Leptogenesis with Light Sterile Neutrinos

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Jets, Particle Production & Transport Properties in Collider & Cosmological Environments

MITP Mainz, 01/08/2014

Outline

- The Baryon Asymmetry of the Universe (BAU) & Particle Physics
- Leptogenesis: Standard Picture
- Closed Time-Path Approach to Leptogenesis
- Leptogenesis with Light Sterile Neutrinos (from Oscillations)
- Discovery Opportunities
- Flavoured Leptogenesis
- Summary

I. Baryon Asymmetry of the Universe and Particle Physics

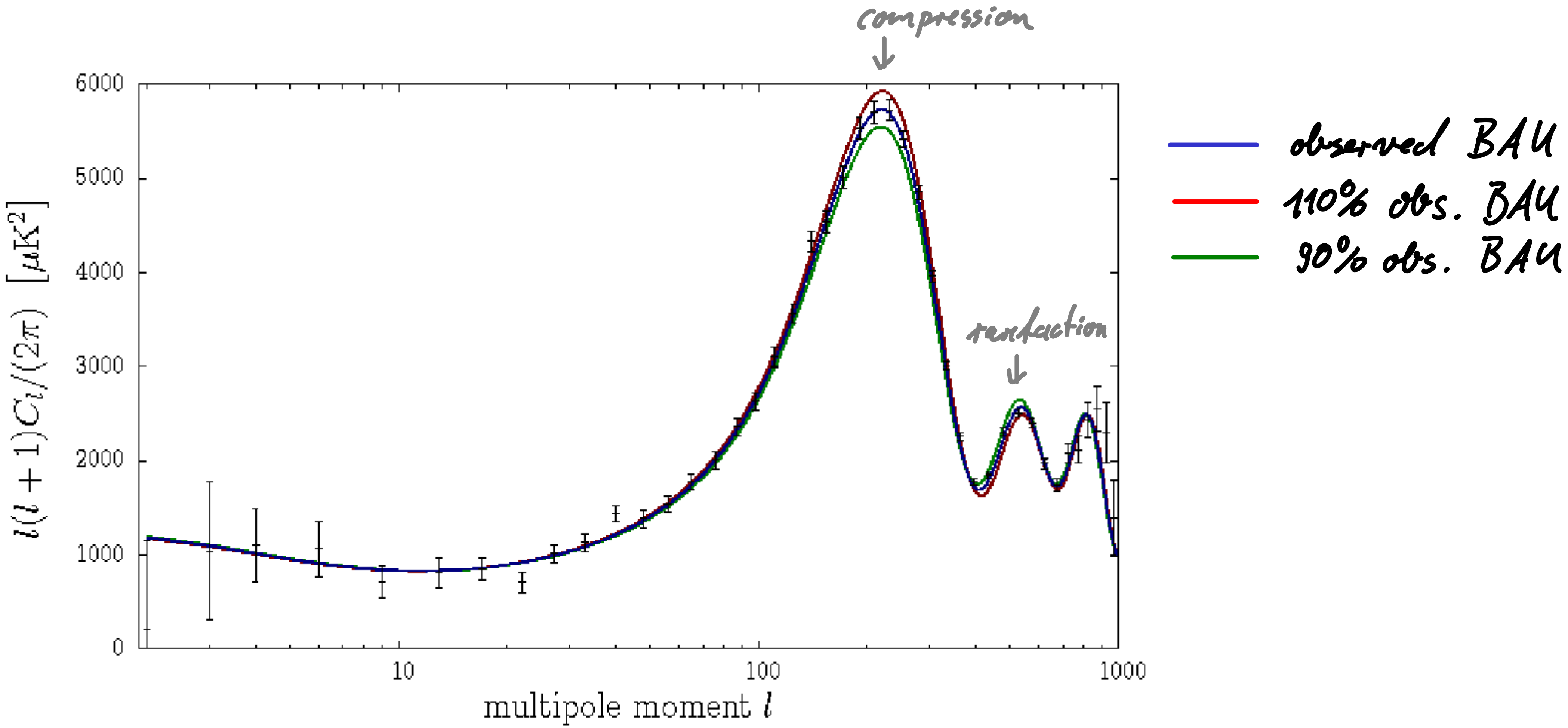
Observations

- Baryon-to-photon ratio $\eta_B = \frac{n_B}{n_\gamma} = \begin{cases} (6.16 \pm 0,15) * 10^{-10} \ (68\% \ c.l.) & \text{CMB} \\ (5.1 - 6,5) * 10^{-10} \ (95\% \ c.l.) & \text{BBN} \end{cases}$

- Big Bang Nucleosynthesis: baryon content controls the chemical potential in the Boltzmann equations that determine the abundances of ^1H, ^2H, ^3He, ^4He, ^7Li

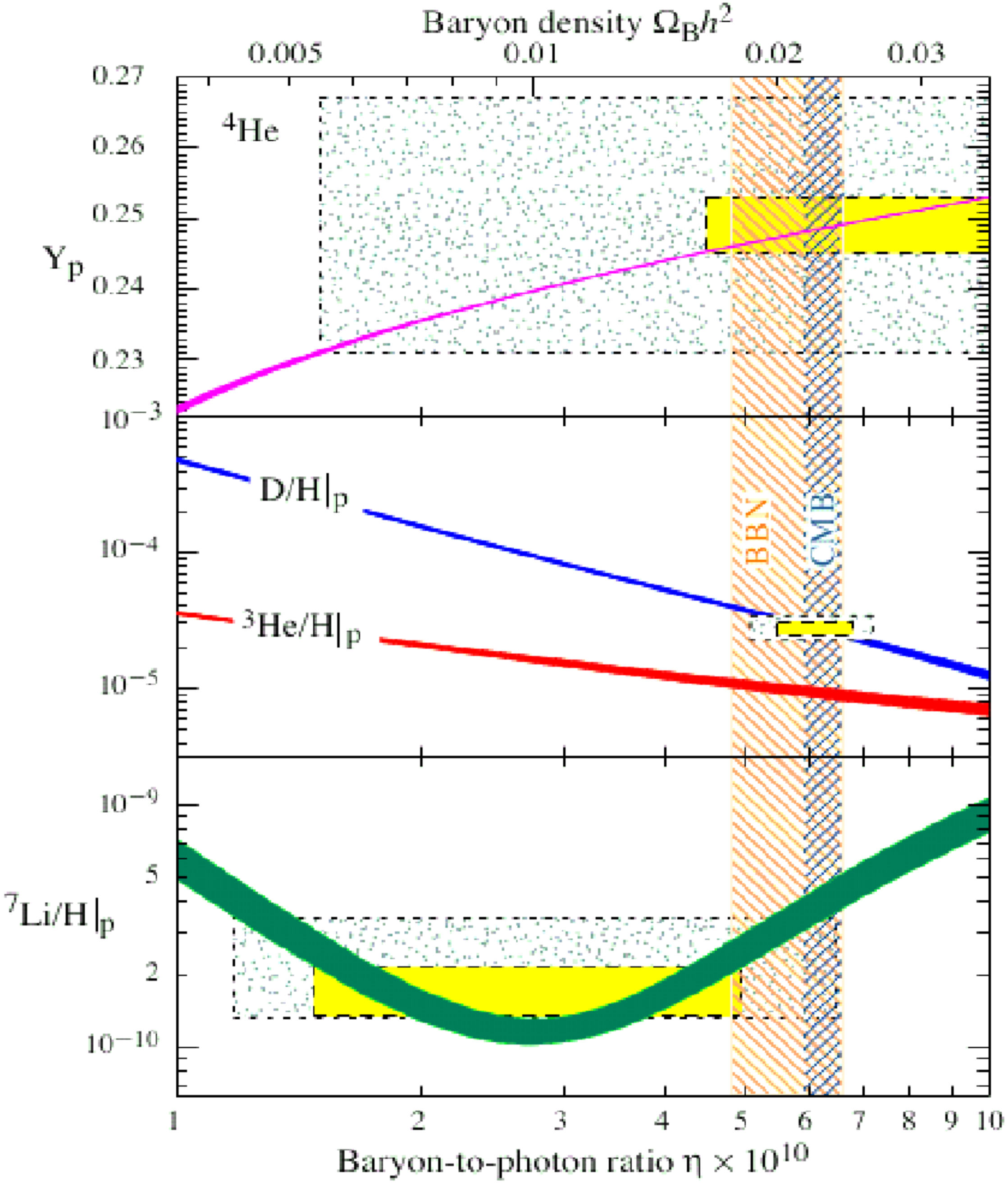
- Cosmic Microwave Background: baryons control the inertia of the baryon-photon-electron fluid oscillating in the gravitational potentials

CMB Acoustic Peaks & the BAU



$$\eta = \frac{n_B}{n_\gamma} = 7{,}04\,\frac{n_B}{n_s} = (6{,}225 \pm 0{,}170) * 10^{-10}$$ [WMAP5 (2007)]

Big Bang Nucleosynthesis & the BAU



Agreement between BBN & CMB

[from Fields & Sarkar (2007)]

Impressive success of Cosmology & the approach of applying Boltzmann equations/kinetic theory & QFT reaction rates in the context of the Early Universe.

Methods for Predictions

- BBN & CMB predictions are based on Boltzmann equations:

$$\nabla_\mu j^\mu_X = \underbrace{\partial_t n_X - \vec{\nabla}\cdot\vec{j}_X + \overset{\substack{\text{Hubble expansion}\\ \downarrow}}{3H n_X}}_{\text{Liouville term}} = \underset{\text{collision term}}{\mathcal{C}_X}$$

particles & interactions:

QED (CMB)

Nuclear Physics (BBN)

$$\mathcal{C}_X = \frac{1}{2E_X}\int \prod_i \frac{d^3p_i}{(2\pi)^3 2E_i}\,\delta^4(p_X + p_{A_1} + \ldots - p_{B_1} - \ldots)\Big\{\overset{\text{"gain term"}}{(1-f_X)(1\pm f_{A_1})\cdot\ldots\cdot f_{B_1}\cdot\ldots\cdot|\mathcal{M}_{B_1B_2\ldots\to XA_1A_2\ldots}|^2}$$
$$\underset{\text{"loss term"}}{- f_X f_{A_1}\cdot\ldots\cdot(1\pm f_{B_1})\cdot\ldots\cdot|\mathcal{M}_{XA_1A_2\ldots\to B_1B_2\ldots}|^2}\Big\}$$

- Goal: Identify the particles & interactions responsible for the BAU, predict the asymmetry & find agreement with observation.
  (... or at least achieve progress toward this goal).

- I.e.: aim to repeat the successful CMB & BBN program
  (also applies to Dark Matter)

Origin of the BAU & the Sakharov Conditions

- Primordial BAU could be imposed as initial condition in all disconnected patches. ⟶ But why?

- Inflationary paradigm: Universe is void of baryons at the end of inflation.

- Sakharov (1967): Three criteria for dynamical generation of the BAU:

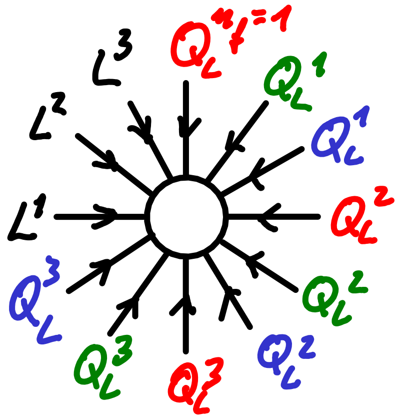
  $\cancel{B}$ Baryon number B violation

  $\cancel{C}$ & $\cancel{CP}$ C & CP violation

  $\cancel{Eq}$ Deviation from thermal equilibrium: circumvent CPT by breaking T, such that C & CP violation become effective

Situation in the Standard Model

- $\cancel{B}$ ✓ Weak Sphalerons ($SU(2)$ instanton/Higgs configurations) violate $B+L$ by six units
  [Belavin, Polyakov, Shvarts, Tyupkin (1975); 't Hooft (1976); Klinkhamer & Manton (1984); Kuzmin, Rubakov, Shaposhnikov (1985); Arnold, McLerran (1987)]



- $\cancel{\text{Eq}}$ ✗ Universe expands & cools, but gauge interactions maintain equilibrium. First order Electroweak Phase Transition requires $m_H < 70\,\text{GeV}$.
  [Kajantie, Laine, Rummukainen, Shaposhnikov (1995)]
  However: many BSM models can still have 1st order PT.

- $\cancel{CP}$ ✗ Lowest order ~~CP~~ operator: Jarlskog determinant:

$$\operatorname{Im}\left(\det[m_u m_u^\dagger, m_d m_d^\dagger]\right) \simeq -2\underset{\hookrightarrow \simeq 3*10^{-5}\ (\text{CKM})}{J}\, m_t^4\, m_b^4\, m_c^2\, m_s^2$$

$$2J\,\frac{m_t^4 m_b^4 m_c^2 m_s^2}{\underset{\hookrightarrow \text{assuming Baryogenesis occurs above the Electroweak scale}}{(100\,\text{GeV})^{12}}} \simeq 3 * 10^{-19}$$

➡ Viable mechanisms (Electroweak Baryogenesis, Leptogenesis, ...) rely on extensions of the Standard Model

II. Leptogenesis: Standard Picture

- See-saw model

$$\mathcal{L} = \frac{1}{2}\bar{N}_i(i\not\partial\delta_{ij} - M_{Nij})N_j + \bar{\ell}_a i\not\partial \ell_a + (\partial^\mu\phi^\dagger)(\partial_\mu\phi)$$

$$- Y^*_{ia}\,\ell_a(\epsilon\phi)^\dagger N_i + Y_{ia}\,\underset{\text{sterile Neutrino}}{\bar{N}_i}\,\underset{\text{Higgs}}{\phi}\,\epsilon\,\underset{\text{SM lepton}}{\ell_a}$$

- Sakharov criteria:

$\cancel{B}$ ✓ : Majorana mass M_{ij} ($N_i = N_i^c$) violates lepton number, communicated to baryon number through weak sphaleron.

