Systematic approach to leptogenesis

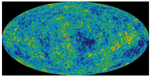
Mathias Garny (CERN)

July 30, 2014

Physics beyond the Standard Model

Collider exp.

Baryon asymmetry



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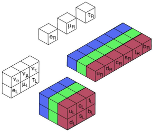

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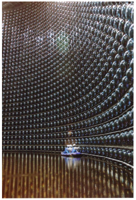

Neutrino exp.

Dark matter

Leptogenesis

Standard Model (SM) extended by three heavy singlet neutrino fields $N_i = N_i^c$, $i = 1, 2, 3$ with Majorana masses $\hat{M} = \mathrm{diag}(M_i)$ in the mass eigenbasis

$$\mathcal{L} \quad = \quad \mathcal{L}_{SM} + \frac{1}{2}\bar{N} i\partial\!\!\!/ N - \frac{1}{2}\bar{N}\hat{M}N - \bar{\ell}\widetilde{\phi} h P_R N - \bar{N} P_L h^\dagger \widetilde{\phi}^\dagger \ell$$

Light neutrino masses via seesaw mechanism

$$m_\nu = -v_{EW}^2 h \hat{M}^{-1} h^T \quad \rightarrow \quad \mathrm{TeV} \lesssim M_i \lesssim M_{GUT} \text{ for } m_e/v_{EW} < h_{ij} < 1$$

Baryogenesis via leptogenesis *Fukugita, Yanagida 86*

- B-violation via L-violating Majorana masses M_i
- CP-violation via Yukawa couplings $\mathrm{Im}[(h^\dagger h)_{ij}] \neq 0$
- Out-of-equilibrium (inverse) decay $N_i \leftrightarrow \ell\phi^\dagger$ and $N_i \leftrightarrow \ell^c\phi$

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$$(\Gamma_i/H)|_{T=M_i} \quad \simeq \quad \left.\frac{(h^\dagger h)_{ii} M_i/8\pi}{1.66 g_* T^2/M_{pl}}\right|_{T=M_i}$$

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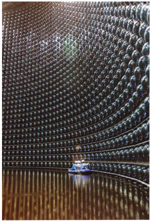
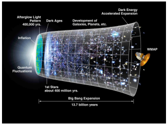
$$(\Gamma_{SM}/H)|_{T=M_i} \quad \sim \quad g^4 M_{pl}/M_i \gg 1 \qquad \text{for } M_i \ll 10^{14}\mathrm{GeV}$$

Leptogenesis

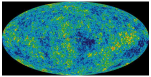


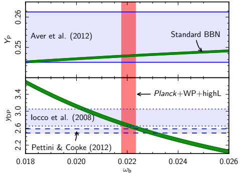
m_ν, θ_{ij}, δ_{CP}

$\leftrightarrow$



$\leftrightarrow$





Planck XVI 1303.5076

$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma} = (6.2 \pm 0.15) \cdot 10^{-10}$$

Relation between neutrino physics and baryon asymmetry depends on

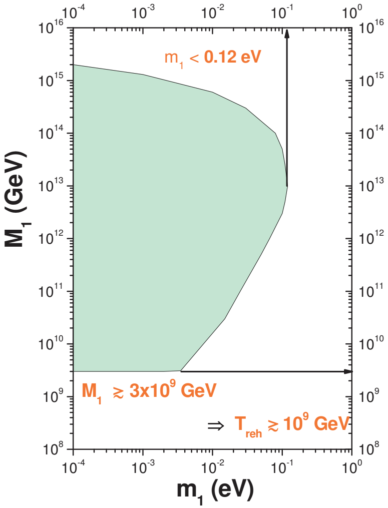
- Model building (seesaw, SO(10), ...)
- Microscopic theory for dynamics

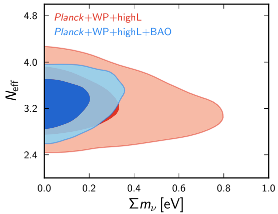
Outline

Systematic approach to leptogenesis

- Vanilla Leptogenesis
- Techniques
- CTP/Kadanoff-Baym approach
- Resonant enhancement

Vanilla Leptogenesis

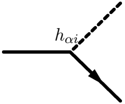




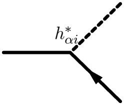
Di Bari 1206.3168 (unflavoured)

Planck 1303.5076

L-violating decay of heavy right-handed neutrino N_i

$\mathcal{M}_{N_i \to \ell_\alpha \phi^\dagger} =$ 

$+ \ldots$

$\mathcal{M}_{N_i \to \ell^c_\alpha \phi} =$ 

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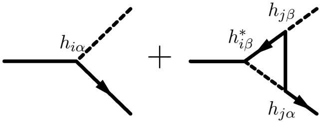
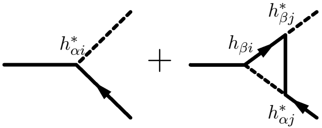
$\mathcal{M}_{N_i \to \ell_\alpha \phi^\dagger} =$  $+ \ldots$

$\mathcal{M}_{N_i \to \ell^c_\alpha \phi} =$  $+ \ldots$

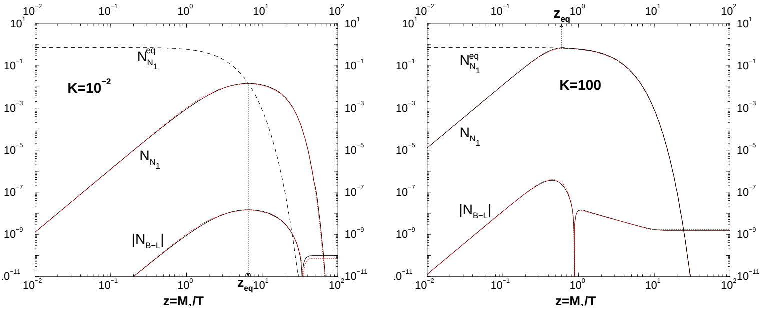
$\Leftrightarrow$ interference of tree and loop processes

$$
\begin{aligned}
\epsilon_i &= \frac{\Gamma_{N_i \to \ell \phi^\dagger} - \Gamma_{N_i \to \ell^c \phi}}{\Gamma_{N_i \to \ell \phi^\dagger} + \Gamma_{N_i \to \ell^c \phi}} \\
&= \sum_{j \neq i} \frac{\mathrm{Im}[(h^\dagger h)^2_{ij}]}{8\pi (h^\dagger h)_{ii}} \left\{ \frac{M_j}{M_i} \left[1 - \left(1 + \frac{M_j^2}{M_i^2} \right) \ln \left(1 + \frac{M_i^2}{M_j^2} \right) \right] + \frac{M_i M_j}{M_i^2 - M_j^2} \right\} \\
\epsilon_1 &\lesssim 10^{-6} \frac{M_1}{10^{10}\mathrm{GeV}} \frac{m_{atm}}{m_{\nu_1} + m_{\nu_3}} \quad \text{unless} \quad \Delta M_N \ll M_N
\end{aligned}
$$

Rate equations

$$\frac{dN_{N_i}}{dt} = -\underbrace{\Gamma_{N_i}}_{\text{equilibration rate}}(N_{N_i} - N_{N_i}^{eq})$$

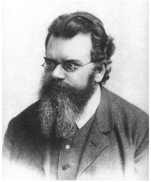
$$\frac{dN_{B-L}}{dt} = \underbrace{\sum_i \Gamma_{\not{C}\not{P},i}(N_{N_i} - N_{N_i}^{eq})}_{\text{source term}} - \underbrace{\Gamma_W N_{B-L}}_{\text{washout term}}$$



Buchmüller, Di Bari, Plümacher 04

Standard Boltzmann approach

$$N_L(t) = \int d^3x \sqrt{|g|} \int \frac{d^3p}{(2\pi)^3} \sum_\ell [f_\ell(t, \mathbf{x}, \mathbf{p}) - f_{\bar{\ell}}(t, \mathbf{x}, \mathbf{p})]$$



$$\begin{aligned} p^\mu \mathcal{D}_\mu f_\ell(t, \mathbf{x}, \mathbf{p}) \; = \; & \sum_i \int d\Pi_{N_i} d\Pi_h \\ & \times (2\pi)^4 \delta(p_\ell + p_h - p_{N_i}) \\ & \times \Big[|\mathcal{M}|^2_{N_i \to \ell\phi^\dagger} \; f_{N_i}(1 - f_\ell)(1 + f_\phi) + \dots \\ & \quad - \; |\mathcal{M}|^2_{\ell\phi^\dagger \to N_i} \; f_\ell f_\phi (1 - f_{N_i}) + \dots \Big] \end{aligned}$$

