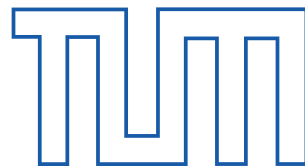# Quarkonium dissociation in a thermal bath

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Heavy particles

Heavy particles

Heavy particles are a window to new physics for they may be sensitive to new fundamental degrees of freedom. Some of these new degrees of freedom may be themselves heavy particles.

Ex. Heavy Majorana neutrinos (talk by S.Biondini, ...).

Heavy particles can also be clean probes of new phenomena emerging in particularly complex environments.

Ex. Heavy quarks and quarkonia in heavy-ion collisions (talk by M.Escobedo, this talk ...).
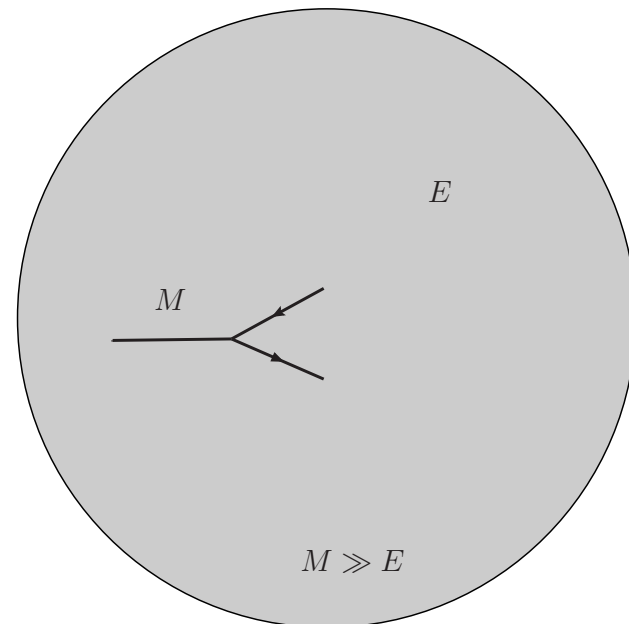
Scales

We call a particle of mass M heavy if

$$M \gg E \text{ (all other energy scales)}$$

This implies that the particle is non-relativistic.
It calls for an EFT treatment at a scale $M \gg \mu \gg E$.

Ex.:

- HQET

- (p)NRQCD, (p)NRQED

- HBET

- Electroweak EFTs for heavy quarks

- ...

Non-relativistic EFTs

The hierarchy $M \gg E$ allows describing the system at a scale $M \gg \mu \gg E$ in terms of a heavy-particle low-energy field H and all other fields that exist at the energy E.

The EFT Lagrangian reads

$$
\begin{aligned}
\mathcal{L} &= H^\dagger i D_0 H + \text{higher dimension operators suppressed in } 1/M \\
&+ \mathcal{L}_{\text{light fields}}
\end{aligned}
$$

- In the heavy-particle sector the Lagrangian is organized as an expansion in $1/M$. Contributions of higher-order operators to physical observables are suppressed by powers of E/M.

- The Lagrangian $\mathcal{L}$ may be computed at $E = 0$.

- The Lagrangian has been written in a reference frame where the heavy particle is at rest up to fluctuations of order E or smaller.

Temperature

A special case is the case of a heavy particle of mass M in a medium characterized by a temperature T such that

$$M \gg T$$

Thermal non-relativistic EFTs

The system is described at a scale $M \gg \mu \gg T$ in terms of the EFT Lagrangian

$$\mathcal{L} = H^\dagger i D_0 H + \text{higher dimension operators suppressed in } 1/M$$
$$+ \mathcal{L}_{\text{light fields}}$$

- In the heavy-particle sector the Lagrangian is organized as an expansion in $1/M$. Contributions of higher-order operators to physical observables are suppressed by powers of T/M.

- The Lagrangian $\mathcal{L}$ may be computed at $T = 0$, i.e. the Wilson coefficients encoding the high-energy modes may be computed in vacuum.

- The Lagrangian has been written in a reference frame where the heavy particle is at rest up to fluctuations of order T or smaller.

Real-time formalism

Temperature is introduced via the partition function.

Sometimes it is useful to work in the real-time formalism.
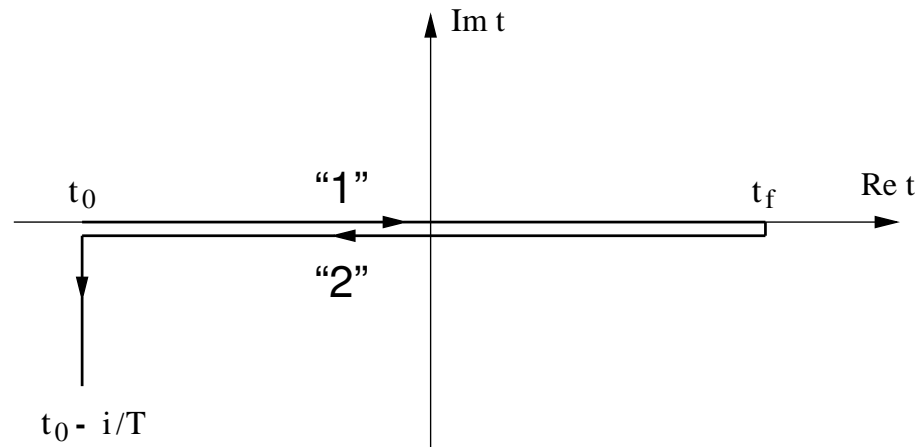
In real time, the degrees of freedom double ("1" and "2"), however, the advantages are

- the framework becomes very close to the one for $T = 0$ EFTs;

- in the heavy-particle sector, the second degrees of freedom, labeled "2", decouple from the physical degrees of freedom, labeled "1".

This usually leads to a simpler treatment with respect to alternative calculations in imaginary time formalism.

Real-time gauge boson propagator

- Gauge boson propagator (in Coulomb gauge):

$$\mathbf{D}_{00}^{(0)}(\vec{k}) = \frac{i}{\vec{k}^2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathbf{D}_{ij}^{(0)}(k) = \left(\delta_{ij} - \frac{k^i k^j}{\vec{k}^2}\right) \left\{ \begin{pmatrix} \dfrac{i}{k^2 + i\epsilon} & \theta(-k^0)\,2\pi\delta(k^2) \\ \theta(k^0)\,2\pi\delta(k^2) & -\dfrac{i}{k^2 - i\epsilon} \end{pmatrix} \right. \\ \left. + 2\pi\delta(k^2)\,n_{\mathrm{B}}(|k^0|) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}$$

where

$$n_{\mathrm{B}}(k^0) = \frac{1}{e^{k^0/T} - 1}$$

Real-time heavy-particle propagator

- The free heavy-particle propagator is proportional to

$$\mathbf{S}^{(0)}(p) = \begin{pmatrix} \dfrac{i}{p^0 + i\epsilon} & 0 \\ 2\pi\delta(p^0) & \dfrac{-i}{p^0 - i\epsilon} \end{pmatrix}$$

Since $[\mathbf{S}^{(0)}(p)]_{12} = 0$, the static quark fields labeled "2" never enter in any physical amplitude, i.e. any amplitude that has the physical fields, labeled "1", as initial and final states.

These properties hold also for interacting heavy particle(s): interactions do not change the nature ("1" or "2") of the interacting fields.

○ Brambilla Ghiglieri Vairo Petreczky PRD 78 (2008) 014017

Thermal widths at weak coupling

We will consider heavy particles interacting weakly with a weakly coupled plasma.

In particular, we will consider a quarkonium formed in heavy-ion collisions of sufficiently high energy that may be described as a Coulombic bound state interacting with a weakly coupled quark-gluon plasma.
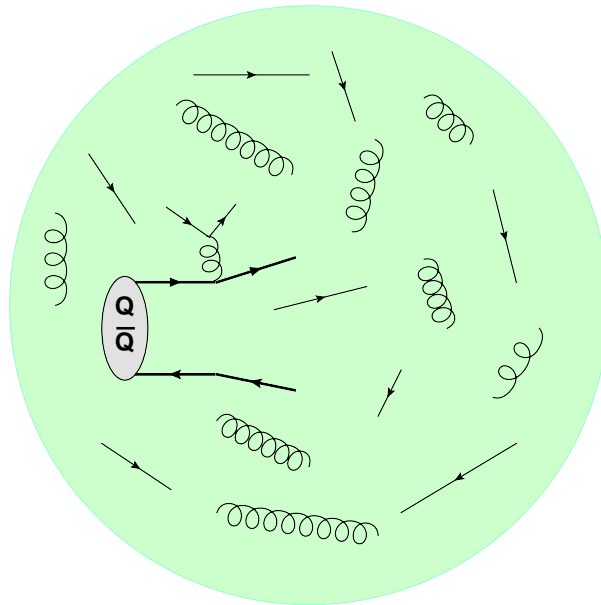
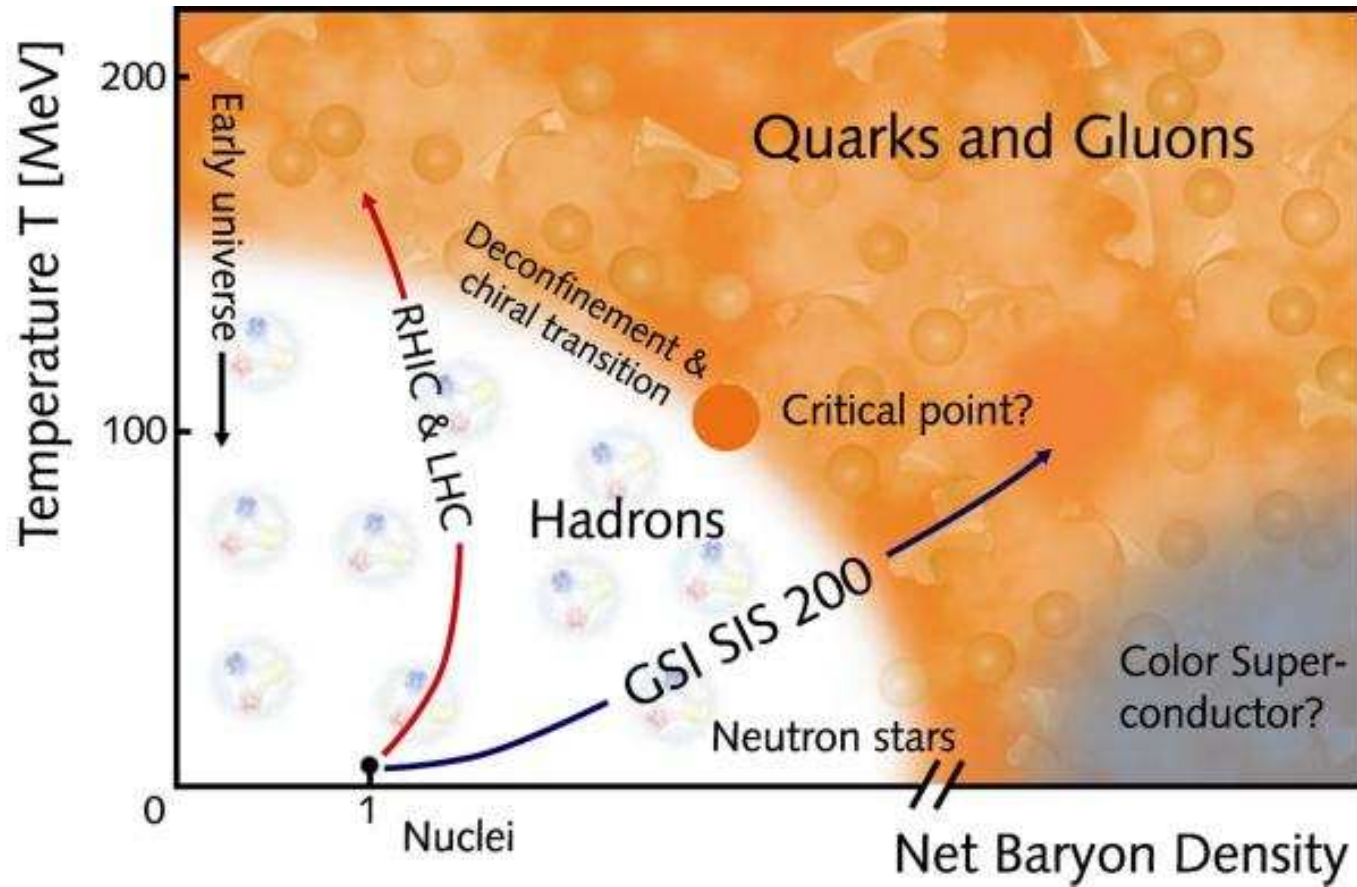We will compute corrections to the width induced by the medium: thermal width, Γ.