$\cancel{CP}$ ✓ : Y_{ia} introduce new phases

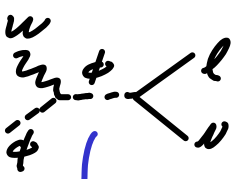
$\cancel{\text{Eq}}$ ✓ : The N_i are gauge singlets, so they only equilibrate through Y_{ia}. Neutrino oscillations point to the ideal region, where lepton-number violation close to equilibrium only for brief period.

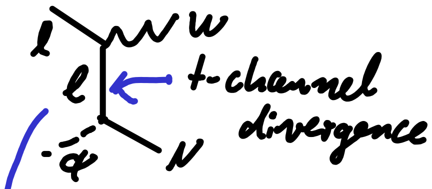
Interaction Rates

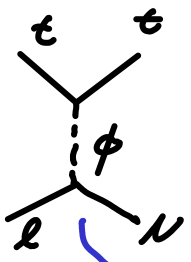
- $T \gg M_N$ (relativistic):

  $1 \leftrightarrow 2$ processes are kinematically suppressed & sub-dominant compared to processes with extra radiation, e.g.

[Bödeker & Besak (2012), BG, Glowna, Schwaller (2013)]



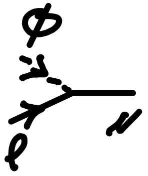




$$\frac{\langle\Gamma_N\rangle}{V} = Y^2\left[\left(\frac{3}{2}g_2^2+\frac{1}{2}g_1^2\right)\left(3{,}08 * 10^{-3} - 3{,}67 * 10^{-4}\log\left(\frac{3}{2}g_2^2+\frac{1}{2}g_1^2\right)\right)T^4 + 5{,}22 * 10^{-4}\, h_t^2\, T^4\right]$$

- $T \sim M_N$: $1 \leftrightarrow 2$ processes become important, but are soon Maxwell-suppressed (non-relativistic)

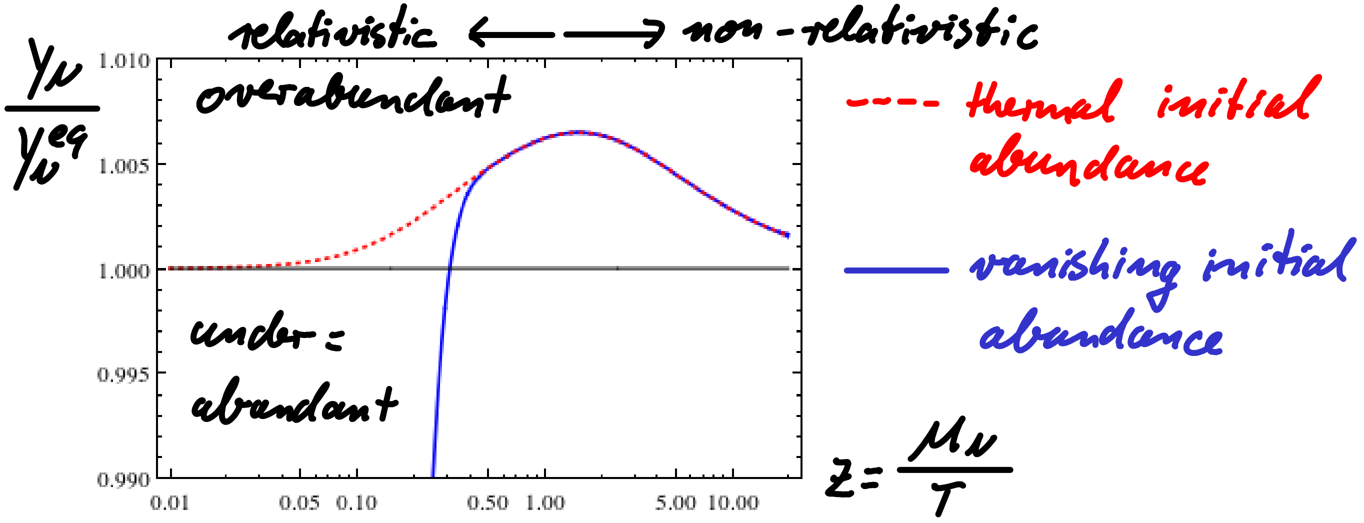
$$\frac{\langle\Gamma_N\rangle}{V} = 2 * \frac{M_N}{8\pi}\int\frac{d^3k}{(2\pi)^3}\frac{M_N}{\sqrt{k^2+M_N^2}}\, e^{-\frac{k^0}{T}} = \frac{M_N^3 T}{8\pi}K_1\left(\frac{M_N}{T}\right) \hat{\approx} 2^{-\frac{7}{2}}\pi^{-\frac{5}{2}} M_N^{\frac{5}{2}} T^{\frac{3}{2}} e^{-\frac{M_N}{T}}$$



Non-Equilibrium Windows:

$Y_N = \frac{n_N}{s}$

sterile neutrino to entropy ratio



Neutrino Production Rate & Self-Energy

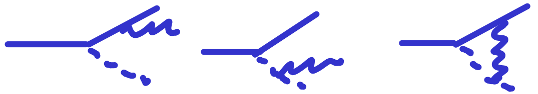
- Spectral self-energy: $\not{\Sigma}_N^{\mathcal{A}} = \frac{1}{2i}\left(\not{\Sigma}_N^{A} - \not{\Sigma}_N^{R}\right)$
  advanced retarded

- Can use Kadanoff-Baym equations (see below) to relate

$$\frac{\langle \Gamma_N \rangle}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k^0} \mathrm{tr}\left[\not{k}\,\not{\Sigma}_N^{\mathcal{A}}\right]\Big|_{k^0=\sqrt{\vec{k}^2+M_N^2}}$$

- Leading order, non-relativistic form of $\not{\Sigma}_N^{\mathcal{A}}$ can be evaluated analytically

- NLO, non-relativistic $\not{\Sigma}_N^{\mathcal{A}}$ ($g_3^2, g_2^2, g_1^2, h_t^2, \lambda$ corrections) can be expanded in $\frac{T^2}{M^2}$. ⚠: soft & collinear divergences



Lodone, Salvio, Strumia (2011); Laine, Schröder (2011); Biondini, Brambilla, Escobedo, Vairo (2013)]

- Relativistic regime – see above [Bödeker et. al. (2010,12)]

- Crossover between relativistic & non-relativistic regimes: Soft & collinear divergences cancel without $\frac{T^2}{M^2}$ expansion
  [Laine (2012,13); BG, Glowna, Herranen (2013)]

Neutrino Oscillations & Non-Equilibrium

- Ideally, want coincidence between equilibration & Maxwell suppression → produce many out-of-equilibrium particles, then rapid freeze-out, to protect asymmetry from washout.

$$H^2 = \frac{8\pi}{3}\frac{\rho}{m_{Pl}^2} = \frac{4\pi^3}{45} \underbrace{g_*}_{\to 106{,}75} \frac{T^4}{m_{Pl}^2}$$ Hubble rate square

$$\Gamma_N = Y^2 \frac{M_N}{8\pi}$$

$$m_* \sim \frac{Y^2 v^2}{2 M_N}$$ see-saw for light neutrino mass m, $v = 246\,\mathrm{GeV}$

$$\Gamma_N = H\big|_{T=M_N} \Longrightarrow m_* \sim 216 \frac{v^2}{m_{Pl}} = 1\,\mathrm{meV}\ (!)$$ independent of M_N !

- Neutrino oscillations point to the best compromise between weak (too little production) and strong (too close to equilibrium) coupling. Typically out-of-equilibrium when relativistic, close-to-equilibrium when Maxwell-suppressed.

- → Large interest in leptogenesis after discovery of oscillations

Leptogenesis in the Strong Washout Regime

- "Standard" scenario: Equations for creation of asymmetry & freeze-out:

$$\frac{dY_\ell}{dz} = \overset{\cancel{CP}}{\varepsilon}\, \overline{\Gamma}\left(Y_N - Y_N^{eq}\right) - W\, Y_\ell$$

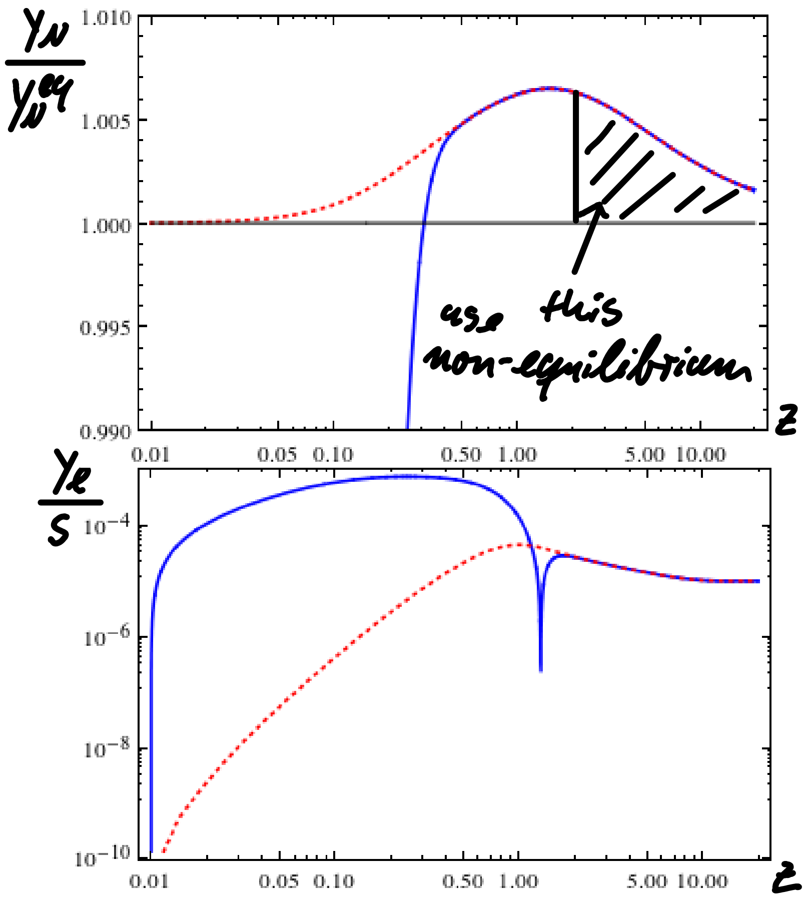
$$\frac{dY_N}{dz} = - \overline{\Gamma}\left(Y_N - Y_N^{eq}\right)$$

$$z = \frac{M_N}{T}$$

$$W = |y|^2 \sqrt{\frac{45}{g_* \pi^3}}\; \frac{3\, m_{Pl}\, e^{-z} z^{\frac{5}{2}}}{2^{\frac{9}{2}} \pi^{\frac{5}{2}} M_N}$$

$$Y_N^{eq} = 2^{-\frac{3}{2}} \pi^{-\frac{7}{2}} \frac{45}{g_*} z^{\frac{3}{2}} e^{-z}$$

$$\overline{\Gamma}_N = |y|^2 \frac{z}{16\pi} \sqrt{\frac{45}{g_* \pi^3}}\; \frac{m_{Pl}}{M_N}$$

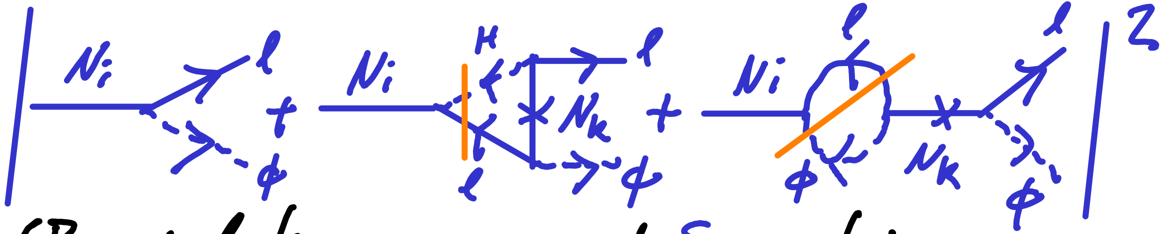


----- thermal initial conditions

—— vanishing initial conditions

Nice feature: Independent of initial conditions.