(Naive) LO rates

- Equilibration $\mathcal{O}(h^2)$ $\quad (N_i \leftrightarrow \ell\phi^\dagger)_{tree}$

$$\Gamma_{N_i} = \Gamma^0_{N_i} \left\langle \frac{M_i}{E_i} \right\rangle_T \qquad \text{where } \Gamma^0_{N_i} = \frac{(h^\dagger h)_{ii} M_i}{8\pi}$$

- Source term $\mathcal{O}(h^4)$ $\quad (N_i \leftrightarrow \ell\phi^\dagger)_{1-loop}, \quad (\ell\phi^\dagger \leftrightarrow \ell^c\phi)_{1-loop}\big|_{RIS}$

$$\Gamma_{\not{C}\not{P},i} = \epsilon_i \Gamma_{N_i} \qquad \epsilon_i = \frac{\Gamma_{N_i\to\ell\phi^\dagger} - \Gamma_{N_i\to\ell^c\phi}}{\Gamma_{N_i\to\ell\phi^\dagger} + \Gamma_{N_i\to\ell^c\phi}}$$

- Washout term $\mathcal{O}(h^2)$ $\quad (\ell\phi^\dagger \to N_i)_{tree}$

$$\Gamma_W = \frac{8}{\pi^2}\left(c_\ell + \frac{c_\phi}{2}\right) N^{eq}_{N_i} \Gamma_{N_i} \qquad \text{where } c_{\ell(\phi)} = -\frac{N_{\ell(\phi)}}{N_{B-L}}$$

Beyond the vanilla case

- Other models (type-II seesaw, susy, . . . huge amount of literature)

  see e.g. review Nir Nardi Davidson 0802.2962

- Compute coefficients more precisely *→ talk by Biondini*
  - Equilibration rate (↔ production rate at LO in N-Yukawa h)
  - Washout
- Take qualitatively new effects into account (extend/scrutinize equations)
  - Active lepton flavor effects *→ talk by Garbrecht*

$$\ell_\tau \leftrightarrow \tau_R \phi \quad \text{vs} \quad (h_{1e}\ell_e + h_{1\mu}\ell_\mu + h_{1\tau}\ell_\tau)\phi \leftrightarrow N_1$$

  - N_i flavor effects (resonant enhancement, oscillations) *→ here+Kartavtsev*

$$\epsilon_{N_i}^{wave} = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi (h^\dagger h)_{ii}} \times \frac{M_1 M_2}{M_2^2 - M_1^2}$$

  - Partial flavor/spectator equilibration
  - Kinetic non-equilibrium for RH neutrinos
- Check the mechanism (interference+nonequilibrium)

Techniques

- Compute coefficients more precisely
  - Add processes to Boltzmann eq. ($tN \leftrightarrow \ell\phi$)
    Include thermal masses in kinematics
  - Map rates to thermal correlation functions/suszeptibilities
    Consistent expansion in g, y_t for $M_N/T \sim g, 1, \gg 1$
- Take qualitatively new effects into account (extend/scrutinize equations)
  - Boltzmann eqs.

$$N_{B-L} \to N_{B-\frac{1}{3}L_\alpha} \qquad N_{N_1} \to N_{N_i}$$

  - Density matrix equations

$$N_{B-L} \to \rho_{\alpha\beta} \qquad N_{N_1} \to \rho_{ij}$$

  - Derive kinetic equations based on CTP/Kadanoff-Baym

$$N_{B-L} \to \langle \ell_\alpha \ell_\beta \rangle \qquad N_{N_1} \to \langle N_i \bar{N}_j \rangle$$

- Check the mechanism (interference+nonequilibrium)
  - Derive source term starting from CTP/Kadanoff-Baym
    . . . or from van Neumann eq.

Production rate

Boltzmann ('naive' NLO in N-yukawa and SM couplings)

$$
\begin{array}{ll}
2 \leftrightarrow 2 & AN \leftrightarrow \ell h, tN \leftrightarrow \ell h, \qquad A = ad_{SU(2)\times U(1)} \\
1 \leftrightarrow 2 & \text{vertex+wavefctn virtual} \\
1 \leftrightarrow 3 & N \leftrightarrow \ell h A, N \leftrightarrow \ell h t
\end{array}
$$

$\rightarrow$ blocking of $N \leftrightarrow \ell\phi$ for $M_N \ll T$ ('LO$\rightarrow$ 0'), instead $\phi \leftrightarrow N\ell$

All orders in SM couplings, leading order in N-yukawa h,

$$\frac{d\Gamma_{N_1}}{d^3k} = \frac{1}{(2\pi)^3 2E_k} 2f_{FD}(E_k)\mathrm{Im}\mathrm{Tr}[\not{k}\Sigma_R(k)]_{k=(E_k+i0^+,\mathbf{k})}$$

- $M_N \gg T$ NLO

  RTF $\mathcal{O}(T^2/M_N^2)$ Lodone, Strumia 1106.2814;
  ITF $\mathcal{O}(T^4/M_N^4)$ Laine 1209.2869;
  NR-EFT Biondini, Brambilla, Escobedo, Vairo 1307.7680

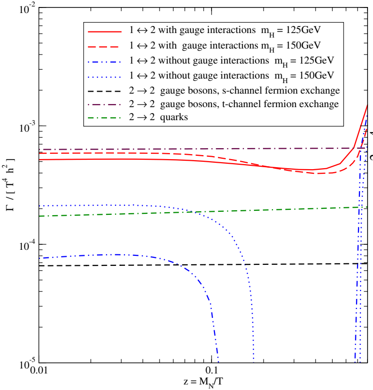
- $M_N \sim T$ NLO

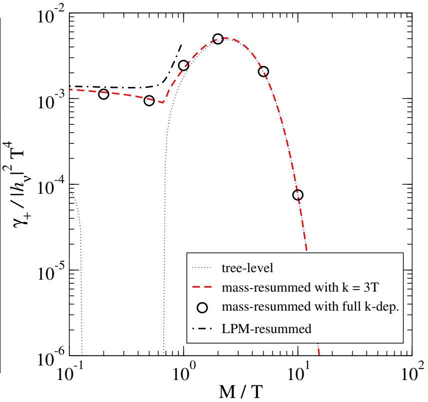
  ITF Laine 1307.4909;
  CTP Garbrecht, Glowna, Herranen 1302.0743

- $M_N \sim gT$ LO LPM $N \leftrightarrow \ell\phi$, $\phi \leftrightarrow N\ell$, $\ell \leftrightarrow N\phi$ $\quad + \quad 2 \rightarrow 2$

  Anisimov, Bodeker, Besak 1012.3784; Besak Bodeker 1208.1288;
  Garbrecht, Glowna, Schwaller 1303.5498

Production rate





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cf. Garbrecht, Glowna, Herranen 1302.0743; Garbrecht, Glowna, Schwaller 1303.5498

Source Term - Double Counting Problem

Naive contribution from decay/inverse decay

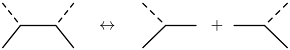
$$|\mathcal{M}|^2_{N_i \to \ell\phi^\dagger} = |\mathcal{M}_0|^2(1+\epsilon_i) \qquad |\mathcal{M}|^2_{\ell\phi^\dagger \to N_i} = |\mathcal{M}_0|^2(1-\epsilon_i)$$

$$|\mathcal{M}|^2_{N_i \to \ell^c\phi} = |\mathcal{M}_0|^2(1-\epsilon_i) \qquad |\mathcal{M}|^2_{\ell^c\phi \to N_i} = |\mathcal{M}_0|^2(1+\epsilon_i)$$