Heavy quarkonia

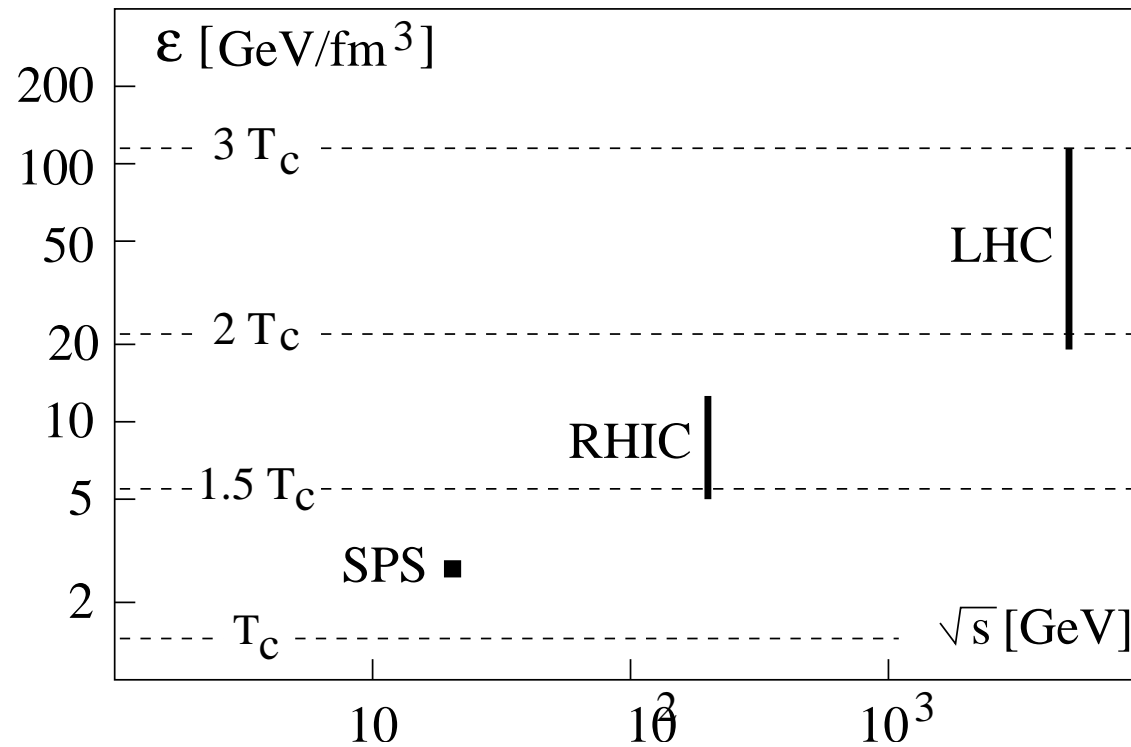# Colour deconfinement

Karsch 2002
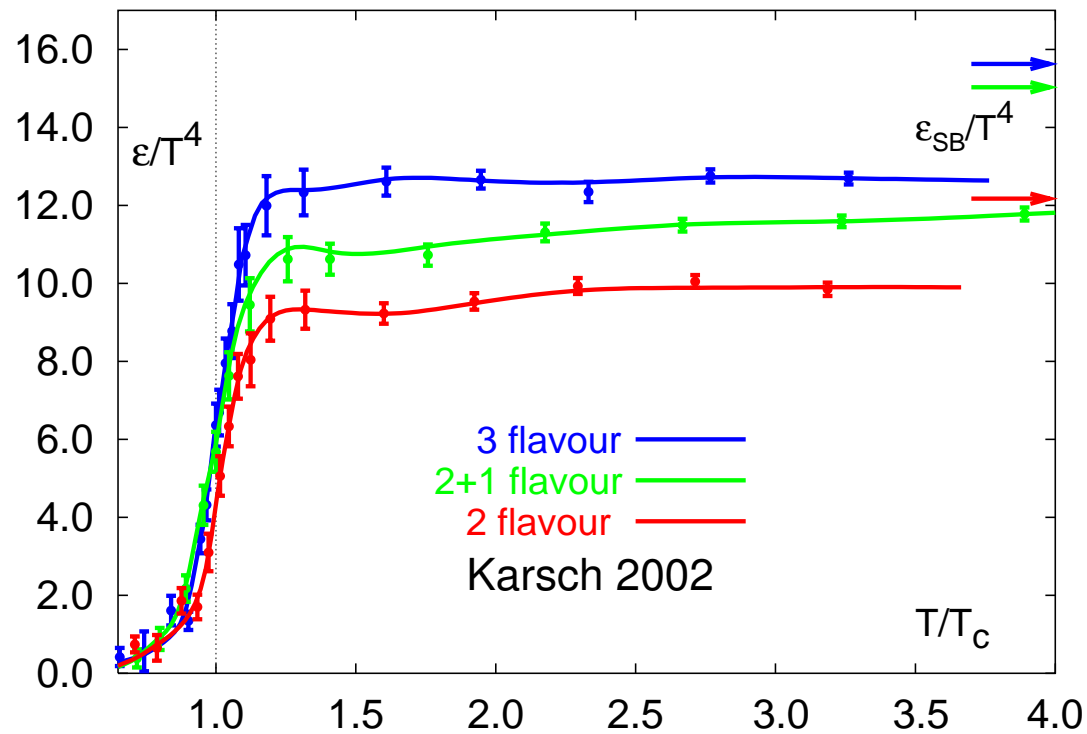
Transition from hadronic matter to a plasma of deconfined quarks and gluons happening at some critical temperature T_c (≈ 170 MeV) as studied in finite temperature lattice QCD.
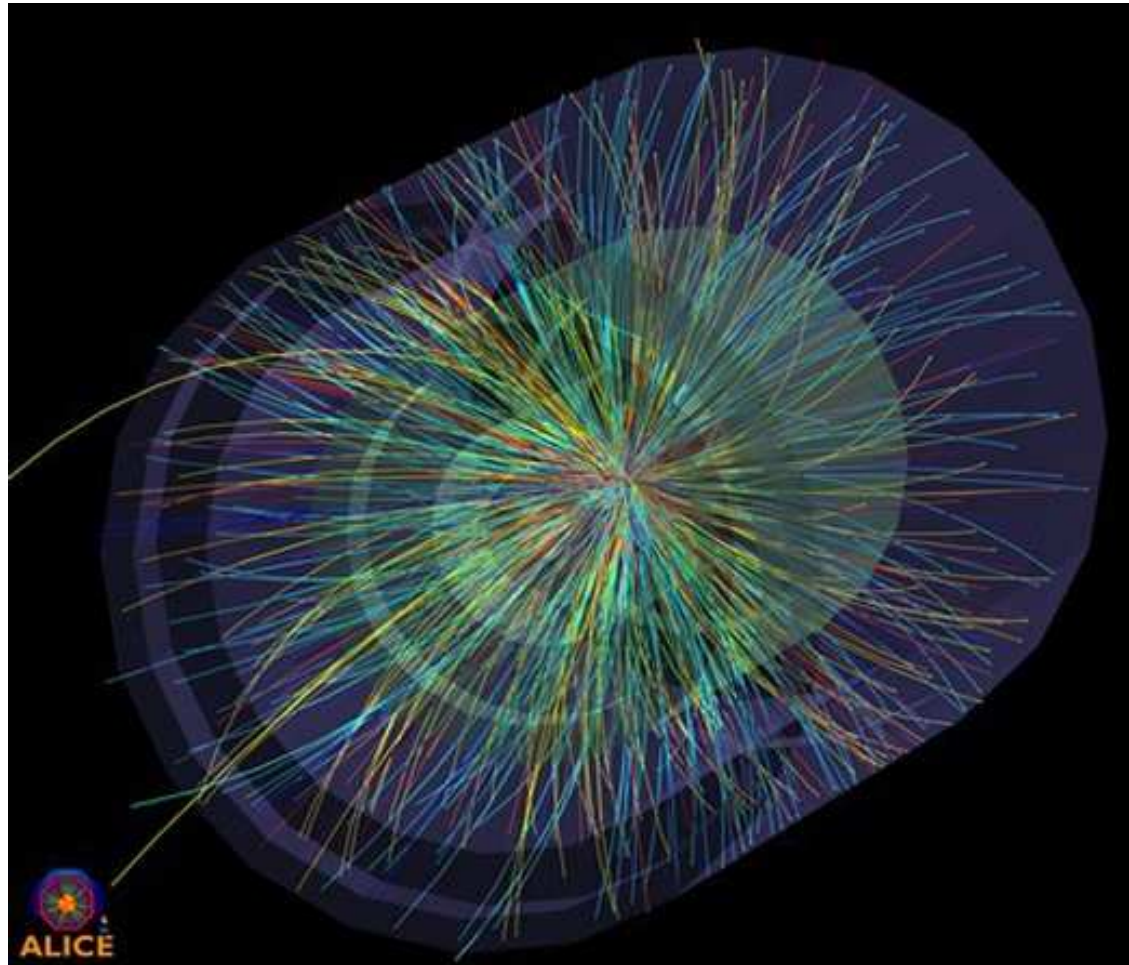
Heavy-ion experiments

High energy densities and temperatures $> T_c$ as explored at the heavy-ion experiments at RHIC and LHC.