Standard Approach to Leptogenesis



CP-violating squared S-matrix elements (quantum)

$\mathcal{L}[f] = \mathcal{C}[f]$

Boltzmann equation (classical)

Lepton Asymmetry

[Fukugita & Yanagida (1986)]

$$\varepsilon_{N_i \to l_a} = \frac{\Gamma_{N_i \to l_a \phi} - \Gamma_{N_i \to \bar{l}_a \bar{\phi}}}{\Gamma_{N_i \to l_a \phi} + \Gamma_{N_i \to \bar{l}_a \bar{\phi}}}$$

$$\varepsilon^{wf}_{N_i \to l_a} = \frac{1}{8\pi} \sum_{j \neq i} \frac{M_i}{M_i^2 - M_j^2} \frac{\mathrm{Im}\left[(Y^\dagger Y^* Y^t)_{aj} M_j Y_{ja} + Y^\dagger Y Y^\dagger M_j Y_{ja} \right]}{[Y^\dagger Y]_{aa}}$$

what if $M_i \to M_j$?

↳ May be interpreted as mixing: Mass eigenstates of N are not exactly CP-even

$$\varepsilon^{vertex}_{N_i \to l_a} = \frac{1}{8\pi} \sum_{j \neq i} \sqrt{\frac{M_j}{M_i}} \left[1 - \left(1 + \frac{M_j}{M_i}\right) \log\left(1 + \frac{M_i}{M_j}\right)\right] \frac{\mathrm{Im}\left[(Y^\dagger Y^* Y^t)_{aj} M_j Y_{ja} \right]}{[Y^\dagger Y]_{aa}}$$

Remarks on Strong Washout Leptogenesis

- Typically considered a "high scale" scenario:

  smaller $M_N \xrightarrow{\text{see-saw}}$ smaller $y \longrightarrow$ smaller ε [Davidson & Ibarra (2002); Buchmüller, Di Bari, Plümacher (2002)]

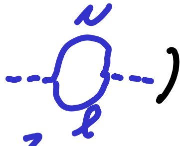
  $M_N > 10^9\,\mathrm{GeV}$ ... and consequences for detection

- Indirect "smoking gun": $0\nu\beta\beta$

- Way around mass bound: resonances, i.e. $M_i \to M_j$

  [Covi, Roulet & Vissani (1996); Flanz, Paschos, Sarkar, Weiss (1997); Pilaftsis (1997)]

  What happens in degenerate case?

- Heavy sterile neutrinos destabilise Higgs mass (

), unless cancellation:

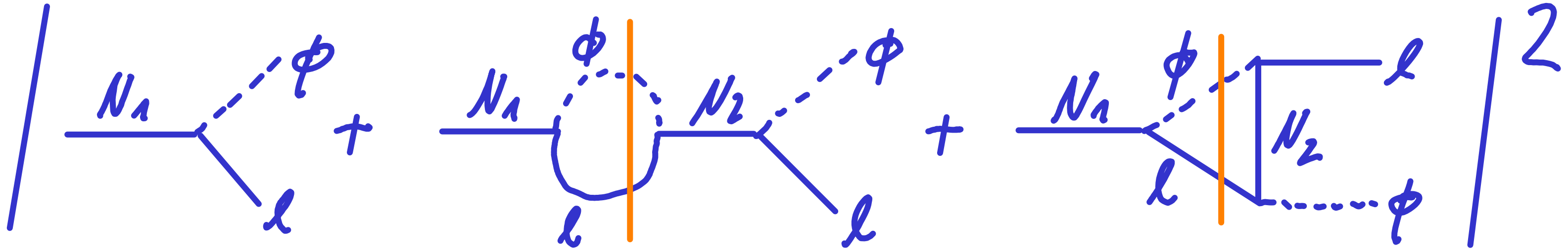
  $$\Delta m_\phi \sim \sqrt{y^2 \frac{M_N^2}{16\pi}} \sim \sqrt{\frac{m_\nu M_N^3}{8\pi v^2}} \sim 6*10^{-9}\left(\frac{M_N}{1\,\mathrm{GeV}}\right)^{\frac{3}{2}}$$

- Weak sphalerons turn lepton into baryon asymmetry (true for all leptogenesis models).

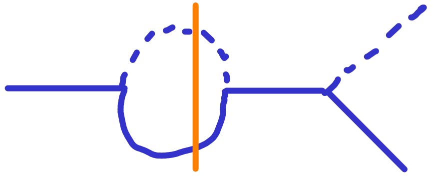
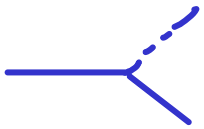
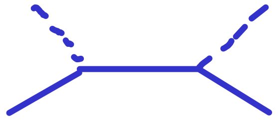
Asymmetry in Boltzmann Approach

Fukugita & Yanagida (1986)
Covi, Roulet, Vissani (1996)
Buchmüller & Plümacher (1996)

- Interference of tree & loop amplitudes ⟶ $\not{CP}$ violation

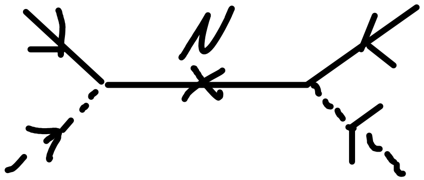


(*)

- $\not{CP}$ violating contributions from discontinuities ⟶ loop momenta where cut particles are on shell (Cutkosky rules)
- Is  an extra process or is it already accounted for by  and  ?
- Including (*) only ⟶ $\not{CP}$ asymmetry generated even in equilibrium

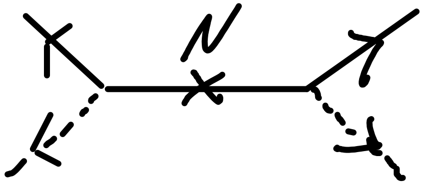
A problem when Using Vacuum S-Matrix Elements for Leptogenesis

$2 \longleftrightarrow 2$



$$|\mathcal{M}_{\ell\phi\to\bar{\ell}\phi^*}|^2$$

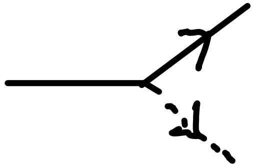
$$=$$



$$|\mathcal{M}_{\bar{\ell}\phi^*\to\ell\phi}|^2$$

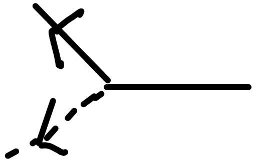
No asymmetry generated

$1 \longleftrightarrow 2$

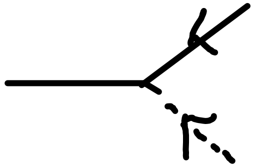


$$|\mathcal{M}_{N\to\ell\phi}|^2 \sim 1+\varepsilon$$

$$\overset{CPT}{=}$$

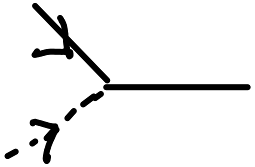


$$|\mathcal{M}_{\bar{\ell}\phi^*\to N}|^2 \sim 1+\varepsilon$$



$$|\mathcal{M}_{N\to\bar{\ell}\phi^*}|^2 \sim 1-\varepsilon$$

$$\overset{CPT}{=}$$



$$|\mathcal{M}_{\ell\phi\to N}|^2 \sim 1-\varepsilon$$

Naive multiplication* suggests asymmetry even in equilibrium:

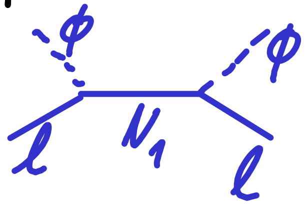
$$\Gamma_{\bar{\ell}\phi^*\to\ell\phi} \sim 1+2\varepsilon$$

*Do not try this at home: The unstable N cannot constitute elements of a unitary S-matrix.

CPT, Unitarity & RIS

- Generation of CP asymmetry in equilibrium ↯ CPT theorem ↯
- N_1 unstable, not an asymptotic state of a unitary S-matrix

  ⟶ Multiplication of matrix elements implicit in Boltzmann equations leads to non-unitary evolution
- Usual fix: subtract Real Intermediate States (RIS) from



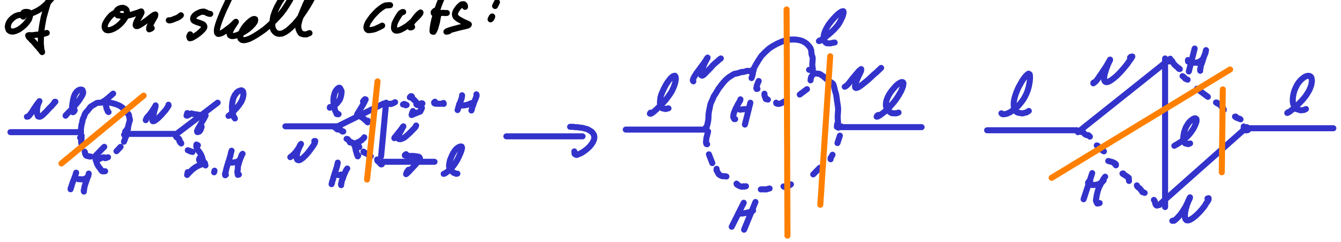
  Kolb & Wolfram (1980)
- Heuristic argument that furthermore leaves unclear how to include quantum statistical (Bose-Einstein / Fermi-Dirac) corrections.

  ⟶ Can be resolved systematically using CTP methods

➤ CTP fixes CPT

III. Closed Time-Path Approach to Leptogenesis

- Obtain kinetic equations for Leptogenesis from one single formalism (rather than combining classical Boltzmann equations with QFT S-matrix elements):
  → Closed-Time-Path / In-In / Schwinger-Keldysh approach
- Inclusive approach based on Green functions only.
  → Crucial for CP violation @ finite T: Avoid over/undercounting of on-shell cuts:



In contrast to usual heuristic Real Intermediate State subtraction.

- Applicable to models beyond those considered here. — Address subtleties and identify new possibilities.

Garny, Hohenegger, Kartavtsev, Lindner (2009-)
Anisimov, Buchmüller, Drewes, Mendizabal (2010-)
Beneke, Fidler, Garbrecht, Herranen, Schwaller (2010-)

Functional Approach (in-out)

- In-out generating functional for time ordered expectation values:

$$Z[J] = N^{-1} \langle vac_{out} | vac_{in} \rangle_J = \int D\phi \, e^{i\int d^4x \left(\mathcal{L} + J(x)\phi(x) \right)}$$

$$\langle T[\phi(x)\phi(y)] \rangle = - \frac{\delta^2}{\delta J(x)\, \delta J(y)} \log Z[J] \Big|_{J=0}$$

The Closed Time Path

Schwinger (1961)
Keldysh (1964)
Calzetta & Hu (1987)

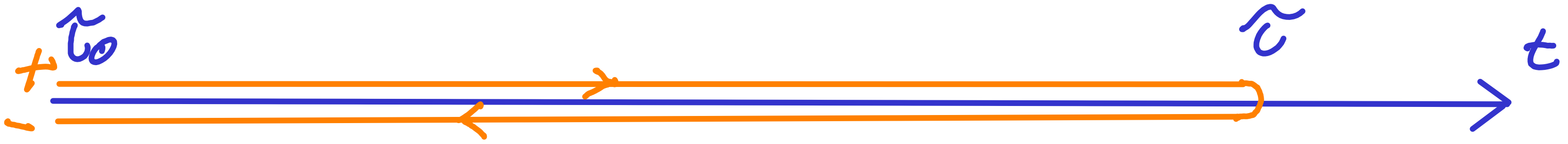
- **In-In generating functional:**

$$\phi_{in}(x) = \phi(\vec{x}, t_0)$$

$$Z[J_+, J_-] = \int \mathcal{D}\psi\, \mathcal{D}\phi_{in}^-\, \mathcal{D}\phi_{in}^+\, \langle \phi_{in}^- | \psi, \tau \rangle_{J_-} \langle \psi, \tau | \phi_{in}^+ \rangle_{J_+} \langle \phi_{in}^- | \rho | \phi_{in}^+ \rangle$$

$$= \int \mathcal{D}\phi^+ \mathcal{D}\phi^-\, e^{i\int d^4x \left\{ \mathcal{L}[\phi^+] + J_+\phi^+ - \mathcal{L}[\phi_-] - J_-\phi^- \right\}} \langle \phi_{in}^- | \rho | \phi_{in}^+ \rangle$$

The Closed Time Path:



- **Path ordered Green functions:**

$$i\Delta_\phi^{ab}(u,v) = -\frac{\delta^2}{\delta J_a(u)\,\delta J_b(v)} \log Z[J_+, J_-]\Big|_{J_\pm = 0} = i\langle \underset{\substack{\uparrow \\ \text{path ordering}}}{\mathcal{C}}[\phi^a(u)\,\phi^b(v)] \rangle$$

Path Ordered Green Functions

$$i\Delta_\phi^{<}(u,v) = i\Delta_\phi^{+-}(u,v) = \langle \phi(v)\,\phi(u)\rangle$$

$$i\Delta_\phi^{>}(u,v) = i\Delta_\phi^{-+}(u,v) = \langle \phi(u)\,\phi(v)\rangle$$

$$i\Delta_\phi^{T}(u,v) = i\Delta_\phi^{++}(u,v) = \langle T[\phi(u)\,\phi(v)]\rangle$$

$$i\Delta_\phi^{\overline{T}}(u,v) = i\Delta_\phi^{--}(u,v) = \langle \overline{T}[\phi(u)\,\phi(v)]\rangle$$

Free propagators – building blocks of perturbation theory:

$$i\Delta_\phi^{<}(p) = 2\pi\,\delta(p^2-m^2)\left[\vartheta(p^0)\,f(\vec{p}) + \vartheta(-p^0)\left(1+\overline{f}(-\vec{p})\right)\right]$$

$$i\Delta_\phi^{>}(p) = 2\pi\,\delta(p^2-m^2)\left[\vartheta(p^0)\left(1+f(\vec{p})\right) + \vartheta(-p^0)\,\overline{f}(-\vec{p})\right]$$

$$i\Delta_\phi^{T}(p) = \frac{i}{p^2-m^2+i\varepsilon} + 2\pi\,\delta(p^2-m^2)\left[\vartheta(p^0)\,f(\vec{p}) + \vartheta(-p^0)\,\overline{f}(-\vec{p})\right]$$

$$i\Delta_\phi^{\overline{T}}(p) = -\frac{i}{p^2-m^2-i\varepsilon} + 2\pi\,\delta(p^2-m^2)\left[\vartheta(p^0)\,\underbrace{f(\vec{p}) + \vartheta(-p^0)\,\overline{f}}_{\text{(anti-) particle distribution functions}}(-\vec{p})\right]$$

Similarly for spin-$\frac{1}{2}$ fermions, gauge bosons ...

