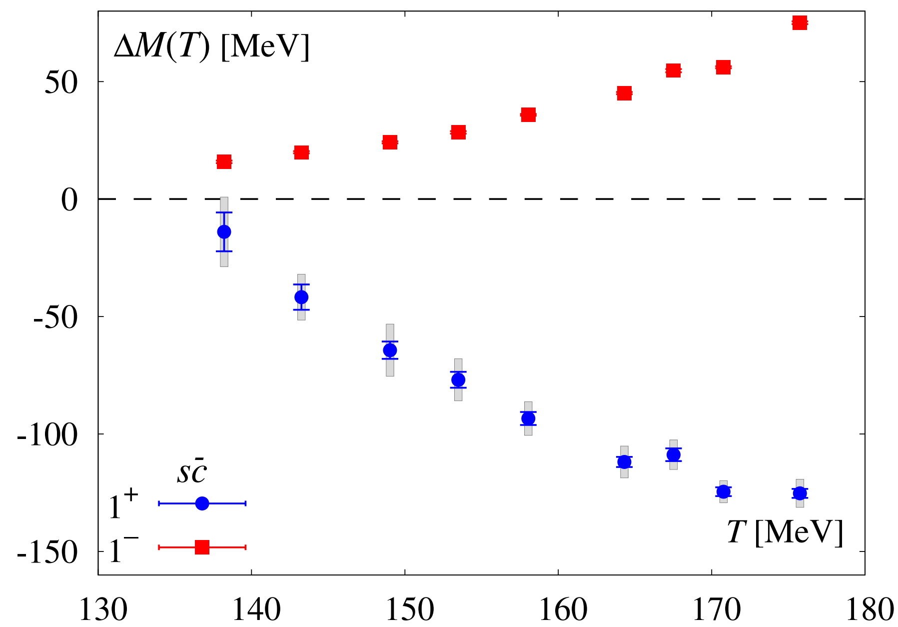
Qualitatively similar behavior of the screening masses for $s\bar{s}$, $s\bar{c}$, $c\bar{c}$ sectors

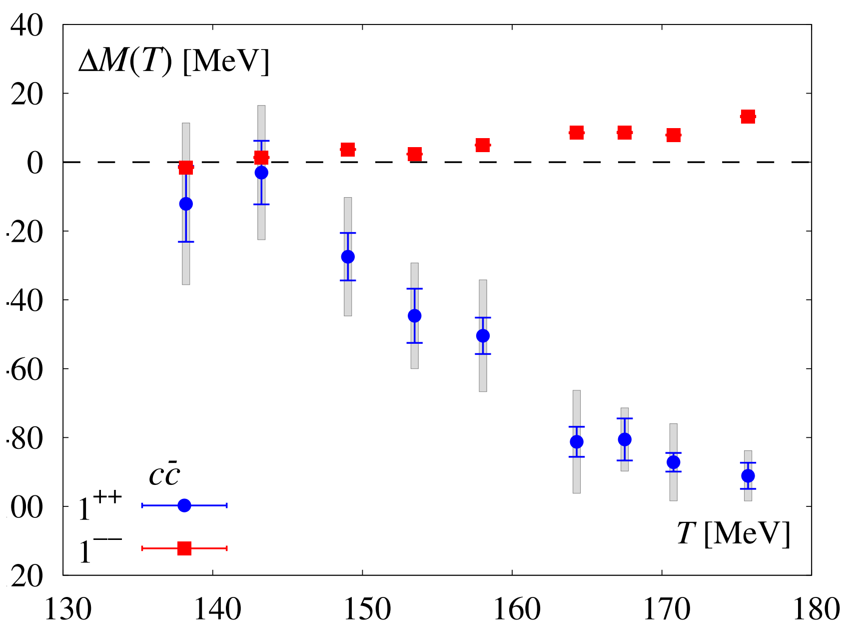
Screening Masses of opposite parity mesons become degenerate at high T (restoration of chiral and axial symmetry)

Screening masses are close to the free limit $\Sigma_{i=1,2}\, (m_i^2+(\pi T)^2)^{1/2}$ at $T>200$ MeV, $T>250$ MeV, $T>300$ MeV for $s\bar{s}$, $s\bar{c}$, $c\bar{c}$ sectors, respectively.