Heavy-ion experiments

We need probes to identify the state of matter (temperature, ...) which is formed.

Quarkonium as a quark-gluon plasma probe

In 1986, Matsui and Satz suggest quarkonium as an ideal quark-gluon plasma probe.

- Heavy quarks are formed early in heavy-ion collisions: $1/M \sim 0.1$ fm $\ll 1$ fm.

- Heavy quarkonium formation will be sensitive to the medium.

- The dilepton signal makes the quarkonium a clean experimental probe.
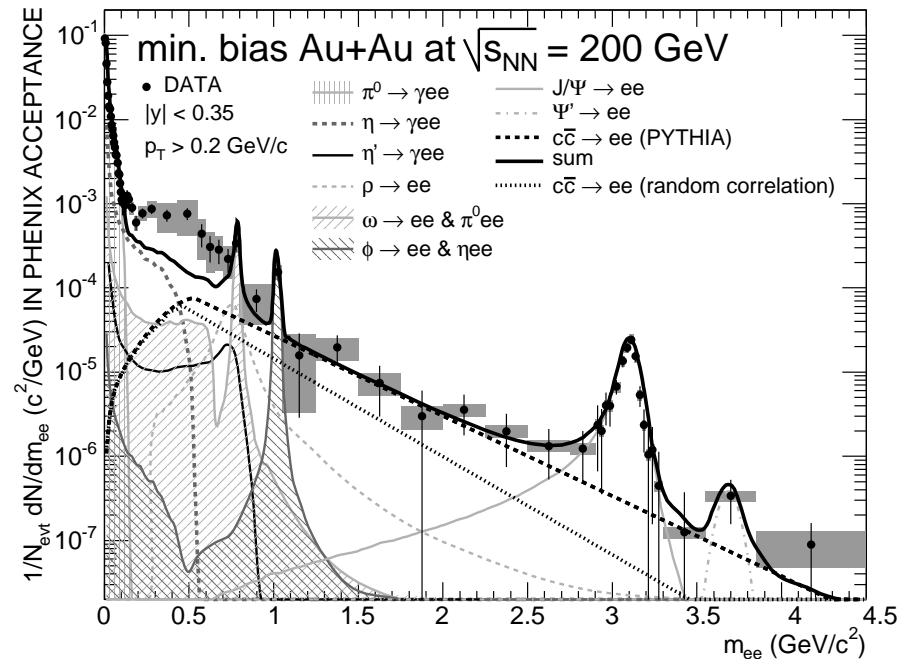
Scales

Quarkonium being a composite system is characterized by several energy scales, these in turn may be sensitive to thermodynamical scales smaller than the temperature:

- the scales of a non-relativistic bound state
  (v is the relative heavy-quark velocity; $v \sim \alpha_{\rm s}$ for a Coulombic bound state):
  M (mass),
  Mv (momentum transfer, inverse distance),
  Mv^2 (kinetic energy, binding energy, potential V), ...

- the thermodynamical scales:
  πT (temperature),
  m_D (Debye mass, i.e. screening of the chromoelectric interactions), ...

The non-relativistic scales are hierarchically ordered: $M \gg Mv \gg Mv^2$
We assume this to be also the case for the thermodynamical scales: $\pi T \gg m_D$
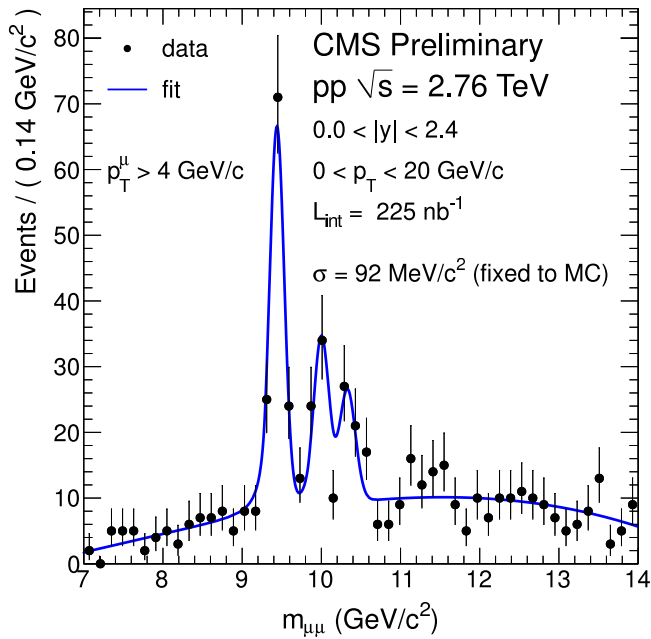
$\Upsilon(1S)$ scales

A weakly coupled quarkonium possibly produced in a weakly coupled plasma is the bottomonium ground state $\Upsilon(1S)$ produced in heavy-ion experiments at the LHC:

$$M_b \approx 5 \text{ GeV} > M_b \alpha_{\text{s}} \approx 1.5 \text{ GeV} > \pi T \approx 1 \text{ GeV} > M_b \alpha_{\text{s}}^2 \approx 0.5 \text{ GeV} \sim m_D \gtrsim \Lambda_{\text{QCD}}$$
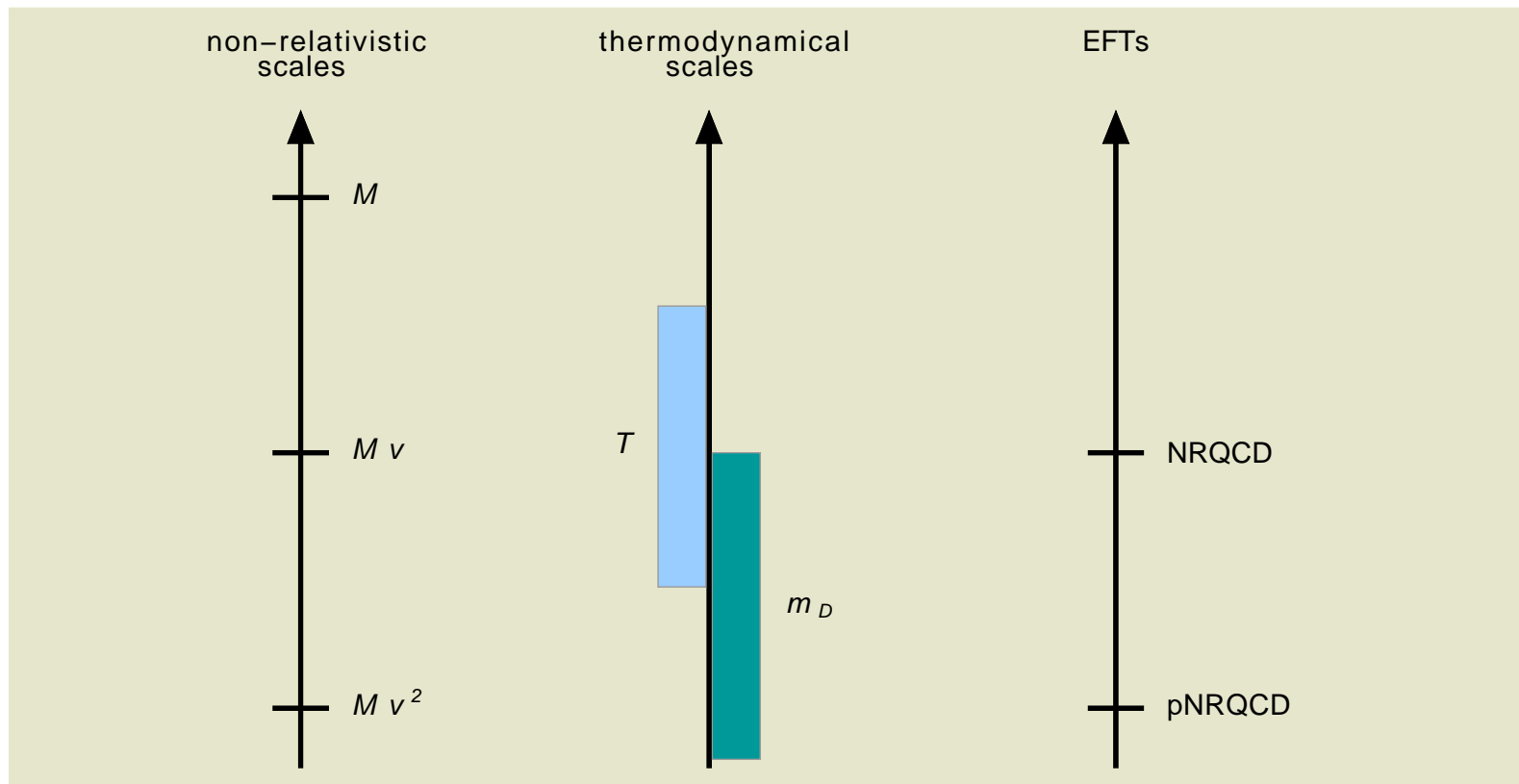
○ Brambilla Escobedo Ghiglieri Soto Vairo JHEP 1009 (2010) 038
  Vairo AIP CP 1317 (2011) 241

Υ suppression at CMS
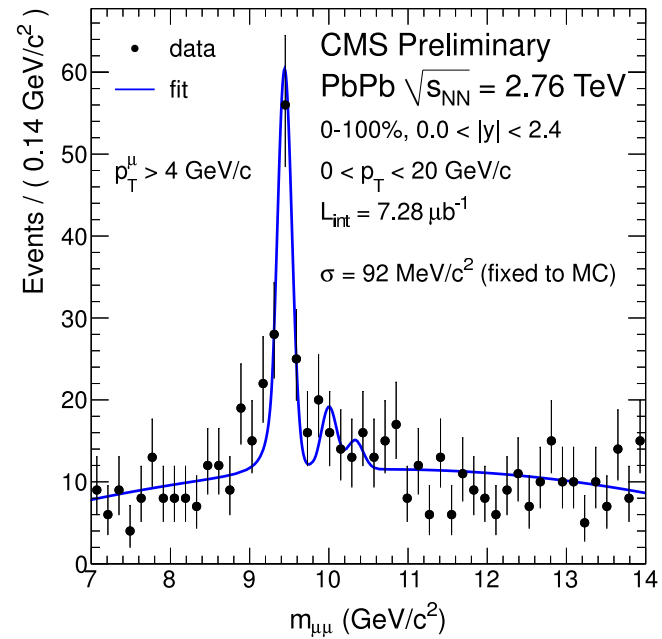
Non-relativistic EFTs of QCD

The existence of a hierarchy of energy scales calls for a description of the system (quarkonium at rest in a thermal bath) in terms of a hierarchy of EFTs.