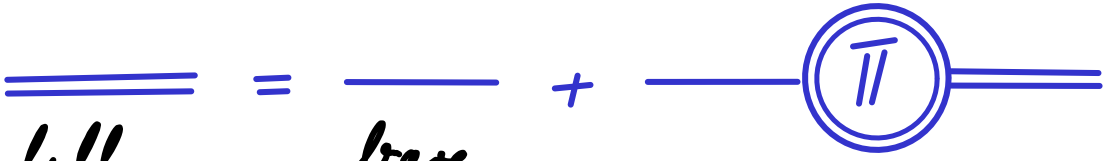
Feynman Rules

- Vertices either + or −, factor −1 for each − vertex
- Connect vertices $a=\pm$ and $b=\pm$ with $i\Delta^{ab}$

Schwinger-Dyson Equations

- $$i\Delta^{ab} = i\Delta^{(0)ab} + cd\, i\Delta^{(0)ac} \odot \Pi^{cd} \odot \Delta^{db}$$

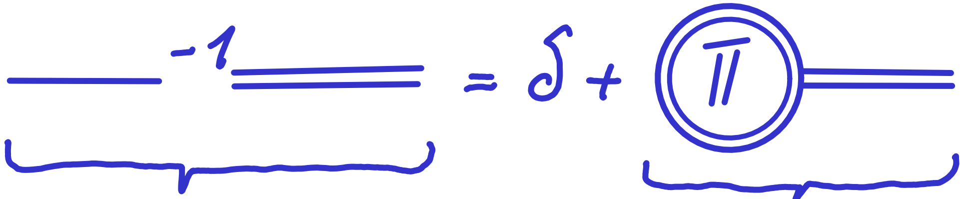
$$A(x,w)\odot B(w,y) = \int d^4w\, A(x,w)\, B(w,y)$$



full propagator

bare propagator

self energy: 1PI, full propagators



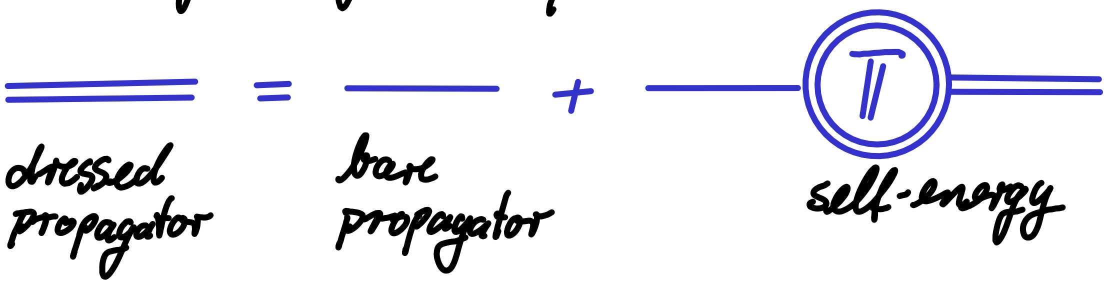
"flow term"

"collision term"

- Kinetic equations in terms of Green functions — no reference to asymptotic states ⟶ by construction, no unitarity problem

Toward Kinetic Theory: Kadanoff-Baym Equations

- Schwinger-Dyson equations:



- The Kadanoff-Baym equations are the $<,>$ -components:

$$(-\partial^2 - m^2)\Delta^{<,>} - \Pi^{H} \odot \Delta^{<,>} - \Pi^{<,>} \odot \Delta^{H} = \frac{1}{2}\left(\Pi^{>} \odot \Delta^{<} - \Pi^{<} \odot \Delta^{>}\right)$$

→ "continuity equation"

collision term

$\odot$: convolution → $[A \odot B](x,y) = \int d^4w \, A(x,w) B(w,y)$

- Remaining linear combination: mass-shell equations

$$(-\partial^2 - m^2) i\Delta^{R,A} - \Pi^{R,A} \odot i\Delta^{R,A} = i\delta^4$$

→ retarded/advanced propagator

Toward Kinetic Equations: Wigner Transformation

- Wigner transformation:

$$A(k,x) = \int d^4\tau\, e^{ik\tau} A\left(x+\frac{\tau}{2}, x-\frac{\tau}{2}\right)$$

→ average coordinate — macroscopic evolution

→ relative coordinate — microscopic (quantum) properties

- For the convolutions, can show that:

$$\int d^4\tau\, e^{ik\tau} \int d^4w\, A\left(x+\frac{\tau}{2}, w\right) B\left(w, x-\frac{\tau}{2}\right) = e^{-i\Diamond}\{A(k,x)\}\{B(k,x)\}$$

where: $\Diamond\{\cdot\}\{\cdot\} = \frac{1}{2}\left(\partial_x^{(1)}\cdot\partial_k^{(2)} - \partial_k^{(1)}\cdot\partial_x^{(2)}\right)$

Gradient Expansion

■ For slowly evolving system, expand in powers of

$$\partial_X \cdot \partial_k \sim H/T \sim T/m_{Pl} \ll 1$$

∂_k ↳ typical momentum scale, i.e. T

∂_X ↳ typical time scale, i.e. Hubble time H^{-1}

$\vec{\nabla}_x \equiv 0$ for spatially homogeneous system

■ Leptogenesis most efficient when

$$\Gamma = y^2 \frac{1}{16\pi} m_N \sim H \quad \text{and} \quad m_N \sim T \Longrightarrow y^2 \frac{1}{16\pi} \sim H/T \ll 1$$

↷ Expand in $\partial_X \cdot \partial_k$ and y^2 $\left(\sim \frac{T}{m_{Pl}}\right)$

m_N, y: Mass & Yukawa coupling of the singlet Majorana neutrino

⚠ Gradient expansion does not apply to terms $\propto \partial_k \delta(k^2 - m_N^2)$

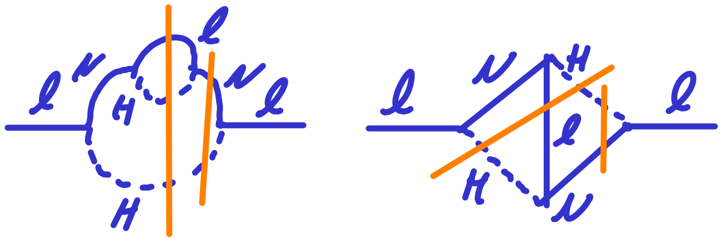
Resummation possible & leads to finite width effects [Garny & BG (2011)]

➡ Perturbation theory works out-of-equilibrium (in contrast to, e.g., Greiner & Leupold (1996))

A Word on Truncations

- The Schwinger Dyson Equations are exact and hence time-reversible
- They are non-Markovian ($\longrightarrow$ "memory integrals")
- Approximations used here:
  - Assume equilibrium of ℓ and ϕ except for chemical potentials $\mu_\ell \neq 0$ and $\mu_\phi \neq 0$.
  - Neglect backreaction of ℓ, ϕ on N.
  - Perturbative expansion.
  - Gradient expansion.
- Result: Markovian, irreversible evolution.
- While the approximations are justified and the results plausible, it may be interesting to formulate Baryogenesis in a reversible manner.

Standard Leptogenesis in CTP Approach



- First-principle derivation of kinetic equations including CP violation.
- No ad-hoc subtraction of RIS necessary.
- Systematic & correct account of quantum statistics.

Garny, Hohenegger, Kartavtsev, Lindner (2009-); Anisimov, Buchmüller, Drewes, Mendizabal (2010-); Beneke, Fidler, Garbrecht, Herranen, Schwaller (2010-)

- Method as a platform for further developments:
  - * Flavoured Leptogenesis
  - * Mixing Leptons & Higgs bosons as sources of CP-violation [BG (2012)]
  - * Assess viability of other scenarios with equilibrium/ non-equilibrium cuts (e.g. soft Leptogenesis) [BG, M.J. Ramsey-Musolf (2013)]
  - * (Resonant) Leptogenesis from mixing & oscillations (see below)
  - * ...

Kinetic Equations With "RH Flavour" → Resonant Leptogenesis

BG, Herranen (2011)

- Take neutral mixing scalars χ as model for mixing right-handed neutrinos (for this talk's sake, have also worked out fermionic case)

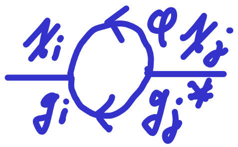
$$\mathcal{L} = (\partial_\mu\varphi)(\partial_\mu\varphi^*) - M_\varphi^2|\varphi|^2 + \frac{1}{2}(\partial_\mu\chi_i)(\partial^\mu\chi_i) - \frac{1}{2}M^2_{\chi_{ij}}\chi_i\chi_j - g_i\chi_i\varphi^2 - g_i^*\chi_i\varphi^{*2}$$

⟶ kinetic equations:

$$2ik^0\partial_t\Delta_\chi^{<,>} - [M^2_\chi, \Delta_\chi^{<,>}] = -\frac{1}{2}\left(\{i\Pi_\chi^>, i\Delta_\chi^<\} - \{i\Pi_\chi^<, i\Delta_\chi^>\}\right)$$

↳ diagonal

$$\Pi_{\chi_{ij}} =$$



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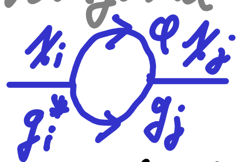


$\hat{\Pi}^{\mathcal{A}}_\chi$: spectral self-energy without coupling constants

- non-equilibrium part of diagonal components of $i\Delta_\chi^{<,>}$:

$$i\delta\Delta_{\chi_{ii}}(k) = 2\pi\delta(k^2 - M^2_{\chi_{ii}})\delta f_{\chi ii}(\vec{k})$$

complies with definition $\delta f_{\chi_{ij}}(\vec{k}) = \int_0^\infty \frac{dk^0}{2\pi} 2k^0 i\delta\Delta_{\chi_{ij}}$ (also valid for $i\neq j$)

- Source term: $\int\frac{d^3p}{(2\pi)^3}\int\frac{dp^0}{2\pi} ip^0\partial_t i\Delta_\varphi^{<,>}(p) = S_\varphi = -g_i g_j^* \int\frac{d^4q}{(2\pi)^4} i\delta\Delta_{\chi_{ij}}\hat{\Pi}^{\mathcal{A}}_\chi$

Resonant Enhancement of off-Diagonal Correlations

- → integrated kinetic equation: same as for density matrix ansatz

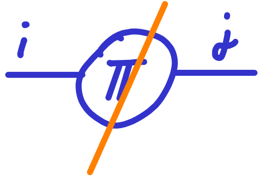
$$\partial_t \delta f + \frac{i}{2k^0}[M^2, \delta f] = -\frac{1}{2}\{\Gamma, \delta f\} \qquad \Gamma \equiv \frac{\Pi^{\mathcal{A}}}{k^0}$$

→ spectral (antihermitian) self-energy

Non-singular in the degenerate case.