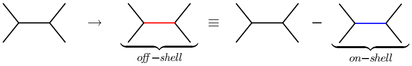
$$\begin{aligned}\frac{dN_{B-L}}{dt} &\propto (|\mathcal{M}|^2_{N_i \to \ell\phi^\dagger} - |\mathcal{M}|^2_{N_i \to \ell^c\phi})N_{N_i} \\ &\quad - (|\mathcal{M}|^2_{\ell\phi^\dagger \to N_i} - |\mathcal{M}|^2_{\ell^c\phi \to N_i})N^{eq}_{N_i} \\ &\propto \epsilon_i(N_{N_i} + N^{eq}_{N_i})\end{aligned}$$

$\Rightarrow$ spurious generation of asymmetry even in equilibrium

Origin: Double Counting Problem



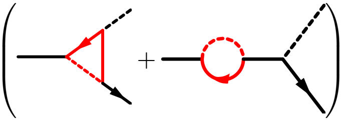
$\rightarrow$ Can be fixed by real intermediate state subtraction in $\ell\phi^\dagger \leftrightarrow \ell^c\phi$ (at least close to equilibrium)



Resonant enhancement

Efficiency of leptogenesis depends on CP-violating parameter, which is one-loop suppressed

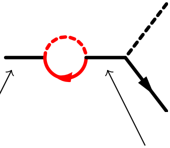
$$\epsilon_{N_i} = \frac{\Gamma(N_i \to \ell\phi^\dagger) - \Gamma(N_i \to \ell^c\phi)}{\Gamma(N_i \to \ell\phi^\dagger) + \Gamma(N_i \to \ell^c\phi)} \propto \mathrm{Im}$$



Self-energy (or 'wave') contribution to CP-violating parameter features a resonant enhancement for a quasi-degenerate spectrum $M_1 \simeq M_2 \ll M_3$

$$\epsilon_{N_i}^{wave} = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi(h^\dagger h)_{ii}} \times \frac{M_1 M_2}{M_2^2 - M_1^2}$$

Flanz Paschos Sarkar 94/96; Covi Roulet Vissani 96;



On-shell initial N_1: $p^2 = M_1^2$ Internal N_2: $\frac{i}{p^2 - M_2^2}$

Resonant enhancement

- *Flanz Paschos Sarkar Weiss 96;* effective Hamiltonian approach

$$\epsilon_{N_i} = -\frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{16\pi(h^\dagger h)_{ii}} \frac{M_1(M_2 - M_1)}{(M_2 - M_1)^2 + M_1^2(\mathrm{Re}(h^\dagger h)_{12}/(16\pi))^2}$$

- *Covi Roulet 96;* CP violating decay of mixing scalar fields described by effective mass matrix; formalism as in Liu Segre 93
- *Pilaftsis 97; Pilaftsis Underwood 03;* Pole mass expansion of resummed propagators

$$\epsilon_{N_i} = \frac{\mathrm{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2)M_i\Gamma_j}{(M_i^2 - M_j^2)^2 + M_i^2\Gamma_j^2}$$

- *Buchmüller Plümacher 97;* Diagonalization of resummed propagators

$$\epsilon_{N_i} = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi(h^\dagger h)_{ii}} \frac{M_1 M_2(M_2^2 - M_1^2)}{(M_2^2 - M_1^2 - \frac{1}{\pi}\Gamma_i M_i \ln \frac{M_2^2}{M_1^2})^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2}$$

- *Rangarajan Mishra 99;* comparison of different approaches
- *Anisimov Broncano Plümacher 05;* Reconciliation of diagonalization approach with the pole mass expansion approach
- Invariant quantity $M_1 M_2(M_2^2 - M_1^2)\mathrm{Im}(h^\dagger h)_{12}^2$ related to CP violation appears in the enumerator

Resonant enhancement

The results can be summarized (neglecting log-corrections) as

$$\epsilon_{N_i} = \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi (h^\dagger h)_{ii}} \times R, \qquad R \equiv \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + A^2}$$

Different calculations correspond to different expressions for the 'regulator' A^2

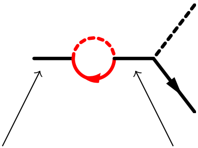
$$A^2 = \begin{cases} \frac{1}{4}(M_1 + M_2)^4 \left(\frac{\mathrm{Re}(h^\dagger h)_{12}}{16\pi} \right)^2 & \textit{Flanz Paschos Sarkar Weiss 96} \\ M_i^2 \Gamma_j^2 & \textit{Pilaftsis 97; Pilaftsis Underwood 03} \\ (M_1\Gamma_1 - M_2\Gamma_2)^2 & \textit{Buchmüller Plümacher 97;} \\ & \textit{Anisimov Broncano Plümacher 05; ...} \\ \dots & \end{cases}$$

The regulator is relevant for determining the maximal possible resonant enhancement, which occurs for $M_2^2 - M_1^2 = \pm A$, and is given by

$$R_{max} = \frac{M_1 M_2}{2|A|}$$

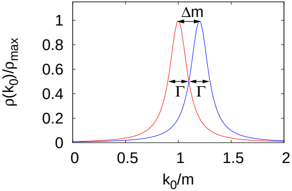
Resonant enhancement

The origin of the regulator is the finite width of N_1 and N_2



Off-shell initial N_1: $p^2 = M_1^2 + iM_1\Gamma_1$ Internal N_2: $\frac{i}{p^2 - M_2^2 - iM_2\Gamma_2}$

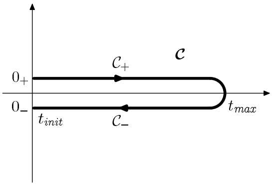
In the maximal resonant case $M_2 - M_1 = \mathcal{O}(\Gamma_i)$, the spectral functions overlap



but deviation from equilibrium is also essential

Closed time path / Schwinger-Keldysh / *in-in* formalism

$$\begin{aligned}\langle \mathcal{O}(t)\rangle &= \mathrm{Tr}\left(\rho\, U_I(t_{init},t)\,\mathcal{O}_I(t)\,U_I(t,t_{init})\right)\\ &= \mathrm{Tr}\left(\rho\ \tilde{T}\left[\exp\left(+i\int_{t_{init}}^{t} dt'\,H_I(t')\right)\right]\mathcal{O}_I(t)\ T\left[\exp\left(-i\int_{t_{init}}^{t} dt'\,H_I(t')\right)\right]\right)\end{aligned}$$



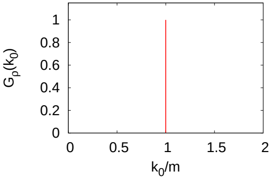
$$\langle \mathcal{O}(t)\rangle = \mathrm{Tr}\left(\rho\ T_{\mathcal{C}}\left[\exp\left(+i\int_{\mathcal{C}} dt'\,H_I(t')\right)\mathcal{O}_I(t)\right]\right)$$

Statistical propagator $S_F^{ij}(x,y) = \langle N_i(x)\bar{N}_j(y) - \bar{N}_j(y)N_i(x)\rangle/2$

Spectral function $S_\rho^{ij}(x,y) = i\langle N_i(x)\bar{N}_j(y) + \bar{N}_j(y)N_i(x)\rangle$

Boltzmann limit

- on-shell quasi-stable particles



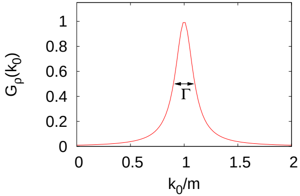
$$S_\rho^{ij}(k) \sim \delta^{ij}\delta(k^2 - m_i^2)$$

- equilibrium-like
  fluctuation-dissipation relation

$$S_F^{ij}(t,k) = \left(\frac{1}{2} - f_k^i(t)\right) S_\rho^{ij}(k)$$

More general

- spectrum with (thermal) width



$$S_\rho^{ij}(t,k) \propto \frac{\delta^{ij} 2k_0 \Gamma_i(t)}{(k^2 - m_{th,i}^2(t))^2 + k_0^2\Gamma_i(t)^2} + \ldots$$

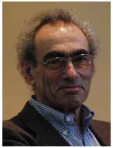
- coherent $N_1 - N_2$ transitions

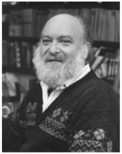
$$S_F^{ij}(t,k) = \begin{pmatrix} S_F^{11} & S_F^{12} \\ S_F^{21} & S_F^{22} \end{pmatrix}$$