For larger temperatures the quarkonium does not form.

NRQCD

NRQCD is obtained by integrating out modes associated with the scale M and possibly with thermal scales larger than Mv.

- The Lagrangian is organized as an expansion in $1/M$:

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2M} + \dots \right) \psi + \chi^\dagger \left(iD_0 - \frac{\mathbf{D}^2}{2M} + \dots \right) \chi + \dots$$
$$+ \mathcal{L}_{\text{light}}$$

$\psi\,(\chi)$ is the field that annihilates (creates) the (anti)fermion.

○ Caswell Lepage PLB 167 (1986) 437
  Bodwin Braaten Lepage PRD 51 (1995) 1125

pNRQCD

pNRQCD is obtained by integrating out modes associated with the scale Mv and possibly with thermal scales larger than Mv^2.

- The degrees of freedom of pNRQCD are quark-antiquark states (color singlet S, color octet O), low energy gluons and light quarks propagating in the medium.

- The Lagrangian is organized as an expansion in $1/M$ and r:

$$
\mathcal{L} = \int d^3r \, \mathrm{Tr} \left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{M} - V_s + \dots \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{M} - V_o + \dots \right) \mathrm{O} \right\}
$$

$$
+ \mathrm{Tr} \left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \, \mathrm{S} + \mathrm{H.c.} \right\} + \frac{1}{2} \mathrm{Tr} \left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \, \mathrm{O} + \mathrm{c.c.} \right\} + \cdots
$$

$$
+ \mathcal{L}_{\mathrm{light}}
$$

- At leading order in r, the singlet S satisfies a Schrödinger equation. The explicit form of the potential depends on the version of pNRQCD.

○ Pineda Soto NP PS 64 (1998) 428

Brambilla Pineda Soto Vairo NPB 566 (2000) 275, RMP 77 (2005) 1423

pNRQCD

- Feynman rules:

$$\underline{} \quad = \theta(t)\, e^{-itH_s} \qquad\qquad ===== \quad = \theta(t)\, e^{-itH_o}\left(e^{-i\int dt\, A^{\mathrm{adj}}}\right)$$

$$= \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\,\mathrm{S} \qquad\qquad = \mathrm{O}^\dagger\{\mathbf{r} \cdot g\mathbf{E},\, \mathrm{O}\}$$

Dissociation mechanisms at LO

A key quantity for describing the observed quarkonium dilepton signal suppression is the quarkonium thermal dissociation width.

Two distinct dissociation mechanisms may be identified at leading order:

- gluodissociation,
  which is the dominant mechanism for $Mv^2 \gg m_D$;

- dissociation by inelastic parton scattering,
  which is the dominant mechanism for $Mv^2 \ll m_D$.

Beyond leading order the two mechanisms are intertwined and distinguishing between them becomes unphysical, whereas the physical quantity is the total decay width.
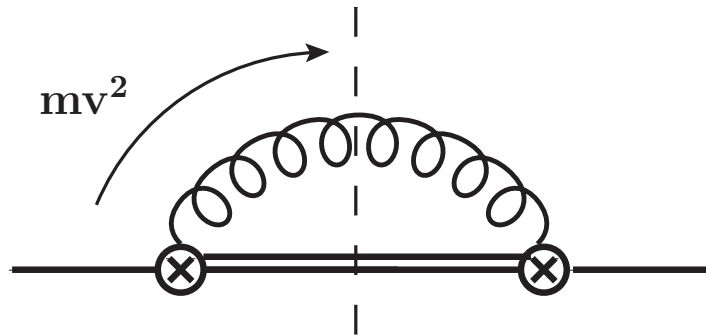
Gluodissociation

Gluodissociation is the dissociation of quarkonium by absorption of a gluon from the medium.
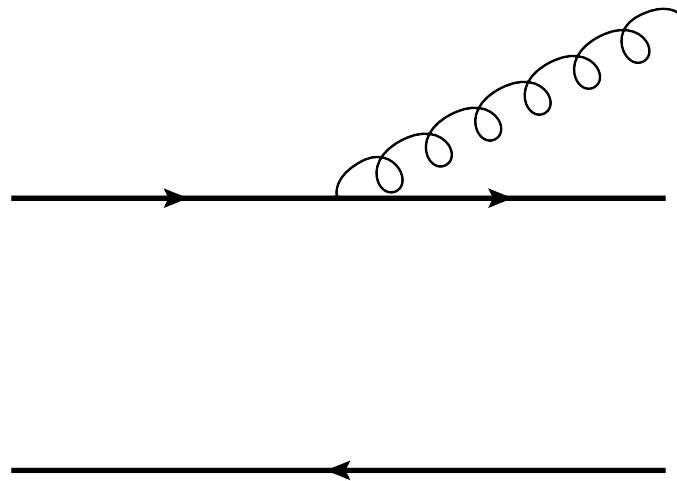
- The exchanged gluon is lightlike or timelike.

- The process happens when the gluon has an energy of order Mv^2.

○ Kharzeev Satz PLB 334 (1994) 155
  Xu Kharzeev Satz Wang PRC 53 (1996) 3051

Gluodissociation

From the optical theorem, the gluodissociation width follows from cutting the gluon propagator in the following pNRQCD diagram
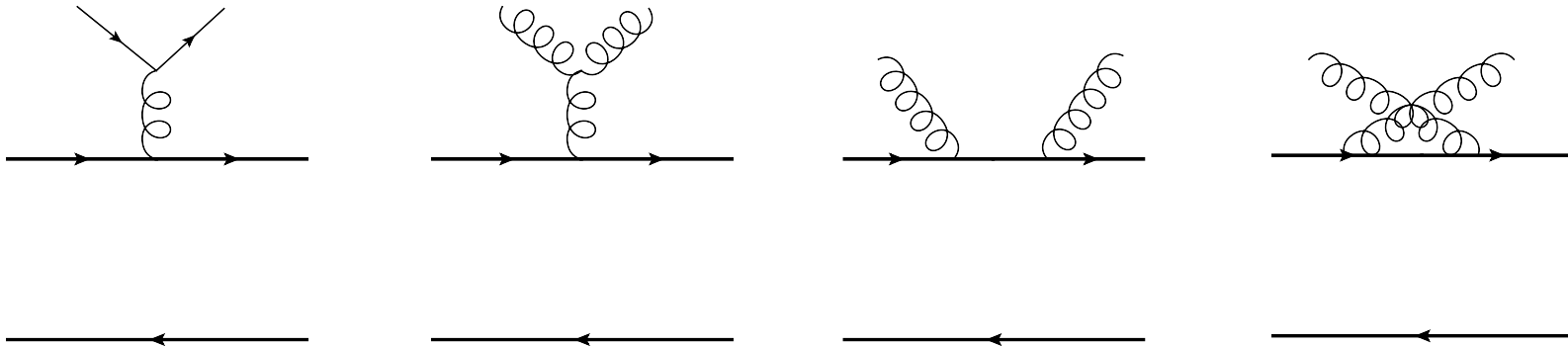

For a quarkonium at rest with respect to the medium, the width has the form

$$\Gamma_{nl} = \int_{q_{\min}} \frac{d^3 q}{(2\pi)^3}\, n_{\mathrm{B}}(q)\, \sigma_{\mathrm{gluo}}^{nl}(q)\,.$$

- $\sigma_{\mathrm{gluo}}^{nl}$ is the in-vacuum cross section $(Q\overline{Q})_{nl} + g \to Q + \overline{Q}$.
- Gluodissociation is also known as singlet-to-octet break up.

○ Brambilla Ghiglieri Vairo Petreczky PRD 78 (2008) 014017

Dissociation by inelastic parton scattering

Dissociation by inelastic parton scattering is the dissociation of quarkonium by scattering with gluons and light-quarks in the medium.
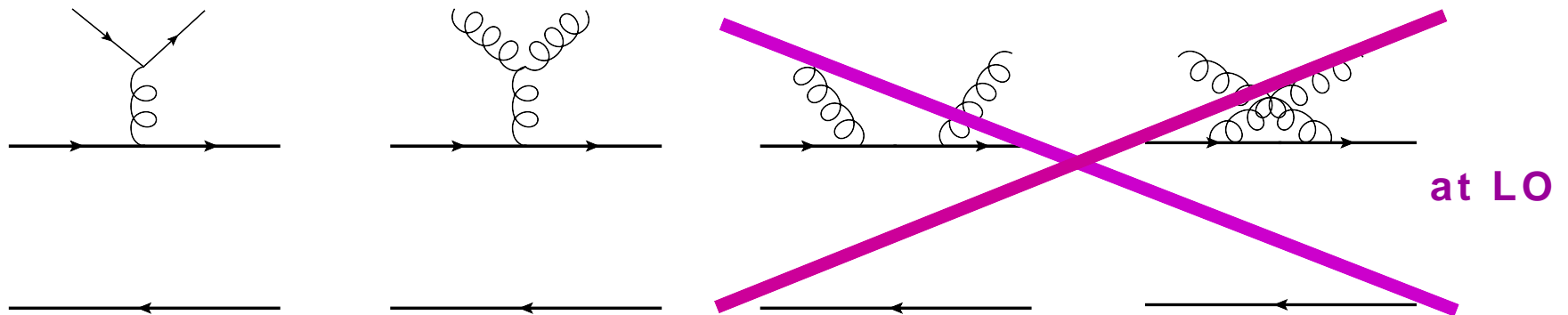

○ Grandchamp Rapp PLB 523 (2001) 60, NPA 709 (2002) 415

- The exchanged gluon is spacelike.