- When neglecting $\partial_t \delta f$, obtain perturbative result for off-diagonal correlation in terms of diagonal:

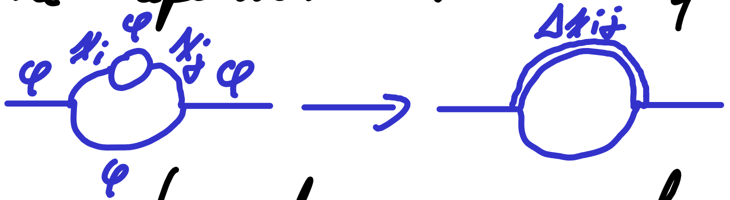
$$\delta f_{ij} = \frac{i\Pi^{\mathcal{A}}_{ij}}{M^2_{ii} - M^2_{jj} \underbrace{- i\Pi^{\mathcal{A}}_{ii} + i\Pi^{\mathcal{A}}_{jj}}_{\text{NLO corrections}}} \delta f_{ii} \equiv$$



in Hamiltonian picture: Covi & Roulet (1996); cf. also Garny, Hohenegger, Kartavtsev & Lindner (2009)

- Method breaks down when $M_{ii} \to M_{jj}$. Then need to resum propagator by solving the time-dependent kinetic equations:

$S_\varphi =$



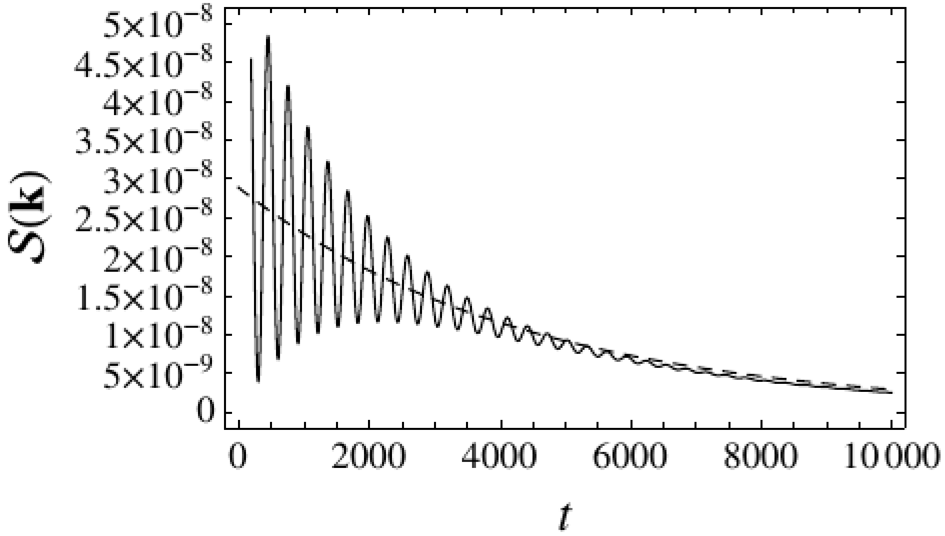
2PI

- Connection between decay asymmetry from wave-function correction & mixing/oscillating density matrix established.

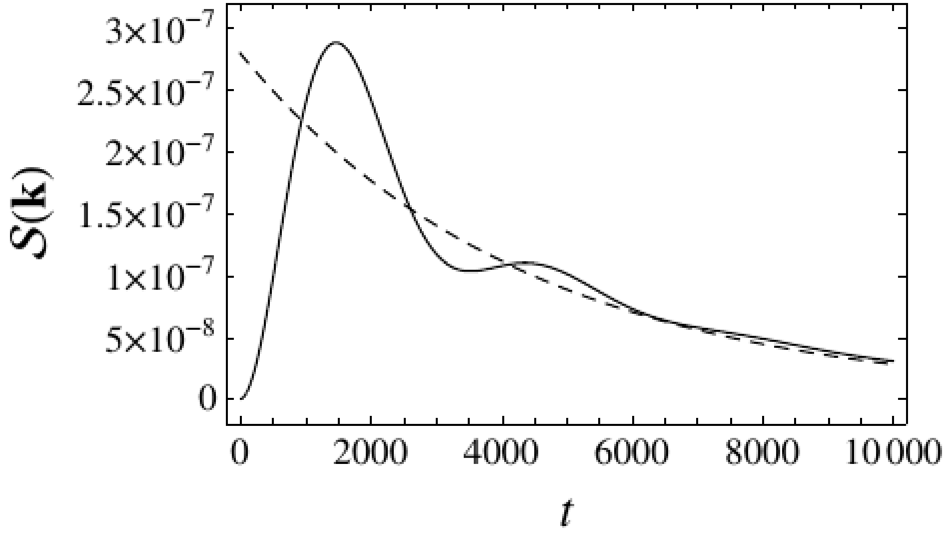
cf. Garny, Hohenegger, Kartavtsev (2011)

Leptogenesis from Mixing Oscillations

- When $\Delta M^2 \sim \Pi^4$ or $\Delta M^2 \ll \Pi^4$, oscillations are important
- Obtain oscillatory solution to kinetic equations



$\Delta M^2 \gg \Pi^4$



$\Delta M^2 \sim \Pi^4$

- Unified approach to standard wave function asymmetry (dashed) and to Leptogenesis from neutrino oscillations (solid)

Overlapping applicability of "standard" and "mixing & oscillation" approaches when $\frac{M_1^2+M_2^2}{2} \gg \Delta M^2 \gg \Pi^4$ — for $\Delta M^2 < \Pi^4$ must take account of oscillations!
⟶ Cf. neutral meson systems.

BG, Herranen (2012)

Spinors

- **Kinetic equations for sterile neutrinos:**

$$\delta f'_{N0h} + a^2 \frac{1}{2k^0} i[M^2, \delta f_{N0h}] + f_N^{eq\,\prime} = -\left(\hat{\Sigma}_N^{A0} + h\hat{k}^i \hat{\Sigma}_N^{Ai}\right)\left(1 - \frac{h|\vec{k}|}{k^0}\right)\{Y^* Y^t, \delta f_{N0h}\}$$
$$- \left(\hat{\Sigma}_N^{A0} - h\hat{k}^i \hat{\Sigma}_N^{Ai}\right)\left(1 + \frac{h|\vec{k}|}{k^0}\right)\{Y Y^t, \delta f_{N0h}\}$$

$$k^0 = \pm\omega(\vec{k}) = \pm\sqrt{\vec{k}^2 + a^2 M^2}$$

Applies to sterile neutrino oscillations in medium (provided $\hat{\Sigma}_N^{A}$ is known) throughout the kinematic regime (relativistic/non-relativistic).

- **Source term:**

$$\frac{d}{d\eta}(n_\ell - \bar{n}_\ell) = \underset{\text{washout}}{W} + \underset{\text{source}}{S}$$

$$S = -Y_i^* Y_j \sum_{h=\pm} \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{k \cdot \hat{\Sigma}_N^{A}}{k^0} (\delta f_{N0hij} - \delta f^*_{N0hij}) + h \frac{\tilde{k} \cdot \hat{\Sigma}_N^{A}}{k^0} (\delta f_{N0hij} + \delta f^*_{N0hij}) \right\}_{k^0 = \omega(\vec{k})}$$

$$\tilde{k} = (|\vec{k}|, k^0 \hat{k})$$

reduces to "standard" contribution in non-relativistic limit

cf. Garny, Hohenegger & Kartavtsev

reduces to "Leptogenesis from oscillations" in ultra-relativistic limit

[Akhmedov, Rubakov, Smirnov (1998); Asaka, Shaposhnikov (2005)]

[Iso, Shimada (2014);
BG, Gautier & Klaric (2014)]

Regulator for the Resonance in the Strong Washout Regime

- Non-relativistic regime: Can average over momentum and deal with number densities n_N instead of distributions f_N:

$$\bar{M}\frac{d}{dz}\delta n_N \pm \frac{ia}{\bar{M}}[M^2, \delta n_N] + a\frac{1}{2}\bar{\Gamma}\{\mathrm{Re}[Y^*, Y^t], \delta n_N\} + \bar{M}\frac{d}{dz}n_N^{eq} = 0$$

neglect this term

↓

estimate:

$$\bar{M} = \frac{M_1+M_2}{2} \qquad M^2 = \begin{pmatrix} M_1^2 & 0 \\ 0 & M_2^2 \end{pmatrix} \qquad a = \frac{T^2}{H}\frac{z}{\bar{M}} \qquad z = \frac{\bar{M}}{T}$$

$$\frac{d}{dt}\delta n_N \sim H\delta n_N \ll \Gamma\delta n_N \longrightarrow \text{strong washout}$$

Vanilla leptogenesis – decay asymmetry:

$$\varepsilon = \frac{\bar{M}}{16\pi}\frac{i(M_1^2-M_2^2)\bar{M}(Y_1Y_2^*-Y_1^*Y_2)(Y_1Y_2^*+Y_1^*Y_2)(|Y_1|^2+|Y_2|^2)}{|Y_1|^2|Y_2|^2(M_1^2-M_2^2)^2-\frac{1}{4}\bar{M}^2\bar{\Gamma}^2(|Y_1|^2+|Y_2|^2)(Y_1Y_2^*-Y_1^*Y_2)^2}$$

- Generalisation to flavoured case has been done – necessary & straightforward.

Regulator for the Resonance in the Strong Washout Regime

- The parameter ε can be as large as one, corresponding to zero eigenvalues in the evolution equation for sterile neutrinos $\longrightarrow$ breakdown of approximation.

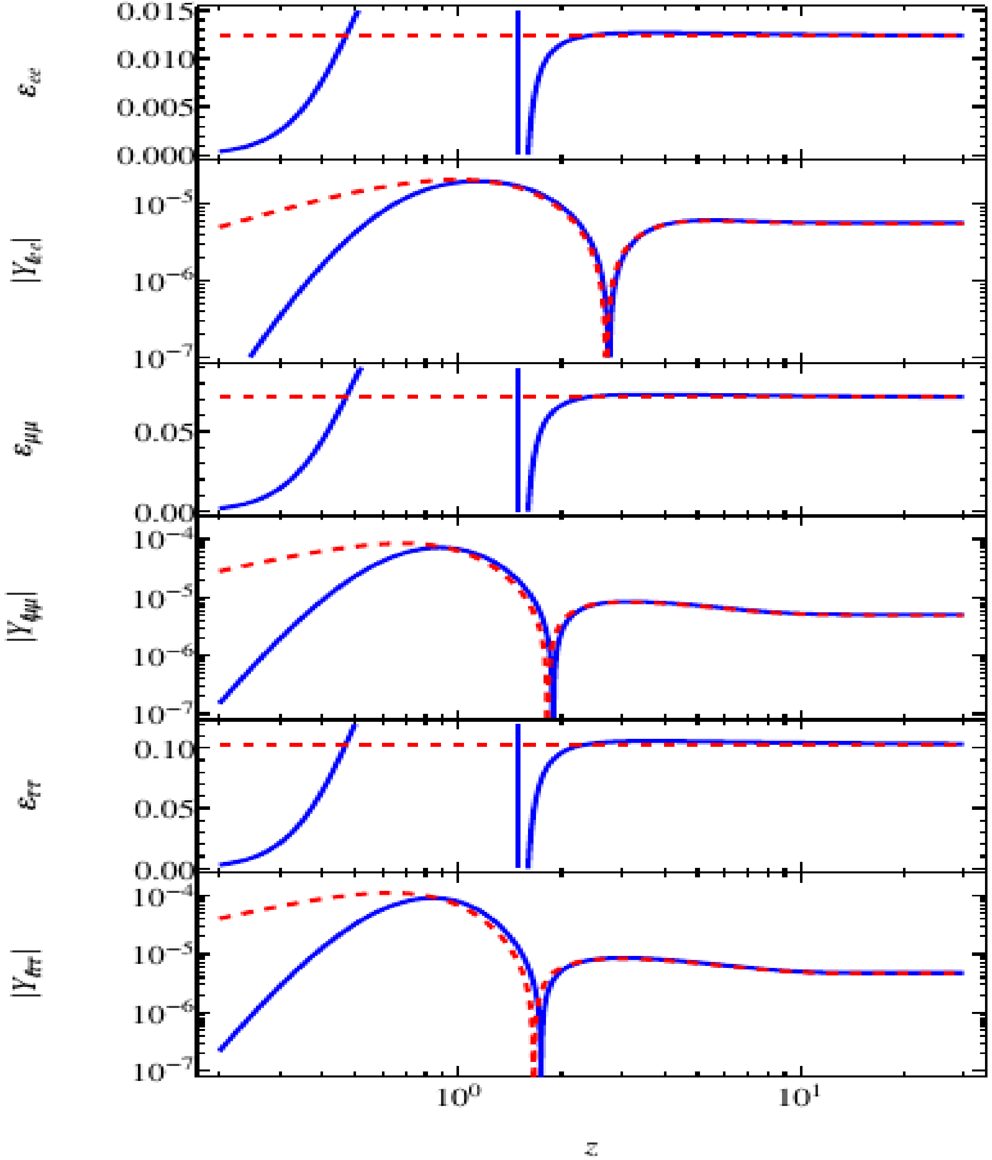
- However, can show that in the SM with 2 sterile neutrinos, above approximation is always valid

Example point:

$\alpha = 0$, $e = \frac{\pi}{4} + 0{,}2i$, $\frac{\Delta M}{\bar{M}^2} = 2 * 10^{-17}\,\mathrm{GeV}^{-1}$

red: based on effective decay asymmetry

blue: full evolution based on right-handed neutrino oscillations



IV. Leptogenesis with Light Sterile Neutrinos
(Leptogenesis from Neutrino Oscillations)

- **Standard** Resonant Leptogenesis: Relevant asymmetry is produced in the **strong washout regime**, where sterile neutrinos are **non-relativistic** ($M_N \gg T$).