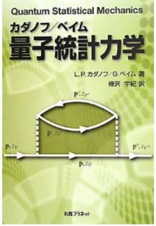
Kadanoff-Baym equations

$$
\begin{aligned}
((i\partial\!\!\!/_x - M_i)\delta^{ik} - \delta\Sigma_N(x)^{ik})S_F^{kj}(x,y) &= \int_0^{x^0} dz^0 \int d^3z\, \Sigma_{N\rho}^{ik}(x,z) S_F^{kj}(z,y) \\
&\quad - \int_0^{y^0} dz^0 \int d^3z\, \Sigma_{NF}^{ik}(x,z) S_\rho^{kj}(z,y) \\
((i\partial\!\!\!/_x - M_i)\delta^{ik} - \delta\Sigma_N(x)^{ik})S_\rho^{kj}(x,y) &= \int_{y^0}^{x^0} dz^0 \int d^3z\, \Sigma_{N\rho}^{ik}(x,z) S_\rho^{kj}(z,y)
\end{aligned}
$$

- Statistical propagator encodes time-evolution of the state
- Spectral function includes off-shell effects self-consistently
- Conserving, non-secular for $\Sigma = \delta\Gamma/\delta S$ (2PI); nPI







CTP/Kadanoff-Baym approach to leptogenesis

$$j_L^\mu(x) = \left\langle \sum_\alpha \bar{\ell}_\alpha(x)\gamma^\mu \ell_\alpha(x) \right\rangle = -\mathrm{tr}\left[\gamma^\mu S_\ell{}^{\alpha\beta}(x,x)\right]$$

Lepton asymmetry

$$n_L(t) = \frac{1}{V}\int_V d^3x\, j_L^0(t,\mathbf{x})$$

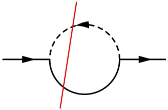
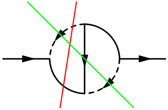
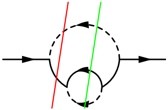
Equation of motion

$$\frac{dn_L}{dt} = \frac{1}{V}\int_V d^3x\,\partial_\mu j_L^\mu(x) = -\frac{1}{V}\int_V d^3x\,\mathrm{tr}\left[\gamma_\mu(\partial_x^\mu + \partial_y^\mu)S_\ell{}^{\alpha\beta}(x,y)\right]_{x=y}$$

Use KB equations for leptons on the right-hand side $\Rightarrow$

$$\begin{aligned}\frac{dn_L}{dt} \quad = \quad & i\int_0^t dt' \int \tfrac{d^3p}{(2\pi)^3}\,\mathrm{tr}\Big[\Sigma_{\ell}{}_{\rho\,\mathbf{p}}^{\alpha\gamma}(t,t')S_{\ell}{}_{F\,\mathbf{p}}^{\gamma\beta}(t',t) - \Sigma_{\ell}{}_{F\,\mathbf{p}}^{\alpha\gamma}(t,t')S_{\ell}{}_{\rho\,\mathbf{p}}^{\gamma\beta}(t',t)\\ & - S_{\ell}{}_{\rho\,\mathbf{p}}^{\alpha\gamma}(t,t')\Sigma_{\ell}{}_{F\,\mathbf{p}}^{\gamma\beta}(t',t) + S_{\ell}{}_{F\,\mathbf{p}}^{\alpha\gamma}(t,t')\Sigma_{\ell}{}_{\rho\,\mathbf{p}}^{\gamma\beta}(t',t)\Big]\end{aligned}$$

CTP/Kadanoff-Baym approach to leptogenesis

			
$N \leftrightarrow \ell\phi^\dagger$ $N \leftrightarrow \ell^c\phi$	$\lvert tree \rvert^2$	tree $\times$ vertex-corr.	tree $\times$ wave-corr.
$\ell\phi^\dagger \leftrightarrow \ell^c\phi$		s $\times$ t	s $\times$ s, t $\times$ t

- unified description of CP-violating decay, inverse decay, scattering
- dn_L/dt vanishes in equilibrium due to KMS relations

$$S_F^{eq} = \frac{1}{2}\tanh\left(\frac{\beta k^0}{2}\right) S_\rho^{eq} \qquad \Sigma_F^{eq} = \frac{1}{2}\tanh\left(\frac{\beta k^0}{2}\right) \Sigma_\rho^{eq}$$

$\Rightarrow$ consistent equations free of double-counting problems

Two strategies

1. Derive kinetic equations

$$S(t,k) = \int ds\, e^{iks} D(t + s/2, t - s/2)$$

Gradient expansion $\partial_t \partial_k \sim \frac{\text{slow}}{\text{fast}} \sim \frac{\Gamma, H, y^2 T, \Delta M}{M, T}$

$$\int dz\, \Sigma(x,z) S(z,y) \to \Sigma(t,k) S(t,k) + \frac{i}{2}\left(\frac{\partial \Sigma}{\partial t}\frac{\partial S}{\partial k} - \frac{\partial \Sigma}{\partial k}\frac{\partial S}{\partial t}\right)$$

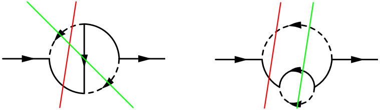
On-shell limit

$$\begin{aligned}
S^{ij}_\rho(t,k) &\to U^{in}(t)\delta^{nm}(\not{k} - M_n)\delta(k^2 - M_n^2(t))U^{\dagger nj}(t) \\
&\to \delta^{ij}(\not{k} - M_{av})\delta(k^2 - M_{av}^2) \\
S^{ij}_F(t,k) &\to \left(\frac{1}{2}\delta^{ij}\delta_{hh'} - f^{ij}_{hh'}(t,\mathbf{k})\right) u_h(\mathbf{k})\bar{u}_{h'}(\mathbf{k})\delta(k^2 - M_{av}^2)
\end{aligned}$$

2. Solve two-time KB-eqs. for some simplified setup to study generation of the asymmetry (thermal bath)

Kinetic equations

$$\partial_t n_L = 16\pi\,(h^\dagger h)_{11} \int \frac{d^3p d^3q d^3k}{(2\pi)^9 8\omega_p k^0 q^0}\,\Theta(p_0)(2\pi)^4\delta(k-p-q)$$
$$\times\,\big(f_N(p)(1-f_\ell(k))(1+f_\phi(q) - (1-f_N(p))f_\ell(k)f_\phi(q)\big)$$
$$\times\,\epsilon_1(t,k,p,q)$$



$$\epsilon_i^{vertex}(t,k,p,q) = \sum_j \frac{Im(h^\dagger h)^2_{ij}}{8\pi(h^\dagger h)_{11}} \int d\Pi_{k_1} d\Pi_{k_2} d\Pi_{k_3}$$
$$\times\,(2\pi)^4\delta(k+k_1+k_2)(2\pi)^4\delta(k_2-k_3+q)$$
$$\times\,\mathrm{tr}[D^\ell_\rho(t,k_1)D^\phi_F(t,k_2)D^{N_j}_h(t,k_3) + \{k_1 \leftrightarrow k_2\}$$
$$+\,D^\ell_h(t,k_1)D^\phi_F(t,k_2)D^{N_j}_\rho(t,k_3) + \{k_1 \leftrightarrow k_2\}$$
$$+\,D^\ell_\rho(t,k_1)D^\phi_h(t,k_2)D^{N_j}_F(t,k_3) - \{k_1 \leftrightarrow k_2\}]$$

$\Rightarrow$ consistent equations w/o need for RIS subtractions

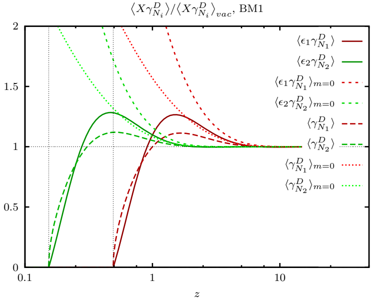
works also for arbitrary $f \neq f_{eq}$

Hierarchical limit $M_1 \ll M_{2,3}$

$$
\begin{aligned}
\partial_t n_L &= 16\pi\epsilon_1 \int_{\mathbf{p},\mathbf{q},\mathbf{q'},\mathbf{k},\mathbf{k'}} \frac{k\cdot k'}{M_1}(2\pi)^4\delta(p-k-q)(2\pi)^4\delta(p-k'-q') \\
&\quad \times (f_{N_1}(p) - f^{eq}_{N_1}(p))(1-f_\ell(k)+f_\phi(q))(1-f_\ell(k')+f_\phi(q'))
\end{aligned}
$$