- External thermal gluons are transverse.

- In the NRQCD power counting, each external transverse gluon is suppressed by T/M.

Dissociation by inelastic parton scattering

Dissociation by inelastic parton scattering is the dissociation of quarkonium by scattering with gluons and light-quarks in the medium.


at LO

○ Grandchamp Rapp PLB 523 (2001) 60, NPA 709 (2002) 415

- The exchanged gluon is spacelike.

- External thermal gluons are transverse.

- In the NRQCD power counting, each external transverse gluon is suppressed by T/M.

Dissociation by inelastic parton scattering

From the optical theorem, the thermal width follows from cutting the gluon self-energy in the following NRQCD diagrams (momentum of the gluon $\gtrsim Mv$)
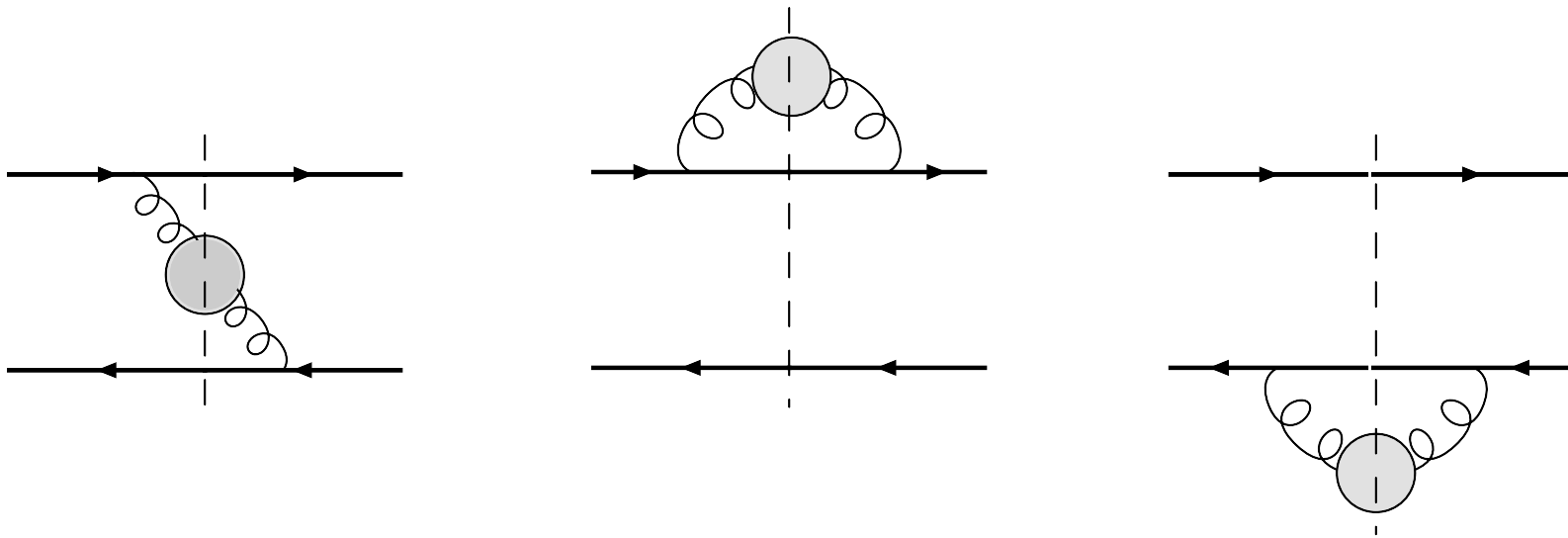
and/or pNRQCD diagram (momentum of the gluon $\ll Mv$)
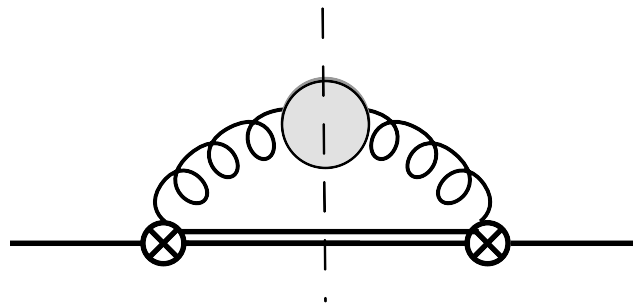
- Dissociation by inelastic parton scattering is also known as Landau damping.

Dissociation by inelastic parton scattering

For a quarkonium at rest with respect to the medium, the thermal width has the form

$$\Gamma_{nl} = \sum_p \int_{q_{\min}} \frac{d^3 q}{(2\pi)^3} f_p(q) \left[1 \pm f_p(q)\right] \sigma_p^{nl}(q)$$

where the sum runs over the different incoming light partons and $f_g = n_{\mathrm{B}}$ or $f_q = n_{\mathrm{F}}$.

- σ_p^{nl} is the in-medium cross section $(Q\overline{Q})_{nl} + p \to Q + \overline{Q} + p$.
- The convolution formula correctly accounts for Pauli blocking in the fermionic case (minus sign).
- The formula differs from the gluodissociation formula.
- The formula differs from the one used so far in the literature, which has been inspired by the gluodissociation formula.
  - Grandchamp Rapp PLB 523 (2001)
    Park Kim Song Lee Wong PRC 76 (2007) 044907, ...

Dissociation temperature

- At weak coupling quarkonium dissociates at a temperature such that $\Gamma \sim E_{\text{binding}}$:

$$\pi T_{\text{dissociation}} \sim mg^{4/3}$$

which in the $\Upsilon(1S)$ case is about 450 MeV.
(Note that $\pi T \approx 1$ GeV is below the dissociation temperature.)

- The interaction is screened when $\langle 1/r \rangle \sim m_D$, hence

$$\pi T_{\text{screening}} \sim mg \gg \pi T_{\text{dissociation}}$$

○ Escobedo Soto PRA 78 (2008) 032520, 82 (2010) 042506
  Laine arXiv:0810.1112

The temperature region $T \gg mv \gg m_D$: $1S$ dissociation by parton scattering

$$\sigma_p^{1S} = \sigma_{cp} f(m_D a_0)$$

$$\sigma_{cq} \equiv 8\pi C_F n_f \, \alpha_s^2 \, a_0^2, \quad \sigma_{cg} \equiv 8\pi C_F N_c \, \alpha_s^2 \, a_0^2$$

$$f(m_D a_0) \equiv \frac{2}{(m_D a_0)^2} \left[1 - 4 \frac{(m_D a_0)^4 - 16 + 8(m_D a_0)^2 \ln\left(4/(m_D a_0)^2\right)}{((m_D a_0)^2 - 4)^3} \right]$$

- $\langle \mathbf{r} | 1S \rangle = 1/(\sqrt{\pi} a_0^{3/2}) \exp(-r/a_0)$, where $a_0 = 2/(m C_F \alpha_s)$.

- The cross section for inelastic parton scattering is not related, even at leading order, with a zero temperature process. The underlying reason is the infrared sensitivity of the cross section at the momentum scale mv. This IR sensitivity is cured by the HTL resummation at the scale m_D and signaled by $\ln(m_D a_0)$.

- The cross section is momentum independent.

Quasi-free approximation

The quasi-free approximation amounts at replacing σ_p^{nl} by $2\,\sigma_p^Q$; σ_p^Q is the free in-vacuum cross section $p + Q \to p + Q$ (with thermal masses for IR regularization).

- The quasi-free approximation amounts at neglecting interference terms between the different heavy-quark lines in the amplitude square. Interference terms are the ones sensitive to the bound state. This corresponds in approximating

$$f(m_D a_0) \approx \frac{2}{(m_D a_0)^2}$$

- The approximation holds only for $m_D \gg 1/a_0$.

- For temperatures where the approximation holds, quarkonium is dissociated. Otherwise, the approximation is largely violated by bound-state effects. E.g. for $m_D \ll 1/a_0$

$$f(m_D a_0) \approx \frac{2}{(m_D a_0)^2} \left[\cancel{1} - \cancel{1} + (m_D a_0)^2 \left(-\frac{3}{4} + \ln(2/(m_D a_0)) \right) + \ldots \right]$$
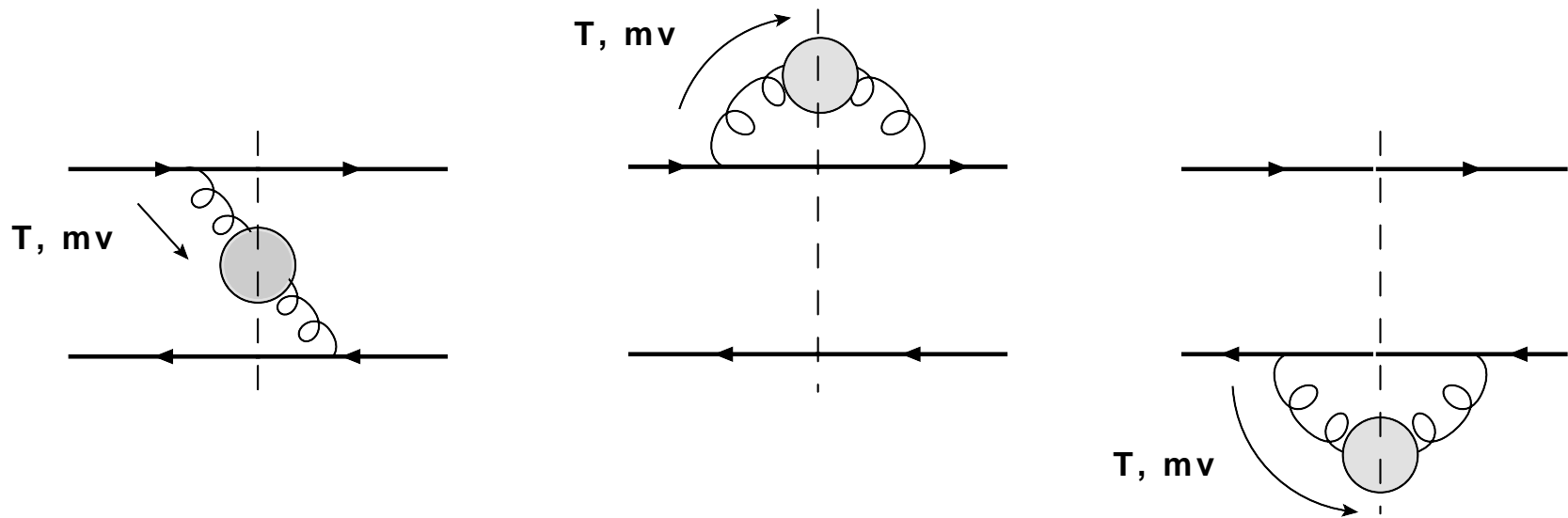
Quasi-free approximation vs full result