  Sterile Neutrino self-energy dominated by vacuum cuts: $\not{\Sigma}_N^{\mathcal{A}} \approx$ 

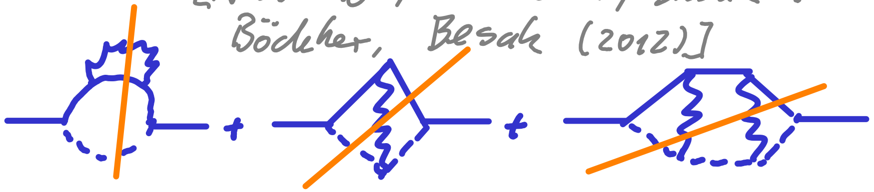
- Leptogenesis from **light sterile neutrinos**: Relevant asymmetry is produced during the first oscillation, where the sterile neutrinos are typically **relativistic** (unless very strongly degenerate):

$$\frac{M_{N_i}^2 - M_{N_j}^2}{T_{osc}} \sim H \sim \frac{T_{osc}^2}{m_{Pl}} \Longrightarrow T_{osc} \sim \left[m_{Pl}\left(M_{N_i}^2 - M_{N_j}^2\right)\right]^{\frac{1}{3}} \overset{\text{no/moderate degeneracy}}{\gg} M_{N_{ij}}$$

Washout negligible around T_{osc}, must remain weak/moderate at lower temperatures.

⟶ For vanishing initial conditions, the deviation of the N around T_{osc} is maximal.

[Anisimov, Bödeker, Besak (2010); Bödeker, Besak (2012)]

$\not{\Sigma}_N^{\mathcal{A}}$ dominated by thermal cuts: $\not{\Sigma}_N^{\mathcal{A}} \approx$  $+ \ldots$

Weak Washout

- If leptogenesis lives on the large initial deviation of the sterile neutrinos for vanishing initial conditions, we may arrive at very different phenomenological perspectives.
- To avoid washout of the asymmetry by close-to-equilibrium neutrinos, have two cards to play (either one or both):

[Akhmedov, Rubakov, Smirnov (1998); Asaka, Shaposhnikov (2005)]

[Drewes, BG (2017); BG (2014)]

If $M_N \ll 100$ GeV, the weak sphaleron freeze-out protects baryon asymmetry before the N equilibrate.

When there are more than two N_i in the game, can couple one flavour (e, μ, τ) so weakly it survives washout.

The Asymmetry in Weak Washout Leptogenesis*

*a.k.a. Leptogenesis from Oscillations

■ Consequences of $M_N \ll T$:

- $2\leftrightarrow 2$ processes determine N_i production & CP-violation rather than $1\leftrightarrow 2$.
- $\omega_{osc} \sim \frac{M_{N_i}^2 - M_{N_j}^2}{T} \ll T \longrightarrow$ can use description in terms of N_i-oscillations
- large non-equilibrium density $Y_N - Y_N^{eq}$ possible & natural.

■ Mixing & oscillating sterile neutrinos: $\delta f_N(k) = f_N(k) - f_N^{eq}(k)$ (matrix-valued distribution function)

$$\frac{\partial}{\partial\eta}\delta f_{Nh} + a^2(\eta)\frac{i}{2k^0}\left[M^2, \delta f_{Nh}\right] + \frac{\partial}{\partial\eta} f^{eq} = -\left\{\mathrm{Re}[YY^\dagger]\frac{\mathrm{tr}[k\hat{\not\Sigma}_N]}{k^0} + ih\,\mathrm{Im}[YY^\dagger]\frac{\mathrm{tr}[\tilde{k}\hat{\not\Sigma}_N]}{k^0}, \delta f_{Nh}\right\}$$

conformal time

helicity

scale-factor (expansion)

source of non-equilibrium

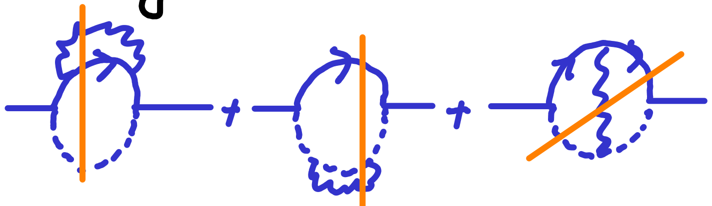
"collision term"

$$\tilde{k} = \left(|\vec{k}|, \frac{\vec{k}}{|\vec{k}|}k^0\right)$$

$\hat{\not\Sigma}_N$ is the reduced (without Yukawas Y) neutrino-self energy.

$\longrightarrow$ Can be computed systematically in Closed-Time-Path formalism/Thermal Field Theory & includes all relevant cuts:

$\hat{\not\Sigma}_N \supset$



etc. [Anisimov, Bödeker, Besak (2010), Bödeker, Besak (2012)]

The Asymmetry in Weak Washout Leptogenesis

- Equation for the production of the lepton asymmetry:

$$\partial_\eta q_{\ell ab} = S_{ab} \sim \int d^4k \, \mathrm{tr}[\not{k} \not{\Delta}_{\ell ab}] \sim \int d^4k$$

propagator for mixing/oscillating sterile neutrinos

flavour $a,b = e,\mu,\tau$

- Most of the asymmetry produced during first few oscillations

Averaged result:

[Drewes, BG (2012)]

$$S_{ab} = \sum_{\substack{i,j\\ i\neq j}} \frac{32}{M_{jj}^2 - M_{ii}^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\sqrt{\vec{k}^2 + M_{ii}^2}}$$

$$* \Big\{ \mathrm{Im}\big[Y_{ai}^\dagger (YY^\dagger)_{ij} Y_{jb}\big] * \Big[(M_{ii}^2 + 2\vec{k}^2)(\hat{\Sigma}_N^{H0\,2} + \hat{\Sigma}_N^{Ai\,2}) - 4|\vec{k}|\sqrt{\vec{k}^2 + M_{ii}^2}\, \hat{\Sigma}_N^{H0}\, \hat{k}^i \hat{\Sigma}_N^{Ai}\Big]$$

$$+ \mathrm{Im}\big[Y_{ai}^\dagger (Y^* Y^t)_{ij} Y_{jb}\big] M_{ii} M_{jj} \hat{\Sigma}_{N\mu}^{H} \hat{\Sigma}_N^{A\mu} \Big\} \delta f_{0hii}(\vec{k})$$

(Standard) lepton-number violating contribution $\sim \frac{M^2}{\Delta M^2}$

$\Rightarrow$ need $\Delta M^2 \ll M^2$ for large enhancement

Lepton number conserving (but flavour violating) contribution $\sim \frac{T^2}{M^2}$

$\Rightarrow$ large enhancement for $\Delta M^2 \ll T \longrightarrow$ smaller/no mass degeneracy needed!

Leptogenesis from Oscillations – Summary Picture

[Akhmedov, Rubakov, Smirnov (1998); Shaposhnikov, Asaka (2005)]

① $T_{osc} \approx (m_{Pl}\, \Delta M^2)^{\frac{1}{3}}$

Sterile neutrinos begin to oscillate and produce $q_{\ell aa}$.

However: $\sum_a q_{\ell aa} = 0 \longrightarrow$ no baryon asymmetry

② $T \gtrsim T_{EW} = 140$ GeV

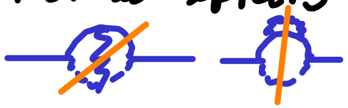
Sterile neutrinos approach equilibrium & absorb parts of the $q_{\ell aa}$, but differently for each a.

Consequence: $\sum_a q_{\ell aa} \neq 0$ and sphalerons create a baryon asymmetry

③ $T \lesssim T_{EW} = 140$ GeV

Sphalerons freeze out during the Electroweak phase transition, such the baryon asymmetry existing at that point is protected against washout from the later equilibration of sterile neutrinos at $T \sim M_N$.

"Standard" Leptogenesis vs. Leptogenesis from Oscillations

	Standard	Oscillations
typical mass range	$M_N > 10^9$ GeV (Davidson/Ibarra)	$10\,\mathrm{MeV} \lesssim M_N \lesssim 100\,\mathrm{GeV}$ (sphaleron freeze-out)
temperature where asymmetry is generated	$T \lesssim M_N$ (strong washout)	$T = (m_{Pl}\,\Delta M_N)^{\frac{1}{3}}$ (begin of oscillations)
deviation from equilibrium	small (strong washout)	maximal for vanishing initial conditions
temperature where washout is important	$T \lesssim M_N$ (strong washout)	$T = T_{sph} \hat{=} 140\,\mathrm{GeV}$
CP violating cut	dominated by vacuum 1→2 	dominated by thermal effects 
resonant enhancement	$\sim \frac{M_1 M_2}{M_1^2 - M_2^2}$	$\sim \frac{T^2}{M_1^2 - M_2^2}$
asymmetry	lepton flavour & lepton number violating	lepton flavour violating → lepton number "hidden in sterile neutrinos"

V. Discovery Opportunities

Where in parameter space Leptogenesis from oscillations works:

Ⓐ With two sterile neutrinos in the game (motivation: minimality, third neutrino needed as Dark Matter...) getting the right Δm_{sol} and Δm_{atm} requires to couple the three SM leptons & sterile neutrinos with similar strength. These couplings are limited by requirement of small washout above T_{EW}

→ upper bound on ν-N mixing

→ small couplings → small initial asymmetry → moderate mass degeneracy needed

→ harder to discover (requires SHIP)

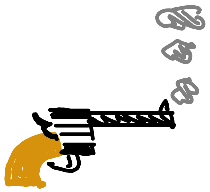
Ⓑ Three sterile neutrinos: can couple one flavour (typically e) weakly, the other two (μ, τ) more strongly

→ initial asymmetry enhanced by large $Y_{i\mu}$, $Y_{i\tau}$. No mass degeneracy needed.

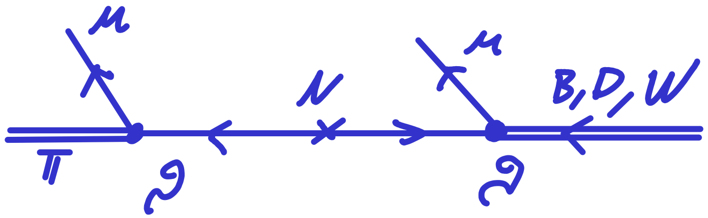
→ protection against washout because of small Y_{ie}.

→ Within b-factory reach because of large $|U_\mu|^2$ (ν_μ-N mixing)

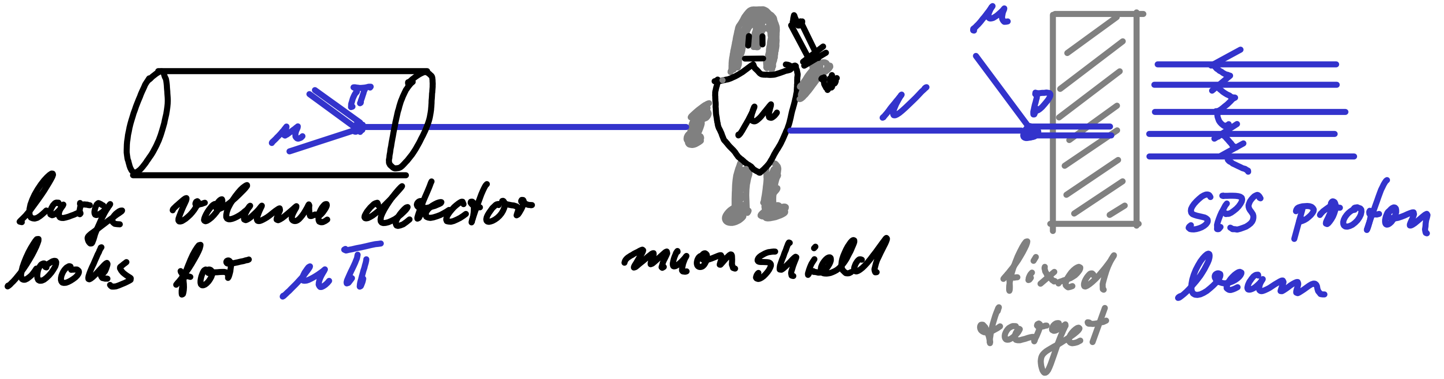
"Direct" Searches



Same sign dileptons



- Search for Hidden Particles (SHIP) proposal:



- Neutrino mixing angles:

[see, e.g. M. Drewes's review (2013)]

→ (unitary) PMNS matrix

$$\underbrace{\mathcal{U}}_{6\times6} = \left[\begin{pmatrix} \mathbb{1}-\frac{1}{2}\theta\theta^{\dagger} & \theta \\ -\theta^{\dagger} & 1-\frac{1}{2}\theta^{\dagger}\theta \end{pmatrix} + \mathcal{O}(\theta^{3})\right]\begin{pmatrix} U_{PMNS} & \\ & U_N^{*} \end{pmatrix}$$

↪ ≈ 𝟙, unless there is large mixing among the N

$$\theta = m_D M_N^{-1} \qquad m_D = h^{t}_{ai}\frac{v}{\sqrt{2}}$$

Model Considered

- Casas-Ibarra parametrisation of see-saw with three sterile neutrinos

$$Y^\dagger = \frac{\sqrt{2}}{V} U_{PMNS} \sqrt{m_\nu} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega_{23} & \sin\omega_{23} \\ 0 & -\sin\omega_{23} & \cos\omega_{23} \end{pmatrix} \begin{pmatrix} \cos\omega_{13} & & \sin\omega_{13} \\ 0 & 1 & 0 \\ -\sin\omega_{13} & 0 & \cos\omega_{13} \end{pmatrix} \begin{pmatrix} \cos\omega_{12} & \sin\omega_{12} & 0 \\ -\sin\omega_{12} & \cos\omega_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \sqrt{M_N}$$

fix $M_1 = 1\,GeV$, $m_1 = 2{,}5 * 10^{-3}\,eV$, & the PMNS angles
$M_2 = 3\,GeV$, $m_2 = 9{,}1 * 10^{-3}\,eV$,
$m_3 = 5 * 10^{-2}\,eV$,

free: M_3, PMNS phases, ω_{ij} (complex!)