Temperature dependence of the meson screening masses

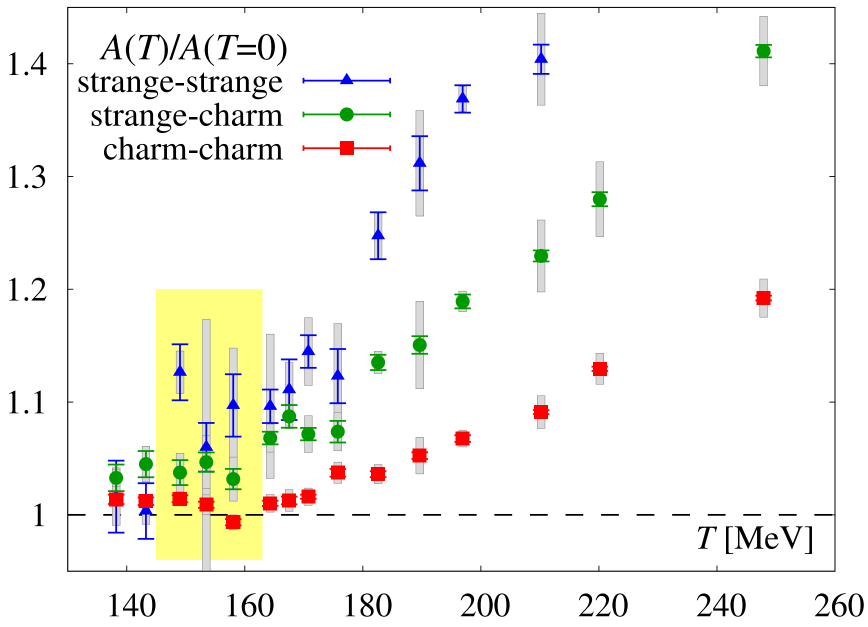


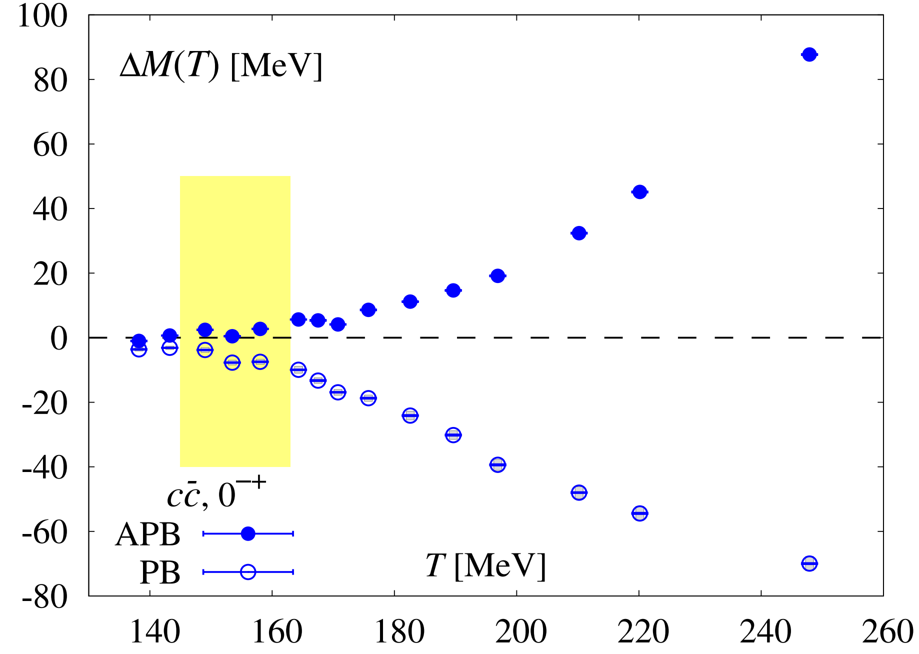




- At low T changes in the meson screening Masses $\Delta M = M_{scr}(T) - M_{T=0}$ are indicative of the changes in meson binding energies
- ΔM is significant already below T_c
- Above the transition temperature the changes in ΔM are comparable to the meson binding energy and are consistent with melting of meson states except for *1S* charmonium