In a weak-coupling framework, the quasi-free approximation is not justified for the whole range of temperatures where a quarkonium can exist.
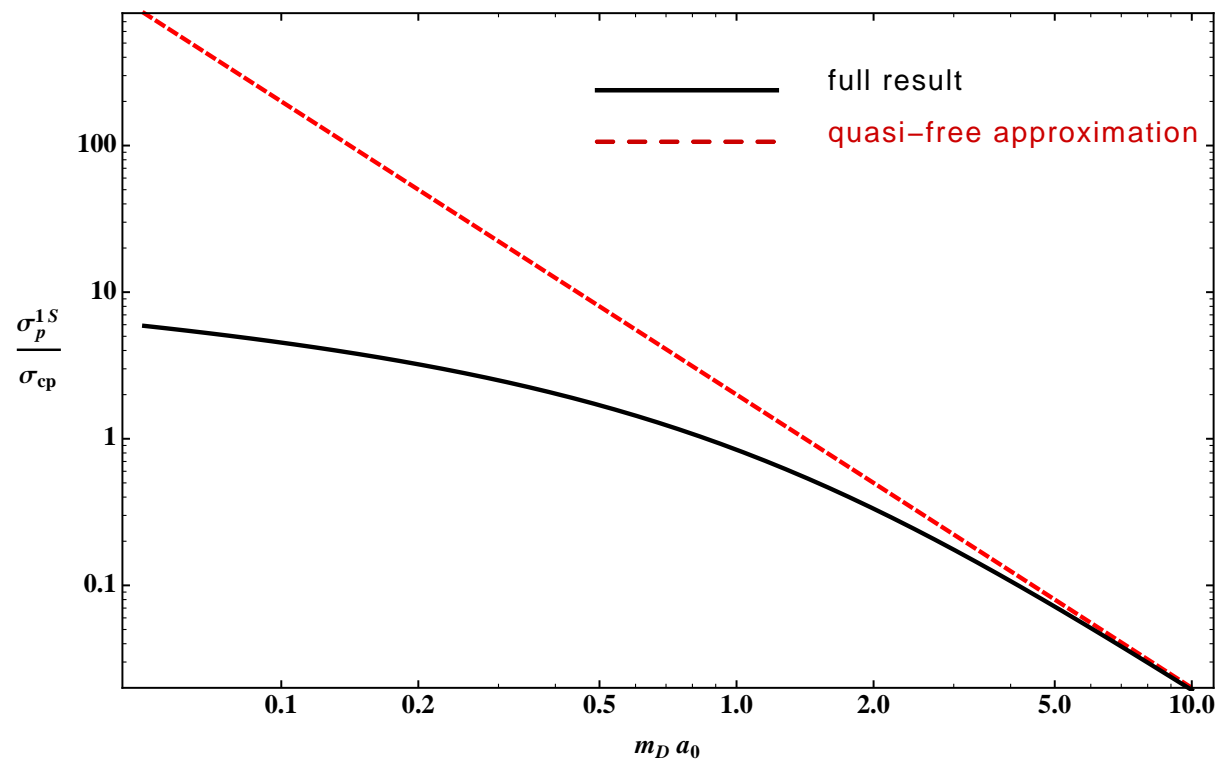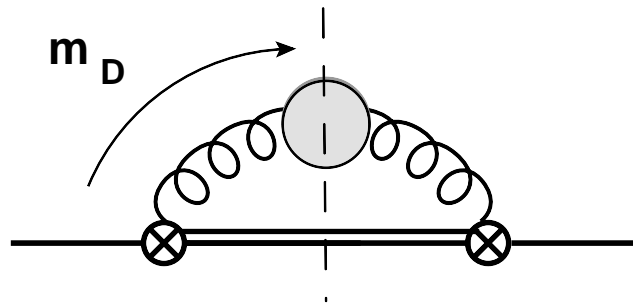
The temperature region $T \sim mv \gg m_D$

The gluon self-energy contributes through the NRQCD diagrams

and the pNRQCD diagram

The temperature region $T \sim mv \gg m_D$: $1S$ dissociation by parton scattering

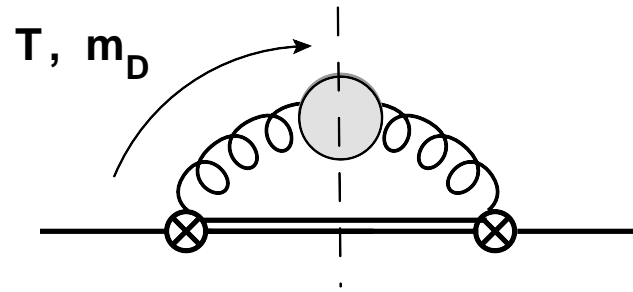$$\sigma_p^{1S} = \sigma_{cp} h_p(m_D a_0, q a_0)$$

$$h_q(m_D a_0, q a_0) \equiv -\ln\left(\frac{(m_D a_0)^2}{4}\right) - \frac{3}{2} + \ln\left(\frac{(q a_0)^2}{1 + (q a_0)^2}\right)$$

$$- \frac{1}{2(q a_0)^2} \ln(1 + (q a_0)^2)$$

$$h_g(m_D a_0, q a_0) \equiv -\ln\left(\frac{(m_D a_0)^2}{4}\right) - \frac{3}{2} + \ln\left(\frac{(q a_0)^2}{1 + (q a_0)^2}\right)$$

$$+ \frac{1}{2(1 + (q a_0)^2)} - \frac{1}{(q a_0)^2} \ln(1 + (q a_0)^2)$$

- The cross section is momentum dependent.

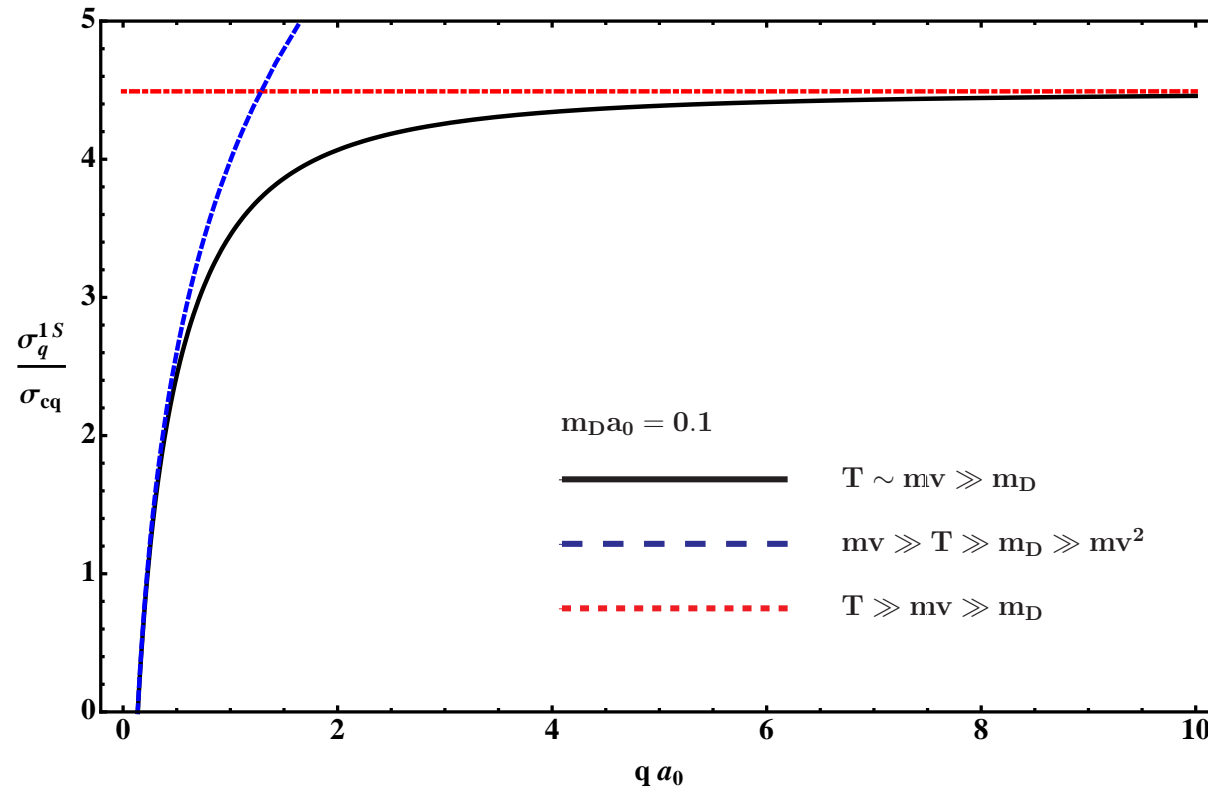## The temperature region $mv \gg T \gg m_D \gg mv^2$: $1S$ dissociation by parton scattering