- For non-degenerate, GeV scale sterile neutrinos

$$T_{osc} \simeq \sqrt[3]{m_{Pl}\,(1GeV)^2} \simeq 2 * 10^6\,GeV$$

cf. Asaka & Shaposhnikov (2005)
BG & Drewes (2012)

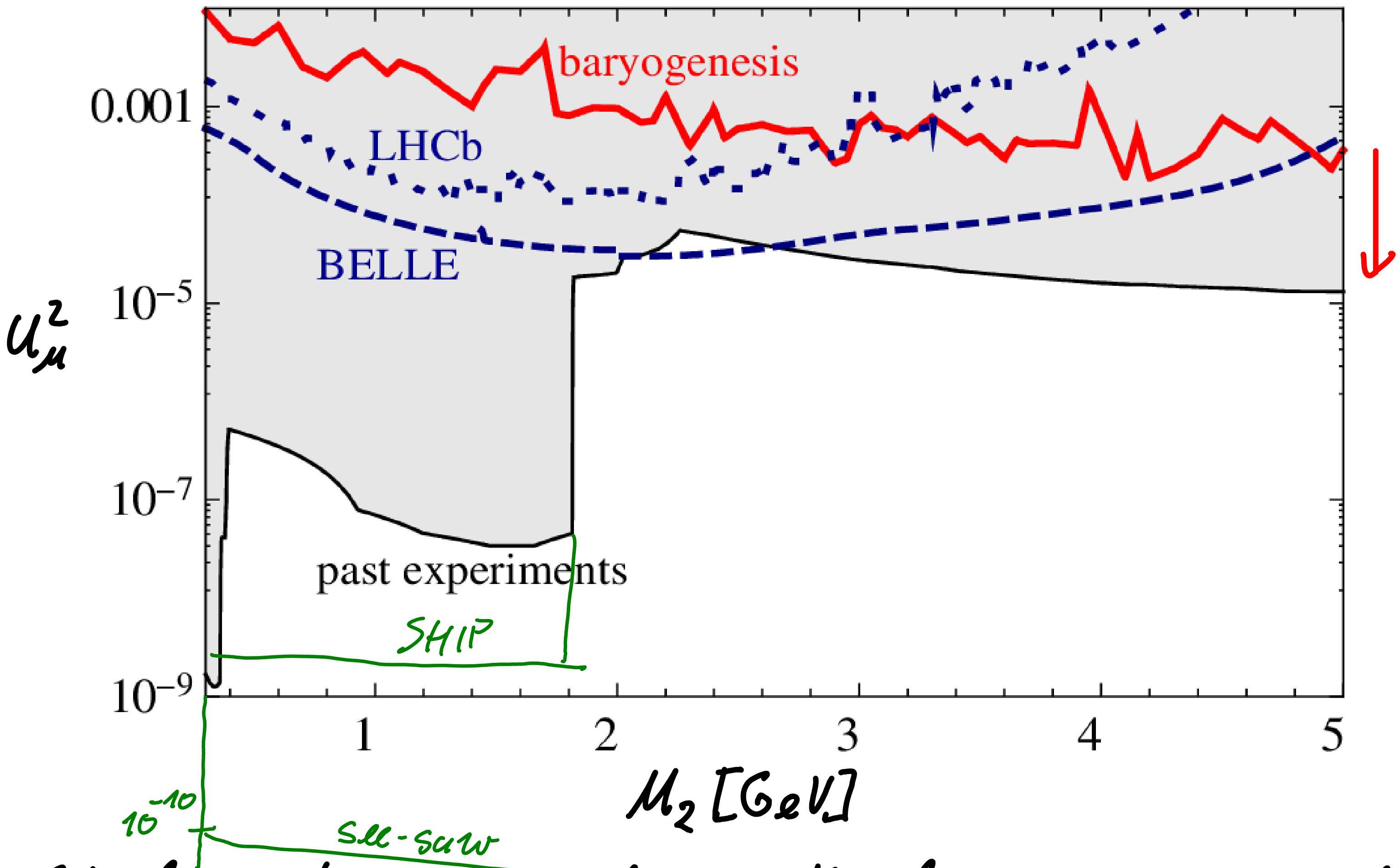
- Initial asymmetry (fast evaluation)

$$\frac{q_{\ell a}}{s} = -i \sum_{\substack{b \\ i \neq j}} \frac{Y^t_{ai} Y_{ib} Y^\dagger_{bj} Y_{ja} - Y^t_{ai} Y^*_{ib} Y^t_{bj} Y_{ja}}{\mathrm{sign}(M^2_{N_i} - M^2_{N_j})} * \left(\frac{m_{Pl}^2}{|M^2_{N_i} - M^2_{N_j}|} \right)^{\frac{2}{3}} * 2{,}4 * 10^{-8}$$

- Evaluate baryon asymmetry after redistribution with the N_i at T_{EW}.

Parameter Scan

- Exclude points inconsistent with upper bound on $Br(\mu \to e\gamma)$ and with $0\nu\beta\beta$ bound $m_{ee} < 0{,}2\,eV$
- Scan $5 * 10^8$ random points of the free parameters



below the red line, there are models that yield at least the observed baryon asymmetry

Canetti, Marco Drewes, BG (2014)

- Sterile neutrinos consistent with baryogenesis can be observed at b and c factories.

Remarks

- The viable points either imply an alignment of Yukawa couplings or a mass degeneracy of order 10^{-3} or a combination of these.
- If $\not{Z}_N\big|_{T=10^6\,\mathrm{GeV}}$ is larger than in the νSM (extra Higgs doublets, ...), we could get along without parametric tweaks altogether.
- No radiative destabilisation of Higgs mass.

Sterile Neutrinos above the Electroweak Scale

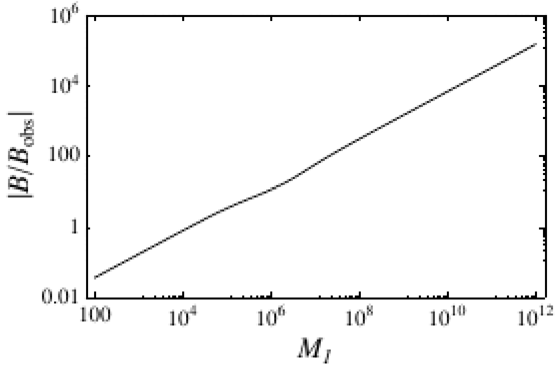
[BG (2014)]

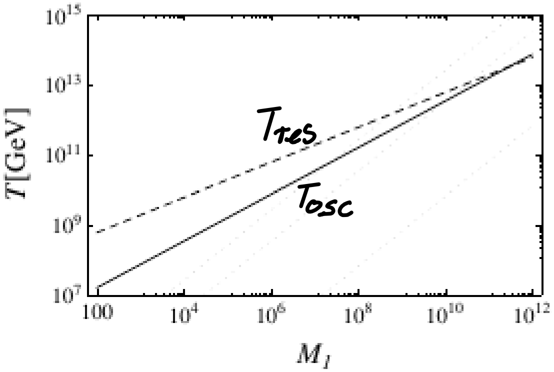
- Instead of freezing out the baryon asymmetry B along with the sphalerons, may consider freeze-out of the $B-L$ asymmetry when $T \lesssim M_N$.
- For sizeable asymmetry to survive, need to couple one active flavour weakly to the sterile neutrinos (cf. Di Bari's studies on N_2-leptogenesis). Well possible, since $m_* \lesssim m_{sol}$.
- Example scenario:

  $\delta = 0{,}2 \quad \alpha_1 = 0 \quad \alpha_2 = 2{,}6$

  $\omega_{23} = 0{,}6 + 1{,}4i; \quad \omega_{13} = 0{,}1 - 1{,}5i \quad \omega_{12} = -1{,}9 - 1{,}0i$

  $M_1 : M_2 : M_3 \longleftrightarrow 1:2:3$

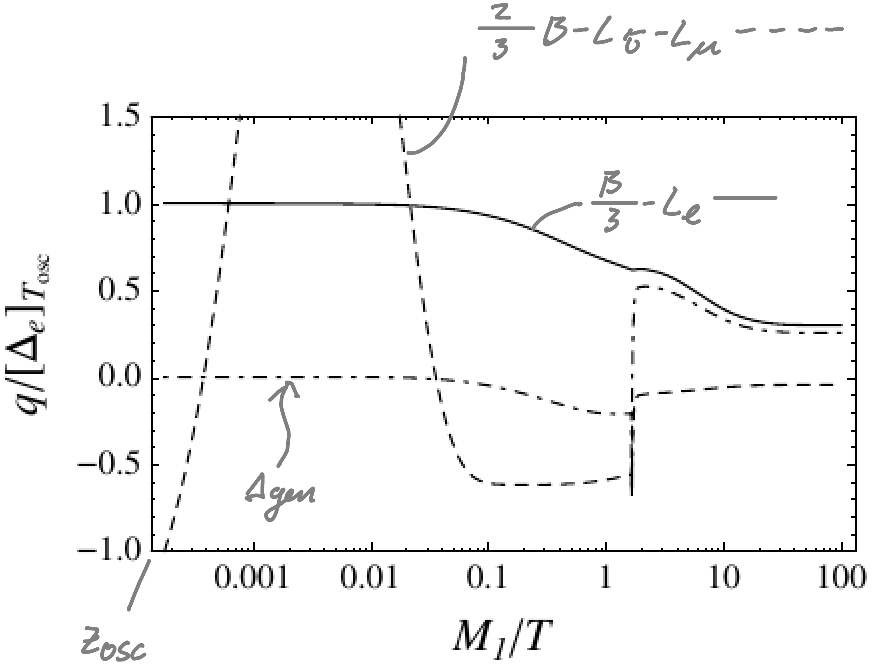




Reheat temperature $\sim 10^8\,\text{GeV}$ viable; no destabilisation of the Electroweak scale.

Sterile Neutrinos above the Electroweak Scale

- Evolution of asymmetries until freeze-out:



$M_1 = 2 * 10^4\,GeV$

$\Delta_{gen} = B - L + (n_{N h=+} - n_{N h=-})$

-·-·-·-

relativistic sterile neutrinos

X-over

non-relativistic sterile neutrinos

VI. Flavoured Leptogenesis

Abada, Davidson, Josse-Michaux, Losada, Riotto (2006)
Nardi, Nir, Roulet, Racker (2006)

- Basis convenient for N_1 decay, where Y_{ia} as in $Y_{ia} N_i \phi \ell_a$ is lower triangular $(N_1\ N_2\ N_3)\begin{pmatrix} x & 0 & 0 \\ x & x & 0 \\ x & x & x \end{pmatrix}\begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix}$ is different in general from lepton flavour basis, where $h_{\tau a}$ as in $h_{\tau a} R_\tau^\dagger \phi^\dagger \ell_a$ is diagonal

  only ℓ_1 couples to N_1 → single flavour leptogenesis, but $\ell_1 = \alpha_e \ell_e + \alpha_\mu \ell_\mu + \alpha_\tau \ell_\tau$ in general

- For $T \lesssim 10^{12}$ GeV (10^9 GeV, 10^4 GeV) $h_{\tau\tau}$ ($h_{\mu\mu}$, h_{ee}) is in equilibrium (interactions faster than expansion rate H)

  In flavour basis: $\begin{pmatrix} q_{ee} & q_{e\mu} & q_{e\tau} \\ q_{\mu e} & q_{\mu\mu} & q_{\mu\tau} \\ q_{\tau e} & q_{\tau\mu} & q_{\tau\tau} \end{pmatrix} \xrightarrow[\text{decoherence}]{\text{complete flavour}} \begin{pmatrix} q_{ee} & 0 & 0 \\ 0 & q_{\mu\mu} & 0 \\ 0 & 0 & q_{\tau\tau} \end{pmatrix}$

  (between 10^4 and 10^9 GeV, only the off-diags including τ evaporate and we may effectively distinguish two flavours)

- So far, calculations in either the fully flavoured or unflavoured regime

- Goal: Calculate asymmetry in intermediate regime (incomplete decoherence)

Importance of Flavour Effects

- No longer maximal coupling to the decaying N → suppression of washout → $O(1)$ enhancement of the asymmetry

- Asymmetries produced in the decays of $N_{2,3}$ not completely washed out by N_1 (different flavour composition)

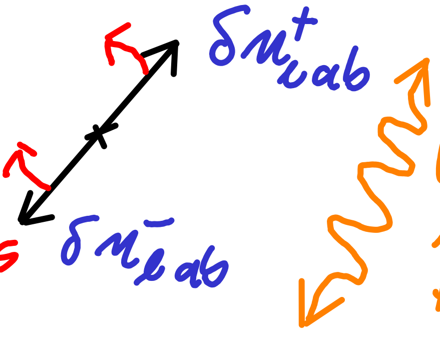
  Engelhard, Grossman, Nardi, Nir (2006); Antusch, Di Bari, Jones & King (2010)