MG, Kartavtsev, Hohenegger, Lindner 09; Beneke, Garbrecht, Herranen, Schwaller 10

$$\frac{\Gamma_{\not{C}\not{P}}}{\Gamma^{Boltzmann}_{\not{C}\not{P}}}$$



Frossard, MG, Hohenegger, Kartavtsev, Mitrouskas 12

Symmetry quantum statistics vs thermal loop corr., important for models where $\epsilon^{vac} = 0$

Garbrecht, Ramsey-Musolf 13

Flavoured leptogenesis

$$\mathcal{L}_{int} = -\bar{\ell}_a \widetilde{\phi} h_{ai} P_R N_i - y_{ab} \bar{e}_{R,a} \phi \ell_b \qquad y_{ab} = \mathrm{diag}(m_e m_\mu m_\tau)/v_{EW}$$

$$q_\ell^{ab} = (\delta n_\ell^+ - \delta n_\ell^-)^{ab},\ \delta n_\ell^{ab} = \int \frac{d^3k}{(2\pi)^3} (f_\ell^{ab} - f_{\ell,eq}^{ab}) = \mu_{ab} \frac{T^2}{12},\ \Xi = \dot{U} U^\dagger$$

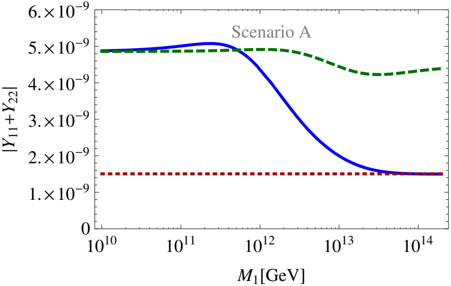
$$\begin{aligned} \partial_t q_\ell + 3 H q_\ell &= S + [\Xi, q] - \{W, q\} - \Gamma_{LR}(y^\dagger y q_\ell + q_\ell^\dagger y^\dagger y - y^\dagger q_R y - y^\dagger q_R^\dagger y) \\ \partial_t q_R + 3 H q_R &= -\Gamma_{LR}(y y^\dagger q_R + q_R^\dagger y y^\dagger - y q_\ell y^\dagger - y q_\ell^\dagger y^\dagger) \end{aligned}$$

Beneke, Garbrecht, Fidler, Herranen, Schwaller 1007.4783

- Γ_{LR}-term decohere $a \neq b$ terms in y=diag basis and equilibrate $q_{\ell,ab}$, $q_{R,ab}$
- Gauge interactions impose $\delta n^+ + \delta n^- = 0 \Rightarrow$ no oscillations

Flavoured leptogenesis

- Unflavoured $\Gamma_{LR} \ll H \Rightarrow$ project on flavor that couples to N_1
- Flavoured $\Gamma_{LR} \gg H \Rightarrow$ project on flavor that couples to τ (and $\perp$ or e, μ)
- Full $\Gamma_{LR} \sim H$, off-diagonal comp. of q_ℓ^{ab} important



Beneke, Garbrecht, Fidler, Herranen, Schwaller 1007.4783; Garbrecht, Glowna, Schwaller 1303.5498

Flavoured leptogenesis

Source term $N_1 \to \ell_a \phi \quad (\mathcal{O}(h^4))$

$$S_{ab} = \sum_j \Big[\underbrace{(h_{a1} h^T_{1c} h^*_{cj} h_{jb} - h^*_{b1} h^\dagger_{1c} h_{cj} h^*_{ja})}_{\mathrm{Tr} \propto \epsilon_1} \underbrace{S^{LNV}}_{\propto M_1 M_j} + \underbrace{(h_{a1} h^\dagger_{1c} h_{cj} h_{jb} - h^*_{b1} h^T_{1c} h^*_{cj} h^*_{ja})}_{\mathrm{Tr} = 0} \underbrace{S^{LFV}}_{\propto T^2} \Big]$$

Garbrecht, Drewes 12

Washout $\ell_a \phi \to N_1$

$$W_{ab} \propto h_{a1} h^\dagger_{1a}$$

- Potentially large effects for ultrarelativistic N's, which decay after sphaleron freeze-out *Garbrecht, Drewes 12, cf. also Pilaftsis et. al. 14*
- Use freedom in h_{ai} to 'hide' asymmetry from washout when $N_{1,2,3}$ have comparable masses ⇒ possibility of GeV-scale leptogenesis w/o need for resonant enhancement *Garbrecht, Drewes 14*

Two strategies

1. Derive kinetic equations

$$S(t,k) = \int ds\, e^{iks} D(t + s/2, t - s/2)$$

Gradient expansion $\partial_t \partial_k \sim \frac{\text{slow}}{\text{fast}} \sim \frac{\Gamma, H, y^2 T}{M, T}$

$$\int dz\, \Sigma(x,z) S(z,y) \to \Sigma(t,k) S(t,k) + \frac{i}{2}\left(\frac{\partial \Sigma}{\partial t}\frac{\partial S}{\partial k} - \frac{\partial \Sigma}{\partial k}\frac{\partial S}{\partial t}\right)$$

On-shell limit

$$\begin{aligned} S_\rho^{ij}(t,k) &\to U^{in}(t)\delta^{nm}(\not{k} - M_n)\delta(k^2 - M_n^2(t))U^{\dagger nj}(t) \\ &\to \delta^{ij}(\not{k} - M_{av})\delta(k^2 - M_{av}^2) \\ S_F^{ij}(t,k) &\to \left(\frac{1}{2}\delta^{ij}\delta_{hh'} - f_{hh'}^{ij}(t,\mathbf{k})\right) u_h(\mathbf{k})\bar{u}_{h'}(\mathbf{k})\delta(k^2 - M_{av}^2) \end{aligned}$$

2. Solve two-time KB-eqs. for some simplified setup to study generation of the asymmetry (thermal bath)

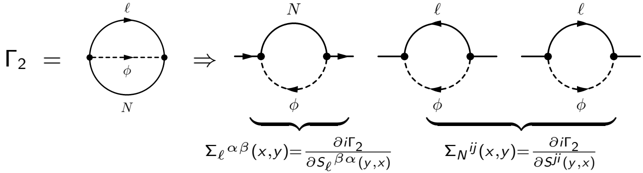
Two-time approach (flavoured for $N_{1,2}$)

- Statistical propagator S_F and spectral function S_ρ are matrices in N_1, N_2, N_3 flavor space. We consider the sub-space N_1, N_2 of the quasi-degenerate states.

$$S^{ij}(x,y) = \langle T_{\mathcal{C}} N_i(x) \bar{N}_j(y) \rangle = \begin{pmatrix} S^{11} & S^{12} \\ S^{21} & S^{22} \end{pmatrix}$$

  $\Rightarrow$ coherent N_1–N_2 transitions out-of-equilibrium

- Self-energies for leptons and for Majorana neutrinos



- Important: lepton self-energy contains full Majorana propagator-matrix

Two-time approach (flavoured for $N_{1,2}$)

- First step: solve KBEs treating lepton and Higgs as a thermal bath (no backreaction); include qualitative damping term for lepton/Higgs (not essential, no consistent treatment of gauge-int. yet;)

MG, Kartavtsev, Hohenegger Annals Phys. 328 (2013) 26

[hierarchical case: Anisimov, Buchmüller, Drewes, Mendizabal Annals Phys. 326 (2011) 1998]

$$
\begin{aligned}
((i\partial\!\!\!/_x - M_i)\delta^{ik} - \delta\Sigma_N(x)^{ik})S_F^{kj}(x,y) &= \int_0^{x^0} dz^0 \int d^3z\, \Sigma_{N\rho}^{\ ik}(x,z) S_F^{kj}(z,y) \\
&\quad - \int_0^{y^0} dz^0 \int d^3z\, \Sigma_{N F}^{\ ik}(x,z) S_\rho^{kj}(z,y) \\
((i\partial\!\!\!/_x - M_i)\delta^{ik} - \delta\Sigma_N(x)^{ik})S_\rho^{kj}(x,y) &= \int_{y^0}^{x^0} dz^0 \int d^3z\, \Sigma_{N\rho}^{\ ik}(x,z) S_\rho^{kj}(z,y)
\end{aligned}
$$

see also Garbrecht, Herranen 1112.5954; Garbrecht Gautier Klaric 1406.4190 for approach with grad. exp.