The gluon self-energy contributes through the pNRQCD diagram

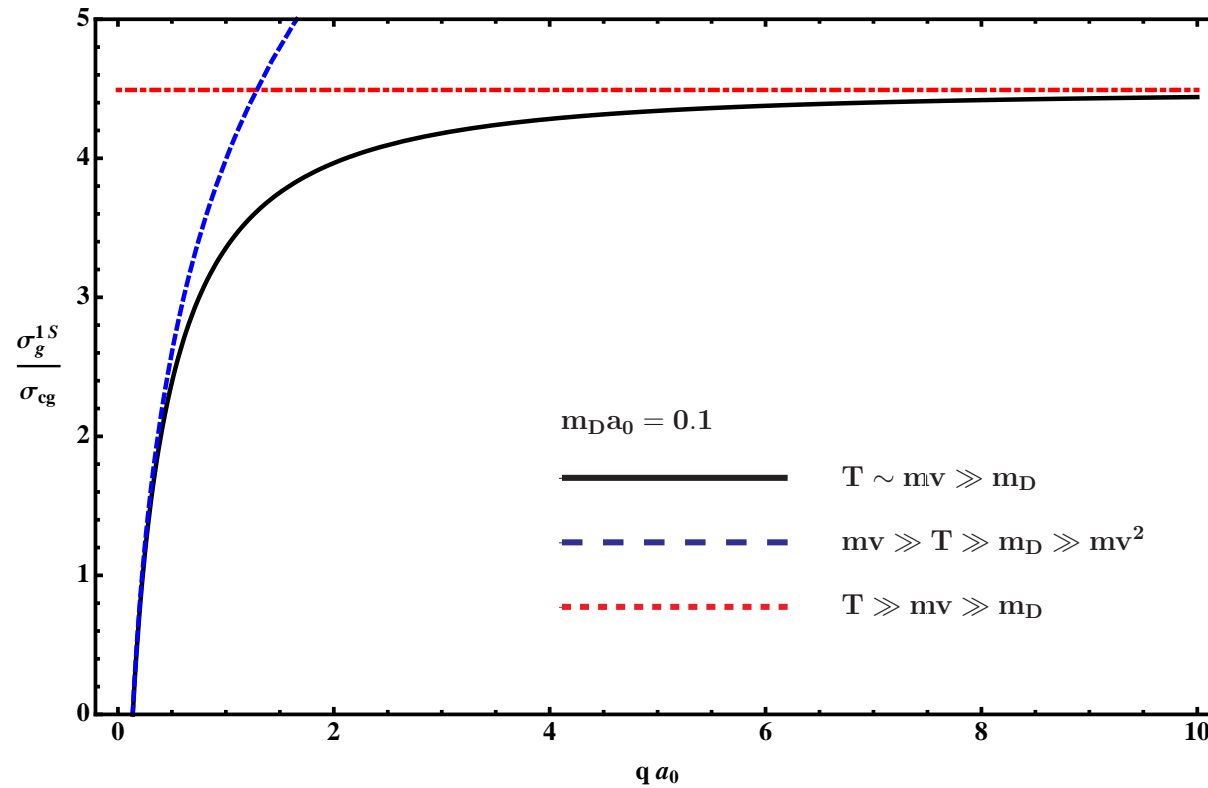leading to dissociation cross sections for $1S$ Coulombic states:

$$\sigma_p^{1S}(q) = \sigma_{cp}\left[\ln\left(\frac{4q^2}{m_D^2}\right) - 2\right]$$

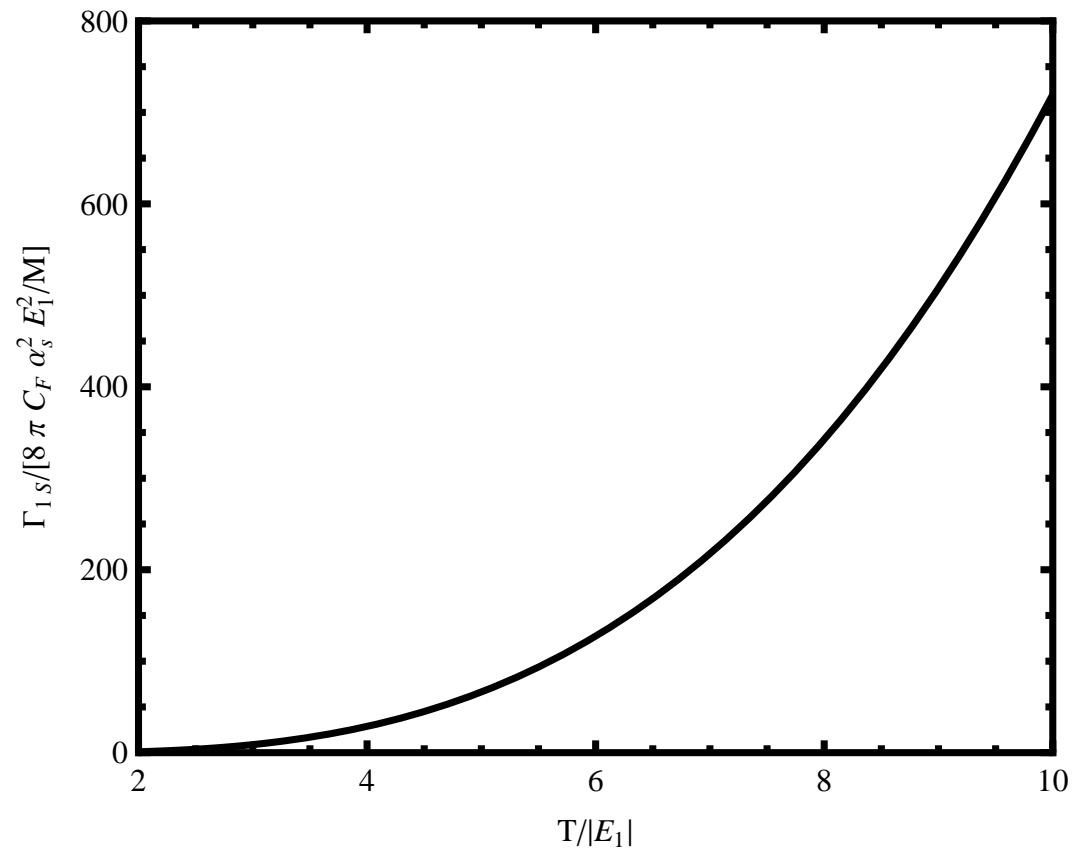# Dissociation by quark inelastic scattering

○ Brambilla Escobedo Ghiglieri Vairo JHEP 05 (2013) 130

Dissociation by gluon inelastic scattering

○ Brambilla Escobedo Ghiglieri Vairo JHEP 05 (2013) 130
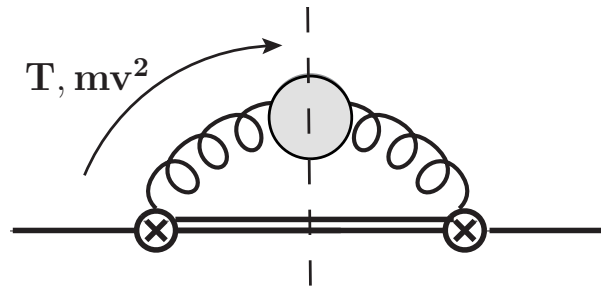
Dissociation by parton inelastic scattering: width

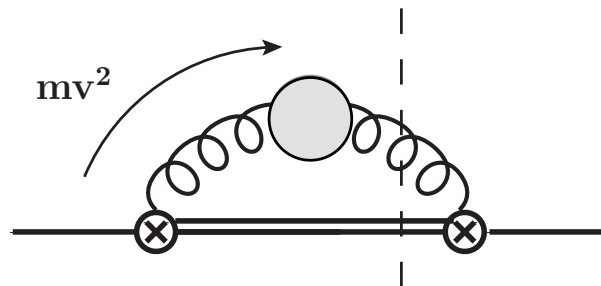$m_D a_0 = 0.5$

$|E_1|/m_D = 0.5$

$n_f = 3$

○ Vairo arXiv:1401.3204

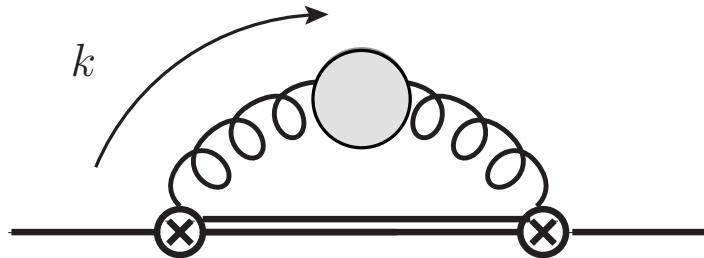The gluon self-energy gives rise to a quarkonium width through:

which contributes to dissociation by inelastic parton scattering, and through

which contributes to gluodissociation.

Integrating out mv^2

The relevant diagram is

where the loop momentum region is $k_0 \sim mv^2$ and $k \sim mv^2$.

- Gluons are HTL gluons.

- Since $k \sim mv^2 \gg m_D$, the HTL propagators can be expanded in $m_D/mv^2 \ll 1$.

Integrating out mv^2: momentum regions

In the loop with transverse gluons, this type of integral appears

$$\int \frac{d^{D-1}k}{(2\pi)^{D-1}} \int_0^\infty \frac{dk_0}{2\pi} \frac{1}{k_0^2 - k^2 - m_D^2 + i\eta} \left(\frac{1}{E - H_o - k_0 + i\eta} + \frac{1}{E - H_o + k_0 + i\eta} \right)$$

which exhibits two momentum regions for $E \sim mv^2$

- off-shell region: $k_0 - k \sim E$, $k_0 \sim E$, $k \sim E$;

- collinear region: $k_0 - k \sim m_D^2/E$, $k_0 \sim E$, $k \sim E$.

In our energy scale hierarchy, the collinear scale is $mv^2 \gg m_D^2/E \gg mv^3$, i.e. it is smaller than m_D by a factor of $m_D/mv^2 \ll 1$ and still larger than the non-perturbative magnetic mass, which is of order $g^2 T$, by a factor $T/mv^2 \gg 1$.
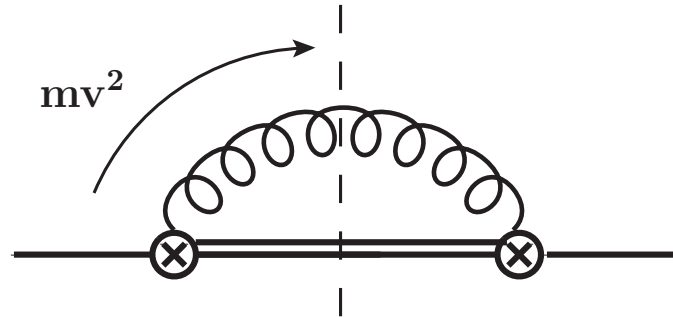
The temperature region $mv \gg T \gg mv^2 \gg m_D$: $1S$ dissociation by parton scattering

The dissociation cross section by parton scattering for $1S$ Coulombic states is

$$\sigma_p^{1S}(q) = \sigma_{cp} \left[\ln \left(\frac{4q^2}{m_D^2} \right) + \ln 2 - 2 \right]$$

The corresponding parton-scattering decay width is of order $\alpha_{\rm s} T \times (m_D/mv)^2$.