Temperature dependence of the meson screening masses

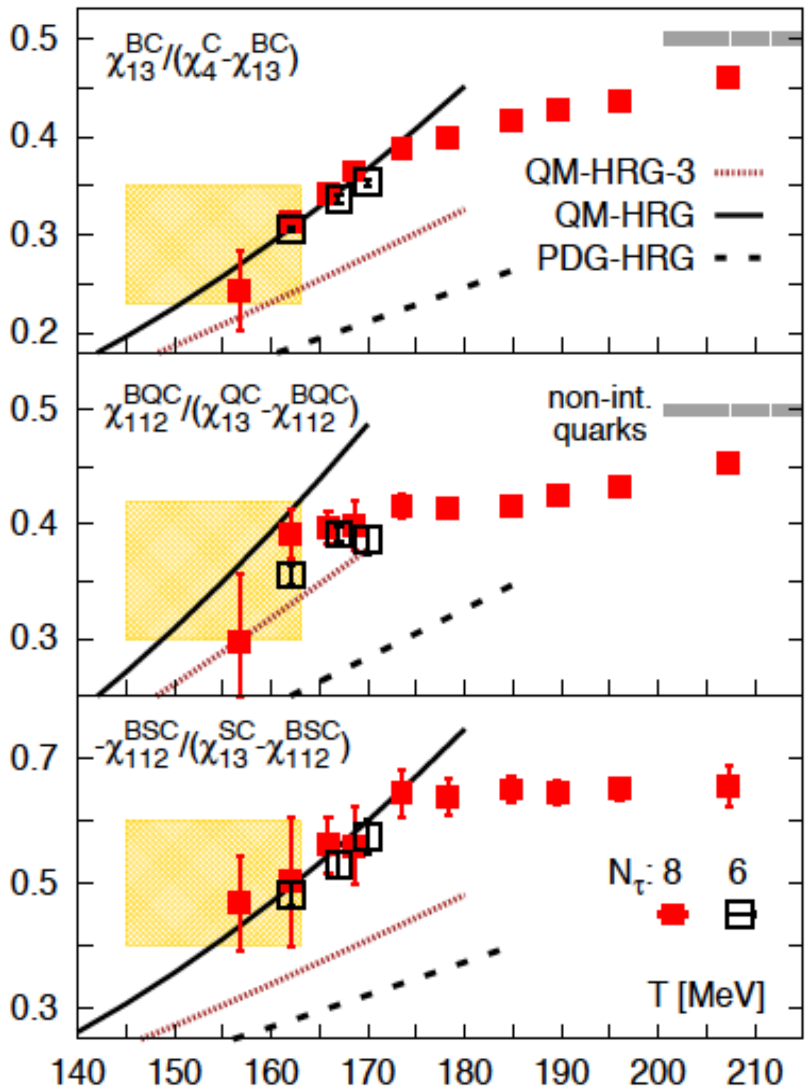


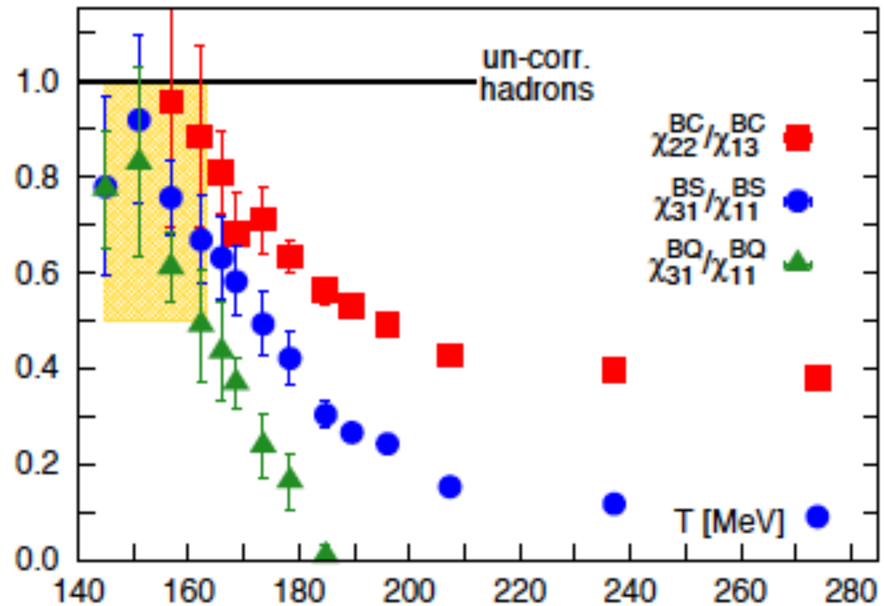


Large change in the amplitude (wave function) of $c\bar{c}$ for *T>200* MeV

Large dependence on the temporal boundary conditions of $c\bar{c}$ correlator for *T>200* MeV

Charm susceptibilities





BNL-BI, arXiv:1404.4043

Susceptibilities involving charm quarks
Can be described in terms of gas of
Charmed hadrons below T_c, but the
Description breaks down above the
crossover region

see S. Sharma for more info

Summary

- Continuum lattice QCD results exist for several quantities and there is an agreement between different lattice collaborations

- The real and imaginary parts of the static energy can be obtained at $T>0$ by analyzing correlation functions of Wilson line in Coulomb gauge :
  1) real part is screened at high T, but is larger than the singlet free energy
  2) the imaginary part is slightly larger compared to HTL result
  3) pNRQCD at strong coupling: $T_{diss}(Y) > 3T_c$

- Temporal meson correlation functions are not very sensitive to the in-medium modification of the spectral functions but the spatial propagators are:
  1) 1S charmonium (J/ψ, η_c) may survive up to $1.4T_c$
  2) All other mesons (φ, D_s, χ_c), melt close to the transition region (confirmed by analysis of charm susceptibilities)