Two-time approach (flavoured for $N_{1,2}$)

- Second step: Lepton asymmetry

$$
\begin{aligned}
n_L(t) \quad = \quad & i(h^\dagger h)_{ji} \int_0^t dt' \int_0^t dt'' \int \tfrac{d^3 p}{(2\pi)^3} \\
& \mathrm{tr}\Big[P_R \underbrace{\left(\delta S^{ij}_{F\,\mathbf{p}}(t', t'') - \delta \bar{S}^{ji}_{F\,\mathbf{p}}(t', t'') \right)}_{\propto \text{ Deviation from equilibrium, CP-violation}} P_L S_{\ell\phi\,\rho\,\mathbf{p}}(t'' - t') \Big]
\end{aligned}
$$

$\delta \bar{S}^{ji}_{F\,\mathbf{p}}(t', t'') = CP\ \delta S^{ij}_{F\,\mathbf{p}}(t'', t')^T (CP)^{-1}$

$\delta S = S - S_{th}$

lepton-Higgs loop $S_{\ell\phi} = S_\ell \Delta_\phi$

Solution of KBE

Retarded and advanced propagators

$$
\begin{aligned}
S_R(x,y) &= \Theta(x^0-y^0)S_\rho(x,y) \\
S_A(x,y) &= -\Theta(y^0-x^0)S_\rho(x,y)
\end{aligned}
$$

The Kadanoff-Baym equation for the statistical propagator can be written as

$$
\begin{aligned}
\int_0^\infty d^4z \left[\left(\left(i\partial\!\!\!/_x - M_i\right)\delta^{ik} - \delta\Sigma_N{}^{ik}(x)\right)\delta(x-z) - \Sigma_N{}_R^{ik}(x,z)\right] S_F^{kj}(z,y) \\
= \int_0^\infty d^4z\, \Sigma_N{}_F^{ik}(x,z) S_A^{kj}(z,y)
\end{aligned}
$$

Special solution of the inhomogeneous equation

$$
S_F^{ij}(x,y)_{inhom} = -\int_0^\infty d^4u S_R^{ik}(x,u)\int_0^\infty d^4z\, \Sigma_N{}_F^{kl}(u,z)S_A^{lj}(z,y)
$$

General solution of the homogeneous equation

$$
S_F^{ij}(x,y)_{hom} = -\int d^3u\, S_R^{ik}(x,(0,\mathbf{u}))\int d^3v\;\, A^{kl}(\mathbf{u},\mathbf{v})S_A^{lj}((0,\mathbf{v}),y)
$$

Solution of KBE

Instantaneous excitation of $N_{1,2}$ at $t=0$, lepton/Higgs = thermal bath

$$S^{ij}_{F\,\mathbf{p}}(t,t') \quad = \quad S^{ij\,th}_{F\,\mathbf{p}}(t-t') + \underbrace{S^{ik}_{R\,\mathbf{p}}(t)\gamma_0\delta S^{kl}_{F\,\mathbf{p}}(0,0)\gamma_0 S^{lj}_{A\,\mathbf{p}}(-t')}_{\equiv\,\delta S^{ij}_{F\,\mathbf{p}}(t,t')}$$

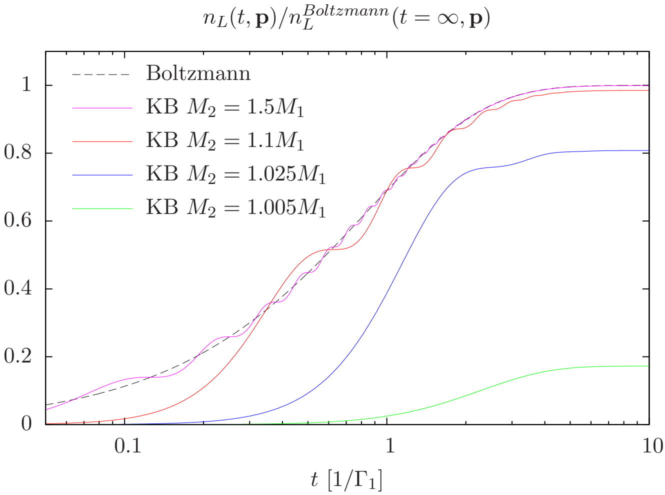
Resummed retarded and advanced propagators

$$\begin{aligned}\left(\left(i\partial\!\!\!/_x - M_i\right)\delta^{ik} - \delta\Sigma_N{}^{ik}(x)\right) S^{ik}_R(x,y) \quad &= \quad i\gamma_0\delta(x-y) \\ &+ \quad \int_{y^0}^{x^0} dz^0 \int d^3z \Sigma_{N\,\rho}^{\,ik}(x,z) S^{kj}_R(z,y)\end{aligned}$$

$$\Sigma_{N\,\rho}^{\,ij}(x,y) \quad = \quad -2\left[(h^\dagger h)_{ij}P_L + (h^\dagger h)_{ji}P_R\right] S_{\ell\phi\,\rho}(x,y)$$

$$S_{\ell\phi\,\rho}(x,y) = S_{\ell\,F}(x,y)\Delta_{\phi\,\rho}(x,y) + S_{\ell\,\rho}(x,y)\Delta_{\phi\,F}(x,y)$$

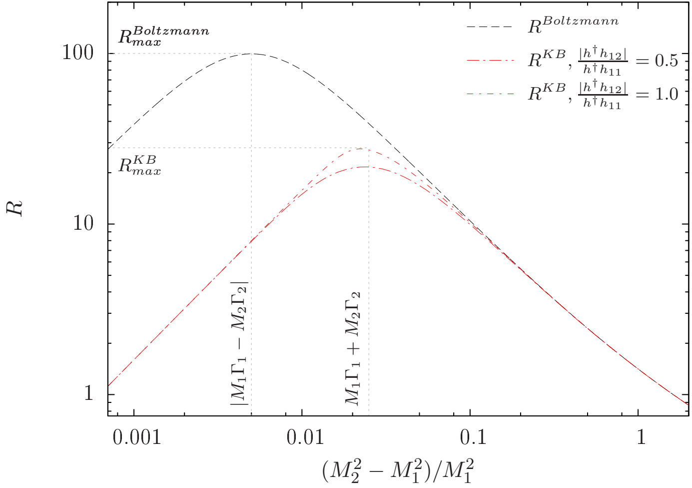
Resonant enhancement



$\Gamma_1 = 0.01M_1, \Gamma_2 = 0.015M_1, \Gamma_{\ell\phi} \to 0$

MG, Kartavtsev, Hohenegger Annals Phys. 328 (2013) 26

Resonant enhancement



$\Gamma_2/\Gamma_1 = 1.5$

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BW approximation

Solve SD equation for $S_{R(A)}(p)$ in Breit-Wigner approximation:

$$S^{ij}_{R(A)}(p) \simeq \frac{Z^{ij}_{1R(A)}}{p^2 - x_1} + \frac{Z^{ij}_{2R(A)}}{p^2 - x_2}$$

with residua $Z^{ij}_{lR(A)}$ and complex poles x_l (basis independent)