The temperature region $mv \gg T \gg mv^2 \gg m_D$: $1S$ gluodissociation at LO

The LO gluodissociation cross section for $1S$ Coulombic states is

$$\sigma_{\text{gluo LO}}^{1S}(q) = \frac{\alpha_s C_F}{3} 2^{10} \pi^2 \rho(\rho+2)^2 \frac{E_1^4}{M q^5} \left(t(q)^2 + \rho^2\right) \frac{\exp\left(\frac{4\rho}{t(q)} \arctan\left(t(q)\right)\right)}{e^{\frac{2\pi\rho}{t(q)}} - 1}$$

where $\rho \equiv 1/(N_c^2 - 1)$, $t(q) \equiv \sqrt{q/|E_1| - 1}$ and $E_1 = -M C_F^2 \alpha_s^2/4$.

○ Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116
  Brezinski Wolschin PLB 707 (2012) 534

Bhanot–Peskin approximation

In the large N_c limit:

$$\sigma_{\text{gluo LO}}^{1S}(q) \xrightarrow[N_c \to \infty]{} 16 \frac{2^9 \pi \alpha_{\text{s}}}{9} \frac{|E_1|^{5/2}}{m} \frac{(q + E_1)^{3/2}}{q^5} = 16\, \sigma_{\text{BP}}^{1S}(q)$$

$$\Gamma_{1S\,\text{LO}} \xrightarrow[N_c \to \infty]{} \int_{q \geq |E_1|} \frac{d^3 q}{(2\pi)^3} n_{\text{B}}(q)\, 16\, \sigma_{\text{BP}}^{1S}(q) = \Gamma_{1S,\text{BP}}$$

The Bhanot–Peskin approximation corresponds to neglecting final state interactions, i.e. the rescattering of a $Q\overline{Q}$ pair in a color octet configuration (recall $V_o = 1/(2N_c) \times \alpha_{\text{s}}/r$).

○ Peskin NPB 156 (1979) 365, Bhanot Peskin NPB 156 (1979) 391

Gluodissociation width vs Bhanot–Peskin width
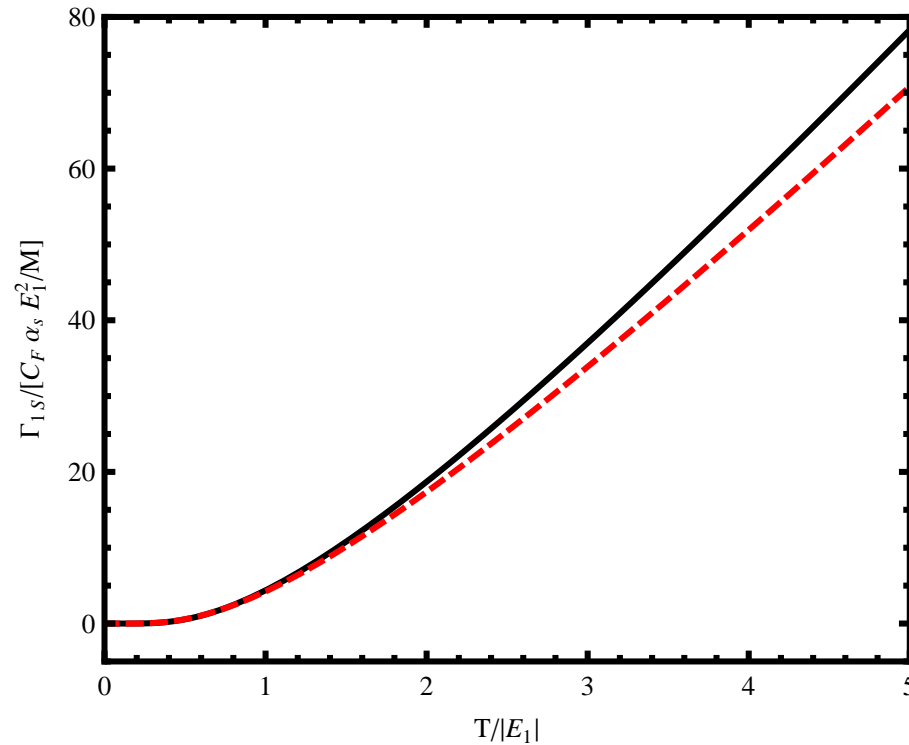
The gluodissociation decay width is of order $\alpha_{\rm s} T \times (mv^2/mv)^2$, i.e., by a factor $(mv^2/m_D)^2$ larger than the dissociation width by inelastic parton scattering. Gluodissociation is the dominant process.

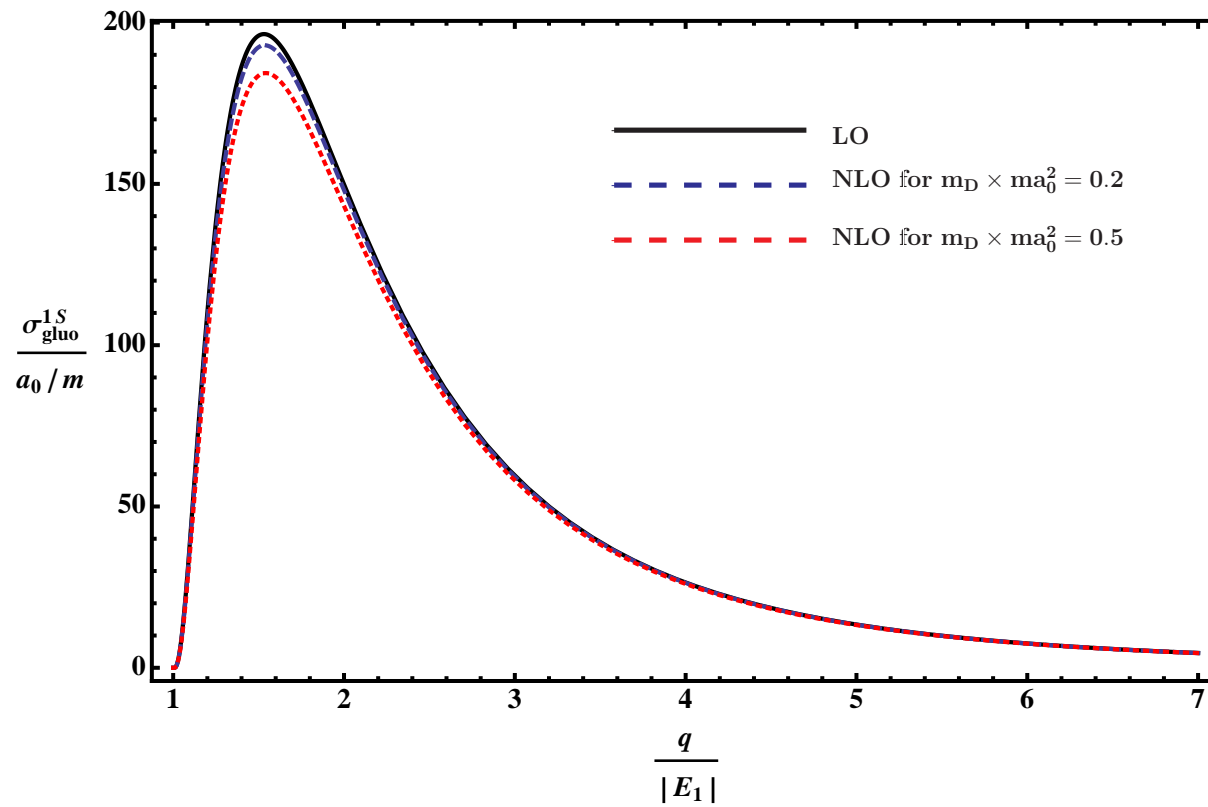○ Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116

The temperature region $mv \gg T \gg mv^2 \gg m_D$: gluodissociation at NLO
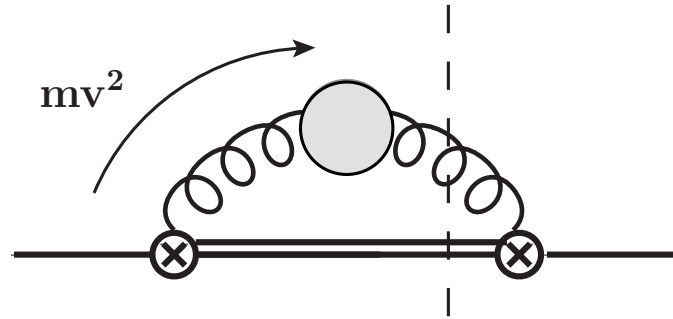
The NLO gluodissociation cross section for $1S$ Coulombic states is

$$\sigma_{\text{gluo}}^{nl}(q) = Z(q/m_D) \, \sigma_{\text{gluo LO}}^{nl}(q)$$

$$Z(q/m_D) = 1 - \frac{m_D^2}{4q^2} \left[\ln(8q^2/m_D^2) - 2 \right]$$

$1S$ gluodissociation at LO vs NLO

○ Brambilla Escobedo Ghiglieri Vairo JHEP 05 (2013) 130

Conclusions

In a framework that makes close contact with modern effective field theories for non relativistic bound states at zero temperature, one can study the dissociation of a quarkonium in a thermal bath of gluons and light quarks.

In a weakly-coupled framework, the situation is the following.

- For $T < E_{\text{bin}}$ the potential coincides with the $T = 0$ potential.

- For $T > E_{\text{bin}}$ the potential gets thermal contributions.

- For $E_{\text{bin}} > m_D$ quarkonium decays dominantly via gluodissociation (aka singlet-to-octet break up).

- For $m_D > E_{\text{bin}}$ quarkonium decays dominantly via inelastic parton scattering (aka Landau damping).

- For $T \sim T_{\text{dissociation}} < T_{\text{screening}}$, quarkonium cannot be formed.

In a strongly-coupled framework, the hierarchy of non-relativistic scales is preserved, whereas the thermodynamical hierarchy may break down. This requires a non-perturbative definition and evaluation of the potential (real and imaginary).