  ↷ Leptogenesis proceeds in several stages & it appears likely that weak washout, partial flavour decoherence play a quantitatively important role

Flavoured Leptogenesis in the CTP approach

- Schwinger-Dyson equations, Green functions straightforwardly decorated with flavour indices
- Need systematic approximations — account for flavour sensitive & flavour blind interactions & dispersion relations
- Flavour blind interactions through $W^{0,\pm}$, B impose $\delta n^{+}_{\ell ab} = -\delta n^{-}_{\ell ab}$ [deviation of (anti-)lepton density from equilibrium]
- Flavour oscillations: $\delta n^{\pm}_{\ell ab} \sim \exp\left[\mp i \# \frac{m_a^{th\,2} - m_b^{th\,2}}{T} t\right]$

thermal masses like to induce flavour oscillations in opposite directions



$W^{0,\pm}$, B like to keep these aligned

Gauge interactions win tug-of-war: oscillations overdamped

Suppression of Flavour Oscillations

Essential dynamics is captured by the toy system

$$\frac{d}{dt}\delta g^+(t) = -i\Delta\omega\,\delta g^+(t) - \Gamma^{bl}\left[\delta g^+(t) + \delta g^-(t)\right]$$

$$\frac{d}{dt}\delta g^-(t) = +i\Delta\omega\,\delta g^-(t) - \Gamma^{bl}\left[\delta g^+(t) + \delta g^-(t)\right]$$

$\Gamma^{bl} \sim g_2^4 T$

$\Delta\omega \sim h_t^2 T \ll \Gamma^{bl}$

⟶ short & long modes: $\delta g_{s,l} \approx \delta g^+ \pm \left(1 \mp i\frac{\Delta\omega}{\Gamma}\right)\delta g^-$

$$\tau_{s,l}^{-1} = \Gamma^{bl} \pm \sqrt{\Gamma^{bl\,2} - \Delta\omega^2}$$

*identify long mode with q_ℓ

*constrain $\delta g^+ + \delta g^- = 0$

$$\tau_s \approx \frac{1}{2\Gamma^{bl}}$$

pair creation/annihilation

$$\tau_l \approx \frac{2\Gamma^{bl}}{\Delta\omega^2} \sim \frac{g^4}{h_t^4 T} \gg \tau^{fl} \sim \frac{1}{g^2 h_t^2 T}$$

flavour oscillations over-damped because of fast pair creation/annihilation

⟶ Flavour sensitive damping dominates the dynamics of off-diagonal densities.

Flavoured Kinetic Equations

$$\frac{\partial q_{\ell ab}}{\partial \eta} = -\sum_c \left[W_{ac}\, q_{\ell cb} + q_{\ell ac}\, W_{cb} \right] + 2 S_{ab} - \Gamma^{\ell\ell}_{\ell ab}$$

washout

source

$$\frac{\partial q_{Rab}}{\partial \eta} = -\Gamma^{\ell\ell}_{Rab}$$

Can work in fixed basis, since oscillations are frozen in.

Take $h = \begin{pmatrix} h_\tau & 0 \\ 0 & 0 \end{pmatrix}$

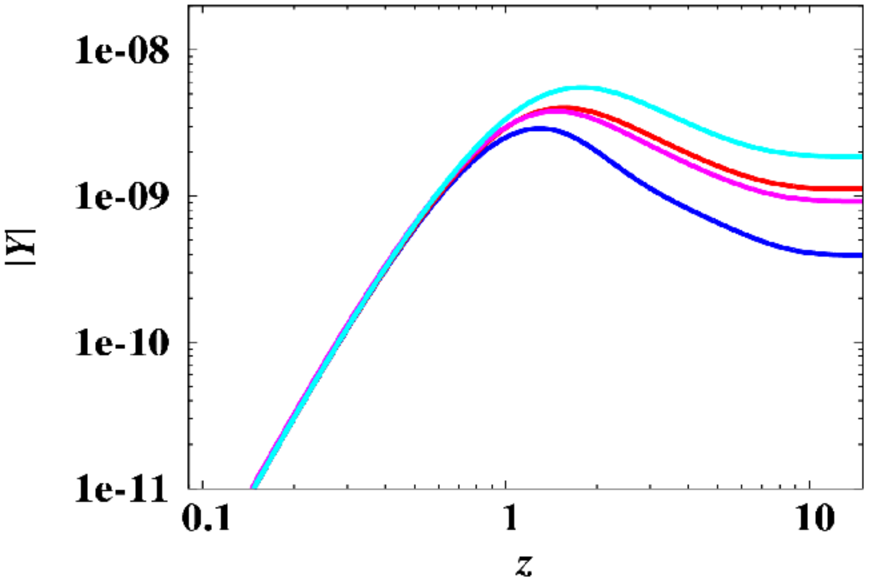
$$\Gamma^{\ell\ell}_{\ell} \sim h_\tau^2 \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} q_\ell + q_\ell \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - 2 \begin{pmatrix} q_{R11} & 0 \\ 0 & 0 \end{pmatrix} \right]$$

$$\Gamma^{\ell\ell}_{R} \sim h_\tau^2 \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} q_R + q_R \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - 2 \begin{pmatrix} q_{\ell 11} & 0 \\ 0 & 0 \end{pmatrix} \right]$$

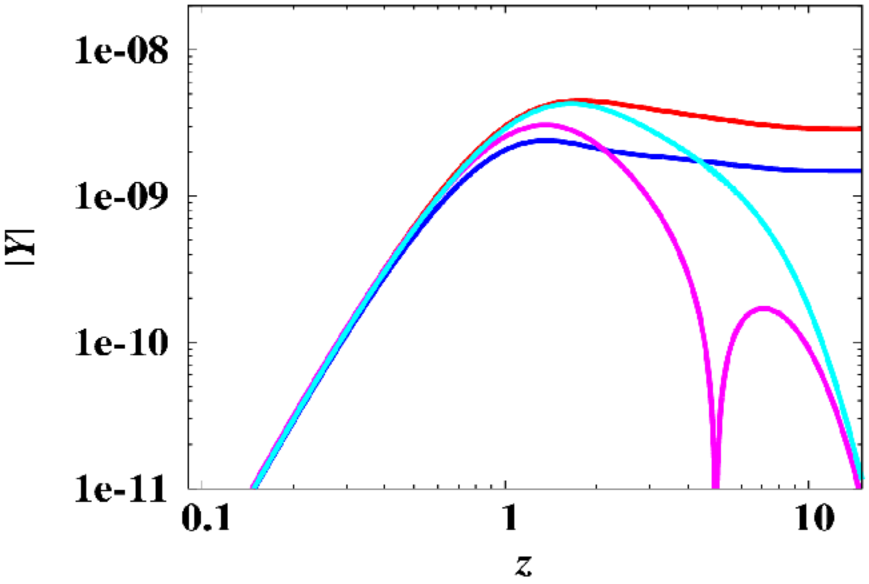
$\longrightarrow$ off-diagonals suppressed for $\Gamma^{\ell\ell}_{\ell,R} \gg H, \Gamma^{ID} = \Gamma_{\ell\phi \to N_1}$

Suppression of the off-Diagonals

$h_\tau = 0$



$h_\tau = 7*10^{-3}$



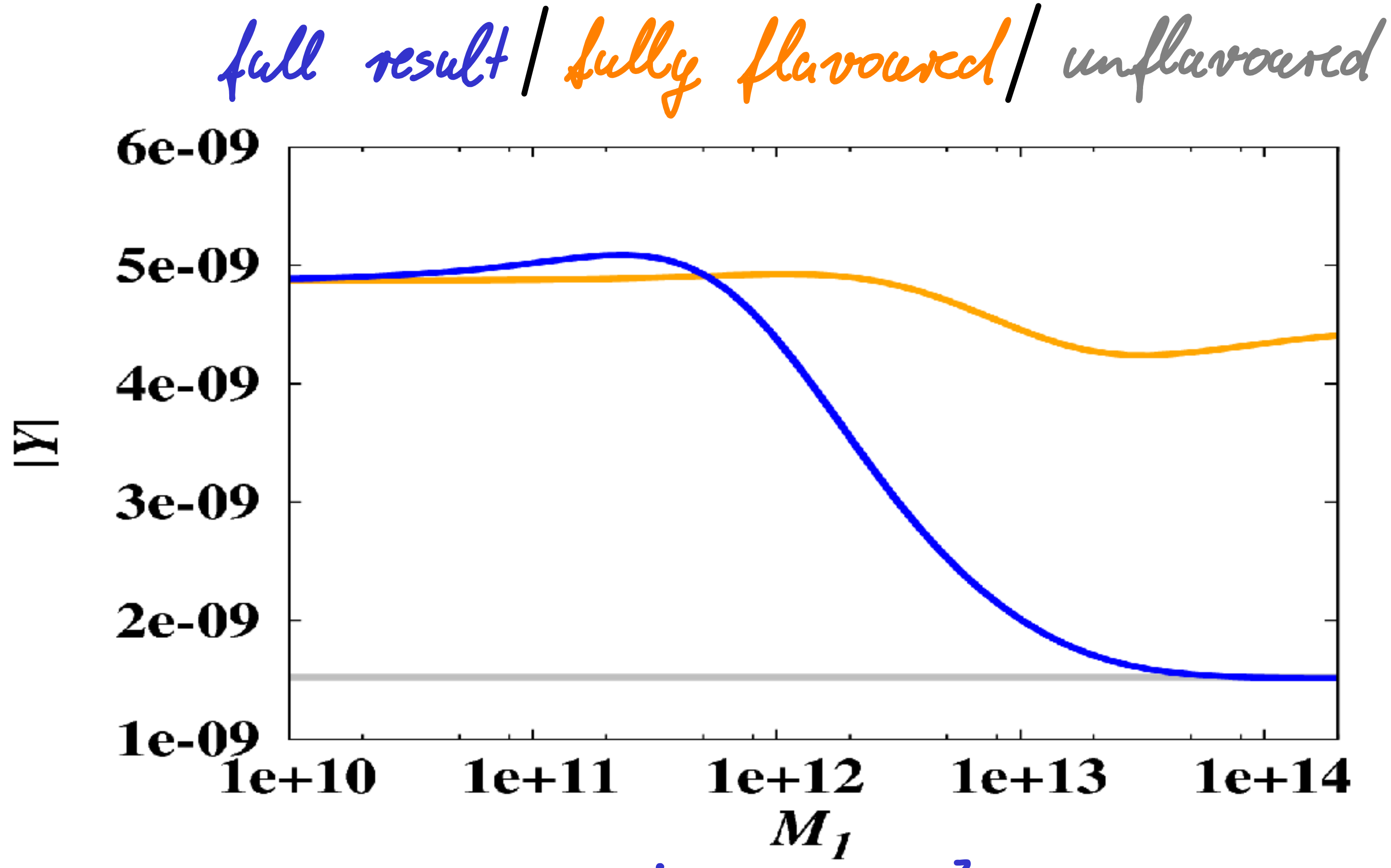
In flavour basis: $\begin{pmatrix} Y_{\ell 11} & Y_{\ell 12} \\ Y_{\ell 21} & Y_{\ell 22} \end{pmatrix}$ lepton number to entropy ratio

$$Y = \begin{array}{c} \left(\begin{array}{cc} 1.4*10^{-2} & 1*10^{-2} \\ i*10^{-1} & 10^{-1} \end{array}\right)\left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\}\text{r.h. neutrino} \\ \begin{array}{cc} \ell_\tau \equiv \ell_1 & \ell_{\mu,e} \equiv \ell_2 \end{array} \end{array}$$

$M_1 = 10^{12}$ GeV

$M_2 = 10^{14}$ GeV

Full Result Interpolates Between Flavoured/Unflavoured Limits



$h_\tau = 7 * 10^{-3}$

$$\left.\begin{array}{l} M_1 \longrightarrow \alpha M_1 \\ Y_{11} \longrightarrow \alpha Y_{11} \\ Y_{12} \longrightarrow \alpha Y_{12} \end{array}\right\} \text{fixed } Y_\ell \text{ in the unflavoured limit}$$

$$Y = \left.\begin{pmatrix} 1.4 * 10^{-2} & 1 * 10^{-2} \\ i * 10^{-1} & 10^{-1} \end{pmatrix}\right\} \text{r.h. neutrino}$$

$$\ell_\tau \equiv \ell_1 \qquad \ell_{\mu,e} \equiv \ell_2$$

VII. Summary

- Baryon Asymmetry of the Universe requires phenomena beyond the Standard Model.
- Well motivated candidate extension: sterile neutrino N.
  → Neutrino oscillations hint that these have favourable out-of-equilibrium behaviour.
- Reliable first-principle theoretical description in terms of the CTP formalism.
- No strong hints to the mass scale before input of theory preference. (Sub)GeV-scale N are however plausible by model-building considerations & viability of leptogenesis.
- In general (especially for high mass), existence of N hard to establish directly.
- High luminosity b, c quark sources & searches for rare decays are a unique possibility to probe the origin of the Baryon Asymmetry of the Universe directly!