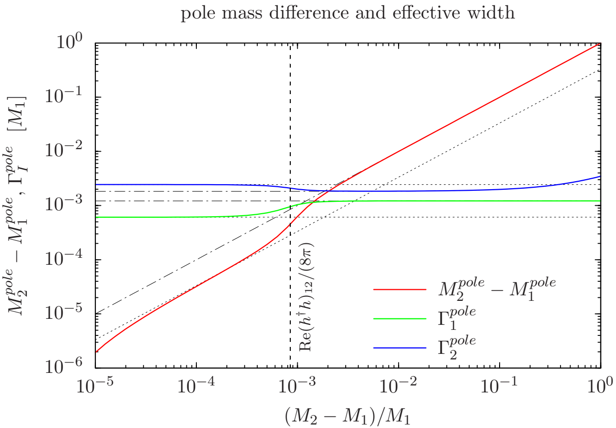
$$x_{1,2} = \frac{(V \pm W)^2}{4Q^2} \equiv \left(\omega_{pl} - i\frac{\Gamma_{pl}}{2}\right)^2 - \mathbf{p}^2$$

where

$$\begin{aligned} V &= \sqrt{(\eta_1 M_1(1+i\gamma_{22}) - \eta_2 M_2(1+i\gamma_{11}))^2 - 4\eta_1\eta_2 M_1 M_2(\mathrm{Re}\gamma_{12})^2} \\ W &= \sqrt{(\eta_1 M_1(1+i\gamma_{22}) + \eta_2 M_2(1+i\gamma_{11}))^2 + 4\eta_1\eta_2 M_1 M_2(\mathrm{Im}\gamma_{12})^2} \\ Q &= \det\Omega_{LR} = \det\Omega_{RL} = (1+i\gamma_{11})(1+i\gamma_{22}) + |\gamma_{12}|^2 \end{aligned}$$

$$\gamma_{ij} \simeq (h^\dagger h)_{ij}\left[\frac{\Theta(p^2)\mathrm{sign}(p_0)}{16\pi}\left(1 + \frac{2}{e^{|p_0|/T} - 1}\right) + i\left(\frac{\ln\frac{|p^2|}{\mu^2}}{16\pi^2} + \frac{T^2}{6p^2} - \frac{T^2}{6\mu^2}\right)\right]$$

BW approximation



Effective masses $M_I^{pole} \equiv \omega_{pI}|_{\mathbf{p}=0}$ and widths $\Gamma_I^{pole} \equiv \Gamma_{pI}|_{\mathbf{p}=0}$ of the sterile Majorana neutrinos extracted from complex poles of resummed ret/adv prop. for $(h^\dagger h)_{11} = 0.03$, $(h^\dagger h)_{22} = 0.045$, $(h^\dagger h)_{12} = 0.03 \cdot e^{i\pi/4}$ and $T = 0.25 M_1$.

Result for the lepton asymmetry

$$n_L(t) = \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, 2q\, 2k}\, (2\pi)^3 \delta(\mathbf{p}-\mathbf{k}-\mathbf{q}) \sum_{I,J=1,2} \sum_{\epsilon_n=\pm 1} F^{\epsilon_n}_{JI} L^{\epsilon_n}_{IJ}(t)$$

- Coefficients F depend on Yukawa couplings, thermal distributions of lepton and Higgs, resummed ret/adv propagators and initial conditions

$$\begin{aligned} F^{\epsilon_n}_{JI} \quad &= \quad \sum_{ijkl=1,2} (h^\dagger h)_{ji} \Big(\big(\tfrac{1}{2} + f_\phi(q)\big) + \epsilon_2 \epsilon_3 \big(\tfrac{1}{2} - f_\ell(k)\big) \Big)\, \mathrm{tr}\Big[P_L(|\mathbf{k}|\gamma_0 + \epsilon_2 \mathbf{k}\gamma) \\ &\quad \times \Big(S^{ik\epsilon_4}_{RI} \gamma_0 \Delta S^{kl}_{F\,\mathbf{p}}(0,0) \gamma_0 S^{lj\epsilon_1}_{AJ} - \bar{S}^{jk\epsilon_4}_{RI} \gamma_0 \Delta \bar{S}^{kl}_{F\,\mathbf{p}}(0,0) \gamma_0 \bar{S}^{li\epsilon_1}_{AJ} \Big) \Big] \end{aligned}$$

- Time-dependence: flavor diagonal and off-diagonal contributions:

$$\begin{aligned} L^{\pm}_{II}(t) \quad &= \quad \frac{1 - e^{-\Gamma_{pI} t}}{\Gamma_{pI}}\, \mathrm{Re}\left(\frac{\Gamma_{\ell\phi}}{(\omega_{pI} - k - q + i\Gamma_{pI}/2)^2 + \Gamma^2_{\ell\phi}/4} \right) + \mathcal{O}(\Gamma^0_{pI}) \\ L^{\pm}_{21}(t) \quad &= \quad \frac{1 - e^{\mp i(\omega_{p1} - \omega_{p2})t} e^{-(\Gamma_{p1} + \Gamma_{p2})t/2}}{\Gamma_{p1} + \Gamma_{p2} \pm 2i(\omega_{p1} - \omega_{p2})} \mathrm{Re}\left(\frac{\Gamma_{\ell\phi}}{(\omega_{p1} - k - q \pm i\Gamma_{p1}/2)^2 + \Gamma^2_{\ell\phi}/4} \right) \end{aligned}$$

Hierarchical limit

$$
\begin{aligned}
n_L(t) &= \frac{\mathrm{Im}[(h^\dagger h)^2_{12}]}{8\pi} \frac{M_1}{M_2} \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_{p1}\, 2q\, 2k}\, 4k\cdot p_1 \\
&\quad (2\pi)^3 \delta(\mathbf{p}-\mathbf{k}-\mathbf{q}) \times \mathrm{Re}\, \frac{\Gamma_{\ell\phi}}{(\omega_{p1}-k-q+i\Gamma_{p1}/2)^2+\Gamma^2_{\ell\phi}/4} \\
&\quad \times (1+f_\phi(q)-f_\ell(k))\, f_{FD}(\omega_{p1}) \frac{1-e^{-\Gamma_{p1}t}}{\Gamma_{p1}}
\end{aligned}
$$

$$
\begin{aligned}
n_L^{Boltzmann}(t) &= \frac{\mathrm{Im}[(h^\dagger h)^2_{12}]}{8\pi} \frac{M_1}{M_2} \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_{p1}\, 2q\, 2k}\, 4k\cdot p_1 \\
&\quad (2\pi)^3 \delta(\mathbf{p}-\mathbf{k}-\mathbf{q}) \times 2\pi\delta(\omega_{p1}-k-q) \\
&\quad \times (1+f_\phi(q)-f_\ell(k))\, f_{FD}(\omega_{p1}) \frac{1-e^{-\Gamma_{p1}t}}{\Gamma_{p1}}\ .
\end{aligned}
$$

The 'width' of lepton and Higgs $\Gamma_{\ell\phi} = \Gamma_\ell + \Gamma_\phi$ leads to a replacement of the on-shell delta function in the Boltzmann equations by a Breit-Wigner curve, in accordance with *Anisimov, Buchmüller, Drewes, Mendizabal Annals Phys. 326 (2011) 1998*

The coherent contributions are suppressed with Γ_{p1}/ω_{p2}

Degenerate case

Analytical result for $\mathrm{Re}(h^\dagger h)_{12} \ll (h^\dagger h)_{ii}$ (mass basis $\sim$ 'int. basis')

$$
\begin{aligned}
n_L(t) \quad = \quad & \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2} \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_p\, 2q\, 2k}\, 4k\cdot p \\
& \times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q})\, \frac{\Gamma_{\ell\phi}}{(\omega_p - k - q)^2 + \Gamma_{\ell\phi}^2/4}\, (1 + f_\phi - f_\ell) f_{FD}(\omega_p) \\
& \times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} - 4\,\mathrm{Re}\left(\frac{1 - e^{-i(\omega_{p1} - \omega_{p2})t} e^{-(\Gamma_{p1} + \Gamma_{p2})t/2}}{\Gamma_{p1} + \Gamma_{p2} + 2i(\omega_{p1} - \omega_{p2})} \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
n_L^{Boltzmann}(t) \quad = \quad & \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2} \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_p\, 2q\, 2k}\, 4k\cdot p \\
& \times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q})\, 2\pi\delta(\omega_p - k - q)\, (1 + f_\phi - f_\ell) f_{FD}(\omega_p) \\
& \times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} \right]
\end{aligned}
$$

Degenerate case

Analytical result for $\mathrm{Re}(h^\dagger h)_{12} \ll (h^\dagger h)_{ii}$ (mass basis $\sim$ 'int. basis')

$$
\begin{aligned}
n_L(t) \quad = \quad & \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2} \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_p\, 2q\, 2k}\, 4k\cdot p \\
& \times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) \frac{\Gamma_{\ell\phi}}{(\omega_p - k - q)^2 + \Gamma_{\ell\phi}^2/4} (1 + f_\phi - f_\ell) f_{FD}(\omega_p) \\
& \times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} - 4\,\mathrm{Re}\left(\frac{1 - e^{-i(\omega_{p1} - \omega_{p2})t} e^{-(\Gamma_{p1} + \Gamma_{p2})t/2}}{\Gamma_{p1} + \Gamma_{p2} + 2i(\omega_{p1} - \omega_{p2})} \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
n_L^{Boltzmann}(t) \quad = \quad & \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2} \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_p\, 2q\, 2k}\, 4k\cdot p \\
& \times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q})\, 2\pi\delta(\omega_p - k - q)\, (1 + f_\phi - f_\ell) f_{FD}(\omega_p) \\
& \times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} \right]
\end{aligned}
$$

- Regulator $M_1\Gamma_1 - M_2\Gamma_2$ is confirmed

Degenerate case

Analytical result for $\mathrm{Re}(h^\dagger h)_{12} \ll (h^\dagger h)_{ii}$ (mass basis $\sim$ 'int. basis')

$$
\begin{aligned}
n_L(t) &= \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2} \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_p\, 2q\, 2k}\, 4k\cdot p \\
&\times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q})\, \frac{\Gamma_{\ell\phi}}{(\omega_p - k - q)^2 + \Gamma_{\ell\phi}^2/4}\, (1 + f_\phi - f_\ell) f_{FD}(\omega_p) \\
&\times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} - 4\,\mathrm{Re}\left(\frac{1 - e^{-i(\omega_{p1} - \omega_{p2})t} e^{-(\Gamma_{p1} + \Gamma_{p2})t/2}}{\Gamma_{p1} + \Gamma_{p2} + 2i(\omega_{p1} - \omega_{p2})} \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
n_L^{Boltzmann}(t) &= \frac{\mathrm{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2} \int \frac{d^3p\, d^3q\, d^3k}{(2\pi)^9\, \omega_p\, 2q\, 2k}\, 4k\cdot p \\
&\times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q})\, 2\pi\delta(\omega_p - k - q)\, (1 + f_\phi - f_\ell) f_{FD}(\omega_p) \\
&\times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} \right]
\end{aligned}
$$

- Regulator $M_1\Gamma_1 - M_2\Gamma_2$ is confirmed
- Additional oscillating contribution due to coherent N_1—N_2 transitions

Resonant enhancement

Resonant enhancement within the Boltzmann approach

$$R^{Boltzmann} \equiv \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + A^2}$$

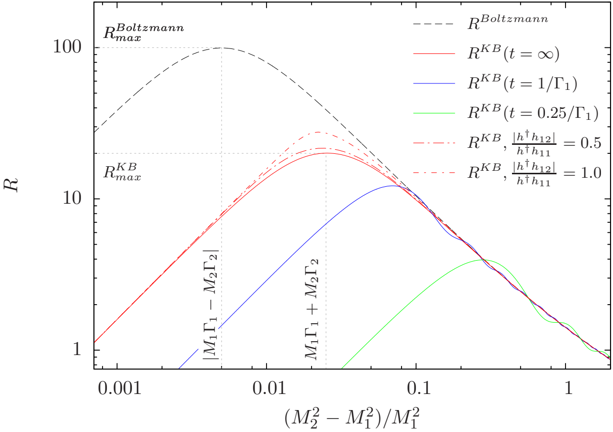
Resonant enhancement within the Kadanoff-Baym approach, including coherent contributions ($|(h^\dagger h)_{12}| \ll (h^\dagger h)_{ii}$)

$$R^{KB}(t) = \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2} \times (1 - f_{coherent}(t))$$

Partial cancellation of Boltzmann- and coherent contribution cuts off the enhancement in the doubly degenerate limit $M_1 \to M_2$ and $\Gamma_1 \to \Gamma_2$

$$R^{KB}(t)|_{t \gtrsim 1/\Gamma_{pl}} \quad \simeq \quad \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1\Gamma_1 + M_2\Gamma_2)^2}$$

Resonant enhancement



Numerical result for $|h^\dagger h_{12}| \sim h^\dagger h_{ii}$ vs BW approximation for $|h^\dagger h_{12}| \ll h^\dagger h_{ii}$

$R^{Boltzmann}_{max} = M_1 M_2/(2|\Gamma_1 M_1 - \Gamma_2 M_2|)$, $R^{KB}_{max} \simeq M_1 M_2/(2(\Gamma_1 M_1 + \Gamma_2 M_2))$

MG, Kartavtsev, Hohenegger Annals Phys. 328 (2013) 26

Conclusions

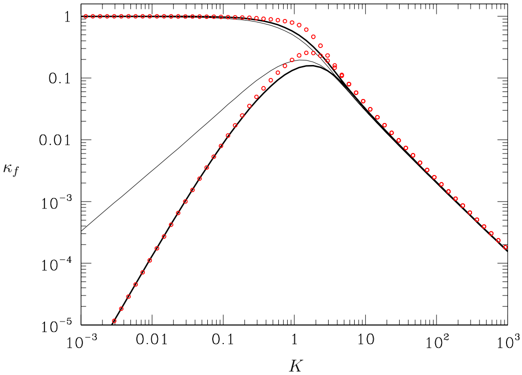
- Lots of progress in theory of leptogenesis
- Washout/Production NLO ($M \gtrsim T$), consistent LO ($M \sim gT$)
- CTP/Kadanoff-Baym helpful to check saturation of resonant enhancement and useful starting point for deriving (flavoured) kinetic equations

- Source term at finite T ?
- Bounds on M_N and m_ν ?
- Other models ?
- Production at NLO for $M \sim gT$?

thank you!

Final asymmetry

$$\eta = \frac{N_B}{N_\gamma} = a_{sph}\frac{N_{B-L}}{N_\gamma} = \frac{3}{4}\frac{a_{sph}}{f}\epsilon_1\kappa_f = 0.0096\epsilon_1\kappa_f$$



$$K = (\Gamma_{N_1}/H)_{T=M_1} = \tilde{m}_1/\mathrm{meV}$$

[N_1-dominated, D+ID, unflavoured] *Buchmüller, Di Bari, Plümacher 04*

$$a_{sph} = \frac{28}{79}, \;\; f = \frac{N_\gamma^{rec}}{N_\gamma^{leptog}} = \frac{2387}{86}$$

BW approximation

- Regime $(M_2 - M_1)/M_1 \gtrsim \mathrm{Re}(h^\dagger h)_{12}/(8\pi)$

$$\begin{aligned} M_i^{pole} &\simeq M_i \,, \\ \Gamma_i^{pole} &\simeq \Gamma_i \equiv \frac{(h^\dagger h)_{ii}}{8\pi} M_i \left(1 + \frac{2}{e^{M_i/T} - 1}\right) \end{aligned}$$

- Regime $(M_2 - M_1)/M_1 \lesssim \mathrm{Re}(h^\dagger h)_{12}/(8\pi)$

$$\begin{aligned} M_i^{pole} &\simeq \frac{M_1 + M_2}{2} \pm \frac{(M_2 - M_1)((h^\dagger h)_{22} - (h^\dagger h)_{11})}{2\sqrt{((h^\dagger h)_{22} - (h^\dagger h)_{11})^2 + 4(\mathrm{Re}(h^\dagger h)_{12})^2}} \,, \\ \Gamma_i^{pole} &\simeq \frac{M_i}{16\pi}\left(1 + \frac{2}{e^{M_i/T} - 1}\right)\Big((h^\dagger h)_{11} + (h^\dagger h)_{22} \\ &\quad \pm \sqrt{((h^\dagger h)_{22} - (h^\dagger h)_{11})^2 + 4(\mathrm{Re}(h^\dagger h)_{12})^2}\Big) \end{aligned}$$

The relation between the mass- and Yukawa coupling matrices at zero and finite temperature is

$$\begin{aligned} M(T) &= (P_L Z(T)^T + P_R Z(T)^\dagger) M(T=0) (P_L Z(T) + P_R Z(T)^*) \,, \\ (h^\dagger h)(T) &= Z(T)^T (h^\dagger h)(T=0) Z(T)^* \,, \end{aligned}$$

where $Z_{ij}(T) \equiv V_{ik}(T)(\delta_{kj} + (h^\dagger h)_{kj} T^2/(6\mu^2))$, $V(T)^\dagger V(T) = 1$